Задача. Крива на Безие $C(u)$ е определена чрез контролните си точки: $P_0(25;0)$, $P_1(50;50)$, $P_2(-50.50)$, $P_3(0;25)$ и $P_4(-25;0)$.

Намерете:

а) $C(0,2)$ като използвате полиномите на Бернщайн;

б) $C(0,2)$ чрез алгоритъма на дьо Кастелжо;

в) подразделете кривата при $u = 0,2$ и намерете контролните полигони на двете дъги;

г) $\dot{C}(0,2)$ и $\ddot{C}(0,2)$

д) да се увеличи степента на $C(u)$ с 1, да се намери и начертае новия контролен полигон;

е) $C^*(0,2)$, ако $C^*(u)$ се получава от $C(u)$ чрез преместване на P_2 в $P_2^*(0,35)$.

Решение:

a) $$C(u) = \sum_{i=0}^{n} B_{n,i}(u) P_i \quad , n=4 \; , u=0{,}2=\frac{2}{10}=\frac{1}{5}$$

$$C(0{,}2) = B_{4,0}(0{,}2)P_0 + B_{4,1}(0{,}2)P_1 + B_{4,2}(0{,}2)P_2 + B_{4,3}(0{,}2)P_3 + B_{4,4}(0{,}2)P_4$$

$$B_{n,i}(u) = \frac{n!}{i!(n-i)!} u^i (1-u)^{n-i}$$

$$B_{4,0}(0{,}2) = \frac{4!}{0!(4-0)!}\left(\frac{1}{5}\right)^0\left(\frac{4}{5}\right)^4 = \frac{256}{625} = 0{,}4096$$

$$B_{4,1}(0{,}2) = \frac{4!}{1!(4-1)!}\left(\frac{1}{5}\right)^1\left(\frac{4}{5}\right)^3 = \frac{256}{625} = 0{,}4096$$

$$B_{4,2}(0{,}2) = \frac{4!}{2!(4-2)!}\left(\frac{1}{5}\right)^2\left(\frac{4}{5}\right)^2 = \frac{96}{625} = 0{,}1536$$

$$B_{4,3}(0{,}2) = \frac{4!}{3!(4-3)!}\left(\frac{1}{5}\right)^3\left(\frac{4}{5}\right)^1 = \frac{16}{625} = 0{,}0256$$

$$B_{4,4}(0{,}2) = \frac{4!}{4!(4-4)!}\left(\frac{1}{5}\right)^4\left(\frac{4}{5}\right)^0 = \frac{1}{625} = 0{,}0016$$

$$C(0{,}2) = 0{,}4096(25;0) + 0{,}4096(50;50) + 0{,}1536(-50;50) + 0{,}0256(0;25) + 0{,}0016(-25;0)$$

$$C(0{,}2) = (10{,}24;0) + (20{,}48;20{,}48) + (-7{,}68;7{,}68) + (0;0{,}64) + (-0{,}04;0)$$

$$C(0{,}2) = (23;28{,}8)$$

б)

$$
\begin{tikzcd}[column sep=small, row sep=tiny]
P_0(25;0) \arrow[dr] & & & & \\
& P_{10}(30;10) \arrow[dr] & & & \\
P_1(50;50) \arrow[ur] \arrow[dr] & & P_{20}(30;18) \arrow[dr] & & \\
& P_{11}(30;50) \arrow[ur] \arrow[dr] & & P_{30}(27,2;24,2) \arrow[dr] & \\
P_2(-50;50) \arrow[ur] \arrow[dr] & & P_{21}(16;49) \arrow[ur] \arrow[dr] & & P_{40}(23;28,8) = C(0,2) \\
& P_{12}(-40;45) \arrow[ur] \arrow[dr] & & P_{31}(6,2;47,2) \arrow[ur] & \\
P_3(0;25) \arrow[ur] \arrow[dr] & & P_{22}(-33;40) \arrow[ur] & & \\
& P_{13}(-5;20) \arrow[ur] & & & \\
P_4(-25;0) \arrow[ur] & & & &
\end{tikzcd}
$$

$$u = 0,2 = \frac{1}{5} \quad ; \quad 1-u = 1-0,2 = 0,8 = \frac{4}{5}$$

$$P_{10} = (1-u)P_0 + uP_1 = \frac{4}{5}(25;0) + \frac{1}{5}(50;50) = (30;10)$$

$$P_{11} = (1-u)P_1 + uP_2 = \frac{4}{5}(50;50) + \frac{1}{5}(-50;50) = (30;50)$$

$$P_{12} = (1-u)P_2 + uP_3 = \frac{4}{5}(-50;50) + \frac{1}{5}(0;25) = (-40;45)$$

$$P_{13} = (1-u)P_3 + uP_4 = \frac{4}{5}(0;25) + \frac{1}{5}(-25;0) = (-5;20)$$

$$P_{20} = (1-u)P_{10} + uP_{11} = \frac{4}{5}(30;10) + \frac{1}{5}(30;50) = (30;18)$$

$$P_{21} = (1-u)P_{11} + uP_{12} = \frac{4}{5}(30;50) + \frac{1}{5}(-40;45) = (16;49)$$

$$P_{22} = (1-u)P_{12} + uP_{13} = \frac{4}{5}(-40;45) + \frac{1}{5}(-5;20) = (-33;40)$$

$$P_{30} = (1-u)P_{20} + uP_{21} = \frac{4}{5}(30;18) + \frac{1}{5}(16;49) = \left(24; \frac{144}{5}\right) + \left(\frac{16}{5}; \frac{49}{5}\right)$$

$$= (24;14,4) + (3,2;9,8) = (27,2;24,2)$$

$P_{31} = (1-u)P_{21} + uP_{22} = 0{,}8(16;49) + 0{,}2(-33;40) =$

$= (12{,}8; 39{,}2) + (-6{,}6; 8) = (6{,}2; 47{,}2)$

$P_{40} = (1-u)P_{30} + uP_{31} = 0{,}8(27{,}2; 24{,}2) + 0{,}2(6{,}2; 47{,}2)$

$= (21{,}76; 19{,}36) + (1{,}24; 9{,}44) = (23; 28{,}8)$

в) Подразделяме кривата при $u = 0{,}2$ на сегменти C_1 и C_2, определени от съответните интервали за параметъра и контролните полигони:

$C_1(u)$: $u \in [0; 0{,}2]$ $[P_0, P_{10}, P_{20}, P_{30}, P_{40}]$

$C_2(u)$: $u \in [0{,}2; 1]$, $[P_{40}, P_{31}, P_{22}, P_{13}, P_4]$.

г) $\dot{C}(0{,}2) = ?$, $\ddot{C}(0{,}2) = ?$

$\dot{C}(u) = n[P_{n-1,1} - P_{n-1,0}]$, $\ddot{C}(u) = n(n-1)[P_{n-2,2} - 2P_{n-2,1} + P_{n-2,0}]$

$\dot{C}(0{,}2) = 4[P_{31} - P_{30}] = 4[(6{,}2; 47{,}2) - (27{,}2; 24{,}2)]$

$= 4(-21; 23) = (-84, 92)$

$\Rightarrow \dot{C}(0{,}2) = (-84; 92)$

$\ddot{C}(0{,}2) = 4.3[P_{22} - 2P_{21} + P_{20}]$

$= 12[(-33; 40) - 2(16; 49) + (30; 18)]$

$= 12[(-33; 40) - (32; 98) + (30; 18)]$

$= 12(-35; -40) = (-420; 480)$

$\Rightarrow \ddot{C}(0{,}2) = (-420; 480)$

д) $n=4 \rightarrow n+1=5$

Новите контролни точки: $Q_0, Q_1, Q_2, Q_3, Q_4, Q_5$

$Q_0 = P_0 \Rightarrow Q_0 = (25;0)$

$Q_5 = P_4 \Rightarrow Q_5 = (-25;0)$

$$Q_i = \frac{i}{n+1} P_{i-1} + \left(1 - \frac{i}{n+1}\right) P_i \quad , i = 1,2,3,4$$

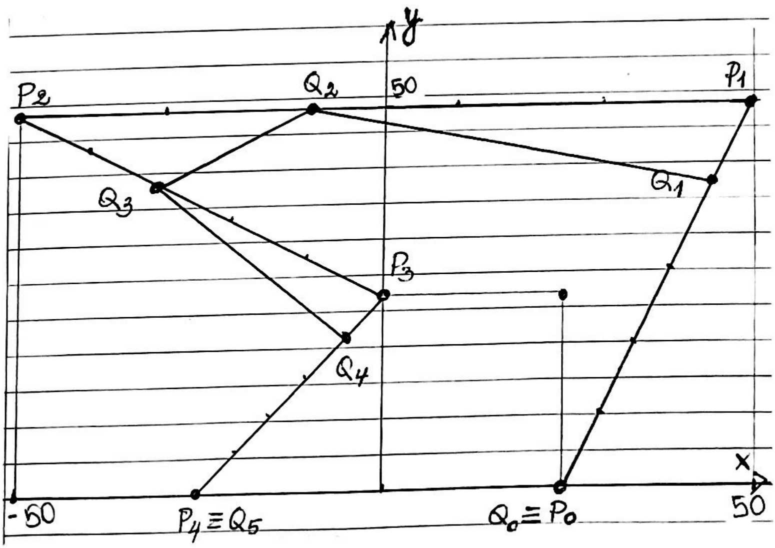
$$Q_1 = \frac{1}{5} P_0 + \left(1 - \frac{1}{5}\right) P_1 = \frac{1}{5}(25;0) + \frac{4}{5}(50;50) = (45;40)$$

$$Q_2 = \frac{2}{5} P_1 + \left(1 - \frac{2}{5}\right) P_2 = \frac{2}{5}(50;50) + \frac{3}{5}(-50;50) = (-10;50)$$

$$Q_3 = \frac{3}{5} P_2 + \left(1 - \frac{3}{5}\right) P_3 = \frac{3}{5}(-50;50) + \frac{2}{5}(0;25) = (-30;40)$$

$$Q_4 = \frac{4}{5} P_3 + \left(1 - \frac{4}{5}\right) P_4 = \frac{4}{5}(0;25) + \frac{1}{5}(-25;0) = (-5;20)$$

$Q_0(25;0), Q_1(45;40), Q_2(-10;50), Q_3(-30;40), Q_4(-5;20), Q_5(-25;0)$



e) $C^*(u) = C(u) + B_{n,j}(u)\vec{V}$

$\vec{V} = P_j^* - P_j$

$\Rightarrow C^*(0{,}2) = C(0{,}2) + B_{4,2}(0{,}2)\vec{V}$

$C(0{,}2) = (23; 28{,}8)$ от б)

$B_{4,2}(0{,}2) = 0{,}1536$ от а)

$\vec{V} = P_j^* - P_j = P_2^* - P_2 = (0; 35) - (-50; 50) = (50; -15)$

$C^*(0{,}2) = (23; 28{,}8) + 0{,}1536\,(50; -15)$

$= (23; 28{,}8) + (7{,}68; 2{,}304)$

$= (30{,}68; 31{,}104)$

$\Rightarrow C^*(0{,}2) = (30{,}68; 31{,}104)$