Задача. Дадени са следните контролни точки: $P_0(0,0)$, $P_1(0,\frac{1}{4})$, $P_2(\frac{1}{2},\frac{1}{2})$, $P_3(1,\frac{1}{4})$, $P_4(1,0)$.

а) Запишете уравнението на кривата на Безие $C(u)$, дефинирана чрез дадените точки.

Решение:

$$B_{n,i}(u) = \frac{n!}{i!(n-i)!} u^i (1-u)^{n-i}, \quad n = 4$$

$$C(u) = B_{4,0}(u)P_0 + B_{4,1}(u)P_1 + B_{4,2}P_2 + B_{4,3}(u)P_3 + B_{4,4}(u)P_4$$

$$B_{4,0}(u) = \frac{4!}{0!(4-0)!} u^0 (1-u)^{4-0} = (1-u)^4$$

$$B_{4,1}(u) = \frac{4!}{1!(4-1)!} u^1 (1-u)^{4-1} = \frac{4.3.2.1}{1.3.2.1} u.(1-u)^3 = 4u(1-u)^3$$

$$B_{4,2}(u) = \frac{4!}{2!(4-2)!} u^2 (1-u)^{4-2} = \frac{4.3.2.1}{2.1.2.1} u^2(1-u)^2 = 6u^2(1-u)^2$$

$$B_{4,3}(u) = \frac{4!}{3!(4-3)!} u^3 (1-u)^{4-3} = \frac{4.3.2.1}{3.2.1.1} u^3(1-u)^1 = 4u^3(1-u)$$

$$B_{4,4}(u) = \frac{4!}{4!(4-4)!} u^4 (1-u)^{4-4} = u^4$$

$C(u) = B_{4,0}(u)P_0 + B_{4,1}(u)P_1 + B_{4,2}(u)P_2 + B_{4,3}(u)P_3 + B_{4,4}(u)P_4$

$C(u) = (1-u)^4(0,0) + 4u(1-u)^3\left(0,\frac{1}{4}\right) + 6u^2(1-u)^2\left(\frac{1}{2},\frac{1}{2}\right) + 4u^3(1-u)\left(1,\frac{1}{4}\right) + u^4(1,0) =$

$= \left(\frac{6u^2(1-u)^2}{2} + 4u^3(1-u) + u^4, \frac{4u(1-u)^3}{4} + \frac{6u^2(1-u)^2}{2} + \frac{4u^3(1-u)}{4}\right) =$

$= \left(3u^2(1-u)^2 + 4u^3(1-u) + u^4, u(1-u)^3 + 3u^2(1-u)^2 + u^3(1-u)\right) =$

$= \left(3u^2 - 2u^3 \;,\; u^4 - 2u^3 + u\right)$

$\Rightarrow C(u) = \left(3u^2 - 2u^3,\; u^4 - 2u^3 + u\right)$ – ур-нието на кривата.

б) Намерете точката от кривата, съответна на $u = 0{,}4$ чрез параметризацията ѝ.

Решение:

$u = 0{,}4$

$C(u) = \left(3u^2 - 2u^3,\; u^4 - 2u^3 + u\right)$ – ур-нието на кривата (параметризацията ѝ).

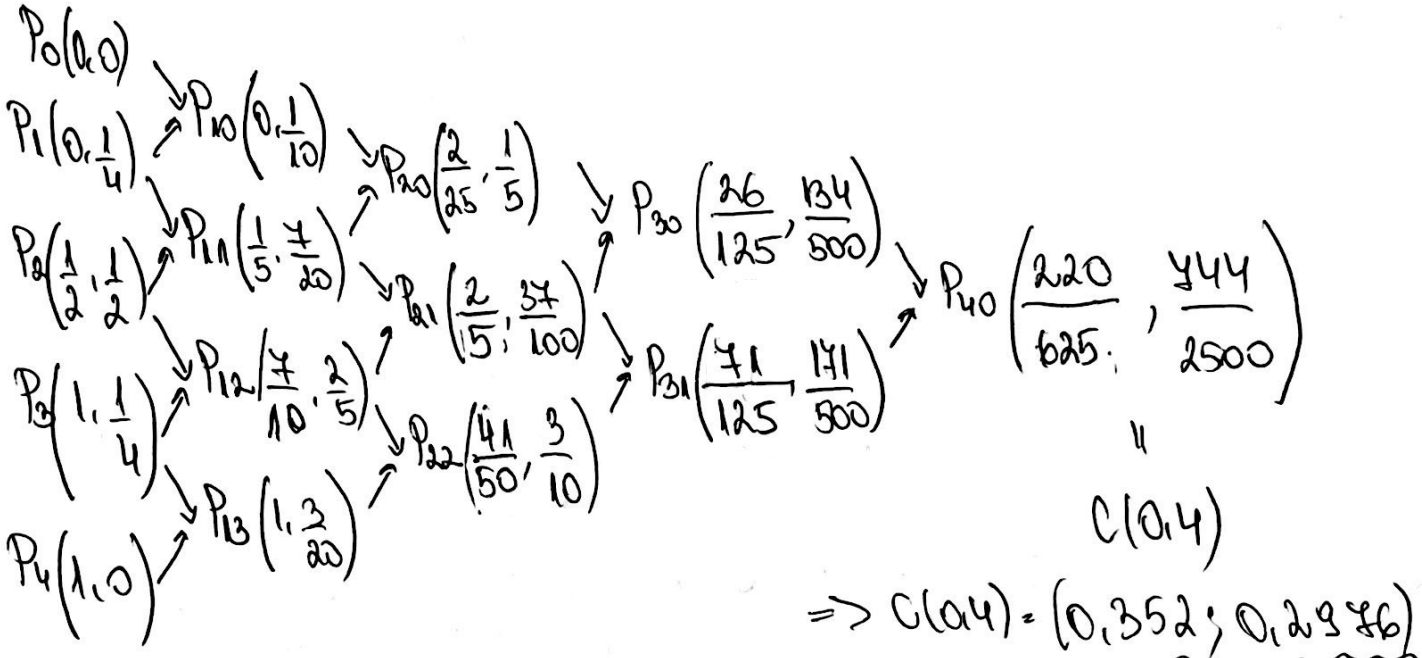
$C(0{,}4) = \left(3(0{,}4)^2 - 2(0{,}4)^3,\; (0{,}4)^4 - 2(0{,}4)^3 + 0{,}4\right) =$

$= \left(3.0{,}16 - 2.0{,}064,\; 0{,}0256 - 2.0{,}064 + 0{,}4\right) =$

$C(0{,}4) = (0{,}352,\; 0{,}2976)$

б) Чрез алгоритъма на фон Каснъно, намерете точка от кривата съответна на $u = 0{,}4$

Решение:



$\Rightarrow C(0{,}4) = (0{,}352;\ 0{,}2976)$

$u = 0{,}4 = \frac{4}{10} = \frac{2}{5}$, $1-u = 1-0{,}4 = 0{,}6 = \frac{6}{10} = \frac{3}{5}$

$P_{10} = (1-u)P_0 + uP_1 = \frac{3}{5}(0,0) + \frac{2}{5}(0,\frac{1}{4}) = (0,\frac{2}{20}) = (0,\frac{1}{10})$

$P_{11} = (1-u)P_1 + uP_2 = \frac{3}{5}(0,\frac{1}{4}) + \frac{2}{5}(\frac{1}{2},\frac{1}{2}) = (\frac{2}{10},\frac{3}{20}+\frac{2}{10}) = (\frac{1}{5},\frac{7}{20})$

$P_{12} = (1-u)P_2 + uP_3 = \frac{3}{5}(\frac{1}{2},\frac{1}{2}) + \frac{2}{5}(1,\frac{1}{4}) = (\frac{3}{10}+\frac{2}{5},\frac{3}{10}+\frac{2}{20}) = (\frac{7}{10},\frac{4}{10}) = (\frac{7}{10},\frac{2}{5})$

$P_{13} = (1-u)P_3 + uP_4 = \frac{3}{5}(1,\frac{1}{4}) + \frac{2}{5}(1,0) = (\frac{3}{5}+\frac{2}{5},\frac{3}{20}) = (1,\frac{3}{20})$

г) Разделете кривата $C(u)$ при $u=0,4$ и подредете контролните точки на двете дъги.

Решение:

$C_1(u): u\in[0;0,4] - P_0(0,0), P_{10}\left(0,\frac{1}{10}\right), P_{20}\left(\frac{2}{25},\frac{1}{5}\right), P_{30}\left(\frac{26}{125},\frac{134}{500}\right), P_{40}\left(\frac{220}{625},\frac{744}{2500}\right)$.

$C_2(u): u\in[0,4;1] - P_{40}\left(\frac{220}{625},\frac{744}{2500}\right), P_{31}\left(\frac{71}{125},\frac{171}{500}\right), P_{22}\left(\frac{41}{50},\frac{3}{10}\right), P_{13}\left(1,\frac{3}{20}\right), P_4(1,0)$

е) Чрез преместване на контролната точка P_2 в $P_2^*\left(\frac{1}{2},\frac{1}{3}\right)$ намерете зависимостта между $C(0,4)$ и $C^*(0,4)$.

Решение: $P_2 \to P_2^*$

$$C^*(0,4) = C(0,4) + B_{4,2}(0,4).\vec{V}$$

$$\vec{V} = P_2^* - P_2 = \left(\frac{1}{2},\frac{1}{3}\right) - \left(\frac{1}{2},\frac{1}{2}\right) = \left(0,-\frac{1}{6}\right)$$

$$B_{4,2}(0,4) = \frac{4!}{2!(4-2)!}(0,4)^2(1-0,4)^{4-2} = \frac{4.3.2.1}{2.1.2.1}\cdot\left(\frac{2}{5}\right)^2\cdot\left(\frac{3}{5}\right)^2 = 6.\frac{4}{25}.\frac{9}{25} = \frac{216}{625}$$

$$\Rightarrow B_{4,2}(0,4) = \frac{216}{625}$$

$$C(0,4) = \left(\frac{220}{625},\frac{744}{2500}\right)$$

$$C^*(0,4) = \left(\frac{220}{625},\frac{744}{2500}\right) + \frac{216}{625}\left(0,-\frac{1}{6}\right) = \left(\frac{220}{625},\frac{744}{2500}\right) + \left(0,-\frac{216}{3750}\right)$$

$$= (0,352; 0,2976) + (0,-0,0576) =$$

$$C^*(0,4) = (0,352; 0,24)$$

4) Намерете $\dot{C}(0{,}4)$ и $\ddot{C}(0{,}4)$.

Решение:

$$\dot{C}(u) = n\left[P_{n-1,1} - P_{n-1,0}\right], \quad \ddot{C}(u) = n.(n-1)\left[P_{n-2,2} - 2P_{n-2,1} + P_{n-2,0}\right]$$

В нашия случай имаме $n = 4$

$$\dot{C}(0{,}4) = 4\left[P_{31} - P_{30}\right] = 4\left[\left(\frac{71}{125}, \frac{171}{125}\right) - \left(\frac{26}{125}, \frac{134}{500}\right)\right] =$$

$$= 4\left[\left(\frac{71}{125} - \frac{26}{125}; \overset{4}{\frac{171}{125}} - \frac{134}{500}\right)\right] = 4\left[\frac{45}{125}; \frac{684}{500} - \frac{134}{500}\right] =$$

$$= 4\left(\frac{9}{25}, \frac{550}{500}\right) = 4\left(\frac{9}{25}, \frac{11}{10}\right) = \left(\frac{36}{25}, \frac{44}{10}\right)$$

$$\Rightarrow \dot{C}(0{,}4) = \left(\frac{36}{25}; \frac{44}{10}\right) = (1{,}44; 4{,}4)$$

$$\ddot{C}(0{,}4) = 4.3\left[P_{22} - 2P_{21} + P_{20}\right] = 12\left[\left(\frac{41}{50}, \frac{3}{10}\right) - 2\left(\frac{2}{5}, \frac{37}{100}\right) + \left(\frac{2}{25}, \frac{1}{5}\right)\right] =$$

$$= 12\left[\left(\frac{41}{50}, \frac{3}{10}\right) - \left(\frac{4}{5}, \frac{74}{100}\right) + \left(\frac{2}{25}, \frac{1}{5}\right)\right]$$

$$= 12\left(\frac{41}{50} - \frac{4}{5} + \frac{2}{25}; \frac{3}{10} - \frac{74}{100} + \frac{1}{5}\right) =$$

$$= 12\left(\frac{1}{10}, \frac{-24}{100}\right) = \left(\frac{12}{10}, \frac{-288}{100}\right)$$

$$\Rightarrow \ddot{C}(0{,}4) = (1{,}2; -2{,}88)$$