Def 13 $f: X \to Y$

$x_1 < x_2$ и $f(x_1) \leq f(x_2)$ → мон. раст.

$x_1 < x_2$ и $f(x_1) \geq f(x_2)$ → мон. нам.

$x_1 < x_2$ и $f(x_1) < f(x_2)$ строг. раст.

$x_1 < x_2$ и $f(x_1) > f(x_2)$ строго нам.

Def 14 Нека ф-ята $f: X \to Y$ удовлетворява условието: за $\forall\, y_0 \in Y$ $\exists$ единствено $x_0 \in X$, така че $f(x_0) = y_0$. Изображението $g: Y \to X$, деф. по този начин, наричаме обратна функция на f и означаваме с f^{-1}

Очевидно $f[f^{-1}(y)] = y$, $y \in Y$ и

$$f[f^{-1}(x)] = x,\ x \in X$$

Функциите f и f^{-1} се наричат взаимно обратни функции

$y = \sin x$, $x \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

$y = \arcsin x$ $\quad x \in [-1, 1]$

$$\sin \underbrace{x}_{\text{угол}} = \text{число}$$

$$\arcsin \underbrace{x}_{\text{число}} = \text{угол}$$

$y = \cos x$, $x \in [0, \pi]$

$y = \arccos x$, $x \in [-1, 1]$

$y = \operatorname{tg} x$, $x \in \left(-\frac{\pi}{2}; \frac{\pi}{2}\right)$

$y = \operatorname{arctg} x$, $x \in (-\infty; +\infty)$

$y = \operatorname{ctg} x$, $x \in (0, \pi)$

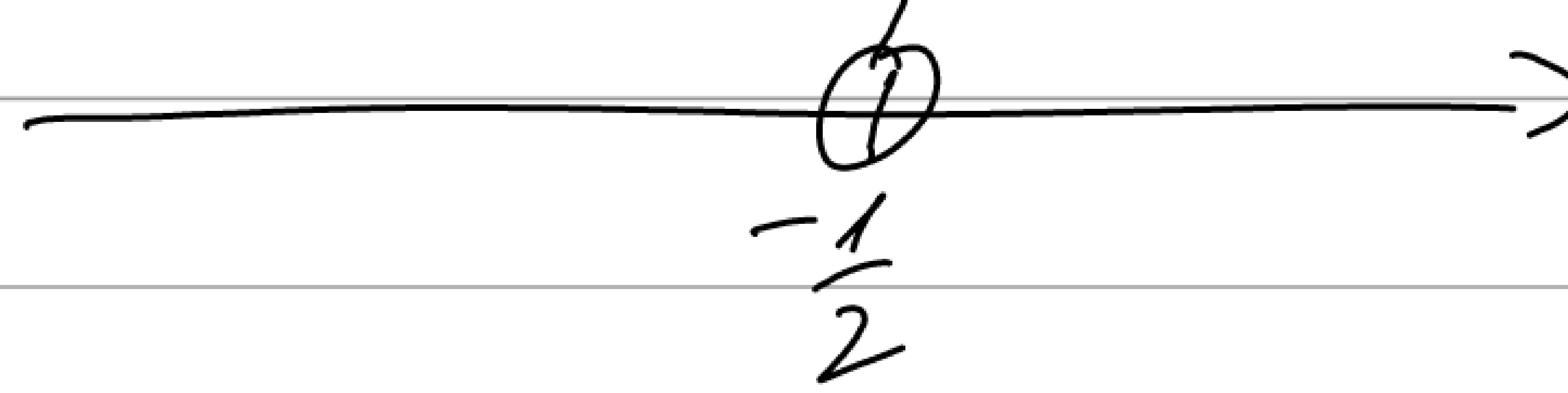
$y = \operatorname{arcctg} x$, $x \in (-\infty; +\infty)$

Деф 16 $f: X \to Y$

- четна $\to f(-x) = f(x)$
- нечетна $\to f(-x) = -f(x)$

① Определете Д.О. $f(x) = \dfrac{x+3}{2x+1}$

Д. О' | $2x+1 \neq 0 \Rightarrow x \neq -\dfrac{1}{2}$



$x \in \left(-\infty; -\frac{1}{2}\right) \cup \left(-\frac{1}{2}; +\infty\right)$

② Определете Д.О. $f(x) = \dfrac{x^2+4}{\underbrace{(x^2+x-2)}\underbrace{(2x-1)}}$

ДО: | $x^2+x-2 \neq 0$ $x \neq 1$ $x \neq -2$
| $2x-1 \neq 0$ $x \neq \dfrac{1}{2}$

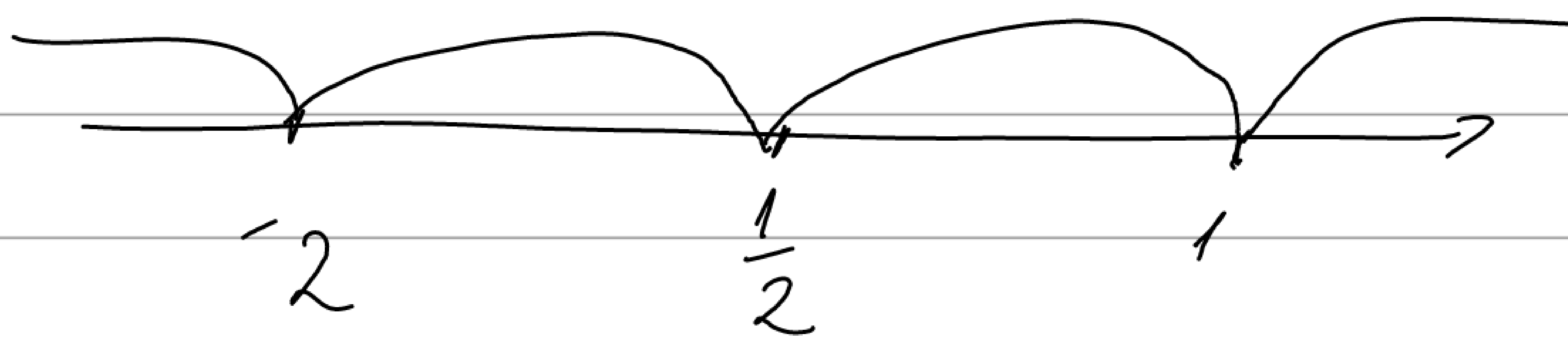
$A \cdot B \neq 0$

$A \neq 0 \cup B \neq 0$

$x^2 + \overbrace{x}^{-1-1} - 2 = 0$

$\underbrace{x^2 - 1} + x - 1 = 0$ $(x-1)(x+1) + 1 \cdot (x-1) = 0$

$(x-1)(x+1+1) = 0 \quad (x-1)(x+2) = 0$



$x \in (-\infty; -2) \cup (-2; \frac{1}{2}) \cup (\frac{1}{2}; 1) \cup (1; +\infty)$

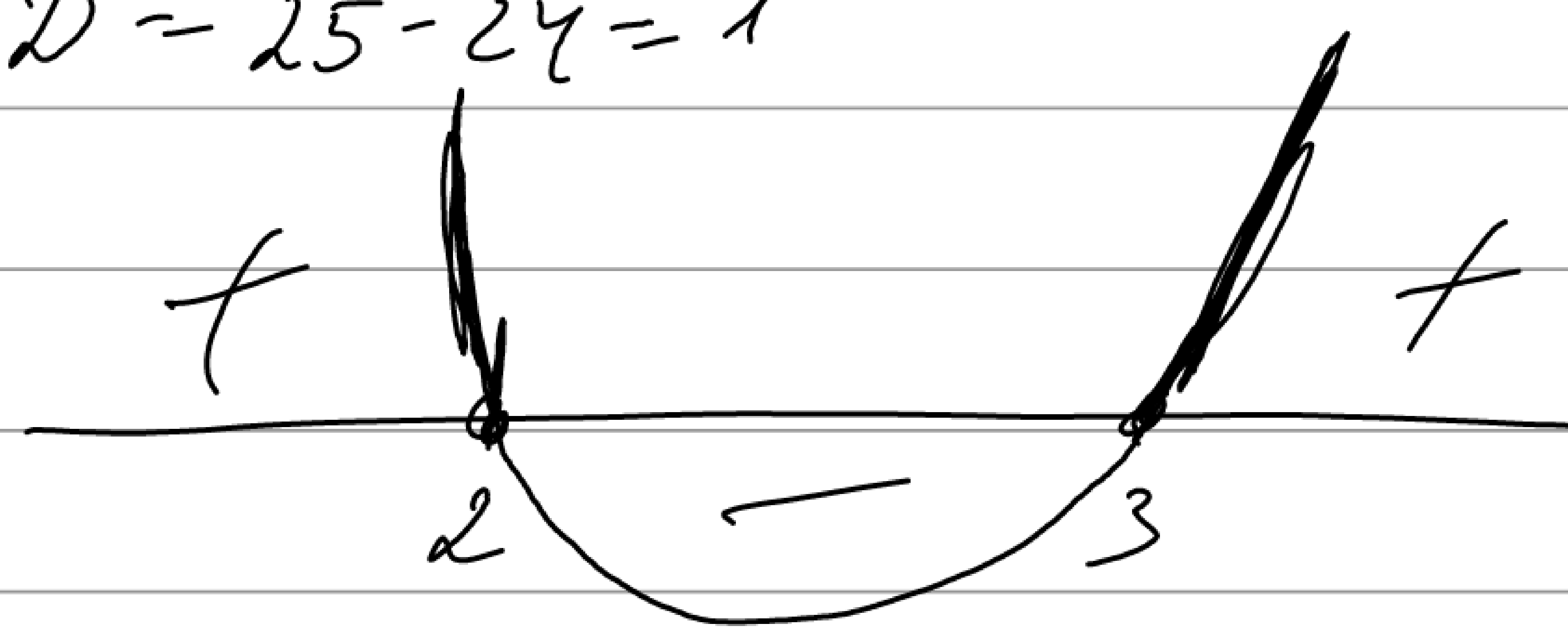
(3) Определете Д.О. $f(x) = \sqrt{x^2 - 5x + 6}$ ≥ 0

Д.О. $x^2 - 5x + 6 \geq 0$

$x^2 - 5x + 6 = 0$

$x_{1,2} = \frac{5 \pm 1}{2}$ $\nearrow x_1 = 3$, $\searrow x_2 = 2$

$D = 25 - 24 = 1$



$x \in (-\infty; 2] \cup [3; +\infty)$

(4) Намерете $f^{-1}(x)$, ако $f(x) = \dfrac{4x-1}{2x+3}$

Р-е: За да намерим $f^{-1}(x)$ трябва да решим у-то $f(y) = x$ относно y

$$y = \frac{4x-1}{2x+3}$$

ДО! $x \neq -\frac{3}{2}$ $x \neq 2$

$y \neq -\frac{3}{2}$

$$\frac{x}{1} = \frac{4y-1}{2y+3}$$

$\frac{a}{b} = \frac{c}{d} \Leftrightarrow ad = bc$

$$4y - 1 = x(2y+3)$$

$$4y - 1 = 2xy + 3x$$

$$4y - 2xy = 3x + 1$$

$$2y(2-x) = 3x+1 \quad /: 2(2-x)$$

$$y = \frac{3x+1}{2(2-x)} = f^{-1}(x)$$

(5) Намерете $f^{-1}(x)$, ако $f(x) = 1 + \sqrt{2+3x}$

Р-е:

$y = 1 + \sqrt{2+3x}$

ДС: $\left| \begin{array}{l} 2+3x \geq 0 \\ 2+3y \geq 0 \end{array} \right.$

$x = 1 + \sqrt{2+3y}$

$\left| \begin{array}{l} x \geq -\frac{2}{3} \\ y \geq -\frac{2}{3} \end{array} \right.$

$x - 1 = \sqrt{2+3y} \quad \uparrow^2$

$(x-1)^2 = 2 + 3y$

$x^2 - 2x + 1 = 2 + 3y$

$3y = x^2 - 2x - 1 \quad /:3$

$y = \frac{1}{3}(x^2 - 2x - 1) = f^{-1}(x)$

(6) Нам. $f^{-1}(x)$, ако $f(x) = e^{2x-1}$

(7) Да се докаже тъждеството

$\sin(\arcsin x) = x$, при $x \in [-1, 1]$

Р-е: От Def 14

$f[f^{-1}(x)] = x \Rightarrow \sin(\arcsin x) = x$

(8) Да се пресм. $\arcsin x$ при $x = 0$

$x = \frac{1}{2}$ и $x = -\frac{\sqrt{3}}{2}$

$\arcsin \frac{1}{2} = \alpha \Rightarrow \sin\alpha = \frac{1}{2}$

$\alpha = 30^\circ = \frac{\pi}{6}$

$\arcsin 0 = \alpha$ $\quad \sin\alpha = 0$

$\alpha = 0$

(9) Да се пресметне $\operatorname{arctg} x$ при

$x = 0,\ \underline{\underline{1}}$ и $-\sqrt{3}$

$\operatorname{arctg} 1 = \alpha$

$\operatorname{tg}\alpha = 1 \Rightarrow \alpha = \frac{\pi}{4}$ т.е.

$\operatorname{arctg} 1 = \frac{\pi}{4}$

(10) Да се докаже тъждеството

$\sin(\arccos x) = \sqrt{1-x^2}$

<u>Р-е</u>!

Нека да положим $\arccos x = \alpha$

$\alpha \in [0, \pi] \Rightarrow \cos\alpha = x$ при

$x \in [-1, 1]$. Тогава

$\sin(\arccos x) = \sin\alpha = |\sin\alpha| = \sqrt{1-\cos^2\alpha} =$

$\sin^2\alpha + \cos^2\alpha = 1$

$\sin^2\alpha = 1-\cos^2\alpha \quad / \sqrt{\ }$

$\sqrt{\sin^2\alpha} = \sqrt{1-\cos^2\alpha}$

$|\sin\alpha| = \sqrt{1-\cos^2\alpha}$

$= \sin\alpha = \sqrt{1-\underbrace{\cos^2\alpha}_{x^2}} \Rightarrow \sin\alpha = \sqrt{1-x^2}$

(11) Да се док. тъжд.

$\cos(\underbrace{\arcsin x}) = \sqrt{1-x^2}$ при

$x \in [-1, 1]$

<u>Р-е</u>! Пол. $\arcsin x = \alpha$ $\quad \alpha \in \left[-\frac{\pi}{2}; \frac{\pi}{2}\right]$

$\Rightarrow \sin\alpha = x$

$|\cos\alpha| = \sqrt{1-\underbrace{\sin^2\alpha}_{x^2}} \Rightarrow \cos(\arcsin x) = \sqrt{1-x^2}$

(12) Да се док. тъждеството $\arcsin x + \arccos x = \frac{\pi}{2}$ при $\arcsin x \in \left[-\frac{\pi}{2}; \frac{\pi}{2}\right]$ и

$\arccos x \in [0, \pi]$

Р-е:

Нека $\arcsin x = \alpha$ и $\alpha \in \left[-\frac{\pi}{2}; \frac{\pi}{2}\right]$

$\Rightarrow \sin\alpha = x$. Нека $\arccos x = \beta \Rightarrow$

$\Rightarrow \cos\beta = x$ за $x \in [-1, 1]$

Това е изпълнено когато $\sin\left(\frac{\pi}{4}\right) = \frac{\sqrt{2}}{2}$

и $\cos\left(\frac{\pi}{4}\right) = \frac{\sqrt{2}}{2}$ $\Rightarrow \alpha = \beta = \frac{\pi}{4}$

$\Rightarrow \alpha + \beta = \arcsin x + \arccos x = \frac{\pi}{4} + \frac{\pi}{4} =$

$= \frac{\pi}{2}$

(13) Да се докаже тъждеството

$\sin(2\operatorname{arctg} x) = \dfrac{2x}{1+x^2}$ при

$x \in (-\infty; +\infty)$ $\alpha \in \left(-\frac{\pi}{2}; \frac{\pi}{2}\right)$

Р-е: Пол. $\operatorname{arctg} x = \alpha \Rightarrow$

$\Rightarrow$ Л.С. $\sin 2\alpha = 2\sin\alpha \cdot \cos\alpha$

От това, че $\operatorname{arctg} x = \alpha \Rightarrow \operatorname{tg}\alpha = x$

Д.С. $\dfrac{2x}{1+x^2} \overset{x=\operatorname{tg}\alpha}{=} \dfrac{2\operatorname{tg}\alpha}{1+\operatorname{tg}^2\alpha} =$

$= \dfrac{2 \cdot \frac{\sin\alpha}{\cos\alpha}}{1+\frac{\sin^2\alpha}{\cos^2\alpha}} = \dfrac{\frac{2\sin\alpha}{\cancel{\cos\alpha}}}{\frac{\cos^2\alpha+\sin^2\alpha}{\cos^2\alpha}}$ $\cos^2\alpha+\sin^2\alpha = 1$ $=$

$= 2\sin\alpha \cos\alpha$

Извод: Л.С. = Д.С. $\Rightarrow$ равенството е тъждество

(14) Да се док. тъждеството:

$$\cos(2\underbrace{\operatorname{arccotg} x}) = \frac{x^2-1}{1+x^2}$$ при

$x \in (-\infty; +\infty)$

Р-е:

Пол. $\operatorname{arccotg} x = \alpha$ $\alpha \in (0, \pi)$

$Л.С = \cos 2\alpha = \underbrace{\cos^2\alpha - \sin^2\alpha}$

Т.к. $\operatorname{arccotg} x = \alpha \Rightarrow \operatorname{cotg}\alpha = x$

$$Д.С: \frac{x^2-1}{1+x^2} \underset{\overline{\quad}}{\overset{x=\operatorname{cotg}\alpha}{=}} \frac{\operatorname{cotg}^2\alpha - 1}{1+\operatorname{cotg}^2\alpha} =$$

$$= \frac{\dfrac{\cos^2\alpha}{\sin^2\alpha} - \overset{\sin^2\alpha}{1}}{\overset{\sin^2\alpha}{\dfrac{1}{1}} + \dfrac{\cos^2\alpha}{\sin^2\alpha}} = \frac{\dfrac{\cos^2\alpha - \sin^2\alpha}{\cancel{\sin^2\alpha}}}{\dfrac{\sin^2\alpha + \cos^2\alpha}{\cancel{\sin^2\alpha}}}$$

$\sin^2\alpha + \cos^2\alpha = 1$

$= \underbrace{\cos^2\alpha - \sin^2\alpha} \Rightarrow Л.С. = Д.С$

и равенството е тъждество.

Граница на функция. Непрекъснати функции

Ако някое аритметично действие няма стойност, то резултатът наричаме неопределеност. Основните неопределености са $\left[\frac{0}{0}\right]$ и $\left[\frac{\infty}{\infty}\right]$. Останалите, които се свеждат до тези две основни неопределености са $[0.\infty]$, $[\infty-\infty]$, $[1^{\infty}]$

① Да се пресм. границата

$$\lim_{x\to -1} \frac{(x^3-2x-1)(x+1)}{x^4+4x^2-5} = \left[\frac{0}{0}\right]$$

$$x^4+4x^2-5=0$$

Пол $x^2 = t$ $\qquad D = 16+20=36$

$$t^2+4t-5=0$$

$$t_{1,2} = \frac{-4\pm 6}{2} \nearrow 1, \searrow -5$$

$$x^4+4x^2-5=(x^2-1)(x^2+5)$$

$$ax^2+bx+c=a(x-x_1)(x-x_2)$$

$$t^2+4t-5=(t-t_1)(t-t_2)$$

$$(t-1)(t+5)$$

$$x^4+4x^2-5=\underbrace{(x-1)(x+1)}_{x^2}(x^2+5)$$

$$(-1)^3-2(-1)-1=0$$

$$\lim_{x\to-1}\frac{(x^3-2x-1)\cancel{(x+1)}}{\underbrace{(x^2+5)}_{6}\underbrace{(x-1)}_{-2}\cancel{(x+1)}}=\frac{0}{-12}=0$$

(2) Да се пресм. гран.:

$$\lim_{x\to1}\frac{x^3+x^2-5x+3}{x^3-x^2-x+1}=\left[\frac{0}{0}\right]$$

$$x^3+x^2-5x+3=x^3+x^2-x-x-3x+3=$$

$$=x^3-x+x^2-x-3x+3=$$

$$=x(x^2-1)+x(x-1)-3(x-1)=$$

$$=x.(x-1)(x+1)+x(x-1)-3(x-1)=$$

$(x-1)[x(x+1)+x-3] =$

$= (x-1)(x^2+x+x-3) =$

$= (x-1)(x^2+2x-3)$

$x^2-1+2x-2 = (x-1)(x+1)+2(x-1) =$

$= (x-1)(x+1+2) =$

$= (x-1)(x+3)$

$x^3+x^2-5x+3 = (x-1)(x-1)(x+3) = (x-1)^2(x+3)$

$\underbrace{x^3-x^2}-\underbrace{x+1} = x^2(x-1)-(x-1) =$

$= (x-1)(x^2-1) =$

$= (x-1)(x-1)(x+1) =$

$= (x-1)^2(x+1)$

$$\lim_{x\to 1} \frac{\cancel{(x-1)^2}(x+3)}{\cancel{(x-1)^2}(x+1)} = \frac{\cancel{4}}{\cancel{2}} = 2$$

③ Да се пресм. гран.

$$\lim_{x\to 1} \frac{x^2-\sqrt{x}}{\sqrt{x}-1} = \left[\frac{0}{0}\right]$$

За доп. чрез изнасяне на $x^?$ в числ. и знам.

$$\lim_{x\to 1}\frac{(x^2-\sqrt{x})(\sqrt{x}+1)\cdot(x^2+\sqrt{x})}{(\sqrt{x}-1)(\sqrt{x}+1)(x^2+\sqrt{x})}=$$

$$=\lim_{x\to 1}\frac{(x^4-x)(\sqrt{x}+1)}{(x-1)(x^2+\sqrt{x})}=$$

$$=\lim_{x\to 1}\frac{x(x^3-1)(\sqrt{x}+1)}{(x-1)(x^2+\sqrt{x})}=$$

$$=\lim_{x\to 1}\frac{x\cancel{(x-1)}(x^2+x+1)(\sqrt{x}+1)}{\cancel{(x-1)}(x^2+\sqrt{x})}=\frac{\cancel{6}}{\cancel{2}}=3$$

4) Da ce presm. gran. $\lim_{x\to+\infty}\frac{\sqrt{x+\sqrt{x+\sqrt{x}}}}{\sqrt{x+1}}=\left[\frac{\infty}{\infty}\right]$

$$\lim_{x\to+\infty}\frac{\sqrt{x+\sqrt{x+x^{\frac{1}{2}}}}}{\sqrt{x(1+\frac{1}{x})}}=\lim_{x\to+\infty}\frac{\sqrt{x+\sqrt{x(1+\frac{1}{\sqrt{x}})}}}{\sqrt{x}\sqrt{1+\frac{1}{x}}}=$$

$$=\lim_{x\to+\infty}\frac{\sqrt{x+\sqrt{x}\cdot\sqrt{1+\frac{1}{\sqrt{x}}}}}{\sqrt{x}\cdot\sqrt{1+\frac{1}{x}}}=$$

$$= \lim_{x \to +\infty} \frac{\sqrt{x\left(1 + \frac{1}{\sqrt{x}}\left(1+\frac{1}{\sqrt{x}}\right)\right)}}{\sqrt{x} \cdot \sqrt{1+\frac{1}{x}}} =$$

$$= \lim_{x \to +\infty} \frac{\cancel{\sqrt{x}}\sqrt{1+\underbrace{\frac{1}{\sqrt{x}}}_{\to 0}\left(1+\frac{1}{\sqrt{x}}\right)} \to 0}{\cancel{\sqrt{x}}\sqrt{1+\frac{1}{x}} \to 0} = \frac{\sqrt{1}}{\sqrt{1}} = 1$$

(5) $$\lim_{x \to +\infty} \frac{\sqrt{x} + \sqrt[3]{x} + \sqrt[4]{x}}{\sqrt{2x+1}}$$

(6) $$\lim_{x \to 4} \frac{\sqrt{1+2x} - 3}{\sqrt{x} - 2}$$

(7) $$\lim_{x \to a} \frac{\sqrt{x} - \sqrt{a} + \sqrt{x-a}}{\sqrt{x^2-a^2}}$$

(8) $$\lim_{x \to -\infty} \left(\sqrt{x^2+x+1} - \sqrt{x^2-x+1}\right)$$

$$\lim_{x \to -\infty} \frac{\left(\sqrt{x^2+x+1} - \sqrt{x^2-x+1}\right)\left(\sqrt{x^2+x+1} + \sqrt{x^2-x+1}\right)}{\left(\sqrt{x^2+x+1} + \sqrt{x^2-x+1}\right)}$$

$$= \lim_{x \to -\infty} \frac{\cancel{x^2} + x + \cancel{1} - \overbrace{(x^2 - x + 1)}^{-x^2 + x - 1}}{\underbrace{\sqrt{x^2}}_{|x|}\sqrt{\left(1 + \frac{1}{x} + \frac{1}{x^2}\right)} + \underbrace{\sqrt{x^2}}_{|x|}\sqrt{\left(1 - \frac{1}{x} + \frac{1}{x^2}\right)}} =$$

$$= \lim_{x \to -\infty} \frac{2\cancel{x}}{\underbrace{\cancel{|x|}}_{-}\left(\sqrt{1 + \underbrace{\frac{1}{x}}_{\to 0} + \underbrace{\frac{1}{x^2}}_{\to 0}} + \sqrt{1 - \underbrace{\frac{1}{x}}_{\to 0} + \underbrace{\frac{1}{x^2}}_{\to 0}}\right)} = -\frac{\cancel{2}}{\cancel{2}} = -1$$

(g) $\lim_{x \to +\infty} \left(\sqrt{x^2 - 2x - 1} - \sqrt{x^2 - 7x + 3}\right)$

$$\lim_{x \to +\infty} \frac{\left(\sqrt{x^2 - 2x - 1} - \sqrt{x^2 - 7x + 3}\right)\left(\sqrt{x^2 - 2x - 1} + \sqrt{x^2 - 7x + 3}\right)}{\sqrt{x^2 - 2x - 1} + \sqrt{x^2 - 7x + 3}} =$$

$$= \lim_{x \to +\infty} \frac{\cancel{x^2} - 2x - 1 - \overbrace{(x^2 - 7x + 3)}^{-x^2 + 7x - 3}}{\underbrace{\sqrt{x^2}}_{|x|}\sqrt{\left(1 - \frac{2}{x} - \frac{1}{x^2}\right)} + \underbrace{\sqrt{x^2}}_{|x|}\sqrt{\left(1 - \frac{7}{x} + \frac{3}{x^2}\right)}} =$$

$$= \lim_{x \to +\infty} \frac{5x - 4}{|x|\left(\sqrt{1 - \frac{2}{x} - \frac{1}{x^2}} + \sqrt{1 - \frac{7}{x} + \frac{3}{x^2}}\right)} =$$

$$= \lim_{x \to +\infty} \frac{\cancel{x}\left(5 - \frac{4}{x}\right)}{\cancel{x}\cdot\left(\sqrt{1 - \frac{2}{x} - \frac{1}{x^2}} + \sqrt{1 - \frac{7}{x} + \frac{3}{x^2}}\right)} = \frac{5}{2}$$

$\frac{4}{x} \to 0$, $\frac{2}{x} \to 0$, $\frac{1}{x^2} \to 0$, $\frac{7}{x} \to 0$, $\frac{3}{x^2} \to 0$

(10) $$\lim_{x \to 0} \frac{\sqrt{1+x} - \sqrt{1-x}}{\sqrt[3]{1+x} - \sqrt[3]{1-x}}$$

(11) $$\lim_{x \to +\infty} \left[\sqrt{(x+a)(x+b)} - x\right]$$

Приложение на основната граница

$$\lim_{x \to 0} \frac{\sin x}{x} = 1$$

(1) Да се пресм. гран. $\lim_{x \to 0} \frac{3\sin 3x}{3x}$

$$= 3 \underbrace{\lim_{x \to 0} \frac{\sin 3x}{3x}}_{1} = \underline{\underline{3}}$$

(2) $\lim\limits_{x \to 0} \dfrac{tg\,x}{x} = \lim\limits_{x \to 0} \dfrac{\frac{\sin x}{\cos x}}{x} =$

$= \lim\limits_{x \to 0} \dfrac{\sin x}{x \cdot \cos x} = \underbrace{\lim\limits_{x \to 0} \dfrac{\sin x}{x}}_{1} \cdot \underbrace{\lim\limits_{x \to 0} \dfrac{1}{\cos x}}_{1}$

$= 1$

(3) $\lim\limits_{x \to 0} x \cdot cotg\, 3x =$

$= \lim\limits_{x \to 0} x \cdot \dfrac{\cos 3x}{\sin 3x} =$

$= \lim\limits_{x \to 0} \dfrac{3x}{3 \sin 3x} \cdot \underbrace{\lim\limits_{x \to 0} \cos 3x}_{1} =$

$= \dfrac{1}{3} \underbrace{\lim\limits_{x \to 0} \left(\dfrac{\sin 3x}{3x} \right)^{-1}}_{1} \cdot \underbrace{\lim\limits_{x \to 0} \cos 3x}_{1} = \dfrac{1}{3}$

(4) $\lim\limits_{x \to 0} \dfrac{\cos 3x - \cos 7x}{x^2} =$

$= \lim\limits_{x \to 0} \dfrac{\left(-2 \cdot \sin \frac{10x}{2} \cdot \sin\left(\frac{-4x}{2}\right)\right)}{x^2} =$

$= +2 \lim\limits_{x \to 0} \dfrac{\sin 5x \cdot \sin 2x}{x^2} =$

$\boxed{\sin(-2x) = -\sin 2x \text{ защото } \sin x \text{ е неч}}$

$= 2 \cdot \lim\limits_{x \to 0} \dfrac{\sin 5x}{x} \cdot \dfrac{\sin 2x}{x} =$

$= 2 \lim\limits_{x \to 0} \dfrac{5 \sin 5x}{5x} \cdot \lim\limits_{x \to 0} \dfrac{2 \sin 2x}{2x} =$

$= 20 \underbrace{\lim\limits_{x \to 0} \dfrac{\sin 5x}{5x}}_{= 1} \cdot \underbrace{\lim\limits_{x \to 0} \dfrac{\sin 2x}{2x}}_{1} = \underline{\underline{20}}$

(5) Да се пресм. гран. $\lim_{x \to 0} \frac{\sin(ax) - \sin(bx)}{x}$

$\sin\alpha - \sin\beta = \dots$ ———

<u>Приложение на основната граница</u>

$$\lim_{x \to 0} (1+x)^{\frac{1}{x}} = e$$

<u>Th 19</u> Нека $\lim_{x \to a} u(x) = \underline{\underline{A}}$, $(A>0)$ и $\lim_{x \to a} v(x) = \underline{\underline{B}}$. Тогава:

- Ако $A \in \mathbb{R}$ и $B \in \mathbb{R}$, то $\exists$ $\lim_{x \to a} (u(x))^{v(x)}$ и $\lim_{x \to a} (u(x))^{v(x)} = A^B$

- Ако $A=1$ $B=\infty$, то $\lim_{x \to a} u^v = e^{\lim_{x \to a} (u-1) \cdot v}$

① Да се пресметне границата

$$\lim_{x\to\infty}\left(\frac{x^2+1}{x^2-1}\right)^{x^2}$$

$$u(x)=\frac{x^2+1}{x^2-1} \qquad v(x)=x^2$$

$$\lim_{x\to\infty}\frac{x^2+1}{x^2-1}=1=A$$

$$\lim_{x\to\infty}x^2=\infty=B$$

$$\lim_{x\to\infty}\left(\frac{x^2+1}{x^2-1}\right)^{x^2}=e^{\lim\limits_{x\to\infty}\left(\frac{x^2+1}{x^2-1}-1\right)\cdot x^2}$$

$$\lim_{x\to\infty}\left(\frac{\cancel{x^2}+1-(\cancel{x^2}+1)}{x^2-1}\right)\cdot x^2=\lim_{x\to\infty}\frac{2x^2}{1\cdot x^2-1}=$$

$$=2$$

$$\lim_{x\to\infty}\left(\frac{x^2+1}{x^2-1}\right)^{x^2}=\underline{\underline{e^2}}$$

② Да се пресмета границата

$$\lim_{x\to 0}\underbrace{\left(\frac{\arcsin x}{x}\right)}_{u(x)}^{\underbrace{2(x+5)}_{v(x)}}$$

$$\lim_{x\to 0}\frac{\arcsin x}{x}$$ Полагаме $\arcsin x = y$

$\Rightarrow \sin y = x \Rightarrow$

$$\lim_{x\to 0}\left(\frac{\arcsin x}{x}\right)=\lim_{y\to 0}\frac{y}{\sin y}=\underline{1}=A$$

$$\lim_{x\to 0}2(x+5)=\underline{10=B}$$

$$\lim_{x\to 0}\left(\frac{\arcsin x}{x}\right)^{2(x+5)}=1^{10}=1$$

③ $$\lim_{x\to\infty}\left(\frac{x+a}{x-a}\right)^{x}$$

$$\lim_{x\to\infty}x=\infty$$

П сл на П

$$\lim_{x\to\infty}\left(\frac{x+a}{x-a}\right)=1$$