Задача. Дадена е крива на Безие, дефинирана чрез контролните точки: $P_0(-1,0)$, $P_1(-3,2)$, $P_2(1,4)$, $P_3(3,2)$, $P_4(-1,-4)$, $P_5(-3,-2)$.

а) Намерете точката от кривата, съответна на $u=\frac{1}{2}$, чрез алгоритъма на де Кастелжо.

Решение:

$$
\begin{array}{llllll}
P_0(-1,0) & & & & & \\
 & P_{10}(-2,1) & & & & \\
P_1(-3,2) & & P_{20}\left(-\frac{3}{2},2\right) & & & \\
 & P_{11}(-1,3) & & P_{30}\left(-\frac{1}{2},\frac{5}{2}\right) & & \\
P_2(1,4) & & P_{21}\left(\frac{1}{2},3\right) & & P_{40}\left(\frac{1}{4},\frac{9}{4}\right) & \\
 & P_{12}(2,3) & & P_{31}(1,2) & & P_{50}\left(\frac{1}{2},\frac{3}{2}\right)=C\left(\frac{1}{2}\right) \\
P_3(3,2) & & P_{22}\left(\frac{3}{2},1\right) & & P_{41}\left(\frac{3}{4},\frac{3}{4}\right) & \\
 & P_{13}(1,-1) & & P_{32}\left(\frac{1}{2},-\frac{1}{2}\right) & & \\
P_4(-1,-4) & & P_{23}\left(-\frac{1}{2},-2\right) & & & \\
 & P_{14}(-2,-3) & & & & \\
P_5(-3,-2) & & & & &
\end{array}
$$

$u=\frac{1}{2}$, $1-u=1-\frac{1}{2}=\frac{1}{2}$

$P_{10}=(1-u)P_0+uP_1=\frac{1}{2}(-1,0)+\frac{1}{2}(-3,2)=\left(-\frac{1}{2}-\frac{3}{2},\frac{2}{2}\right)=(-2,1)$

$P_{11}=(1-u)P_1+uP_2=\frac{1}{2}(-3,2)+\frac{1}{2}(1,4)=\left(-\frac{3}{2}+\frac{1}{2},\frac{2}{2}+\frac{4}{2}\right)=(-1,3)$

$P_{12}=(1-u)P_2+uP_3=\frac{1}{2}(1,4)+\frac{1}{2}(3,2)=\left(\frac{1}{2}+\frac{3}{2},\frac{4}{2}+\frac{2}{2}\right)=(2,3)$

$P_{13}=(1-u)P_3+uP_4=\frac{1}{2}(3,2)+\frac{1}{2}(-1,-4)=\left(\frac{3}{2}-\frac{1}{2},\frac{2}{2}-\frac{4}{2}\right)=(1,-1)$

$P_{14}=(1-u)P_4+uP_5=\frac{1}{2}(-1,-4)+\frac{1}{2}(-3,-2)=\left(-\frac{1}{2}-\frac{3}{2},-\frac{4}{2},-\frac{2}{2}\right)=(-2,-3)$

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б) Пресметнете коефициентите на Безие при $u=\frac{1}{2}$ и $C(u=\frac{1}{2})$ чрез тях.

Решение:

$$C(u)=B_{5,0}(u)P_0+B_{5,1}(u)P_1+B_{5,2}(u)P_2+B_{5,3}(u)P_3+B_{5,4}(u)P_4+B_{5,5}(u)P_5$$

$$B_{n,i}(u)=\frac{n!}{i!(n-i)!}u^i(1-u)^{n-i}$$

$$B_{5,0}\left(\frac{1}{2}\right)=\frac{5!}{0!(5-0)!}\cdot\left(\frac{1}{2}\right)^0\left(1-\frac{1}{2}\right)^{5-0}=\left(\frac{1}{2}\right)^5=\frac{1}{32}$$

$$B_{5,1}\left(\frac{1}{2}\right)=\frac{5!}{1!(5-1)!}\left(\frac{1}{2}\right)^1\left(1-\frac{1}{2}\right)^{5-1}=\frac{5.\cancel{4.3.2.1}}{1.\cancel{4.3.2.1}}\cdot\frac{1}{2}\left(\frac{1}{2}\right)^4=\frac{5}{2}\cdot\frac{1}{16}=\frac{5}{32}$$

$$B_{5,2}\left(\frac{1}{2}\right)=\frac{5!}{2!(5-2)!}\left(\frac{1}{2}\right)^2\left(1-\frac{1}{2}\right)^{5-2}=\frac{5.\cancel{4}.\cancel{3.2.1}}{\cancel{2.1.3.2.1}}\cdot\frac{1}{4}\cdot\left(\frac{1}{2}\right)^3=\frac{10}{4}\cdot\frac{1}{8}=\frac{10}{32}$$

$$B_{5,3}\left(\frac{1}{2}\right)=\frac{5!}{3!(5-3)!}\left(\frac{1}{2}\right)^3\left(1-\frac{1}{2}\right)^{5-3}=\frac{5.4.3.2.1}{3.2.1.2.1}\cdot\frac{1}{8}\cdot\left(\frac{1}{2}\right)^2=\frac{10}{8}\cdot\frac{1}{4}=\frac{10}{32}$$

$$B_{5,4}\left(\frac{1}{2}\right)=\frac{5!}{4!(5-4)!}\left(\frac{1}{2}\right)^4\left(1-\frac{1}{2}\right)^{5-4}=\frac{5.\cancel{4.3.2.1}}{\cancel{4.3.2.1}.1}\cdot\frac{1}{16}\cdot\frac{1}{2}=\frac{5}{32}$$

$$B_{5,5}\left(\frac{1}{2}\right)=\frac{5!}{5!(5-5)!}\left(\frac{1}{2}\right)^5\cdot\left(1-\frac{1}{2}\right)^{5-5}=\left(\frac{1}{2}\right)^5=\frac{1}{32}$$

$$C\left(\frac{1}{2}\right)=B_{5,0}\left(\frac{1}{2}\right)P_0+B_{5,1}\left(\frac{1}{2}\right)P_1+B_{5,2}\left(\frac{1}{2}\right)P_2+B_{5,3}\left(\frac{1}{2}\right)P_3+B_{5,4}\left(\frac{1}{2}\right)P_4+B_{5,5}\left(\frac{1}{2}\right)P_5$$

$$C\left(\frac{1}{2}\right)=\frac{1}{32}(-1,0)+\frac{5}{32}(-3,2)+\frac{10}{32}(1,4)+\frac{10}{32}(3,2)+\frac{5}{32}(-1,-4)+\frac{1}{32}(-3,-2)\Rightarrow$$

$$C\left(\frac{1}{2}\right)=\left(\frac{-1}{32}-\frac{15}{32}+\frac{10}{32}+\frac{30}{32}-\frac{5}{32}-\frac{3}{32},\ \frac{10}{32}+\frac{40}{32}+\frac{20}{32}-\frac{20}{32}-\frac{2}{32}\right)=\left(\frac{1}{2},\frac{3}{2}\right)$$

6) Увеличете степента на кривата с единица и намерете новите контролни точки и начертайте новия контролен полигон.

Решение:

$C(u)$: $P_0(-1,0)$, $P_1(-3,2)$, $P_2(1,4)$, $P_3(3,2)$, $P_4(-1,-4)$, $P_5(-3,-2)$

$n=5 \to 6$

$D(u)$: $Q_0, Q_1, Q_2, Q_3, Q_4, Q_5, Q_6$

$Q_0 = P_0 \Rightarrow Q_0(-1,0)$

$Q_6 = P_5 \Rightarrow Q_6(-3,-2)$

$$Q_i = \frac{i}{n+1}P_{i-1} + \left(1-\frac{i}{n+1}\right)P_i \quad , i=1,2,3,4,5$$

$$Q_1 = \frac{1}{6}P_0 + \left(1-\frac{1}{6}\right)P_1 = \frac{1}{6}(-1,0) + \frac{5}{6}(-3,2) = \left(-\frac{1}{6}-\frac{15}{6}, \frac{10}{6}\right) = \left(-\frac{16}{6}, \frac{10}{6}\right) = \left(-\frac{8}{3}, \frac{5}{3}\right)$$

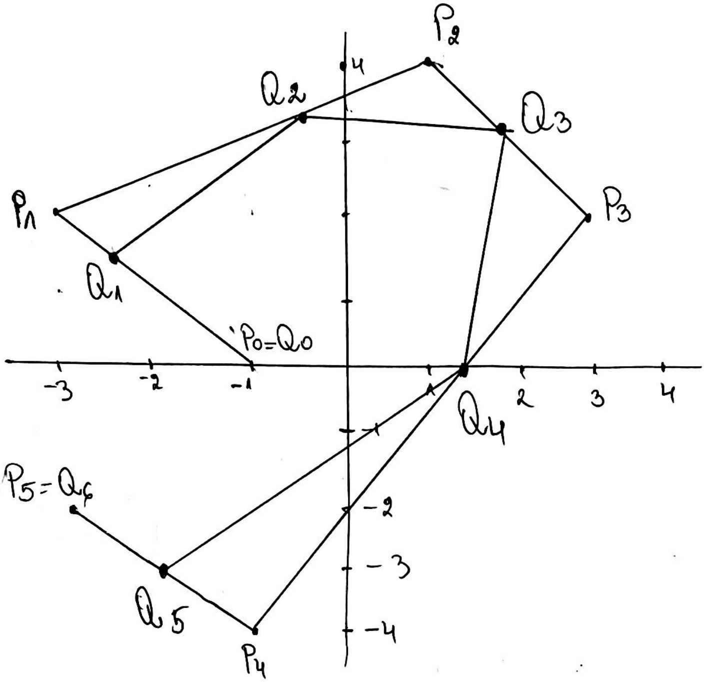
$$Q_2 = \frac{2}{6}P_1 + \left(1-\frac{2}{6}\right)P_2 = \frac{2}{6}(-3,2) + \frac{4}{6}(1,4) = \left(-\frac{6}{6}+\frac{4}{6}, \frac{4}{6}+\frac{16}{6}\right) = \left(-\frac{2}{6}, \frac{20}{6}\right) = \left(-\frac{1}{3}, \frac{10}{3}\right)$$

$$Q_3 = \frac{3}{6}P_2 + \left(1-\frac{3}{6}\right)P_3 = \frac{3}{6}(1,4) + \frac{3}{6}(3,2) = \left(\frac{3}{6}+\frac{9}{6}, \frac{12}{6}+\frac{6}{6}\right) = \left(\frac{12}{6}, \frac{18}{6}\right) = (2,3)$$

$$Q_4 = \frac{4}{6}P_3 + \left(1-\frac{4}{6}\right)P_4 = \frac{4}{6}(3,2) + \frac{2}{6}(-1,-4) = \left(\frac{12}{6}-\frac{2}{6}, \frac{8}{6}-\frac{8}{6}\right) = \left(\frac{10}{6}, 0\right) = \left(\frac{5}{3}, 0\right)$$

$$Q_5 = \frac{5}{6}P_4 + \left(1-\frac{5}{6}\right)P_5 = \frac{5}{6}(-1,-4) + \frac{1}{6}(-3,-2) = \left(-\frac{5}{6}-\frac{3}{6}, -\frac{15}{6}-\frac{2}{6}\right) = \left(-\frac{8}{6}, -\frac{17}{6}\right) = \left(-\frac{4}{3}, -\frac{17}{6}\right)$$

$D(u)$: $Q_0(-1,0)$, $Q_1\left(-\frac{8}{3}, \frac{5}{3}\right)$, $Q_2\left(-\frac{1}{3}, \frac{10}{3}\right)$, $Q_3(2,3)$, $Q_4\left(\frac{5}{3}, 0\right)$, $Q_5\left(-\frac{4}{3}, -\frac{17}{6}\right)$, $Q_6(-3,-2)$



Чертеж: $Q_0 = P_0$

Q_1 лежи на рамото $P_0 P_1$

Q_2 лежи на рамото $P_1 P_2$

Q_3 лежи на рамото $P_2 P_3$

Q_4 лежи на рамото $P_3 P_4$

Q_5 лежи на рамото $P_4 P_5$

$Q_6 = P_5$

Забележете ефекта на отрязване на върховете на стария контролен полигон – P_1, P_2, P_3, P_4

г) Чрез преместване на контролната точка P_4 в $P_4^*(0, 1)$ намерете зависимостта между $C\left(\frac{1}{2}\right)$ и $C^*\left(\frac{1}{2}\right)$.

Решение:

$$C^*(u) = C(u) + B_{n,j}(u)\vec{V}$$

$$\Rightarrow C^*\left(\frac{1}{2}\right) = C\left(\frac{1}{2}\right) + B_{5,4}\left(\frac{1}{2}\right)\vec{V}$$

$$C\left(\frac{1}{2}\right) = \left(\frac{1}{2}, \frac{3}{2}\right)$$

$$B_{5,4}\left(\frac{1}{2}\right) = \frac{5!}{4!\,(5-4)!}\left(\frac{1}{2}\right)^4\left(1-\frac{1}{2}\right)^{5-4} = \frac{5.4.3.2.1}{4.3.2.1.1}\cdot\frac{1}{16}\cdot\frac{1}{2} = \frac{5}{32}$$

$$\vec{V} = P_4^* - P_4 = (0,1) - (-1,-4) = (1,5)$$

$$C^*\left(\frac{1}{2}\right) = C\left(\frac{1}{2}\right) + B_{5,4}\left(\frac{1}{2}\right)\vec{V} \Rightarrow C^*\left(\frac{1}{2}\right) = \left(\frac{1}{2}, \frac{3}{2}\right) + \frac{5}{32}(1,5)$$

$$C^*\left(\frac{1}{2}\right) = \left(\frac{1}{2}, \frac{3}{2}\right) + \left(\frac{5}{32}, \frac{25}{32}\right) = \left(\frac{21}{32}, \frac{73}{32}\right)$$

$$\Rightarrow C^*\left(\frac{1}{2}\right) = \left(\frac{21}{32}, \frac{73}{32}\right)$$

2) Намерете $\dot{c}\left(\frac{1}{2}\right)$ и $\ddot{c}\left(\frac{1}{2}\right)$

Решение:

$$\dot{C}(u) = n\left[P_{n-1,1} - P_{n-1,0}\right]$$

$$\dot{c}\left(\frac{1}{2}\right) = 5\left[P_{41} - P_{40}\right] = 5\left[\left(\frac{3}{4}, \frac{3}{4}\right) - \left(\frac{1}{4}, \frac{9}{4}\right)\right]$$

$$= 5\left(\frac{3}{4} - \frac{1}{4}, \frac{3}{4} - \frac{9}{4}\right) = 5\left(\frac{2}{4}, \frac{-6}{4}\right) = 5\left(\frac{1}{2}, \frac{-3}{2}\right)$$

$$\Rightarrow \dot{c}\left(\frac{1}{2}\right) = \left(\frac{5}{2}, \frac{-15}{2}\right)$$

$$\ddot{C}(u) = n(n-1)\left[P_{n-2,2} - 2P_{n-2,1} + P_{n-2,0}\right]$$

$$\ddot{C}\left(\frac{1}{2}\right) = 5.4\left[P_{32} - 2P_{31} + P_{30}\right]$$

$$= 20\left[\left(\frac{1}{2}, -\frac{1}{2}\right) - 2(1,2) + \left(-\frac{1}{2}, \frac{5}{2}\right)\right]$$

$$= 20\left(-2, -2\right) = (-40, -40)$$

$$\Rightarrow \ddot{c}\left(\frac{1}{2}\right) = (-40, -40)$$

Задача. Дадена е крива на Безие $C(u)$ дефинирана чрез контролния полигон

$P_0(0,-2)$, $P_1(-2,-4)$, $P_2(-2,2)$, $P_3(6,10)$, $P_4(2,0)$, $P_5(0,-2)$

Определете вида на непрекъснатостта в точката на свързване.

Решение:

Имаме $\underline{C(0) = C(1) = P_0 = P_5 = (0,-2)}$

Пресмятаме първите производни в точката на свързване

$\dot{C}(0) = n[P_1 - P_0] \Rightarrow \dot{C}(0) = 5[P_1 - P_0] = 5[(-2,-4)-(0,-2)]$

$\Rightarrow \dot{C}(0) = 5(-2,-2) = \underline{(-10,-10)}$

$\dot{C}(1) = n[P_5 - P_4] = 5[(0,-2)-(2,0)] = 5(-2,-2) = \underline{(-10,-10)}$

$\dot{C}(0) = \dot{C}(1) \Rightarrow$ съществува C^1 и G^1-непр.

Пресмятаме вторите производни

$\ddot{C}(0) = n(n-1)[P_2 - 2P_1 + P_0]$

$\ddot{C}(0) = 5.4\,[(-2,2) - 2(-2,-4) + (0,-2)] = \underline{(40,160)}$

$\ddot{C}(1) = n(n-1)[P_n - 2P_{n-1} + P_{n-2}]$

$\ddot{C}(1) = 5.4\,[P_5 - 2P_4 + P_3] = 20[(0,-2) - 2(2,0) + (6,10)]$

$\Rightarrow \underline{\ddot{C}(1) = (40,160)}$

$\ddot{C}(0) = \ddot{C}(1) \Rightarrow$ съществува C^2 и G^2 непрекъснатост.

$$\varkappa_{c(0)} = \frac{|\dot{c}(0) \times \ddot{c}(0)|}{|\dot{c}(0)|^3} = \varkappa_{c(1)} = \frac{|\dot{c}(1) \times \ddot{c}(1)|}{|\dot{c}(1)|^3}$$

$\dot{c}(0) = (-10, -10) \Rightarrow \dot{c}(0) = (-10, -10, 0)$

$\ddot{c}(0) = (40, 160) \Rightarrow \ddot{c}(0) = (40, 160, 0)$

$$\dot{c}(0) \times \ddot{c}(0) = \left(\begin{vmatrix} -10 & 0 \\ 160 & 0 \end{vmatrix}; -\begin{vmatrix} -10 & 0 \\ 40 & 0 \end{vmatrix}; \begin{vmatrix} -10 & -10 \\ 40 & 160 \end{vmatrix} \right) = (0, 0, -1200)$$

$|\dot{c}(0) \times \ddot{c}(0)| = \sqrt{0^2 + 0^2 + (-1200)^2} = 1200$

$|\dot{c}(0)| = \sqrt{(-10)^2 + (-10)^2 + 0^2} = 10\sqrt{2}$

$$\varkappa_{c(0)} = \frac{|\dot{c}(0) \times \ddot{c}(0)|}{|\dot{c}(0)|^3} = \frac{1200}{(10\sqrt{2})^3} = \frac{1200}{1000 \cdot 2\sqrt{2}} = \frac{3\sqrt{2}}{10}$$

$\dot{c}(1) = (-10, -10) \Rightarrow \dot{c}(1) = (-10, -10, 0)$

$\ddot{c}(1) = (40, 160) \Rightarrow \ddot{c}(1) = (40, 160, 0)$

$\dot{c}(1) \times \ddot{c}(1) = (0, 0, -1200)$; $|\dot{c}(1) \times \ddot{c}(1)| = 1200$

$|\dot{c}(1)| = 10\sqrt{2}$

$$\varkappa_{c(1)} = \frac{|\dot{c}(1) \times \ddot{c}(1)|}{|\dot{c}(1)|^3} = \frac{1200}{(10\sqrt{2})^3} = \frac{3\sqrt{2}}{10}$$

$\Rightarrow \varkappa_{c(0)} = \varkappa_{c(1)} \Rightarrow$ съществува $\varkappa$-цеир.