Интерполационен полином на Лагранж

Генериране на данни

In[1]:=
```
xt = Table[7 + i * 0.17, {i, -7, 7}]
```

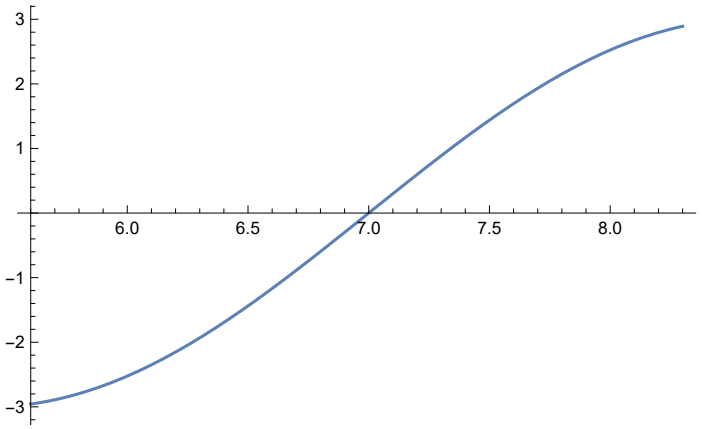
Out[1]=
```
{5.81, 5.98, 6.15, 6.32, 6.49, 6.66, 6.83, 7., 7.17, 7.34, 7.51, 7.68, 7.85, 8.02, 8.19}
```

In[6]:=
```
f[x_] := 3 Sin[x - 7]
yt = f[xt]
```

Out[7]=
```
{-2.78511, -2.55632, -2.25384, -1.88638, -1.46453, -1.00046, -0.507547,
 0., 0.507547, 1.00046, 1.46453, 1.88638, 2.25384, 2.55632, 2.78511}
```

In[5]:=
```
grf = Plot[f[x], {x, 5.6, 8.3}]
```

Out[5]=



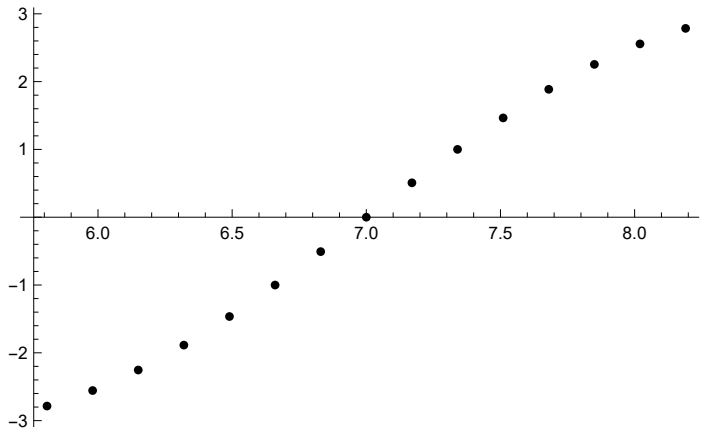
In[8]:=
```
n = Length[xt]
points = Table[{xt[[i]], yt[[i]]}, {i, 1, n}]
```

Out[8]=
```
15
```

Out[9]=
```
{{5.81, -2.78511}, {5.98, -2.55632}, {6.15, -2.25384},
 {6.32, -1.88638}, {6.49, -1.46453}, {6.66, -1.00046}, {6.83, -0.507547},
 {7., 0.}, {7.17, 0.507547}, {7.34, 1.00046}, {7.51, 1.46453},
 {7.68, 1.88638}, {7.85, 2.25384}, {8.02, 2.55632}, {8.19, 2.78511}}
```

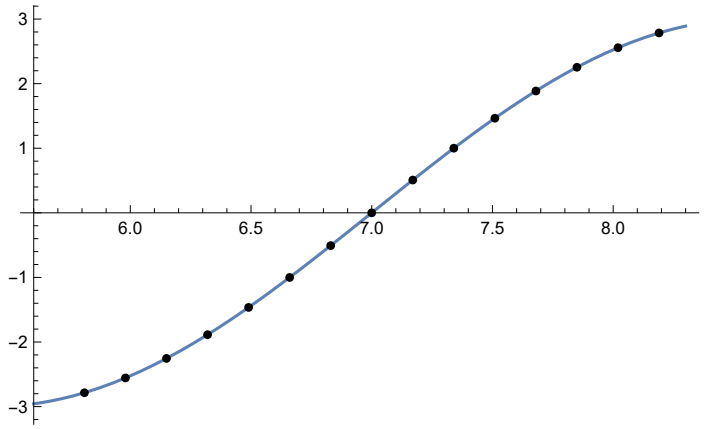
In[11]:=
```
grp = ListPlot[points, PlotStyle → Black]
```

Out[11]=



In[12]:=
```
Show[grf, grp]
```

Out[12]=



Линейна интерполация

In[13]:=
```
L1[x_] := -2.2538 * (x - 6.32)/(6.15 - 6.32) - 1.886 * (x - 6.15)/(6.32 - 6.15)
```

In[14]:=
```
Expand[L1[x]]
```

Out[14]=

$$-15.5595 + 2.16353\,x$$

Проверка на интерполационните условия

In[15]:=
```
L1[6.15]
L1[6.32]
```

Out[15]=

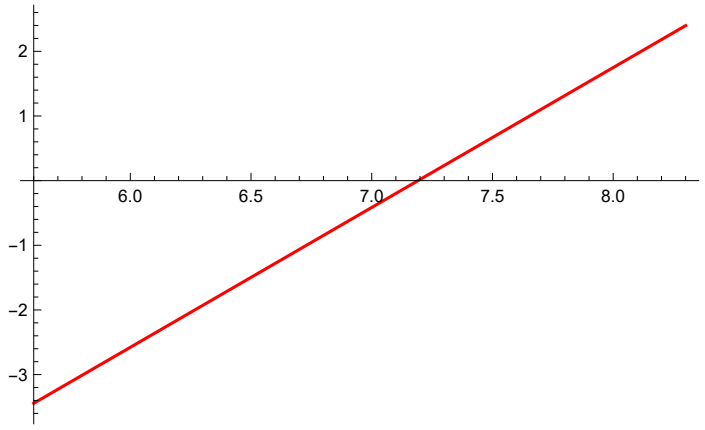
$$-2.2538$$

Out[16]=

$$-1.886$$

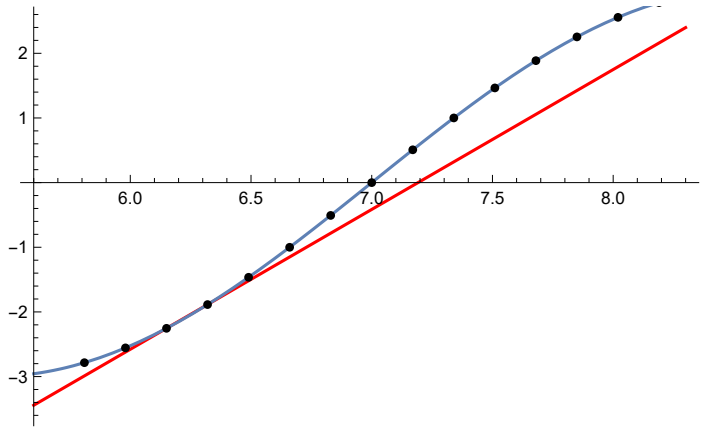
In[17]:=
```
grL1 = Plot[L1[x], {x, 5.6, 8.3}, PlotStyle → Red]
```

Out[17]=



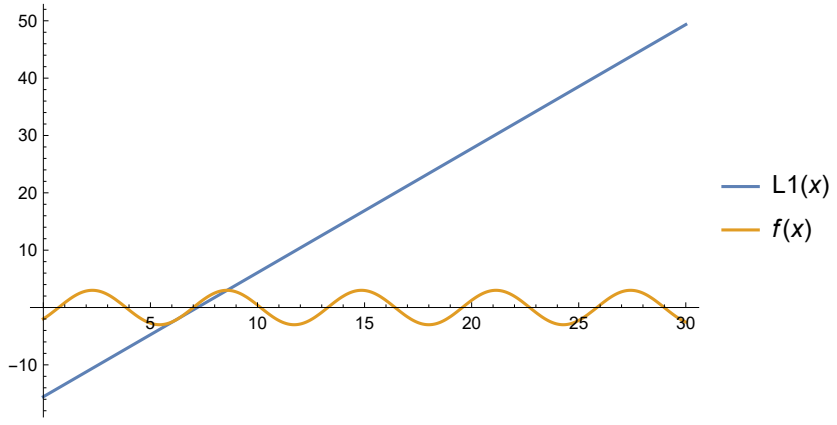
In[18]:=
```
Show[grL1, grf, grp]
```

Out[18]=



In[19]:=
```
Plot[{L1[x], f[x]}, {x, 0, 30}, PlotLegends → "Expressions"]
```

Out[19]=



Пресмятане на приближена стойност

In[20]:=
```
L1[6.18]
```

Out[20]=

−2.18889

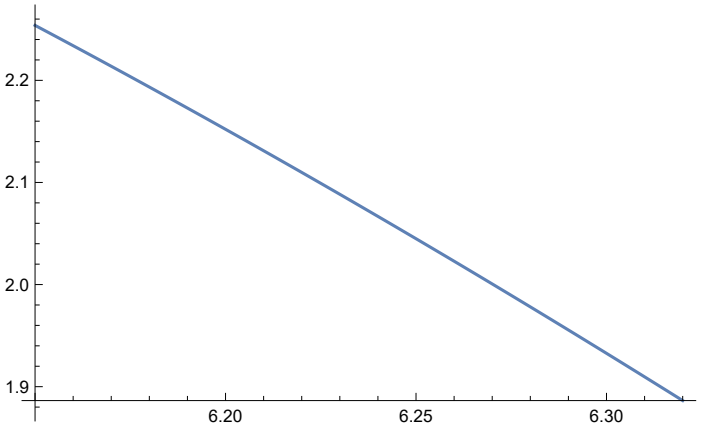
за сравнение с истинската стойност

In[21]:=
```
f[6.18]
```
Out[21]=
```
-2.19344
```

Оценка на грешката

Теоретична грешка

In[22]:=
```
Plot[Abs[f''[x]], {x, 6.15, 6.32}]
```
Out[22]=



In[23]:=
```
M2 = Abs[f''[6.15]]
```
Out[23]=
```
2.25384
```

In[24]:=
```
R1[x_] := M2/2! Abs[(x - 6.15) (x - 6.32)]
```

In[25]:=
```
R1[6.18]
```
Out[25]=
```
0.00473307
```

Истинска грешка

In[26]:=
```
Abs[L1[6.18] - f[6.18]]
```
Out[26]=
```
0.00454337
```

Квадратична интерполация

In[27]:=
```
L2[x_] := -2.556 * (x - 6.15) (x - 6.32)/((5.98 - 6.15) (5.98 - 6.32)) -
  2.2538 * (x - 5.98) (x - 6.32)/((6.15 - 5.98) (6.15 - 6.32)) - 1.886 * (x - 5.98) (x - 6.15)/((6.32 - 5.98) (6.32 - 6.15))
```

In[28]:=
```
Expand[L2[x]]
```
Out[28]=
```
28.5537 - 11.9893 x + 1.13495 x^2
```

Проверка на интерполационните условия

In[29]:=
```
L2[5.98]
L2[6.15]
L2[6.32]
```

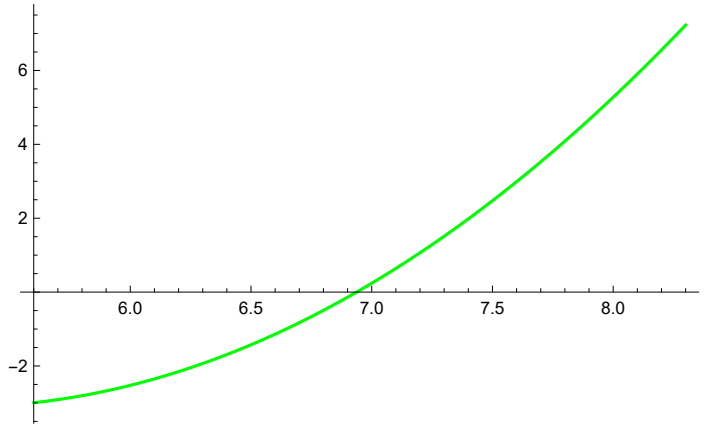
Out[29]=
```
-2.556
```

Out[30]=
```
-2.2538
```

Out[31]=
```
-1.886
```

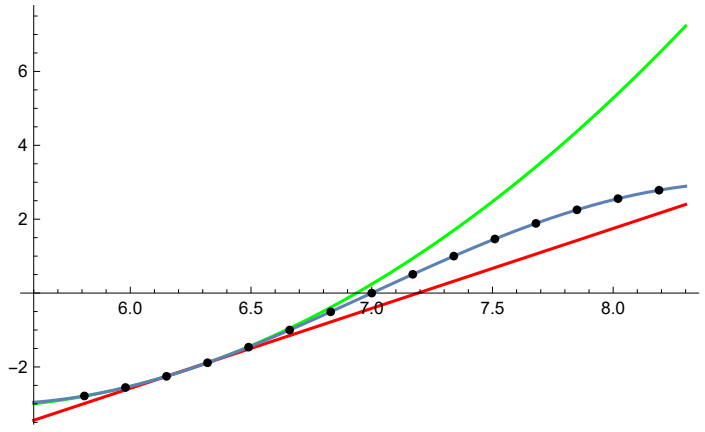
In[32]:=
```
grL2 = Plot[L2[x], {x, 5.6, 8.3}, PlotStyle → Green]
```

Out[32]=



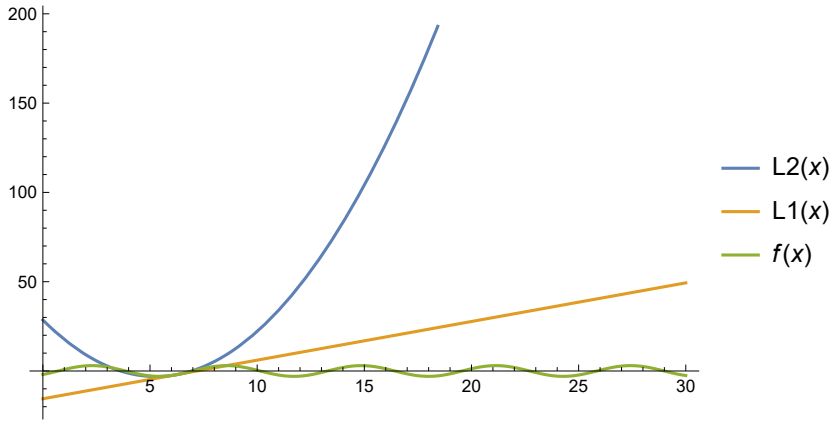
In[33]:=
```
Show[grL2, grL1, grf, grp]
```

Out[33]=



In[34]:=
```
Plot[{L2[x], L1[x], f[x]}, {x, 0, 30}, PlotLegends → "Expressions"]
```

Out[34]=



Пресмятане на приближена стойност

In[35]:=
```
L2[6.18]
```

Out[35]=

−2.19366

за сравнение с истинската стойност

In[ ]:=
```
f[6.18]
```

Out[ ]=

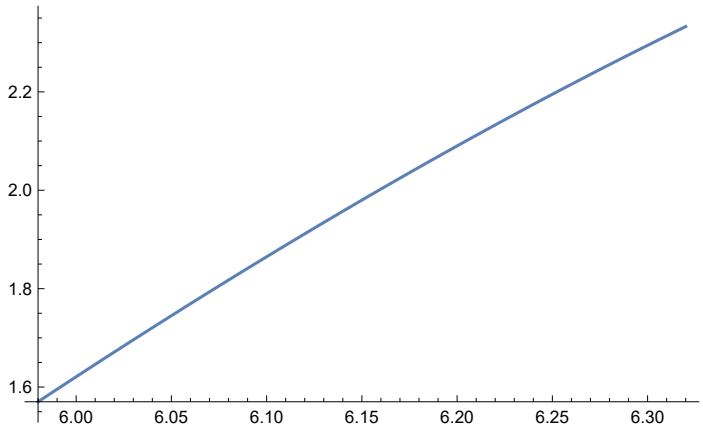
−2.19344

Оценка на грешката

Теоретична грешка

In[37]:=
```
Plot[Abs[f'''[x]], {x, 5.98, 6.32}]
```

Out[37]=



In[38]:=
```
M3 = Abs[f'''[6.32]]
```

Out[38]=

2.33272

In[39]:=
```
R2[x_] := M3/3! Abs[(x - 5.98) (x - 6.15) (x - 6.32)]
```

In[40]:=
```
R2[6.18]
```
Out[40]=
```
0.000326581
```

Истинска грешка

In[41]:=
```
Abs[L2[6.18] - f[6.18]]
```
Out[41]=
```
0.00022341
```

Екстраполация

In[42]:=
```
L1[30]
```
Out[42]=
```
49.3464
```

In[43]:=
```
L2[30]
```
Out[43]=
```
690.329
```

In[44]:=
```
f[30.]
```
Out[44]=
```
-2.53866
```

In[46]:=
```
R1[30]
```
Out[46]=
```
636.449
```

In[47]:=
```
R2[30]
```
Out[47]=
```
5274.17
```