

Числено диференциране

In[123]:=

```
xt = Table[4 + i * (0.1), {i, 0, 10}];  
f[x_] := 2 Sin[x + 3]  
yt = f[xt]
```

Out[125]=

```
{1.31397, 1.45794, 1.58734, 1.70087, 1.79742,  
1.876, 1.93584, 1.97634, 1.99709, 1.99788, 1.97872}
```

In[126]:=

```
h = 0.1
```

Out[126]=

```
0.1
```

In[127]:=

```
n = Length[xt]
```

Out[127]=

```
11
```

Формули с точност $O(h)$ - първи порядък

Първа производна

In[128]:=

```
yp1 = Table[ $\frac{yt[[i + 1]] - yt[[i]]}{h}$ , {i, 1, n - 1}]
```

Out[128]=

```
{1.43965, 1.29398, 1.13538, 0.96543, 0.785838,  
0.598394, 0.404971, 0.207502, 0.00795993, -0.191662}
```

In[129]:=

```
AppendTo[yp1,  $\frac{yt[[n]] - yt[[n - 1]]}{h}$ ]
```

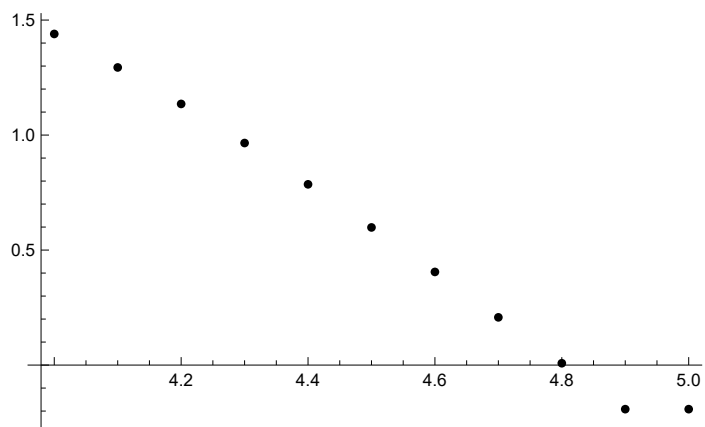
Out[129]=

```
{1.43965, 1.29398, 1.13538, 0.96543, 0.785838,  
0.598394, 0.404971, 0.207502, 0.00795993, -0.191662, -0.191662}
```

In[130]:=

```
pointsyp1 = Table[{xt[[i]], yp1[[i]]}, {i, 1, n}];
gryp1 = ListPlot[pointsyp1, PlotStyle -> Black]
```

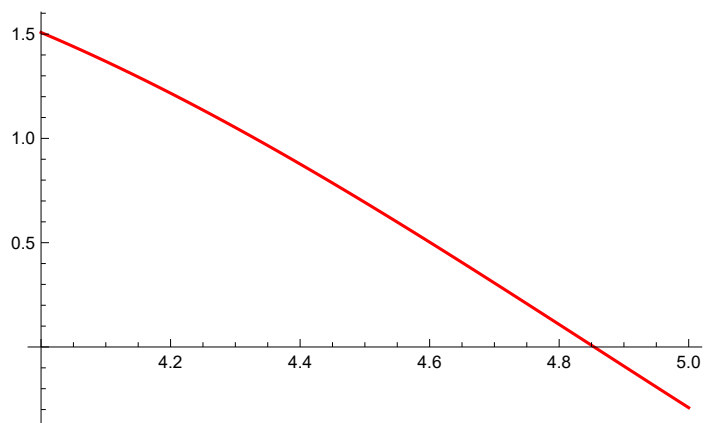
Out[131]=



In[132]:=

```
grfyp = Plot[f'[x], {x, xt[[1]], xt[[n]]}, PlotStyle -> Red]
```

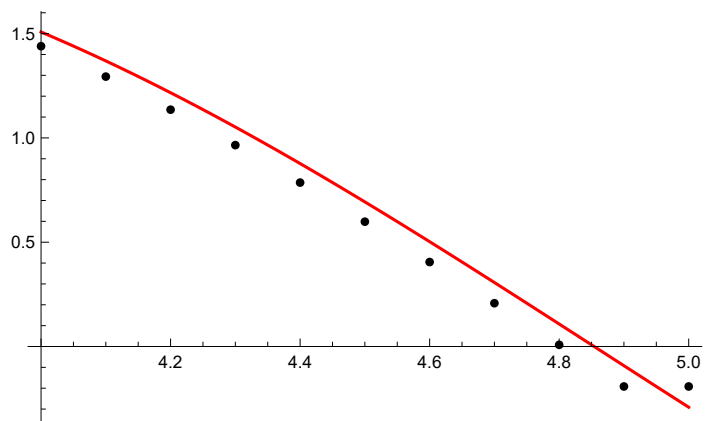
Out[132]=



In[133]:=

```
Show[grfyp, gryp1]
```

Out[133]=



Формули с точност $\mathcal{O}(h^2)$ - втори порядък

Първа производна

попълваме средните точки

In[134]:=

```
yp2 = Table[ $\frac{yt[[i + 1]] - yt[[i - 1]]}{2 h}$ , {i, 2, n - 1}]
```

Out[134]=

```
{1.36681, 1.21468, 1.0504, 0.875634, 0.692116, 0.501683, 0.306237, 0.107731, -0.091851}
```

допълваме производната в десния край (последната)

In[135]:=

```
AppendTo[yp2,  $\frac{yt[[n - 2]] - 4 yt[[n - 1]] + 3 yt[[n]]}{2 h}$ ]
```

Out[135]=

```
{1.36681, 1.21468, 1.0504, 0.875634, 0.692116,  
0.501683, 0.306237, 0.107731, -0.091851, -0.291473}
```

допълваме производната в левия край (първата)

In[136]:=

```
PrependTo[yp2,  $\frac{-3 yt[[1]] + 4 yt[[2]] - yt[[3]]}{2 h}$ ]
```

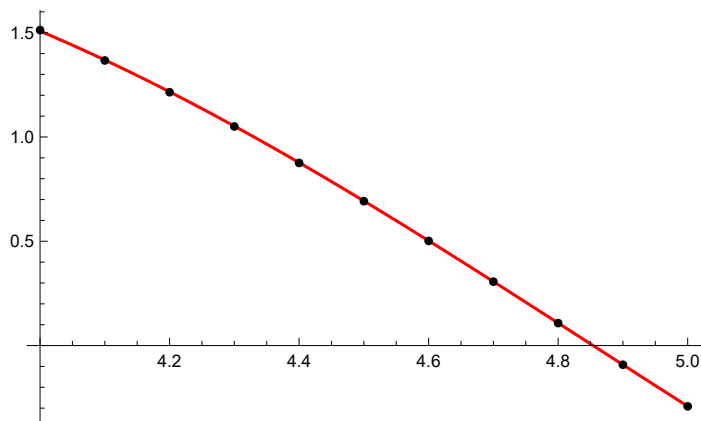
Out[136]=

```
{1.51249, 1.36681, 1.21468, 1.0504, 0.875634,  
0.692116, 0.501683, 0.306237, 0.107731, -0.091851, -0.291473}
```

In[137]:=

```
pointsyp2 = Table[{xt[[i]], yp2[[i]]}, {i, 1, n}];  
gryp2 = ListPlot[pointsyp2, PlotStyle -> Black];  
grfyp = Plot[f'[x], {x, xt[[1]], xt[[n]]}, PlotStyle -> Red];  
Show[grfyp, gryp2]
```

Out[140]=



Втора производна

попълваме средните точки

In[141]:=

$$ypp2 = \text{Table}\left[\frac{yt[[i+1]] - 2\,yt[[i]] + yt[[i-1]]}{h^2}, \{i, 2, n-1\}\right]$$

Out[141]=

```
{-1.45672, -1.58601, -1.69946, -1.79592,
-1.87444, -1.93423, -1.97469, -1.99542, -1.99622}
```

In[142]:=

f''[xt]

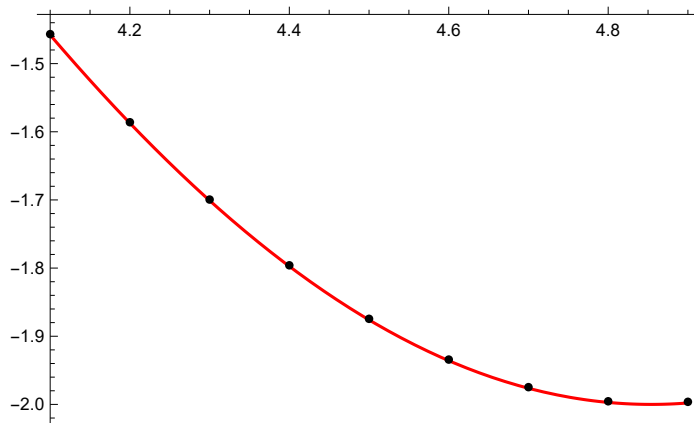
Out[142]=

```
{-1.31397, -1.45794, -1.58734, -1.70087, -1.79742,
-1.876, -1.93584, -1.97634, -1.99709, -1.99788, -1.97872}
```

In[143]:=

```
pointsypp2 = Table[{xt[[i+1]], ypp2[[i]]}, {i, 1, n-2}];
grypp2 = ListPlot[pointsypp2, PlotStyle -> Black];
grfypp = Plot[f''[x], {x, xt[[2]], xt[[n-1]]}, PlotStyle -> Red];
Show[grfypp, grypp2]
```

Out[146]=



Числено диференциране - повишаваме точността чрез сгъстяване на мрежата

In[147]:=

```
xt = Table[4 + i * (0.01), {i, 0, 100}];  
f[x_] := 2 Sin[x + 3]  
yt = f[xt]
```

Out[149]=

```
{1.31397, 1.32899, 1.34386, 1.35861, 1.37322, 1.38769, 1.40202, 1.41622, 1.43027, 1.44417,  
1.45794, 1.47156, 1.48503, 1.49835, 1.51152, 1.52454, 1.53741, 1.55013, 1.56269,  
1.57509, 1.58734, 1.59942, 1.61135, 1.62312, 1.63472, 1.64616, 1.65744, 1.66855,  
1.67949, 1.69027, 1.70087, 1.71131, 1.72157, 1.73167, 1.74159, 1.75133, 1.7609,  
1.7703, 1.77952, 1.78856, 1.79742, 1.8061, 1.8146, 1.82292, 1.83105, 1.83901,  
1.84678, 1.85436, 1.86176, 1.86897, 1.876, 1.88284, 1.88949, 1.89595, 1.90222,  
1.9083, 1.9142, 1.91989, 1.9254, 1.93072, 1.93584, 1.94077, 1.9455, 1.95004, 1.95439,  
1.95854, 1.96249, 1.96625, 1.96981, 1.97317, 1.97634, 1.97931, 1.98208, 1.98465,  
1.98702, 1.9892, 1.99117, 1.99295, 1.99453, 1.99591, 1.99709, 1.99807, 1.99885,  
1.99942, 1.9998, 1.99998, 1.99996, 1.99974, 1.99932, 1.9987, 1.99788, 1.99686,  
1.99564, 1.99422, 1.99261, 1.99079, 1.98877, 1.98655, 1.98414, 1.98153, 1.97872}
```

In[150]:=

```
h = 0.01
```

Out[150]=

```
0.01
```

In[151]:=

```
n = Length[xt]
```

Out[151]=

```
101
```

Формули с точност $O(h)$ - първи порядък

Първа производна

In[152]:=

```
yp1 = Table[ $\frac{yt[[i + 1]] - yt[[i]]}{h}$ , {i, 1, n - 1}]
```

Out[152]=

```
{1.50121, 1.48792, 1.47448, 1.4609, 1.44716, 1.43329, 1.41927, 1.4051, 1.3908, 1.37636,
1.36178, 1.34707, 1.33222, 1.31723, 1.30212, 1.28687, 1.2715, 1.256, 1.24037, 1.22462,
1.20875, 1.19275, 1.17664, 1.16041, 1.14406, 1.1276, 1.11102, 1.09434, 1.07754,
1.06064, 1.04363, 1.02652, 1.0093, 0.991988, 0.974572, 0.957059, 0.93945, 0.921747,
0.903952, 0.886067, 0.868093, 0.850032, 0.831886, 0.813657, 0.795347, 0.776957,
0.758489, 0.739946, 0.721329, 0.702639, 0.683879, 0.665051, 0.646156, 0.627197,
0.608175, 0.589092, 0.56995, 0.550751, 0.531497, 0.51219, 0.492832, 0.473425, 0.45397,
0.43447, 0.414926, 0.395341, 0.375716, 0.356054, 0.336356, 0.316624, 0.296861,
0.277068, 0.257248, 0.237401, 0.217531, 0.197639, 0.177728, 0.157798, 0.137853,
0.117894, 0.0979237, 0.0779432, 0.0579549, 0.0379608, 0.017963, -0.00203672,
-0.0220362, -0.0420335, -0.0620265, -0.0820134, -0.101992, -0.121961, -0.141917,
-0.161859, -0.181785, -0.201692, -0.22158, -0.241445, -0.261287, -0.281102}
```

In[153]:=

```
AppendTo[yp1,  $\frac{yt[[n]] - yt[[n - 1]]}{h}$ ]
```

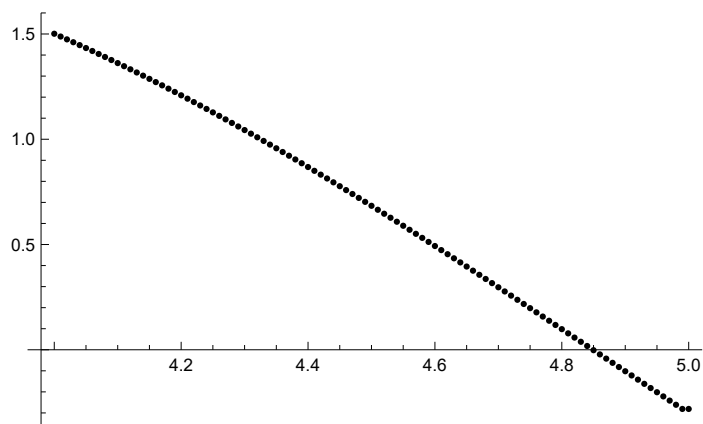
Out[153]=

```
{1.50121, 1.48792, 1.47448, 1.4609, 1.44716, 1.43329, 1.41927, 1.4051, 1.3908, 1.37636,
1.36178, 1.34707, 1.33222, 1.31723, 1.30212, 1.28687, 1.2715, 1.256, 1.24037, 1.22462,
1.20875, 1.19275, 1.17664, 1.16041, 1.14406, 1.1276, 1.11102, 1.09434, 1.07754, 1.06064,
1.04363, 1.02652, 1.0093, 0.991988, 0.974572, 0.957059, 0.93945, 0.921747, 0.903952,
0.886067, 0.868093, 0.850032, 0.831886, 0.813657, 0.795347, 0.776957, 0.758489,
0.739946, 0.721329, 0.702639, 0.683879, 0.665051, 0.646156, 0.627197, 0.608175,
0.589092, 0.56995, 0.550751, 0.531497, 0.51219, 0.492832, 0.473425, 0.45397, 0.43447,
0.414926, 0.395341, 0.375716, 0.356054, 0.336356, 0.316624, 0.296861, 0.277068,
0.257248, 0.237401, 0.217531, 0.197639, 0.177728, 0.157798, 0.137853, 0.117894,
0.0979237, 0.0779432, 0.0579549, 0.0379608, 0.017963, -0.00203672, -0.0220362,
-0.0420335, -0.0620265, -0.0820134, -0.101992, -0.121961, -0.141917, -0.161859,
-0.181785, -0.201692, -0.22158, -0.241445, -0.261287, -0.281102, -0.281102}
```

In[154]:=

```
pointsyp1 = Table[{xt[[i]], yp1[[i]]}, {i, 1, n}];
gryp1 = ListPlot[pointsyp1, PlotStyle -> Black]
```

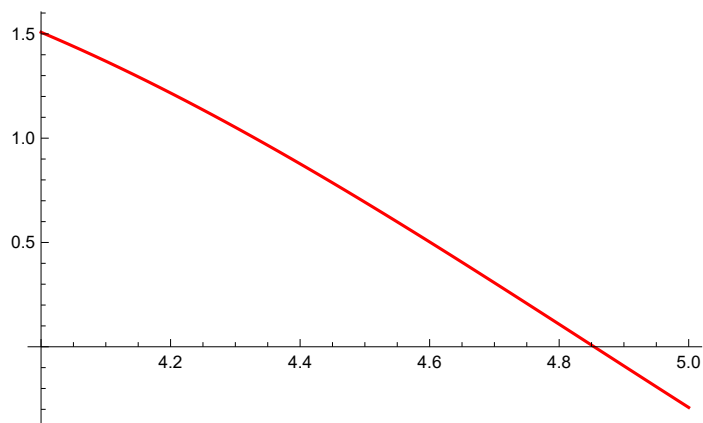
Out[155]=



In[156]:=

```
grfyp = Plot[f'[x], {x, xt[[1]], xt[[n]]}, PlotStyle -> Red]
```

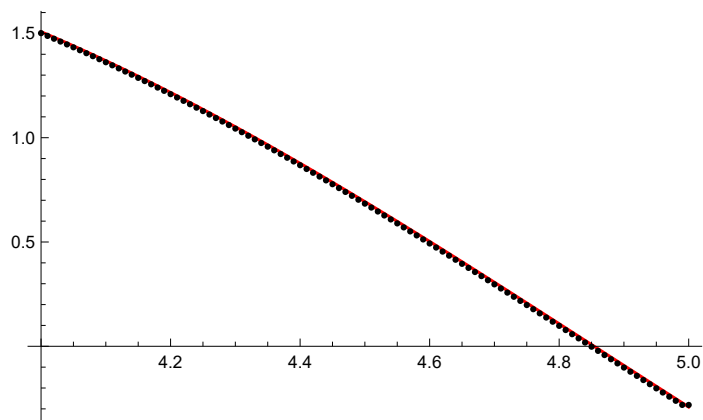
Out[156]=



In[157]:=

```
Show[grfyp, gryp1]
```

Out[157]=



Формули с точност $\alpha(h^2)$ - втори порядък

Първа производна

попълваме средните точки

In[158]:=

```
yp2 = Table[ $\frac{yt[[i + 1]] - yt[[i - 1]]}{2 h}$ , {i, 2, n - 1}]
```

Out[158]=

```
{1.49456, 1.4812, 1.46769, 1.45403, 1.44022, 1.42628, 1.41219, 1.39795, 1.38358, 1.36907,
1.35442, 1.33964, 1.32472, 1.30967, 1.29449, 1.27918, 1.26375, 1.24818, 1.23249,
1.21668, 1.20075, 1.18469, 1.16852, 1.15223, 1.13583, 1.11931, 1.10268, 1.08594,
1.06909, 1.05214, 1.03508, 1.01791, 1.00065, 0.98328, 0.965816, 0.948255, 0.930599,
0.91285, 0.89501, 0.87708, 0.859063, 0.840959, 0.822772, 0.804502, 0.786152,
0.767723, 0.749218, 0.730637, 0.711984, 0.693259, 0.674465, 0.655604, 0.636677,
0.617686, 0.598633, 0.579521, 0.560351, 0.541124, 0.521844, 0.502511, 0.483128,
0.463697, 0.44422, 0.424698, 0.405133, 0.385528, 0.365885, 0.346205, 0.32649,
0.306743, 0.286965, 0.267158, 0.247324, 0.227466, 0.207585, 0.187684, 0.167763,
0.147826, 0.127874, 0.107909, 0.0879334, 0.0679491, 0.0479579, 0.0279619, 0.00796311,
-0.0120365, -0.0320348, -0.05203, -0.07202, -0.0920027, -0.111976, -0.131939,
-0.151888, -0.171822, -0.191739, -0.211636, -0.231513, -0.251366, -0.271194}
```

допълваме производната в десния край (последната)

In[159]:=

```
AppendTo[yp2,  $\frac{yt[[n - 2]] - 4 yt[[n - 1]] + 3 yt[[n]]}{2 h}$ ]
```

Out[159]=

```
{1.49456, 1.4812, 1.46769, 1.45403, 1.44022, 1.42628, 1.41219, 1.39795, 1.38358, 1.36907,
1.35442, 1.33964, 1.32472, 1.30967, 1.29449, 1.27918, 1.26375, 1.24818, 1.23249,
1.21668, 1.20075, 1.18469, 1.16852, 1.15223, 1.13583, 1.11931, 1.10268, 1.08594,
1.06909, 1.05214, 1.03508, 1.01791, 1.00065, 0.98328, 0.965816, 0.948255, 0.930599,
0.91285, 0.89501, 0.87708, 0.859063, 0.840959, 0.822772, 0.804502, 0.786152, 0.767723,
0.749218, 0.730637, 0.711984, 0.693259, 0.674465, 0.655604, 0.636677, 0.617686,
0.598633, 0.579521, 0.560351, 0.541124, 0.521844, 0.502511, 0.483128, 0.463697,
0.44422, 0.424698, 0.405133, 0.385528, 0.365885, 0.346205, 0.32649, 0.306743, 0.286965,
0.267158, 0.247324, 0.227466, 0.207585, 0.187684, 0.167763, 0.147826, 0.127874,
0.107909, 0.0879334, 0.0679491, 0.0479579, 0.0279619, 0.00796311, -0.0120365,
-0.0320348, -0.05203, -0.07202, -0.0920027, -0.111976, -0.131939, -0.151888,
-0.171822, -0.191739, -0.211636, -0.231513, -0.251366, -0.271194, -0.291009}
```

допълваме производната в левия край (първата)

In[160]:=

$$\text{PrependTo}[\text{yp2}, \frac{-3 \text{yt}[[1]] + 4 \text{yt}[[2]] - \text{yt}[[3]]}{2 h}]$$

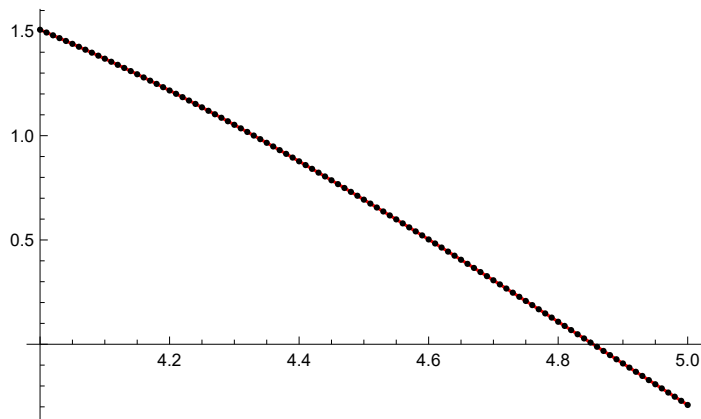
Out[160]=

```
{1.50785, 1.49456, 1.4812, 1.46769, 1.45403, 1.44022, 1.42628, 1.41219,
 1.39795, 1.38358, 1.36907, 1.35442, 1.33964, 1.32472, 1.30967, 1.29449,
 1.27918, 1.26375, 1.24818, 1.23249, 1.21668, 1.20075, 1.18469, 1.16852,
 1.15223, 1.13583, 1.11931, 1.10268, 1.08594, 1.06909, 1.05214, 1.03508,
 1.01791, 1.00065, 0.98328, 0.965816, 0.948255, 0.930599, 0.91285, 0.89501,
 0.87708, 0.859063, 0.840959, 0.822772, 0.804502, 0.786152, 0.767723, 0.749218,
 0.730637, 0.711984, 0.693259, 0.674465, 0.655604, 0.636677, 0.617686, 0.598633,
 0.579521, 0.560351, 0.541124, 0.521844, 0.502511, 0.483128, 0.463697, 0.44422,
 0.424698, 0.405133, 0.385528, 0.365885, 0.346205, 0.32649, 0.306743, 0.286965,
 0.267158, 0.247324, 0.227466, 0.207585, 0.187684, 0.167763, 0.147826, 0.127874,
 0.107909, 0.0879334, 0.0679491, 0.0479579, 0.0279619, 0.00796311, -0.0120365,
 -0.0320348, -0.05203, -0.07202, -0.0920027, -0.111976, -0.131939, -0.151888,
 -0.171822, -0.191739, -0.211636, -0.231513, -0.251366, -0.271194, -0.291009}
```

In[161]:=

```
pointsyp2 = Table[{xt[[i]], yp2[[i]]}, {i, 1, n}];
gryp2 = ListPlot[pointsyp2, PlotStyle -> Black];
grfyp = Plot[f'[x], {x, xt[[1]], xt[[n]]}, PlotStyle -> Red];
Show[grfyp, gryp2]
```

Out[164]=



Втора производна

попълваме средните точки

In[165]:=

$$\text{ypp2} = \text{Table}\left[\frac{\text{yt}[[i + 1]] - 2 \text{yt}[[i]] + \text{yt}[[i - 1]]}{h^2}, \{i, 2, n - 1\}\right]$$

Out[165]=

```
{-1.32897, -1.34385, -1.3586, -1.37321, -1.38768, -1.40201, -1.4162, -1.43025,
-1.44416, -1.45793, -1.47154, -1.48501, -1.49834, -1.51151, -1.52453, -1.5374,
-1.55011, -1.56267, -1.57508, -1.58732, -1.59941, -1.61134, -1.6231, -1.63471,
-1.64615, -1.65742, -1.66853, -1.67948, -1.69025, -1.70086, -1.7113, -1.72156,
-1.73165, -1.74157, -1.75132, -1.76089, -1.77028, -1.7795, -1.78854, -1.7974,
-1.80608, -1.81458, -1.8229, -1.83104, -1.83899, -1.84676, -1.85435, -1.86174,
-1.86896, -1.87598, -1.88282, -1.88947, -1.89594, -1.90221, -1.90829, -1.91418,
-1.91988, -1.92539, -1.9307, -1.93582, -1.94075, -1.94549, -1.95003, -1.95437,
-1.95852, -1.96247, -1.96623, -1.96979, -1.97315, -1.97632, -1.97929,
-1.98206, -1.98463, -1.98701, -1.98918, -1.99116, -1.99293, -1.99451,
-1.99589, -1.99707, -1.99805, -1.99883, -1.99941, -1.99979, -1.99997,
-1.99995, -1.99973, -1.99931, -1.99869, -1.99787, -1.99685, -1.99563,
-1.99421, -1.99259, -1.99077, -1.98875, -1.98654, -1.98412, -1.98151}
```

In[166]:=

f''[xt]

Out[166]=

```
{-1.31397, -1.32899, -1.34386, -1.35861, -1.37322, -1.38769, -1.40202, -1.41622,
-1.43027, -1.44417, -1.45794, -1.47156, -1.48503, -1.49835, -1.51152, -1.52454,
-1.53741, -1.55013, -1.56269, -1.57509, -1.58734, -1.59942, -1.61135,
-1.62312, -1.63472, -1.64616, -1.65744, -1.66855, -1.67949, -1.69027,
-1.70087, -1.71131, -1.72157, -1.73167, -1.74159, -1.75133, -1.7609, -1.7703,
-1.77952, -1.78856, -1.79742, -1.8061, -1.8146, -1.82292, -1.83105, -1.83901,
-1.84678, -1.85436, -1.86176, -1.86897, -1.876, -1.88284, -1.88949, -1.89595,
-1.90222, -1.9083, -1.9142, -1.91989, -1.9254, -1.93072, -1.93584, -1.94077,
-1.9455, -1.95004, -1.95439, -1.95854, -1.96249, -1.96625, -1.96981, -1.97317,
-1.97634, -1.97931, -1.98208, -1.98465, -1.98702, -1.9892, -1.99117, -1.99295,
-1.99453, -1.99591, -1.99709, -1.99807, -1.99885, -1.99942, -1.9998, -1.99998,
-1.99996, -1.99974, -1.99932, -1.9987, -1.99788, -1.99686, -1.99564, -1.99422,
-1.99261, -1.99079, -1.98877, -1.98655, -1.98414, -1.98153, -1.97872}
```

In[167]:=

```

pointsypp2 = Table[{xt[[i + 1]], ypp2[[i]]}, {i, 1, n - 2}];
grypp2 = ListPlot[pointsypp2, PlotStyle -> Black];
grfypp = Plot[f''[x], {x, xt[[2]], xt[[n - 1]]}, PlotStyle -> Red];
Show[grfypp, grypp2]

```

Out[170]=

