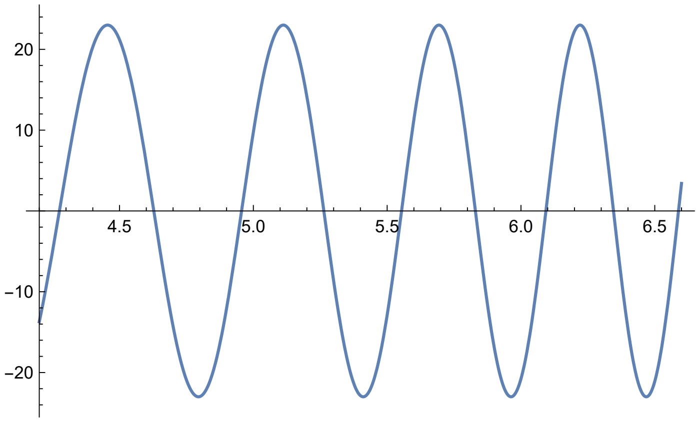
Интерполационен полином на Лагранж

Генериране на данни

In[∘]:=
```
xt = Table[5 + i * 0.15, {i, -3, 9}]
```
Out[∘]=
```
{4.55, 4.7, 4.85, 5., 5.15, 5.3, 5.45, 5.6, 5.75, 5.9, 6.05, 6.2, 6.35}
```
In[∘]:=
```
n = Length[xt]
```
Out[∘]=
```
13
```
In[∘]:=
```
f[x_] := 23 Cos[x^2 - 1]
yt = f[xt]
```
Out[∘]=
```
{15.1287, -14.2763, -19.8288, 9.75612, 21.275, -8.66902,
 -20.9238, 11.3257, 18.3575, -16.8677, -11.5441, 22.2322, -1.20905}
```
In[∘]:=
```
grf = Plot[f[x], {x, 4.2, 6.6}]
```
Out[∘]=

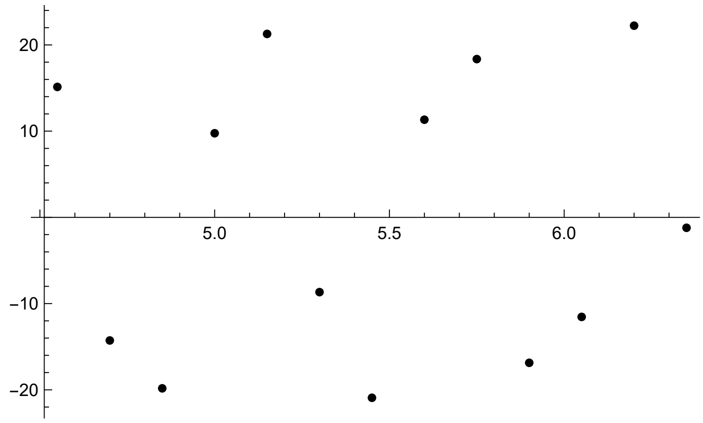


In[∘]:=
```
points = Table[{xt[[i]], yt[[i]]}, {i, 1, n}]
```
Out[∘]=
```
{{4.55, 15.1287}, {4.7, -14.2763}, {4.85, -19.8288}, {5., 9.75612}, {5.15, 21.275},
 {5.3, -8.66902}, {5.45, -20.9238}, {5.6, 11.3257}, {5.75, 18.3575},
 {5.9, -16.8677}, {6.05, -11.5441}, {6.2, 22.2322}, {6.35, -1.20905}}
```

In[ ]:=

```
grp = ListPlot[points, PlotStyle → Black]
```

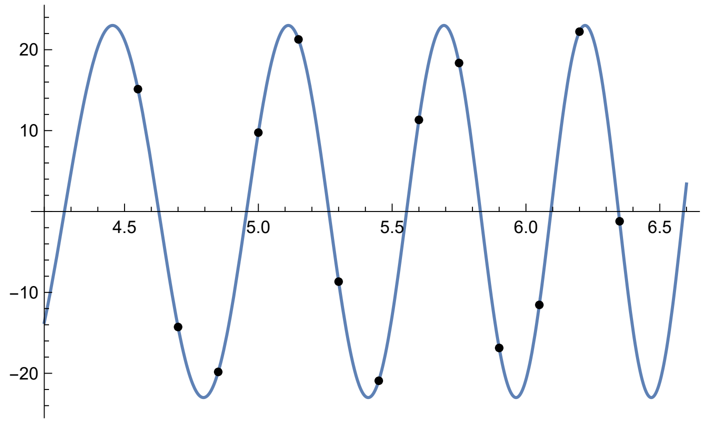
Out[ ]=



In[ ]:=

```
Show[grf, grp]
```

Out[ ]=



Линейна интерполация

In[ ]:=

$$L1[x\_] := 21.275 * \frac{x - 5.3}{5.15 - 5.3} - 8.66902 * \frac{x - 5.15}{5.3 - 5.15}$$

Проверка на интерполационните условия

In[ ]:=

```
L1[5.15]
L1[5.3]
```

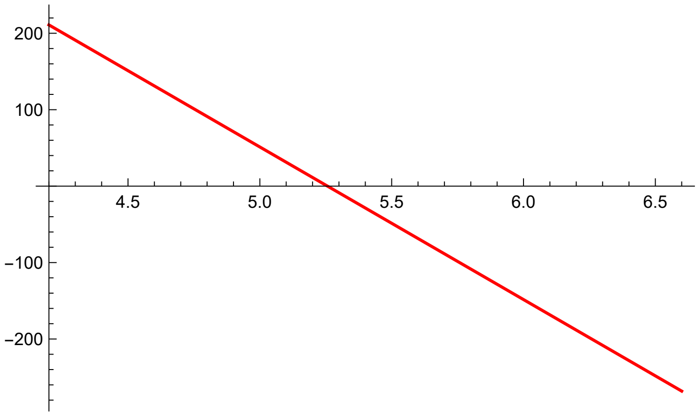
Out[ ]=

21.275

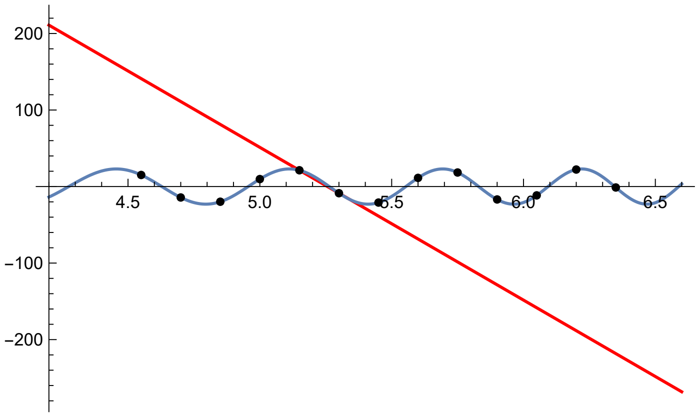
Out[ ]=

−8.66902

In[◦]:=
```
grL1 = Plot[L1[x], {x, 4.2, 6.6}, PlotStyle → Red]
```
Out[◦]=



In[◦]:=
```
Show[grL1, grf, grp]
```
Out[◦]=



Намиране на приближена стойност в т. s = 5.21

In[◦]:=
```
L1[5.21]
```
Out[◦]=

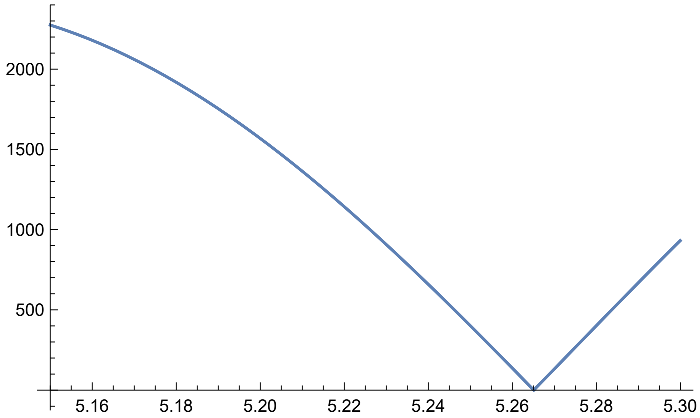
9.29739

Оценка на грешката

Теоретична грешка

In[ ]:=
```
Plot[Abs[f''[x]], {x, 5.15, 5.3}]
```

Out[ ]=



In[ ]:=
```
M2 = Abs[f''[5.15]]
```

Out[ ]=

2274.54

In[ ]:= $R1[x\_] := \frac{M2}{2!} \text{Abs}[(x - 5.15)\ (x - 5.3)]$

In[ ]:=
```
R1[5.21]
```

Out[ ]=

6.14127

Истинската грешка - за сравнение

In[ ]:=
```
Abs[L1[5.21] - f[5.21]]
```

Out[ ]=

2.90893

Квадратична интерполация

In[ ]:=

$$L2[x\_] := 9.756 * \frac{(x - 5.15)\ (x - 5.3)}{(5 - 5.15)\ (5 - 5.3)} + 21.275 * \frac{(x - 5)\ (x - 5.3)}{(5.15 - 5)\ (5.15 - 5.3)} - 8.66902 * \frac{(x - 5)\ (x - 5.15)}{(5.3 - 5)\ (5.3 - 5.15)}$$

In[ ]:=
```
Expand[L2[x]]
```

Out[ ]=

$-24\,100.3 + 9429.01\, x - 921.4\, x^2$

Проверка на интерполационните условия

In[∘]:=
```
L2[5.]
L2[5.15]
L2[5.3]
```

Out[∘]=

9.756

Out[∘]=

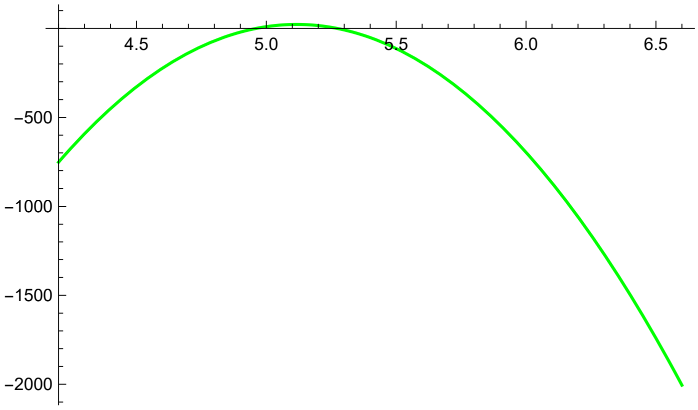
21.275

Out[∘]=

-8.66902

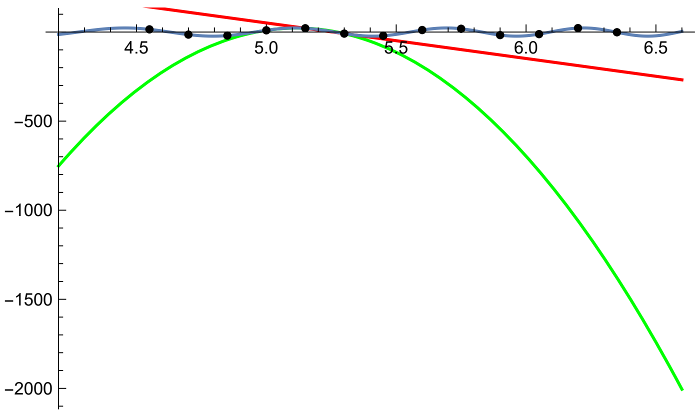
In[∘]:=
```
grL2 = Plot[L2[x], {x, 4.2, 6.6}, PlotStyle → Green]
```

Out[∘]=

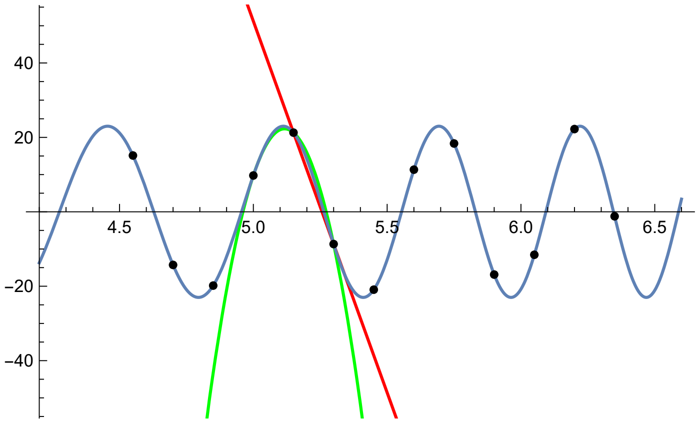


In[∘]:=
```
Show[grL2, grL1, grf, grp]
```

Out[∘]=



In[•]:=
```
Show[grL2, grL1, grf, grp, PlotRange → {-50, 50}]
```
Out[•]=



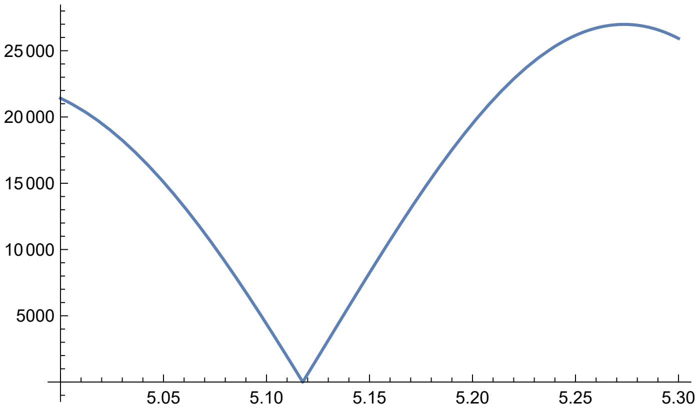
Намиране на приближена стойност в т. s = 5.21

In[•]:=
```
L2[5.21]
```
Out[•]=
```
14.273
```

Оценка на грешката

Теоретична грешка

In[•]:=
```
Plot[Abs[f'''[x]], {x, 5., 5.3}]
```
Out[•]=



In[•]:=
```
M3 = 30000
```
Out[•]=
```
30000
```

In[•]:= $R2[x\_] := \frac{M3}{3!} \text{Abs}[(x - 5)\ (x - 5.15)\ (x - 5.3)]$

In[•]:=
```
R2[5.21]
```
Out[•]=
```
5.67
```

Истинската грешка - за сравнение

In[*]:=
```
Abs[L2[5.21] - f[5.21]]
```
Out[*]=

2.06663

При екстраполация

линейна интерполация

In[*]:=
```
L1[12]
```
Out[*]=

-1346.17

In[*]:=
```
f[12.]
```
Out[*]=

1.32256

In[*]:=
```
R1[12]
```
Out[*]=

52 195.1

квадратична интерполация

In[*]:=
```
L2[12]
```
Out[*]=

-43 633.8

In[*]:=
```
f[12.]
```
Out[*]=

1.32256

In[*]:=
```
R2[12]
```
Out[*]=

1.60633×10^{6}