The Chinese University of Hong Kong, Shenzhen
School of Data Science

MAT3220 · Homework 1
Due: 23:59, September 22

Instructions:

- Homework problems must be carefully and clearly answered to receive full credit. Complete sentences that establish a clear logical progression are highly recommended.
- You must submit your assignment in Blackboard. Please upload a pdf file with codes. The file name should be in the format **last name-first name-student ID-hw1**, e.g. **Zhang-San-123456789**.
- The homework must be written in English.
- Late submission will not be graded.
- Each student **must not copy** homework solutions from another student or from any other source.
- For those questions that ask you to write MATLAB/Python/other codes to solve the problem. Please attach the code to the homework. You also need to clearly state (write or type) the optimal solution and the optimal value you obtained. However, you do not need to attach the outputs in the command window of MATLAB/Python/others.

Problem 1 Coerciveness and Global Minimizers (20pts).
Determine whether the following functions are coercive and fnd their global minimizers, if any.

(a) $f(x, y, z) = x^2 - 2xy + y^2 + z^2$

(b) $f(x, y) = x^4 + y^4 - 4xy$

Problem 2 Quadratic Optimization Problems (30pts).
Consider the unconstrained quadratic optimization problem.

$$\min\ f(x) = \tfrac{1}{2}x^T Hx + g^T x + b$$

where $x \in \mathbb{R}^n$ and $H \in \mathbb{R}^{n\times n}$ is symmetric, $g \in \mathbb{R}^n$, $b \in \mathbb{R}$.

(a) Each of the following functions can be written in the form $f(x) = \frac{1}{2}x^T Hx + g^T x + b$, For each of these functions what are H matrix and g vector.

(i) $f(x) = 3x_1^2 + 4x_2^2 - 2x_1x_2 + 4x_1 + 5x_2 - 6$
(ii) $f(x) = 5x_1^2 + 2x_2^2 - 4x_1x_2 + 3x_1 - 2$
(iii) $f(x) = (3x_1 - 2)^2 + (4x_2 - 3)^2 + (5x_3 - 1)^2$

(b) Let x^* be a local minimum of $f(x)$, what are the necessary optimality conditions of $f(x^*)$?

(c) Given H and g

$$H = \begin{pmatrix} 2 & 3 & 1 \\ 3 & 10 & 2 \\ 1 & 2 & 4 \end{pmatrix}, \qquad g = \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix}.$$

Does there exists a vector $v \in \mathbb{R}^3$ such that $f(tv) \to -\infty$ when $t \to \infty$? If yes, construct v.

Problem 3 Optimality Condition (30pts).
Consider the function

$$f(x_1, x_2) = e^{x_1^2 + x_2^2 (1-x_1)^3}$$

Prove that

(a) The point $z = (0, 0)$ is the unique stationary point of the function f.

(b) The point z is also a local minimal of f.

(c) The function f is bounded from below, but z is not the global minimal of f and f does not have any global minimal.

Problem 4 Coding: Nonlinear Least Square (20pts).
Generate 50 points $\{(x_i, y_i), i = 1, 2, \cdots, 50\}$ through the following code. (Type them by yourself to avoid the error caused by indentation.)

```
randn('seed',314);
x=linspace(0,1,50);
y=2*x.^2-3*x+1+0.05*randn(size(x));
```

Or you could generate your own random sample through the same quadratic rule with your favorite language. Find the quadratic function $f(x) = ax^2 + bx + c$ that best fits the points in the least squares sense.

(a) What are the values of a,b,c that found by the least squares solution?

(b) Plot the points along with the derived quadratic function. The resulting plot should look like the one in Figure 1. You can apply Matlab/Python or any other programs to generate the picture.

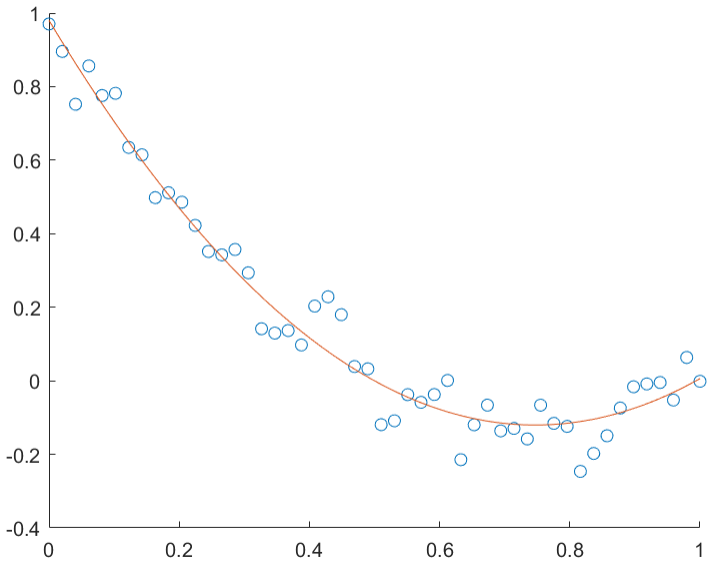

Figure 1: 50 points and their best quadratic least squares fit.