

Задача. Дадени са контролните точки  
 $P_0(0,0); P_1(-6,-3); P_2(-6,0); P_3(-12,12); P_4(0,12)$  и  $P_5(12,12)$   
 и възловия вектор  $U = \{0[3]; 0,1; 0,2; 0,8; 1[3]\}$ .

За каква дефинираната Б-сплайн крива  $C(u)$  от  
 втора степен да се:

- а) изчислят всички ненулеви Б-сплайн функции  $N_{i,2}(u)$   
 за  $u=0,3$ ;
- б) подадете нов възел  $t=0,3$  и нов;
- в) камерите  $C(0,3)$  чрез алгоритъма на де Боор.
- г) камерите  $C(0,8)$  чрез алгоритъма на де Боор.

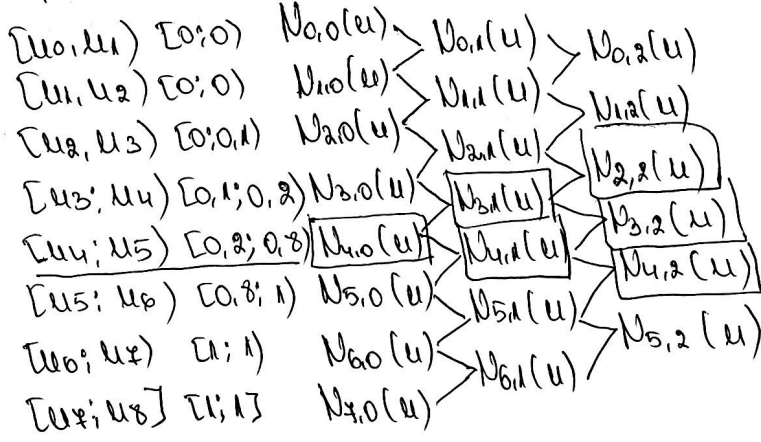
Решение:

а)  $p=2$

$$u_0 = u_1 = u_2 \mid u_3 \mid u_4 \mid u_5 \mid u_6 = u_7 = u_8$$

$$0 \mid 0,1 \mid 0,2 \mid 0,8 \mid 1$$

$N_{i,2}(0,3) = ?$



$u=0,3 \in [0,2; 0,8)$

1.  $N_{4,0}(0,3) = 1, u \in [0,2; 0,8)$

$$N_{i,p}(u) = \frac{u - u_i}{u_{i+p} - u_i} N_{i,p-1}(u) + \frac{u_{i+p+1} - u}{u_{i+p+1} - u_{i+1}} N_{i+1,p-1}(u)$$

$$2. N_{3,1}(0,3) = ? \quad \text{u} \quad N_{4,1}(0,3) = ?$$

$$N_{3,1}(0,3) = \frac{u_5 - u}{u_5 - u_4} \quad N_{3,0}(0,3) = \frac{0,8 - 0,3}{0,8 - 0,2} \cdot 1 = \frac{0,5}{0,6} = \frac{5}{6}$$

$$\Rightarrow N_{3,1}(0,3) = \frac{5}{6}$$

$$N_{3,1}(0,3) + N_{4,1}(0,3) = 1$$

$$N_{4,1}(0,3) = 1 - N_{3,1}(0,3) = 1 - \frac{5}{6} = \frac{1}{6}$$

$$\Rightarrow N_{4,1}(0,3) = \frac{1}{6}$$

$$3. N_{2,2}(0,3) = ? , \quad N_{3,2}(0,3) = ? , \quad N_{4,2}(0,3) = ?$$

$$N_{2,2}(0,3) = \frac{u_5 - u}{u_5 - u_3} \quad N_{3,1}(0,3) = \frac{0,8 - 0,3}{0,8 - 0,1} \cdot \frac{5}{6} = \frac{0,5}{0,7} \cdot \frac{5}{6} = \frac{5}{7} \cdot \frac{5}{6}$$

$$= \frac{5}{7} \cdot \frac{5}{6} = \frac{25}{42}$$

$$\Rightarrow N_{2,2}(0,3) = \frac{25}{42} = 0,59$$

$$N_{4,2}(0,3) = \frac{u - u_4}{u_6 - u_4} \cdot N_{4,1}(0,3) = \frac{0,3 - 0,2}{1 - 0,2} \cdot \frac{1}{6} = \frac{0,1}{0,8} \cdot \frac{1}{6} = \frac{1}{8} \cdot \frac{1}{6} = \frac{1}{48}$$

$$\Rightarrow N_{4,2}(0,3) = \frac{1}{48} = 0,02$$

$$N_{2,2}(0,3) + N_{3,2}(0,3) + N_{4,2}(0,3) = 1$$

$$N_{3,2}(0,3) = 1 - N_{2,2}(0,3) - N_{4,2}(0,3) = 1 - 0,59 - 0,02 = 0,39$$

$$\Rightarrow N_{3,2}(0,3) = 0,39$$

$$\begin{aligned}
 C(0,3) &= N_{2,2}(0,3)P_2 + N_{3,2}(0,3)P_3 + N_{4,2}(0,3)P_4 \\
 &= 0,59(-6,0) + 0,39(-12,12) + 0,02(0,12) \\
 &= (-3,54 - 4,68; 4,68 + 0,24) \\
 &= (-8,22; 4,92)
 \end{aligned}$$

$$\Rightarrow C(0,3) = (-8,22; 4,92)$$

g)

$$\begin{array}{c|c|c|c|c}
 u_0 = u_1 = u_2 & u_3 & u_4 & u_5 & u_6 = u_7 = u_8 \\
 0 & 0,1 & 0,2 & 0,8 & 1
 \end{array}$$

$$t = 0,3 \in [u_4, u_5) \rightarrow \begin{matrix} P_4, P_3, P_2 \\ P_i \end{matrix}$$

$P_0$

$P_1$

$$\begin{array}{c}
 P_2 \\
 P_3 \\
 P_4 \\
 P_5
 \end{array}
 \begin{array}{l}
 > Q_3\left(-\frac{54}{4}, \frac{24}{4}\right) \\
 > Q_4\left(-\frac{21}{2}, 12\right) \\
 \equiv R_5
 \end{array}
 > R_4(8,18; 4,86) = C(0,3)$$

$$Q_3 = (1 - a_3)P_2 + a_3P_3, \quad a_i = \frac{t - u_i}{u_{i+p} - u_i}$$

$$a_3 = \frac{t - u_3}{u_5 - u_3} = \frac{0,3 - 0,1}{0,8 - 0,1} = \frac{0,2}{0,7} = \frac{2}{7}$$

$$Q_3 = \left(1 - \frac{2}{7}\right)P_2 + \frac{2}{7}P_3 = \frac{5}{7}(-6,0) + \frac{2}{7}(-12,12) = \left(-\frac{30}{7}, -\frac{24}{7}; \frac{24}{7}\right)$$

$$Q_3 = \left(-\frac{54}{7}; \frac{24}{7}\right)$$

$$Q_4 = (1 - a_4)P_3 + a_4P_4$$

$$a_4 = \frac{t - u_4}{u_6 - u_4} = \frac{0,3 - 0,2}{1 - 0,2} = \frac{0,1}{0,8} = \frac{1}{8}$$

$$Q_4 = \left(1 - \frac{1}{8}\right)P_3 + \frac{1}{8}P_4 = \frac{7}{8}(-12, 12) + \frac{1}{8}(0, 12) = \left(-\frac{84}{8}; \frac{84}{8} + \frac{12}{8}\right)$$

$$Q_4 = \left(-\frac{84}{8}, \frac{96}{8}\right) = \left(-\frac{21}{2}, 12\right)$$

Преглед простог буревсаке

$$V_0 = V_1 = V_2 \mid V_3 \mid V_4 \mid V_5 \mid V_6 \mid V_7 = V_8 = V_9$$

$$0 \mid 0,1 \mid 0,2 \mid 0,3 \mid 0,8 \mid 1$$

$$\begin{array}{c|c|c|c|c|c|c} P_0' & P_1' & P_2' & P_3' & P_4' & P_5' & P_6' \\ \hline P_0 & P_1 & P_2 & Q_3 & Q_4 & P_4 & P_5 \end{array}$$

$$t = 0,3 \in [V_5; V_6) \rightarrow P_5', P_4', P_3'$$

$$R_4 = (1 - a_4)Q_3 + a_4Q_4$$

$$a_4 = \frac{t - V_4}{V_6 - V_4} = \frac{0,3 - 0,2}{0,8 - 0,2} = \frac{0,1}{0,6} = \frac{1}{6}$$

$$R_4 = \left(1 - \frac{1}{6}\right)\left(-\frac{54}{7}, \frac{24}{7}\right) + \frac{1}{6}\left(-\frac{21}{2}, 12\right)$$

$$= \frac{5}{6}\left(-\frac{54}{7}, \frac{24}{7}\right) + \frac{1}{6}\left(-\frac{21}{2}, 12\right)$$

$$= \left(\frac{-240}{42} - \frac{21}{12}; \frac{120}{42} + \frac{12}{6}\right) = (-6,43 - 1,75; 2,86 + 2)$$

$$R_4 = (8,18; 4,86)$$

$$R_5 = (1 - 0.5)P_4' + 0.5P_5'$$

$$0.5 = \frac{t - u_5}{u_4 - u_5} = \frac{0.3 - 0.3}{1 - 0.3} = 0$$

$$R_5 = (1 - 0)P_4' + 0P_5' = P_4' \equiv Q_4$$

$$\Rightarrow R_5 \equiv Q_4$$

Из второто външно име

$$w_0 = w_1 = w_2 \mid w_3 \mid w_4 \mid w_5 = w_6 \mid w_7 \mid w_8 = w_9 = w_{10}$$

$$0 \mid 0.1 \mid 0.2 \mid 0.3 \mid 0.8 \mid 1$$

$$\begin{array}{c|c|c|c|c|c|c|c} P_0'' & P_1'' & P_2'' & P_3'' & P_4'' & P_5'' & P_6'' & P_7'' \\ \hline P_0 & P_1 & P_2 & Q_3 & R_4 & Q_4 & P_4 & P_5 \end{array}$$

$$b) C(0.3) = R_4(8.18; 4.86)$$

г)  $t = 0.8$  с все възел, то ще то външно име възел, за да може кратността на възел за обхваща със степеня  $P$ .

$$t = 0.8 \in (u_5, u_6) \rightarrow P_5, P_4, P_3$$

$$Q_4 = (1 - 0.4)P_3 + 0.4P_4$$

$$0.4 = \frac{t - u_4}{u_6 - u_4} = \frac{0.8 - 0.2}{1 - 0.2} = \frac{0.6}{0.8} = \frac{6}{8}$$

$$Q_4 = \left(1 - \frac{6}{8}\right)P_3 + \frac{6}{8}P_4 = \frac{2}{8}(-12, 12) + \frac{6}{8}(0, 12)$$

$$= \left(-\frac{24}{8}, \frac{24}{8} + \frac{42}{8}\right) = \left(-\frac{24}{8}, \frac{36}{8}\right) = (-3, 12)$$

$$Q_5 = (1 - 0.5)P_4 + 0.5P_5$$

$$0.5 = \frac{t - u_5}{u_4 - u_5} = \frac{0.8 - 0.8}{1 - 0.8} = 0$$

$$\Rightarrow Q_5 = (1-0)P_4 + 0 \cdot P_5 = P_4$$

$$\Rightarrow C(0,8) = Q_4(-3,12)$$

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