Метод на най-малките квадрати

Генериране на данни

In[1]:=
```
xt = Table[2 + s * 0.17, {s, -20, 3}]
```

Out[1]=
```
{-1.4, -1.23, -1.06, -0.89, -0.72, -0.55, -0.38, -0.21, -0.04, 0.13, 0.3,
 0.47, 0.64, 0.81, 0.98, 1.15, 1.32, 1.49, 1.66, 1.83, 2., 2.17, 2.34, 2.51}
```

In[2]:=
```
f[x_] := -2 Cos[π x]
yt = f[xt]
```

Out[3]=
```
{0.618034, 1.50022, 1.96457, 1.88176, 1.27485, 0.312869, -0.736249, -1.58031,
 -1.98423, -1.83551, -1.17557, -0.188217, 0.851559, 1.65416, 1.99605, 1.78201,
 1.07165, 0.0628215, -0.963507, -1.72148, -2., -1.72148, -0.963507, 0.0628215}
```

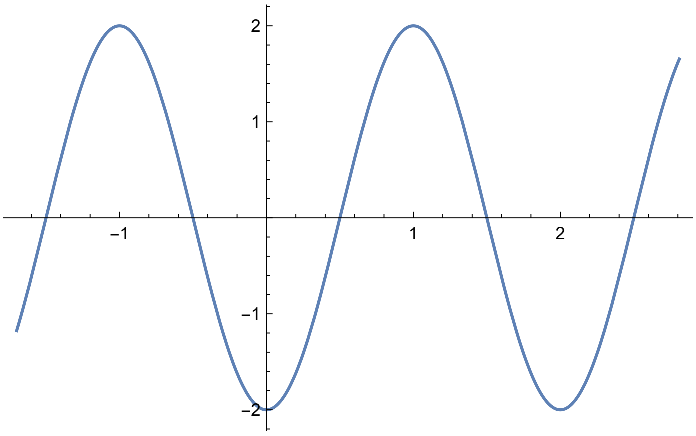
In[4]:=
```
M = Length[xt]
```

Out[4]= 24

Визуализация

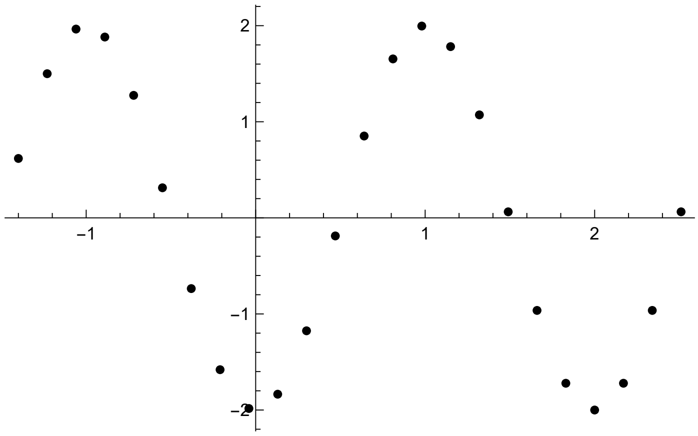
In[5]:=
```
grf = Plot[f[x], {x, xt〚1〛 - 0.3, xt〚M〛 + 0.3}]
```

Out[5]=



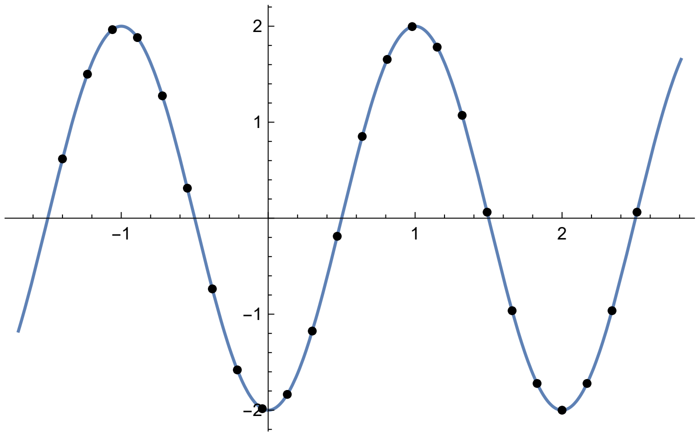
In[6]:=
```
points = Table[{xt[[i]], yt[[i]]}, {i, 1, M}];
grp = ListPlot[points, PlotStyle → Black]
```

Out[7]=



In[8]:=
```
Show[grf, grp]
```

Out[8]=



Линейна регресия

Попълване на таблицата

In[9]:= xt^2

Out[9]=
```
{1.96, 1.5129, 1.1236, 0.7921, 0.5184, 0.3025, 0.1444,
 0.0441, 0.0016, 0.0169, 0.09, 0.2209, 0.4096, 0.6561, 0.9604,
 1.3225, 1.7424, 2.2201, 2.7556, 3.3489, 4., 4.7089, 5.4756, 6.3001}
```

In[10]:=
```
yt * xt
```

Out[10]=
```
{-0.865248, -1.84527, -2.08245, -1.67477, -0.917891, -0.172078, 0.279775, 0.331865,
 0.0793692, -0.238616, -0.352671, -0.0884618, 0.544997, 1.33987, 1.95613, 2.04932,
 1.41458, 0.0936041, -1.59942, -3.15032, -4., -3.73562, -2.25461, 0.157682}
```

Пресмятаме сумите

In[11]:= $\sum_{i=1}^{M} xt[[i]]$

Out[11]=

13.32

In[12]:= $\sum_{i=1}^{M} yt[[i]]$

Out[12]=

0.163324

In[13]:= $\sum_{i=1}^{M} xt[[i]]^2$

Out[13]=

40.6276

In[14]:= $\sum_{i=1}^{M} yt[[i]] * xt[[i]]$

Out[14]=

-14.7302

Решаваме СЛАУ

In[15]:= A = $\begin{pmatrix} 24 & 13.32 \\ 13.32 & 40.628 \end{pmatrix}$; b = {0.163, -14.730};

```
LinearSolve[A, b]
```

Out[16]=

{0.25428, -0.445924}

Съставяме полинома

In[17]:=
```
P1[x_] := -0.446 x + 0.254
```

таен коз

In[25]:=
```
Fit[points, {x, 1}, x]
```

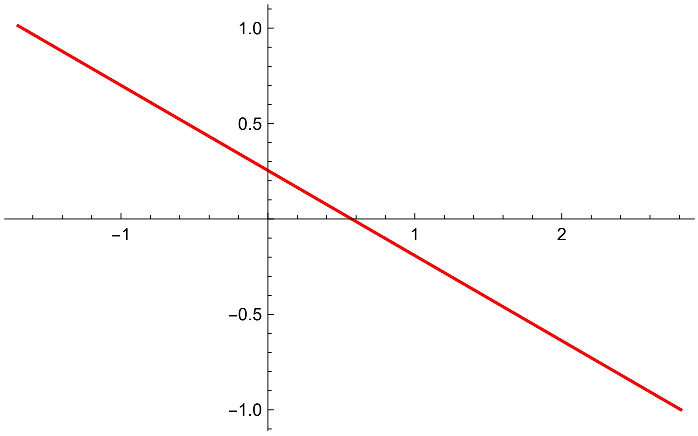
Out[25]=

0.254303 - 0.445942 x

Визуализация

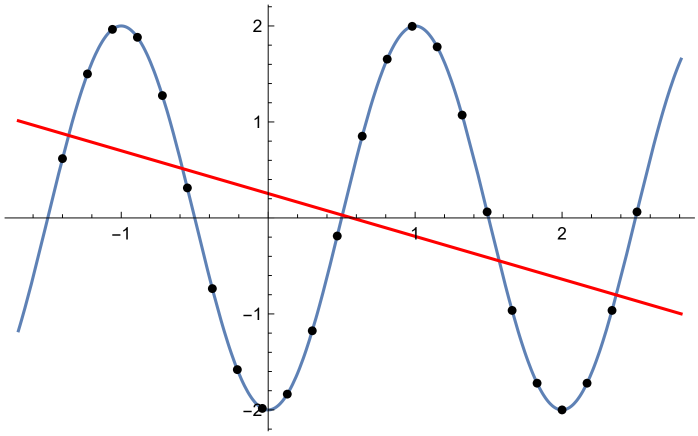
In[18]:=
```
grP1 = Plot[P1[x], {x, xt[[1]] - 0.3, xt[[M]] + 0.3}, PlotStyle → Red]
```

Out[18]=



In[20]:=
```
Show[grf, grp, grP1]
```

Out[20]=



Пресмятане на приближена стойност на функция (апроксимация)

Приближена стойност

In[21]:=
```
P1[2.]
```

Out[21]=

```
-0.638
```

Истинска стойност (за сравнение)

In[22]:=
```
f[2.]
```

Out[22]=

```
-2.
```

Оценка на грешката

Теоретична грешка (средноквадратична)

In[23]:= $\sqrt{\sum_{i=1}^{M}(yt[[i]]-P1[xt[[i]]])^2}$

Out[23]=

```
6.36163
```

Истинска грешка

In[24]:= **Abs[f[2.] - P1[2.]]**

Out[24]=

```
1.362
```

Квадратична регресия

Попълване на таблицата

In[∘]:= xt^2

Out[∘]=

```
{1.96, 1.5129, 1.1236, 0.7921, 0.5184, 0.3025, 0.1444,
 0.0441, 0.0016, 0.0169, 0.09, 0.2209, 0.4096, 0.6561, 0.9604,
 1.3225, 1.7424, 2.2201, 2.7556, 3.3489, 4., 4.7089, 5.4756, 6.3001}
```

In[∘]:= **yt * xt**

Out[∘]=

```
{-0.865248, -1.84527, -2.08245, -1.67477, -0.917891, -0.172078, 0.279775, 0.331865,
 0.0793692, -0.238616, -0.352671, -0.0884618, 0.544997, 1.33987, 1.95613, 2.04932,
 1.41458, 0.0936041, -1.59942, -3.15032, -4., -3.73562, -2.25461, 0.157682}
```

In[26]:= xt^3

Out[26]=

```
{-2.744, -1.86087, -1.19102, -0.704969, -0.373248, -0.166375, -0.054872,
 -0.009261, -0.000064, 0.002197, 0.027, 0.103823, 0.262144, 0.531441, 0.941192,
 1.52087, 2.29997, 3.30795, 4.5743, 6.12849, 8., 10.2183, 12.8129, 15.8133}
```

In[27]:= xt^4

Out[27]=

{3.8416, 2.28887, 1.26248, 0.627422, 0.268739, 0.0915063, 0.0208514, 0.00194481,
 2.56×10^{-6}, 0.00028561, 0.0081, 0.0487968, 0.167772, 0.430467, 0.922368,
 1.74901, 3.03596, 4.92884, 7.59333, 11.2151, 16., 22.1737, 29.9822, 39.6913}

In[28]:= $yt * xt^2$

Out[28]=

```
{1.21135, 2.26969, 2.2074, 1.49054, 0.660881, 0.0946429, -0.106314, -0.0696917,
 -0.00317477, -0.0310201, -0.105801, -0.0415771, 0.348798, 1.0853, 1.91701,
 2.35671, 1.86725, 0.13947, -2.65504, -5.76508, -8., -8.1063, -5.27578, 0.395782}
```

Пресмятаме сумите

In[◦]:= $\sum_{i=1}^{M} xt[[i]]$

Out[◦]= 13.32

In[◦]:= $\sum_{i=1}^{M} yt[[i]]$

Out[◦]= 0.163324

In[◦]:= $\sum_{i=1}^{M} xt[[i]]^2$

Out[◦]= 40.6276

In[◦]:= $\sum_{i=1}^{M} yt[[i]] * xt[[i]]$

Out[◦]= -14.7302

In[29]:= $\sum_{i=1}^{M} xt[[i]]^3$

Out[29]= 59.4392

In[30]:= $\sum_{i=1}^{M} xt[[i]]^4$

Out[30]= 146.351

In[31]:= $\sum_{i=1}^{M} yt[[i]] * xt[[i]]^2$

Out[31]= -14.115

Решаваме СЛАУ

In[32]:= $A = \begin{pmatrix} 24 & 13.32 & 40.628 \\ 13.32 & 40.628 & 59.439 \\ 40.628 & 59.439 & 146.351 \end{pmatrix}$; b = {0.163, -14.730, -14.115};

LinearSolve[A, b]

Out[33]= {0.193735, -0.508334, 0.0562265}

In[45]:=
```
A = ( M                  ∑_{i=1}^M xt[[i]]    ∑_{i=1}^M xt[[i]]^2
      ∑_{i=1}^M xt[[i]]    ∑_{i=1}^M xt[[i]]^2  ∑_{i=1}^M xt[[i]]^3
      ∑_{i=1}^M xt[[i]]^2  ∑_{i=1}^M xt[[i]]^3  ∑_{i=1}^M xt[[i]]^4 );
b = {∑_{i=1}^M yt[[i]], ∑_{i=1}^M yt[[i]] * xt[[i]], ∑_{i=1}^M yt[[i]] * xt[[i]]^2};
a = LinearSolve[A, b]
```

$$A = \begin{pmatrix} M & \sum_{i=1}^{M} xt[[i]] & \sum_{i=1}^{M} xt[[i]]^2 \\ \sum_{i=1}^{M} xt[[i]] & \sum_{i=1}^{M} xt[[i]]^2 & \sum_{i=1}^{M} xt[[i]]^3 \\ \sum_{i=1}^{M} xt[[i]]^2 & \sum_{i=1}^{M} xt[[i]]^3 & \sum_{i=1}^{M} xt[[i]]^4 \end{pmatrix};$$

Out[46]=

{0.19375, −0.508363, 0.0562356}

Съставяме полинома

In[36]:= `P2[x_] := 0.056226 x^2 - 0.5083 x + 0.1937`

In[47]:= `P2[x_] := a[[1]] + a[[2]] x + a[[3]] x^2`

In[48]:= `P2[x]`

Out[48]=

$0.19375 - 0.508363\,x + 0.0562356\,x^2$

таен коз

In[35]:= `Fit[points, {x^2, x, 1}, x]`

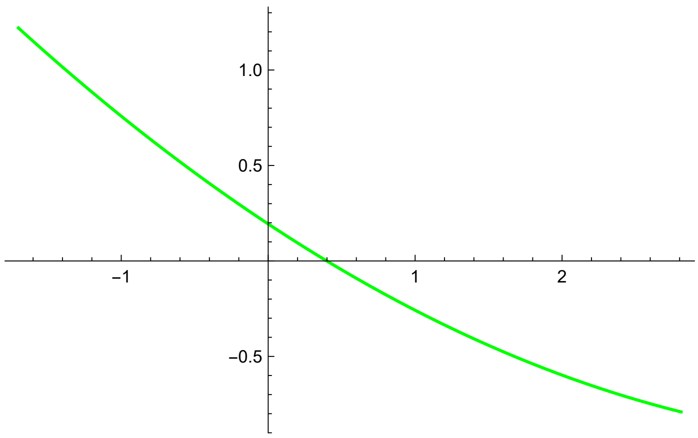
Out[35]=

$0.19375 - 0.508363\,x + 0.0562356\,x^2$

Визуализация

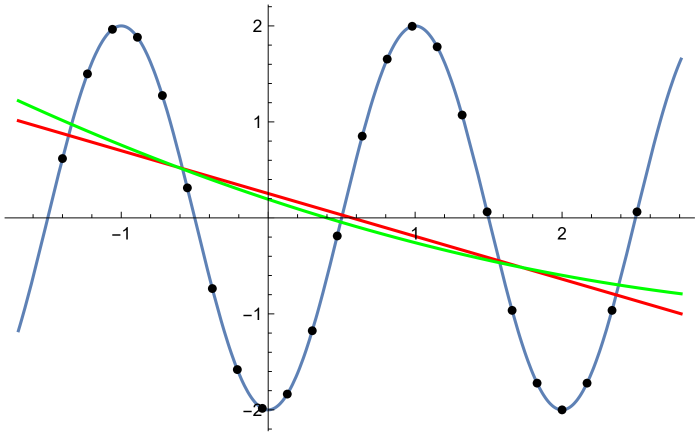
In[37]:= `grP2 = Plot[P2[x], {x, xt[[1]] - 0.3, xt[[M]] + 0.3}, PlotStyle → Green]`

Out[37]=



In[38]:= `Show[grf, grp, grP1, grP2]`

Out[38]=



Пресмятане на приближена стойност на функция (апроксимация)

Приближена стойност

In[39]:= `P2[2.]`

Out[39]=

`-0.597996`

Истинска стойност (за сравнение)

In[ ]:= `f[2.]`

Out[ ]=

`-2.`

Оценка на грешката

Теоретична грешка (средноквадратична)

In[40]:= $\sqrt{\sum_{i=1}^{M} (yt[[i]] - P2[xt[[i]]])^2}$

Out[40]=

`6.35252`

Истинска грешка

In[42]:= `Abs[f[2.] - P2[2.]]`

Out[42]=

`1.402`

Кубична регресия

Попълване на таблицата

In[∘]:= xt^2

Out[∘]=

{1.96, 1.5129, 1.1236, 0.7921, 0.5184, 0.3025, 0.1444, 0.0441, 0.0016, 0.0169, 0.09, 0.2209, 0.4096, 0.6561, 0.9604, 1.3225, 1.7424, 2.2201, 2.7556, 3.3489, 4., 4.7089, 5.4756, 6.3001}

In[∘]:= yt * xt

Out[∘]=

{-0.865248, -1.84527, -2.08245, -1.67477, -0.917891, -0.172078, 0.279775, 0.331865, 0.0793692, -0.238616, -0.352671, -0.0884618, 0.544997, 1.33987, 1.95613, 2.04932, 1.41458, 0.0936041, -1.59942, -3.15032, -4., -3.73562, -2.25461, 0.157682}

In[∘]:= xt^3

Out[∘]=

{-2.744, -1.86087, -1.19102, -0.704969, -0.373248, -0.166375, -0.054872, -0.009261, -0.000064, 0.002197, 0.027, 0.103823, 0.262144, 0.531441, 0.941192, 1.52087, 2.29997, 3.30795, 4.5743, 6.12849, 8., 10.2183, 12.8129, 15.8133}

In[∘]:= xt^4

Out[∘]=

{3.8416, 2.28887, 1.26248, 0.627422, 0.268739, 0.0915063, 0.0208514, 0.00194481, 2.56×10^{-6}, 0.00028561, 0.0081, 0.0487968, 0.167772, 0.430467, 0.922368, 1.74901, 3.03596, 4.92884, 7.59333, 11.2151, 16., 22.1737, 29.9822, 39.6913}

In[∘]:= $yt * xt^2$

Out[∘]=

{1.21135, 2.26969, 2.2074, 1.49054, 0.660881, 0.0946429, -0.106314, -0.0696917, -0.00317477, -0.0310201, -0.105801, -0.0415771, 0.348798, 1.0853, 1.91701, 2.35671, 1.86725, 0.13947, -2.65504, -5.76508, -8., -8.1063, -5.27578, 0.395782}

добавяме необходимото

Пресмятаме сумите

In[∘]:= $\sum_{i=1}^{M} xt[[i]]$

Out[∘]=

13.32

In[∘]:= $\sum_{i=1}^{M} yt[[i]]$

Out[∘]=

0.163324

In[∘]:= $\sum_{i=1}^{M} xt[[i]]^2$

Out[∘]=

40.6276

In[∘]:= $\sum_{i=1}^{M} yt[[i]] * xt[[i]]$

Out[∘]=

-14.7302

In[∘]:= $\sum_{i=1}^{M} xt[[i]]^3$

Out[∘]=

59.4392

In[∘]:= $\sum_{i=1}^{M} xt[[i]]^4$

Out[∘]=

146.351

In[∘]:= $\sum_{i=1}^{M} yt[[i]] * xt[[i]]^2$

Out[∘]=

-14.115

добавяме необходимото

Решаваме СЛАУ

In[49]:= $A = \begin{pmatrix} M & \sum_{i=1}^{M} xt[[i]] & \sum_{i=1}^{M} xt[[i]]^2 & \sum_{i=1}^{M} xt[[i]]^3 \\ \sum_{i=1}^{M} xt[[i]] & \sum_{i=1}^{M} xt[[i]]^2 & \sum_{i=1}^{M} xt[[i]]^3 & \sum_{i=1}^{M} xt[[i]]^4 \\ \sum_{i=1}^{M} xt[[i]]^2 & \sum_{i=1}^{M} xt[[i]]^3 & \sum_{i=1}^{M} xt[[i]]^4 & \sum_{i=1}^{M} xt[[i]]^5 \\ \sum_{i=1}^{M} xt[[i]]^3 & \sum_{i=1}^{M} xt[[i]]^4 & \sum_{i=1}^{M} xt[[i]]^5 & \sum_{i=1}^{M} xt[[i]]^6 \end{pmatrix};$

$b = \left\{\sum_{i=1}^{M} yt[[i]], \sum_{i=1}^{M} yt[[i]] * xt[[i]], \sum_{i=1}^{M} yt[[i]] * xt[[i]]^2, \sum_{i=1}^{M} yt[[i]] * xt[[i]]^3\right\};$

```
a = LinearSolve[A, b]
```

Out[50]=

{-0.229343, 0.0384226, 0.63879, -0.349882}

Съставяме полинома

In[51]:= $P3[x\_] := a[[1]] + a[[2]]\, x + a[[3]]\, x^2 + a[[4]]\, x^3$

In[52]:= P3[x]

Out[52]=

$-0.229343 + 0.0384226\, x + 0.63879\, x^2 - 0.349882\, x^3$

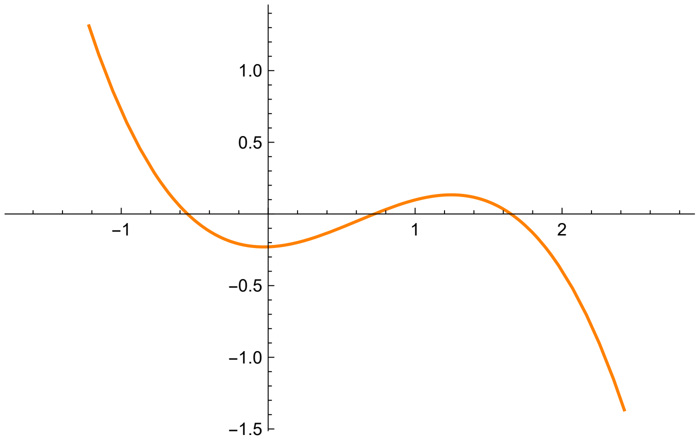
таен коз

In[53]:=
```
Fit[points, {x^3, x^2, x, 1}, x]
```
Out[53]=

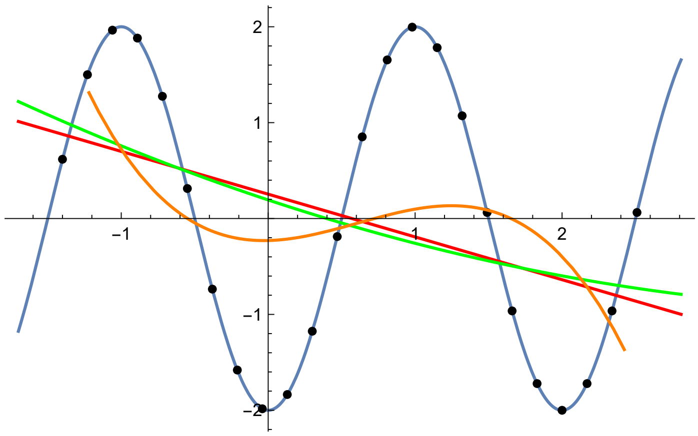
$-0.229343 + 0.0384226\, x + 0.63879\, x^2 - 0.349882\, x^3$

Визуализация

In[54]:=
```
grP3 = Plot[P3[x], {x, xt[[1]] - 0.3, xt[[M]] + 0.3}, PlotStyle → Orange]
```
Out[54]=



In[55]:=
```
Show[grf, grp, grP1, grP2, grP3]
```
Out[55]=



Пресмятане на приближена стойност на функция (апроксимация)

Приближена стойност

In[56]:=
```
P3[2.]
```
Out[56]=

−0.396398

Истинска стойност (за сравнение)

In[ ]:=
```
f[2.]
```
Out[ ]=

−2.

Оценка на грешката

Теоретична грешка (средноквадратична)

In[57]:=

$$\sqrt{\sum_{i=1}^{M} (yt[[i]] - P3[xt[[i]]])^2}$$

Out[57]=

```
5.96919
```

Истинска грешка

In[58]:=

```
Abs[f[2.] - P3[2.]]
```

Out[58]=

```
1.6036
```