Best Estimate

Methodology for Calculating the Weighted Average of Several Measurements, Each Having an Estimated Uncertainty, to Minimize the Uncertainty of the Result

References: (1) *The Statistical Analysis of Experimental Data,* John Mandel, National Bureau of Standards, Dover Publications

(2) ANSI/ASME American National Standard PTC 19.1-1985, *Measurement Uncertainty*

(3) ANS Paper, *Traceability of Thermal Power Measurements, Part 2 Modified Venturi Tubes*

Description

The methodology described in this appendix treats measurements of the same variable, specifically, total feedwater flow, from disparate sources. A "best estimate" of the variable is obtained by summing the weighted measurements from each disparate source. The method minimizes the error in the estimate by using a weighting factor for each measurement that is equal to the reciprocal of its variance, divided by the sum of the reciprocals of the variances of all instruments whose measurements are being used. [The variance of a measurement is the square of its estimated uncertainty.]

The method is described in most statistics texts, for example, reference (1), Chapter 7. It is also endorsed by the ASME for use in thermal performance tests, reference (2).

As noted in reference (1), the combination of the estimates and their uncertainties in this methodology relies on the independence of the individual measurements. There can be no systematic errors among the measurements; they must be diverse. The four variables used in the example below meet this requirement.

Table A-1, below, has been prepared to illustrate the application of the methodology.

Table A-1: Weighting Factors for Best Estimate of Feedwater Mass Flow, Sample Case

Measurement	Estimated Uncertainty $2\sigma_i$	Basis for Uncertainty	Reciprocal of Variance $1/(2\sigma_i)^2$	Weighting Factor, $w_i = \frac{(1/2\sigma_i)^2}{\sum(1/(2\sigma_i)^2}$
W_{noz} Total FW Flows, Nozzles (uncorrected)	± 1.4%	Note (a)	0.5102	0.2726
W_{LEFM} Total FW Flows, LEFM	±1.0%	Note (b)	1.0000	0.5344
W_{SF} Total, Steam Flows	± 3.0%	Note (c)	0.1111	0.0594
P_1 Turbine First Stage Pressure	± 2.0%	Note (c)	0.2500	0.1336
SUM	--	--	1.8713	1.0000

Note (a) Based on a comparison of 62 nozzle based measurements with chordal LEFMs, reference (3)
Note (b) Based on the site specific uncertainty analysis
Note (c) See discussion in text.

The measurements listed in Table A-1 are given as examples only. Obviously, the sensitivity of the result to inaccuracies in any single measurement is reduced as the number of valid measurements is increased; other instruments can also be used to form a reasonably accurate measurement of total feedwater flow.

For example, some plants are equipped with measurements of net condensate flow and heater drain pump discharge flow; the sum of these flows, less feed pump packing leakage and minimum flow valve leakage equals feed flow in the steady state.

Additionally many plants are equipped with feed pump flow instruments that are used to control the minimum flow valves for these pumps. If these instruments are ranged appropriately, and the minimum flow valves are leak tight, at or near full power the sum of these measurements less packing leakage also equals feed flow in the steady state.

It is important to remember that the variable being estimated is total feedwater flow. The turbine first stage pressure estimate of Table A-1 merits discussion on this score. A numerical relationship between turbine mass flow and first stage (impulse chamber) pressure in psia can be determined using the turbine vendor's thermal kit for the applicable turbine configuration. The actual pressure at rated flow, when the turbine is new, typically matches the value in the thermal kit to within ± 1%. Allowing for the potential for increased diaphragm leakage over time and for measurement instrument uncertainties the measurement yields an estimate of turbine mass flow within about ± 1.5%. But total feedwater flow equals the sum of turbine steam flow, hot reheater steam flow and blowdown flow. The reheater steam flow, usually about 5% of the total feed flow, if it is measured at all, is rarely measured accurately. Likewise the measurement of blowdown flow, usually 1 to 2% of total feedwater flow, is subject to large uncertainties. The ± 2% uncertainty of Table A-1 reflects all of these factors for a specific plant for which a best estimate analysis was carried out.

Likewise the determination of feedwater flow from a steam flow measurement is subject to significant uncertainties. Steam flow nozzles are typically not calibrated. They are often not dimensionally controlled to the degree necessary for an accurate flow measurement. In many plants their material of construction is carbon steel. With throat velocities often in the 400 feet/second range, they are therefore subject to erosion-corrosion. Depending on specifics of the erosion pattern, erosion may cause their calibration to shift in either direction. The estimated steam flow measurement uncertainty in Table A-1 was prepared for a specific plant and accounts quantitatively for these factors. It also accounts for steam density and blowdown flow uncertainties.

Calculation of Estimated Mass Flow

For the sample measurements of Table A-1, the best estimate of the normalized mass flow is calculated using the following equation:

$$W_{est} = w_{noz} \times W_{noz} + w_{LEFM} \times W_{LEFM} + w_{SF} \times W_{SF} + w_{P1} \times P1$$

$$W_{est} = 0.2726 \times W_{noz} + 0.5344 \times W_{LEFM} + 0.0594 \times W_{SF} + 0.1336 \times P1$$

Normalized data for the nozzle flow, LEFM flow, steam flow, and first stage pressure for each date under analysis are substituted for the variables in the above equation to determine the best estimate of the normalized mass flow for that date. All data are normalized to the nominal full power mass flow.

Uncertainty of the Best Estimate

From reference (1), the uncertainty in the best estimate of the mass flow, $2\sigma_{est}$, for the example of Table A-1 is given by:

$$2\sigma_{est} = \{1/\sum [1/(2\sigma_i)^2]\}^{1/2} = (1/\ 1.8713)^{1/2} = \pm\ 0.73\%$$

Determining that the Error in a Variable Exceeds its Assigned Uncertainty

Effective detection of errors in any of the variables making up the best estimate of feedwater flow requires that data be trended over time. If an unaccounted bias in the LEFM is to be detected by best estimate trending, it is desirable to start the trend at the commissioning of the LEFM. When the trend data carries over several fuel cycles, sudden changes in the difference between any instrument and the best estimate following a refueling may indicate (1) a variation due to recalibration of that instrument or (2) a change in the material condition of the system in which the instrument is located that affects the instrument's calibration.

The difference between any variable and the best estimate at any time can be a qualitative indicator of the bias in the measurement of that variable. Such biases may be present at the outset of the trend. For example, following its commissioning the indication of an LEFM might differ from that of the venturis by, say, 0.7%. This difference *might well be due to a bias in the venturi indication due to* fouling; however since the indications of both instruments, uncorrected, contribute to the best estimate, a difference between the LEFM and the best estimate will likely be present at commissioning. Such a difference should not be viewed as necessarily an error in the error in the LEFM (though it might be). However a substantial *change* in the indication of the LEFM relative to the best estimate can be indicative of a net change in the biases of the LEFM.

What is an acceptable change? With the instruments described in Table A-1, there is a 95% probability that the LEFM indication will lie within a band given by the root sum squares of its assigned uncertainty (±1%) and that of the best estimate (± 0.73%) or ± 1.24%.

This last figure should not be viewed as an acceptance band but if, for example, the difference between the LEFM indication and the best estimate was, say, 0.3% at commissioning and, following, say, a refueling, if the discrepancy approached 1% and persisted in this range, such behavior should be viewed as indicative of an LEFM error, not necessarily outside its design basis, but larger than probable if the instrument and its calibration are in accordance with its design.* In the absence of other indications of LEFM error, the likely cause of this hypothetical discrepancy would be a change in the axial velocity profile outside the range on which the LEFM's Profile Factor and its uncertainty are based.

* The same criterion applies to any of the other components of the best estimate