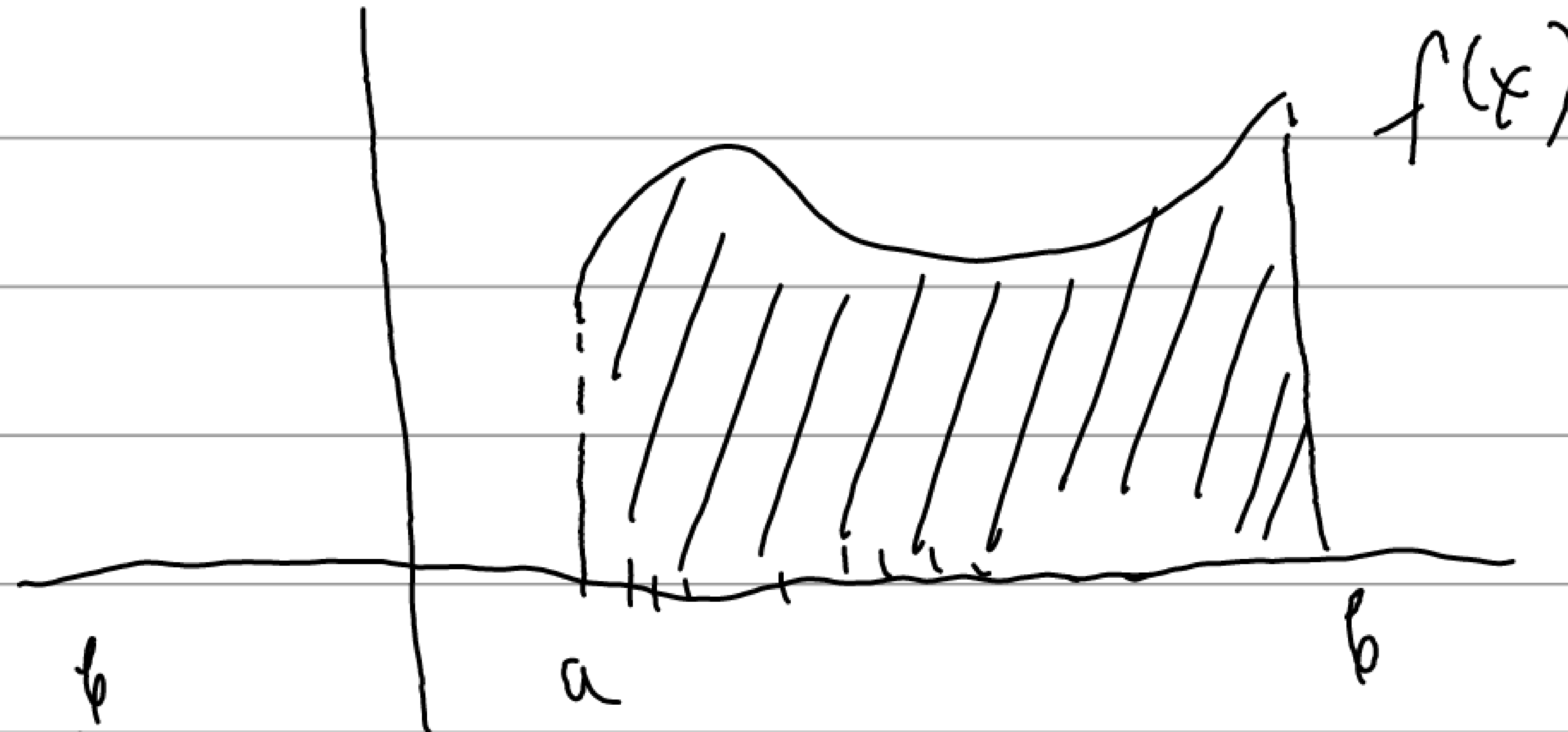

$$\int_a^b f(x)\,dx = S_{\text{крив. тр.}}$$

$$\int f(x)\,dx = F(x) + \cancel{C}$$

$$\int_a^b f(x)\,dx = F(b) - F(a) = \text{число}$$

1) $$\int_a^b f(x)\,dx = \int_a^c f(x)\,dx + \int_c^b f(x)\,dx$$

2) $$\int_a^b [C_1 f(x) + C_2 g(x)]\,dx = C_1 \int_a^b f(x)\,dx + C_2 \int_a^b g(x)\,dx$$

3) f - интегр. $[-a, a]$

$$\int_{-a}^{a} f(x)\,dx = \begin{cases} 0, & f(x) \text{ - нечетна} \\ 2\int_{0}^{a} f(x)\,dx, & f(x) \text{ - четна} \end{cases}$$

4) f - периодична с период T и е интегр. в $[0, T]$, то тя е интегрируема и в $[a, a+T]$ за $\forall a \in \mathbb{R}$

$$\int_{a}^{a+T} f(x)\,dx = \int_{0}^{T} f(x)\,dx$$

<u>Забележка</u>! Често се избира $a = -\frac{T}{2}$ за да може да се приложи св-во 3)

① $J = \int\limits_{-\pi/2}^{\pi/2} \sqrt{\cos x - \cos^3 x}\; dx \Rightarrow$

$f(x) = \sqrt{\cos x - \cos^3 x}$ $\rightarrow$ четна

$$J = 2 \int\limits_{0}^{\pi/2} \sqrt{\cos x - \cos^3 x}\; dx =$$

$$= 2 \int\limits_{0}^{\pi/2} \sqrt{\cos x \,(1 - \cos^2 x)}\; dx =$$

$$= 2 \int\limits_{0}^{\pi/2} \sqrt{\cos x \cdot \sin^2 x}\; dx =$$

$$= 2 \int\limits_{0}^{\pi/2} |\sin x| \cdot \cos^{\frac{1}{2}} x \; dx =$$

$$= -2 \int\limits_{0}^{\pi/2} \cos^{\frac{1}{2}} x \; d\cos x =$$

$$= -2 \cdot \frac{\cos^{\frac{3}{2}} x}{\frac{3}{2}} \Bigg|_{0}^{\frac{\pi}{2}} = \frac{4}{3}$$

② $\displaystyle\int_{-1/2}^{1/2} \cos x \cdot \ln \frac{1+x}{1-x}\, dx$

$$f(x) = \cos x \cdot \ln\left(\frac{1+x}{1-x}\right)$$

$$f(-x) = \cos(-x) \cdot \ln\left(\frac{1-x}{1+x}\right) =$$

$$= \cos x \cdot \ln\left(\frac{1+x}{1-x}\right)^{-1} =$$

$$= -\cos x \cdot \ln\left(\frac{1+x}{1-x}\right)$$

т.е. $f(-x) = -f(x) \to f(x)$ нечетна

3) $\Rightarrow J = 0$

③ $\displaystyle J = \int_{-3}^{3} \frac{x^2 \cdot \sin 2x}{x^2+1}$

$$f(-x) = \frac{(-x)^2 \cdot \sin((-)2x)}{(-x)^2+1} = -f(x)$$

3) $\Rightarrow J = 0$

④ Да се пресметне интеграла

$$J = \int_0^{8\pi} \sqrt[3]{\sin x} \cdot \cos^3 x \, dx =$$

$$= 4 \int_{-\pi}^{\pi} \sqrt[3]{\sin x} \cdot \cos^3 x \, dx \quad \to J = 0$$

$$\int_0^{8\pi} = \int_0^{2\pi} + \int_{2\pi}^{4\pi} + \int_{4\pi}^{6\pi} + \int_{6\pi}^{8\pi} =$$

⑤ $J = \int_0^{100\pi} \sqrt{1-\cos 2x} \, dx$

$$\int \sqrt{1-\cos 2x} \, dx = \int \sqrt{\sin^2 x + \cos^2 x - (\cos^2 x - \sin^2 x)} \, dx$$

$$= \int \sqrt{\sin^2 x + \cancel{\cos^2 x} - \cancel{\cos^2 x} + \sin^2 x} = \sqrt{2} \int \sqrt{\sin^2 x} \, dx$$

$$= \sqrt{2} \int |\sin x| \, dx = 100\sqrt{2} \cos x \Big|_0^{\pi}$$

Интегриране по части

$$\int_a^b u(x)\,dv(x) = u(x).v(x)\Big|_a^b - \int_a^b v(x)\,du(x)$$

① $\int_0^1 x \ln(1+x^2)\,dx =$

$= \frac{1}{2}\int_0^1 \ln(1+x^2)\,d x^2 =$

$\frac{1}{2}x^2 \cdot \ln(1+x^2)\Big|_0^1 - \frac{1}{2}\int_0^1 x^2\,d\ln(1+x^2) =$

$= \frac{1}{2}x^2 \cdot \ln(1+x^2)\Big|_0^1 - \frac{1}{\cancel{2}}\int_0^1 x^2 \cdot \frac{1}{1+x^2} \cdot \cancel{2}x\,dx =$

$= \frac{1}{2}x^2 \cdot \ln(1+x^2)\Big|_0^1 - \int_0^1 \frac{x^3}{1+x^2}\,dx$

$$\int \frac{x^3 - x + x}{x^2+1}\,dx = \int \frac{x\cancel{(x^2+1)}}{\cancel{x^2+1}}\,dx - \int \frac{x\,dx}{x^2+1} =$$

$$= \frac{1}{2}x^2 - \frac{1}{2}\ln\left(x^2+1\right) + C$$

$$= \frac{1}{2}x^2 \cdot \ln\left(1+x^2\right)\Big|_0^1 - \frac{1}{2}x^2\Big|_0^1 + \ln\left(x^2+1\right)\Big|_0^1 =$$

$$= \frac{1}{2}\ln 2 - \frac{1}{2} + \frac{1}{2}\ln 2 - \frac{1}{2}\underbrace{\ln 1}_{0} =$$

$$= \ln 2 - \frac{1}{2}$$

(2) $$\int_0^1 \underbrace{\arcsin x}_{u(x)}\, \underbrace{dx}_{v(x)} = x \cdot \arcsin x\Big|_0^1 - \int_0^1 x\, d\arcsin x$$

$$= x \cdot \arcsin x\Big|_0^1 - \int_0^1 \frac{x}{\sqrt{1-x^2}}\,dx$$

$$= x \cdot \arcsin x \Big|_0^1 + \frac{1}{2}\int_0^1 \frac{d(1-x^2)}{(1-x^2)^{\frac{1}{2}}} =$$

$$= x \cdot \arcsin x \Big|_0^1 + \frac{1}{2}\int_0^1 (1-x^2)^{-\frac{1}{2}} d(1-x^2) =$$

$$= x \cdot \arcsin x \Big|_0^1 + \cancel{\frac{1}{2}} \frac{(1-x^2)^{\frac{1}{2}}}{\cancel{\frac{1}{2}}} \Big|_0^1 =$$

$$= x \cdot \arcsin x \Big|_0^1 + \sqrt{(1-x^2)} \Big|_0^1 =$$

$$= 1 \cdot \arcsin 1 - 1 = \boxed{\frac{\pi}{2} - 1}$$

$$③\ J = \int_0^b e^{ax} \sin bx \, dx = \qquad \boxed{\frac{1}{a}\int e^{ax} \, dax = \frac{1}{a} e^{ax}}$$

$$= \frac{1}{a}\int_0^b \sin bx \, d\, e^{ax} =$$

$$= \frac{1}{a} e^{ax} \cdot \sin bx \Big|_0^b - \frac{1}{a}\int_0^b e^{ax} \, d \sin bx =$$

$$= \frac{1}{a} e^{ax} \sin bx \Big|_0^b - \frac{b}{a}\int_0^b e^{ax} \cdot \cos bx \, dx =$$

$$= \frac{1}{a} e^{ax} \sin bx \Big|_0^b - \frac{b}{a^2}\int_0^b \cos bx \, d\, e^{ax} =$$

$$= \frac{1}{a} e^{ax} \sin bx \Big|_0^b - \frac{b}{a^2}\Big(e^{ax} \cdot \cos bx \Big|_0^b -$$

$$- \int_0^b e^{ax} \, d \underbrace{\cos bx} \Big) =$$

$$= \frac{1}{a} e^{ax}\sin bx\Big|_0^b - \frac{b}{a^2}\left(e^{ax}\cdot\cos bx\Big|_0^b + b\underbrace{\int_0^b e^{ax}\cdot\sin bx\,dx}_{J}\right)$$

$$J = \frac{1}{a}e^{ax}\sin bx\Big|_0^b - \frac{b}{a^2}\cdot e^{ax}\cdot\cos bx\Big|_0^b - \frac{b^2}{a^2}\cdot J$$

$$J + \frac{b^2}{a^2}\cdot J = \frac{1}{a}e^{ax}\sin bx\Big|_0^b - \frac{b}{a^2}\cdot e^{ax}\cdot\cos bx\Big|_0^b$$

$$J\left(1+\frac{b^2}{a^2}\right) = \;\|\; - \;\|\; - \;|$$

$$J = \frac{e^{ax}\sin bx}{\cancel{a}}\cdot\frac{a^{\cancel{2}}}{a^2+b^2}\Big|_0^b - \frac{b\cdot e^{ax}\cdot\cos bx}{\cancel{a^2}}\cdot\frac{\cancel{a^2}}{a^2+b^2}\Big|_0^b$$

$$= \frac{a\cdot e^{ax}\cdot\sin bx}{a^2+b^2}\Big|_0^b - \frac{b\cdot e^{ax}\cdot\cos bx}{a^2+b^2}\Big|_0^b$$

$$= \frac{a\cdot e^{ab}\cdot\sin b^2}{a^2+b^2} - \left(\frac{b\cdot e^{ab}\cdot\cos b^2}{a^2+b^2} - \frac{b}{a^2+b^2}\right)$$

(4) $\int\limits_{1}^{16} \operatorname{arctg}\sqrt{\sqrt{x}-1}\,dx$

Решаване на опр. интеграли чрез субституции

$$J = \int\limits_{3}^{8} \frac{x\,dx}{\sqrt{1+x}} = \int\limits_{3}^{8} x\,(1+x)^{-\frac{1}{2}}\,dx$$

$\boxed{x = t^2 - 1}$ $\quad dx = d(t^2-1) = 2t\,dt$

$$\int x^m (a+bx^n)^p\,dx \qquad m, n \text{ и } p \in \mathbb{Q}$$

p, $\frac{m+1}{n}$ или $\frac{m+1}{n} + p$ е цяло

I

II

III

Интеграл от бин диференциал.

x	3	8
t	2	3
	α	β

$x = t^2 - 1$

$t^2 - 1 = 3 \qquad t^2 - 1 = 8$

$t_{1,2} = \pm 2 \qquad t_{1,2} = \pm 3$

$$\boxed{\begin{array}{l} \varphi(t) = t^2 - 1 : [2, 3] \rightarrow [3; 8] \\ \varphi(t) : [\alpha_1, \beta] \rightarrow [a, b] \end{array}}$$

$\varphi(2) = 2^2 - 1 = 3$

$\varphi(3) = 3^2 - 1 = 8$

$\varphi(2,5) = 2,5^2 - 1$

$$\int_2^3 \frac{(t^2 - 1) \cdot 2t}{\sqrt{\cancel{1} + t^2 - \cancel{1}}} dt = 2\int_2^3 \frac{(t^2-1)\cancel{t}}{\cancel{t}} dt =$$

$$= 2\int_2^3 t^2 - 2\int_2^3 dt = \text{---}$$

② $J = \int_{-\sqrt{3}}^{\sqrt{3}} \sqrt{4 - x^2}\, dx$

$x = 2\sin t$

$dx = d\, 2\sin t = 2\cos t\, dt$

$x = 2\sin t$

a	b
$-\sqrt{3}$	$\sqrt{3}$
$-\frac{\pi}{3}$	$\frac{\pi}{3}$
α	β

$-\sqrt{3} = 2\sin t$ $\qquad \sin t = -\frac{\sqrt{3}}{2} = -\frac{\pi}{3}$

$\sin t = \frac{\sqrt{3}}{2} = \frac{\pi}{3}$

$\int_{-\pi/3}^{\pi/3} \sqrt{4(1-\sin^2 t)}\; 2\cos t\, dt =$

$= 4\int_{-\pi/3}^{\pi/3} \cos^2 t\, dt = \frac{4}{2}\int_{-\pi/3}^{\pi/3} (1 + \cos 2t)\, dt$

$$= 2\int_{-\pi/3}^{\pi/3} dt + \frac{2}{2}\int_{-\pi/3}^{\pi/3} \cos 2t \, d2t =$$

$$= 2 \cdot t\Big|_{-\pi/3}^{\pi/3} + \sin 2t\Big|_{-\pi/3}^{\pi/3} =$$

$$= \_\_ \quad - \quad -- \quad -$$

① $\displaystyle\int_{\pi/4}^{\pi/3} \frac{x}{\sin^2 x}\,dx$ ② $\displaystyle\int_{-\pi/2}^{-\pi/4} \frac{\cos^3 x}{\sqrt[3]{\sin x}}\,dx$

③ $\displaystyle\int_{0}^{\pi/2} \frac{dx}{1+\sin x+\cos x}$

④ $\displaystyle\int_{0}^{\frac{1}{2}} \arcsin x \, dx$

⑤ $\displaystyle\int_{\sqrt{e}}^{e} \frac{dx}{x \cdot \sqrt{2\ln x - \ln^2 x}}$