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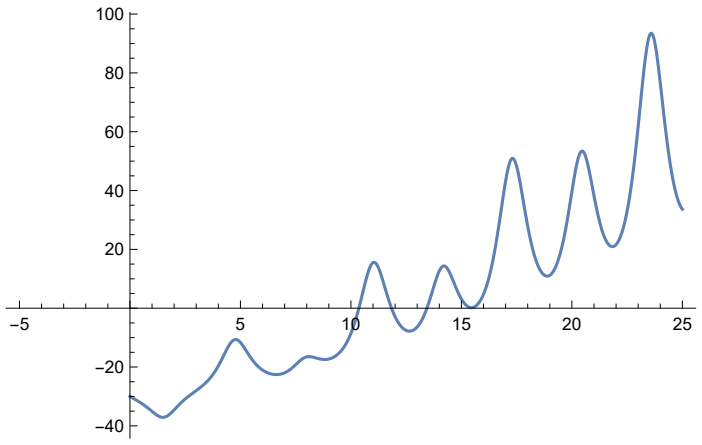
Задача 1:

Условие: $\frac{\sqrt{x^3}-(1+4+4)\sin(x)}{1+\cos^2(x)} - 3(4+4+2) = 0$

а)

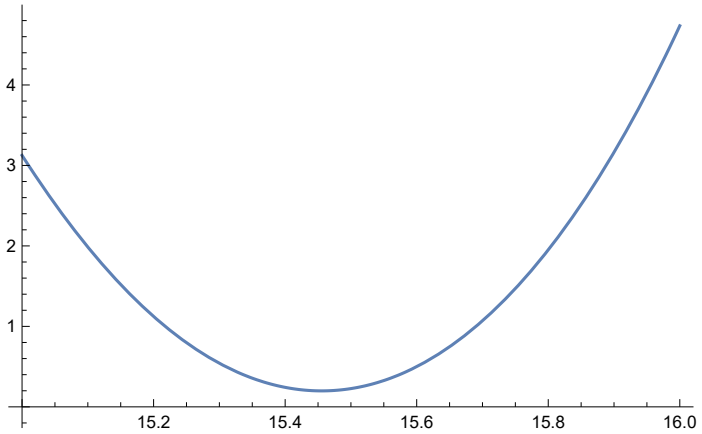
```
In[•]:= f[x_] := (Sqrt[x^3] - (1 + 4 + 4) Sin[x])/(1 + Cos[x]^2) - 3 (4 + 4 + 2)

In[•]:= Plot[f[x], {x, -5, 25}]
```

Out[•]=



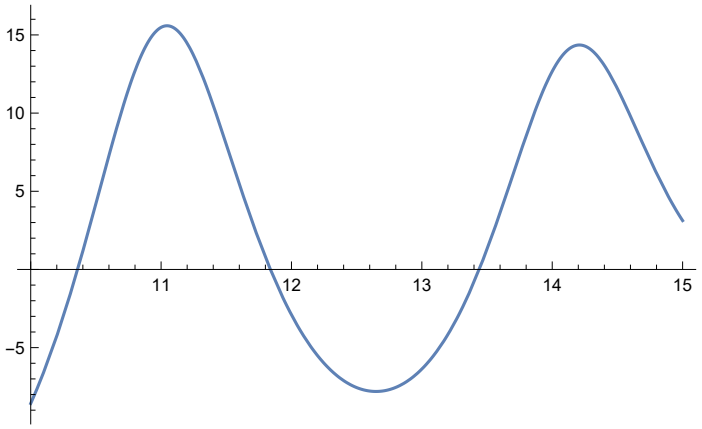
In[•]:= **Plot[f[x], {x, 15, 16}]**

Out[•]=



In[•]:= **Plot[f[x], {x, 10, 15}]**

Out[•]=

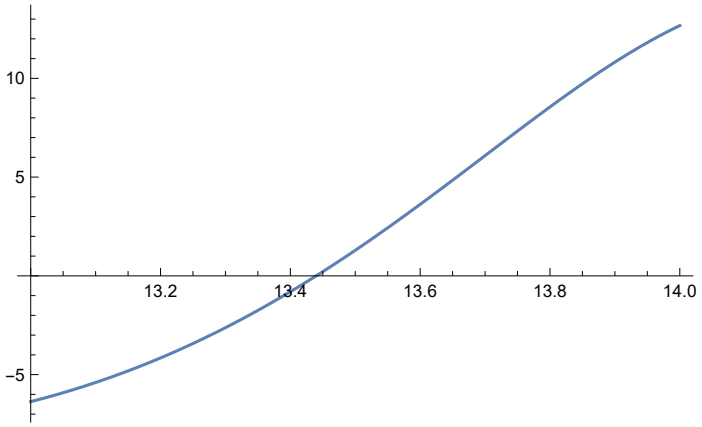


Брой корени: 2

б)

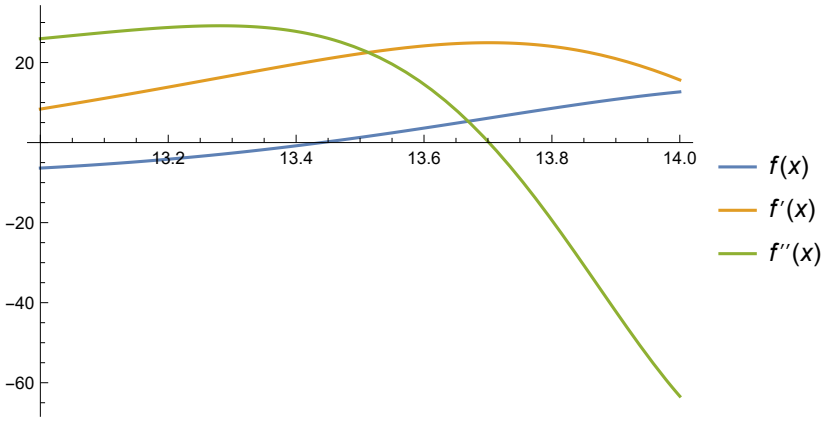
In[•]:= **Plot[f[x], {x, 13, 14}]**

Out[•]=



In[•]:=
```
Plot[{f[x], f'[x], f''[x]}, {x, 13, 14}, PlotLegends → "Expressions"]
```

Out[•]=



In[•]:=
```
f[13.]
```

Out[•]=
```
-6.36873
```

In[•]:=
```
f[14.]
```

Out[•]=
```
12.6699
```

Извод:

(1) Функцията f(x) е непрекъсната, защото е сума от непрекъснати функции

(2). f(13) = -6.36873 < 0

f(14) = 12.6699 > 0

Функцията има различни знаци в двата края на разглеждания интервал [13; 14].

Следователно от (1) и (2) следва, че в интервала [13; 14] функцията има поне един корен.

в) 4 итерации

In[•]:=
```
func[x_] := Sqrt[x^3] - (1 + 4 + 4) Sin[x] / (1 + Cos[x]^2) - 3 (4 + 4 + 2)
```

```
For[
 (*Начални условия*)
 n = 0; a = 13.; b = 14.,
 n ≤ 4, n++,
 (*Тяло на цикъла*)
 Print["|", "n = ", n, "| a_n = ", a, "| b_n = ", b,
  "| m_n = ", m = (a + b)/2, "| f(m_n) = ", func[m], "| ε_n = ", (b - a)/2, "|"];
 If[func[m] < 0, a = m, b = m]
]
```

|n = 0| a_n = 13.| b_n = 14.| m_n = 13.5| f(m_n) = 1.29267| ε_n = 0.5|

|n = 1| a_n = 13.| b_n = 13.5| m_n = 13.25| f(m_n) = -3.42634| ε_n = 0.25|

|n = 2| a_n = 13.25| b_n = 13.5| m_n = 13.375| f(m_n) = -1.2856| ε_n = 0.125|

|n = 3| a_n = 13.375| b_n = 13.5| m_n = 13.4375| f(m_n) = -0.0481029| ε_n = 0.0625|

|n = 4| a_n = 13.4375| b_n = 13.5| m_n = 13.4688| f(m_n) = 0.610015| ε_n = 0.03125|

г)

Приближението е m_n = 13.4688 с точност 0.03125.

д)

In[•]:=
$$\text{func}[x\_] := \frac{\sqrt{x^3} - (1+4+4)\,\text{Sin}[x]}{1+\text{Cos}[x]^2} - 3\,(4+4+2)$$

```
epszad = 0.0000001;
eps = 100;
For[
 (*Начални условия*)
 n = 0; a = 13.; b = 14.,
 eps > epszad, n++,
 (*Тяло на цикъла*)
 Print["|", "n = ", n, "| a_n = ", SetPrecision[a, 10],
  "| b_n = ", SetPrecision[b, 10], "| m_n = ", SetPrecision[m = (a + b)/2, 10],
  "| f(m_n) = ", func[m], "| ε_n = ", eps = (b - a)/2, "|"];
 If[func[m] < 0, a = m, b = m]
]
```

$|n = 0|\ a_n = 13.00000000|\ b_n = 14.00000000|\ m_n = 13.50000000|\ f(m_n) = 1.29267|\ \varepsilon_n = 0.5|$

$|n = 1|\ a_n = 13.00000000|\ b_n = 13.50000000|\ m_n = 13.25000000|\ f(m_n) = -3.42634|\ \varepsilon_n = 0.25|$

$|n = 2|\ a_n = 13.25000000|\ b_n = 13.50000000|\ m_n = 13.37500000|\ f(m_n) = -1.2856|\ \varepsilon_n = 0.125|$

$|n = 3|\ a_n = 13.37500000|\ b_n = 13.50000000$
$|\ m_n = 13.43750000|\ f(m_n) = -0.0481029|\ \varepsilon_n = 0.0625|$

$|n = 4|\ a_n = 13.43750000|\ b_n = 13.50000000|\ m_n = 13.46875000|\ f(m_n) = 0.610015|\ \varepsilon_n = 0.03125|$

$|n = 5|\ a_n = 13.43750000|\ b_n = 13.46875000$
$|\ m_n = 13.45312500|\ f(m_n) = 0.277795|\ \varepsilon_n = 0.015625|$

$|n = 6|\ a_n = 13.43750000|\ b_n = 13.45312500$
$|\ m_n = 13.44531250|\ f(m_n) = 0.114045|\ \varepsilon_n = 0.0078125|$

$|n = 7|\ a_n = 13.43750000|\ b_n = 13.44531250$
$|\ m_n = 13.44140625|\ f(m_n) = 0.0327697|\ \varepsilon_n = 0.00390625|$

$|n = 8|\ a_n = 13.43750000|\ b_n = 13.44140625$
$|\ m_n = 13.43945313|\ f(m_n) = -0.00771711|\ \varepsilon_n = 0.00195313|$

$|n = 9|\ a_n = 13.43945313|\ b_n = 13.44140625$
$|\ m_n = 13.44042969|\ f(m_n) = 0.0125137|\ \varepsilon_n = 0.000976563|$

$|n = 10|\ a_n = 13.43945313|\ b_n = 13.44042969$
$|\ m_n = 13.43994141|\ f(m_n) = 0.00239513|\ \varepsilon_n = 0.000488281|$

$|n = 11|\ a_n = 13.43945313|\ b_n = 13.43994141$
$|\ m_n = 13.43969727|\ f(m_n) = -0.00266178|\ \varepsilon_n = 0.000244141|$

$|n = 12|\ a_n = 13.43969727|\ b_n = 13.43994141$
$|\ m_n = 13.43981934|\ f(m_n) = -0.000133523|\ \varepsilon_n = 0.00012207|$

$|n = 13|\ a_n = 13.43981934|\ b_n = 13.43994141$
$|\ m_n = 13.43988037|\ f(m_n) = 0.00113075|\ \varepsilon_n = 0.0000610352|$

$|n = 14|\ a_n = 13.43981934|\ b_n = 13.43988037$
$|\ m_n = 13.43984985|\ f(m_n) = 0.000498603|\ \varepsilon_n = 0.0000305176|$

$|n = 15|\ a_n = 13.43981934|\ b_n = 13.43984985$
$|\ m_n = 13.43983459|\ f(m_n) = 0.000182537|\ \varepsilon_n = 0.0000152588|$

$|n = 16|\ a_n = 13.43981934|\ b_n = 13.43983459$
$|\ m_n = 13.43982697|\ f(m_n) = 0.0000245063|\ \varepsilon_n = 7.62939\times10^{-6}|$

$|n = 17|\ a_n = 13.43981934|\ b_n = 13.43982697$
$|\ m_n = 13.43982315|\ f(m_n) = -0.0000545085|\ \varepsilon_n = 3.8147\times10^{-6}|$

$|n = 18|\ a_n = 13.43982315|\ b_n = 13.43982697$
$|\ m_n = 13.43982506|\ f(m_n) = -0.0000150011|\ \varepsilon_n = 1.90735\times10^{-6}|$

$|n = 19|\ a_n = 13.43982506|\ b_n = 13.43982697$
$|\ m_n = 13.43982601|\ f(m_n) = 4.75261\times10^{-6}|\ \varepsilon_n = 9.53674\times10^{-7}|$

$|n = 20|\ a_n = 13.43982506|\ b_n = 13.43982601$
$|\ m_n = 13.43982553|\ f(m_n) = -5.12425\times10^{-6}|\ \varepsilon_n = 4.76837\times10^{-7}|$

$|n = 21|\ a_n = 13.43982553|\ b_n = 13.43982601$
$|\ m_n = 13.43982577|\ f(m_n) = -1.85822\times10^{-7}|\ \varepsilon_n = 2.38419\times10^{-7}|$

$|n = 22|\ a_n = 13.43982577|\ b_n = 13.43982601$
$|\ m_n = 13.43982589|\ f(m_n) = 2.28339\times10^{-6}|\ \varepsilon_n = 1.19209\times10^{-7}|$

$|n = 23|\ a_n = 13.43982577|\ b_n = 13.43982589$
$|\ m_n = 13.43982583|\ f(m_n) = 1.04879\times10^{-6}|\ \varepsilon_n = 5.96046\times10^{-8}|$

Извод: за достигане на точност 0.0000001 за нужни 23 итерации

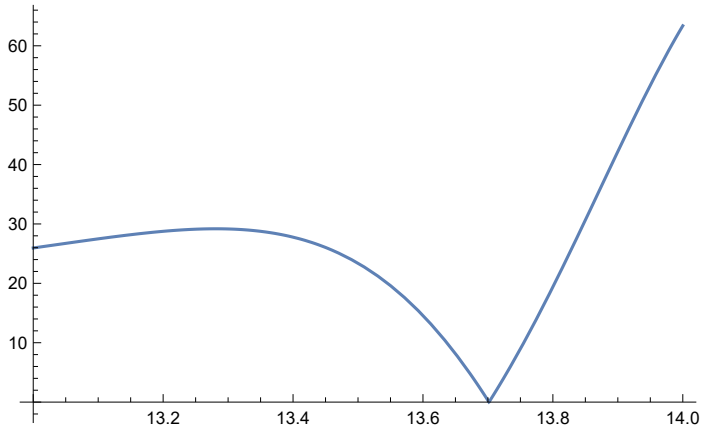
e)

Определяне на постоянните величини

$M_2 = ?,\ m_1 = ?$

In[•]:=
```
Plot[Abs[f''[x]], {x, 13, 14}]
```

Out[•]=



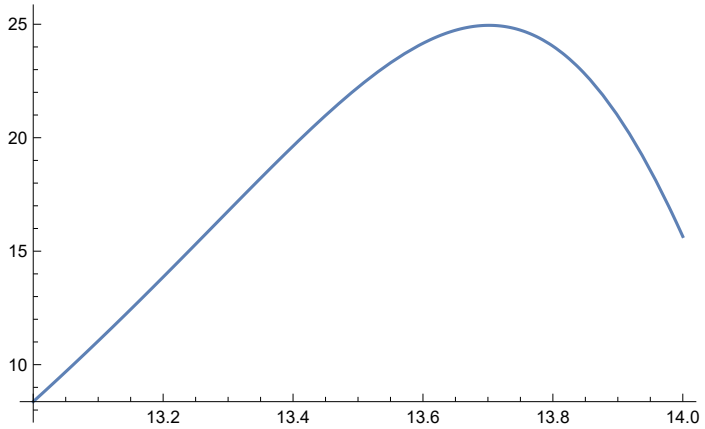
In[•]:=
```
M2 = Abs[f''[14.]]
```

Out[•]=

63.3692

In[•]:=
```
Plot[Abs[f'[x]], {x, 13, 14}]
```

Out[•]=



In[•]:=
```
m1 = Abs[f'[13.]]
```

Out[•]=

8.36955

In[•]:=
$$P = \frac{M2}{2\ m1}$$

Out[•]=

3.7857

Извършване на итерациите

In[•]:=
```
f[x_] := (Sqrt[x^3] - (1 + 4 + 4) Sin[x])/(1 + Cos[x]^2) - 3 (4 + 4 + 2)

x0 = 0.72;
M2 = Abs[f''[0.72]];
m1 = Abs[f'[0.7]];
P = M2/(2 m1);
Print["n = ", 0, " x_n = ", x0, "f(x_n) = ", f[x0], "f'(x_n) = ", f'[x0]]
For[n = 1, n ≤ 4, n++,
 x1 = x0 - f[x0]/f'[x0];
 eps = P * Abs[x1 - x0]^2;
 x0 = x1;
 Print["n = ", n, " x_n = ", x0,
  " f(x_n) = ", f[x0], " f'(x_n) = ", f'[x0], " ε_n= ", eps]
]
```

```
n = 0 x_n = 0.72f(x_n) = -33.4012f'(x_n) = -5.66413
n = 1 x_n = -5.17696 f(x_n) = -36.7009 + 9.80986 i f'(x_n) = -7.82898 + 3.70258 i ε_n= 10.5219
n = 2 x_n = -9.49222 - 0.787806 i f(x_n) =
 -28.2119 - 13.3591 i f'(x_n) = -7.1677 + 1.24059 i ε_n= 5.8222
n = 3 x_n = -13.0005 - 3.25882 i f(x_n) = -30.0125 + 0.529339 i f'(x_n) = 0.399645 -
 0.250541 i ε_n= 5.57163
n = 4 x_n = 41.5059 +
 29.5872 i f(x_n) = -30. + 2.00257×10^-12 i f'(x_n) = -2.00257×10^-12 - 1.57087×10^-12 i ε_n= 1225.38
```

In[•]:=
```
Precision[eps]
```

Out[•]=
```
MachinePrecision
```

In[•]:=
```
% // N
```

Out[•]=
```
15.9546
```