- 1.A coin is thrown 3 times .what is the probability that atleast one head is obtained?
- 2: Find the probability of getting a numbered card when a card is drawn from the pack of 52 cards.
- 3. There are 5 green 7 red balls. Two balls are selected one by one without replacement. Find the probability that first is green and second is red.
- 4. What is the probability of getting a sum of 7 when two dice are thrown?
- 5. 1 card is drawn at random from the pack of 52 cards.
- (i) Find the Probability that it is an honor card.
- (ii) It is a face card.

# **EXERCISES**

- **1.2.1** Suppose  $S = \{1, 2, 3\}$ , with  $P(\{1\}) = 1/2$ ,  $P(\{2\}) = 1/3$ , and  $P(\{3\}) = 1/6$ .
- (a) What is  $P(\{1, 2\})$ ?
- (b) What is  $P(\{1, 2, 3\})$ ?
- (c) List all events A such that P(A) = 1/2.
- **1.2.2** Suppose  $S = \{1, 2, 3, 4, 5, 6, 7, 8\}$ , with  $P(\{s\}) = 1/8$  for  $1 \le s \le 8$ .
- (a) What is  $P(\{1, 2\})$ ?

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- (b) What is  $P(\{1, 2, 3\})$ ?
- (c) How many events A are there such that P(A) = 1/2?
- **1.2.3** Suppose  $S = \{1, 2, 3\}$ , with  $P(\{1\}) = 1/2$  and  $P(\{1, 2\}) = 2/3$ . What must  $P(\{2\})$  be?
- **1.2.4** Suppose  $S = \{1, 2, 3\}$ , and we try to define P by  $P(\{1, 2, 3\}) = 1$ ,  $P(\{1, 2\}) = 0.7$ ,  $P(\{1, 3\}) = 0.5$ ,  $P(\{2, 3\}) = 0.7$ ,  $P(\{1\}) = 0.2$ ,  $P(\{2\}) = 0.5$ ,  $P(\{3\}) = 0.3$ . Is P a valid probability measure? Why or why not?

# Section 1.3: Properties of Probability Models

- **1.2.8** Suppose  $S = \{1, 2, 3\}$ , and  $P(\{1, 2\}) = 1/3$ , and  $P(\{2, 3\}) = 2/3$ . Compute  $P(\{1\})$ ,  $P(\{2\})$ , and  $P(\{3\})$ .
- **1.2.9** Suppose  $S = \{1, 2, 3, 4\}$ , and  $P(\{1\}) = 1/12$ , and  $P(\{1, 2\}) = 1/6$ , and  $P(\{1, 2, 3\}) = 1/3$ . Compute  $P(\{1\})$ ,  $P(\{2\})$ ,  $P(\{3\})$ , and  $P(\{4\})$ .
- **1.2.10** Suppose  $S = \{1, 2, 3\}$ , and  $P(\{1\}) = P(\{3\}) = 2 P(\{2\})$ . Compute  $P(\{1\})$ ,  $P(\{2\})$ , and  $P(\{3\})$ .
- **1.2.11** Suppose  $S = \{1, 2, 3\}$ , and  $P(\{1\}) = P(\{2\}) + 1/6$ , and  $P(\{3\}) = 2P(\{2\})$ . Compute  $P(\{1\})$ ,  $P(\{2\})$ , and  $P(\{3\})$ .
- **1.2.12** Suppose  $S = \{1, 2, 3, 4\}$ , and  $P(\{1\}) 1/8 = P(\{2\}) = 3 P(\{3\}) = 4 P(\{4\})$ . Compute  $P(\{1\})$ ,  $P(\{2\})$ ,  $P(\{3\})$ , and  $P(\{4\})$ .

# Probability and Statistics.pdf

Chapter 1: Probability Models

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# **EXERCISES**

- **1.3.1** Suppose  $S = \{1, 2, ..., 100\}$ . Suppose further that  $P(\{1\}) = 0.1$ .
- (a) What is the probability  $P(\{2, 3, 4, ..., 100\})$ ?
- (b) What is the smallest possible value of  $P(\{1, 2, 3\})$ ?
- **1.3.2** Suppose that Al watches the six o'clock news 2/3 of the time, watches the eleven o'clock news 1/2 of the time, and watches both the six o'clock and eleven o'clock news 1/3 of the time. For a randomly selected day, what is the probability that Al watches only the six o'clock news? For a randomly selected day, what is the probability that Al watches neither news?
- **1.3.3** Suppose that an employee arrives late 10% of the time, leaves early 20% of the time, and both arrives late *and* leaves early 5% of the time. What is the probability that on a given day that employee will either arrive late *or* leave early (or both)?
- **1.3.4** Suppose your right knee is sore 15% of the time, and your left knee is sore 10% of the time. What is the largest possible percentage of time that at least one of your knees is sore? What is the smallest possible percentage of time that at least one of your knees is sore?
- **1.3.5** Suppose a fair coin is flipped five times in a row.
- (a) What is the probability of getting all five heads?
- (b) What is the probability of getting at least one tail?
- **1.3.6** Suppose a card is chosen uniformly at random from a standard 52-card deck.
- (a) What is the probability that the card is a jack?
- (b) What is the probability that the card is a club?
- (c) What is the probability that the card is both a jack and a club?
- (d) What is the probability that the card is either a jack or a club (or both)?
- **1.3.7** Suppose your team has a 40% chance of winning or tying today's game and has a 30% chance of winning today's game. What is the probability that today's game will be a tie?
- **1.3.8** Suppose 55% of students are female, of which 4/5 (44%) have long hair, and 45% are male, of which 1/3 (15% of all students) have long hair. What is the probability that a student chosen at random will either be female or have long hair (or both)?

1st lesson "BINOMIAL DISTRIBUTION", Bernoulli scheme

Task1: Calculate the probability of getting exactly five heads when a coin is tossed 9 times.

Task2: Calculate the probability of getting exactly 7 heads when a coin is tossed 10 times.

Task3: Consider an experiment where a biased coin (with probability of heads (p = 0.6)) is tossed 5 times. What is the probability of getting exactly 3 heads?

Task4: If a basketball player takes 8 independent free throws, with a probability of 0.7 of getting a basket on each shot, what is the probability that she gets exactly 6 baskets?

# Task5:

A student is given a multiple choice exam with 10 questions, each question with five possible answers. He guesses randomly for each question.

- (a) What's P(he will get exactly 6 questions correct)?
- b) What is the probability he will get at least 6?
- c) What is the expected number of correct answers, and what's the standard deviation?

## Task 6:

According to technical control data, 5% of manufactured machines need additional adjustment. Find the probability that out of 6 manufactured machines, 5 need additional adjustment.

# Task 7:

5 shots are fired at the target. The probability of a hit with each shot is ¼. Find the probability that there will be at least three holes in the target, but also no more than four holes. Find the most probable number of hits and the corresponding probability.

# Task 8:

The probability of the shooter hitting the target is 0.5. Find the probability that with 8 shots the target will be hit from 5 to 7 times.

#### Task 9:

5 computers were purchased for the computing laboratory, and the probability of failure for one computer is 0.1. What is the probability that *more than two computers* will have to be replaced.

#### Task 10:

There are 7 customers in the store. Everyone can make a purchase with a probability of 0.4. Find the probability that no more than two people will make a purchase.

# **Task 11:**

The TV studio has 7 TVs. For each TV, the probability that it is currently on is 0.7. Find the probability that it is currently on:

- a) three TVs;
- b) at least one TV;
- c) at least 2(two) TVs.

#### Task12:

There are 10 white and 5 black balls in each of the eight urns. One ball was removed from each urn. Which is more possibility?

- the appearance of 3 black and 5 white balls or 2 black and 6 white balls?

## Poisson distribution—>

#### ### Task 1:

\*\*Problem:\*\* A coffee shop receives 8 customers per hour on average. What is the probability that exactly 5 customers will arrive in the next hour?

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#### ### Task 2:

\*\*Problem:\*\* A machine breaks down on average 4 times per month. What is the probability that it will break down exactly 2 times in a given month?

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# ### Task 3:

\*\*Problem:\*\* A bookstore sells an average of 12 books per day. What is the probability that they will sell no books on a particular day?

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# ### Task 4:

\*\*Problem:\*\* The average number of emails a person receives per hour is 5. What is the probability that a person will receive more than 3 emails in the next hour?

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# ### Task 5:

\*\*Problem:\*\* The average number of accidents at a busy intersection is 2 per month. What is the probability that there will be exactly 1 accident in the next month?

# Task 6:

Problem: A nuclear plant experiences a failure on average once every 5 years. Assuming the failures follow a Poisson process, what is the probability that the plant will experience at least 3 failures in the next 10 years?

# Task 7:

Problem: A rare disease occurs with a frequency of 1 case per 50,000 people. In a city of 200,000 people, what is the probability that exactly 3 people will contract the disease?

# Task 8:

Problem: A factory produces a certain type of machine part. Defective parts occur on average 0.5 times per hour. In a typical 8-hour shift, what is the probability that there will be more than 1 defective parts!?

Task 9:

Problem: On average, 2.5 accidents happen per month at a particular intersection. What is the probability that there will be no more than 1 accident in the next two months?

Task 10:

Problem: A lab technician averages 2 errors per 100 samples tested. What is the probability that the technician will make exactly 5 errors while testing 200 samples?

**Task 11:** 

Problem: A server experiences an average of 3 system failures per day. What is the probability that in a 5-day period, there will be exactly 20 system failures?

Task 12:

Problem: A city experiences an average of 7 earthquakes per year. What is the probability that the city will experience fewer than 5 earthquakes in a 3-year period?