Задача. Дадена е крива на Безие, дефинирана чрез контролните точки:

$P_0(0,0), P_1(-2,-2), P_2(-2,2), P_3(2,2), P_4(2,-2), P_5(0,0)$

а) Намерете точката от кривата съответна на $u=\frac{1}{4}$, чрез алгоритъма на де Кастелжо. (Начертайте мрежата на де Кастелжо с мерна единица 2см).

Решение: $u=\frac{1}{4}$, $1-u=1-\frac{1}{4}=\frac{3}{4}$

$P_0(0,0), P_1(-2,-2), P_2(-2,2), P_3(2,2), P_4(2,-2), P_5(0,0)$

$\Downarrow$

$n=5$

(степента на кривата)

$P_0(0,0)$					
$P_1(-2,-2)$	$P_{10}\left(-\frac{2}{4},-\frac{2}{4}\right)$				
$P_2(-2,2)$	$P_{11}\left(-\frac{8}{4},-\frac{4}{4}\right)$	$P_{20}\left(-\frac{14}{16},-\frac{10}{16}\right)$			
$P_3(2,2)$	$P_{12}\left(-\frac{4}{4},\frac{8}{4}\right)$	$P_{21}\left(-\frac{28}{16},-\frac{4}{16}\right)$	$P_{30}\left(-\frac{70}{64},-\frac{34}{64}\right)$		
$P_4(2,-2)$	$P_{13}\left(\frac{8}{4},\frac{4}{4}\right)$	$P_{22}\left(-\frac{4}{16},\frac{28}{16}\right)$	$P_{31}\left(-\frac{88}{64},\frac{16}{64}\right)$	$P_{40}\left(\frac{-298}{256},-\frac{86}{256}\right)$	
$P_5(0,0)$	$P_{14}\left(\frac{6}{4},-\frac{6}{4}\right)$	$P_{23}\left(\frac{32}{16},\frac{6}{16}\right)$	$P_{32}\left(\frac{20}{64},-\frac{78}{64}\right)$	$P_{41}\left(\frac{-244}{256},\frac{-30}{256}\right)$	$P_{50}\left(\frac{-1138}{1024},-\frac{288}{1024}\right) = C\left(\frac{1}{4}\right)$

$P_{10}=(1-u)P_0+uP_1=\frac{3}{4}(0,0)+\frac{1}{4}(-2,-2)=\left(-\frac{1}{2},-\frac{1}{2}\right)=(-0,5,-0,5)$

$P_{11}=\frac{3}{4}P_1+\frac{1}{4}P_2=\frac{3}{4}(-2,-2)+\frac{1}{4}(-2,2)=\left(-\frac{8}{4},-\frac{4}{4}\right)$

$P_{12}=\frac{3}{4}P_2+\frac{1}{4}P_3=\frac{3}{4}(-2,2)+\frac{1}{4}(2,2)=\left(-\frac{4}{4},\frac{8}{4}\right)$

$P_{13}=\frac{3}{4}P_3+\frac{1}{4}P_4=\frac{3}{4}(2,2)+\frac{1}{4}(2,-2)=\left(\frac{8}{4},\frac{4}{4}\right)$

$P_{14}=\frac{3}{4}P_4+\frac{1}{4}P_5=\frac{3}{4}(2,-2)+\frac{1}{4}(0,0)=\left(\frac{6}{4},-\frac{6}{4}\right)$

б) Подразделете кривата $C(u)$ при $u=\frac{1}{4}$, като подредите контролните точки на двете ръце в правилния ред.

Решение:

$C_1(u): u\in\left[0,\frac{1}{4}\right] - P_0(0,0), P_{10}\left(\frac{-2}{4},\frac{-2}{4}\right), P_{20}\left(\frac{-14}{16},\frac{-10}{16}\right), P_{30}\left(\frac{-70}{64},\frac{-34}{64}\right), P_{40}\left(\frac{-298}{256},\frac{-82}{256}\right), P_{50}\left(\frac{-1138}{1024},\frac{-78}{1024}\right)$

$C_2(u): u\in\left[\frac{1}{4},1\right] - P_{50}\left(\frac{-1138}{1024},\frac{-288}{1024}\right), P_{41}\left(\frac{-244}{256},\frac{-30}{256}\right), P_{32}\left(\frac{20}{64},\frac{-78}{64}\right), P_{23}\left(\frac{32}{16},\frac{6}{16}\right), P_{14}\left(\frac{6}{4},\frac{-6}{4}\right), P_5(0,0).$

c) Определете степента на дадената крива и я увеличете с единица. Намерете новите контролни точки и начертайте новия контролен полигон.

Решение:

$P_0(0,0), P_1(-2,-2), P_2(-2,2), P_3(2,2), P_4(2,-2), P_5(0,0)$

$n=5 \rightarrow 6.$

$Q_0 = P_0 \Rightarrow Q_0(0,0)$

$Q_6 = P_5 \Rightarrow Q_6(0,0)$

$$Q_i = \frac{i}{n+1}P_{i-1} + \left(1-\frac{i}{n+i}\right)P_i \quad , \quad i=1,2,3\ldots 5.$$

$$Q_1 = \frac{1}{6}P_0 + \left(1-\frac{1}{6}\right)P_1 = \frac{1}{6}(0,0) + \frac{5}{6}(-2,-2) = \left(\frac{-10}{6},\frac{-10}{6}\right) = \left(\frac{-5}{3},\frac{-5}{3}\right)$$

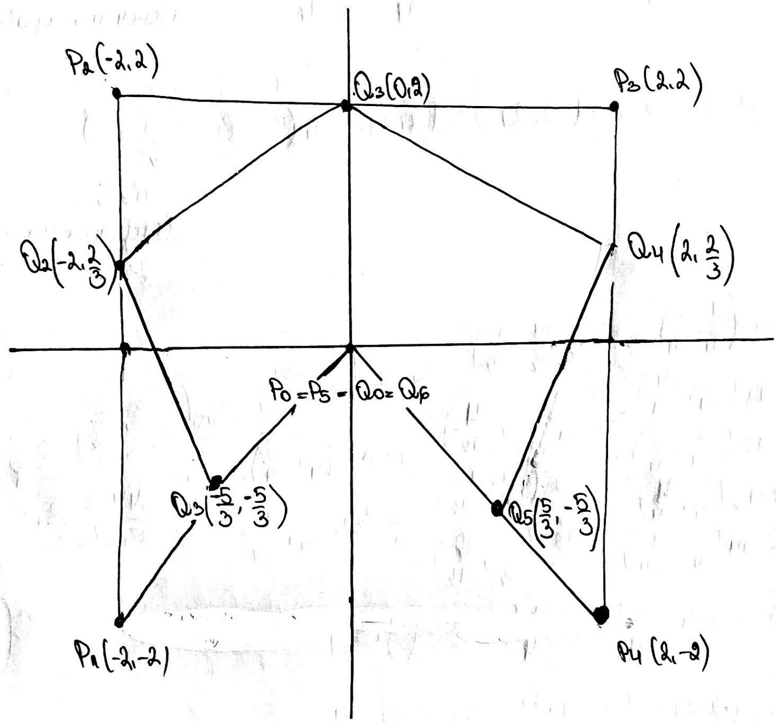
$$Q_2 = \frac{2}{6}P_1 + \left(1-\frac{2}{6}\right)P_2 = \frac{2}{6}(-2,-2) + \frac{4}{6}(-2,2) = \left(\frac{-4}{6}-\frac{8}{6},\frac{-4}{6}+\frac{8}{6}\right) = \left(\frac{-12}{6},\frac{4}{6}\right) = \left(-2,\frac{2}{3}\right)$$

$$Q_3 = \frac{3}{6}P_2 + \left(1-\frac{3}{6}\right)P_3 = \frac{3}{6}(-2,2) + \frac{3}{6}(2,2) = \left(\frac{-6}{6}+\frac{6}{6},\frac{6}{6}+\frac{6}{6}\right) = \left(0,\frac{12}{6}\right) = (0,2)$$

$$Q_4 = \frac{4}{6}P_3 + \left(1-\frac{4}{6}\right)P_4 = \frac{4}{6}(2,2) + \frac{2}{6}(2,-2) = \left(\frac{8}{6}+\frac{4}{6},\frac{8}{6}-\frac{4}{6}\right) = \left(\frac{12}{6},\frac{4}{6}\right) = \left(2,\frac{2}{3}\right)$$

$$Q_5 = \frac{5}{6}P_4 + \left(1-\frac{1}{6}\right)P_5 = \frac{5}{6}(2,-2) + \frac{1}{6}(0,0) = \left(\frac{10}{6},-\frac{10}{6}\right) = \left(\frac{5}{3},-\frac{5}{3}\right)$$

$D(u): Q_0(0,0), Q_1\left(-\frac{5}{3}, \frac{-5}{3}\right), Q_2\left(-2, \frac{2}{3}\right), Q_3\left(0, 2\right), Q_4\left(2, \frac{2}{3}\right), Q_5\left(\frac{5}{3}, \frac{-5}{3}\right), Q_6\left(0, 0\right)$



d) Проверете дали дадената крива е C^1-, C^2-, G^1-, G^2-) и кривинно непрекъсната в (0,0).

$P_0(0,0)$, $P_1(-2,-2)$, $P_2(-2,2)$, $P_3(2,2)$, $P_4(2,-2)$, $P_5(0,0)$

$u \in [0,1]$, $n=5$.

1. $C(0) = C(1) = P_0(0,0) = P_5(0,0)$.

2. ? $\exists$ C^1-непрекъснатост $\iff$? $\dot{C}(0) = \dot{C}(1)$

$\dot{C}(0) = 5.[P_1 - P_0] = 5.[(-2,-2)-(0,0)] = 5.(-2,-2) = (-10,-10)$

$\dot{C}(1) = 5.[P_5 - P_4] = 5[(0,0)-(2,-2)] = 5(-2,2) = (-10,10)$

$\Rightarrow \dot{C}(0) \neq \dot{C}(1) \Rightarrow \nexists$ C^1-непрекъснатост в точката на съединяване.

3. ? $\exists$ G^1-непр. $\iff$ $\dot{C}(0) \uparrow\uparrow \dot{C}(1)$

$\dot{C}(0) = \lambda \dot{C}(1) \Rightarrow (-10,-10) \neq \lambda(-10,10) \Rightarrow$ $\dot{C}(0)$ и $\dot{C}(1)$ не са еднопосочно колинеарни

$\Rightarrow \nexists$ G^1-непр.

4. ? C^2-непр. $\iff$? $\ddot{C}(0) = \ddot{C}(1)$

$\ddot{C}(0) = 5.4\,[P_2 - 2P_1 + P_0] = 20\,[(-2,2) - 2(-2,-2) + (0,0)] = (40,120)$

$\ddot{C}(1) = 5.4\,[P_5 - 2P_4 + P_3] = 20\,[(0,0) - 2(2,-2) + (2,2)] = (-40,120)$

$\Rightarrow \ddot{C}(0) \neq \ddot{C}(1) \Rightarrow \nexists$ C^2-непрекъснатост. $\Rightarrow \nexists$ G^2-непр.

5. ? $\varkappa$-непрек. $\iff$? $\varkappa_C(0) = \varkappa_C(1)$

$$\varkappa_C(0) = \frac{|\dot{C}(0) \times \ddot{C}(0)|}{|\dot{C}(0)|^3} = \varkappa_C(1) = \frac{|\dot{C}(1) \times \ddot{C}(1)|}{|\dot{C}(1)|^3}$$

$\dot{c}(0) = (-10, -10) \quad \longrightarrow \quad \dot{c}(0) = (-10, -10, 0)$

$\ddot{c}(0) = (40, 120) \quad \longrightarrow \quad \ddot{c}(0) = (40, 120, 0)$

$|\dot{c}(0)| = \sqrt{(-10)^2 + (-10)^2 + 0^2} = 10\sqrt{2}$

$\dot{c}(0) \times \ddot{c}(0) = \left(\begin{vmatrix} -10 & 0 \\ 120 & 0 \end{vmatrix} ; - \begin{vmatrix} -10 & 0 \\ 40 & 0 \end{vmatrix} ; \begin{vmatrix} -10 & -10 \\ 40 & 120 \end{vmatrix} \right) = (0, 0, -800)$

$|\dot{c}(0) \times \ddot{c}(0)| = \sqrt{0^2 + 0^2 + (-800)^2} = 800$

$\varkappa_c(0) = \dfrac{|\dot{c}(0) \times \ddot{c}(0)|}{|\dot{c}(0)|^3} = \dfrac{800}{(10\sqrt{2})^3} = \dfrac{\sqrt{2}}{5}$ //

$\dot{c}(1) = (-10, 10) \quad \longrightarrow \quad \dot{c}(1) = (-10, 10, 0)$

$\ddot{c}(1) = (-40, 120) \quad \longrightarrow \quad \ddot{c}(1) = (-40, 120, 0)$

$|\dot{c}(1)| = \sqrt{(-10)^2 + 10^2 + 0^2} = 10\sqrt{2}$

$\dot{c}(1) \times \ddot{c}(1) = \left(\begin{vmatrix} 10 & 0 \\ 120 & 0 \end{vmatrix} ; - \begin{vmatrix} -10 & 0 \\ -40 & 0 \end{vmatrix} ; \begin{vmatrix} -10 & 10 \\ -40 & 120 \end{vmatrix} \right) = (0, 0, -800)$

$|\dot{c}(1) \times \ddot{c}(1)| = 800$

$\varkappa_c(1) = \dfrac{|\dot{c}(1) \times \ddot{c}(1)|}{|\dot{c}(1)|^3} = \dfrac{800}{(10\sqrt{2})^3} = \dfrac{\sqrt{2}}{5}$ //

$\Rightarrow \varkappa_c(0) = \varkappa_c(1) \Rightarrow \exists\, \varkappa$-непр.