

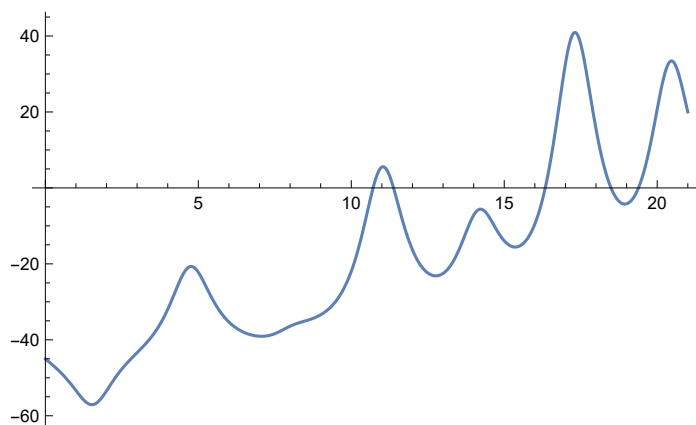
Изпит КЧМ

зад1

In[11]:=
$$f[x_] := \frac{\sqrt{x^3} - 14 * \text{Sin}[x]}{1 + \text{Cos}[x]^2} - 45$$

In[17]:= `Plot[f[x], {x, 0, 21}]`

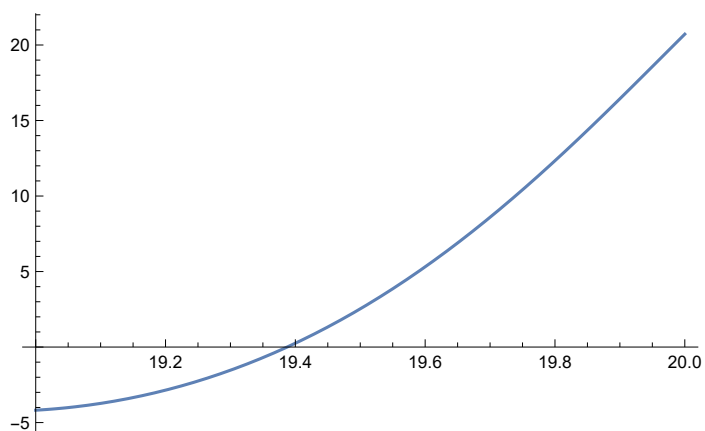
Out[17]=



Izvod: 5 Korena

In[19]:= `Plot[f[x], {x, 19, 20}]`

Out[19]=



```
In[22]:= f[19.]
```

```
Out[22]= -4.18114
```

```
In[23]:= f[20.]
```

```
Out[23]= 20.7175
```

```
In[29]:= f[x_] := 
$$\frac{\sqrt{x^3} - 14 * \text{Sin}[x]}{1 + \text{Cos}[x]^2} - 45$$

```

```
In[30]:= a = 19.; b = 20.;
```

```
In[31]:= For[n = 0, n ≤ 3, n++,
```

```
Print["n = ", n, " an = ", a, " bn = ", b,
```

```
" mn = ", m =  $\frac{a+b}{2}$ , " f(mn) = ", f[m], " εn = ",  $\frac{b-a}{2}$ ];
```

```
If[f[m] > 0, b = m, a = m]
```

```
]
```

```
n = 0 an = 19. bn = 20. mn = 19.5 f(mn) = 2.53021 εn = 0.5
```

```
n = 1 an = 19. bn = 19.5 mn = 19.25 f(mn) = -2.2511 εn = 0.25
```

```
n = 2 an = 19.25 bn = 19.5 mn = 19.375 f(mn) = -0.238705 εn = 0.125
```

```
n = 3 an = 19.375 bn = 19.5 mn = 19.4375 f(mn) = 1.0486 εn = 0.0625
```

```
In[32]:= f[x_] := 
$$\frac{\sqrt{x^3} - 14 * \text{Sin}[x]}{1 + \text{Cos}[x]^2} - 45$$

```

```
In[33]:= a = 19.; b = 20.;
```

```
In[34]:= epszad = 0.0000001;
```

```
eps = Infinity; (*стойност по-голяма от зададената грешка*)
```

```
For[n = 0, eps > epszad, n++,
```

```
Print["n = ", n, " an = ", a, " bn = ", b,
```

```
" mn = ", m =  $\frac{a+b}{2}$ , " f(mn) = ", f[m], " εn = ", eps =  $\frac{b-a}{2}$ ];
```

```
If[f[m] > 0, b = m, a = m]
```

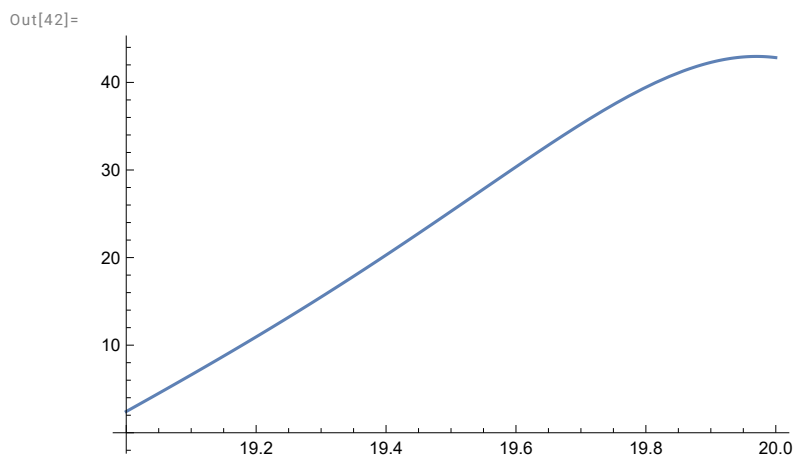
```
]
```

$n = 0$ $a_n = 19.$ $b_n = 20.$ $m_n = 19.5$ $f(m_n) = 2.53021$ $\varepsilon_n = 0.5$
 $n = 1$ $a_n = 19.$ $b_n = 19.5$ $m_n = 19.25$ $f(m_n) = -2.2511$ $\varepsilon_n = 0.25$
 $n = 2$ $a_n = 19.25$ $b_n = 19.5$ $m_n = 19.375$ $f(m_n) = -0.238705$ $\varepsilon_n = 0.125$
 $n = 3$ $a_n = 19.375$ $b_n = 19.5$ $m_n = 19.4375$ $f(m_n) = 1.0486$ $\varepsilon_n = 0.0625$
 $n = 4$ $a_n = 19.375$ $b_n = 19.4375$ $m_n = 19.4063$ $f(m_n) = 0.380945$ $\varepsilon_n = 0.03125$
 $n = 5$ $a_n = 19.375$ $b_n = 19.4063$ $m_n = 19.3906$ $f(m_n) = 0.0651594$ $\varepsilon_n = 0.015625$
 $n = 6$ $a_n = 19.375$ $b_n = 19.3906$ $m_n = 19.3828$ $f(m_n) = -0.0882577$ $\varepsilon_n = 0.0078125$
 $n = 7$ $a_n = 19.3828$ $b_n = 19.3906$ $m_n = 19.3867$ $f(m_n) = -0.011921$ $\varepsilon_n = 0.00390625$
 $n = 8$ $a_n = 19.3867$ $b_n = 19.3906$ $m_n = 19.3887$ $f(m_n) = 0.0265262$ $\varepsilon_n = 0.00195313$
 $n = 9$ $a_n = 19.3867$ $b_n = 19.3887$ $m_n = 19.3877$ $f(m_n) = 0.00727932$ $\varepsilon_n = 0.000976563$
 $n = 10$ $a_n = 19.3867$ $b_n = 19.3877$ $m_n = 19.3872$ $f(m_n) = -0.00232666$ $\varepsilon_n = 0.000488281$
 $n = 11$ $a_n = 19.3872$ $b_n = 19.3877$ $m_n = 19.3875$ $f(m_n) = 0.00247488$ $\varepsilon_n = 0.000244141$
 $n = 12$ $a_n = 19.3872$ $b_n = 19.3875$ $m_n = 19.3873$ $f(m_n) = 0.0000737473$ $\varepsilon_n = 0.00012207$
 $n = 13$ $a_n = 19.3872$ $b_n = 19.3873$ $m_n = 19.3873$ $f(m_n) = -0.00112655$ $\varepsilon_n = 0.0000610352$
 $n = 14$ $a_n = 19.3873$ $b_n = 19.3873$ $m_n = 19.3873$ $f(m_n) = -0.000526422$ $\varepsilon_n = 0.0000305176$
 $n = 15$ $a_n = 19.3873$ $b_n = 19.3873$ $m_n = 19.3873$ $f(m_n) = -0.000226343$ $\varepsilon_n = 0.0000152588$
 $n = 16$ $a_n = 19.3873$ $b_n = 19.3873$ $m_n = 19.3873$ $f(m_n) = -0.0000762993$ $\varepsilon_n = 7.62939 \times 10^{-6}$
 $n = 17$ $a_n = 19.3873$ $b_n = 19.3873$ $m_n = 19.3873$ $f(m_n) = -1.27638 \times 10^{-6}$ $\varepsilon_n = 3.8147 \times 10^{-6}$
 $n = 18$ $a_n = 19.3873$ $b_n = 19.3873$ $m_n = 19.3873$ $f(m_n) = 0.0000362354$ $\varepsilon_n = 1.90735 \times 10^{-6}$
 $n = 19$ $a_n = 19.3873$ $b_n = 19.3873$ $m_n = 19.3873$ $f(m_n) = 0.0000174795$ $\varepsilon_n = 9.53674 \times 10^{-7}$
 $n = 20$ $a_n = 19.3873$ $b_n = 19.3873$ $m_n = 19.3873$ $f(m_n) = 8.10153 \times 10^{-6}$ $\varepsilon_n = 4.76837 \times 10^{-7}$
 $n = 21$ $a_n = 19.3873$ $b_n = 19.3873$ $m_n = 19.3873$ $f(m_n) = 3.41257 \times 10^{-6}$ $\varepsilon_n = 2.38419 \times 10^{-7}$
 $n = 22$ $a_n = 19.3873$ $b_n = 19.3873$ $m_n = 19.3873$ $f(m_n) = 1.0681 \times 10^{-6}$ $\varepsilon_n = 1.19209 \times 10^{-7}$
 $n = 23$ $a_n = 19.3873$ $b_n = 19.3873$ $m_n = 19.3873$ $f(m_n) = -1.04144 \times 10^{-7}$ $\varepsilon_n = 5.96046 \times 10^{-8}$

In[40]:= $\text{Log2}\left[\frac{20. - 19.}{0.0000001}\right] - 1$

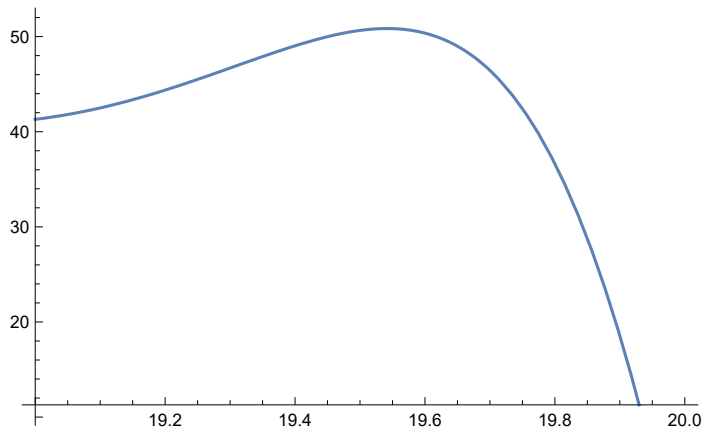
Out[40]=
22.2535

In[42]:= $\text{Plot}[f'[x], \{x, 19., 20.\}]$



```
In[43]:= Plot[f''[x], {x, 19., 20.}]
```

```
Out[43]=
```



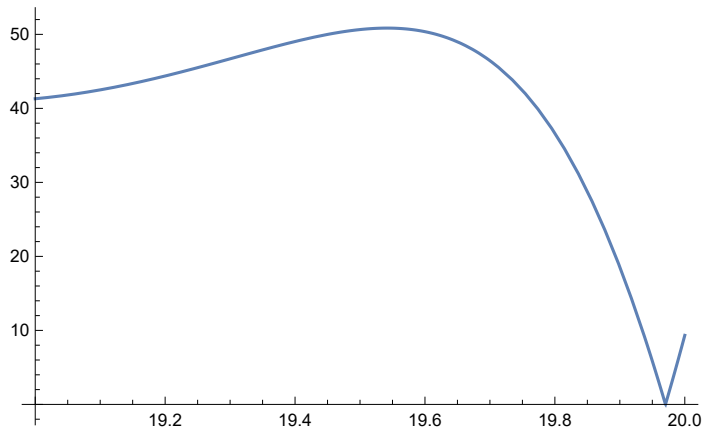
```
In[44]:= x0 = 19.
```

```
Out[44]=
```

```
19.
```

```
In[45]:= Plot[Abs[f''[x]], {x, 19., 20.}]
```

```
Out[45]=
```



```
In[49]:= M2 = Abs[f''[20.]]
```

```
Out[49]=
```

```
9.33048
```

```
In[47]:= m1 = Abs[f'[19.]]
```

```
Out[47]=
```

```
2.42418
```

```
In[50]:= P =  $\frac{M2}{2 m1}$ 
```

```
Out[50]=
```

```
1.92446
```

```

In[97]:= f[x_] := 
$$\frac{\sqrt{x^3 - 14} \sin[x]}{1 + \cos[x]^2} - 45$$

x0 = 19.;
M2 = Abs[f''[20.]];
m1 = Abs[f'[19.]];
P = 
$$\frac{M2}{2 m1}$$
;
epszad = 0.0000001;
eps = Infinity;
Print["n = ", 0, " xn = ", x0, " f(xn) = ", f[x0], " f'(xn) = ", f'[x0]]
For[n = 0, eps > epszad, n++,
  x1 = x0 - 
$$\frac{f[x0]}{f'[x0]}$$
;
  eps = P * Abs[x1 - x0]^2;
  x0 = x1;
  Print["n = ", n, " xn = ", x0,
    " f(xn) = ", f[x0], " f'(xn) = ", f'[x0], " εn = ", eps]
]
n = 0 xn = 19. f(xn) = -4.18114 f'(xn) = 2.42418
n = 0 xn = 20.7248 f(xn) = 29.3156 f'(xn) = -28.8815 εn = 5.72489
n = 1 xn = 21.7398 f(xn) = 5.50297 f'(xn) = -1.94961 εn = 1.98274
n = 2 xn = 24.5624 f(xn) = 30.6763 f'(xn) = -42.8066 εn = 15.3323
n = 3 xn = 25.279 f(xn) = 18.2003 f'(xn) = 6.02293 εn = 0.988312
n = 4 xn = 22.2572 f(xn) = 11.2876 f'(xn) = 25.4495 εn = 17.5731
n = 5 xn = 21.8137 f(xn) = 5.49152 f'(xn) = 1.64363 εn = 0.378578
n = 6 xn = 18.4726 f(xn) = 0.346913 f'(xn) = -20.1732 εn = 21.4826
n = 7 xn = 18.4898 f(xn) = 0.00701639 f'(xn) = -19.3585 εn = 0.000569117
n = 8 xn = 18.4901 f(xn) = 3.09693×10-6 f'(xn) = -19.3414 εn = 2.5281×10-7
n = 9 xn = 18.4901 f(xn) = 5.8975×10-13 f'(xn) = -19.3414 εn = 4.93397×10-14

```

Zad 2

In[106]:=

$$A = \begin{pmatrix} 1 + \frac{2}{7} & 0.05 & -0.1 \\ 0.2 & 1 - \frac{1}{11} & 0 \\ 0.5 & 0 & 1 + \frac{3}{13} \end{pmatrix}; \quad b = \{9, 5, 5\};$$

```
Print["За сравнение, точното решение е ", LinearSolve[A, b]]
```

За сравнение, точното решение е {6.94213, 3.97273, 1.24226}

In[108]:=

```

n = Length[A];
IM = IdentityMatrix[n];
B = IM - A;
c = b;
Print["Итерационният процес е  $x^{(k+1)} =$ ", B // MatrixForm, " $. x^{(k)} +$ ", c // MatrixForm]

```

$$\text{Итерационният процес е } x^{(k+1)} = \begin{pmatrix} -\frac{2}{7} & -0.05 & 0.1 \\ -0.2 & \frac{1}{11} & 0 \\ -0.5 & 0 & -\frac{3}{13} \end{pmatrix} \cdot x^{(k)} + \begin{pmatrix} 9 \\ 5 \\ 5 \end{pmatrix}$$

Normi

$$\text{Max}\left[\text{Table}\left[\sum_{j=1}^n \text{Abs}[B[[i, j]]], \{i, n\}\right]\right] \text{ (*purva*)}$$

Out[113]=

0.730769

$$\text{Max}\left[\text{Table}\left[\sum_{i=1}^n \text{Abs}[B[[i, j]]], \{j, n\}\right]\right] \text{ (*vtora*)}$$

Out[114]=

0.985714

$$\sqrt{\sum_{i=1}^n \sum_{j=1}^n B[[i, j]]^2} \text{ (*tretia*)}$$

Out[115]=

0.667571

In[123]:=

$$A = \begin{pmatrix} 1 + \frac{2}{7} & 0.05 & -0.1 \\ 0.2 & 1 - \frac{1}{11} & 0 \\ 0.5 & 0 & 1 + \frac{3}{13} \end{pmatrix}; \quad b = \{9, 5, 5\};$$

```
Print["За сравнение, точното решение е ", LinearSolve[A, b]]
```

За сравнение, точното решение е {6.94213, 3.97273, 1.24226}

In[125]:=

```

n = Length[A];
IM = IdentityMatrix[n];
B = IM - A;
c = b;
Print["Итерационният процес е  $x^{(k+1)} =$ ", B // MatrixForm, " $. x^{(k)} +$ ", c // MatrixForm]

```

$$\text{Итерационният процес е } x^{(k+1)} = \begin{pmatrix} -\frac{2}{7} & -0.05 & 0.1 \\ -0.2 & \frac{1}{11} & 0 \\ -0.5 & 0 & -\frac{3}{13} \end{pmatrix} \cdot x^{(k)} + \begin{pmatrix} 9 \\ 5 \\ 5 \end{pmatrix}$$

In[139]:=

x = {6, 1, 15};

In[135]:=

$$\text{normB} = \sqrt{\sum_{i=1}^n \sum_{j=1}^n B[i, j]^2}$$

Out[135]=

0.667571

In[140]:=

normx0 = Norm[x, 1];

normc = Norm[c, 1];

For[k = 0, k ≤ 5, k++,

$$\text{Print}\left["k = ", k, " \ x^{(k)} = ", x, " \ \varepsilon_k = ", \text{eps} = \text{normB}^k \left(\text{normx0} + \frac{\text{normc}}{1 - \text{normB}} \right)\right];$$

x = B.x + c

]

k = 0 x^(k) = {6, 1, 15} ε_k = 79.1551k = 1 x^(k) = {8.73571, 3.89091, -1.46154} ε_k = 52.8417k = 2 x^(k) = {6.16338, 3.60658, 0.969421} ε_k = 35.2756k = 3 x^(k) = {7.15565, 4.09519, 1.6946} ε_k = 23.549k = 4 x^(k) = {6.92023, 3.94116, 1.03112} ε_k = 15.7206k = 5 x^(k) = {6.92885, 3.97424, 1.30194} ε_k = 10.4946

In[143]:=

$$\frac{\text{Log}\left[\frac{10^{-4}}{\text{normx0} + \frac{\text{normc}}{1 - \text{normB}}}\right]}{\text{Log}[\text{normB}]}$$

Out[143]=

33.6091

Proverka

In[152]:=

$$A = \begin{pmatrix} 1 + \frac{2}{7} & 0.05 & -0.1 \\ 0.2 & 1 - \frac{1}{11} & 0 \\ 0.5 & 0 & 1 + \frac{3}{13} \end{pmatrix}; \quad b = \{9, 5, 5\};$$

Print["За сравнение, точното решение е ", LinearSolve[A, b]]

За сравнение, точното решение е {6.94213, 3.97273, 1.24226}

In[154]:=

n = Length[A];

IM = IdentityMatrix[n];

B = IM - A;

c = b;

Print["Итерационният процес е x^(k+1) = ", B // MatrixForm, ". x^(k) + ", c // MatrixForm]

$$\text{Итерационният процес е } x^{(k+1)} = \begin{pmatrix} -\frac{2}{7} & -0.05 & 0.1 \\ -0.2 & \frac{1}{11} & 0 \\ -0.5 & 0 & -\frac{3}{13} \end{pmatrix} \cdot x^{(k)} + \begin{pmatrix} 9 \\ 5 \\ 5 \end{pmatrix}$$

In[159]:=

x = {6, 1, 15};

$$\text{In}[*]:= \text{normB} = \sqrt{\sum_{i=1}^n \sum_{j=1}^n B[i, j]^2}$$

In[160]:=

```

normx0 = Norm[x, 1];
normc = Norm[c, 1];
For[k = 0, k ≤ 40, k++,
  Print["k = ", k, " x(k) = ", x, " εk = ", eps = normBk (normx0 +  $\frac{\text{normc}}{1 - \text{normB}}$ ) ];
  x = B.x + c
]

k = 0 x(k) = {6, 1, 15} εk = 79.1551
k = 1 x(k) = {8.73571, 3.89091, -1.46154} εk = 52.8417
k = 2 x(k) = {6.16338, 3.60658, 0.969421} εk = 35.2756
k = 3 x(k) = {7.15565, 4.09519, 1.6946} εk = 23.549
k = 4 x(k) = {6.92023, 3.94116, 1.03112} εk = 15.7206
k = 5 x(k) = {6.92885, 3.97424, 1.30194} εk = 10.4946
k = 6 x(k) = {6.95181, 3.97553, 1.23513} εk = 7.00592
k = 7 x(k) = {6.9385, 3.97105, 1.23906} εk = 4.67695
k = 8 x(k) = {6.94292, 3.9733, 1.24481} εk = 3.1222
k = 9 x(k) = {6.94212, 3.97262, 1.24127} εk = 2.08429
k = 10 x(k) = {6.94203, 3.97272, 1.24249} εk = 1.39141
k = 11 x(k) = {6.94218, 3.97275, 1.24226} εk = 0.928868
k = 12 x(k) = {6.94211, 3.97272, 1.24224} εk = 0.620086
k = 13 x(k) = {6.94213, 3.97273, 1.24227} εk = 0.413951
k = 14 x(k) = {6.94213, 3.97273, 1.24226} εk = 0.276342
k = 15 x(k) = {6.94212, 3.97273, 1.24226} εk = 0.184478
k = 16 x(k) = {6.94213, 3.97273, 1.24226} εk = 0.123152
k = 17 x(k) = {6.94213, 3.97273, 1.24226} εk = 0.0822129
k = 18 x(k) = {6.94213, 3.97273, 1.24226} εk = 0.054883
k = 19 x(k) = {6.94213, 3.97273, 1.24226} εk = 0.0366383
k = 20 x(k) = {6.94213, 3.97273, 1.24226} εk = 0.0244587
k = 21 x(k) = {6.94213, 3.97273, 1.24226} εk = 0.0163279
k = 22 x(k) = {6.94213, 3.97273, 1.24226} εk = 0.0109001
k = 23 x(k) = {6.94213, 3.97273, 1.24226} εk = 0.00727657
k = 24 x(k) = {6.94213, 3.97273, 1.24226} εk = 0.00485763
k = 25 x(k) = {6.94213, 3.97273, 1.24226} εk = 0.00324281
k = 26 x(k) = {6.94213, 3.97273, 1.24226} εk = 0.00216481
k = 27 x(k) = {6.94213, 3.97273, 1.24226} εk = 0.00144516
k = 28 x(k) = {6.94213, 3.97273, 1.24226} εk = 0.00096475

```


$k = 29 \quad x^{(k)} = \{6.94213, 3.97273, 1.24226\} \quad \varepsilon_k = 0.00064404$
 $k = 30 \quad x^{(k)} = \{6.94213, 3.97273, 1.24226\} \quad \varepsilon_k = 0.000429943$
 $k = 31 \quad x^{(k)} = \{6.94213, 3.97273, 1.24226\} \quad \varepsilon_k = 0.000287017$
 $k = 32 \quad x^{(k)} = \{6.94213, 3.97273, 1.24226\} \quad \varepsilon_k = 0.000191605$
 $k = 33 \quad x^{(k)} = \{6.94213, 3.97273, 1.24226\} \quad \varepsilon_k = 0.00012791$
 $k = 34 \quad x^{(k)} = \{6.94213, 3.97273, 1.24226\} \quad \varepsilon_k = 0.0000853889$
 $k = 35 \quad x^{(k)} = \{6.94213, 3.97273, 1.24226\} \quad \varepsilon_k = 0.0000570032$
 $k = 36 \quad x^{(k)} = \{6.94213, 3.97273, 1.24226\} \quad \varepsilon_k = 0.0000380537$
 $k = 37 \quad x^{(k)} = \{6.94213, 3.97273, 1.24226\} \quad \varepsilon_k = 0.0000254036$
 $k = 38 \quad x^{(k)} = \{6.94213, 3.97273, 1.24226\} \quad \varepsilon_k = 0.0000169587$
 $k = 39 \quad x^{(k)} = \{6.94213, 3.97273, 1.24226\} \quad \varepsilon_k = 0.0000113211$
 $k = 40 \quad x^{(k)} = \{6.94213, 3.97273, 1.24226\} \quad \varepsilon_k = 7.55766 \times 10^{-6}$

In[147]:=

$$\frac{\text{Log}\left[\frac{10^{-7}}{\text{normx}0 + \frac{\text{normc}}{1 - \text{normB}}}\right]}{\text{Log}[\text{normB}]}$$

Out[147]=

50.3745

Zad4

In[199]:=

```

a = 0.; b = 0.5;
x = a;
y = 4.;
f[x_, y_] := y - 10 * Sin[x];

```

In[203]:=

```
n = 16;
```

In[204]:=

```
h = (b - a) / n;
```

In[205]:=

```

Print["Мрежата е с n = ", n, " и стъпка h = ", h]
(*Изчисляваме теоретичната грешка*)
Print["Теоретичната локална грешка е ", h^2]
Print["Теоретичната глобална грешка е ", h]
(*намираме неизвестните стойности за y_i*)
For[i = 0, i <= n, i++,
  Print["i = ", i, " x_i = ", x, " y_i = ", y, " f_i = ", f[x, y]];
  y = y + h * f[x, y];
  x = x + h
]

```

Мрежата е с $n = 16$ и стъпка $h = 0.03125$

Теоретичната локална грешка е 0.000976563

Теоретичната глобална грешка е 0.03125

$i = 0 \quad x_i = 0. \quad y_i = 4. \quad f_i = 4.$

$i = 1 \quad x_i = 0.03125 \quad y_i = 4.125 \quad f_i = 3.81255$

$i = 2 \quad x_i = 0.0625 \quad y_i = 4.24414 \quad f_i = 3.61955$

$i = 3 \quad x_i = 0.09375 \quad y_i = 4.35725 \quad f_i = 3.42113$

$i = 4 \quad x_i = 0.125 \quad y_i = 4.46416 \quad f_i = 3.21742$

$i = 5 \quad x_i = 0.15625 \quad y_i = 4.56471 \quad f_i = 3.00856$

$i = 6 \quad x_i = 0.1875 \quad y_i = 4.65872 \quad f_i = 2.79469$

$i = 7 \quad x_i = 0.21875 \quad y_i = 4.74606 \quad f_i = 2.57596$

$i = 8 \quad x_i = 0.25 \quad y_i = 4.82656 \quad f_i = 2.35252$

$i = 9 \quad x_i = 0.28125 \quad y_i = 4.90007 \quad f_i = 2.12451$

$i = 10 \quad x_i = 0.3125 \quad y_i = 4.96646 \quad f_i = 1.89208$

$i = 11 \quad x_i = 0.34375 \quad y_i = 5.02559 \quad f_i = 1.65539$

$i = 12 \quad x_i = 0.375 \quad y_i = 5.07732 \quad f_i = 1.4146$

$i = 13 \quad x_i = 0.40625 \quad y_i = 5.12153 \quad f_i = 1.16986$

$i = 14 \quad x_i = 0.4375 \quad y_i = 5.15809 \quad f_i = 0.921325$

$i = 15 \quad x_i = 0.46875 \quad y_i = 5.18688 \quad f_i = 0.669164$

$i = 16 \quad x_i = 0.5 \quad y_i = 5.20779 \quad f_i = 0.413535$

Tochno reshenie

Obshto

In[188]:=

```
Clear[x, y]
DSolve[y'[x] == y[x] - 10 * Sin[x], y[x], x]
```

Out[189]=

```
{ {y[x] -> e^x C[1] + 5 (Cos[x] + Sin[x]) } }
```

Chastno

In[190]:=

```
Clear[x, y]
DSolve[{y'[x] == y[x] - 10 * Sin[x], y[0] == 4}, y[x], x]
```

Out[191]=

```
{ {y[x] -> -e^x + 5 Cos[x] + 5 Sin[x] } }
```

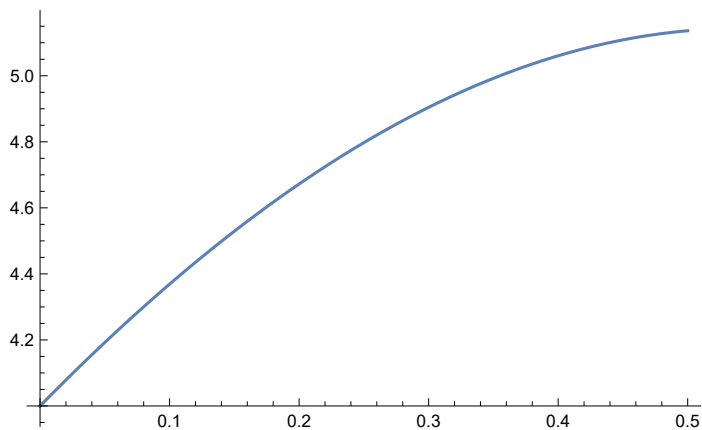
In[193]:=

```
yt[x_] := -e^x + 5 Cos[x] + 5 Sin[x]
```

In[194]:=

Plot[yt[x], {x, 0., 0.5}]

Out[194]=



Modificiran Metod na Oller

In[217]:=

```

a = 0.; b = 0.5;
x = a;
y = 4.;
f[x_, y_] := y - 10 * Sin[x];

```

In[209]:=

points = {{x, y}};In[]:= **n = 16;**

In[221]:=

$$h = \frac{b - a}{n};$$

In[226]:=

Print["Мрежата е с n = ", n, " и стъпка h = ", h]

(*Изчисляваме теоретичната грешка*)

Print["Теоретичната локална грешка е ", h^3]**Print**["Теоретичната глобална грешка е ", h^2]

Мрежата е с n = 16 и стъпка h = 0.03125

Теоретичната локална грешка е 0.0000305176

Теоретичната глобална грешка е 0.000976563

In[222]:=

```

For[i = 0, i ≤ n, i++,
  x12 = x +  $\frac{h}{2}$ ;
  y12 = y +  $\frac{h}{2}$  * f[x, y];
  Print["i = ", i, " xi = ", x, " yi = ", y, " fi = ",
    f[x, y], " xi+1/2 = ", x12, " yi+1/2 = ", y12, " fi+1/2 = ", f[x12, y12],
    " yточно = ", yt[x], " истинска грешка = ", Abs[y - yt[x]]];
  y = y + h * f[x12, y12];
  x = x + h;
  AppendTo[points, {x, y}]
]
(*визуализация на резултатите*)
gryt = Plot[yt[x], {x, 0., 0.5}, PlotStyle → Red];
grp = ListPlot[points, PlotStyle → Black];
Show[gryt, grp]

```

$i = 0$ $x_i = 0$. $y_i = 4$. $f_i = 4$. $x_{i+1/2} = 0.015625$
 $y_{i+1/2} = 4.0625$ $f_{i+1/2} = 3.90626$ $y_{\text{точно}} = 4$. истинска грешка = 0.

$i = 1$ $x_i = 0.03125$ $y_i = 4.12207$ $f_i = 3.80962$ $x_{i+1/2} = 0.046875$ $y_{i+1/2} = 4.1816$ $f_{i+1/2} = 3.71302$ $y_{\text{точно}} = 4.12204$ истинска грешка = 0.0000305563

$i = 2$ $x_i = 0.0625$ $y_i = 4.2381$ $f_i = 3.61351$ $x_{i+1/2} = 0.078125$ $y_{i+1/2} = 4.29456$ $f_{i+1/2} = 3.51411$ $y_{\text{точно}} = 4.23804$ истинска грешка = 0.0000626239

$i = 3$ $x_i = 0.09375$ $y_i = 4.34792$ $f_i = 3.41179$ $x_{i+1/2} = 0.109375$ $y_{i+1/2} = 4.40123$ $f_{i+1/2} = 3.30966$ $y_{\text{точно}} = 4.34782$ истинска грешка = 0.0000962306

$i = 4$ $x_i = 0.125$ $y_i = 4.45134$ $f_i = 3.2046$ $x_{i+1/2} = 0.140625$ $y_{i+1/2} = 4.50142$ $f_{i+1/2} = 3.0998$ $y_{\text{точно}} = 4.45121$ истинска грешка = 0.000131405

$i = 5$ $x_i = 0.15625$ $y_i = 4.54821$ $f_i = 2.99206$ $x_{i+1/2} = 0.171875$ $y_{i+1/2} = 4.59496$ $f_{i+1/2} = 2.88466$ $y_{\text{точно}} = 4.54805$ истинска грешка = 0.000168177

$i = 6$ $x_i = 0.1875$ $y_i = 4.63836$ $f_i = 2.77433$ $x_{i+1/2} = 0.203125$ $y_{i+1/2} = 4.68171$ $f_{i+1/2} = 2.6644$ $y_{\text{точно}} = 4.63815$ истинска грешка = 0.000206576

$i = 7$ $x_i = 0.21875$ $y_i = 4.72162$ $f_i = 2.55153$ $x_{i+1/2} = 0.234375$ $y_{i+1/2} = 4.76149$ $f_{i+1/2} = 2.43914$ $y_{\text{точно}} = 4.72138$ истинска грешка = 0.000246633

$i = 8$ $x_i = 0.25$ $y_i = 4.79784$ $f_i = 2.32381$ $x_{i+1/2} = 0.265625$ $y_{i+1/2} = 4.83415$ $f_{i+1/2} = 2.20903$ $y_{\text{точно}} = 4.79756$ истинска грешка = 0.000288381

$i = 9$ $x_i = 0.28125$ $y_i = 4.86688$ $f_i = 2.09131$ $x_{i+1/2} = 0.296875$ $y_{i+1/2} = 4.89955$ $f_{i+1/2} = 1.97422$ $y_{\text{точно}} = 4.86655$ истинска грешка = 0.000331852

$i = 10$ $x_i = 0.3125$ $y_i = 4.92857$ $f_i = 1.85419$ $x_{i+1/2} = 0.328125$ $y_{i+1/2} = 4.95754$ $f_{i+1/2} = 1.73486$ $y_{\text{точно}} = 4.92819$ истинска грешка = 0.000377081

$i = 11$ $x_i = 0.34375$ $y_i = 4.98279$ $f_i = 1.61259$ $x_{i+1/2} = 0.359375$ $y_{i+1/2} = 5.00798$ $f_{i+1/2} = 1.49109$ $y_{\text{точно}} = 4.98236$ истинска грешка = 0.000424103

$i = 12$ $x_i = 0.375$ $y_i = 5.02938$ $f_i = 1.36666$ $x_{i+1/2} = 0.390625$ $y_{i+1/2} = 5.05074$ $f_{i+1/2} = 1.24307$ $y_{\text{точно}} = 5.02891$ истинска грешка = 0.000472953

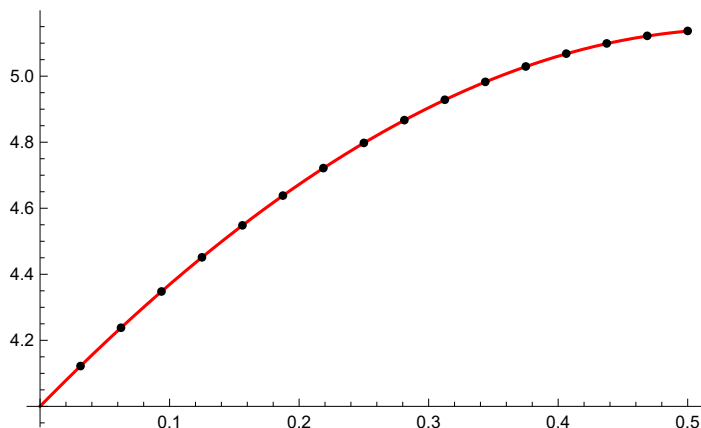
$i = 13$ $x_i = 0.40625$ $y_i = 5.06823$ $f_i = 1.11655$ $x_{i+1/2} = 0.421875$ $y_{i+1/2} = 5.08567$ $f_{i+1/2} = 0.990957$ $y_{\text{точно}} = 5.0677$ истинска грешка = 0.00052367

$i = 14$ $x_i = 0.4375$ $y_i = 5.0992$ $f_i = 0.862433$ $x_{i+1/2} = 0.453125$ $y_{i+1/2} = 5.11267$ $f_{i+1/2} = 0.734898$ $y_{\text{точно}} = 5.09862$ истинска грешка = 0.000576293

$i = 15$ $x_i = 0.46875$ $y_i = 5.12216$ $f_i = 0.604447$ $x_{i+1/2} = 0.484375$ $y_{i+1/2} = 5.13161$ $f_{i+1/2} = 0.475052$ $y_{\text{точно}} = 5.12153$ истинска грешка = 0.000630861

$i = 16$ $x_i = 0.5$ $y_i = 5.13701$ $f_i = 0.342751$ $x_{i+1/2} = 0.515625$ $y_{i+1/2} = 5.14236$ $f_{i+1/2} = 0.211575$ $y_{\text{точно}} = 5.13632$ истинска грешка = 0.000687417

Out[225]=



Zad3

In[229]:=

xt = Table[9 + q * (0.5), {q, -5, 5}]

Out[229]=

{6.5, 7., 7.5, 8., 8.5, 9., 9.5, 10., 10.5, 11., 11.5}

In[230]:=

bigN = Length[xt]

Out[230]=

11

In[231]:=

f[x_] := x - 3 * Cos[x]

In[232]:=

yt = f[xt]

Out[232]=

{3.57024, 4.73829, 6.46009, 8.4365, 10.306,
11.7334, 12.4915, 12.5172, 11.9266, 10.9867, 10.0501}

In[233]:=

xt²

Out[233]=

{42.25, 49., 56.25, 64., 72.25, 81., 90.25, 100., 110.25, 121., 132.25}

In[234]:=

yt * xt

Out[234]=

{23.2065, 33.1681, 48.4507, 67.492, 87.6013,
105.601, 118.669, 125.172, 125.229, 120.854, 115.576}

In[235]:=

xt³

Out[235]=

{274.625, 343., 421.875, 512., 614.125, 729., 857.375, 1000., 1157.63, 1331., 1520.88}

In[236]:=

xt⁴

Out[236]=

{1785.06, 2401., 3164.06, 4096., 5220.06,
6561., 8145.06, 10000., 12155.1, 14641., 17490.1}

In[237]:=

yt * xt²

Out[237]=

{150.843, 232.176, 363.38, 539.936, 744.611,
950.405, 1127.36, 1251.72, 1314.91, 1329.39, 1329.12}

In[238]:=

 $\sum_{i=1}^{\text{bigN}} \text{xt}[[i]]$

Out[238]=

99.

In[239]:=

$$\sum_{i=1}^{\text{bigN}} y_t[[i]]$$

Out[239]=

103.217

In[240]:=

$$\sum_{i=1}^{\text{bigN}} x_t[[i]]^2$$

Out[240]=

918.5

In[241]:=

$$\sum_{i=1}^{\text{bigN}} y_t[[i]] * x_t[[i]]$$

Out[241]=

971.02

In[242]:=

$$\sum_{i=1}^{\text{bigN}} x_t[[i]]^3$$

Out[242]=

8761.5

In[243]:=

$$\sum_{i=1}^{\text{bigN}} x_t[[i]]^4$$

Out[243]=

85 658.4

In[244]:=

$$\sum_{i=1}^{\text{bigN}} y_t[[i]] * x_t[[i]]^2$$

Out[244]=

9333.86

In[250]:=

$$A = \begin{pmatrix} 11 & 99. & 918.5 \\ 99. & 918.5 & 8761.5 \\ 918.5 & 8761.5 & 85\,658.4 \end{pmatrix}; \quad b = \{103.217, 971.02, 9333.86\};$$

LinearSolve[A, b]

Out[251]=

{-65.7874, 15.6095, -0.782209}

In[253]:=

P2[x_] := -0.782209 x² + 15.6095 x - 65.7874

In[254]:=

```
grP2 = Plot[P2[x], {x, xt[[1]] - 0.2, xt[[bigN]] + 0.2}, PlotStyle -> Green]
```

Out[254]=

