Calculations of Radon Emanation for the Bulk of LZ and Surface Contaminants

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Radon Emanation Calculations

1. Calculate emanation from bulk contamination from Ra226 inferred from HPGe measurements already taken
   - This yields a lower limit on emanation since it ignores possible surface contamination
2. Calculate emanation from surface contamination at 50 nBq/cm^2 (rough expected sensitivity) for extremes of thin or thick contamination layer

Recoil Ejection vs Diffusion

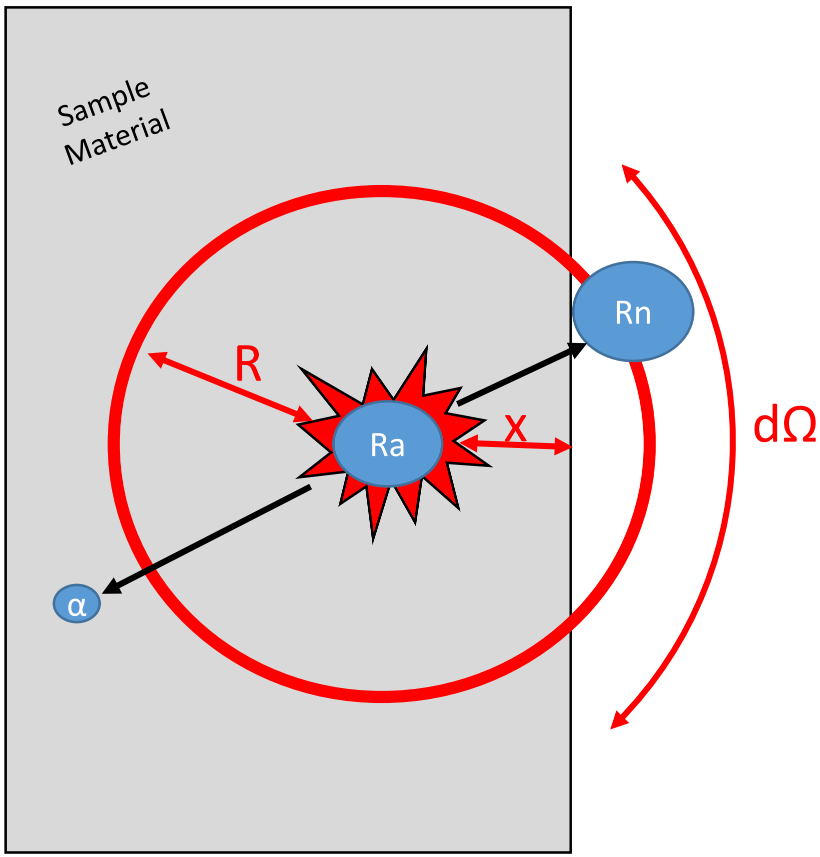
Recoil Ejection

- After α-decay, the recoiling radon nucleus can be ejected from the material
- Range of ejected radon nucleus is different in each material

Diffusion

- Higher contribution at warmer temperatures
- Diffusion length is different in each material
- Negligible in most metals, but important for plastics

Recoil Ejection Calculation



- Integrating dΩ/SA gives the percentage of particles being ejected from depth x:

$$\% = \frac{1}{2} - \frac{x}{2R}$$

- $R_{eff} = \frac{R}{4}$ is the range needed to create the same yield of ejected radon if 100% of the Radon escaped
- The ejected flux of radon is therefore $\Phi_E = R_{eff} \cdot \rho$

Density of radon in the material from Ra226 decay

Diffusion Calculation

- Radon concentration u(x,t) is described by the following equation:

$$\underbrace{\frac{\delta u(x,t)}{\delta t} = \overset{\text{Diffusion constant}}{D}\,\frac{\delta^2 u(x,t)}{\delta x^2}}_{\text{General Diffusion Equation}} - \overset{\text{Radon Decay}}{\lambda u(x,t)} + \underset{\substack{\text{Radon production}\\ \text{from Ra226 decay}}}{\rho}$$

- For steady state and short outgassing:

$$\Phi_D = \rho\sqrt{D/\lambda}$$

Summary of Materials:

		Radon Ejection		Warm Diffusion (room temp)			Cold Diffusion (Liquid xenon temp)		
Material	Ra-226 Density (mBq/cm^3)	Recoil Depth (cm)	Recoil Production (mBq/cm^2)	Diffusion Length (cm)	Diffusion Production (mBq/cm^2)	Total (mBq/cm^2)	Diffusion Length (cm)	Diffusion Production (mBq/cm^2)	Total (mBq/cm^2)
PTFE 8764	4.4E-05	1.01E-07	4.46E-11	2.58E-02	1.14E-06	1.14E-06	4.35E-06	1.91E-10	2.36E-10
Titanium	1.68	6.03E-07	1.014E-6	0	0	1.014E-6	0	0	1.014E-6
Steel		3.55E-7		0	0		0	0	
Kapton (Cirlex)	2.72E-2	1.43E-6	3.88E-8	2.92E-4	7.94E-6	7.97E-6	7.22E-10	1.96E-11	3.88E-8
Alumina (Ceramic)	7.23E-2	5.67E-7	4.10E-8						
BaTiO3		5.13E-7							

Current LZ Estimated Backgrounds:

- PTFE 8764 - 9.00E-7 mBq/cm^3
- Titanium- 1.00E-6 mBq/cm^3

Alpha Screening – Two Extremes

1. All contamination is at the surface of the material
   - The relationship between alpha and radon emanation is 1-to-1
   - $\Phi_{Rn} = \Phi_{\alpha}$
2. Contamination is evenly distributed up to the depth that α particles lose 10% of their energy.
   - A higher percentage of alpha particles are escaping compared to radon since they have a longer range.
   - Using titanium as an example: $\Phi_{Rn} = \Phi_{\alpha}(.0085)$

- Our alpha screening has a sensitivity of $\Phi_{\alpha} \approx 50\,\frac{nBq}{cm^2}$

Conclusion

- For the best case scenario, we compare this thick contamination limit to the bulk emanation of titanium:

$$\Phi_{Total} = 1.0\,\frac{nBq}{cm^2} + 0.4\,\frac{nBq}{cm^2} = 1.4\,\frac{nBq}{cm^2}$$

- Comparing the thin contamination limit to the bulk emanation gives:

$$\Phi_{Total} = 1.0\,\frac{nBq}{cm^2} + 50\,\frac{nBq}{cm^2} = 51\,\frac{nBq}{cm^2}$$

- This shows that surface screening is not sensitive enough to replace radon emanation assay assuming the contamination layer is thin, but is useful for limiting possible radon emanation assuming a thick contamination layer

Appendix: Detailed Diffusion Calculation

- Radon concentration u(x,t) described by following equation:

Diffusion constant

Radon Decay

$$\underbrace{\frac{\delta u(x,t)}{\delta t} = D\,\frac{\delta^2 u(x,t)}{\delta x^2}}_{\text{General Diffusion Equation}} - \lambda u(x,t) + \rho$$

Radon production
from Ra226 decay

- For steady state outgassing the concentration doesn't depend on time:

$$0 = D\,\frac{\delta^2 u(x)}{\delta x^2} - \lambda u(x) + \rho$$

- Solving this DE with boundary conditions gives:

$$u(x) = \frac{-\rho}{\lambda}\left[1 - \frac{1 - e^{-H\sqrt{\lambda/D}}}{2\sinh(H\sqrt{\lambda/D})}\right]e^{-x\sqrt{\lambda/D}} + \frac{\rho}{\lambda}$$

Appendix: Detailed Diffusion Calculation

- Flux of radon diffusing from the surface given by boundary derivatives:

$$\Phi_D = D\left[\frac{d\,u(x)}{dx}\right] = \rho\sqrt{D/\lambda}\,[1 - \frac{1 - e^{-H\sqrt{\lambda/D}}}{\sinh(H\sqrt{\lambda/D})}]$$

- Assuming that the Radon outgassing length is much shorter than the thickness of the material ($\sqrt{D/\lambda}$<<H), the flux simplifies to be:

$$\Phi_D = \rho\sqrt{D/\lambda}$$

- D depends on temperature. We determine Φ_D at lab room temperature (warm) and at liquid xenon temperature (cold) so that we can compare our direct emanation results with how the materials will act in LZ.

Appendix: Alpha Screening – Two Extremes

- To find the relationship between the flux of α particles and radon we first integrate over the depth of the surface contamination to find the density of alpha particles:

$$\Phi_\alpha = \int_0^{R_{10\%}} \rho_\alpha \, \Omega_\alpha(x) \, dx$$

$$\Phi_\alpha = \rho_\alpha \int_0^{R_{10\%}} \left[\frac{1}{2} - \frac{x}{2\, R_\alpha}\right] dx$$

Here I have used the range of α particles in titanium

R_α is found from SRIM and $R_{10\%}$ is just 10% of R_α

$$\Phi_\alpha = \rho_\alpha (6.807\text{E-5cm})$$

Appendix: Alpha Screening – Two Extremes

- We know that for every α particle produced we get a radon particle too:

$$\rho_{\alpha} = {}^{\Phi_{\alpha}}\!/_{(6.807\text{E}{-}5\text{cm})} = \rho_{Rn}$$

- With the radon density we can find the flux of radon:

$$\Phi_{Rn} = \rho_{Rn} R_{eff}$$

$R_{eff} = R_{Rn}/4$

$$\Phi_{Rn} = \frac{\Phi_{\alpha}}{(6.807\text{E}{-}5\text{cm})}(6.025\text{E}{-}7\text{cm})$$

$$\Phi_{Rn} = \Phi_{\alpha}(.0085)$$