Метод на най-малките квадрати (МНМК)

Генериране на данни

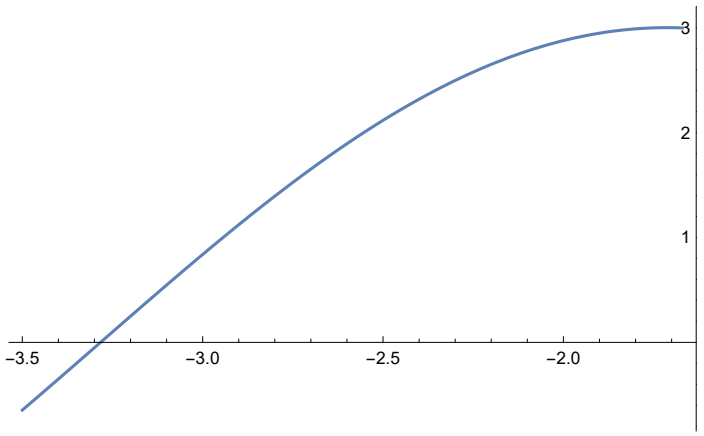
```
In[1]:= xt = Table[-2 + q * (0.13), {q, -10, 1}]
Out[1]= {-3.3, -3.17, -3.04, -2.91, -2.78, -2.65, -2.52, -2.39, -2.26, -2.13, -2., -1.87}

In[4]:= bigN = Length[xt]
Out[4]= 12

In[2]:= f[x_] := 3 * Sin[x - 3]
        yt = f[xt]
Out[3]= {-0.0504417, 0.338831, 0.722386, 1.09375, 1.44666,
         1.77515, 2.07368, 2.33722, 2.56131, 2.74218, 2.87677, 2.96282}
```

Визуализация

графика на функцията (която НЕ знаем)

```
In[6]:= grf = Plot[f[x], {x, xt[[1]] - 0.2, xt[[bigN]] + 0.2}]
```

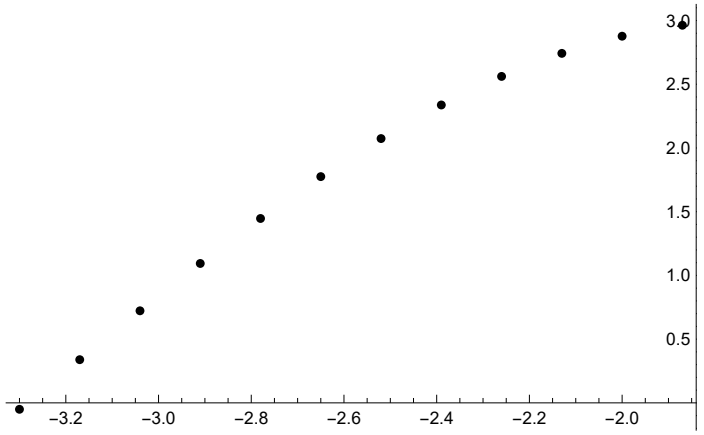
Out[6]=



графика на точките (които знаем)

In[7]:=
```
points = Table[{xt[[i]], yt[[i]]}, {i, 1, bigN}];
grp = ListPlot[points, PlotStyle → Black]
```

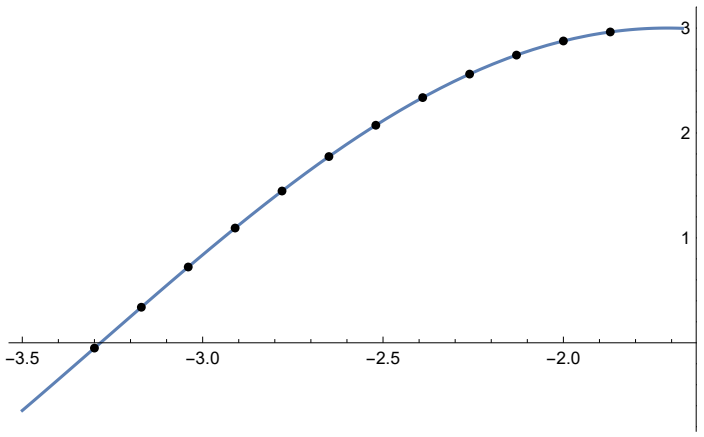
Out[8]=



двете графики на едно

In[9]:=
```
Show[grf, grp]
```

Out[9]=



Линейна регресия

За попълване на таблицата с междинните резултати

In[10]:=
```
xt^2
```

Out[10]=
```
{10.89, 10.0489, 9.2416, 8.4681, 7.7284,
 7.0225, 6.3504, 5.7121, 5.1076, 4.5369, 4., 3.4969}
```

In[11]:=
```
yt * xt
```

Out[11]=
```
{0.166458, -1.0741, -2.19605, -3.18281, -4.0217, -4.70414,
 -5.22567, -5.58595, -5.78857, -5.84085, -5.75355, -5.54046}
```

Сумите

In[12]:=
$$\sum_{i=1}^{bigN} xt[[i]]$$

Out[12]=
$$-31.02$$

In[13]:=
$$\sum_{i=1}^{bigN} yt[[i]]$$

Out[13]=
$$20.8803$$

In[14]:=
$$\sum_{i=1}^{bigN} xt[[i]]^2$$

Out[14]=
$$82.6034$$

In[15]:=
$$\sum_{i=1}^{bigN} yt[[i]] * xt[[i]]$$

Out[15]=
$$-48.7474$$

Решаваме СЛАУ

In[16]:=
```
A = {{12, -31.02}, {-31.02, 82.6034}}; b = {20.8803, -48.7474};
LinearSolve[A, b]
```

Out[17]=
$$\{7.33229, 2.16335\}$$

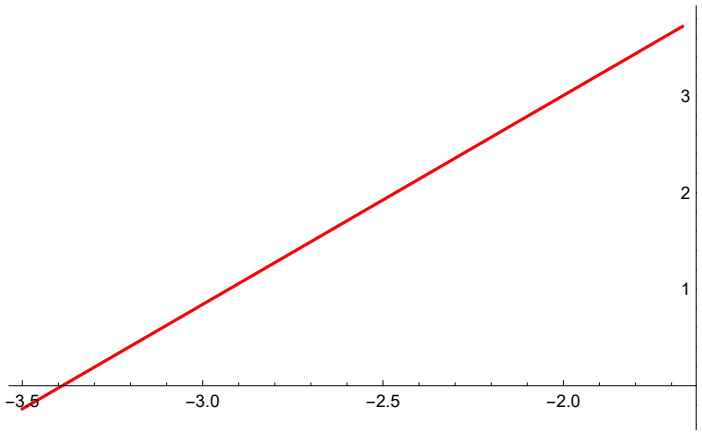
Съставяме полинома

In[19]:=
```
P1[x_] := 2.16335 x + 7.33229
```

Визуализация

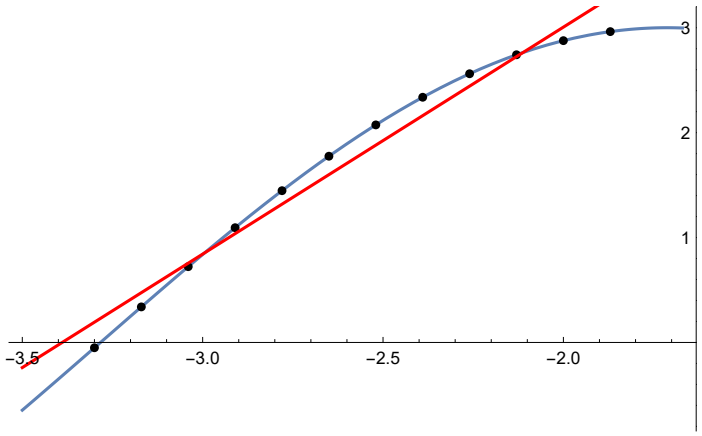
In[20]:=
```
grP1 = Plot[P1[x], {x, xt[[1]] - 0.2, xt[[bigN]] + 0.2}, PlotStyle → Red]
```

Out[20]=



In[21]:=
```
Show[grf, grp, grP1]
```

Out[21]=



Пресмятаме приближена стойност на функцията

In[22]:=
```
P1[-2.5]
```

Out[22]=

```
1.92392
```

Оценка на грешката

Истинска грешка (за сравнение)

In[23]:=
```
Abs[P1[-2.5] - f[-2.5]]
```

Out[23]=

```
0.192706
```

Теоретична грешка (средноквадратична)

In[24]:=
$$\sqrt{\sum_{i=1}^{bigN} (yt[[i]] - P1[xt[[i]]])^2}$$

Out[24]=
```
0.577383
```

Квадратична регресия

За попълване на таблицата с междинните резултати

In[•]:=
```
xt^2
```

Out[•]=
```
{10.89, 10.0489, 9.2416, 8.4681, 7.7284,
 7.0225, 6.3504, 5.7121, 5.1076, 4.5369, 4., 3.4969}
```

In[•]:=
```
yt * xt
```

Out[•]=
```
{0.166458, -1.0741, -2.19605, -3.18281, -4.0217, -4.70414,
 -5.22567, -5.58595, -5.78857, -5.84085, -5.75355, -5.54046}
```

In[25]:=
```
xt^3
```

Out[25]=
```
{-35.937, -31.855, -28.0945, -24.6422, -21.485,
 -18.6096, -16.003, -13.6519, -11.5432, -9.6636, -8., -6.5392}
```

In[26]:=
```
xt^4
```

Out[26]=
```
{118.592, 100.98, 85.4072, 71.7087, 59.7282,
 49.3155, 40.3276, 32.6281, 26.0876, 20.5835, 16., 12.2283}
```

In[27]:=
```
yt * xt^2
```

Out[27]=
```
{-0.54931, 3.40488, 6.67601, 9.26198, 11.1803,
 12.466, 13.1687, 13.3504, 13.0822, 12.441, 11.5071, 10.3607}
```

Сумите

In[•]:=
$$\sum_{i=1}^{bigN} xt[[i]]$$

Out[•]=
```
-31.02
```

In[•]:=
$$\sum_{i=1}^{bigN} yt[[i]]$$

Out[•]=
```
20.8803
```

In[•]:=
$$\sum_{i=1}^{bigN} xt[[i]]^2$$

Out[•]=
82.6034

In[•]:=
$$\sum_{i=1}^{bigN} yt[[i]] * xt[[i]]$$

Out[•]=
−48.7474

In[28]:=
$$\sum_{i=1}^{bigN} xt[[i]]^3$$

Out[28]=
−226.024

In[29]:=
$$\sum_{i=1}^{bigN} xt[[i]]^4$$

Out[29]=
633.587

In[30]:=
$$\sum_{i=1}^{bigN} yt[[i]] * xt[[i]]^2$$

Out[30]=
116.35

Решаваме СЛАУ

In[31]:=
```
A = {{12, -31.02, 82.6034}, {-31.02, 82.6034, -226.02}, {82.6034, -226.02, 633.58}}; b = {20.8803, -48.7474, 116.35};
LinearSolve[A, b]
```

Out[32]=
{1.71917, −2.31602, −0.866702}

In[33]:=
```
Fit[points, {1, x, x^2}, x]
```

Out[33]=
$1.34238 - 2.61503\,x - 0.924255\,x^2$

точно въвеждане с изразите:

In[52]:=
$$A = \begin{pmatrix} bigN & \sum_{i=1}^{bigN} xt[[i]] & \sum_{i=1}^{bigN} xt[[i]]^2 \\ \sum_{i=1}^{bigN} xt[[i]] & \sum_{i=1}^{bigN} xt[[i]]^2 & \sum_{i=1}^{bigN} xt[[i]]^3 \\ \sum_{i=1}^{bigN} xt[[i]]^2 & \sum_{i=1}^{bigN} xt[[i]]^3 & \sum_{i=1}^{bigN} xt[[i]]^4 \end{pmatrix};$$

$$b = \left\{\sum_{i=1}^{bigN} yt[[i]], \sum_{i=1}^{bigN} yt[[i]] * xt[[i]], \sum_{i=1}^{bigN} yt[[i]] * xt[[i]]^2\right\};$$

```
LinearSolve[A, b]
```

Out[53]=
{1.34238, −2.61503, −0.924255}

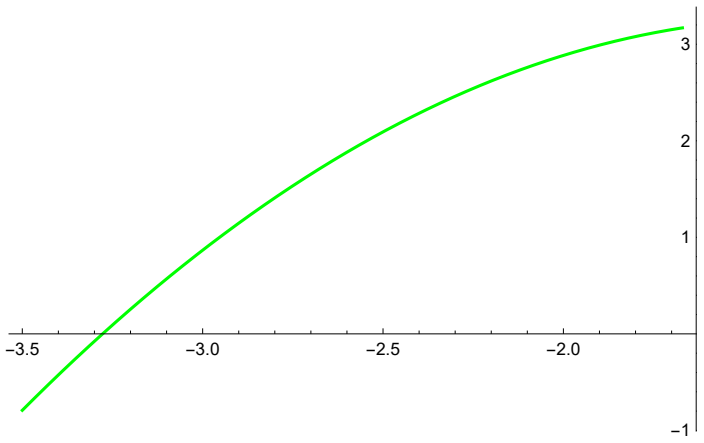
Съставяме полинома

In[34]:=
```
P2[x_] := -0.8667 x^2 - 2.31602 x + 1.7191
```

Визуализация

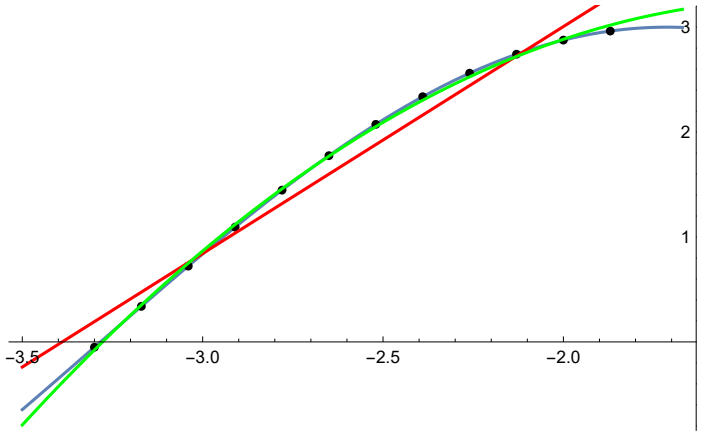
In[37]:=
```
grP2 = Plot[P2[x], {x, xt[[1]] - 0.2, xt[[bigN]] + 0.2}, PlotStyle → Green]
```

Out[37]=



In[38]:=
```
Show[grf, grp, grP1, grP2]
```

Out[38]=



Пресмятаме приближена стойност на функцията

In[39]:=
```
P2[-2.5]
```

Out[39]=

```
2.09228
```

за сравнение истинската стойност е

In[40]:=
```
f[-2.5]
```

Out[40]=

```
2.11662
```

Оценка на грешката

Истинска грешка (за сравнение)

In[41]:=
```
Abs[P2[-2.5] - f[-2.5]]
```

Out[41]=
```
0.024346
```

Теоретична грешка (среднoквадратична)

In[42]:=
```
Sqrt[Sum[(yt[[i]] - P2[xt[[i]]])^2, {i, 1, bigN}]]
```

Out[42]=
```
0.0948914
```

Кубична регресия

За попълване на таблицата с междинните резултати

In[•]:=
```
xt^2
```

Out[•]=
```
{10.89, 10.0489, 9.2416, 8.4681, 7.7284,
 7.0225, 6.3504, 5.7121, 5.1076, 4.5369, 4., 3.4969}
```

In[•]:=
```
yt * xt
```

Out[•]=
```
{0.166458, -1.0741, -2.19605, -3.18281, -4.0217, -4.70414,
 -5.22567, -5.58595, -5.78857, -5.84085, -5.75355, -5.54046}
```

In[•]:=
```
xt^3
```

Out[•]=
```
{-35.937, -31.855, -28.0945, -24.6422, -21.485,
 -18.6096, -16.003, -13.6519, -11.5432, -9.6636, -8., -6.5392}
```

In[•]:=
```
xt^4
```

Out[•]=
```
{118.592, 100.98, 85.4072, 71.7087, 59.7282,
 49.3155, 40.3276, 32.6281, 26.0876, 20.5835, 16., 12.2283}
```

In[•]:=
```
yt * xt^2
```

Out[•]=
```
{-0.54931, 3.40488, 6.67601, 9.26198, 11.1803,
 12.466, 13.1687, 13.3504, 13.0822, 12.441, 11.5071, 10.3607}
```

In[43]:=
```
xt^5
```

Out[43]=
```
{-391.354, -320.108, -259.638, -208.672, -166.044,
 -130.686, -101.626, -77.9811, -58.9579, -43.8428, -32., -22.8669}
```

In[44]:= xt^6

Out[44]=

$$\{1291.47, 1014.74, 789.299, 607.237, 461.603, 346.318, 256.096, 186.375, 133.245, 93.3851, 64., 42.7612\}$$

In[45]:= $yt * xt^3$

Out[45]=

$$\{1.81272, -10.7935, -20.2951, -26.9524, -31.0813, -33.0348, -33.1851, -31.9075, -29.5657, -26.4993, -23.0142, -19.3745\}$$

Сумите

In[•]:= $\sum_{i=1}^{bigN} xt[[i]]$

Out[•]=

$$-31.02$$

In[•]:= $\sum_{i=1}^{bigN} yt[[i]]$

Out[•]=

$$20.8803$$

In[•]:= $\sum_{i=1}^{bigN} xt[[i]]^2$

Out[•]=

$$82.6034$$

In[•]:= $\sum_{i=1}^{bigN} yt[[i]] * xt[[i]]$

Out[•]=

$$-48.7474$$

In[•]:= $\sum_{i=1}^{bigN} xt[[i]]^3$

Out[•]=

$$-226.024$$

In[•]:= $\sum_{i=1}^{bigN} xt[[i]]^4$

Out[•]=

$$633.587$$

In[•]:= $\sum_{i=1}^{bigN} yt[[i]] * xt[[i]]^2$

Out[•]=

$$116.35$$

In[46]:=
$$\sum_{i=1}^{bigN} xt[[i]]^5$$

Out[46]=
$$-1813.78$$

In[47]:=
$$\sum_{i=1}^{bigN} xt[[i]]^6$$

Out[47]=
$$5286.53$$

In[48]:=
$$\sum_{i=1}^{bigN} yt[[i]] * xt[[i]]^3$$

Out[48]=
$$-283.891$$

Решаваме СЛАУ

In[57]:=
$$A = \begin{pmatrix} bigN & \sum_{i=1}^{bigN} xt[[i]] & \sum_{i=1}^{bigN} xt[[i]]^2 & \sum_{i=1}^{bigN} xt[[i]]^3 \\ \sum_{i=1}^{bigN} xt[[i]] & \sum_{i=1}^{bigN} xt[[i]]^2 & \sum_{i=1}^{bigN} xt[[i]]^3 & \sum_{i=1}^{bigN} xt[[i]]^4 \\ \sum_{i=1}^{bigN} xt[[i]]^2 & \sum_{i=1}^{bigN} xt[[i]]^3 & \sum_{i=1}^{bigN} xt[[i]]^4 & \sum_{i=1}^{bigN} xt[[i]]^5 \\ \sum_{i=1}^{bigN} xt[[i]]^3 & \sum_{i=1}^{bigN} xt[[i]]^4 & \sum_{i=1}^{bigN} xt[[i]]^5 & \sum_{i=1}^{bigN} xt[[i]]^6 \end{pmatrix};$$

$$b = \left\{\sum_{i=1}^{bigN} yt[[i]], \sum_{i=1}^{bigN} yt[[i]] * xt[[i]], \sum_{i=1}^{bigN} yt[[i]] * xt[[i]]^2, \sum_{i=1}^{bigN} yt[[i]] * xt[[i]]^3\right\};$$

```
a = LinearSolve[A, b]
```

Out[58]=
$$\{-4.71921, -9.91613, -3.80018, -0.370848\}$$

за сравнение:

In[56]:=
```
Fit[points, {1, x, x^2, x^3}, x]
```

Out[56]=
$$-4.71921 - 9.91613\,x - 3.80018\,x^2 - 0.370848\,x^3$$

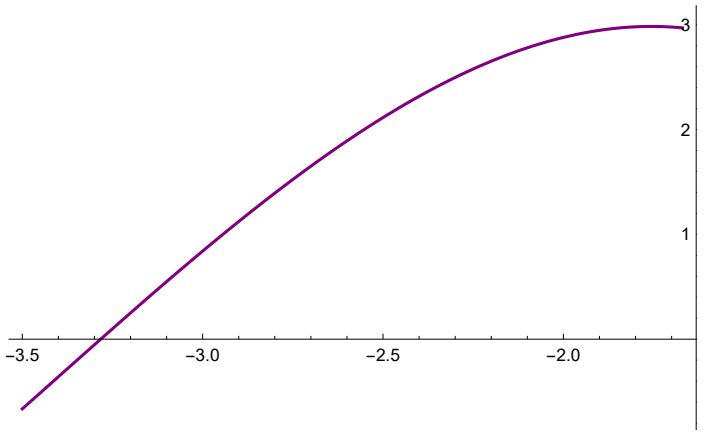
Съставяме полинома

In[59]:=
```
P3[x_] := a[[1]] + a[[2]] x + a[[3]] x^2 + a[[4]] x^3
```

Визуализация

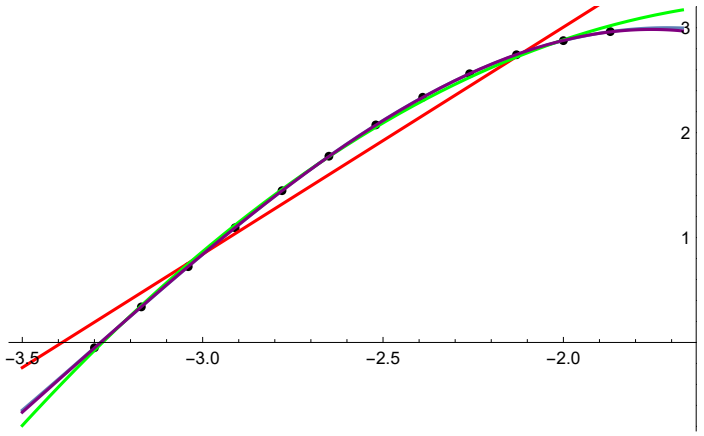
In[60]:=
```
grP3 = Plot[P3[x], {x, xt[[1]] - 0.2, xt[[bigN]] + 0.2}, PlotStyle → Purple]
```

Out[60]=



In[61]:=
```
Show[grf, grp, grP1, grP2, grP3]
```

Out[61]=



Пресмятаме приближена стойност на функцията

In[62]:=
```
P3[-2.5]
```

Out[62]=

```
2.11446
```

за сравнение истинската стойност е

In[•]:=
```
f[-2.5]
```

Out[•]=

```
2.11662
```

Оценка на грешката

Истинска грешка (за сравнение)

In[63]:=
```
Abs[P3[-2.5] - f[-2.5]]
```

Out[63]=
```
0.00215715
```

Теоретична грешка (среднoквадратична)

In[64]:=
$$\sqrt{\sum_{i=1}^{bigN} (yt[[i]] - P3[xt[[i]]])^2}$$

Out[64]=
```
0.00688749
```