

Метод на разполовяването

Задача: Дадено ни е уравнението:

$$x^4 - 12x^3 + 77\sin x - 32 = 0$$

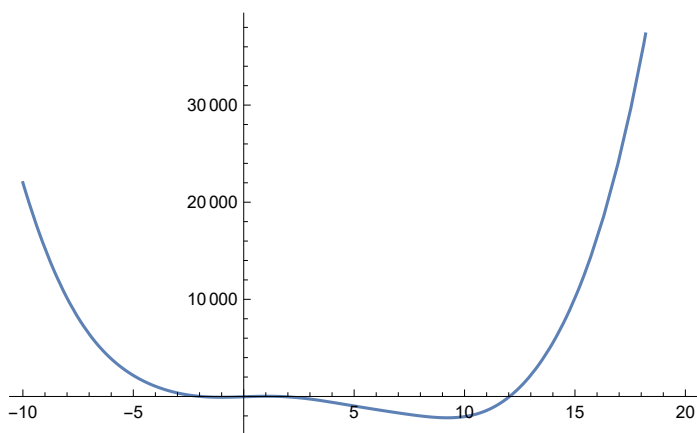
1. Да се визуализира функцията
2. Да се определи броя на корените
3. Да се локализира един от тях
4. Да се уточни локализирания корен **по метода на разполовяването**
5. Оценка на грешката
6. Колко итерации биха били необходими за достигане на точност 0.0001 **по метода на разполовяването** за избрания по време на локализацията интервал?

```
In[*]:= f[x_] := x^4 - 12 x^3 + 77 Sin[x] - 32
```

1. Да се визуализира функцията

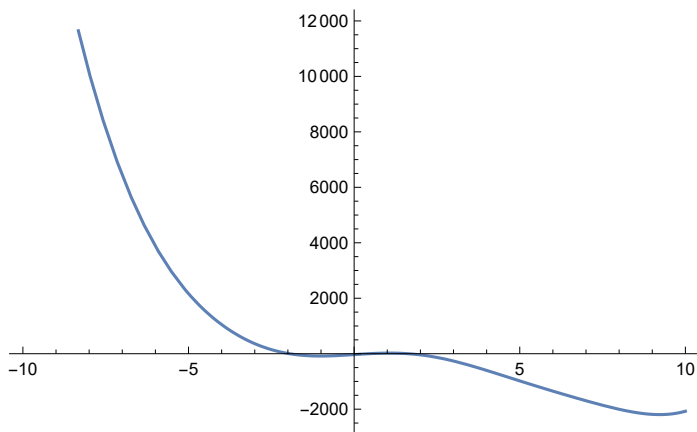
```
In[*]:= Plot[f[x], {x, -10, 20}]
```

Out[*]=



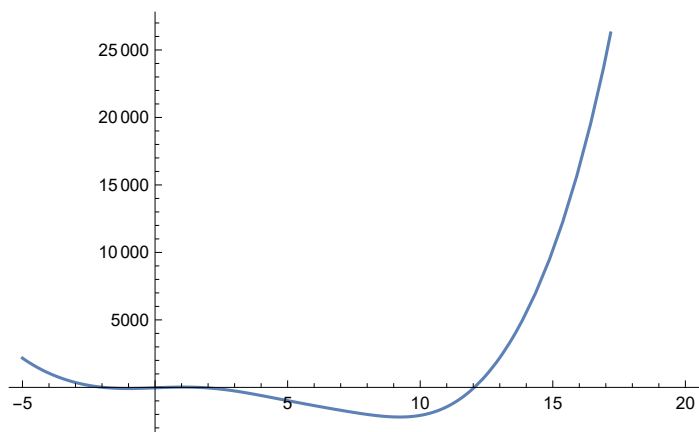
```
In[*]:= Plot[f[x], {x, -10, 10}]
```

Out[*]=



```
In[ ]:= Plot[f[x], {x, -5, 20}]
```

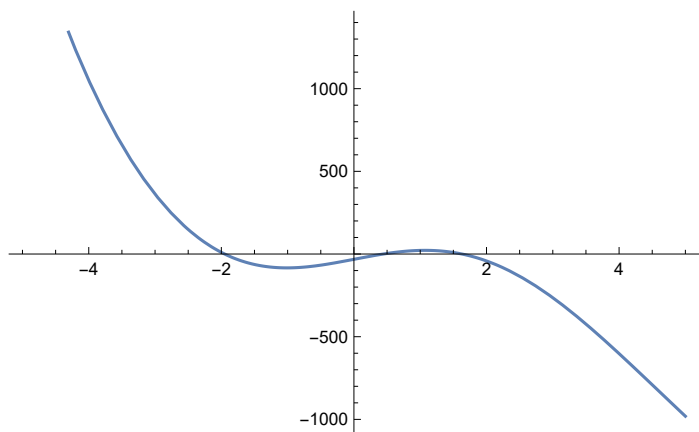
```
Out[ ]:=
```



2. Да се определи броя на корените

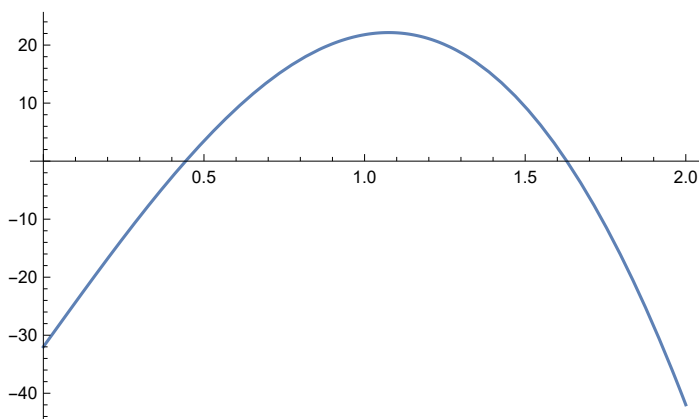
```
In[ ]:= Plot[f[x], {x, -5, 5}]
```

```
Out[ ]:=
```



```
In[ ]:= Plot[f[x], {x, 0, 2}]
```

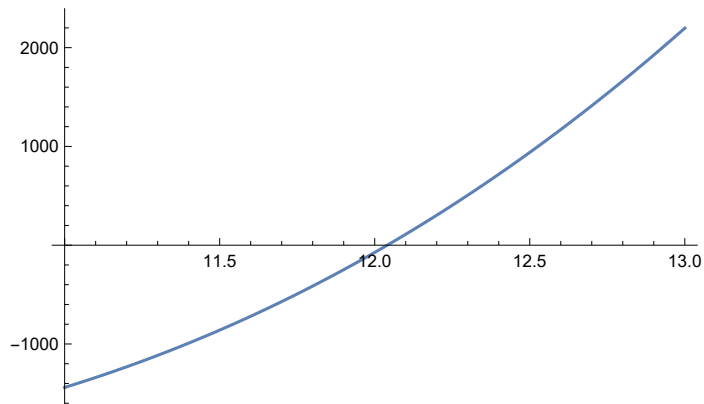
```
Out[ ]:=
```



3. Да се локализира един от тях

```
In[*]:= Plot[f[x], {x, 11, 13}]
```

```
Out[*]=
```



```
In[*]:= f[11.]
```

```
Out[*]=
```

```
-1440.
```

```
In[*]:= f[13.]
```

```
Out[*]=
```

```
2197.35
```

Извод:

(1) Функцията $f(x)$ е непрекъсната, защото е сума от непрекъснати функции (полином и синус)

(2) $f(11) = -1440 < 0$

$f(13) = 2197.35... > 0$

Функцията има различни знаци в двата края на разглеждания интервал $[11; 13]$.

Следователно от (1) и (2) следва, че в интервала $[11; 13]$ функцията има поне един корен.

4. Да се уточни локализирания корен по метода на разполовяването

Уточнение за съставяне на цикли и условни преходи

```
For[]
```

```
In[*]:= For[i = 0, i < 4, i++, Print[i]]
```

```
0
```

```
1
```

```
2
```

```
3
```

```

In[*]:= i
Out[*]=
4

In[*]:= If[i < 2, Print["малко"], Print["голямо"]]
        ГОЛЯМО

In[*]:= For[i = 0, i < 4, i++,
            (*ТЯЛОТО НА ЦИКЪЛА*)
            Print[i];
            If[i < 2, Print["малко"], Print["голямо"]]
        ]
0
малко
1
малко
2
ГОЛЯМО
3
ГОЛЯМО

```

Съставяне на програмен код

ОСНОВЕН КОД

```

In[*]:= f[x_] := x4 - 12 x3 + 77 Sin[x] - 32

For[
    (*начални условия*)
    n = 0; a = 11.; b = 13.,
    n ≤ 3, n++,
    (*тяло на цикъла*)
    Print["n = ", n, " an = ", a, " bn = ", b,
        " mn = ", m =  $\frac{a+b}{2}$ , " f(mn) = ", f[m], " εn = ",  $\frac{b-a}{2}$ ];
    If[f[m] < 0, a = m, b = m]
]

n = 0 an = 11. bn = 13. mn = 12. f(mn) = -73.3161 εn = 1.
n = 1 an = 12. bn = 13. mn = 12.5 f(mn) = 939.456 εn = 0.5
n = 2 an = 12. bn = 12.5 mn = 12.25 f(mn) = 403.61 εn = 0.25
n = 3 an = 12. bn = 12.25 mn = 12.125 f(mn) = 157.928 εn = 0.125

In[*]:= f[12.]
Out[*]=
-73.3161

```

с повече итерации

```
In[*]:= f[x_] := x4 - 12 x3 + 77 Sin[x] - 32
```

```
For[
  (*начални условия*)
  n = 0; a = 11.; b = 13.,
  n ≤ 20, n++,
  (*тяло на цикъла*)
  Print["n = ", n, " an = ", a, " bn = ", b,
    " mn = ", m =  $\frac{a+b}{2}$ , " f(mn) = ", f[m], " εn = ",  $\frac{b-a}{2}$ ];
  If[f[m] < 0, a = m, b = m]
]
```

```
n = 0 an = 11. bn = 13. mn = 12. f(mn) = -73.3161 εn = 1.
n = 1 an = 12. bn = 13. mn = 12.5 f(mn) = 939.456 εn = 0.5
n = 2 an = 12. bn = 12.5 mn = 12.25 f(mn) = 403.61 εn = 0.25
n = 3 an = 12. bn = 12.25 mn = 12.125 f(mn) = 157.928 εn = 0.125
n = 4 an = 12. bn = 12.125 mn = 12.0625 f(mn) = 40.5193 εn = 0.0625
n = 5 an = 12. bn = 12.0625 mn = 12.0313 f(mn) = -16.8428 εn = 0.03125
n = 6 an = 12.0313 bn = 12.0625 mn = 12.0469 f(mn) = 11.7269 εn = 0.015625
n = 7 an = 12.0313 bn = 12.0469 mn = 12.0391 f(mn) = -2.58576 εn = 0.0078125
n = 8 an = 12.0391 bn = 12.0469 mn = 12.043 f(mn) = 4.5636 εn = 0.00390625
n = 9 an = 12.0391 bn = 12.043 mn = 12.041 f(mn) = 0.987181 εn = 0.00195313
n = 10 an = 12.0391 bn = 12.041 mn = 12.04 f(mn) = -0.799724 εn = 0.000976563
n = 11 an = 12.04 bn = 12.041 mn = 12.0405 f(mn) = 0.0936195 εn = 0.000488281
n = 12 an = 12.04 bn = 12.0405 mn = 12.0403 f(mn) = -0.35308 εn = 0.000244141
n = 13 an = 12.0403 bn = 12.0405 mn = 12.0404 f(mn) = -0.129737 εn = 0.00012207
n = 14 an = 12.0404 bn = 12.0405 mn = 12.0405 f(mn) = -0.0180604 εn = 0.0000610352
n = 15 an = 12.0405 bn = 12.0405 mn = 12.0405 f(mn) = 0.0377791 εn = 0.0000305176
n = 16 an = 12.0405 bn = 12.0405 mn = 12.0405 f(mn) = 0.00985925 εn = 0.0000152588
n = 17 an = 12.0405 bn = 12.0405 mn = 12.0405 f(mn) = -0.0041006 εn = 7.62939 × 10-6
n = 18 an = 12.0405 bn = 12.0405 mn = 12.0405 f(mn) = 0.00287932 εn = 3.8147 × 10-6
n = 19 an = 12.0405 bn = 12.0405 mn = 12.0405 f(mn) = -0.000610646 εn = 1.90735 × 10-6
n = 20 an = 12.0405 bn = 12.0405 mn = 12.0405 f(mn) = 0.00113433 εn = 9.53674 × 10-7
```

с повече итерации и да се изписва с повече знаци

```
In[*]:= f[x_] := x^4 - 12 x^3 + 77 Sin[x] - 32
```

```
For[
  (*начални условия*)
  n = 0; a = 11.; b = 13.,
  n ≤ 20, n++,
  (*тяло на цикъла*)
  Print["n = ", n, " a_n = ", SetPrecision[a, 10], " b_n = ", SetPrecision[b, 10],
    " m_n = ", SetPrecision[m =  $\frac{a+b}{2}$ , 10], " f(m_n) = ", f[m], " ε_n = ",  $\frac{b-a}{2}$ ];
  If[f[m] < 0, a = m, b = m]
]
```

n = 0 a_n = 11.00000000 b_n = 13.00000000 m_n = 12.00000000 f(m_n) = -73.3161 ε_n = 1.
n = 1 a_n = 12.00000000 b_n = 13.00000000 m_n = 12.50000000 f(m_n) = 939.456 ε_n = 0.5
n = 2 a_n = 12.00000000 b_n = 12.50000000 m_n = 12.25000000 f(m_n) = 403.61 ε_n = 0.25
n = 3 a_n = 12.00000000 b_n = 12.25000000 m_n = 12.12500000 f(m_n) = 157.928 ε_n = 0.125
n = 4 a_n = 12.00000000 b_n = 12.12500000 m_n = 12.06250000 f(m_n) = 40.5193 ε_n = 0.0625
n = 5 a_n = 12.00000000 b_n = 12.06250000 m_n = 12.03125000 f(m_n) = -16.8428 ε_n = 0.03125
n = 6 a_n = 12.03125000 b_n = 12.06250000 m_n = 12.04687500 f(m_n) = 11.7269 ε_n = 0.015625
n = 7 a_n = 12.03125000 b_n = 12.04687500 m_n = 12.03906250 f(m_n) = -2.58576 ε_n = 0.0078125
n = 8 a_n = 12.03906250 b_n = 12.04687500 m_n = 12.04296875 f(m_n) = 4.5636 ε_n = 0.00390625
n = 9 a_n = 12.03906250 b_n = 12.04296875 m_n = 12.04101563 f(m_n) = 0.987181 ε_n = 0.00195313
n = 10 a_n = 12.03906250 b_n = 12.04101563 m_n = 12.04003906 f(m_n) = -0.799724 ε_n = 0.000976563
n = 11 a_n = 12.04003906 b_n = 12.04101563 m_n = 12.04052734 f(m_n) = 0.0936195 ε_n = 0.000488281
n = 12 a_n = 12.04003906 b_n = 12.04052734 m_n = 12.04028320 f(m_n) = -0.35308 ε_n = 0.000244141
n = 13 a_n = 12.04028320 b_n = 12.04052734 m_n = 12.04040527 f(m_n) = -0.129737 ε_n = 0.00012207
n = 14 a_n = 12.04040527 b_n = 12.04052734
m_n = 12.04046631 f(m_n) = -0.0180604 ε_n = 0.0000610352
n = 15 a_n = 12.04046631 b_n = 12.04052734 m_n = 12.04049683 f(m_n) = 0.0377791 ε_n = 0.0000305176
n = 16 a_n = 12.04046631 b_n = 12.04049683
m_n = 12.04048157 f(m_n) = 0.00985925 ε_n = 0.0000152588
n = 17 a_n = 12.04046631 b_n = 12.04048157
m_n = 12.04047394 f(m_n) = -0.0041006 ε_n = 7.62939×10^{-6}
n = 18 a_n = 12.04047394 b_n = 12.04048157 m_n = 12.04047775 f(m_n) = 0.00287932 ε_n = 3.8147×10^{-6}
n = 19 a_n = 12.04047394 b_n = 12.04047775
m_n = 12.04047585 f(m_n) = -0.000610646 ε_n = 1.90735×10^{-6}
n = 20 a_n = 12.04047585 b_n = 12.04047775
m_n = 12.04047680 f(m_n) = 0.00113433 ε_n = 9.53674×10^{-7}

5. Оценка на грешката

```
In[*]:= Log2[ $\frac{13-11}{0.0001}$ ] - 1
```

```
Out[*]=
```

```
13.2877
```

Извод: Броят на необходимите итерации е 14

```
In[*]:= f[x_] := x4 - 12 x3 + 77 Sin[x] - 32
```

```
For[
  (*начални условия*)
  n = 0; a = 11.; b = 13.,
  n ≤ 14, n++,
  (*ТЯЛО на цикъла*)
  Print["n = ", n, " an = ", SetPrecision[a, 10], " bn = ", SetPrecision[b, 10],
    " mn = ", SetPrecision[m =  $\frac{a+b}{2}$ , 10], " f(mn) = ", f[m], " εn = ",  $\frac{b-a}{2}$ ];
  If[f[m] < 0, a = m, b = m]
]
```

```
n = 0 an = 11.00000000 bn = 13.00000000 mn = 12.00000000 f(mn) = -73.3161 εn = 1.
n = 1 an = 12.00000000 bn = 13.00000000 mn = 12.50000000 f(mn) = 939.456 εn = 0.5
n = 2 an = 12.00000000 bn = 12.50000000 mn = 12.25000000 f(mn) = 403.61 εn = 0.25
n = 3 an = 12.00000000 bn = 12.25000000 mn = 12.12500000 f(mn) = 157.928 εn = 0.125
n = 4 an = 12.00000000 bn = 12.12500000 mn = 12.06250000 f(mn) = 40.5193 εn = 0.0625
n = 5 an = 12.00000000 bn = 12.06250000 mn = 12.03125000 f(mn) = -16.8428 εn = 0.03125
n = 6 an = 12.03125000 bn = 12.06250000 mn = 12.04687500 f(mn) = 11.7269 εn = 0.015625
n = 7 an = 12.03125000 bn = 12.04687500 mn = 12.03906250 f(mn) = -2.58576 εn = 0.0078125
n = 8 an = 12.03906250 bn = 12.04687500 mn = 12.04296875 f(mn) = 4.5636 εn = 0.00390625
n = 9 an = 12.03906250 bn = 12.04296875 mn = 12.04101563 f(mn) = 0.987181 εn = 0.00195313
n = 10 an = 12.03906250 bn = 12.04101563 mn = 12.04003906 f(mn) = -0.799724 εn = 0.000976563
n = 11 an = 12.04003906 bn = 12.04101563 mn = 12.04052734 f(mn) = 0.0936195 εn = 0.000488281
n = 12 an = 12.04003906 bn = 12.04052734 mn = 12.04028320 f(mn) = -0.35308 εn = 0.000244141
n = 13 an = 12.04028320 bn = 12.04052734 mn = 12.04040527 f(mn) = -0.129737 εn = 0.00012207
n = 14 an = 12.04040527 bn = 12.04052734
      mn = 12.04046631 f(mn) = -0.0180604 εn = 0.0000610352
```

цикъл при достигане на нужната точност (със стоп-критерий)

```
In[*]:= f[x_] := x4 - 12 x3 + 77 Sin[x] - 32
```

```
epszad = 0.0001;
```

```
eps = 100;
```

```
For[
```

```
  (*начални условия*)
```

```
  n = 0; a = 11.; b = 13.,
```

```
  eps > epszad, n++,
```

```
  (*тяло на цикъла*)
```

```
  Print["n = ", n, " an = ", SetPrecision[a, 10], " bn = ", SetPrecision[b, 10],
```

```
    " mn = ", SetPrecision[m =  $\frac{a+b}{2}$ , 10], " f(mn) = ", f[m], " εn = ", eps =  $\frac{b-a}{2}$ ];
```

```
  If[f[m] < 0, a = m, b = m]
```

```
]
```

```
n = 0 an = 11.00000000 bn = 13.00000000 mn = 12.00000000 f(mn) = -73.3161 εn = 1.
```

```
n = 1 an = 12.00000000 bn = 13.00000000 mn = 12.50000000 f(mn) = 939.456 εn = 0.5
```

```
n = 2 an = 12.00000000 bn = 12.50000000 mn = 12.25000000 f(mn) = 403.61 εn = 0.25
```

```
n = 3 an = 12.00000000 bn = 12.25000000 mn = 12.12500000 f(mn) = 157.928 εn = 0.125
```

```
n = 4 an = 12.00000000 bn = 12.12500000 mn = 12.06250000 f(mn) = 40.5193 εn = 0.0625
```

```
n = 5 an = 12.00000000 bn = 12.06250000 mn = 12.03125000 f(mn) = -16.8428 εn = 0.03125
```

```
n = 6 an = 12.03125000 bn = 12.06250000 mn = 12.04687500 f(mn) = 11.7269 εn = 0.015625
```

```
n = 7 an = 12.03125000 bn = 12.04687500 mn = 12.03906250 f(mn) = -2.58576 εn = 0.0078125
```

```
n = 8 an = 12.03906250 bn = 12.04687500 mn = 12.04296875 f(mn) = 4.5636 εn = 0.00390625
```

```
n = 9 an = 12.03906250 bn = 12.04296875 mn = 12.04101563 f(mn) = 0.987181 εn = 0.00195313
```

```
n = 10 an = 12.03906250 bn = 12.04101563 mn = 12.04003906 f(mn) = -0.799724 εn = 0.000976563
```

```
n = 11 an = 12.04003906 bn = 12.04101563 mn = 12.04052734 f(mn) = 0.0936195 εn = 0.000488281
```

```
n = 12 an = 12.04003906 bn = 12.04052734 mn = 12.04028320 f(mn) = -0.35308 εn = 0.000244141
```

```
n = 13 an = 12.04028320 bn = 12.04052734 mn = 12.04040527 f(mn) = -0.129737 εn = 0.00012207
```

```
n = 14 an = 12.04040527 bn = 12.04052734
```

```
  mn = 12.04046631 f(mn) = -0.0180604 εn = 0.0000610352
```