Метод на най-малките квадрати (МНМК)

Задача: (**a** и **b** са съответно предпоследната и последната цифра от факултетния номер)

1. Да се състави таблицата (x_k, g(x_k)), където

x_k = -b + k(0.1), k = $\overline{0, 10}$,   g(x) = $e^{\frac{(a+1)x}{10}}$

Търси се апроксимацията в точката s = -b + (0.17)a + 0.01. За тази цел:

2. Да се построи полином на ленейна регресия по получената таблица.
3. Да се построи полином на квадратична регресия по получената таблица.
4. Да се построи полином на кубична регресия по получената таблица.
5. Да се пресметне апроксимацията, използвайки всеки един от построените полиноми (общо 3).
6. Да се оцени грешката за всяка от получените апроксимации.
7. Да се направи сравнение между трите резултата.

Генериране на данни

In[452]:=

```
xt = Table[-9 + k * 0.1, {k, 0, 10}]
```

Out[452]=

```
{-9., -8.9, -8.8, -8.7, -8.6, -8.5, -8.4, -8.3, -8.2, -8.1, -8.}
```

In[453]:=

```
f[x_] := e^(2 x/10)
yt = f[xt]
```

Out[454]=

```
{0.165299, 0.168638, 0.172045, 0.17552, 0.179066,
 0.182684, 0.186374, 0.190139, 0.19398, 0.197899, 0.201897}
```

In[455]:=

```
P = Length[xt]
```

Out[455]=

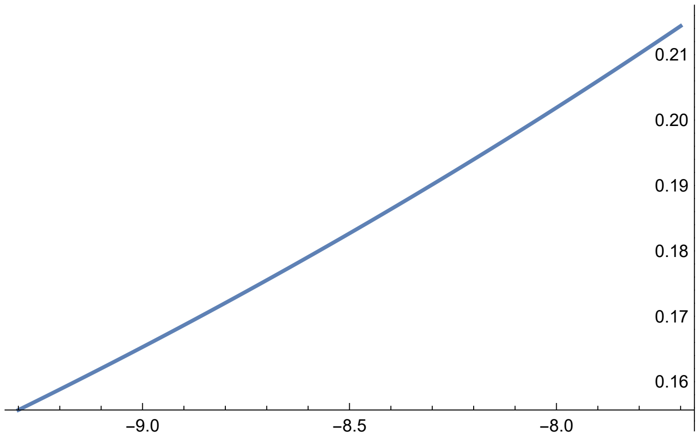
```
11
```

Визуализация

In[456]:=

```
grf = Plot[f[x], {x, xt[[1]] - 0.3, xt[[P]] + 0.3}]
```

Out[456]=



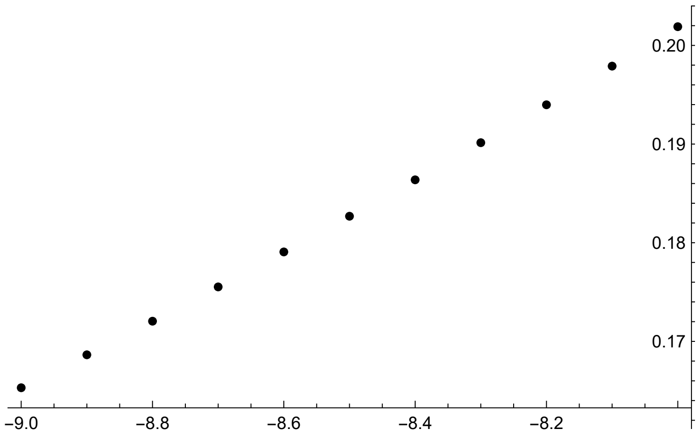
In[457]:=

```
points = Table[{xt[[i]], yt[[i]]}, {i, 1, P}];
```

In[458]:=

```
grp = ListPlot[points, PlotStyle → Black]
```

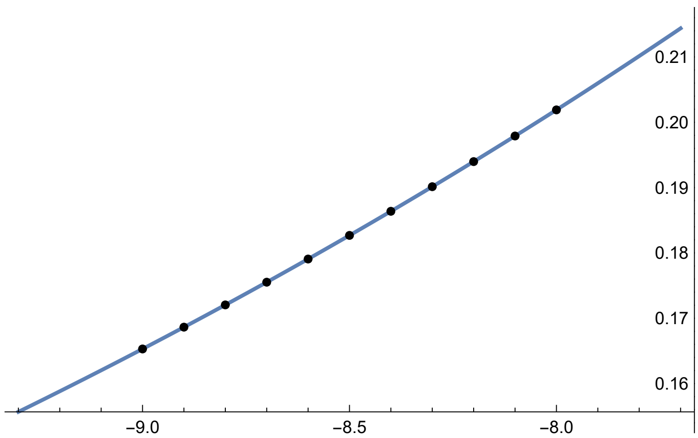
Out[458]=



In[459]:=

```
Show[grf, grp]
```

Out[459]=



Линейна регресия

Попълваме таблицата

In[460]:=

xt^2

Out[460]=

{81., 79.21, 77.44, 75.69, 73.96, 72.25, 70.56, 68.89, 67.24, 65.61, 64.}

In[461]:=

yt * xt

Out[461]=

{-1.48769, -1.50088, -1.51399, -1.52703, -1.53997,
-1.55281, -1.56554, -1.57815, -1.59064, -1.60298, -1.61517}

Намиране на сумите

In[462]:=

$$\sum_{i=1}^{P} xt[[i]]$$

Out[462]=

-93.5

In[463]:=

$$\sum_{i=1}^{P} yt[[i]]$$

Out[463]=

2.01354

In[464]:=

$$\sum_{i=1}^{P} xt[[i]]^2$$

Out[464]=

795.85

In[465]:=

$$\sum_{i=1}^{P} yt[[i]] * xt[[i]]$$

Out[465]=

-17.0749

Решаваме системата

In[466]:=

$$A = \begin{pmatrix} P & \sum_{i=1}^{P} xt[[i]] \\ \sum_{i=1}^{P} xt[[i]] & \sum_{i=1}^{P} xt[[i]]^2 \end{pmatrix};\ b = \left\{\sum_{i=1}^{P} yt[[i]],\ \sum_{i=1}^{P} yt[[i]] * xt[[i]]\right\};$$

In[467]:=

```
LinearSolve[A, b]
```

Out[467]=

{0.49398, 0.0365801}

Съставяме полинома

In[468]:=

```
P1[x_] := 0.49398 + 0.0365801 x
```

Таен коз (възможност за самопроверка)

In[469]:=

```
Fit[points, {1, x}, x]
```

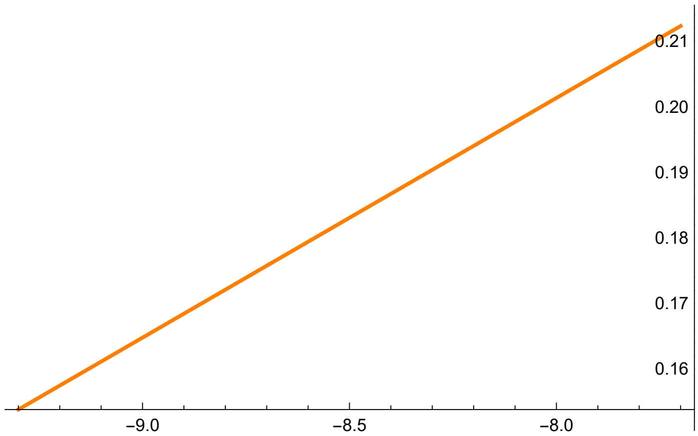
Out[469]=

```
0.49398 + 0.0365801 x
```

In[470]:=

```
grfP1 = Plot[P1[x], {x, xt[[1]] - 0.3, xt[[P]] + 0.3}, PlotStyle → Orange]
```

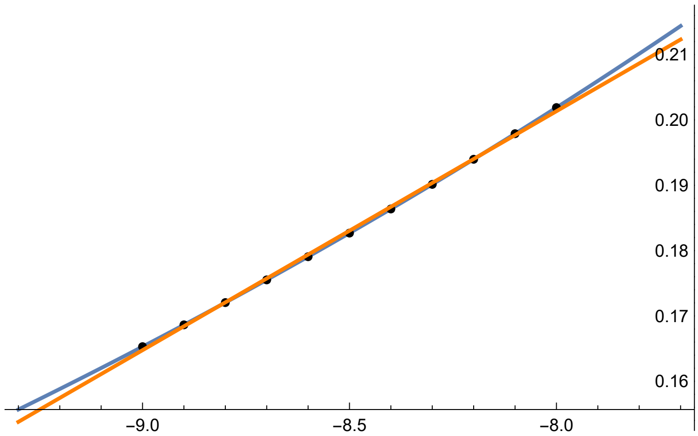
Out[470]=



In[471]:=

```
Show[grf, grp, grfP1]
```

Out[471]=



In[527]:=

```
P1[-8.25]
```

Out[527]=

```
0.192194
```

За сравнение истинската стойност

In[528]:=

```
f[-8.25]
```

Out[528]=

```
0.19205
```

Оценка на грешката

Теоретична грешка (средноквадратична)

In[550]:=

$$\sqrt{\sum_{i=1}^{p} (yt[[i]] - P1[xt[[i]]])^2}$$

Out[550]=

0.00107128

Истинска грешка

In[551]:=

```
Abs[f[-5.97] - P1[-5.97]]
```

Out[551]=

0.02741

Квадратична регресия

Попълваме таблицата

In[531]:=

xt^2

Out[531]=

{81., 79.21, 77.44, 75.69, 73.96, 72.25, 70.56, 68.89, 67.24, 65.61, 64.}

In[477]:=

```
yt * xt
```

Out[477]=

{-1.48769, -1.50088, -1.51399, -1.52703, -1.53997,
-1.55281, -1.56554, -1.57815, -1.59064, -1.60298, -1.61517}

In[478]:=

xt^3

Out[478]=

{-729., -704.969, -681.472, -658.503, -636.056,
-614.125, -592.704, -571.787, -551.368, -531.441, -512.}

In[479]:=

xt^4

Out[479]=

{6561., 6274.22, 5996.95, 5728.98, 5470.08,
5220.06, 4978.71, 4745.83, 4521.22, 4304.67, 4096.}

In[480]:=

$yt * xt^2$

Out[480]=

{13.3892, 13.3578, 13.3232, 13.2851, 13.2437,
13.1989, 13.1505, 13.0987, 13.0432, 12.9841, 12.9214}

Намиране на сумите

In[481]:=

$$\sum_{i=1}^{p} xt[[i]]$$

Out[481]=

-93.5

In[482]:=

$$\sum_{i=1}^{p} yt[[i]]$$

Out[482]=

2.01354

In[483]:=

$$\sum_{i=1}^{p} xt[[i]]^2$$

Out[483]=

795.85

In[484]:=

$$\sum_{i=1}^{p} yt[[i]] * xt[[i]]$$

Out[484]=

-17.0749

In[485]:=

$$\sum_{i=1}^{p} xt[[i]]^3$$

Out[485]=

-6783.43

In[486]:=

$$\sum_{i=1}^{p} xt[[i]]^4$$

Out[486]=

57 897.7

In[487]:=

$$\sum_{i=1}^{p} yt[[i]] * xt[[i]]^2$$

Out[487]=

144.996

Решаваме системата

In[488]:=

$$A = \begin{pmatrix} P & \sum_{i=1}^{P} xt[[i]] & \sum_{i=1}^{P} xt[[i]]^2 \\ \sum_{i=1}^{P} xt[[i]] & \sum_{i=1}^{P} xt[[i]]^2 & \sum_{i=1}^{P} xt[[i]]^3 \\ \sum_{i=1}^{P} xt[[i]]^2 & \sum_{i=1}^{P} xt[[i]]^3 & \sum_{i=1}^{P} xt[[i]]^4 \end{pmatrix};$$

$$b = \left\{\sum_{i=1}^{P} yt[[i]],\ \sum_{i=1}^{P} yt[[i]] * xt[[i]],\ \sum_{i=1}^{P} yt[[i]] * xt[[i]]^2\right\};$$

In[489]:=

```
LinearSolve[A, b]
```

LinearSolve: Result for LinearSolve of badly conditioned matrix

{{11., −93.5, 795.85}, {−93.5, 795.85, −6783.43}, {795.85, −6783.43, 57897.7}} may contain significant numerical errors.

Out[489]=

```
{0.757812, 0.0987443, 0.00365672}
```

Таен коз (възможност за самопроверка)

In[490]:=

$$Fit[points,\ \{1,\ x,\ x^2\},\ x]$$

Out[490]=

$$0.757812 + 0.0987443\, x + 0.00365672\, x^2$$

Съставяме полинома

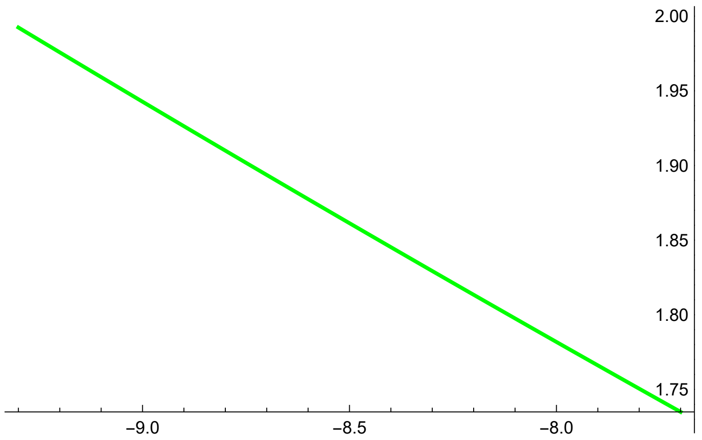
In[534]:=

$$P2[x\_] := 0.757812 - 0.0987443\, x + 0.00365672\, x^2$$

In[535]:=

```
grfP2 = Plot[P2[x], {x, xt[[1]] - 0.3, xt[[P]] + 0.3}, PlotStyle → Green]
```

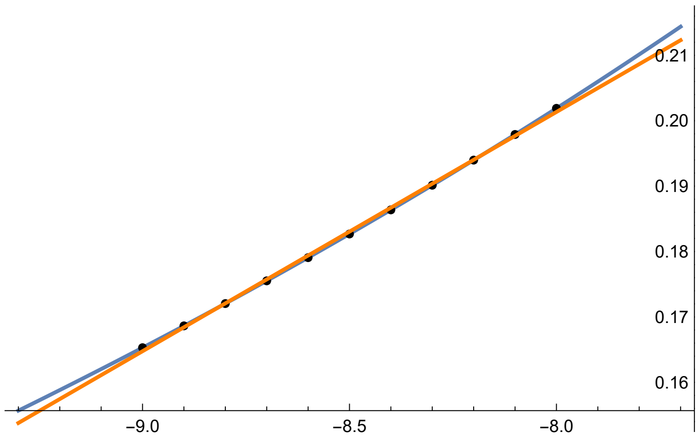
Out[535]=



In[536]:=

```
Show[grf, grp, grfP1, grfP2]
```

Out[536]=



In[537]:=

```
P2[-8.25]
```

Out[537]=

```
1.82134
```

За сравнение истинската стойност

In[538]:=

```
f[-8.25]
```

Out[538]=

```
0.19205
```

Оценка на грешката

Теоретична грешка (средноквадратична)

In[548]:=

$$\sqrt{\sum_{i=1}^{p} (yt[[i]] - P2[xt[[i]]])^2}$$

Out[548]=

```
5.57131
```

Истинска грешка

In[549]:=

```
Abs[f[-8.25] - P2[-8.25]]
```

Out[549]=

```
1.62929
```

Кубична регресия

Попълваме таблицата

In[498]:=

```
xt^2
```

Out[498]=

```
{81., 79.21, 77.44, 75.69, 73.96, 72.25, 70.56, 68.89, 67.24, 65.61, 64.}
```

In[499]:=

```
yt * xt
```

Out[499]=

```
{-1.48769, -1.50088, -1.51399, -1.52703, -1.53997,
 -1.55281, -1.56554, -1.57815, -1.59064, -1.60298, -1.61517}
```

In[500]:=

```
xt^3
```

Out[500]=

```
{-729., -704.969, -681.472, -658.503, -636.056,
 -614.125, -592.704, -571.787, -551.368, -531.441, -512.}
```

In[501]:=

```
xt^4
```

Out[501]=

```
{6561., 6274.22, 5996.95, 5728.98, 5470.08,
 5220.06, 4978.71, 4745.83, 4521.22, 4304.67, 4096.}
```

In[502]:=

```
yt * xt^2
```

Out[502]=

```
{13.3892, 13.3578, 13.3232, 13.2851, 13.2437,
 13.1989, 13.1505, 13.0987, 13.0432, 12.9841, 12.9214}
```

In[503]:=

```
yt * xt^3
```

Out[503]=

```
{-120.503, -118.885, -117.244, -115.581, -113.896,
 -112.191, -110.465, -108.719, -106.954, -105.171, -103.371}
```

In[504]:=

```
xt^5
```

Out[504]=

```
{-59 049., -55 840.6, -52 773.2, -49 842.1, -47 042.7,
 -44 370.5, -41 821.2, -39 390.4, -37 074., -34 867.8, -32 768.}
```

In[505]:=

```
xt^6
```

Out[505]=

```
{531 441., 496 981., 464 404., 433 626., 404 567.,
 377 150., 351 298., 326 940., 304 007., 282 430., 262 144.}
```

Намиране на сумите

In[506]:=

$$\sum_{i=1}^{P} xt[[i]]$$

Out[506]=

-93.5

In[507]:=

$$\sum_{i=1}^{P} yt[[i]]$$

Out[507]=

2.01354

In[508]:=

$$\sum_{i=1}^{P} xt[[i]]^2$$

Out[508]=

795.85

In[509]:=

$$\sum_{i=1}^{P} yt[[i]] * xt[[i]]$$

Out[509]=

-17.0749

In[510]:=

$$\sum_{i=1}^{P} xt[[i]]^3$$

Out[510]=

-6783.43

In[511]:=

$$\sum_{i=1}^{P} xt[[i]]^4$$

Out[511]=

57 897.7

In[512]:=

$$\sum_{i=1}^{P} yt[[i]] * xt[[i]]^2$$

Out[512]=

144.996

In[513]:=

$$\sum_{i=1}^{P} xt[[i]]^5$$

Out[513]=

-494 840.

In[514]:=

$$\sum_{i=1}^{P} xt[[i]]^6$$

Out[514]=

4.23499×10^6

In[515]:=

$$\sum_{i=1}^{P} yt[[i]] * xt[[i]]^3$$

Out[515]=

-1232.98

Решаваме системата

In[516]:=

$$A = \begin{pmatrix} P & \sum_{i=1}^{P} xt[[i]] & \sum_{i=1}^{P} xt[[i]]^2 & \sum_{i=1}^{P} xt[[i]]^3 \\ \sum_{i=1}^{P} xt[[i]] & \sum_{i=1}^{P} xt[[i]]^2 & \sum_{i=1}^{P} xt[[i]]^3 & \sum_{i=1}^{P} xt[[i]]^4 \\ \sum_{i=1}^{P} xt[[i]]^2 & \sum_{i=1}^{P} xt[[i]]^3 & \sum_{i=1}^{P} xt[[i]]^4 & \sum_{i=1}^{P} xt[[i]]^5 \\ \sum_{i=1}^{P} xt[[i]]^3 & \sum_{i=1}^{P} xt[[i]]^4 & \sum_{i=1}^{P} xt[[i]]^5 & \sum_{i=1}^{P} xt[[i]]^6 \end{pmatrix};$$

$$b = \left\{\sum_{i=1}^{P} yt[[i]], \sum_{i=1}^{P} yt[[i]] * xt[[i]], \sum_{i=1}^{P} yt[[i]] * xt[[i]]^2, \sum_{i=1}^{P} yt[[i]] * xt[[i]]^3\right\};$$

In[517]:=

```
LinearSolve[A, b]
```

LinearSolve: Result for LinearSolve of badly conditioned matrix

{{11., −93.5, 795.85, −6783.43}, {−93.5, 795.85, −6783.43, 57897.7}, {795.85, −6783.43, 57897.7, −494840.}, {−6783.43, 57897.7, −494840., 4.23499×10^6}} may contain significant numerical errors.

Out[517]=

{0.907126, 0.15153, 0.0098719, 0.000243733}

Таен коз (възможност за самопроверка)

In[518]:=

$$\text{Fit}\left[\text{points}, \{1, x, x^2, x^3\}, x\right]$$

Out[518]=

$0.907125 + 0.15153\,x + 0.00987189\,x^2 + 0.000243732\,x^3$

Съставяме полинома

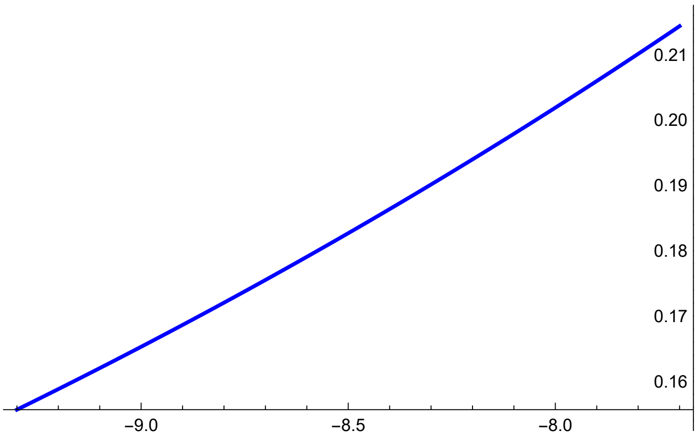
In[541]:=

$$P3[x\_] := 0.907125 + 0.15153\,x + 0.00987189\,x^2 + 0.000243732\,x^3$$

In[542]:=

```
grfP3 = Plot[P3[x], {x, xt〚1〛 - 0.3, xt〚P〛 + 0.3}, PlotStyle → Blue]
```

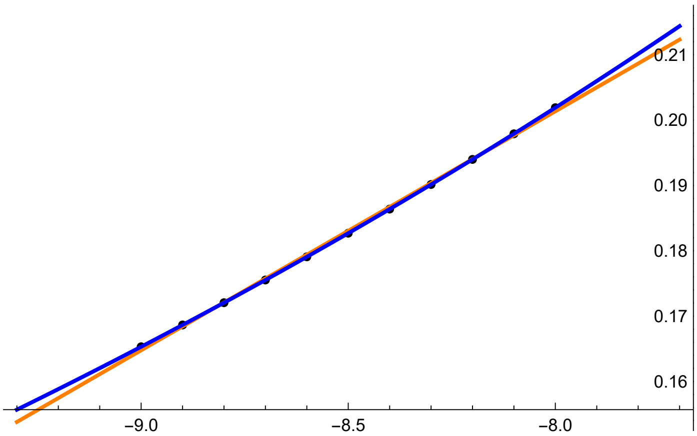
Out[542]=



In[543]:=

```
Show[grf, grp, grfP1, grfP2, grfP3]
```

Out[543]=



Намиране на приближена стойност (s = -b + (0.17)a + 0.01)

In[522]:=

```
s = -9 + 0.17 + 0.01
```

Out[522]=

-8.82

In[544]:=

```
P3[-8.25]
```

Out[544]=

0.192049

За сравнение истинската стойност

In[545]:=

```
f[-8.25]
```

Out[545]=

0.19205

Оценка на грешката

Теоретична грешка (средноквадратична)

In[546]:=

$$\sqrt{\sum_{i=1}^{P} (yt[[i]] - P3[xt[[i]]])^2}$$

Out[546]=

4.3115×10^{-6}

Истинска грешка

In[547]:=

```
Abs[f[-8.25] - P3[-8.25]]
```

Out[547]=

1.22181×10^{-6}