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Зад. 1

Б) $\begin{vmatrix} x = -2x + 2y - z \\ y = -6x + 2y - 2z \\ z = -6x - 2y - z \end{vmatrix}$

$(\lambda_1 = 2, \lambda_2 = 0, \lambda_3 = 1$

$A - \lambda E = \begin{pmatrix} -2-\lambda & 2 & -1 \\ -6 & 2-\lambda & -2 \\ -6 & -2 & -1-\lambda \end{pmatrix}$

I $\lambda_1 = 2$

$(A - \lambda_1 E) h_1 = 0$

$\begin{pmatrix} 0 & 2 & -1 \\ -6 & 4 & -2 \\ -6 & -2 & 1 \end{pmatrix} \begin{pmatrix} \alpha \\ \beta \\ \gamma \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$

$\begin{vmatrix} 2\beta - \gamma = 0 \\ -6\alpha + 4\beta - 2\gamma = 0 \\ -6\alpha - 2\beta + \gamma = 0 \end{vmatrix} +$

$-6\alpha + 2\beta - 2\beta + \gamma - \gamma = 0$

$-6\alpha = 0$

$\alpha = 0 \Rightarrow -6\alpha - 2\beta + \gamma = 0$

$-2\beta + \gamma = 0$

$\gamma = 2\beta$

$h_1 = \begin{pmatrix} \alpha \\ \beta \\ \gamma \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \\ 2 \\ 1 \end{pmatrix}$

Зад. 2

б)
$$\begin{cases} \dot x = 3x + 2y - 4z \\ \dot y = x + 4y - z \\ \dot z = 3x + 6y - 4z \end{cases}$$

$(\lambda_1 = -1, \ \lambda_2 = \lambda_3 = 2)$

$$A = \begin{pmatrix} 3 & 2 & -4 \\ 1 & 4 & -1 \\ 3 & 6 & -4 \end{pmatrix}$$

I $\lambda_1 = -1$

$$A - \lambda E = \begin{pmatrix} 3+1 & 2 & -4 \\ 1 & 5 & -1 \\ 3 & 6 & -3 \end{pmatrix}$$

$h_1 (A - \lambda_1 E) = 0$

$$h_1 = \begin{pmatrix} \alpha \\ \beta \\ \gamma \end{pmatrix}$$

$$\begin{cases} 4\alpha + 2\beta - 4\gamma = 0 \\ \alpha + 5\beta - \gamma = 0 \quad | \cdot (3) \ \text{П} \\ 3\alpha + 6\beta - 3\gamma = 0 \end{cases} +$$

$-3\alpha + 3\alpha - 15\beta + 6\beta + 3\gamma - 3\gamma = 0$

$-9\beta = 0 \qquad -\beta = 0$

$4\alpha - 2\cdot 0 - 4\gamma = 0$

$\alpha = \gamma \qquad \Rightarrow$

$$h_1 = \begin{pmatrix} \alpha \\ 0 \\ \alpha \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}$$

при $\alpha = 1$

$x_1 = h_1 \cdot e^{\lambda_1 t} = \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} \cdot e^t$

II. $\lambda_2 = \lambda_3 = 2$

$$A = \begin{pmatrix} 3 & 2 & -4 \\ 1 & 4 & -1 \\ 3 & 6 & -4 \end{pmatrix}$$

$(A - \lambda E) \cdot h_{2/3} = 0$

$$(A - \lambda E) = \begin{pmatrix} 1 & 2 & -4 \\ 1 & 2 & -1 \\ 3 & 6 & -6 \end{pmatrix}$$

$$\begin{cases} \alpha + 2\beta - 4\gamma = 0 \\ \alpha + 2\beta - \gamma = 0 \quad |\cdot(-1) \\ 3\alpha + 6\beta - 6\gamma = 0 \end{cases} +$$

$\cancel{\alpha} - \cancel{\alpha} - 2\beta + 2\beta - 4\gamma + \gamma = 0$ $\qquad \alpha + 2\beta = 0$

$\qquad\qquad\qquad\qquad -3\gamma = 0$ $\qquad\qquad \alpha = 2\beta$

$\qquad\qquad\qquad\qquad \gamma = 0$

$$h_2 = h_3 = \begin{pmatrix} 2\beta \\ \beta \\ 0 \end{pmatrix} = \begin{pmatrix} 2 \\ 1 \\ 0 \end{pmatrix}$$

при $\beta = 1$

Образува се серия и правя $\Rightarrow$ $\begin{cases} (A - 1 \cdot E) \cdot h_1 = 0 \\ (A - E) \cdot h_2 = h_1 \end{cases}$

$X = C_1 X_1 + C_2 X_2 + C_3 X_3$

$\downarrow$

$(A-E)^2 \cdot h_2 = (A-E)^2 \cdot h_1 = 0$