1 this is the group $\mathbb{Z}_7$

a $4+4 = 8 \mod 7 = 1$

b $3 \times 5 = 15 \mod 7 = 1$

c want to find n s.t $3+n = 7 \Rightarrow n = 4$

2) As S is the group $\mathbb{Z}_7$ and 7 is prime it is a group from the definition

1. Closure

this is obvious as $n+m \mod 7 = p \mod 7$ which is in the group

$$y = (n+m) \mod 7 \Rightarrow$$

$$y = (n+m)7 + r$$

$$y = n7 + t + m7 + (r-t) \qquad t \in S$$

$$\Rightarrow y = t \mod 7 + (r-t) \mod 7$$

2 Associativity

$$(a \mod 7 + b \mod 7) + c \mod 7$$

$$= a \mod 7 + (b \mod 7 + c \mod 7).$$

3 Identity 0 as $0 \mod 7 = 0$

& $0 + n \ \forall\, n \in S$

4 In verse

want $m \; \forall n$ s.t $\quad n \bmod 7 + m \bmod 7 = 0 \bmod 7$

$\Rightarrow \; l7 + n \; + \; \lambda 7 + m$

$7(l + \lambda) + n + m$

and if $\quad m = 7 - n$

$7(l+\lambda) + n + 7 - n$

$7(l + \lambda + 1) \equiv 0 \bmod 7$

and all $\quad 7 - n \in S$

3 $\quad -13 = n5 + m \qquad$ let $\; n = -3$

$-13 = (-3)(5) + m$

$-13 + 15 = m$

$2 = m$

4 $\quad$ 3rd degree as

if $\; g = x^3 - x^2 + 4x - 12$

$g = \sum \alpha_i x^i \qquad \sup(i)$ s.t $\alpha_i \neq 0 \quad = 3$

$x^3 - x^2 + 4x - 12$

as @ $x = 2$

$2^3 - (2)^2 + 4(2) - 12$

$8 - 4 + 8 - 12 = 0$

$(x - 2)$ is a root

2 is a pos root