Задача. Намерете асимптотичните линии на $S: \vec{r}(u\cos v, u\sin v, v^2)$ в произволна точка и в т. $P(u=-1, v=1)$.

Решение:

$\vec{r}_u(\cos v, \sin v, 0)$ $\qquad \vec{r}_u \times \vec{r}_v = (2v\sin v, -2v\cos v, u)$

$\vec{r}_v(-u\sin v, u\cos v, 2v)$

$\vec{r}_{uu}(0, 0, 0)$ $\qquad |\vec{r}_u \times \vec{r}_v| = \sqrt{4v^2\sin^2 v + 4v^2\cos^2 v + u^2}$

$\vec{r}_{vv}(-u\cos v, -u\sin v, 2)$ $\Rightarrow |\vec{r}_u \times \vec{r}_v| = \sqrt{u^2+4v^2}$

$\vec{r}_{uv}(-\sin v, \cos v, 0)$

$$\vec{N} = \frac{\vec{r}_u \times \vec{r}_v}{|\vec{r}_u \times \vec{r}_v|} = \frac{(2v\sin v, -2v\cos v, u)}{\sqrt{u^2+4v^2}} = \left(\frac{2v\sin v}{\sqrt{u^2+4v^2}}, \frac{-2v\cos v}{\sqrt{u^2+4v^2}}, \frac{u}{\sqrt{u^2+4v^2}}\right)$$

Означаваме $\sqrt{u^2+4v^2} = *$

$$h_{11} = \vec{N}\vec{r}_{uu} = \frac{2v\sin v}{*}\cdot 0 - \frac{2v\cos v}{*}\cdot 0 + \frac{u}{*}\cdot 0 = 0$$

$$h_{12} = \vec{N}\vec{r}_{uv} = \frac{2v\sin v}{\sqrt{u^2+4v^2}}\cdot(-\sin v) - \frac{2v\cos v}{\sqrt{u^2+4v}}\cdot\cos v + \frac{u}{\sqrt{u^2+4v^2}}\cdot 0 = \frac{-2v}{\sqrt{u^2+4v^2}}$$

$$h_{22} = \frac{2v\sin v}{\sqrt{u^2+4v^2}}\cdot(-u\cos v) - \frac{2v\cos v}{\sqrt{u^2+4v^2}}\cdot(-u\sin v) + \frac{u}{\sqrt{u^2+4v^2}}\cdot 2 = \frac{2u}{\sqrt{u^2+4v^2}}$$

$$h = h_{11}h_{22} - h_{12}^2 = -\left(\frac{2u}{\sqrt{u^2+4v^2}}\right)^2 = -\frac{4u^2}{(u^2+4v^2)} < 0 \Rightarrow 2 \text{ асимпт. линии.}$$

$$\underline{II}(du, dv) = h_{11}du^2 + 2h_{12}du\,dv + h_{22}dv^2 = 0$$

$$2\left(\frac{-2v}{\sqrt{u^2+4v^2}}\right)du\,dv + \frac{2u}{\sqrt{u^2+4v^2}}dv^2 = 0 \quad /\cdot\left(\sqrt{u^2+4v^2}\right)$$

$$-2dv(2v\,du - u\,dv) = 0$$

$dv = 0 \quad /\int$

$a_1: V = C_1$

$2v\,du = u\,dv$

$$2\frac{du}{u} = \frac{dv}{v} \quad \Big/\int$$

$a_2: 2\ln u = \ln v + \ln C_2 \Rightarrow a_2: u^2 = v\cdot C_2$

$a_1: V = C_1$ $\qquad$ $a_2: U^2 = V \cdot C_2$

B T. $P(U = -1, V = 1)$

$a_1: 1 = C_1 \Rightarrow C_1 = 1$ $\qquad$ $a_2: (-1)^2 = 1 \cdot C_2 \Rightarrow C_2 = 1$

$\Rightarrow a_1^P: V = 1$ $\qquad$ $\Rightarrow a_2^P: U^2 = V$