Calculus II: Final Exam Practice Problems

April 17, 2024

These are practice problems for the final exam. **These will be graded and counted as a quiz grade.** Turn your solutions in on separate sheets of paper, **stapled**, by the final day of class, April 26.

1. Compute $\dfrac{\mathrm{d}}{\mathrm{d}x}\log(\sin^2(x)+1)$.

2. Find $\lim\limits_{x\to\infty}\dfrac{\log(e^x+1)}{x}$.

3. Compute $\displaystyle\int\frac{x\sin(x^2)}{\cos(x^2)}\mathrm{d}x$

4. Compute $\displaystyle\int\frac{\sin(x)}{\pi^2+\cos^2(x)}\mathrm{d}x$

5. Compute $\displaystyle\int\frac{e^{\tan^{-1}(x)}}{x^2+1}\mathrm{d}x$

6. Compute $\lim\limits_{x\to 0}\dfrac{\tan^{-1}(x)}{x}$

7. Compute $\lim\limits_{x\to 0}x\log(x)$

8. Compute $\displaystyle\int_0^\infty xe^{-(x^2)}\mathrm{d}x$

9. Compute $\displaystyle\int_0^\infty xe^{-x}\mathrm{d}x$

10. Compute $\displaystyle\int_0^\infty\frac{2}{1+x^2}\mathrm{d}x$

11. Compute $\displaystyle\int_0^1\frac{x^2}{x^2+1}\mathrm{d}x$

12. Compute $\int x^2 \log(x) \mathrm{d}x$

13. Compute $\int \frac{2x^2}{(x^2+1)(x-1)} \mathrm{d}x$

14. Compute $\int_0^{\pi} x^2 \sin(x) \mathrm{d}x$

15. Show whether the series $\sum_{n=1}^{\infty} \frac{n^2}{n^3+1}$ converges.

16. Show whether the series $\sum_{n=1}^{\infty} \frac{n}{n^3+1}$ converges.

17. Show whether the series $\sum_{n=1}^{\infty} e^{-n} \cdot 2^n \cdot n^2$ converges.

18. Find the Taylor series for $f(x) = \frac{1}{x}$ centered at $a = 1$

19. Find the Taylor series for $f(x) = e^{-(x^2)}$ centered at $a = 0$.

20. Find the third-order Taylor polynomial for $f(x) = \sin(x)$ centered at $a = 0$. Give an upper-bound on the error of this approximation for $0 \leq x \leq \frac{\pi}{2}$.

21. Find the third-order Taylor polynomial for $f(x) = \log(x+1)$ centered at $a = 0$. Give an upper-bound on the error of this approximation for $0 \leq x \leq 1$.

22. Use the second-order Taylor polynomial for $f(x) = \cos(x)$ centered at $a = 0$ to find an approximate solution to $\cos(x) = \frac{1}{2}$ with no calculator.