Метод на най-малките квадрати (МНМК)

Задача: (**a** и **b** са съответно предпоследната и последната цифра от факултетния номер)

1. Да се състави таблицата $(x_k, \mathrm{g}(x_k))$, където

$x_k =$ -b + k(0.1), k = $\overline{0,\ 10}$, g(x) = $e^{\frac{(a+1)\,x}{10}}$

Търси се апроксимацията в точката s = -b + (0.17)a + 0.01. За тази цел:

2. Да се построи полином на ленейна регресия по получената таблица.
3. Да се построи полином на квадратична регресия по получената таблица.
4. Да се построи полином на кубична регресия по получената таблица.
5. Да се пресметне апроксимацията, използвайки всеки един от построените полиноми (общо 3).
6. Да се оцени грешката за всяка от получените апроксимации.
7. Да се направи сравнение между трите резултата.

Генериране на данни

In[237]:=

```
xt = Table[-8 + k * 0.1, {k, 0, 10}]
```

Out[237]=

```
{-8., -7.9, -7.8, -7.7, -7.6, -7.5, -7.4, -7.3, -7.2, -7.1, -7.}
```

In[238]:=

```
f[x_] := e^(1 x/10)
yt = f[xt]
```

Out[239]=

```
{0.449329, 0.453845, 0.458406, 0.463013, 0.467666,
 0.472367, 0.477114, 0.481909, 0.486752, 0.491644, 0.496585}
```

In[240]:=

```
P = Length[xt]
```

Out[240]=

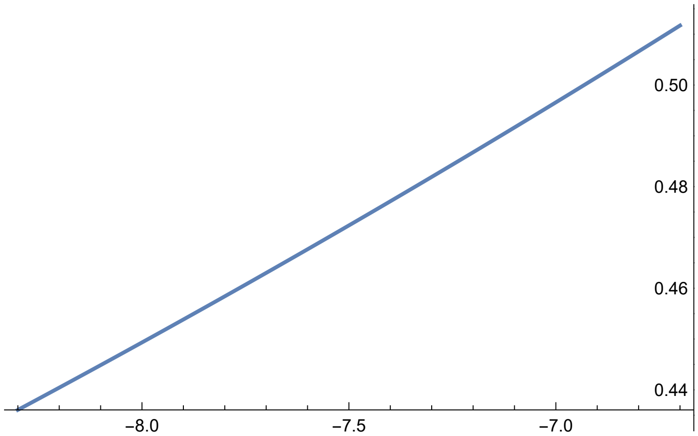
```
11
```

Визуализация

In[241]:=

```
grf = Plot[f[x], {x, xt⟦1⟧ - 0.3, xt⟦P⟧ + 0.3}]
```

Out[241]=



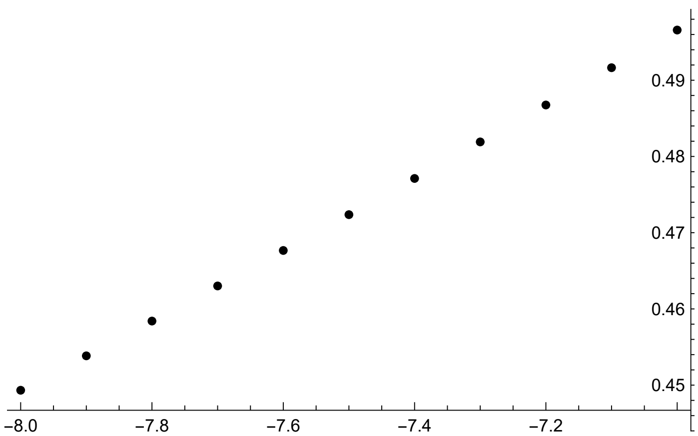
In[242]:=

```
points = Table[{xt⟦i⟧, yt⟦i⟧}, {i, 1, P}];
```

In[243]:=

```
grp = ListPlot[points, PlotStyle → Black]
```

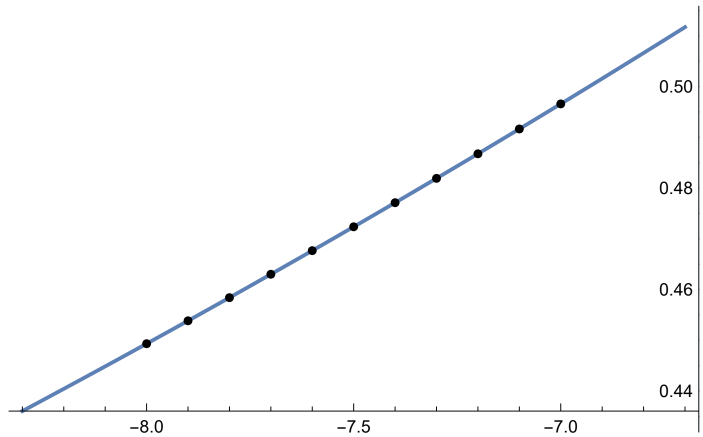
Out[243]=



In[244]:=

```
Show[grf, grp]
```

Out[244]=



Линейна регресия

Попълваме таблицата

In[245]:=

xt^2

Out[245]=

```
{64., 62.41, 60.84, 59.29, 57.76, 56.25, 54.76, 53.29, 51.84, 50.41, 49.}
```

In[246]:=

```
yt * xt
```

Out[246]=

```
{-3.59463, -3.58537, -3.57557, -3.5652, -3.55426,
 -3.54275, -3.53064, -3.51794, -3.50462, -3.49067, -3.4761}
```

Намиране на сумите

In[247]:=

$\sum_{i=1}^{P} xt[[i]]$

Out[247]=

-82.5

In[248]:=

$\sum_{i=1}^{P} yt[[i]]$

Out[248]=

5.19863

In[249]:=

$$\sum_{i=1}^{P} xt[[i]]^2$$

Out[249]=

619.85

In[250]:=

$$\sum_{i=1}^{P} yt[[i]] * xt[[i]]$$

Out[250]=

-38.9378

Решаваме системата

In[251]:=

$$A = \begin{pmatrix} P & \sum_{i=1}^{P} xt[[i]] \\ \sum_{i=1}^{P} xt[[i]] & \sum_{i=1}^{P} xt[[i]]^2 \end{pmatrix}; \; b = \left\{\sum_{i=1}^{P} yt[[i]], \; \sum_{i=1}^{P} yt[[i]] * xt[[i]]\right\};$$

In[252]:=

```
LinearSolve[A, b]
```

Out[252]=

{0.826983, 0.0472507}

Съставяме полинома

In[253]:=

```
P1[x_] := 0.49398 + 0.0365801 x
```

Таен коз (възможност за самопроверка)

In[254]:=

```
Fit[points, {1, x}, x]
```

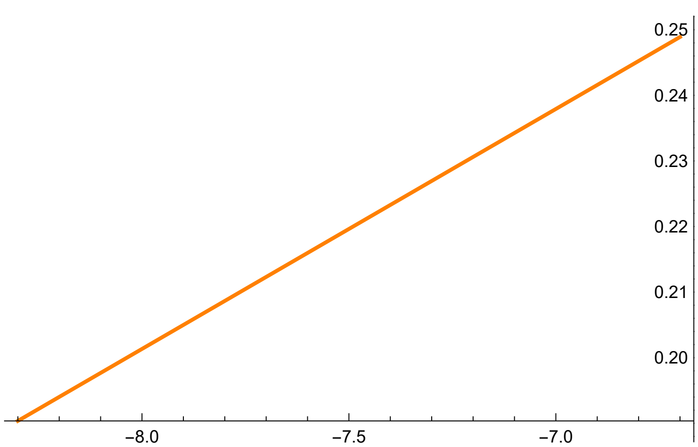
Out[254]=

0.826983 + 0.0472507 x

In[255]:=

```
grfP1 = Plot[P1[x], {x, xt[[1]] - 0.3, xt[[P]] + 0.3}, PlotStyle → Orange]
```

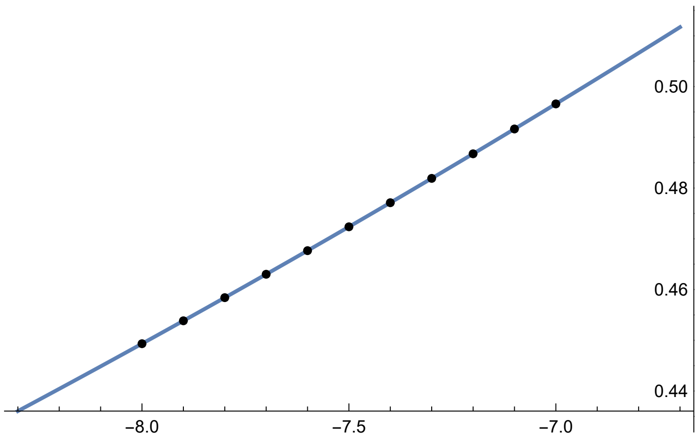
Out[255]=



In[256]:=

```
Show[grf, grp, grfP1]
```

Out[256]=



In[257]:=

```
P1[-8.25]
```

Out[257]=

```
0.192194
```

За сравнение истинската стойност

In[258]:=

```
f[-8.25]
```

Out[258]=

```
0.438235
```

Оценка на грешката

Теоретична грешка (средноквадратична)

In[259]:=

$$\sqrt{\sum_{i=1}^{p} (yt[[i]] - P1[xt[[i]]])^2}$$

Out[259]=

```
0.839093
```

Истинска грешка

In[260]:=

```
Abs[f[-5.97] - P1[-5.97]]
```

Out[260]=

```
0.274864
```

Квадратична регресия

Попълваме таблицата

In[261]:=

xt^2

Out[261]=

{64., 62.41, 60.84, 59.29, 57.76, 56.25, 54.76, 53.29, 51.84, 50.41, 49.}

In[262]:=

yt * xt

Out[262]=

{-3.59463, -3.58537, -3.57557, -3.5652, -3.55426,
-3.54275, -3.53064, -3.51794, -3.50462, -3.49067, -3.4761}

In[263]:=

xt^3

Out[263]=

{-512., -493.039, -474.552, -456.533, -438.976,
-421.875, -405.224, -389.017, -373.248, -357.911, -343.}

In[264]:=

xt^4

Out[264]=

{4096., 3895.01, 3701.51, 3515.3, 3336.22,
3164.06, 2998.66, 2839.82, 2687.39, 2541.17, 2401.}

In[265]:=

$yt * xt^2$

Out[265]=

{28.7571, 28.3245, 27.8894, 27.452, 27.0124,
26.5706, 26.1268, 25.6809, 25.2332, 24.7838, 24.3327}

Намиране на сумите

In[266]:=

$$\sum_{i=1}^{P} xt[[i]]$$

Out[266]=

-82.5

In[267]:=

$$\sum_{i=1}^{P} yt[[i]]$$

Out[267]=

5.19863

In[268]:=

$$\sum_{i=1}^{P} xt[[i]]^2$$

Out[268]=

619.85

In[269]:=

$$\sum_{i=1}^{P} yt[[i]] * xt[[i]]$$

Out[269]=

-38.9378

In[270]:=

$$\sum_{i=1}^{P} xt[[i]]^3$$

Out[270]=

-4665.38

In[271]:=

$$\sum_{i=1}^{P} xt[[i]]^4$$

Out[271]=

35176.1

In[272]:=

$$\sum_{i=1}^{P} yt[[i]] * xt[[i]]^2$$

Out[272]=

292.163

Решаваме системата

In[273]:=

$$A = \begin{pmatrix} P & \sum_{i=1}^{P} xt[[i]] & \sum_{i=1}^{P} xt[[i]]^2 \\ \sum_{i=1}^{P} xt[[i]] & \sum_{i=1}^{P} xt[[i]]^2 & \sum_{i=1}^{P} xt[[i]]^3 \\ \sum_{i=1}^{P} xt[[i]]^2 & \sum_{i=1}^{P} xt[[i]]^3 & \sum_{i=1}^{P} xt[[i]]^4 \end{pmatrix}; \; b = \left\{\sum_{i=1}^{P} yt[[i]], \; \sum_{i=1}^{P} yt[[i]] * xt[[i]], \; \sum_{i=1}^{P} yt[[i]] * xt[[i]]^2\right\};$$

In[274]:=

```
LinearSolve[A, b]
```

Out[274]=

{0.959627, 0.0826855, 0.00236232}

Таен коз (възможност за самопроверка)

In[275]:=

$$\text{Fit}\left[\text{points}, \left\{1, x, x^2\right\}, x\right]$$

Out[275]=

$$0.959627 + 0.0826855\,x + 0.00236232\,x^2$$

Съставяме полинома

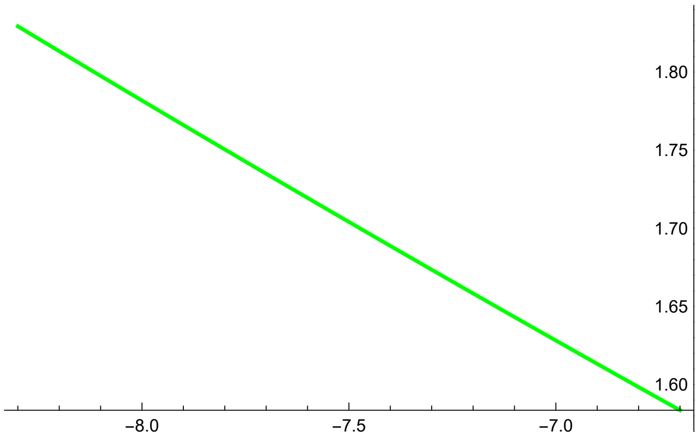
In[276]:=

```
P2[x_] := 0.757812 - 0.0987443 x + 0.00365672 x^2
```

In[277]:=

```
grfP2 = Plot[P2[x], {x, xt⟦1⟧ - 0.3, xt⟦P⟧ + 0.3}, PlotStyle → Green]
```

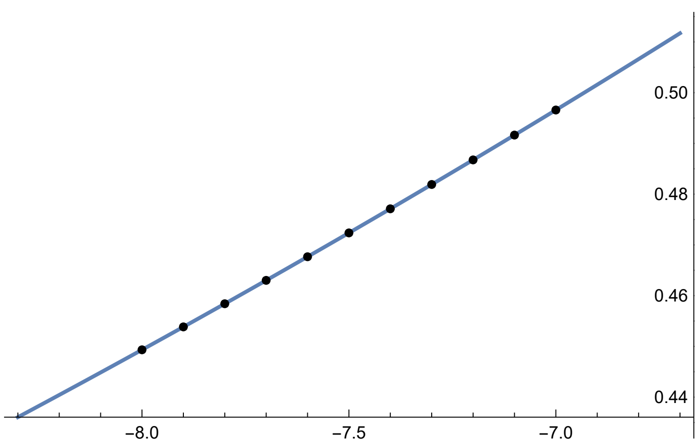
Out[277]=



In[278]:=

```
Show[grf, grp, grfP1, grfP2]
```

Out[278]=



In[279]:=

```
P2[-8.25]
```

Out[279]=

```
1.82134
```

За сравнение истинската стойност

In[280]:=

```
f[-8.25]
```

Out[280]=

```
0.438235
```

Оценка на грешката

Теоретична грешка (средноквадратична)

In[281]:=

$$\sqrt{\sum_{i=1}^{P} (yt[[i]] - P2[xt[[i]]])^2}$$

Out[281]=

4.091

Истинска грешка

In[282]:=

```
Abs[f[-8.25] - P2[-8.25]]
```

Out[282]=

1.3831

Кубична регресия

Попълваме таблицата

In[283]:=

xt^2

Out[283]=

```
{64., 62.41, 60.84, 59.29, 57.76, 56.25, 54.76, 53.29, 51.84, 50.41, 49.}
```

In[284]:=

```
yt * xt
```

Out[284]=

```
{-3.59463, -3.58537, -3.57557, -3.5652, -3.55426,
 -3.54275, -3.53064, -3.51794, -3.50462, -3.49067, -3.4761}
```

In[285]:=

xt^3

Out[285]=

```
{-512., -493.039, -474.552, -456.533, -438.976,
 -421.875, -405.224, -389.017, -373.248, -357.911, -343.}
```

In[286]:=

xt^4

Out[286]=

```
{4096., 3895.01, 3701.51, 3515.3, 3336.22,
 3164.06, 2998.66, 2839.82, 2687.39, 2541.17, 2401.}
```

In[287]:=

```
yt * xt^2
```

Out[287]=

{28.7571, 28.3245, 27.8894, 27.452, 27.0124,
 26.5706, 26.1268, 25.6809, 25.2332, 24.7838, 24.3327}

In[288]:=

```
yt * xt^3
```

Out[288]=

{-230.056, -223.763, -217.537, -211.381, -205.294,
 -199.28, -193.338, -187.471, -181.679, -175.965, -170.329}

In[289]:=

```
xt^5
```

Out[289]=

{-32 768., -30 770.6, -28 871.7, -27 067.8, -25 355.3,
 -23 730.5, -22 190.1, -20 730.7, -19 349.2, -18 042.3, -16 807.}

In[290]:=

```
xt^6
```

Out[290]=

{262 144., 243 087., 225 200., 208 422., 192 700.,
 177 979., 164 206., 151 334., 139 314., 128 100., 117 649.}

Намиране на сумите

In[291]:=

$$\sum_{i=1}^{P} xt[[i]]$$

Out[291]=

-82.5

In[292]:=

$$\sum_{i=1}^{P} yt[[i]]$$

Out[292]=

5.19863

In[293]:=

$$\sum_{i=1}^{P} xt[[i]]^2$$

Out[293]=

619.85

In[294]:=

$$\sum_{i=1}^{P} yt[[i]] * xt[[i]]$$

Out[294]=

-38.9378

In[295]:=

$$\sum_{i=1}^{P} xt[[i]]^3$$

Out[295]=

-4665.38

In[296]:=

$$\sum_{i=1}^{P} xt[[i]]^4$$

Out[296]=

35176.1

In[297]:=

$$\sum_{i=1}^{P} yt[[i]] * xt[[i]]^2$$

Out[297]=

292.163

In[298]:=

$$\sum_{i=1}^{P} xt[[i]]^5$$

Out[298]=

-265683.

In[299]:=

$$\sum_{i=1}^{P} xt[[i]]^6$$

Out[299]=

2.01014×10^6

In[300]:=

$$\sum_{i=1}^{P} yt[[i]] * xt[[i]]^3$$

Out[300]=

-2196.09

Решаваме системата

In[301]:=

$$A = \begin{pmatrix} P & \sum_{i=1}^{P} xt[[i]] & \sum_{i=1}^{P} xt[[i]]^2 & \sum_{i=1}^{P} xt[[i]]^3 \\ \sum_{i=1}^{P} xt[[i]] & \sum_{i=1}^{P} xt[[i]]^2 & \sum_{i=1}^{P} xt[[i]]^3 & \sum_{i=1}^{P} xt[[i]]^4 \\ \sum_{i=1}^{P} xt[[i]]^2 & \sum_{i=1}^{P} xt[[i]]^3 & \sum_{i=1}^{P} xt[[i]]^4 & \sum_{i=1}^{P} xt[[i]]^5 \\ \sum_{i=1}^{P} xt[[i]]^3 & \sum_{i=1}^{P} xt[[i]]^4 & \sum_{i=1}^{P} xt[[i]]^5 & \sum_{i=1}^{P} xt[[i]]^6 \end{pmatrix};$$

$$b = \left\{\sum_{i=1}^{P} yt[[i]],\ \sum_{i=1}^{P} yt[[i]] * xt[[i]],\ \sum_{i=1}^{P} yt[[i]] * xt[[i]]^2,\ \sum_{i=1}^{P} yt[[i]] * xt[[i]]^3\right\};$$

In[302]:=

```
LinearSolve[A, b]
```

LinearSolve: Result for LinearSolve of badly conditioned matrix {{11., −82.5, 619.85, −4665.38}, {−82.5, 619.85, −4665.38, 35176.1}, {619.85, −4665.38, 35176.1, −265683.}, {−4665.38, 35176.1, −265683., 2.01014 × 10^6}} may contain significant numerical errors.

Out[302]=

```
{0.992741, 0.0959589, 0.00413397, 0.0000787398}
```

Таен коз (възможност за самопроверка)

In[303]:=

```
Fit[points, {1, x, x^2, x^3}, x]
```

Out[303]=

$0.992741 + 0.0959589\,x + 0.00413398\,x^2 + 0.0000787402\,x^3$

Съставяме полинома

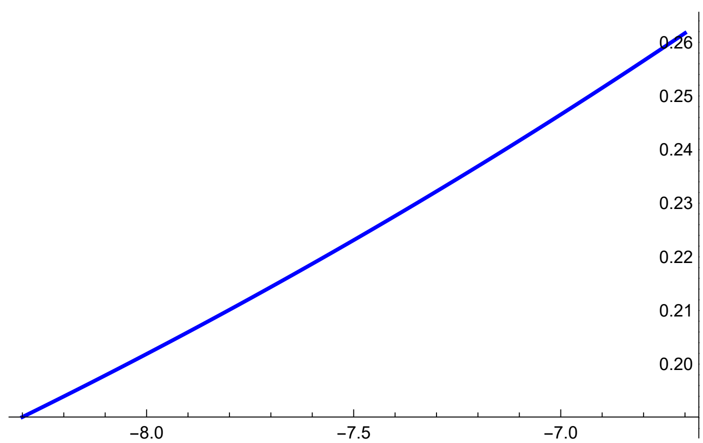
In[304]:=

```
P3[x_] := 0.907125 + 0.15153 x + 0.00987189 x^2 + 0.000243732 x^3
```

In[305]:=

```
grfP3 = Plot[P3[x], {x, xt[[1]] - 0.3, xt[[P]] + 0.3}, PlotStyle → Blue]
```

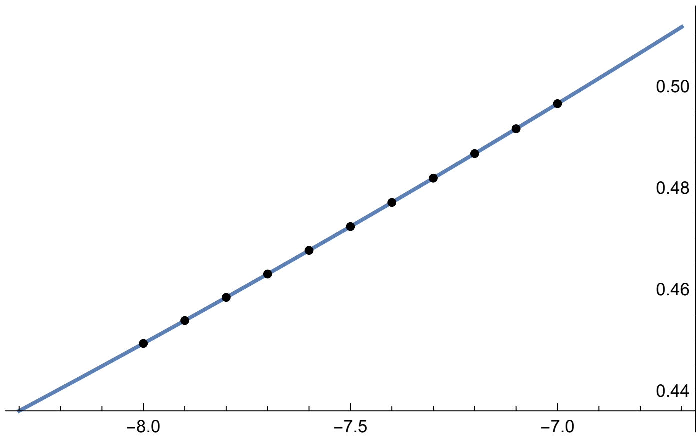
Out[305]=



In[306]:=

```
Show[grf, grp, grfP1, grfP2, grfP3]
```

Out[306]=



Намиране на приближена стойност (s = -b + (0.17)a + 0.01)

In[307]:=

```
s = -8 + 0.17 + 0.01
```

Out[307]=

```
-7.82
```

In[308]:=

```
P3[-8.25]
```

Out[308]=

```
0.192049
```

За сравнение истинската стойност

In[309]:=

```
f[-8.25]
```

Out[309]=

```
0.438235
```

Оценка на грешката

Теоретична грешка (средноквадратична)

In[310]:=

$$\sqrt{\sum_{i=1}^{P} (yt[[i]] - P3[xt[[i]]])^2}$$

Out[310]=

```
0.825992
```

Истинска грешка

In[311]:=

```
Abs[f[-8.25] - P3[-8.25]]
```

Out[311]=

```
0.246186
```