Задача 15 Да се пресметне границата

$$\lim_{n\to\infty}\sum_{k=0}^{n}\frac{1}{(2k+1)(2k+3)}$$

Р-е:

$$\frac{1}{(2k+1)(2k+3)}=\underbrace{\frac{A}{2k+1}+\frac{B}{2k+3}}_{(2k+1)(2k+3)}=$$

$2k+3$ $\quad$ $2k+1$

$$=\frac{A(2k+3)+B(2k+1)}{(2k+1)(2k+3)}=$$

$$=\frac{2A\cdot k+3A+2Bk+B}{(2k+1)(2k+3)}=$$

$$=\frac{(2A+2B)\cdot k+(3A+B)}{(2k+1)(2k+3)}=\frac{1+0\cdot k}{(2k+1)(2k+3)}$$

$$\left|\begin{array}{l}2A+2B=0 \quad /:2 \\ 3A+B=1\end{array}\right. \qquad \begin{array}{l}A+B=0 \\ A=-B\end{array}$$

$$-3B+B=1 \quad => \quad B=-\frac{1}{2} \qquad A=\frac{1}{2}$$

$$\frac{1}{(2k+1)(2k+3)} = \frac{\frac{1}{2}}{2k+1} - \frac{\frac{1}{2}}{2k+3} =$$

$$= \frac{1}{4k+2} - \frac{1}{4k+6}$$

$$\underbrace{\lim_{n\to\infty} \sum_{k=0}^{n} \left(\frac{1}{4k+2} - \frac{1}{4k+6} \right)} =$$

$$= \lim_{n\to\infty} \left(\left(\frac{1}{2} - \cancel{\frac{1}{6}} \right) + \left(\cancel{\frac{1}{6}} - \cancel{\frac{1}{10}} \right) + \left(\cancel{\frac{1}{10}} - \cancel{\frac{1}{14}} \right) + \cancel{\cdots} + \left(\cancel{\frac{1}{4n+2}} - \frac{1}{4n+6} \right) \right) =$$

$$= \lim_{n\to\infty} \underbrace{\left(\overset{2n+3}{\frac{1}{2}} - \overset{1}{\frac{1}{4n+6}} \right)}_{2(2n+3)} =$$

$$= \lim_{n\to\infty} \frac{2n+3-1}{4n+6} = \lim_{n\to\infty} \frac{2n+2}{4n+6} = \frac{\cancel{2}}{\cancel{4}}\ \frac{1}{2}$$

Задача 16 Да се пресметне границата

$$\lim_{n\to\infty} \frac{1}{n^2} \sum_{k=1}^{n} k$$

Р-е:

$$\lim_{n\to\infty} \frac{1}{n^2} \cdot \left(\underbrace{1+2+3+\dots+n}_{\frac{n(n+1)}{2}} \right)$$

$\frac{+}{\cdot}$ $a_1 = 1$ $d = 1$

$$S_n = \frac{2a_1+(n-1)d}{2} \cdot n$$

$$S_n = \frac{2+n-1}{2} \cdot n = \frac{n(n+1)}{2}$$

$$\lim_{n\to\infty} \frac{1}{n^2} \frac{n(n+1)}{2} = \lim_{n\to\infty} \frac{n+1}{2n} = \frac{1}{2}$$

Задача 17 $\lim_{n\to\infty} \frac{1}{n^3} \sum_{k=1}^{n} k^2$

$$\sum_{k=1}^{n} k^2 = 1^2 + 2^2 + 3^2 + 4^2 + \dots + n^2$$

Задача 18 Да се пресметне границата

$$\lim_{n\to\infty} \frac{n\cdot\sqrt[3]{5n^2}+\sqrt[4]{9n^8+1}}{(n+\sqrt{n})\left(\sqrt{7-n+n^2}\right)} = \qquad n^1\cdot n^{\frac{2}{3}} \qquad 1+\frac{2}{3}=\frac{5}{3}$$

$$= \lim_{n\to\infty} \frac{n^1\cdot\sqrt[3]{5}\cdot n^{\frac{2}{3}}+\sqrt[4]{n^8\left(9+\frac{1}{n^8}\right)}}{n\left(1+\frac{1}{\sqrt{n}}\right)\left(\sqrt{n^2\left(\frac{7}{n^2}-\frac{1}{n}+1\right)}\right)} =$$

$$= \lim_{n\to\infty} \frac{n^{\frac{5}{3}}\cdot\sqrt[3]{5}+n^2\sqrt[4]{9+\frac{1}{n^8}}}{n^2\left(1+\frac{1}{\sqrt{n}}\right)\left(\sqrt{\frac{7}{n^2}-\frac{1}{n}+1}\right)} =$$

$$= \lim_{n\to\infty} \frac{\cancel{n^2}\left(\underbrace{\frac{\sqrt[3]{5}}{\sqrt[6]{n}}}_{\to 0}+\sqrt[4]{9+\underbrace{\frac{1}{n^8}}_{\to 0}}\right)}{\cancel{n^2}\left(1+\underbrace{\frac{1}{\sqrt{n}}}_{\to 0}\right)\left(\sqrt{\underbrace{\frac{7}{n^2}}_{\to 0}-\underbrace{\frac{1}{n}}_{\to 0}+1}\right)} =$$

$$= \frac{\sqrt[4]{9}}{\sqrt{1}} = \sqrt[4]{9}$$

19 Задача $\lim\limits_{n\to\infty} \dfrac{\sqrt{n-1}-\sqrt{n^2+1}}{\sqrt[3]{3n^3+3}+\sqrt[4]{n^5+1}} =$

$$= \lim_{n\to\infty} \frac{\sqrt{n\left(1-\frac{1}{n}\right)}-\sqrt{n^2\left(1+\frac{1}{n^2}\right)}}{\sqrt[3]{n^3\left(3+\frac{3}{n^3}\right)}+\sqrt[4]{n^5\left(1+\frac{1}{n^5}\right)}} =$$

$$= \lim_{n\to\infty} \frac{n^{\frac{1}{2}}\cdot\sqrt{\left(1-\frac{1}{n}\right)}-n\sqrt{1+\frac{1}{n^2}}}{n\sqrt[3]{\left(3+\frac{3}{n^3}\right)}+n^{\frac{5}{4}}\sqrt[4]{\left(1+\frac{1}{n^5}\right)}} =$$

$$= \lim_{n\to\infty} \frac{\cancel{n}\left(\overbrace{\frac{1}{\sqrt{n}}\cdot\sqrt{1-\frac{1}{n}}}^{\to 0}-\sqrt{1+\overbrace{\frac{1}{n^2}}^{\to 0}}\right)}{n^{5/4}\left(\frac{1}{\sqrt[4]{n}}\cdot\sqrt[3]{3+\underbrace{\frac{3}{n^3}}_{\to 0}}+\sqrt[4]{1+\underbrace{\frac{1}{n^5}}_{\to 0}}\right)} = 0$$

20 зад $\lim\limits_{n\to\infty} \dfrac{n\sqrt[5]{n}-\sqrt[3]{27n^6+n^2}}{(n+\sqrt[4]{n})\sqrt{9+n^2}}$

21 зад $\lim\limits_{n\to\infty} \dfrac{\sqrt{2n^3+1}}{n+2\sqrt{n^3}}$

22 зад. Да се пресм. гран.

$$\lim_{n\to\infty} \frac{\sqrt{2n^2+1}+n}{\sqrt[3]{n^3+7}-1} =$$

$$\lim_{n\to\infty} \frac{\sqrt{n^2\left(2+\frac{1}{n^2}\right)}+n}{\sqrt[3]{n^3\left(1+\frac{7}{n^3}\right)}-1} =$$

$$= \lim_{n\to\infty} \frac{n\sqrt{2+\frac{1}{n^2}}+n}{n\sqrt[3]{1+\frac{7}{n^3}}-1} =$$

$$= \lim_{n\to\infty} \frac{\cancel{n}\left(\sqrt{2+\frac{1}{n^2}}+1\right)}{\cancel{n}\left(\sqrt[3]{1+\frac{7}{n^3}}-\frac{1}{n}\right)} = \frac{\sqrt{2}+1}{\sqrt[3]{1}} =$$

$\frac{1}{n^2}\to 0$, $\frac{7}{n^3}\to 0$, $\frac{1}{n}\to 0$

$$= \sqrt{2}+1$$

23 зад. Да се пресм. гран.:

$$\lim_{n\to\infty} \left(\sqrt{2n+3}-\sqrt{2n-1}\right) = [\infty-\infty]$$

$$\lim_{n\to\infty} \frac{\left(\sqrt{2n+3}-\sqrt{2n-1}\right)\cdot\left(\sqrt{2n+3}+\sqrt{2n-1}\right)}{\sqrt{2n+3}+\sqrt{2n-1}} =$$

$$=\lim_{n\to\infty} \frac{\left(\sqrt{2n+3}\right)^2-\left(\sqrt{2n-1}\right)^2}{\sqrt{2n+3}+\sqrt{2n-1}} =$$

$$=\lim_{n\to\infty} \frac{2n+3-(2n-1)}{\sqrt{2n+3}+\sqrt{2n-1}} =$$

$$=\lim_{n\to\infty} \frac{\cancel{2n}+3-\cancel{2n}+1}{\sqrt{n\left(2+\frac{3}{n}\right)}+\sqrt{n\left(2-\frac{1}{n}\right)}} =$$

$$\sqrt{n} \qquad \sqrt{n}$$

$$=\lim_{n\to\infty} \frac{4}{\sqrt{n}\left(\sqrt{2+\underset{\to 0}{\frac{3}{n}}}+\sqrt{2-\underset{\to 0}{\frac{1}{n}}}\right)} = 0$$

<u>Зад. 24</u> $$\lim_{n\to\infty} \frac{\left(7n-\sqrt{49n^2-3}\right)\cdot\left(7n+\sqrt{49n^2-3}\right)}{\left(7n+\sqrt{49n^2-3}\right)} =$$

$$= \lim_{n\to\infty} \frac{49n^2 - (49n^2+3)}{7n + \sqrt{49n^2+3}} =$$

$$= \lim_{n\to\infty} \frac{\cancel{49n^2} - \cancel{49n^2} - 3}{n\left(7 + \sqrt{49 + \frac{3}{n^2}}\right)} = 0$$

Зад. 25 Да се пресм. гран.

$$\lim_{n\to\infty} \left(\sqrt{3n^2+n-2} - \sqrt{3n^2+2n+3}\right)$$

$$\lim_{n\to\infty} \frac{\left(\sqrt{3n^2+n-2} - \sqrt{3n^2+2n+3}\right)\left(\sqrt{3n^2+n-2} + \sqrt{3n^2+2n+3}\right)}{\sqrt{3n^2+n-2} + \sqrt{3n^2+2n+3}} =$$

$$= \lim_{n\to\infty} \frac{3n^2+n-2-(3n^2+2n+3)}{n\left(\sqrt{3+\frac{1}{n}-\frac{2}{n^2}} + \sqrt{3+\frac{2}{n}+\frac{3}{n^2}}\right)} =$$

$$= \lim_{n\to\infty} \frac{\cancel{3n^2}+n-2-\cancel{3n^2}-2n-3}{n\left(\sqrt{3+\frac{1}{n}-\frac{2}{n^2}} + \sqrt{3+\frac{2}{n}+\frac{3}{n^2}}\right)} =$$

$$= \lim_{n\to\infty} \frac{-n-5}{n\left(\sqrt{3+\frac{1}{n}-\frac{2}{n^2}} + \sqrt{3+\frac{2}{n}+\frac{3}{n^2}}\right)} =$$

$$\lim_{n\to\infty} \frac{\cancel{n}\left(-1-\underbrace{\frac{5}{n}}_{\to 0}\right)}{\cancel{n}\left(\sqrt{3+\underbrace{\frac{1}{n}}_{\to 0}-\underbrace{\frac{2}{n^2}}_{\to 0}}+\sqrt{3+\underbrace{\frac{2}{n}}_{\to 0}+\underbrace{\frac{3}{n^2}}_{\to 0}}\right)} =$$

$$= \frac{-1}{2\sqrt{3}}$$

26 зад. Да се пресм. гран.

$$\lim_{n\to\infty} n\left(\sqrt{n^2+1}-n\right)$$

27 зад. Да се пресм. гран.

$$\lim_{n\to\infty} \frac{1}{n\left(\sqrt{n^2-1}-n\right)}$$

За решаването на следващите граници ще използваме, че $\lim_{n\to\infty} q^n = 0$ при $|q| < 1$ и $\lim_{n\to\infty} q^n = +\infty$ при $|q| > 1$

Зад. 28 Да се пресм. границата

$$\lim_{n\to\infty} \frac{2^{n+1}+3^{n+1}}{2^n+3^n} =$$

$$= \lim_{n\to\infty} \frac{2\cdot 2^n + 3\cdot 3^n}{2^n+3^n}$$

$$\lim_{n\to\infty} \frac{\cancel{3^n}\left(2\cdot\frac{2^n}{3^n} + \frac{3\cdot \cancel{3^n}}{\cancel{3^n}}\right)}{\cancel{3^n}\left(\frac{2^n}{3^n}+\frac{\cancel{3^n}}{\cancel{3^n}}\right)} =$$

$$= \lim_{n\to\infty} \frac{2\cdot\underbrace{\left(\frac{2}{3}\right)^n}_{\to 0} + 3}{\underbrace{\left(\frac{2}{3}\right)^n}_{\to 0} + 1} = \frac{3}{1} = 3$$

Зад. 29 Да се пресм. гран.

$$\lim_{n\to\infty} \frac{6^{n+2}+5^{n+1}}{6^n+5^n} =$$

$$= \lim_{n\to\infty} \frac{36\cdot 6^n + 5\cdot 5^n}{6^n+5^n}$$

$$= \lim_{n\to\infty} \frac{\cancel{6^n}\left(36 + 5\cdot\left(\frac{5}{6}\right)^n\right)}{\cancel{6^n}\left(1 + \left(\frac{5}{6}\right)^n\right)} = 36$$

$\left(\frac{5}{6}\right)^n \to 0$

Зад. 30 Да се пресм. гран.

$$\lim_{n\to\infty} \frac{3^n + 5^n}{7^n} =$$

$$= \lim_{n\to\infty} \frac{\cancel{7^n}\left(\left(\frac{3}{7}\right)^n + \left(\frac{5}{7}\right)^n\right)}{\cancel{7^n}} = 0$$

$\left(\frac{3}{7}\right)^n \to 0$, $\left(\frac{5}{7}\right)^n \to 0$

Зад. 31 Да се пресм. гран.

$$\lim_{n\to\infty} \frac{1 + 3^{n+1}}{1 - 3^n}$$

Зад. 32 $\lim_{n\to\infty} \frac{2^n + 3^n}{4^n}$

Зад. 33 $\lim_{n\to\infty} \frac{2^{n+1} - 3^n + 6^n}{2^{n+3} + 3^{n+2} + 6^{n+1}}$

Th 12 Ако $\lim_{n\to\infty} a_n = a$, $\lim_{n\to\infty} b_n = b$

и $a_n \leq b_n$ за безброй много стойности на n, то $a \leq b$

Th 13 Ако за три редици са изпълнени неравенствата $a_n \leq c_n \leq b_n$ за всяко n и $\lim_{n\to\infty} a_n = \lim_{n\to\infty} b_n = c$,

то съществува $\lim_{n\to\infty} c_n = c$

34 зад Да се намери границата на редицата с общ член c_n, ако
$$c_n = \sum_{\nu=1}^{n} \frac{n}{n^2+\nu}$$

$$c_n = \frac{n}{n^2+1} + \frac{n}{n^2+2} + \frac{n}{n^2+3} + \dots + \frac{n}{n^2+n}$$

$$\Rightarrow a_n = \underbrace{\frac{n}{n^2+n} + \frac{n}{n^2+n} + \dots + \frac{n}{n^2+n}}_{n \text{ - брой}}$$

$a_n \leq c_n$

$$\Rightarrow b_n = \underbrace{\frac{n}{n^2+1} + \frac{n}{n^2+1} + \dots + \frac{n}{n^2+1}}_{n}$$

$$\quad \vee| \qquad \vee \qquad \qquad \vee$$

$$c_n = \frac{n}{n^2+1} + \frac{n}{n^2+2} + \dots + \frac{n}{n^2+n}$$

$$\underline{\underline{b_n \geq c_n}}$$

$$a_n \leq c_n \leq b_n$$

$$\lim_{n\to\infty} a_n = C \quad \| T13 \quad \lim_{n\to\infty} b_n = C$$

$$\Downarrow$$

$$\lim_{n\to\infty} c_n = C$$

$$\lim_{n\to\infty} a_n = \lim_{n\to\infty} n \cdot \frac{n}{n^2+1} = \lim_{n\to\infty} \frac{n^2}{n^2+1} = 1$$

$$\lim_{n\to\infty} b_n = \lim_{n\to\infty} n \frac{n}{n^2+1} = \lim_{n\to\infty} \frac{n^2}{n^2+1} = 1$$

$$\overset{T13}{\Longrightarrow}$$

$$\exists \lim_{n\to\infty} c_n = 1$$

Зад. 35 Да се намери границата на редицата с общ член c_n, ако

$$c_n = \sum_{\nu=1}^{n} \sqrt[3]{\frac{n}{2n^4+\nu}}$$

Р-е:

$$c_n = \sqrt[3]{\frac{n}{2n^4+1}} + \sqrt[3]{\frac{n}{2n^4+2}} + \dots + \sqrt[3]{\frac{n}{2n^4+n}}$$

$$a_n = \underbrace{\sqrt[3]{\frac{n}{2n^4+n}} + \sqrt[3]{\frac{n}{2n^4+n}} + \dots + \sqrt[3]{\frac{n}{2n^4+n}}}_{n \text{ брой пъти}}$$

$$b_n = \sqrt[3]{\frac{n}{2n^4+1}} + \sqrt[3]{\frac{n}{2n^4+1}} + \dots + \sqrt[3]{\frac{n}{2n^4+1}}$$

$$\Rightarrow \quad a_n \le c_n \le b_n$$

$$\lim_{n\to\infty} a_n = \lim_{n\to\infty} n \cdot \sqrt[3]{\frac{n}{2n^4+n}} =$$

$$= \lim_{n\to\infty} \sqrt[3]{\frac{1\cancel{n^4}}{\cancel{n^4}\left(2+\frac{1}{n^3}\right)}} = \sqrt[3]{\frac{1}{2}} = \frac{1}{\sqrt[3]{2}}, \qquad \frac{1}{n^3} \to 0$$

$$\lim_{n\to\infty} b_n = \lim_{n\to\infty} \sqrt[3]{n\frac{n}{2n^4+n}} =$$

$$= \lim_{n\to\infty} \sqrt[3]{\frac{n^4}{2n^4+n}} = \sqrt[3]{\frac{1}{2}} = \frac{1}{\sqrt[3]{2}}$$

$$\underset{\frac{1}{\sqrt[3]{2}}}{\overset{\parallel}{a_n}} \le c_n \le \underset{\frac{1}{\sqrt[3]{2}}}{\overset{\parallel}{b_n}} \qquad \text{T}k3$$

$$\Rightarrow \lim_{n\to\infty} c_n = \frac{1}{\sqrt[3]{2}}$$

<u>Th 14</u> Редицата $\{a_n\}_{n=1}^{\infty}$, където $a_n = \left(1+\frac{1}{n}\right)^n$, $n \in \mathbb{N}$ е растяща и ограничена

<u>Th 15</u> Нека $\{a_n\}_{n=1}^{\infty}$ е редица удовлетворяваща $a_n \neq 0$, $a_n \neq -1$ и $\lim_{n\to\infty} |a_n| = \infty$, тогава $\lim_{n\to\infty}\left(1+\frac{1}{a_n}\right)^{a_n} = e$

Corr. 1 За $\forall k \in \mathbb{Z}$ е в сила равенството $\lim\limits_{n\to\infty}\left(1+\frac{k}{n}\right)^n = e^k$

Зад. 36 Да се пресм. гран.?

$$\lim_{n\to\infty}\left(\frac{n+2}{n}\right)^n =$$

$$= \lim_{n\to\infty}\left(\frac{\cancel{n}}{\cancel{n}}+\frac{2}{n}\right)^n = \lim_{n\to\infty}\left(1+\frac{\overset{\in\mathbb{Z}}{2}}{n}\right)^n = e^2$$

Зад. 37 Да се пресм. границата

$$\lim_{n\to\infty}\left(1-\frac{2}{\underbrace{2n+3}_{a_n}}\right)^{n} = a_n$$

$$= \lim_{n\to\infty}\left(\left(1-\frac{2}{2n+3}\right)^{2n}\right)^{\frac{1}{2}} =$$

$$= \lim_{n\to\infty}\left(\left(1-\frac{2}{2n+3}\right)^{2n+3-3}\right)^{\frac{1}{2}} =$$

$$= \underbrace{\lim_{n\to\infty}\left(\left(1-\frac{2}{2n+3}\right)^{2n+3}\right)^{\frac{1}{2}}}_{(1)} \cdot \underbrace{\lim_{n\to\infty}\left(\left(1-\frac{2}{2n+3}\right)^{-3}\right)^{\frac{1}{2}}}_{(2)}$$

(1) $\lim\limits_{n\to\infty}\left(\left(1+\frac{(-2)}{2n+3}\right)^{2n+3}\right)^{\frac{1}{2}} = \left(e^{-2}\right)^{\frac{1}{2}} = \frac{1}{e}$

$$\sqrt{\frac{1}{e^2}} = \frac{1}{e}$$

(2) $\lim\limits_{n\to\infty}\left(\left(\overset{2n+3}{\underbrace{\frac{1}{1}-\frac{2}{2n+3}}}\right)^{-3}\right)^{\frac{1}{2}} =$

$$\lim_{n\to\infty}\left(\left(\overset{2n+3}{\underbrace{\frac{2n+3-2}{2n+3}}_{\frac{2n+1}{2n+3}}}\right)^{-3}\right)^{\frac{1}{2}} = 1^{-\frac{3}{2}} = 1$$

$$\lim_{n\to\infty}\left(1-\frac{2}{2n+3}\right)^{n} = \frac{1}{e}$$

$$\lim_{n\to\infty}\left(\underbrace{\frac{1}{1}-\frac{2}{2n+3}}\right)^{n} = \lim_{n\to\infty}\left(\frac{2n+1}{2n+3}\right)^{n} =$$

$\notin \mathbb{Z}$

$$= \lim_{n\to\infty}\frac{\left(\cancel{2n}\left(1+\frac{1}{2n}\right)\right)^{n}}{\left(\cancel{2n}\left(1+\frac{3}{2n}\right)\right)^{n}} = \lim_{n\to\infty}\frac{\left(1+\frac{\frac{1}{2}}{n}\right)^{n}}{\left(1+\frac{\frac{3}{2}}{n}\right)^{n}}$$

$$= \frac{e^{\frac{1}{2}}}{e^{\frac{3}{2}}} = \frac{1}{e^{\frac{3}{2}} \cdot e^{-\frac{1}{2}}} = \frac{1}{e}$$

Зад. 38 Да се пресм. гран.

$$\lim_{n\to\infty}\left(\frac{2n^2-n-1}{2n^2-n-6}\right)^n$$

$ax^2+bx+c = a(x-x_1)(x-x_2)$, където x_1 и x_2 са корени на кв. у-е.

$ax^2+bx+c=0$

$$2n^2-n-1 = 2(n-1)\left(\frac{n}{1}+\frac{1}{2}\right) = \cancel{2}\,\frac{(n-1)(2n+1)}{\cancel{2}}$$

$2n^2-n-1=0$

$$n_{1,2} = \frac{1\pm 3}{4} \quad n_1 = 1, \quad n_2 = -\frac{1}{2}$$

$D = 1+8 = 9$

$$2n^2-n-6 = 2(n-2)\left(\frac{n}{1}+\frac{3}{2}\right) = \cancel{2}\,\frac{(n-2)(2n+3)}{\cancel{2}}$$

$2n^2-n-6=0$

$$n_{1,2} = \frac{1\pm 7}{4} \quad n_1 = 2, \quad n_2 = -\frac{3}{2}$$

$D = 1+48 = 49$

$$\lim_{n\to\infty}\left(\frac{(n-1)(2n+1)}{(n-2)(2n+3)}\right)^n =$$

$$= \underbrace{\lim_{n\to\infty}\left(\frac{n-1}{n-2}\right)^n}_{(1)} \cdot \underbrace{\lim_{n\to\infty}\left(\frac{2n+1}{2n+3}\right)^n}_{(2)}$$

(1) $$\lim_{n\to\infty}\left(\frac{\cancel{n}\left(1-\frac{1}{n}\right)}{\cancel{n}\left(1-\frac{2}{n}\right)}\right)^n = \lim_{n\to\infty}\frac{\left(1-\frac{1}{n}\right)^n}{\left(1-\frac{2}{n}\right)^n} =$$

$$= \frac{e^{-1}}{e^{-2}} = e^{-1}\cdot e^{2} = \underline{\underline{e}}$$

(2) $$\lim_{n\to\infty}\left(\frac{2n+1}{2n+3}\right)^n =$$

$$\lim_{n\to\infty}\left(\frac{2n+3-2}{2n+3}\right)^n = \lim_{n\to\infty}\left(\cancel{\frac{2n+3}{2n+3}}-\frac{2}{2n+3}\right)^n$$

$$\lim_{n\to\infty}\left(1-\frac{2}{2n+3}\right)^n$$ → гран. от зад. 37 $\Rightarrow$

$$(2) = \frac{1}{e}$$

$$\lim_{n\to\infty}\left(\frac{2n^2-n-1}{2n^2-n-6}\right)^n = \cancel{e} \; \frac{1}{\cancel{e}} = 1$$

(1) (2) зад. 37

Зад. 39 $$\lim_{n\to\infty}\left(\frac{n-1}{n+1}\right)^{\frac{n}{2}} =$$

$$= \lim_{n\to\infty}\left(\left(\frac{\cancel{n}\left(1-\frac{1}{n}\right)}{\cancel{n}\left(1+\frac{1}{n}\right)}\right)^n\right)^{\frac{1}{2}} =$$

$$= \sqrt{\frac{e^{-1}}{e^{1}}} = \sqrt{\frac{1}{e^2}} = \frac{1}{e}$$

Зад. 40 $$\lim_{n\to\infty}\left(\frac{n^2-1}{n^2-n-6}\right)^n =$$

$$= \lim_{n\to\infty}\left(\frac{(n-1)(n+1)}{(n-3)(n+2)}\right)^n =$$

$$= \lim_{n\to\infty}\left(\frac{n-1}{n-3}\right)^n \cdot \lim_{n\to\infty}\left(\frac{n+1}{n+2}\right)^n =$$

$$= \underline{\lim_{n\to\infty}\left(\frac{\cancel{n}\left(1-\frac{1}{n}\right)}{\cancel{n}\left(1-\frac{3}{n}\right)}\right)^n} \cdot \lim_{n\to\infty}\left(\frac{\cancel{n}\left(1+\frac{1}{n}\right)}{\cancel{n}\left(1+\frac{2}{n}\right)}\right)^n$$

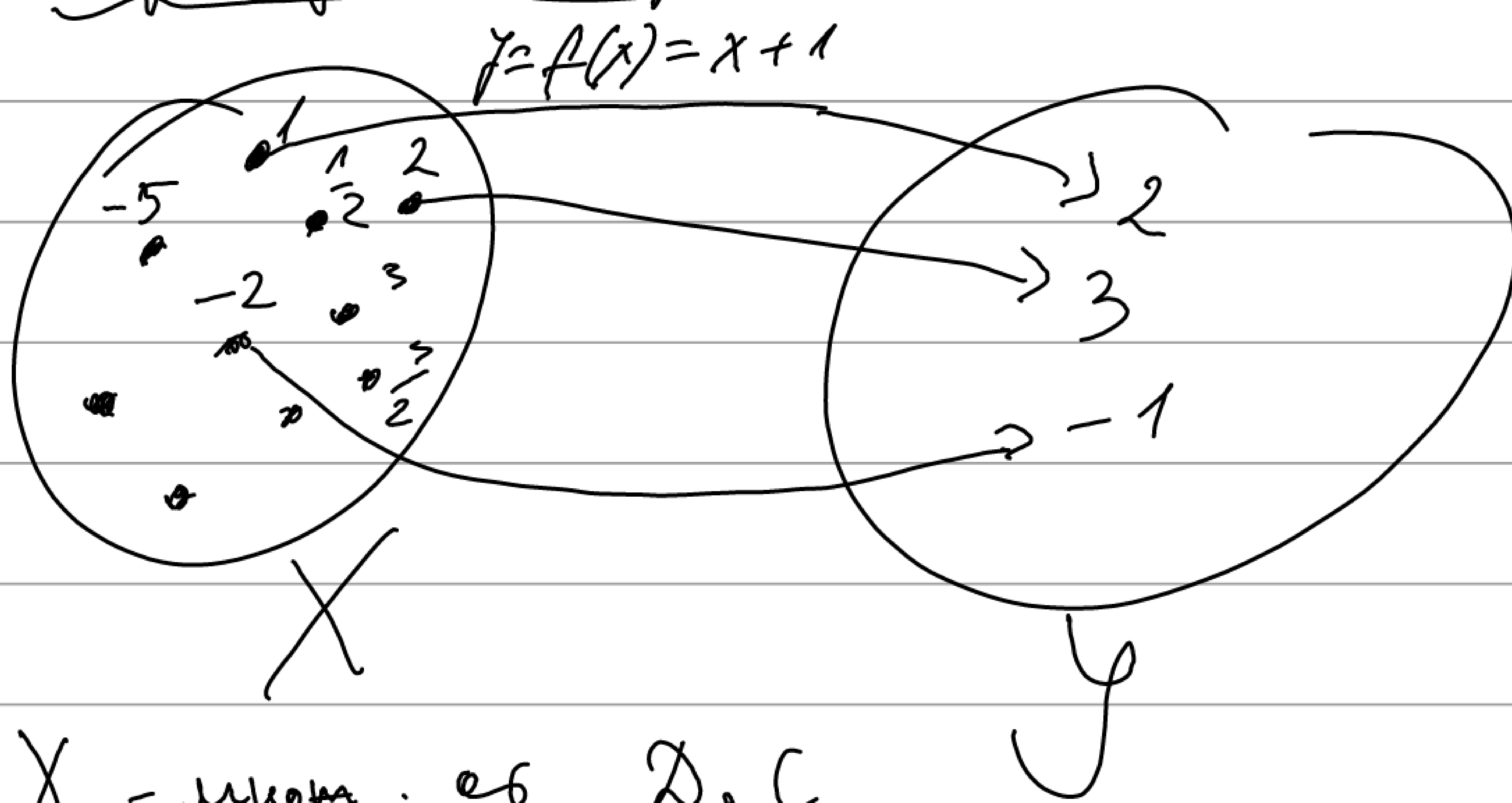
$$= \frac{e^{-1}}{e^{-3}} \cdot \frac{e^{1}}{e^{2}} = \frac{e^{-1+1}}{e^{-3+2}} = \frac{e^{0}}{e^{-1}} = \frac{1}{e^{-1}} = e$$

Задача 41 $\lim\limits_{n\to\infty} \left(\frac{n^2-5n+6}{n^2+5n+6}\right)^{n}$

Задача 42 $\lim\limits_{n\to\infty} \left(\frac{n^2-4n+3}{n^2+3n+2}\right)^{n}$

Задача 43 $\lim\limits_{n\to\infty} \left(\frac{2n^2+2}{2n^2+1}\right)^{n^2}$

Функции. Образи функции



X – множ. от Д.С

Y – множ. от функционалните стойности

Def 12 Нек $X \subset \mathbb{R}$ и $Y \subset \mathbb{R}$. Ако за $\forall x \in X$ по някакъв закон (правило) f се съпоставя единствено число $y = f(x) \in Y$, казваме, че в множеството X е определена функцията $f: X \to Y$, областта от стойностите на която е подмножество на множеството Y. Множеството X се нарича дефиниционна област на ф-та f, а променливата x се нарича аргумент. Числото $y = f(x)$ се нарича стойност на функцията f при стойност на аргумента, равна на x.

Основни класове функции

1) Полиномни функции:

$f(x) = a_n x^n + a_{n-1} x^{n-1} + \dots + a_1 x + a_0$, където $a_k \in \mathbb{R}$, за $k = 0, 1, 2, \dots n$

2.) Дробно рационална функция

$$f(x) = \frac{a_n x^n + a_{n-1} x^{n-1} + \dots + a_1 x + a_0}{b_m x^m + b_{m-1} x^{m-1} + \dots + b_1 x + b_0},$$

където $a_k, b_j \in \mathbb{R}$, за $\forall k = 0, 1, \dots n$
и $j = 0, 1, 2 \dots m$

3) Степенна функция
$f(x) = x^{\alpha}$, където $\alpha \in \mathbb{R}$

4. Показателна функция:
$f(x) = (\alpha)^x$, където $\alpha > 0$ и $\alpha \neq 1$

5) Логаритмична функция
$f(x) = \log_{\alpha} x$, където $\alpha > 0$ и $\alpha \neq 1$

6). Тригонометрични функции!
$f(x) = \sin(x)$, $f(x) = \cos(x)$ $f(x) = tg(x)$
$f(x) = cotg(x)$