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Курсовая работа 22

Загл. 1

$$\text{a) } \begin{cases} \dot{x} = -5x - 2y - 2z \\ \dot{y} = 10x + 4y + 2z \\ \dot{z} = 2x + y + 3z \end{cases}$$

$$(\lambda_1 = 1, \lambda_2 = 2, \lambda_3 = -1)$$

$$A = \begin{pmatrix} -5 & -2 & -2 \\ 10 & 4 & 2 \\ 2 & 1 & 3 \end{pmatrix} \Rightarrow A - \lambda E = \begin{pmatrix} -5-\lambda & -2 & -2 \\ 10 & 4-\lambda & 2 \\ 2 & 1 & 3-\lambda \end{pmatrix}$$

$$\lambda_1 = 1$$

$$(A - \lambda_1 E) h_1 = 0$$

$$\begin{pmatrix} -6 & -2 & -2 \\ 10 & 3 & 2 \\ 2 & 1 & 2 \end{pmatrix} \begin{pmatrix} \alpha \\ \beta \\ \gamma \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$$\begin{cases} -6\alpha - 2\beta - 2\gamma = 0 \\ 10\alpha + 3\beta + 2\gamma = 0 \\ 2\alpha + \beta + 2\gamma = 0 \end{cases} \Rightarrow \begin{cases} 4\alpha + \beta = 0 \Rightarrow \beta = -4\alpha \\ -2\alpha + 2\gamma = 0 \end{cases}$$

$$h_1 = \begin{pmatrix} \alpha \\ \beta \\ \gamma \end{pmatrix} = \begin{pmatrix} \alpha \\ -4\alpha \\ \alpha \end{pmatrix}$$

$$\begin{aligned} -2\alpha + 2\gamma &= 0 \\ 2\gamma &= 2\alpha \\ \gamma &= \alpha \end{aligned}$$

$$\alpha = 1 \Rightarrow$$

$$h_1 = \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} \Rightarrow x_1 = h_1 \cdot e^{\lambda_1 t} = \begin{pmatrix} 1 \\ -4 \\ 1 \end{pmatrix} \cdot e^t$$

3A9.2

u) $x' = -3x + y - 2z$

$y' = 4x + y$

$z' = 4x + 2z$

$(\lambda_1 = 1, \lambda_2 = \lambda_3 = -1)$

1) $h_1 \begin{pmatrix} d_1 \\ \beta_1 \\ f_1 \end{pmatrix}, \lambda_1 = 1$

$(A - \lambda_1 B) \cdot h_1 = 0$

$\begin{pmatrix} -4 & 1 & -2 \\ 4 & 0 & 0 \\ 4 & 0 & 0 \end{pmatrix} \cdot \begin{pmatrix} d_1 \\ \beta_1 \\ f_1 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$

$-4d_1 + \beta_1 + 2f_1 = 0$

$4d_1 = 0$

$4d_1 = 0$

$\begin{pmatrix} \beta_1 = 2f_1 \\ d_1 = 0 \end{pmatrix} \quad h_1 = \begin{pmatrix} 0 \\ -2f_1 \\ f_1 \end{pmatrix} = \begin{pmatrix} 0 \\ -2 \\ 1 \end{pmatrix}$

2) $h_2, \lambda_3 = -1$

$(A - \lambda_3 B) \cdot h_2 = 0$

$x_1 = \begin{pmatrix} 0 \\ -2 \\ 1 \end{pmatrix} e^t$

$\begin{pmatrix} -2 & 1 & -2 \\ 4 & 2 & 0 \\ 4 & 0 & 2 \end{pmatrix} \cdot \begin{pmatrix} d_2 \\ \beta_2 \\ f_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$

$-2d_2 + \beta_2 - 2f_2 = 0$

$4d_2 + \beta_2 = 0$

$4d_2 + 2f_2 = 0$

$-2d_2 + \beta_2 - 2f_2 = 0$

$\beta_2 = 2d_2$

$f_2 = -2d_2$

$-4d_2 + 4d_2 = 0 \dots ?$

$$\text{3A9, 3}$$

$$b) \begin{cases} \dot{x} = 3x - 6y + \frac{1}{\cos^3 3t} \\ \dot{y} = 3x - 3y \end{cases}$$

$$A = \begin{pmatrix} 3 & -6 \\ 3 & 3 \end{pmatrix} \quad K = \begin{pmatrix} \cos^3 3t \end{pmatrix}$$

$$A - \lambda E = \begin{pmatrix} 3-\lambda & -6 \\ 3 & 3-\lambda \end{pmatrix}$$

$$0 = |A - \lambda E| = (3-\lambda)(3-\lambda) + 18 = 9 - 3\lambda + 3\lambda + \lambda^2 + 18 = \lambda^2 + 9 \Rightarrow \lambda_{1,2} = \pm 3i$$

$$\lambda_1 = 3i$$

$$(A - \lambda_1 E) \cdot h = 0$$

$$\begin{pmatrix} 3-3i & -6 \\ 3 & 3-3i \end{pmatrix} \begin{pmatrix} \alpha \\ \beta \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \Leftrightarrow \begin{cases} (3-i)\alpha - 6\beta = 0 & | :3 \\ 3\alpha - 3(1+i)\beta = 0 & | :3 \end{cases}$$

$$(1-i)\alpha - 2\beta = 0$$

$$-\alpha + (1+i)\beta = 0 \Rightarrow \alpha = (1+i)\beta$$

$$(1-i)(1+i)\beta - 2\beta = 0$$

$$(1^2 - i^2)\beta - 2\beta = 0$$

$$2\beta - 2\beta = 0$$

$$h_1 = \begin{pmatrix} (1+i)\beta \\ \beta \end{pmatrix} \Leftrightarrow \beta = 1 \quad x_1 = \begin{pmatrix} 1+i \\ 1 \end{pmatrix} e^{it}$$

$$= \begin{pmatrix} 1+i \\ 1 \end{pmatrix} (\cos x + i \sin x)$$