Числено диференциране

Създаваме табличната функция

```
In[47]:= a = 5.; b = 6;
         n = 10;
             b - a
         h = ─────── ;
               n
         xt = Table[a + i * h, {i, 0, n}]
```
```
Out[50]=
         {5., 5.1, 5.2, 5.3, 5.4, 5.5, 5.6, 5.7, 5.8, 5.9, 6.}
```

```
In[51]:= Length[xt]
```
```
Out[51]=
         11
```

```
In[52]:= f[x_] := 4 Sin[π x - 3]
         yt = f[xt]
```
```
Out[53]=
         {0.56448, 1.76055, 2.78429, 3.53548, 3.94059,
          3.95997, 3.59172, 2.87189, 1.87094, 0.686846, -0.56448}
```

Намиране на производните с точност O(h) - първи порядък на грешката

Първа производна

```
In[54]:= z = Table[ (yt[[i + 1]] - yt[[i]]) / h , {i, 1, n}]
```
```
Out[54]=
         {11.9607, 10.2374, 7.5119, 4.05113, 0.193808,
          -3.68249, -7.19831, -10.0095, -11.8409, -12.5133}
```

добавяме последния елемент

```
In[55]:= AppendTo[z, (yt[[n + 1]] - yt[[n]]) / h ]
```
```
Out[55]=
         {11.9607, 10.2374, 7.5119, 4.05113, 0.193808,
          -3.68249, -7.19831, -10.0095, -11.8409, -12.5133, -12.5133}
```

Оценка на грешката

теоретична грешка

In[56]:= **h**

Out[56]=

0.1

истинска грешка

In[57]:= **z (∗приближена стойност∗)**

Out[57]=

{11.9607, 10.2374, 7.5119, 4.05113, 0.193808,
 −3.68249, −7.19831, −10.0095, −11.8409, −12.5133, −12.5133}

In[58]:= **f'[xt] (∗истинска стойност∗)**

Out[58]=

{12.4406, 11.2837, 9.02231, 5.87773, 2.15779,
 −1.77337, −5.53093, −8.74709, −11.107, −12.3797, −12.4406}

In[59]:= **Abs[z − f'[xt]]**

Out[59]=

{0.479909, 1.04637, 1.51041, 1.82659, 1.96398,
 1.90912, 1.66738, 1.26243, 0.733898, 0.13353, 0.0726438}

Визуализация

In[60]:= **points = Table[{xt〚i〛, z〚i〛}, {i, 1, n + 1}];**
grp = ListPlot[points, PlotStyle → Black];
grf = Plot[f'[x], {x, a, b}, PlotStyle → Red];
Show[grf, grp]

Out[63]=

Намиране на производните с точност $O(h^2)$ - втори порядък на грешката

Първа производна

попълваме вътрешните стойности

In[64]:= **z = Table$\left[\dfrac{yt[\![i+1]\!] - yt[\![i-1]\!]}{2\,h}, \{i, 2, n\}\right]$**

Out[64]=
$\{11.099,\ 8.87463,\ 5.78152,\ 2.12247,\ -1.74434,\ -5.4404,\ -8.60392,\ -10.9252,\ -12.1771\}$

добавяме последния елемент

In[65]:= **AppendTo$\left[z,\ \dfrac{yt[\![n-1]\!] - 4\,yt[\![n]\!] + 3\,yt[\![n+1]\!]}{2\,h}\right]$**

Out[65]=
$\{11.099,\ 8.87463,\ 5.78152,\ 2.12247,\ -1.74434,$
$-5.4404,\ -8.60392,\ -10.9252,\ -12.1771,\ -12.8494\}$

добавяме първия елемент

In[66]:= **PrependTo$\left[z,\ \dfrac{-3\,yt[\![1]\!] + 4\,yt[\![2]\!] - yt[\![3]\!]}{2\,h}\right]$**

Out[66]=
$\{12.8224,\ 11.099,\ 8.87463,\ 5.78152,\ 2.12247,$
$-1.74434,\ -5.4404,\ -8.60392,\ -10.9252,\ -12.1771,\ -12.8494\}$

Оценка на грешката

теоретична грешка

In[67]:= **h^2**

Out[67]=
0.01

истинска грешка

In[68]:= **z (*приближена стойност*)**

Out[68]=
$\{12.8224,\ 11.099,\ 8.87463,\ 5.78152,\ 2.12247,$
$-1.74434,\ -5.4404,\ -8.60392,\ -10.9252,\ -12.1771,\ -12.8494\}$

In[69]:= **f'[xt] (*истинска стойност*)**

Out[69]=
$\{12.4406,\ 11.2837,\ 9.02231,\ 5.87773,\ 2.15779,$
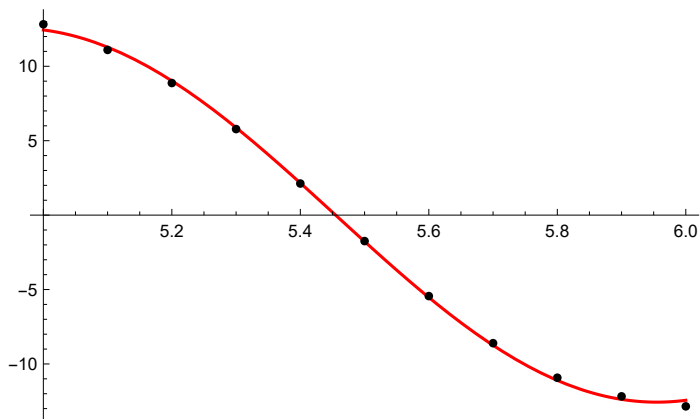$-1.77337,\ -5.53093,\ -8.74709,\ -11.107,\ -12.3797,\ -12.4406\}$

In[70]:= **Abs[z - f'[xt]]**

Out[70]=
$\{0.381766,\ 0.184696,\ 0.14768,\ 0.0962087,\ 0.0353195,$
$0.0290271,\ 0.0905323,\ 0.143176,\ 0.181804,\ 0.202636,\ 0.40881\}$

Визуализация

```
In[71]:= points = Table[{xt[[i]], z[[i]]}, {i, 1, n + 1}];
        grp = ListPlot[points, PlotStyle → Black];
        grf = Plot[f'[x], {x, a, b}, PlotStyle → Red];
        Show[grf, grp]
```

Out[74]=

Втора производна

попълваме вътрешните стойности

$$In[75]:= \ z = Table\left[\frac{yt[[i-1]] - 2\,yt[[i]] + yt[[i+1]]}{h^2}, \{i, 2, n\}\right]$$

Out[75]=

{-17.2335, -27.2545, -34.6077, -38.5732, -38.7629, -35.1583, -28.1121, -18.314, -6.72332}

добавяме последния елемент

добавяме първия елемент

Оценка на грешката

теоретична грешка

```
In[76]:= h²
```

Out[76]=

0.01

истинска грешка

```
In[77]:= z (*приближена стойност*)
```

Out[77]=

{-17.2335, -27.2545, -34.6077, -38.5732, -38.7629, -35.1583, -28.1121, -18.314, -6.72332}
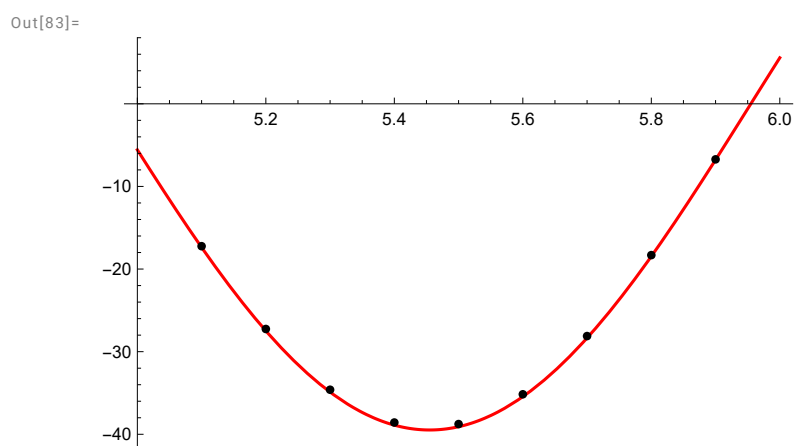
```
In[78]:= f''[xt] (*истинска стойност*)
```

Out[78]=

{-5.57119, -17.3759, -27.4798, -34.8938, -38.8921, -39.0833, -35.4489, -28.3444, -18.4654, -6.77889, 5.57119}

In[79]:= `Table[Abs[z〚i〛 - f''[xt〚i + 1〛]], {i, 1, n - 1}]`

Out[79]=

$\{0.142442, 0.22527, 0.286047, 0.318824,$
$0.320392, 0.290598, 0.232358, 0.151373, 0.0555711\}$

Визуализация

In[80]:= `points = Table[{xt〚i + 1〛, z〚i〛}, {i, 1, n - 1}];`
`grp = ListPlot[points, PlotStyle → Black];`
`grf = Plot[f''[x], {x, a, b}, PlotStyle → Red];`
`Show[grf, grp]`

Out[83]=

Числено диференциране - увеличаване на точността чрез сгъстяване на мрежата

Създаваме табличната функция

```
In[84]:= a = 5.; b = 6;
        n = 100;
        h = (b - a)/n;
        xt = Table[a + i * h, {i, 0, n}]
```

```
Out[87]=
{5., 5.01, 5.02, 5.03, 5.04, 5.05, 5.06, 5.07, 5.08, 5.09, 5.1, 5.11, 5.12, 5.13, 5.14,
 5.15, 5.16, 5.17, 5.18, 5.19, 5.2, 5.21, 5.22, 5.23, 5.24, 5.25, 5.26, 5.27, 5.28,
 5.29, 5.3, 5.31, 5.32, 5.33, 5.34, 5.35, 5.36, 5.37, 5.38, 5.39, 5.4, 5.41, 5.42, 5.43,
 5.44, 5.45, 5.46, 5.47, 5.48, 5.49, 5.5, 5.51, 5.52, 5.53, 5.54, 5.55, 5.56, 5.57,
 5.58, 5.59, 5.6, 5.61, 5.62, 5.63, 5.64, 5.65, 5.66, 5.67, 5.68, 5.69, 5.7, 5.71,
 5.72, 5.73, 5.74, 5.75, 5.76, 5.77, 5.78, 5.79, 5.8, 5.81, 5.82, 5.83, 5.84, 5.85,
 5.86, 5.87, 5.88, 5.89, 5.9, 5.91, 5.92, 5.93, 5.94, 5.95, 5.96, 5.97, 5.98, 5.99, 6.}
```

```
In[88]:= Length[xt]
```

```
Out[88]=
101
```

```
In[89]:= f[x_] := 4 Sin[π x - 3]
        yt = f[xt]
```

```
Out[90]=
{0.56448, 0.688587, 0.812015, 0.934641, 1.05634, 1.17701, 1.29651, 1.41473,
 1.53155, 1.64686, 1.76055, 1.8725, 1.9826, 2.09075, 2.19683, 2.30074, 2.40239,
 2.50166, 2.59846, 2.6927, 2.78429, 2.87312, 2.95912, 3.0422, 3.12227, 3.19927,
 3.27311, 3.34371, 3.41102, 3.47496, 3.53548, 3.5925, 3.64598, 3.69586,
 3.74209, 3.78463, 3.82343, 3.85846, 3.88969, 3.91707, 3.94059, 3.96022,
 3.97594, 3.98774, 3.9956, 3.99952, 3.99949, 3.99552, 3.9876, 3.97575, 3.95997,
 3.94029, 3.91671, 3.88927, 3.858, 3.82291, 3.78406, 3.74146, 3.69518, 3.64525,
 3.59172, 3.53465, 3.47409, 3.4101, 3.34274, 3.27209, 3.19821, 3.12117, 3.04105,
 2.95793, 2.87189, 2.78302, 2.6914, 2.59712, 2.50028, 2.40097, 2.2993, 2.19535,
 2.08924, 1.98107, 1.87094, 1.75896, 1.64525, 1.52992, 1.41307, 1.29483,
 1.17532, 1.05464, 0.932922, 0.810284, 0.686846, 0.56273, 0.438059, 0.312955,
 0.187543, 0.0619454, -0.0637131, -0.189309, -0.314718, -0.439816, -0.56448}
```

Намиране на производните с точност O(h) - първи

порядък на грешката

Първа производна

In[91]:= $z = \text{Table}\left[\dfrac{yt[\![i+1]\!] - yt[\![i]\!]}{h}, \{i, 1, n\}\right]$

Out[91]=

{12.4107, 12.3428, 12.2626, 12.1704, 12.0661, 11.95, 11.822, 11.6824, 11.5313,
 11.3687, 11.195, 11.0102, 10.8145, 10.6082, 10.3914, 10.1644, 9.92727, 9.68039,
 9.42395, 9.15821, 8.88344, 8.59989, 8.30787, 8.00764, 7.69951, 7.38378, 7.06076,
 6.73078, 6.39415, 6.05121, 5.7023, 5.34777, 4.98795, 4.62322, 4.25392, 3.88042,
 3.50309, 3.12231, 2.73845, 2.35188, 1.96299, 1.57216, 1.17979, 0.786245, 0.391928,
 −0.00277685, −0.397478, −0.791788, −1.18532, −1.57767, −1.96848, −2.35733,
 −2.74387, −3.12769, −3.50843, −3.8857, −4.25914, −4.62838, −4.99305, −5.35279,
 −5.70725, −6.05608, −6.39893, −6.73547, −7.06535, −7.38827, −7.70389, −8.01192,
 −8.31203, −8.60394, −8.88736, −9.16201, −9.42762, −9.68393, −9.93067, −10.1676,
 −10.3945, −10.6112, −10.8174, −11.0129, −11.1975, −11.3711, −11.5335, −11.6845,
 −11.8239, −11.9517, −12.0677, −12.1718, −12.2638, −12.3438, −12.4116, −12.4671,
 −12.5103, −12.5412, −12.5597, −12.5659, −12.5596, −12.5409, −12.5098, −12.4664}

добавяме последния елемент

In[92]:= $\text{AppendTo}\left[z, \dfrac{yt[\![n+1]\!] - yt[\![n]\!]}{h}\right]$

Out[92]=

{12.4107, 12.3428, 12.2626, 12.1704, 12.0661, 11.95, 11.822, 11.6824, 11.5313,
 11.3687, 11.195, 11.0102, 10.8145, 10.6082, 10.3914, 10.1644, 9.92727, 9.68039,
 9.42395, 9.15821, 8.88344, 8.59989, 8.30787, 8.00764, 7.69951, 7.38378, 7.06076,
 6.73078, 6.39415, 6.05121, 5.7023, 5.34777, 4.98795, 4.62322, 4.25392, 3.88042,
 3.50309, 3.12231, 2.73845, 2.35188, 1.96299, 1.57216, 1.17979, 0.786245, 0.391928,
 −0.00277685, −0.397478, −0.791788, −1.18532, −1.57767, −1.96848, −2.35733, −2.74387,
 −3.12769, −3.50843, −3.8857, −4.25914, −4.62838, −4.99305, −5.35279, −5.70725,
 −6.05608, −6.39893, −6.73547, −7.06535, −7.38827, −7.70389, −8.01192, −8.31203,
 −8.60394, −8.88736, −9.16201, −9.42762, −9.68393, −9.93067, −10.1676, −10.3945,
 −10.6112, −10.8174, −11.0129, −11.1975, −11.3711, −11.5335, −11.6845, −11.8239,
 −11.9517, −12.0677, −12.1718, −12.2638, −12.3438, −12.4116, −12.4671, −12.5103,
 −12.5412, −12.5597, −12.5659, −12.5596, −12.5409, −12.5098, −12.4664, −12.4664}

Оценка на грешката

теоретична грешка

In[93]:= h

Out[93]=

0.01

истинска грешка

In[94]:= **z (*приближена стойност*)**

Out[94]=

{12.4107, 12.3428, 12.2626, 12.1704, 12.0661, 11.95, 11.822, 11.6824, 11.5313,
11.3687, 11.195, 11.0102, 10.8145, 10.6082, 10.3914, 10.1644, 9.92727, 9.68039,
9.42395, 9.15821, 8.88344, 8.59989, 8.30787, 8.00764, 7.69951, 7.38378, 7.06076,
6.73078, 6.39415, 6.05121, 5.7023, 5.34777, 4.98795, 4.62322, 4.25392, 3.88042,
3.50309, 3.12231, 2.73845, 2.35188, 1.96299, 1.57216, 1.17979, 0.786245, 0.391928,
-0.00277685, -0.397478, -0.791788, -1.18532, -1.57767, -1.96848, -2.35733, -2.74387,
-3.12769, -3.50843, -3.8857, -4.25914, -4.62838, -4.99305, -5.35279, -5.70725,
-6.05608, -6.39893, -6.73547, -7.06535, -7.38827, -7.70389, -8.01192, -8.31203,
-8.60394, -8.88736, -9.16201, -9.42762, -9.68393, -9.93067, -10.1676, -10.3945,
-10.6112, -10.8174, -11.0129, -11.1975, -11.3711, -11.5335, -11.6845, -11.8239,
-11.9517, -12.0677, -12.1718, -12.2638, -12.3438, -12.4116, -12.4671, -12.5103,
-12.5412, -12.5597, -12.5659, -12.5596, -12.5409, -12.5098, -12.4664, -12.4664}

In[95]:= **f'[xt] (*истинска стойност*)**

Out[95]=

{12.4406, 12.3788, 12.3047, 12.2185, 12.1203, 12.01, 11.888, 11.7542, 11.6087, 11.4519,
11.2837, 11.1044, 10.9142, 10.7131, 10.5015, 10.2796, 10.0475, 9.80544, 9.55374,
9.29261, 9.02231, 8.7431, 8.45527, 8.15909, 7.85486, 7.54288, 7.22346, 6.8969,
6.56354, 6.2237, 5.87773, 5.52594, 5.16871, 4.80638, 4.4393, 4.06784, 3.69237,
3.31325, 2.93086, 2.54558, 2.15779, 1.76787, 1.3762, 0.983178, 0.589183, 0.194607,
-0.200161, -0.594731, -0.988714, -1.38172, -1.77337, -2.16326, -2.55102,
-2.93626, -3.31861, -3.69767, -4.07309, -4.44449, -4.81151, -5.17377, -5.53093,
-5.88263, -6.22853, -6.56828, -6.90155, -7.228, -7.54732, -7.8592, -8.16332,
-8.45938, -8.74709, -9.02617, -9.29635, -9.55735, -9.80891, -10.0508, -10.2828,
-10.5046, -10.716, -10.9169, -11.107, -11.2862, -11.4542, -11.6109, -11.7561,
-11.8898, -12.0117, -12.1217, -12.2198, -12.3058, -12.3797, -12.4414, -12.4908,
-12.5279, -12.5526, -12.5649, -12.5648, -12.5523, -12.5274, -12.4902, -12.4406}

In[96]:= **Abs[z - f'[xt]]**

Out[96]=

{0.0299, 0.0360137, 0.042092, 0.0481287, 0.0541178, 0.0600536, 0.0659301, 0.0717416,
0.0774822, 0.0831464, 0.0887285, 0.0942231, 0.0996247, 0.104928, 0.110128,
0.115219, 0.120196, 0.125055, 0.12979, 0.134397, 0.138872, 0.143209, 0.147405,
0.151456, 0.155357, 0.159105, 0.162696, 0.166127, 0.169393, 0.172492, 0.175421,
0.178177, 0.180757, 0.183159, 0.18538, 0.187418, 0.189271, 0.190937, 0.192415,
0.193702, 0.194799, 0.195704, 0.196415, 0.196933, 0.197256, 0.197384, 0.197318,
0.197057, 0.196601, 0.195952, 0.195109, 0.194073, 0.192846, 0.191429, 0.189823,
0.188029, 0.18605, 0.183888, 0.181543, 0.17902, 0.17632, 0.173446, 0.170401,
0.167187, 0.163809, 0.160269, 0.156571, 0.152718, 0.148715, 0.144565, 0.140272,
0.13584, 0.131275, 0.12658, 0.12176, 0.11682, 0.111765, 0.1066, 0.101329, 0.095958,
0.0904926, 0.0849378, 0.0792992, 0.0735824, 0.0677929, 0.0619365, 0.056019,
0.0500463, 0.0440241, 0.0379585, 0.0318554, 0.0257209, 0.019561, 0.0133818,
0.0071894, 0.000989898, 0.00521058, 0.0114059, 0.01759, 0.0237567, 0.0258074}

Визуализация

```
In[97]:= points = Table[{xt[[i]], z[[i]]}, {i, 1, n + 1}];
       grp = ListPlot[points, PlotStyle → Black];
       grf = Plot[f'[x], {x, a, b}, PlotStyle → Red];
       Show[grf, grp]
```

Out[100]=
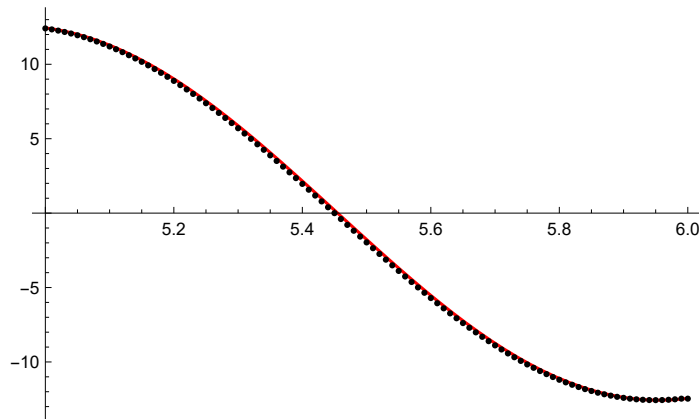
Намиране на производните с точност $O(h^2)$ - втори порядък на грешката

Първа производна

попълваме вътрешните стойности

```
In[101]:=
       z = Table[ (yt[[i + 1]] - yt[[i - 1]]) / (2 h), {i, 2, n}]
```

Out[101]=

{12.3767, 12.3027, 12.2165, 12.1183, 12.0081, 11.886, 11.7522, 11.6068, 11.45, 11.2819, 11.1026, 10.9124, 10.7114, 10.4998, 10.2779, 10.0458, 9.80383, 9.55217, 9.29108, 9.02082, 8.74167, 8.45388, 8.15775, 7.85357, 7.54164, 7.22227, 6.89577, 6.56246, 6.22268, 5.87676, 5.52504, 5.16786, 4.80559, 4.43857, 4.06717, 3.69176, 3.3127, 2.93038, 2.54516, 2.15743, 1.76758, 1.37598, 0.983016, 0.589086, 0.194575, -0.200128, -0.594633, -0.988552, -1.38149, -1.77307, -2.1629, -2.5506, -2.93578, -3.31806, -3.69707, -4.07242, -4.44376, -4.81072, -5.17292, -5.53002, -5.88167, -6.2275, -6.5672, -6.90041, -7.22681, -7.54608, -7.85791, -8.16197, -8.45799, -8.74565, -9.02469, -9.29482, -9.55577, -9.8073, -10.0491, -10.2811, -10.5029, -10.7143, -10.9151, -11.1052, -11.2843, -11.4523, -11.609, -11.7542, -11.8878, -12.0097, -12.1197, -12.2178, -12.3038, -12.3777, -12.4393, -12.4887, -12.5258, -12.5505, -12.5628, -12.5627, -12.5502, -12.5254, -12.4881}

добавяме последния елемент

In[102]:=

$$\text{AppendTo}\left[z, \frac{yt[\![n-1]\!] - 4\,yt[\![n]\!] + 3\,yt[\![n+1]\!]}{2\,h}\right]$$

Out[102]=

{12.3767, 12.3027, 12.2165, 12.1183, 12.0081, 11.886, 11.7522, 11.6068, 11.45, 11.2819, 11.1026, 10.9124, 10.7114, 10.4998, 10.2779, 10.0458, 9.80383, 9.55217, 9.29108, 9.02082, 8.74167, 8.45388, 8.15775, 7.85357, 7.54164, 7.22227, 6.89577, 6.56246, 6.22268, 5.87676, 5.52504, 5.16786, 4.80559, 4.43857, 4.06717, 3.69176, 3.3127, 2.93038, 2.54516, 2.15743, 1.76758, 1.37598, 0.983016, 0.589086, 0.194575, -0.200128, -0.594633, -0.988552, -1.38149, -1.77307, -2.1629, -2.5506, -2.93578, -3.31806, -3.69707, -4.07242, -4.44376, -4.81072, -5.17292, -5.53002, -5.88167, -6.2275, -6.5672, -6.90041, -7.22681, -7.54608, -7.85791, -8.16197, -8.45799, -8.74565, -9.02469, -9.29482, -9.55577, -9.8073, -10.0491, -10.2811, -10.5029, -10.7143, -10.9151, -11.1052, -11.2843, -11.4523, -11.609, -11.7542, -11.8878, -12.0097, -12.1197, -12.2178, -12.3038, -12.3777, -12.4393, -12.4887, -12.5258, -12.5505, -12.5628, -12.5627, -12.5502, -12.5254, -12.4881, -12.4447}

добавяме първия елемент

In[103]:=

$$\text{PrependTo}\left[z, \frac{-3\,yt[\![1]\!] + 4\,yt[\![2]\!] - yt[\![3]\!]}{2\,h}\right]$$

Out[103]=

{12.4447, 12.3767, 12.3027, 12.2165, 12.1183, 12.0081, 11.886, 11.7522, 11.6068, 11.45, 11.2819, 11.1026, 10.9124, 10.7114, 10.4998, 10.2779, 10.0458, 9.80383, 9.55217, 9.29108, 9.02082, 8.74167, 8.45388, 8.15775, 7.85357, 7.54164, 7.22227, 6.89577, 6.56246, 6.22268, 5.87676, 5.52504, 5.16786, 4.80559, 4.43857, 4.06717, 3.69176, 3.3127, 2.93038, 2.54516, 2.15743, 1.76758, 1.37598, 0.983016, 0.589086, 0.194575, -0.200128, -0.594633, -0.988552, -1.38149, -1.77307, -2.1629, -2.5506, -2.93578, -3.31806, -3.69707, -4.07242, -4.44376, -4.81072, -5.17292, -5.53002, -5.88167, -6.2275, -6.5672, -6.90041, -7.22681, -7.54608, -7.85791, -8.16197, -8.45799, -8.74565, -9.02469, -9.29482, -9.55577, -9.8073, -10.0491, -10.2811, -10.5029, -10.7143, -10.9151, -11.1052, -11.2843, -11.4523, -11.609, -11.7542, -11.8878, -12.0097, -12.1197, -12.2178, -12.3038, -12.3777, -12.4393, -12.4887, -12.5258, -12.5505, -12.5628, -12.5627, -12.5502, -12.5254, -12.4881, -12.4447}

Оценка на грешката

теоретична грешка

In[104]:=

$$h^2$$

Out[104]=

0.0001

истинска грешка

In[105]:=

z(*приближена стойност*)

Out[105]=

{12.4447, 12.3767, 12.3027, 12.2165, 12.1183, 12.0081, 11.886, 11.7522, 11.6068,
11.45, 11.2819, 11.1026, 10.9124, 10.7114, 10.4998, 10.2779, 10.0458, 9.80383,
9.55217, 9.29108, 9.02082, 8.74167, 8.45388, 8.15775, 7.85357, 7.54164, 7.22227,
6.89577, 6.56246, 6.22268, 5.87676, 5.52504, 5.16786, 4.80559, 4.43857, 4.06717,
3.69176, 3.3127, 2.93038, 2.54516, 2.15743, 1.76758, 1.37598, 0.983016, 0.589086,
0.194575, −0.200128, −0.594633, −0.988552, −1.38149, −1.77307, −2.1629, −2.5506,
−2.93578, −3.31806, −3.69707, −4.07242, −4.44376, −4.81072, −5.17292, −5.53002,
−5.88167, −6.2275, −6.5672, −6.90041, −7.22681, −7.54608, −7.85791, −8.16197,
−8.45799, −8.74565, −9.02469, −9.29482, −9.55577, −9.8073, −10.0491, −10.2811,
−10.5029, −10.7143, −10.9151, −11.1052, −11.2843, −11.4523, −11.609, −11.7542,
−11.8878, −12.0097, −12.1197, −12.2178, −12.3038, −12.3777, −12.4393, −12.4887,
−12.5258, −12.5505, −12.5628, −12.5627, −12.5502, −12.5254, −12.4881, −12.4447}

In[106]:=

f'[xt](*истинска стойност*)

Out[106]=

{12.4406, 12.3788, 12.3047, 12.2185, 12.1203, 12.01, 11.888, 11.7542, 11.6087, 11.4519,
11.2837, 11.1044, 10.9142, 10.7131, 10.5015, 10.2796, 10.0475, 9.80544, 9.55374,
9.29261, 9.02231, 8.7431, 8.45527, 8.15909, 7.85486, 7.54288, 7.22346, 6.8969,
6.56354, 6.2237, 5.87773, 5.52594, 5.16871, 4.80638, 4.4393, 4.06784, 3.69237,
3.31325, 2.93086, 2.54558, 2.15779, 1.76787, 1.3762, 0.983178, 0.589183, 0.194607,
−0.200161, −0.594731, −0.988714, −1.38172, −1.77337, −2.16326, −2.55102,
−2.93626, −3.31861, −3.69767, −4.07309, −4.44449, −4.81151, −5.17377, −5.53093,
−5.88263, −6.22853, −6.56828, −6.90155, −7.228, −7.54732, −7.8592, −8.16332,
−8.45938, −8.74709, −9.02617, −9.29635, −9.55735, −9.80891, −10.0508, −10.2828,
−10.5046, −10.716, −10.9169, −11.107, −11.2862, −11.4542, −11.6109, −11.7561,
−11.8898, −12.0117, −12.1217, −12.2198, −12.3058, −12.3797, −12.4414, −12.4908,
−12.5279, −12.5526, −12.5649, −12.5648, −12.5523, −12.5274, −12.4902, −12.4406}

In[107]:=

Abs[z − f'[xt]]

Out[107]=

{0.00407764, 0.00203613, 0.00202394, 0.00200977, 0.0019936, 0.00197547, 0.00195539,
0.00193339, 0.00190947, 0.00188367, 0.00185601, 0.00182651, 0.00179522, 0.00176215,
0.00172735, 0.00169084, 0.00165266, 0.00161285, 0.00157145, 0.0015285, 0.00148404,
0.00143811, 0.00139077, 0.00134205, 0.00129201, 0.00124069, 0.00118815, 0.00113444,
0.00107961, 0.00102371, 0.000966799, 0.000908937, 0.000850177, 0.000790578,
0.000730199, 0.0006691, 0.00060734, 0.000544981, 0.000482083, 0.000418711, 0.000354925,
0.000290788, 0.000226365, 0.000161718, 0.000096912, 0.00003201, 0.0000329235,
0.0000978245, 0.000162629, 0.000227273, 0.000291693, 0.000355825, 0.000419605,
0.000482972, 0.000545862, 0.000608213, 0.000669964, 0.000731054, 0.000791422,
0.00085101, 0.000909757, 0.000967607, 0.0010245, 0.00108039, 0.0011352, 0.0011889,
0.00124142, 0.00129272, 0.00134275, 0.00139144, 0.00143877, 0.00148467, 0.00152911,
0.00157204, 0.00161342, 0.00165321, 0.00169136, 0.00172785, 0.00176263, 0.00179567,
0.00182694, 0.00185641, 0.00188404, 0.00190982, 0.00193371, 0.00195569, 0.00197574,
0.00199384, 0.00200998, 0.00202413, 0.00203628, 0.00204643, 0.00205455, 0.00206065,
0.00206471, 0.00206674, 0.00206672, 0.00206467, 0.00206058, 0.00205445, 0.00410513}

Визуализация

In[108]:=
```
points = Table[{xt〚i〛, z〚i〛}, {i, 1, n + 1}];
grp = ListPlot[points, PlotStyle → Black];
grf = Plot[f'[x], {x, a, b}, PlotStyle → Red];
Show[grf, grp]
```

Out[111]=
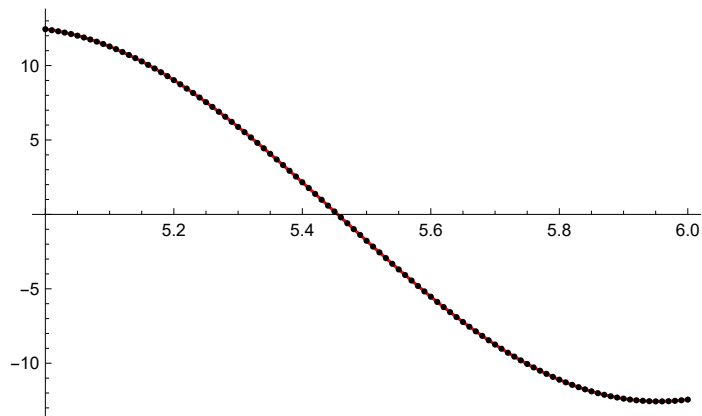
Втора производна

попълваме вътрешните стойности

In[112]:=

$$z = Table\left[\frac{yt〚i-1〛 - 2\,yt〚i〛 + yt〚i+1〛}{h^2}, \{i, 2, n\}\right]$$

Out[112]=
```
{−6.79552, −8.01361, −9.22378, −10.4248, −11.6156, −12.7949, −13.9616, −15.1146,
 −16.2525, −17.3745, −18.4793, −19.5659, −20.6332, −21.6801, −22.7056, −23.7087,
 −24.6884, −25.6437, −26.5737, −27.4775, −28.3542, −29.2029, −30.0228,
 −30.8131, −31.5729, −32.3016, −32.9984, −33.6627, −34.2937, −34.8909,
 −35.4536, −35.9814, −36.4736, −36.9299, −37.3497, −37.7327, −38.0784,
 −38.3865, −38.6568, −38.8889, −39.0826, −39.2377, −39.3542, −39.4318,
 −39.4704, −39.4702, −39.4309, −39.3528, −39.2358, −39.0801, −38.8859,
 −38.6532, −38.3824, −38.0738, −37.7275, −37.3441, −36.9237, −36.467, −35.9742,
 −35.446, −34.8827, −34.2851, −33.6536, −32.9888, −32.2916, −31.5624, −30.8022,
 −30.0115, −29.1912, −28.3421, −27.465, −26.5608, −25.6304, −24.6748, −23.6947,
 −22.6913, −21.6655, −20.6183, −19.5507, −18.4639, −17.3588, −16.2366, −15.0984,
 −13.9453, −12.7784, −11.599, −10.408, −9.20681, −7.99652, −6.77834, −5.55346,
 −4.32311, −3.08849, −1.85082, −0.611327, 0.628771, 1.86825, 3.10588, 4.34045}
```

добавяме последния елемент

добавяме първия елемент

Оценка на грешката

теоретична грешка

In[113]:=

h^2

Out[113]=

0.0001

истинска грешка

In[114]:=

z(*приближена стойност*)

Out[114]=

{−6.79552, −8.01361, −9.22378, −10.4248, −11.6156, −12.7949, −13.9616, −15.1146,
 −16.2525, −17.3745, −18.4793, −19.5659, −20.6332, −21.6801, −22.7056, −23.7087,
 −24.6884, −25.6437, −26.5737, −27.4775, −28.3542, −29.2029, −30.0228,
 −30.8131, −31.5729, −32.3016, −32.9984, −33.6627, −34.2937, −34.8909,
 −35.4536, −35.9814, −36.4736, −36.9299, −37.3497, −37.7327, −38.0784,
 −38.3865, −38.6568, −38.8889, −39.0826, −39.2377, −39.3542, −39.4318,
 −39.4704, −39.4702, −39.4309, −39.3528, −39.2358, −39.0801, −38.8859,
 −38.6532, −38.3824, −38.0738, −37.7275, −37.3441, −36.9237, −36.467, −35.9742,
 −35.446, −34.8827, −34.2851, −33.6536, −32.9888, −32.2916, −31.5624, −30.8022,
 −30.0115, −29.1912, −28.3421, −27.465, −26.5608, −25.6304, −24.6748, −23.6947,
 −22.6913, −21.6655, −20.6183, −19.5507, −18.4639, −17.3588, −16.2366, −15.0984,
 −13.9453, −12.7784, −11.599, −10.408, −9.20681, −7.99652, −6.77834, −5.55346,
 −4.32311, −3.08849, −1.85082, −0.611327, 0.628771, 1.86825, 3.10588, 4.34045}

In[115]:=

f''[xt](*истинска стойност*)

Out[115]=

{−5.57119, −6.79608, −8.01426, −9.22454, −10.4257, −11.6166, −12.796, −13.9628,
 −15.1158, −16.2539, −17.3759, −18.4808, −19.5675, −20.6349, −21.6818, −22.7074,
 −23.7106, −24.6904, −25.6458, −26.5759, −27.4798, −28.3566, −29.2053, −30.0253,
 −30.8156, −31.5755, −32.3043, −33.0011, −33.6654, −34.2965, −34.8938,
 −35.4565, −35.9843, −36.4766, −36.9329, −37.3528, −37.7358, −38.0815,
 −38.3897, −38.6599, −38.8921, −39.0858, −39.241, −39.3574, −39.435, −39.4737,
 −39.4734, −39.4342, −39.356, −39.239, −39.0833, −38.8891, −38.6564, −38.3856,
 −38.0769, −37.7306, −37.3471, −36.9268, −36.47, −35.9772, −35.4489, −34.8856,
 −34.2879, −33.6563, −32.9916, −32.2942, −31.565, −30.8047, −30.014, −29.1936,
 −28.3444, −27.4673, −26.563, −25.6325, −24.6768, −23.6967, −22.6932, −21.6673,
 −20.62, −19.5523, −18.4654, −17.3603, −16.238, −15.0997, −13.9465, −12.7795,
 −11.5999, −10.4089, −9.20757, −7.99718, −6.77889, −5.55392, −4.32347,
 −3.08874, −1.85097, −0.611377, 0.628823, 1.8684, 3.10614, 4.34081, 5.57119}

In[116]:=

```
Table[Abs[z〚i〛 - f''[xt〚i + 1〛]], {i, 1, n - 1}]
```

Out[116]=

{0.000558937, 0.000659125, 0.000758663, 0.000857452, 0.000955394, 0.00105239,
 0.00114836, 0.00124318, 0.00133678, 0.00142907, 0.00151994, 0.00160931, 0.00169709,
 0.0017832, 0.00186755, 0.00195006, 0.00203064, 0.00210921, 0.00218571, 0.00226005,
 0.00233216, 0.00240196, 0.0024694, 0.0025344, 0.0025969, 0.00265683, 0.00271415,
 0.00276878, 0.00282068, 0.0028698, 0.00291609, 0.0029595, 0.00299998, 0.00303751,
 0.00307204, 0.00310354, 0.00313197, 0.00315732, 0.00317955, 0.00319864,
 0.00321457, 0.00322733, 0.00323691, 0.00324329, 0.00324647, 0.00324645,
 0.00324322, 0.0032368, 0.00322718, 0.00321437, 0.00319839, 0.00317926, 0.00315698,
 0.0031316, 0.00310312, 0.00307158, 0.003037, 0.00299944, 0.00295891, 0.00291546,
 0.00286913, 0.00281997, 0.00276803, 0.00271336, 0.00265601, 0.00259604, 0.0025335,
 0.00246847, 0.002401, 0.00233116, 0.00225902, 0.00218465, 0.00210812, 0.00202952,
 0.00194891, 0.00186638, 0.001782, 0.00169587, 0.00160806, 0.00151867, 0.00142778,
 0.00133548, 0.00124186, 0.00114701, 0.00105104, 0.000954023, 0.000856068,
 0.000757267, 0.00065772, 0.000557524, 0.000456776, 0.000355579, 0.00025403,
 0.000152232, 0.0000502819, 0.0000517168, 0.000153665, 0.000255461, 0.000357005}

Визуализация

In[117]:=

```
points = Table[{xt〚i + 1〛, z〚i〛}, {i, 1, n - 1}];
grp = ListPlot[points, PlotStyle → Black];
grf = Plot[f''[x], {x, a, b}, PlotStyle → Red];
Show[grf, grp]
```

Out[120]=