Метод на разполовяването

Задача: Да се реши уравнението (1. Да се намери броя на корените, 2. Да се уточни най-малкия корен по **метода на разполовяването**, 3. Да се направи оценка на грешката)

$$x^3 + 45 \cos x + 6x - 76 = 0$$

Да се изчисли предварително броят на стъпките (итерациите) за достигане на точност 0.00001 за определения по време на локализацията интервал по **метода на разполовяването**.

Графично представяне на функцията

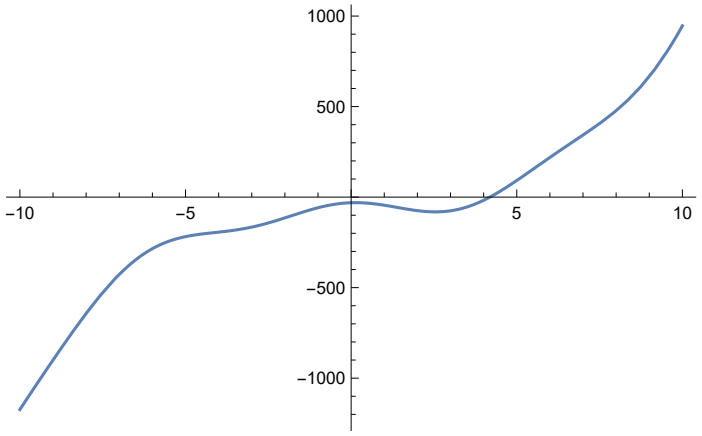
Дефиниция на функция

In[1]:=
```
f[x_] := x^3 + 45 Cos[x] + 6 x - 76
```

Графика на функция

In[•]:=
```
Plot[f[x], {x, -10, 10}]
```

Out[•]=



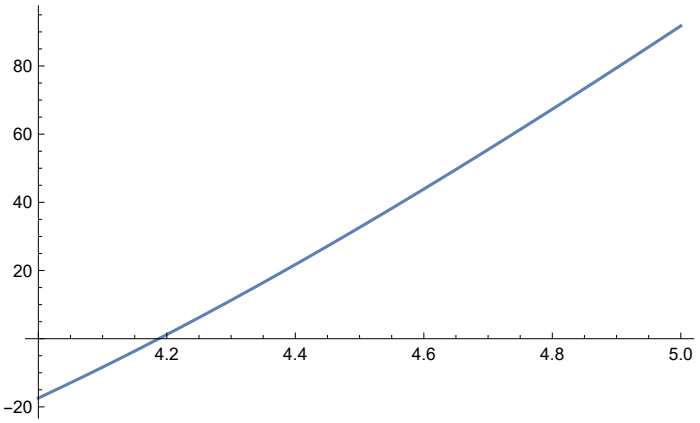
Plot[function , { var , min , max }]

Извод: Уравнението има един корен.

Локализация на корен

In[•]:=
```
Plot[f[x], {x, 4, 5}]
```

Out[•]=



In[•]:=
```
f[4]
```

Out[•]=

$12 + 45 \cos[4]$

In[•]:=
```
f[4.]
```

Out[•]=

```
-17.414
```

In[•]:=
```
f[5.]
```

Out[•]=

```
91.7648
```

Извод:

Функцията f(x) е непрекъсната, защото е сума от непрекъснати функции (полиним и косинус).

f(4) = -17.4... < 0

f(5) = 91.76... > 0

Функцията има различни знаци в двата края на разглеждания интервал [4; 5].

Следователно в този интервал [4; 5] функцията има корен.

Уточняване на корен

Уточнение за цикли и условни преходи

```
For[]
```

In[2]:=
```
For[i = 0, i < 4, i++, Print[i]]
```

```
0
1
2
3
```

In[3]:=
```
If[i < 2, Print["малко"], Print["голямо"]]
```
```
голямо
```

In[4]:=
```
i
```
Out[4]=
```
4
```

In[6]:=
```
For[i = 0, i < 4, i++,
 (*body*)
 Print[i];
 If[i < 2, Print["малко"], Print["голямо"]]
]
```
```
0
малко
1
малко
2
голямо
3
голямо
```

In[11]:=
```
Clear[if]
if[i_] := If[i < 2, Print["малко"], Print["голямо"]]
For[i = 0, i < 4, i++,
 (*body*)
 Print[i];
 if[i]
]
```
```
0
малко
1
малко
2
голямо
3
голямо
```

програмен код за метода на разполовяването

In[45]:=
```
f[x_] := x^3 + 45 Cos[x] + 6 x - 76
a = 4.;
b = 5.;
For[n = 0, n ≤ 3, n++,
 Print["n = ", n, " a = ", a, " b = ", b,
  " m = ", m = (a + b)/2, " f(m) = ", f[m], " ε = ", (b - a)/2];
 If[f[m] < 0, a = m, b = m]
]
```

```
n = 0 a = 4. b = 5. m = 4.5 f(m) = 32.6392 ε = 0.5
n = 1 a = 4. b = 4.5 m = 4.25 f(m) = 6.19169 ε = 0.25
n = 2 a = 4. b = 4.25 m = 4.125 f(m) = -5.99908 ε = 0.125
n = 3 a = 4.125 b = 4.25 m = 4.1875 f(m) = 0.00320478 ε = 0.0625
```

In[44]:=
```
f[4.5]
```

Out[44]=
```
32.6392
```

Извод: На третата стъпка получихме приближено решение 4.18... с грешка 0.06...

Оценка на грешката

Пускаме повече на брой итерации

In[49]:=
```
f[x_] := x^3 + 45 Cos[x] + 6 x - 76
a = 4.;
b = 5.;
For[n = 0, n ≤ 30, n++,
 Print["n = ", n, " a = ", a, " b = ", b,
  " m = ", m = (a + b)/2, " f(m) = ", f[m], " ε = ", (b - a)/2];
 If[f[m] < 0, a = m, b = m]
]
```

n = 0 a = 4. b = 5. m = 4.5 f(m) = 32.6392 ε = 0.5

n = 1 a = 4. b = 4.5 m = 4.25 f(m) = 6.19169 ε = 0.25

n = 2 a = 4. b = 4.25 m = 4.125 f(m) = −5.99908 ε = 0.125

n = 3 a = 4.125 b = 4.25 m = 4.1875 f(m) = 0.00320478 ε = 0.0625

n = 4 a = 4.125 b = 4.1875 m = 4.15625 f(m) = −3.02171 ε = 0.03125

n = 5 a = 4.15625 b = 4.1875 m = 4.17188 f(m) = −1.51514 ε = 0.015625

n = 6 a = 4.17188 b = 4.1875 m = 4.17969 f(m) = −0.757428 ε = 0.0078125

n = 7 a = 4.17969 b = 4.1875 m = 4.18359 f(m) = −0.377476 ε = 0.00390625

n = 8 a = 4.18359 b = 4.1875 m = 4.18555 f(m) = −0.187227 ε = 0.00195313

n = 9 a = 4.18555 b = 4.1875 m = 4.18652 f(m) = −0.0920338 ε = 0.000976563

n = 10 a = 4.18652 b = 4.1875 m = 4.18701 f(m) = −0.0444202 ε = 0.000488281

n = 11 a = 4.18701 b = 4.1875 m = 4.18726 f(m) = −0.0206091 ε = 0.000244141

n = 12 a = 4.18726 b = 4.1875 m = 4.18738 f(m) = −0.00870252 ε = 0.00012207

n = 13 a = 4.18738 b = 4.1875 m = 4.18744 f(m) = −0.00274896 ε = 0.0000610352

n = 14 a = 4.18744 b = 4.1875 m = 4.18747 f(m) = 0.00022789 ε = 0.0000305176

n = 15 a = 4.18744 b = 4.18747 m = 4.18745 f(m) = −0.00126054 ε = 0.0000152588

n = 16 a = 4.18745 b = 4.18747 m = 4.18746 f(m) = −0.000516327 ε = 7.62939×10^{-6}

n = 17 a = 4.18746 b = 4.18747 m = 4.18747 f(m) = −0.000144219 ε = 3.8147×10^{-6}

n = 18 a = 4.18747 b = 4.18747 m = 4.18747 f(m) = 0.0000418353 ε = 1.90735×10^{-6}

n = 19 a = 4.18747 b = 4.18747 m = 4.18747 f(m) = −0.0000511918 ε = 9.53674×10^{-7}

n = 20 a = 4.18747 b = 4.18747 m = 4.18747 f(m) = -4.67821×10^{-6} ε = 4.76837×10^{-7}

n = 21 a = 4.18747 b = 4.18747 m = 4.18747 f(m) = 0.0000185786 ε = 2.38419×10^{-7}

n = 22 a = 4.18747 b = 4.18747 m = 4.18747 f(m) = 6.95018×10^{-6} ε = 1.19209×10^{-7}

n = 23 a = 4.18747 b = 4.18747 m = 4.18747 f(m) = 1.13598×10^{-6} ε = 5.96046×10^{-8}

n = 24 a = 4.18747 b = 4.18747 m = 4.18747 f(m) = -1.77111×10^{-6} ε = 2.98023×10^{-8}

n = 25 a = 4.18747 b = 4.18747 m = 4.18747 f(m) = -3.17564×10^{-7} ε = 1.49012×10^{-8}

n = 26 a = 4.18747 b = 4.18747 m = 4.18747 f(m) = 4.0921×10^{-7} ε = 7.45058×10^{-9}

n = 27 a = 4.18747 b = 4.18747 m = 4.18747 f(m) = 4.58226×10^{-8} ε = 3.72529×10^{-9}

n = 28 a = 4.18747 b = 4.18747 m = 4.18747 f(m) = -1.35871×10^{-7} ε = 1.86265×10^{-9}

n = 29 a = 4.18747 b = 4.18747 m = 4.18747 f(m) = -4.50241×10^{-8} ε = 9.31323×10^{-10}

n = 30 a = 4.18747 b = 4.18747 m = 4.18747 f(m) = 3.9924×10^{-10} ε = 4.65661×10^{-10}

печатаме с повече знака:

In[57]:=
```
f[x_] := x^3 + 45 Cos[x] + 6 x - 76
a = 4.;
b = 5.;
For[n = 0, n ≤ 30, n++,
 Print["n = ", n, " a = ", SetPrecision[a, 12], " b = ", SetPrecision[b, 12],
  " m = ", SetPrecision[m = (a + b)/2, 12], " f(m) = ", f[m], " ε = ", (b - a)/2];
 If[f[m] < 0, a = m, b = m]
]
```

```
n = 0 a = 4.00000000000 b = 5.00000000000 m = 4.50000000000 f(m) = 32.6392 ε = 0.5
n = 1 a = 4.00000000000 b = 4.50000000000 m = 4.25000000000 f(m) = 6.19169 ε = 0.25
n = 2 a = 4.00000000000 b = 4.25000000000 m = 4.12500000000 f(m) = -5.99908 ε = 0.125
n = 3 a = 4.12500000000 b = 4.25000000000 m = 4.18750000000 f(m) = 0.00320478 ε = 0.0625
n = 4 a = 4.12500000000 b = 4.18750000000 m = 4.15625000000 f(m) = -3.02171 ε = 0.03125
n = 5 a = 4.15625000000 b = 4.18750000000 m = 4.17187500000 f(m) = -1.51514 ε = 0.015625
n = 6 a = 4.17187500000 b = 4.18750000000 m = 4.17968750000 f(m) = -0.757428 ε = 0.0078125
n = 7 a = 4.17968750000 b = 4.18750000000 m = 4.18359375000 f(m) = -0.377476 ε = 0.00390625
n = 8 a = 4.18359375000 b = 4.18750000000 m = 4.18554687500 f(m) = -0.187227 ε = 0.00195313
n = 9 a = 4.18554687500 b = 4.18750000000
  m = 4.18652343750 f(m) = -0.0920338 ε = 0.000976563
n = 10 a = 4.18652343750 b = 4.18750000000
  m = 4.18701171875 f(m) = -0.0444202 ε = 0.000488281
n = 11 a = 4.18701171875 b = 4.18750000000
  m = 4.18725585938 f(m) = -0.0206091 ε = 0.000244141
n = 12 a = 4.18725585938 b = 4.18750000000
  m = 4.18737792969 f(m) = -0.00870252 ε = 0.00012207
n = 13 a = 4.18737792969 b = 4.18750000000
  m = 4.18743896484 f(m) = -0.00274896 ε = 0.0000610352
n = 14 a = 4.18743896484 b = 4.18750000000
  m = 4.18746948242 f(m) = 0.00022789 ε = 0.0000305176
n = 15 a = 4.18743896484 b = 4.18746948242
  m = 4.18745422363 f(m) = -0.00126054 ε = 0.0000152588
n = 16 a = 4.18745422363 b = 4.18746948242
  m = 4.18746185303 f(m) = -0.000516327 ε = 7.62939×10^-6
n = 17 a = 4.18746185303 b = 4.18746948242
  m = 4.18746566772 f(m) = -0.000144219 ε = 3.8147×10^-6
n = 18 a = 4.18746566772 b = 4.18746948242
  m = 4.18746757507 f(m) = 0.0000418353 ε = 1.90735×10^-6
n = 19 a = 4.18746566772 b = 4.18746757507
  m = 4.18746662140 f(m) = -0.0000511918 ε = 9.53674×10^-7
n = 20 a = 4.18746662140 b = 4.18746757507
  m = 4.18746709824 f(m) = -4.67821×10^-6 ε = 4.76837×10^-7
n = 21 a = 4.18746709824 b = 4.18746757507
  m = 4.18746733665 f(m) = 0.0000185786 ε = 2.38419×10^-7
```

n = 22 a = 4.18746709824 b = 4.18746733665
m = 4.18746721745 f(m) = 6.95018×10^{-6} ε = 1.19209×10^{-7}

n = 23 a = 4.18746709824 b = 4.18746721745
m = 4.18746715784 f(m) = 1.13598×10^{-6} ε = 5.96046×10^{-8}

n = 24 a = 4.18746709824 b = 4.18746715784
m = 4.18746712804 f(m) = -1.77111×10^{-6} ε = 2.98023×10^{-8}

n = 25 a = 4.18746712804 b = 4.18746715784
m = 4.18746714294 f(m) = -3.17564×10^{-7} ε = 1.49012×10^{-8}

n = 26 a = 4.18746714294 b = 4.18746715784
m = 4.18746715039 f(m) = 4.0921×10^{-7} ε = 7.45058×10^{-9}

n = 27 a = 4.18746714294 b = 4.18746715039
m = 4.18746714666 f(m) = 4.58226×10^{-8} ε = 3.72529×10^{-9}

n = 28 a = 4.18746714294 b = 4.18746714666
m = 4.18746714480 f(m) = -1.35871×10^{-7} ε = 1.86265×10^{-9}

n = 29 a = 4.18746714480 b = 4.18746714666
m = 4.18746714573 f(m) = -4.50241×10^{-8} ε = 9.31323×10^{-10}

n = 30 a = 4.18746714573 b = 4.18746714666
m = 4.18746714620 f(m) = 3.9924×10^{-10} ε = 4.65661×10^{-10}

Пресмятане с предварително зададена точност (стоп-критерий)

In[67]:=
```
f[x_] := x^3 + 45 Cos[x] + 6 x - 76
a = 4.;
b = 5.;
epszad = 0.00001;
eps = 10; (*произволна стойност по-голяма от предварително зададената грешка*)
For[n = 0, eps > epszad, n++,
 Print["n = ", n, " a = ", SetPrecision[a, 12], " b = ", SetPrecision[b, 12],
  " m = ", SetPrecision[m = (a + b)/2, 12], " f(m) = ", f[m], " ε = ", eps = (b - a)/2];
 If[f[m] < 0, a = m, b = m]
]
```

n = 0 a = 4.00000000000 b = 5.00000000000 m = 4.50000000000 f(m) = 32.6392 ε = 0.5

n = 1 a = 4.00000000000 b = 4.50000000000 m = 4.25000000000 f(m) = 6.19169 ε = 0.25

n = 2 a = 4.00000000000 b = 4.25000000000 m = 4.12500000000 f(m) = −5.99908 ε = 0.125

n = 3 a = 4.12500000000 b = 4.25000000000 m = 4.18750000000 f(m) = 0.00320478 ε = 0.0625

n = 4 a = 4.12500000000 b = 4.18750000000 m = 4.15625000000 f(m) = −3.02171 ε = 0.03125

n = 5 a = 4.15625000000 b = 4.18750000000 m = 4.17187500000 f(m) = −1.51514 ε = 0.015625

n = 6 a = 4.17187500000 b = 4.18750000000 m = 4.17968750000 f(m) = −0.757428 ε = 0.0078125

n = 7 a = 4.17968750000 b = 4.18750000000 m = 4.18359375000 f(m) = −0.377476 ε = 0.00390625

n = 8 a = 4.18359375000 b = 4.18750000000 m = 4.18554687500 f(m) = −0.187227 ε = 0.00195313

n = 9 a = 4.18554687500 b = 4.18750000000
m = 4.18652343750 f(m) = −0.0920338 ε = 0.000976563

n = 10 a = 4.18652343750 b = 4.18750000000
m = 4.18701171875 f(m) = −0.0444202 ε = 0.000488281

n = 11 a = 4.18701171875 b = 4.18750000000
m = 4.18725585938 f(m) = −0.0206091 ε = 0.000244141

n = 12 a = 4.18725585938 b = 4.18750000000
m = 4.18737792969 f(m) = −0.00870252 ε = 0.00012207

n = 13 a = 4.18737792969 b = 4.18750000000
m = 4.18743896484 f(m) = −0.00274896 ε = 0.0000610352

n = 14 a = 4.18743896484 b = 4.18750000000
m = 4.18746948242 f(m) = 0.00022789 ε = 0.0000305176

n = 15 a = 4.18743896484 b = 4.18746948242
m = 4.18745422363 f(m) = −0.00126054 ε = 0.0000152588

n = 16 a = 4.18745422363 b = 4.18746948242
m = 4.18746185303 f(m) = −0.000516327 ε = 7.62939×10^{-6}

Определяне на броя на итерациите за достигане на зададената точност:

In[73]:=
$$\mathbf{Log2}\left[\frac{\mathbf{5-4}}{\mathbf{0.00001}}\right]\mathbf{-1}$$

Out[73]=

15.6096