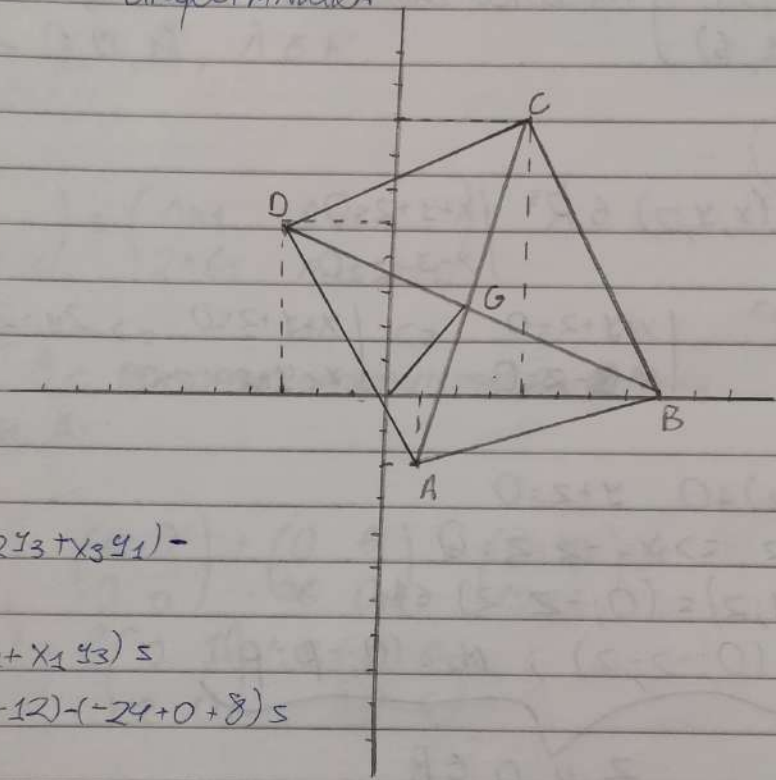
1

Фак. номер 2001261008

ПУ „Паисий Хилендарски", ФМИ Задачи за самостоятелни упражнения по ПААГ за спец. Информатика

Зад. 1



а) $\frac{1}{2}(x_1y_2+x_2y_3+x_3y_1)-$

$(x_2y_1+x_3y_2+x_1y_3)=$

$=\frac{1}{2}(0+64+(-12)-(-24+0+8)=$

$=\frac{1}{2}(52-(-16)=\frac{1}{2}.68=34$

A, B, C не са колинеарни

б) $AB \parallel CD \Rightarrow AD \parallel BC \rightarrow ABCD$ е успоредник

$\overrightarrow{AB}(8-1;0-3)=\overrightarrow{DC}(4-(-3);5-8)$, т.е $ABCD$ е успоредник

в) $\overrightarrow{OG}=\frac{1}{4}(\overrightarrow{OA}+\overrightarrow{OB}+\overrightarrow{OC}+\overrightarrow{OD})$

$G=\left(\frac{1+8+4-3}{4};\frac{-3+0+8+5}{4}\right) \Leftrightarrow G=\left(\frac{10}{4};\frac{10}{4}\right) \Rightarrow G\left(\frac{5}{2};\frac{5}{2}\right)$

Зад. 2

а) $(1, -1, 1)$
$(1, 0, 5)$
$(2, -1, 6)$
$\}$ линейно зависима

б) $(2, 1, 0)$
$(0, -2, 4)$
$(1, 0, 1)$
$\}$ линейно зависима

Зад. 3

а) $M = \{(x, y, z) \in \mathbb{R}^3 \mid \begin{cases} x+y+z=0 \\ x-y-z=0 \end{cases}$

$M \leq \mathbb{R}^3$ $\quad \begin{cases} x+y+z=0 \\ x-y-z=0 \end{cases} \Leftrightarrow \begin{cases} x+y+z=0 \\ x=y+z \end{cases} \Leftrightarrow 2y+2z=0$

$2(y+z)=0 \quad y+z=0$
$y=-z \Leftrightarrow x=-z+z=0$
$(x;y;z)=(0;-z;z) \in M$
$\underbrace{m_1=(0;-z;z) \; ; \; m_2=(0;-p;p)}_{z \text{ и } p \in \mathbb{R}}$

$\left.\begin{array}{l} m_1+m_2=(0+0; -(z+p); z+p) \in M \\ \lambda \cdot m_1=(\lambda \cdot 0; -\lambda \cdot z; \lambda \cdot z) \in M \end{array}\right\} \Rightarrow$ M е векторно пространство

б) $A = \begin{pmatrix} a & b \\ 2b & a+c \end{pmatrix}$; $a, b, c \in \mathbb{R}$

1) $A \leq M_{2\times2}(\mathbb{R})$ $\left\{ \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}, a_{ij} \in \mathbb{R} \right\}$

2) $\begin{pmatrix} 1 & 2 \\ 4 & 4 \end{pmatrix} \in A$, $(a=1; b=2; c=3) \Rightarrow A \neq 0$

3) $\begin{pmatrix} a_1 & b_1 \\ 2b_1 & a_1+c_1 \end{pmatrix} \in A$; $\begin{pmatrix} a_2 & b_2 \\ 2b_2 & a_2+c_2 \end{pmatrix} \in A$

$$\begin{pmatrix} a_1 & b_1 \\ 2b_1 & a_1+c_1 \end{pmatrix} + \begin{pmatrix} a_2 & b_2 \\ 2b_2 & a_2+c_2 \end{pmatrix} = \begin{pmatrix} a_1+a_2 & b_1+b_2 \\ 2(b_1+b_2) & (a_1+a_2)+(c_1+c_2) \end{pmatrix} \in A$$

4) $\begin{pmatrix} a_1 & b_1 \\ 2b_1 & a_1+c_1 \end{pmatrix} \in A, \lambda \in R$

$$\lambda \cdot \begin{pmatrix} a_1 & b_1 \\ 2b_1 & a_1+c_1 \end{pmatrix} = \begin{pmatrix} \lambda a_1 & \lambda b_1 \\ 2\lambda b_1 & \lambda a_1+\lambda c_1 \end{pmatrix}$$

от 1) до 4) $\Rightarrow$ A е векторно пространство на $M_{2\times 2}$

РАЗМЕРНОСТ НА A:

$$A = \begin{pmatrix} a & b \\ 2b & a+c \end{pmatrix} = \begin{pmatrix} a & 0 \\ 0 & a \end{pmatrix} + \begin{pmatrix} 0 & b \\ 2b & 0 \end{pmatrix} + \begin{pmatrix} 0 & 0 \\ 0 & c \end{pmatrix} =$$

$$= a\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} + b\begin{pmatrix} 0 & 1 \\ 2 & 0 \end{pmatrix} + c\begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$$

$$\left[e_1 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}; e_2 = \begin{pmatrix} 0 & 1 \\ 2 & 0 \end{pmatrix}; e_3 = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \right]$$

$\lambda_1 \cdot e_1 + \lambda_2 \cdot e_2 + \lambda_3 \cdot e_3 = 0$

$$\lambda_1 \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} + \lambda_2 \begin{pmatrix} 0 & 1 \\ 2 & 0 \end{pmatrix} + \lambda_3 \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$$

$$\begin{pmatrix} \lambda_1 & 0 \\ 0 & \lambda_1 \end{pmatrix} + \begin{pmatrix} 0 & \lambda_2 \\ 2\lambda_2 & 0 \end{pmatrix} + \begin{pmatrix} 0 & 0 \\ 0 & \lambda_3 \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$$

$$\begin{pmatrix} \lambda_1 & \lambda_2 \\ 2\lambda_2 & \lambda_1+\lambda_3 \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$$

$$\left|\begin{array}{l}\lambda_1 = 0 \\ \lambda_2 = 0 \\ 2\lambda_2 = 0 \\ \lambda_1 + \lambda_3 = 0\end{array}\right\} \lambda_1 = \lambda_2 = \lambda_3 = 0 \Rightarrow \dim A = 3$$

B) $A = \begin{pmatrix} a & b \\ 2 & a-c \end{pmatrix}$; $a, b, c \in R$

1) $A \leq M_{2\times2}(R)$ $\left[\begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}, a_{ij} \in R\right]$

2) $\begin{pmatrix} 1 & 2 \\ 2 & -2 \end{pmatrix} \in A$, $(a=1; b=2; c=3) \Rightarrow A \neq 0$

3) $\begin{pmatrix} a_1 & b_1 \\ 2 & a_1-c_1 \end{pmatrix} \in A$, $\begin{pmatrix} a_2 & b_2 \\ 2 & a_2-c_2 \end{pmatrix} \in A$

$$\begin{pmatrix} a_1 & b_1 \\ 2 & a_1-c_1 \end{pmatrix} + \begin{pmatrix} a_2 & b_2 \\ 2 & a_2-c_2 \end{pmatrix} = \begin{pmatrix} a_1+a_2 & b_1+b_2 \\ 4 & (a_1+a_2)-(c_1+c_2) \end{pmatrix} \notin A$$

$\Rightarrow$ A не е векторно подпространство, няма нужда от размерност

Зад 4

$A(2,1,-3)$ $D(2,2,0)$

$B(4,-2,4)$ $E(0,1,1)$

$C(0,1,0)$

Доказателство:

$$AB \begin{cases} \ni A(2,1,-3) \\ \parallel \overrightarrow{AB}(2,-3,7) \end{cases}$$

2

Фак. номер 2001261008

$$\begin{cases} x = 2+2t \\ y = 1-3t \\ z = -3+8t \end{cases}$$

$$CDE: \begin{vmatrix} x-0 & y-1 & z-0 \\ 2-0 & 2-1 & 0-0 \\ 0-0 & 1-1 & 1-0 \end{vmatrix} =$$

$$= \begin{vmatrix} x & y-1 & z \\ 2 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix} = x-2(y-1)=0$$

$CDE: x-2y+2=0$

$AB \cap CDE$

$(2+2t)-2(1-3t)+2=0$

$2+2t-2+6t+2=0$

$8t=-2 \qquad t=-\frac{1}{4}$

$$T.M \begin{cases} x = 2+2\left(-\frac{1}{4}\right) = \frac{3}{2} \\ y = 1-3\left(-\frac{1}{4}\right) = \frac{7}{4} \\ z = -3+8\left(-\frac{1}{4}\right) = -\frac{14}{8} \end{cases}$$

Зад. 5

$\vec{a}\cdot\vec{b} = |\vec{a}|\cdot|\vec{b}|\cdot\cos\sphericalangle(\vec{a},\vec{b}) = 2\cdot3\cdot\cos 60^\circ = \cancel{2}\cdot3\cdot\frac{1}{\cancel{2}} = 3$

$\vec{a}^2 = |\vec{a}|^2 = 2^2 = 4$

$\vec{b}^2 = |\vec{b}|^2 = 3^2 = 9$

$(\vec{a}+\vec{b})(\vec{a}-3\vec{b}) = \vec{a}^2 - 3\vec{a}\vec{b} + \vec{b}\cdot\vec{a} - 3\vec{b}^2 =$

$= 4-3\cdot3+3-3\cdot9 = 4-9+3-27 = -29$

Зад. 6

$\vec{a}(1,-1,0),\ \vec{b}(-2,0,3)$

$\vec{p} = a+b = (1,-1,0)+(-2,0,3) = (-1,-1,3)$

$\vec{q} = 3\vec{a} - \vec{b} = 3\cdot(1,-1,0) - (-2,0,3) = (3,-3,0) - (-2,0,3) = (5,-3,3)$

$|\vec{p}|^2 = (-1)^2 + (-1)^2 + 3^2 = 11 \Rightarrow |\vec{p}| = \sqrt{11}$

$|\vec{q}|^2 = 5^2 + (-3)^2 + (-3)^2 = 43 \Rightarrow |\vec{q}| = \sqrt{43}$

$\vec{p}\cdot\vec{q} = (-1,-1,3)\cdot(5,-3,-3) = (-1)\cdot 5 + (-1)\cdot(-3) + 3\cdot(-3) = -5+3-9 = -11$

$$\cos\angle(\vec{p},\vec{q}) = \frac{\vec{p}\cdot\vec{q}}{|\vec{p}|\cdot|\vec{q}|} = \frac{-11}{\sqrt{11}\cdot\sqrt{43}} = -\sqrt{\frac{11}{43}}$$

Зад. 7

а) $\begin{vmatrix} 1 & -3 \\ 2 & 4 \end{vmatrix} = (1\cdot 4) - (-3\cdot 2) = 10$

б) $\begin{vmatrix} 1 & 2 & 3 \\ 0 & 0 & 7 \\ 2 & 1 & -3 \end{vmatrix} \cdot(-2) = \begin{vmatrix} 1 & 2 & 3 \\ 0 & 0 & 7 \\ 0 & -3 & -9 \end{vmatrix} = \begin{vmatrix} 1 & 2 & 3 \\ 0 & -3 & -9 \\ 0 & 0 & -7 \end{vmatrix}$

$\begin{vmatrix} 1 & 2 & 3 \\ 0 & 0 & 7 \\ 2 & 1 & -3 \end{vmatrix} = \begin{vmatrix} 1 & 2 & 3 \\ 0 & -3 & -9 \\ 0 & 0 & 7 \end{vmatrix} = 1\cdot(-3)\cdot(-7) = 21$

в) $\begin{vmatrix} 1 & 0 & 3 & 2 \\ -1 & 0 & 1 & 3 \\ 2 & 0 & -5 & 8 \\ 4 & 1 & 0 & 2 \end{vmatrix} \cdot(1) = \begin{vmatrix} 1 & 0 & 3 & 2 \\ 0 & 0 & 4 & 5 \\ 2 & 0 & -5 & 8 \\ 4 & 1 & 0 & 2 \end{vmatrix} \cdot(-2) = \begin{vmatrix} 1 & 0 & 3 & 2 \\ 0 & 0 & 4 & 5 \\ 0 & 0 & -11 & 4 \\ 4 & 1 & 0 & 2 \end{vmatrix} \cdot(-4) =$

$= \begin{vmatrix} 1 & 0 & 3 & 2 \\ 0 & 0 & 4 & 5 \\ 0 & 0 & -11 & 4 \\ 0 & 1 & -12 & -6 \end{vmatrix} = \begin{vmatrix} 1 & 0 & 3 & 2 \\ 0 & 1 & -12 & -6 \\ 0 & 0 & -11 & 4 \\ 0 & 0 & -4 & -5 \end{vmatrix} \cdot\left(\frac{-4}{11}\right) = \begin{vmatrix} 1 & 0 & 3 & 2 \\ 0 & 1 & -12 & -6 \\ 0 & 0 & -11 & 4 \\ 0 & 0 & 0 & -\frac{11}{11} \end{vmatrix}$

$$\begin{vmatrix} 1 & 0 & 3 & 2 \\ -1 & 0 & 1 & 3 \\ 2 & 0 & -5 & 8 \\ 4 & 1 & 0 & 2 \end{vmatrix} = \begin{vmatrix} 1 & 0 & 3 & 2 \\ 0 & 1 & -12 & -6 \\ 0 & 0 & -11 & 4 \\ 0 & 0 & 0 & -\frac{71}{11} \end{vmatrix} = 1\cdot 1\cdot(-11)\cdot\left(-\frac{71}{11}\right) = 71$$

г) $$\begin{vmatrix} 1 & 2 & 3 & 4 \\ 3 & 6 & 9 & 12 \\ 7 & 4 & 2 & 1 \\ 3 & 5 & 6 & -1 \end{vmatrix} \cdot(-3) = \begin{vmatrix} 1 & 2 & 3 & 4 \\ 0 & 0 & 0 & 0 \\ 7 & 4 & 2 & 1 \\ 3 & 5 & 6 & -1 \end{vmatrix}$$

$$\begin{vmatrix} 1 & 2 & 3 & 4 \\ 3 & 6 & 9 & 12 \\ 7 & 4 & 2 & 1 \\ 3 & 5 & 6 & -1 \end{vmatrix} = \begin{vmatrix} 1 & 2 & 3 & 4 \\ 0 & 0 & 0 & 0 \\ 7 & 4 & 2 & 1 \\ 3 & 5 & 6 & -1 \end{vmatrix} = 0$$

зад 9 X

зад. 10

а) $$\begin{cases} x_1 + x_2 - x_3 = 0 \\ x_1 - x_2 + x_3 = 0 \\ -x_1 + x_2 + x_3 = 0 \end{cases} \to \left(\begin{array}{ccc|c} 1 & 1 & -1 & 0 \\ 1 & -1 & 1 & 0 \\ -1 & 1 & 1 & 0 \end{array}\right) \begin{array}{l} \\ -1 \\ +1 \end{array} \sim$$

$$\sim \left(\begin{array}{ccc|c} 1 & 1 & -1 & 0 \\ 0 & -2 & 2 & 0 \\ 0 & 2 & 0 & 0 \end{array}\right) -1 \sim \left(\begin{array}{ccc|c} 1 & 1 & -1 & 0 \\ 0 & -2 & 2 & 0 \\ 0 & 0 & 2 & 0 \end{array}\right)$$

$$\Leftrightarrow \begin{cases} x_1 + x_2 - x_3 = 0 \to x_1 + 0 - 0 = 0 \to x_1 = 0 \\ -2x_2 + 2x_3 = 0 \to -2x_2 + 0 = 0 \to x_2 = 0 \\ 2x_3 = 0 \to x_3 = 0 \end{cases}$$

краен отговор $(0,0,0)$

б) $\begin{cases} 3x_1 + x_2 + 2x_3 = 1 \\ x_1 + x_3 = 2 \\ 6x_1 + x_2 + 5x_3 = 3 \end{cases} \to \left(\begin{array}{ccc|c} 3 & 1 & 2 & 1 \\ 1 & 0 & 1 & 2 \\ 6 & 1 & 5 & 3 \end{array}\right) \sim \left(\begin{array}{ccc|c} 1 & 0 & 1 & 2 \\ 3 & 1 & 2 & 1 \\ 6 & 1 & 5 & 3 \end{array}\right) \begin{matrix} -3 \\ -6 \end{matrix} \sim$

$\sim \left(\begin{array}{ccc|c} 1 & 0 & 1 & 2 \\ 0 & 1 & -1 & -5 \\ 0 & 1 & -1 & -4 \end{array}\right) -1 \sim \left(\begin{array}{ccc|c} 1 & 0 & 1 & 2 \\ 0 & 1 & -1 & -5 \\ 0 & 0 & 0 & -4 \end{array}\right)$

$\Rightarrow \begin{cases} x_1 + x_3 = 2 \\ x_2 - x_3 = -5 \\ 0.x_3 = -4 \end{cases} \Leftrightarrow$ нема решение

в) $\begin{cases} 3x_1 + 2x_2 - x_3 - 2x_4 = 1 \\ 5x_1 + x_2 - 3x_3 + x_4 = 3 \\ 2x_1 - x_2 - 2x_3 - 2x_4 = 4 \end{cases} \to \left(\begin{array}{cccc|c} 3 & 2 & -1 & -2 & 1 \\ 5 & 1 & -3 & 1 & 3 \\ 2 & -1 & -2 & -2 & 4 \end{array}\right) \begin{matrix} -1 \\ -2 \end{matrix} \sim$

$\sim \left(\begin{array}{cccc|c} 1 & 3 & 1 & 0 & -3 \\ 1 & 3 & 1 & 3 & -5 \\ 2 & -1 & -2 & -2 & 4 \end{array}\right) \begin{matrix} -1 \\ -2 \end{matrix} \sim \left(\begin{array}{cccc|c} 1 & 3 & 1 & 0 & 3 \\ 0 & 0 & 0 & 3 & -2 \\ 0 & -7 & -4 & -2 & 2 \end{array}\right) \sim$

$\sim \left(\begin{array}{cccc|c} 1 & 3 & 1 & 0 & -3 \\ 0 & -7 & -4 & -2 & 2 \\ 0 & 0 & 0 & 3 & -2 \end{array}\right)$

$\Rightarrow \begin{cases} x_1 + 3x_2 + x_3 = -3 \\ -7x_2 - 4x_3 - 2x_4 = 2 \\ 3x_4 = -2 \to x_4 = -\frac{2}{3} \end{cases}$

$\Rightarrow -7x_2 - 4x_3 + \frac{4}{3} = 2$

$-4x_3 = 2 - \frac{4}{3} + 7x_2 = \frac{2}{3} + 7x_2 \quad | : (-4)$

$x_3 = -\frac{1}{6} - \frac{7}{4}x_2$

$x_1 = -\frac{17}{6} - \frac{5}{4}x_2$

Краен отговор: $x_1 = -\frac{17}{6} - \frac{5}{4}x_2$; $x_2 \in R$;
$x_3 = -\frac{1}{6} - \frac{7}{4}x_2$; $x_4 = -\frac{2}{3}$

3

Факномер 2001261008

г) $\begin{cases} x_1+2x_2-x_3+x_4=4 \\ -x_1+x_2+x_4=1 \\ 3x_1+6x_2-3x_3+3x_4=12 \\ 2x_1-x_2+3x_3-x_4=2 \end{cases} \to \left(\begin{array}{cccc|c} 1 & 2 & -1 & 1 & 4 \\ -1 & 1 & 0 & 1 & 1 \\ 3 & 6 & -3 & 3 & 12 \\ 2 & -1 & 3 & 1 & 2 \end{array}\right) \begin{matrix} +1 \\ -3 \\ -2 \end{matrix} \sim$

$\sim \left(\begin{array}{cccc|c} 1 & 2 & -1 & 1 & 4 \\ 0 & 3 & -1 & 2 & 5 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & -5 & 5 & -3 & -6 \end{array}\right) 2 \sim \left(\begin{array}{cccc|c} 1 & 2 & -1 & 1 & 4 \\ 0 & 3 & -1 & 2 & 5 \\ 0 & 1 & 3 & 1 & 4 \end{array}\right) \sim$

$\sim \left(\begin{array}{cccc|c} 1 & 2 & -1 & 1 & 4 \\ 0 & 1 & 3 & 1 & 4 \\ 0 & 3 & -1 & 2 & 5 \end{array}\right) -3 \sim \left(\begin{array}{cccc|c} 1 & 2 & -1 & 1 & 4 \\ 0 & 1 & 3 & 1 & 4 \\ 0 & 0 & -10 & -1 & -7 \end{array}\right)$

$\Rightarrow \begin{cases} x_1+2x_2-x_3+x_4=4 \\ x_2+3x_3+x_4=4 \\ -10x_3+x_4=-7 \to x_4=-10x_3+7 \end{cases}$

$x_2+3x_3-10x_3+7=4 \to x_2=7x_3-3$

$x_1+2(7x_3-3)-x_3+(-10x_3+7)=4$

$x_1+3x_3+1=4 \to x_1=3-3x_3$

Краен отговор $\Rightarrow x_1=3-3x_3;\ x_2=7x_3-3$

$x_3 \in R;\ x_4=-10x_3+7$

Зад. 11

а) $\underbrace{\begin{pmatrix} 2 & 1 \\ 2 & 1 \end{pmatrix}}_{A} \cdot X = \underbrace{\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}}_{B}$

$\det A = \begin{vmatrix} 2 & 1 \\ 2 & 1 \end{vmatrix} = 2-2=0 \Rightarrow$ не съществува $A^{-1} \Rightarrow$ няма решение

Зад. 12

а) $l \begin{cases} \ni A(-1,-1) \\ \measuredangle 45^\circ \text{ с } g: 2x+3y-6=0 \end{cases}$

$y = k \cdot x + n \rightarrow$ урав. на g: $3y = -2x+6$

$y = -\frac{2}{3}x + 2$

$y = k_1 \cdot x + n_1$ и $y = k_2 \cdot x + n_2$

$tg\,\varphi = \frac{k_1 - k_2}{1 + k_1 \cdot k_2} \rightarrow \varphi = 45^\circ \quad k_g = -\frac{2}{3}$

I сл. $tg\,45^\circ = \frac{k - (-\frac{2}{3})}{1 + (-\frac{2}{3})}$

$1 = \frac{3k+2}{3-2k} \Rightarrow 3k+2 = 3-2k \rightarrow k = \frac{1}{5}$

$\Rightarrow l: y = \frac{1}{5}x + n \rightarrow -1 = -\frac{1}{5} + n \Rightarrow n = -\frac{4}{5}$

$\Leftrightarrow l: y = \frac{1}{5}x - \frac{4}{5} \rightarrow x - 5y - 4 = 0$

II сл. $tg\,45^\circ = \frac{-\frac{2}{3} - k}{1 + (-\frac{2}{3})k}$

$1 = \frac{-2-3x}{3-2k} \Rightarrow 3-2k = -2-3k \rightarrow k = -5$

$\Rightarrow y = -5x + n \rightarrow 1 = 5 + n \rightarrow n = -6$

$\Rightarrow y = -5x - 6$

б) $l \begin{cases} \ni B(-1,3) \\ \parallel p: 3x+4y-8=0 \end{cases}$

$l \parallel p \Rightarrow l: 3x + 4y + c = 0$

$3 \cdot (-1) + 4 \cdot 3 + c = 0 \Rightarrow c = -9$

$\Rightarrow l: 3x + 4y - 9 = 0$

в) $g \begin{cases} \ni C(1,3) \\ \perp l: 3x+4y+11=0 \end{cases}$

$g \perp l \Rightarrow g \begin{cases} \ni C(1,3) \\ \parallel \vec{n}(3,4) \end{cases} \Rightarrow \frac{x-1}{3} = \frac{y-3}{4} \Leftrightarrow 4(x-1)=3(y-3) \Rightarrow$

$\Rightarrow g: 4x-3y+5=0$

г) $l \begin{cases} \ni D(1,0) \\ \ni E(2,2) \end{cases} \Rightarrow l \parallel \vec{DE}(1,2)$

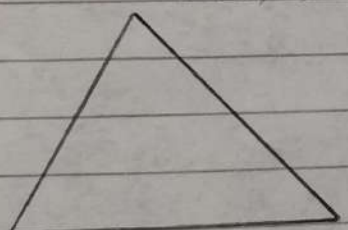
$\frac{x-1}{1}=\frac{y-0}{2} \Leftrightarrow 2(x-1)=y$

$\Rightarrow l: 2x-y-2=0$

зад 13

$\triangle ABC; A(-1;-2); B(3;-1); C(0;4)$

$C(0;4)$



$A(-1;-2)$ $B(3;-1)$

а) правата AB $\begin{cases}\ni A(-1;-2) \\ \ni B(3;-1)\end{cases} \Rightarrow \frac{x+1}{3+1} = \frac{y+2}{-1+2}$

$\Rightarrow \frac{x+1}{4}=\frac{y+2}{1} \Rightarrow x+1=4y+8 \Leftrightarrow$

$\Rightarrow x-4y-7=0 \to AB$

$BC \begin{cases}\ni B(3;-1) \\ \ni C(0;4)\end{cases} \Rightarrow \frac{x-3}{0-3}=\frac{y+1}{4+1} \Rightarrow \frac{x-3}{-3}=\frac{y+1}{5} \Rightarrow 5x-15=-3y-3 \Leftrightarrow$

$\Leftrightarrow BC: 5x+3y-12=0$

$AC \begin{cases}\ni A(-1;-2) \\ \ni C(0;4)\end{cases} \Rightarrow \frac{x+1}{0+1}=\frac{y+2}{4+2} \Rightarrow \frac{x+1}{1}=\frac{y+2}{6} \Leftrightarrow 6x+6=y+2$

$\Rightarrow AC: 6x-y+4=0$

б) $x_M = \frac{x_B+x_C}{2} = \frac{3+0}{2} = \frac{3}{2}$ $\quad y_M = \frac{y_B+y_C}{2} = \frac{-1+4}{2} = \frac{3}{2}$

$\Rightarrow m_a \begin{cases} \ni A(-1;-2) \\ \ni M\left(\frac{3}{2};\frac{3}{2}\right) \end{cases} \Rightarrow \dfrac{x+1}{\frac{3}{2}+1} = \dfrac{y+2}{\frac{3}{2}+2} \Rightarrow \dfrac{x+1}{\frac{5}{2}} = \dfrac{y+2}{\frac{7}{2}} \Rightarrow$

$\Rightarrow \frac{7}{2}(x+1) = \frac{5}{2}(y+2) \Leftrightarrow 7x+7 = 5y+10 \Leftrightarrow m_a: 7x-5y-3=0$

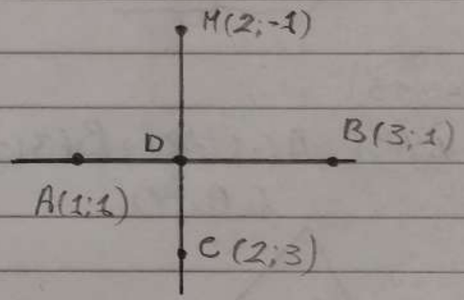
в) височината през върха A: $h_a \begin{cases} \ni A(-1;-2) \\ \perp \vec{BC}(-3;5) \end{cases}$

$-3x+5y+c=0 \to -3\cdot(-1)+5\cdot(-2)+c=0 \Rightarrow c=7$

$\Rightarrow h_a: -3x+5y+7=0 \;|\cdot(-1) \Rightarrow h_a: 3x-5y-7=0$

зад. 14

$A(1;1);\ B(3;1);\ M(2;-1)$



а) права $AB \begin{cases} \ni A(1;1) \\ \ni B(3;1) \end{cases} \Leftrightarrow$

$\Leftrightarrow \dfrac{x-1}{3-1} = \dfrac{y-1}{1-1} \Rightarrow \dfrac{x-1}{2} = \dfrac{y-1}{0} \Rightarrow$

$\Rightarrow 0(x-1) = 2(y-1) \Leftrightarrow AB: 2y-2=0 \;|:2 \Leftrightarrow AB: y-1=0$

б) 1) $l \begin{cases} \ni M(2;-1) \\ \perp \vec{AB}(2;0) \end{cases} \Rightarrow l: 2\cdot x+0\cdot y+c=0$

$2\cdot 2+c=0 \Rightarrow c=-4$

$\Rightarrow l: 2x-4=0 \;|:2 \Leftrightarrow l: x-2=0$

2) $l \cap AB = D \Leftrightarrow \left| \begin{array}{l} y-1=0 \\ x-2=0 \end{array} \right. \Leftrightarrow \begin{array}{l} y=1 \\ x=2 \end{array} \Rightarrow D(2,1)$

3) D - среда на $MC \Rightarrow x_D = x_M + x_C \Rightarrow 2 = \dfrac{2+x_C}{2} \Rightarrow x_C = 2$

$y_D = \dfrac{y_M+y_C}{2} \Rightarrow 1 = \dfrac{-1+y_C}{2} \Leftrightarrow y_C = 3 \Rightarrow C(2;3)$

в) $S_{ABC} = \frac{1}{2}\cdot AB \cdot CD$

$|\vec{AB}| = \sqrt{(3-1)^2+(1-1)^2} = 2 \quad |\vec{CD}| = \sqrt{(2-2)^2+(1-3)^2} = 2$

$\Rightarrow S_{ABC} = \frac{1}{2}\cdot 2\cdot 2 \to S_{ABC} = 2\,см^2$

Фак. номер 2001261008

Зад. 8

а) $\begin{pmatrix} 1 & 3 \\ 4 & 5 \end{pmatrix} \to A^{(-1)} = \begin{pmatrix} a & b \\ c & d \end{pmatrix}^{(-1)} = \frac{1}{|A|} \cdot \begin{pmatrix} a_{11} & a_{21} \\ a_{12} & a_{22} \end{pmatrix} = \frac{1}{ad-bc} \cdot \begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$

$\begin{pmatrix} 1 & 3 \\ 4 & 5 \end{pmatrix}^{(-1)} = \frac{1}{1 \cdot 5 - 3 \cdot 4} \cdot \begin{pmatrix} 5 & -3 \\ -4 & 1 \end{pmatrix} = \begin{pmatrix} -\frac{5}{7} & \frac{3}{5} \\ \frac{4}{7} & -\frac{-1}{7} \end{pmatrix}$

б) $\begin{pmatrix} -1 & 1 & 1 \\ 1 & -1 & 1 \\ 1 & 1 & -1 \end{pmatrix} \quad \left(\begin{array}{ccc|ccc} -1 & 1 & 1 & 1 & 0 & 0 \\ 1 & -1 & 1 & 0 & 1 & 0 \\ 1 & 1 & -1 & 0 & 0 & 1 \end{array}\right)$

$\left(\begin{array}{ccc|ccc} -1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & -2 & -1 & -1 & 0 \\ 0 & -2 & 0 & -1 & 0 & -1 \end{array}\right)$ $R_3 \leftrightarrow R_2$

$\left(\begin{array}{ccc|ccc} -1 & 1 & 1 & 1 & 0 & 0 \\ 0 & -2 & 0 & -1 & 0 & -1 \\ 0 & 0 & -2 & -1 & -1 & 0 \end{array}\right) \quad \left(\begin{array}{ccc|ccc} -2 & 0 & 2 & 1 & 0 & -1 \\ 0 & -2 & 0 & -1 & 0 & -1 \\ 0 & 0 & -4 & -2 & -2 & 0 \end{array}\right)$

$\begin{pmatrix} -1 & 1 & 1 \\ 1 & -1 & 1 \\ 1 & 1 & -1 \end{pmatrix}^{(-1)} = \frac{1}{4} \begin{pmatrix} 0 & -2 & -2 \\ -2 & 0 & -2 \\ -2 & -2 & 0 \end{pmatrix} = \begin{pmatrix} 0 & \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & 0 & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} & 0 \end{pmatrix}$