Home work | Vibes Class | H. Ryott Glazer

16-1) The electric field associated with a plane EM wave is given by

$E_x = 0$
$E_y = 0$
$E_z = E_0 \sin k(x+ct)$, where $E_0 = 234\,\mu V/m$, $k = 9.72 \times 10^6\ m^{-1}$

a) In what direction is the wave propogatig?

For a plane wave, always $\vec{E} \perp \vec{B} \perp \vec{v} \perp \vec{E}$, $\vec{E} \parallel \ddot{x}$

Given:

$$\vec{E} = (234\,\mu V \cdot m^{-1}) \sin\left[(9.72 \times 10^6\ m^{-1})(x+ct)\right] \hat{z}$$

since a plane wave dependent on x in the y-plane propogates in the positive x-direction, this wave ought to propagate in the negative x-direction intuitively. However, this is physics, and we don't operate on vibes alone. How can I show this mathemtically? We know that $\hat{E} \times \hat{B} = \hat{v}$ and we know $\vec{E}$. We want to know $\hat{v}$. Since $\vec{E}$ & $\vec{B}$ are intrinsically linked, we can find that

$$\vec{B} = \frac{E_0}{c} \sin k(x+ct)\, \hat{y}$$

* I know I haven't proven that $\vec{B}$ is not in the $-\hat{y}$ plane, but I don't know how I can do that. Please provide feedback on this.

We know this because it depends on the x-variable and thus cannot be in the x-direction/plane.

$\because \hat{E} \times \hat{B} = \hat{v}$, and $\hat{z} \times \hat{y} = -\hat{x}$, we are propagating in the negative x direction.

b) Write expressions for all 3 components

$$\vec{E} = (234\,\mu V \cdot m^{-1}) \sin\left[(9.72 \times 10^6\ m^{-1})(x+ct)\right] \hat{z}$$

$$\vec{B} = (780\ fV s \cdot m^{-2}) \sin\left[(9.72 \times 10^6\ m^{-1})(x+ct)\right] \hat{y}$$

$$\vec{v} = -(3 \times 10^8\ m \cdot sec^{-1})\, \hat{x}$$

$3\overline{)234}$ = 78; -24, 210, -210, 0

$$\frac{10^{-6}\ V \cdot m^{-1}}{10^8\ m \cdot s^{-1}} = 10^{-14}\ V \cdot m^{-2} \cdot s$$

$10^{-15} = fV \cdot s \cdot m^{-2}$

m 3
μ 6
n 9
p 12
f 15

c) What is the wave's wavelength?

$\because k = \frac{2\pi}{\lambda}$, and $k = 9.72 \times 10^6\ m^{-1}$, $\lambda = \frac{2\pi}{9.72 \times 10^6} \approx \frac{2}{3} \times 10^{-6} \approx 6.46 \times 10^{-7}\ m$

(and $\therefore \lambda = 2\pi/k$)

HOMEWORK!
~ VIBES CLASS ~

H. Ryott Glynn
15 April 2028

16-2) Suppose there exists a plane electromagnetic wave with an electric field polarized in the x-direction and traveling in the negative z direction through free space. It has a frequency of 91.3 MHz and an amplitude of 7.2Ø mV·m⁻¹.

GIVENS

$\lambda = 91.3$ MHz
$A = 7.20\ \text{mV}\cdot\text{m}^{-1}$

a) Write down the complete expression for the E-field.

Given $\lambda = 91.3\text{MHz}$, $\lambda = \frac{2\pi}{k} \Rightarrow k = \frac{2\pi}{\lambda}$, $A = 7.2Ø\ \text{mV}\cdot\text{m}^{-1} = E_m$ $\quad \hat{x} \times -\hat{y} = -\hat{z}$

$\lambda = \frac{v}{f}$ $\quad v = c$

$$\vec{E} = (7.20\ \text{mV}\cdot\text{m}^{-1})\sin[(191.6\ \text{m}^{-1})(x - ct)]\hat{\imath}$$

$$\frac{3\times10^{8}\ \text{m}\cdot\text{sec}^{-1}}{91.3\times10^{6}\ \text{sec}^{-1}} = 32.8\ \text{mm} = \lambda$$

$$k = \frac{2\pi}{\lambda} = \frac{2\pi}{32.8\text{mm}} = 191.6\ \text{m}^{-1}$$

b) Write the corresponding expression for the $\vec{B}$ field.

$E_m = B_m c \Rightarrow B_m = \frac{E_m}{c}$, $E_m = 7.20\ \text{mV}\cdot\text{m}^{-1}$, $B_m = 24.0\ \text{pV}\cdot\text{s}\cdot\text{m}^{-2}$

$$\vec{B}_m = (-24.0\ \text{pV}\cdot\text{s}\cdot\text{m}^{-2})\sin[(191.6\ \text{m}^{-1})(x - ct)]\hat{\jmath}$$

c) What is the wavelength?

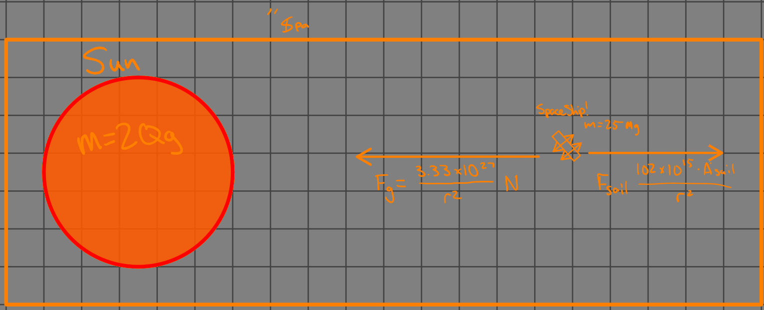
$\lambda = 32.8\ \text{mm}$ (see work in part a)

HOMEWORK
~ VIBES CLASS ~

16-3) What is the minimum surface area of a solar sail adequate enough to prevent my spaceship (mass = 25 metric tonnes) from falling into the Sun?

$m_{ship} = 25\ Mg$ $\quad m_{sun} = 2\ Qg$ (1 quettagram $= 1\times10^{30}$ grams) $\quad G = 6.67430\times10^{-11}\ N\cdot m^2/kg^2$

$F_g = G\frac{m_1 m_2}{r^2}$



We need $F_{radpressure} > F_g$

6.67430×10^{-11} $\quad F_g = \frac{333.175\times10^{25}}{r^2} \approx \frac{3.33\times10^{27}}{r^2}$

$36 - 11 = 25$

Assuming light scatters equally in all directions from Sol, we will only receive a small fraction of light from the Sun. However, its isotropic, so that doesn't really matter.

$F_{sail} \geq 3.33\times10^{27}/r^2$ N

$F_{sail} = UA = \varepsilon_0 E^2 A = \varepsilon_0 E^2_{rms} A$

$$\boxed{\begin{array}{c}\text{According to Wikipedia}\\ 3.846\times10^{26}\ W_{TOT} = P_{Sun,TOT}\end{array}}$$

* If I were truly commander, I would just know this

assume Sun is a point

$I = \frac{3.846\times10^{26}\ W}{4\pi r^2}$

$I = \frac{P}{A}$

$F = UA =$

$\langle F\rangle = \frac{IA}{c}$

$3.846 / 3 = 1.282$

$\frac{(3.846\times10^{26}\ W)\cdot(A_{sail})}{4\pi r^2\cdot(3\times10^8\ m/s)} \approx \frac{1.282\times10^{18}\ N\cdot A_{sail}\ m^2}{4\pi r^2\ m^2}$

J/s

$\cancel{W} = kg\, m^2 s^{-3} \longleftrightarrow kg\, m\, s^{-2}$

$\frac{A_{sail}}{A_{TOT} = 4\pi r^2} = A_{\%}$

$A_{\%}\cdot P_{TOT}$

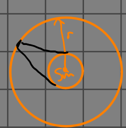
$1.282 / 4 = 0.3205$

$\frac{320.5\times10^{15}\ N\cdot A_{sail}\ m^2}{\pi r^2\ m^2}$

$\frac{102\times10^{15}\ N\cdot A_{sail}\ (m^2)}{r^2\ (m^2)}$

$\frac{102\ PN\cdot A_{sail}}{r^2}$

Watt = 1 J·sec^{-1}



take area of solar sail as a percentage of the total inner surface area of a sphere surrounding the Sun

$F_{sail} = \frac{102\times10^{15}\ N\cdot A_{sail}}{r^2}$

$F_g = \frac{3.33\times10^{27}}{r^2}$ N, $F_{sail} = \frac{102\times10^{15}\cdot A_{sail}}{r^2}$ N

$F_g \leq F_{sail}$ or we die.

$\frac{3.33\times10^{27}}{r^2} \leq \frac{102\times10^{15}\cdot A_{sail}}{r^2} \Longleftrightarrow 3.33\times10^{27} = 102\times10^{15}\cdot A_{sail} \longrightarrow A_{sail} = 32.65\times10^{9}\ m^2$

If the sail is a black body, absorbing 100% of radiation, the sail would need to be $32.65\times10^9\ m^2$ or more.

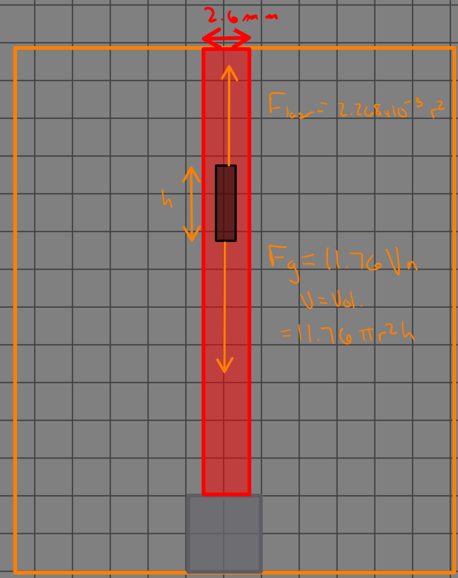
If the sail were perfectly reflective, the area would need to be at least $16.33\times10^9\ m^2$

However, these are ideal situations. In real life, solar sails reflect ~90% of light, and this would require $\geq 18.14 \times 10^9\ m^2$ of solar sail IFF an angle of 0° was maintained at all times, and longer in the other scenario.

It is unlikely we would be carrying 20,000 sq km of solar sail material, so we would likely die screaming "FUCK MICROSOFT!".

16-9

A $\underline{4.60\ W}$ laser beam with diameter $\underline{2.60\ mm}$ is levitating a smaller perfectly reflective cylinder $\underline{w/\ \rho = 1.20\ g/cm^3}$. What is its height?



$|F_{laser}| = |F_g|$

$\begin{array}{r} 6.25 \\ +.51 \\ \hline 6.76 \end{array}$ $A_{90} = \dfrac{6.76\times10^{-6}\, m^2}{r^2}$

$\dfrac{4.60\ W}{6.76\times10^{-6}\, m^2} = 680\ kW/m^2$

lasers are not isotropic.

$I = \langle \vec{S} \rangle = \dfrac{P}{A} = \dfrac{c\langle F \rangle}{A}$

$I = 680\ kW/m^2$

$\langle F \rangle = \dfrac{IA}{c} = \dfrac{680\times10^{3}\, W/m^2}{3\times10^{8}\, m/s} \cdot r^2 = 2.268\times10^{-3} \cdot r^2\, N$

$J \cdot \cancel{s^{-1}} \cdot m^{-2} \cdot m^{-1} \cdot \cancel{s} = J \cdot m^{-3}$

$F_{laser} = F_{gravity}$

$2.268\times10^{-3} \cdot \cancel{r^2}\, N = 11.76\, \pi \cancel{r^2} h\ N$

$h = \dfrac{2.268\times10^{-3}}{11.76\pi} = 61.38\times10^{-6}\, m = 61.38\ \mu m$

The height of the cylinder is $61.38\ \mu m$