Метод на най-малките квадрати (МНМК)

Генериране на данни

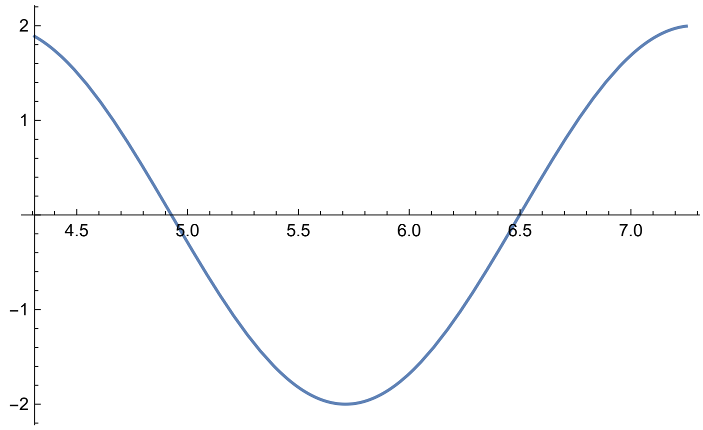
In[17]:=
```
xt = Table[5 + t * 0.13, {t, -3, 15}]
```
Out[17]=
```
{4.61, 4.74, 4.87, 5., 5.13, 5.26, 5.39, 5.52, 5.65,
 5.78, 5.91, 6.04, 6.17, 6.3, 6.43, 6.56, 6.69, 6.82, 6.95}
```

In[18]:=
```
f[x_] := 2 Cos[2 x - 2]
yt = f[xt]
```
Out[19]=
```
{1.18471, 0.730649, 0.22747, -0.291, -0.789909, -1.23572,
 -1.59847, -1.85376, -1.98445, -1.98174, -1.84582, -1.58583,
 -1.21923, -0.770676, -0.270319, 0.24821, 0.750054, 1.20148, 1.57214}
```

In[20]:=
```
P = Length[xt]
```
Out[20]=
```
19
```

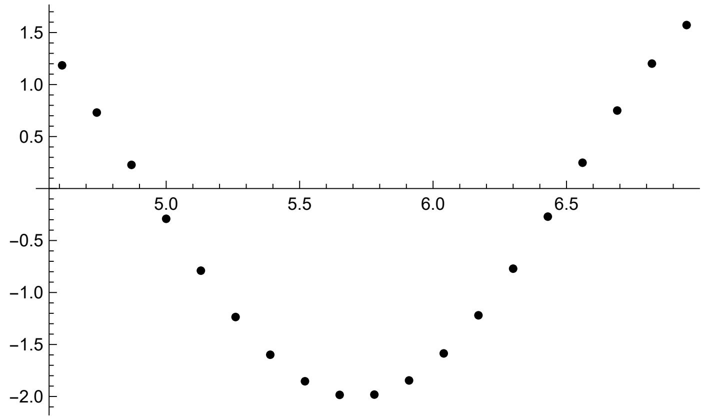
Визуализация

In[21]:=
```
grf = Plot[f[x], {x, xt[[1]] - 0.3, xt[[P]] + 0.3}]
```
Out[21]=



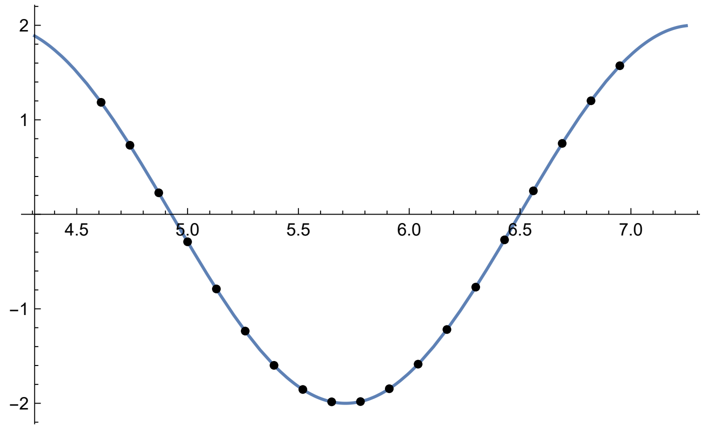
In[22]:=
```
points = Table[{xt⟦i⟧, yt⟦i⟧}, {i, 1, P}];
grp = ListPlot[points, PlotStyle → Black]
```

Out[23]=



In[24]:=
```
Show[grf, grp]
```

Out[24]=



Линейна регресия

Попълваме таблицата

In[25]:= xt^2

Out[25]=

```
{21.2521, 22.4676, 23.7169, 25., 26.3169, 27.6676, 29.0521, 30.4704, 31.9225, 33.4084,
 34.9281, 36.4816, 38.0689, 39.69, 41.3449, 43.0336, 44.7561, 46.5124, 48.3025}
```

In[26]:=
```
yt * xt
```

Out[26]=

```
{5.46153, 3.46328, 1.10778, -1.455, -4.05223, -6.49989,
 -8.61573, -10.2328, -11.2121, -11.4545, -10.9088, -9.57838,
 -7.52264, -4.85526, -1.73815, 1.62826, 5.01786, 8.19409, 10.9264}
```

Намиране на сумите

In[27]:= $\sum_{i=1}^{p} xt[[i]]$

Out[27]= 109.82

In[28]:= $\sum_{i=1}^{p} yt[[i]]$

Out[28]= -9.51221

In[29]:= $\sum_{i=1}^{p} xt[[i]]^2$

Out[29]= 644.393

In[30]:= $\sum_{i=1}^{p} yt[[i]] * xt[[i]]$

Out[30]= -52.3263

Решаваме системата

In[31]:= $A = \begin{pmatrix} 19 & 109.82 \\ 109.82 & 644.393 \end{pmatrix}$; b = {-9.512, -52.326};

LinearSolve[A, b]

Out[32]= {-2.09264, 0.275433}

Съставяме полинома

In[33]:= P1[x_] := -2.093 + 0.275 x

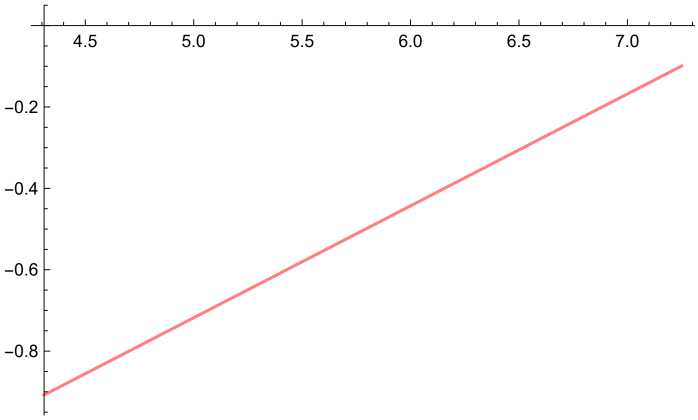
таен коз (възможност за самопроверка)

In[40]:= Fit[points, {1, x}, x]

Out[40]= -2.09324 + 0.275535 x

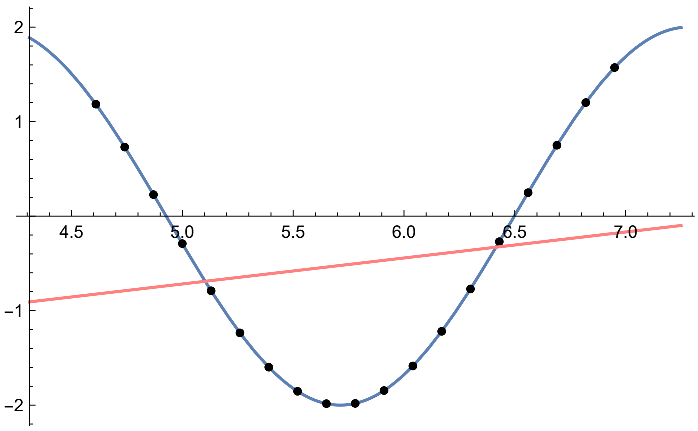
In[34]:=
```
grP1 = Plot[P1[x], {x, xt[[1]] - 0.3, xt[[P]] + 0.3}, PlotStyle → Pink]
```

Out[34]=



In[35]:=
```
Show[grf, grp, grP1]
```

Out[35]=



Намиране на приближена стойност (апроксимация)

In[36]:=
```
P1[4]
```

Out[36]=
```
-0.993
```

за сравнение истинската стойност

In[37]:=
```
f[4.]
```

Out[37]=
```
1.92034
```

Оценка на грешката

Теоретична грешка (средноквадратична)

In[38]:=

$$\sqrt{\sum_{i=1}^{P} (yt[[i]] - P1[xt[[i]]])^2}$$

Out[38]=
```
5.02027
```

Истинска грешка

In[39]:= `Abs[f[4.] - P1[4]]`

Out[39]=

2.91334

Квадратична регресия

Попълваме таблицата

In[ ]:= xt^2

Out[ ]=

{21.2521, 22.4676, 23.7169, 25., 26.3169, 27.6676, 29.0521, 30.4704, 31.9225, 33.4084, 34.9281, 36.4816, 38.0689, 39.69, 41.3449, 43.0336, 44.7561, 46.5124, 48.3025}

In[ ]:= `yt * xt`

Out[ ]=

{5.46153, 3.46328, 1.10778, -1.455, -4.05223, -6.49989, -8.61573, -10.2328, -11.2121, -11.4545, -10.9088, -9.57838, -7.52264, -4.85526, -1.73815, 1.62826, 5.01786, 8.19409, 10.9264}

In[41]:= xt^3

Out[41]=

{97.9722, 106.496, 115.501, 125., 135.006, 145.532, 156.591, 168.197, 180.362, 193.101, 206.425, 220.349, 234.885, 250.047, 265.848, 282.3, 299.418, 317.215, 335.702}

In[42]:= xt^4

Out[42]=

{451.652, 504.793, 562.491, 625., 692.579, 765.496, 844.025, 928.445, 1019.05, 1116.12, 1219.97, 1330.91, 1449.24, 1575.3, 1709.4, 1851.89, 2003.11, 2163.4, 2333.13}

In[43]:= $yt * xt^2$

Out[43]=

{25.1777, 16.4159, 5.39488, -7.275, -20.788, -34.1894, -46.4388, -56.4849, -63.3486, -66.2069, -64.4711, -57.8534, -46.4147, -30.5881, -11.1763, 10.6814, 33.5695, 55.8837, 75.9383}

Намиране на сумите

In[ ]:= $\sum_{i=1}^{P} xt[[i]]$

Out[ ]=

109.82

In[ ]:= $\sum_{i=1}^{P} yt[[i]]$

Out[ ]=

-9.51221

In[∘]:= $\sum_{i=1}^{P} xt[[i]]^2$

Out[∘]= 644.393

In[∘]:= $\sum_{i=1}^{P} yt[[i]] * xt[[i]]$

Out[∘]= -52.3263

In[44]:= $\sum_{i=1}^{P} xt[[i]]^3$

Out[44]= 3835.95

In[45]:= $\sum_{i=1}^{P} xt[[i]]^4$

Out[45]= 23146.

In[46]:= $\sum_{i=1}^{P} yt[[i]] * xt[[i]]^2$

Out[46]= -282.174

Решаваме системата

In[47]:= $A = \begin{pmatrix} 19 & 109.82 & 644.393 \\ 109.82 & 644.393 & 3835.95 \\ 644.393 & 3835.95 & 23146 \end{pmatrix}$; b = {-9.512, -52.326, -282.174};

LinearSolve[A, b]

Out[48]= {80.6781, -28.8073, 2.51589}

записваме в общ вид

In[61]:= $A = \begin{pmatrix} P & \sum_{i=1}^{P} xt[[i]] & \sum_{i=1}^{P} xt[[i]]^2 \\ \sum_{i=1}^{P} xt[[i]] & \sum_{i=1}^{P} xt[[i]]^2 & \sum_{i=1}^{P} xt[[i]]^3 \\ \sum_{i=1}^{P} xt[[i]]^2 & \sum_{i=1}^{P} xt[[i]]^3 & \sum_{i=1}^{P} xt[[i]]^4 \end{pmatrix}$;

$b = \left\{\sum_{i=1}^{P} yt[[i]], \sum_{i=1}^{P} yt[[i]] * xt[[i]], \sum_{i=1}^{P} yt[[i]] * xt[[i]]^2\right\}$;

a = LinearSolve[A, b]

Out[62]= {80.7295, -28.8245, 2.5173}

Съставяме полинома

In[50]:= $P2[x\_] := 80.678 + -28.807\, x + 2.516\, x^2$

In[65]:=
```
P2[x_] := a[[1]] + a[[2]] x + a[[3]] x^2
P2[x]
```
Out[66]=

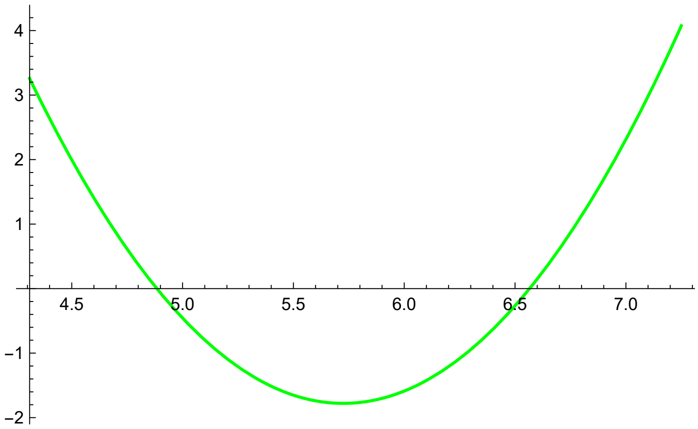
$80.7295 - 28.8245\, x + 2.5173\, x^2$

таен коз (възможност за самопроверка)

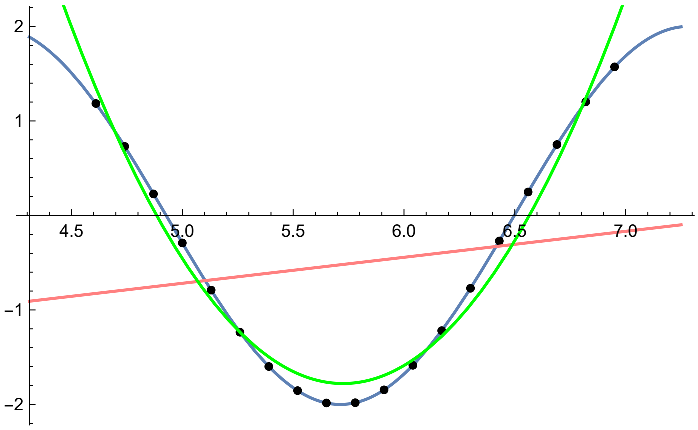
In[49]:=
```
Fit[points, {1, x, x^2}, x]
```
Out[49]=

$80.7295 - 28.8245\, x + 2.5173\, x^2$

In[51]:=
```
grP2 = Plot[P2[x], {x, xt[[1]] - 0.3, xt[[P]] + 0.3}, PlotStyle → Green]
```
Out[51]=



In[52]:=
```
Show[grf, grp, grP1, grP2]
```
Out[52]=



Намиране на приближена стойност (апроксимация)

стойност извън разглеждания интервал

In[53]:=
```
P2[4]
```
Out[53]=

5.706

за сравнение истинската стойност

In[54]:= `f[4.]`

Out[54]= 1.92034

стойност вътре в разглеждания интервал

In[55]:= `P2[5.3]`

Out[55]= -1.32466

за сравнение истинската стойност

In[56]:= `f[5.3]`

Out[56]= -1.35744

Оценка на грешката

Теоретична грешка (средноквадратична)

In[57]:= $\sqrt{\sum_{i=1}^{p} (yt[[i]] - P2[xt[[i]]])^2}$

Out[57]= 0.806867

Истинска грешка

In[59]:= `Abs[f[4.] - P2[4]]`

Out[59]= 3.78566

In[60]:= `Abs[f[5.3] - P2[5.3]]`

Out[60]= 0.0327801

Кубична регресия

Попълваме таблицата

In[ ]:= xt^2

Out[ ]= {21.2521, 22.4676, 23.7169, 25., 26.3169, 27.6676, 29.0521, 30.4704, 31.9225, 33.4084, 34.9281, 36.4816, 38.0689, 39.69, 41.3449, 43.0336, 44.7561, 46.5124, 48.3025}

In[]:= yt * xt

Out[]=

{5.46153, 3.46328, 1.10778, -1.455, -4.05223, -6.49989, -8.61573, -10.2328, -11.2121, -11.4545, -10.9088, -9.57838, -7.52264, -4.85526, -1.73815, 1.62826, 5.01786, 8.19409, 10.9264}

In[]:= xt^3

Out[]=

{97.9722, 106.496, 115.501, 125., 135.006, 145.532, 156.591, 168.197, 180.362, 193.101, 206.425, 220.349, 234.885, 250.047, 265.848, 282.3, 299.418, 317.215, 335.702}

In[]:= xt^4

Out[]=

{451.652, 504.793, 562.491, 625., 692.579, 765.496, 844.025, 928.445, 1019.05, 1116.12, 1219.97, 1330.91, 1449.24, 1575.3, 1709.4, 1851.89, 2003.11, 2163.4, 2333.13}

In[]:= $yt * xt^2$

Out[]=

{25.1777, 16.4159, 5.39488, -7.275, -20.788, -34.1894, -46.4388, -56.4849, -63.3486, -66.2069, -64.4711, -57.8534, -46.4147, -30.5881, -11.1763, 10.6814, 33.5695, 55.8837, 75.9383}

допълваме необходимото

Намиране на сумите

In[]:= $\sum_{i=1}^{P} xt[[i]]$

Out[]=

109.82

In[]:= $\sum_{i=1}^{P} yt[[i]]$

Out[]=

-9.51221

In[]:= $\sum_{i=1}^{P} xt[[i]]^2$

Out[]=

644.393

In[]:= $\sum_{i=1}^{P} yt[[i]] * xt[[i]]$

Out[]=

-52.3263

In[]:= $\sum_{i=1}^{P} xt[[i]]^3$

Out[]=

3835.95

In[∘]:=
$$\sum_{i=1}^{P} xt[[i]]^4$$
Out[∘]=
23146.

In[∘]:=
$$\sum_{i=1}^{P} yt[[i]] * xt[[i]]^2$$
Out[∘]=
-282.174

допълваме необходимото

Решаваме системата

записваме в общ вид

In[67]:=
$$A = \begin{pmatrix} P & \sum_{i=1}^{P} xt[[i]] & \sum_{i=1}^{P} xt[[i]]^2 & \sum_{i=1}^{P} xt[[i]]^3 \\ \sum_{i=1}^{P} xt[[i]] & \sum_{i=1}^{P} xt[[i]]^2 & \sum_{i=1}^{P} xt[[i]]^3 & \sum_{i=1}^{P} xt[[i]]^4 \\ \sum_{i=1}^{P} xt[[i]]^2 & \sum_{i=1}^{P} xt[[i]]^3 & \sum_{i=1}^{P} xt[[i]]^4 & \sum_{i=1}^{P} xt[[i]]^5 \\ \sum_{i=1}^{P} xt[[i]]^3 & \sum_{i=1}^{P} xt[[i]]^4 & \sum_{i=1}^{P} xt[[i]]^5 & \sum_{i=1}^{P} xt[[i]]^6 \end{pmatrix};$$

$$b = \left\{\sum_{i=1}^{P} yt[[i]], \sum_{i=1}^{P} yt[[i]] * xt[[i]], \sum_{i=1}^{P} yt[[i]] * xt[[i]]^2, \sum_{i=1}^{P} yt[[i]] * xt[[i]]^3\right\};$$

a = LinearSolve[A, b]

LinearSolve: Result for LinearSolve of badly conditioned matrix {{19., 109.82, 644.393, 3835.95}, {109.82, 644.393, 3835.95, 23146.}, {≪1≫}, {3835.95, 23146., 141427., 874141.}} may contain significant numerical errors.

Out[68]=
{128.634, -54.1524, 6.93941, -0.255023}

Съставяме полинома

In[70]:=
$$P3[x\_] := a[[1]] + a[[2]]\, x + a[[3]]\, x^2 + a[[4]]\, x^3$$
P3[x]

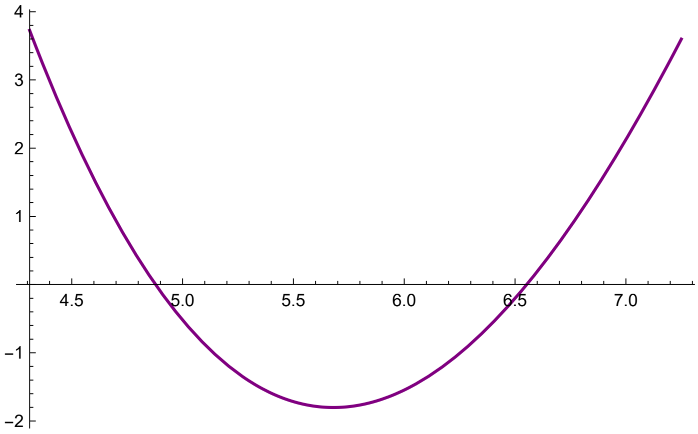
Out[71]=
$$128.634 - 54.1524\, x + 6.93941\, x^2 - 0.255023\, x^3$$

таен коз (възможност за самопроверка)

In[69]:=
$$\mathrm{Fit}\left[points, \{1, x, x^2, x^3\}, x\right]$$
Out[69]=
$$128.634 - 54.1524\, x + 6.93941\, x^2 - 0.255023\, x^3$$

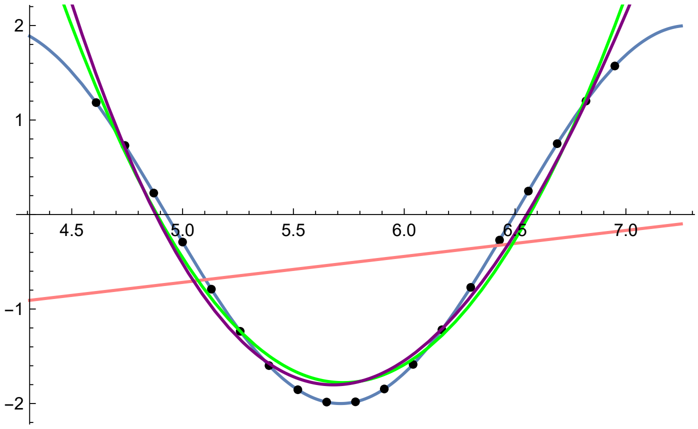
In[72]:= `grP3 = Plot[P3[x], {x, xt[[1]] - 0.3, xt[[P]] + 0.3}, PlotStyle → Purple]`

Out[72]=



In[73]:= `Show[grf, grp, grP1, grP2, grP3]`

Out[73]=



Намиране на приближена стойност (апроксимация)

стойност извън разглеждания интервал

In[74]:= `P3[4]`

Out[74]=

6.73399

за сравнение истинската стойност

In[ ]:= `f[4.]`

Out[ ]=

1.92034

стойност вътре в разглеждания интервал

In[75]:= `P3[5.3]`

Out[75]=

-1.41228

за сравнение истинската стойност

In[◦]:= f[5.3]

Out[◦]=

−1.35744

Оценка на грешката

Теоретична грешка (средноквадратична)

In[76]:= $\sqrt{\sum_{i=1}^{P} (yt[[i]] - P3[xt[[i]]])^2}$

Out[76]=

0.744355

Истинска грешка

In[77]:= Abs[f[4.] - P3[4]]

Out[77]=

4.81364

In[78]:= Abs[f[5.3] - P3[5.3]]

Out[78]=

0.0548399