CS/MATH 375, Fall 2024 — HOMEWORK # 5
Due : Sept. 27th at 10:00pm on Canvas

Instructions

- **Report:** In general, your report needs to read coherently. That is, start off by answering question 1. Fully answer the question, and provide all the information needed to understand your answer. If Matlab code or output is part of the question, include that code or output (e.g., screenshot) alongside your narrative answer. If discussion is required for a question, include that. Overall, your report is your narrative explanation of what was done, your answers to the specific questions, and how you arrived at your answers. *Your report should include your Matlab scripts, code output, and any figures.*

- **What to hand in:** Submission must be one **single PDF** document, containing your entire report, submitted on Canvas.

- **Partners:** You are allowed to (even encouraged) to **work in pairs**. If you work with a partner, only one member of the group should need to submit a report. On Canvas, both partners sho uld join a group (numbered 1 through 15). Then either member can upload the report for the entire group. Groups of more than 2 students are not allowed.

- **Typesetting:** If you write your answers by hand, then make sure that your handwriting is readable. Otherwise, I cannot grade it.

- **Plots:** All plots/figures in the report must be generated in Matlab or Python and not hand drawn (unless otherwise specified in the homework question).

  In general, make sure to (1) title figures, (2) label both axes, (3) make the curves nice and thick to be easily readable, and (4) include a legend for the plotted data sets. The font-size of all text in your figures must be large and easily readable.

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1. *Cost of Gaussian Elimination*
   Assume that an ancient computer solves a 1000-variable, upper-triangular, linear system by back substitution in 0.5 seconds. Estimate the time needed to solve a general (full) system by Gaussian Elimination (forward elimination + back substitution). Use the counts from the lectures: $2n^3/3 + \mathcal{O}(n^2)$ for elimination and n^2 for back substitution.

2. *Gaussian Elimination with and without partial pivoting*

   (a) Write a Matlab script that uses naive Gaussian Elimination to solve the linear system
   $$\begin{bmatrix} a & 1 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 1+a \\ 2 \end{bmatrix}$$

for $a = 10^{-2k}, k = 1, 2, ..., 10$. The exact solution is $x = [1 \quad 1]^T$, regardless of the value of a. Place the solutions your script produces in a table. How does the accuracy of your numerical solution behave as k gets bigger? Explain.

(b) Next, change your Matlab code to carry out partial-pivoting. What is the solution now? Give a table of solution values. Explain your result and the accuracy relative to part (a).

3. *Gaussian Elimination with scaled partial pivoting* Using scaled partial pivoting *without* actually moving data in the matrix, show the steps required to solve the following system of equations. Calculate the scale vector (called s in lecture). Show how the pivot row is selected at each step, and carry out the computations. At each step, include the index vector (called ℓ in lecture).

$$
\begin{aligned}
2x_1 - x_2 + 3x_3 + 4x_4 &= 15 \\
4x_1 + 4x_2 + 7x_4 &= 11 \\
2x_1 + x_2 + x_3 + 3x_4 &= 7 \\
6x_1 + 5x_2 + 4x_3 + 17x_4 &= 31
\end{aligned}
$$