Числено диференциране

Табличната функция

съставяне на равномерна мрежа

```
In[77]:= a = 3.; b = 4;
       n = 10;
       h = (b - a)/n;
       xt = Table[a + i * h, {i, 0, n}]
```

```
Out[80]=
       {3., 3.1, 3.2, 3.3, 3.4, 3.5, 3.6, 3.7, 3.8, 3.9, 4.}
```

```
In[81]:= f[x_] := π Cos[2 x - 3]
       yt = f[xt]
```

```
Out[82]=
       {-3.11015, -3.13624, -3.03729, -2.81725, -2.4849,
        -2.05348, -1.5402, -0.965515, -0.352338, 0.274886, 0.891151}
```

```
In[83]:= Length[xt]
```

```
Out[83]=
       11
```

Намиране на производните с точност O(h) - първи порядък на грешката

Първа производна

```
In[84]:= z = Table[(yt[[i + 1]] - yt[[i]])/h, {i, 1, n}]
```

```
Out[84]=
       {-0.260824, 0.989494, 2.20036, 3.32351,
        4.31416, 5.13282, 5.74685, 6.13177, 6.27224, 6.16265}
```

добавяме последния елемент

```
In[85]:= AppendTo[z, (yt[[n + 1]] - yt[[n]])/h]
```

```
Out[85]=
       {-0.260824, 0.989494, 2.20036, 3.32351, 4.31416,
        5.13282, 5.74685, 6.13177, 6.27224, 6.16265, 6.16265}
```

Оценка на грешката

теоретична грешка

In[86]:= **h**

Out[86]=

0.1

истинската грешка

In[87]:= **z (*приближени стойност*)**

Out[87]=

{−0.260824, 0.989494, 2.20036, 3.32351, 4.31416,
5.13282, 5.74685, 6.13177, 6.27224, 6.16265, 6.16265}

In[88]:= **f'[xt] (*точни стойност*)**

Out[88]=

{−0.886683, 0.366776, 1.60561, 2.78044, 3.84442,
4.75513, 5.47627, 5.97909, 6.24354, 6.25909, 6.0251}
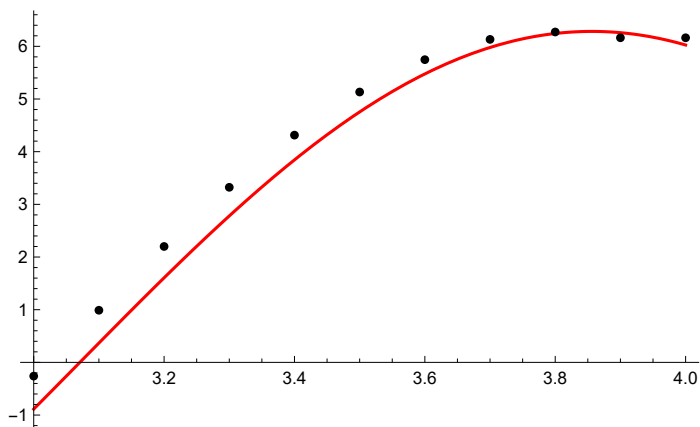
In[89]:= **Abs[z − f'[xt]]**

Out[89]=

{0.625859, 0.622719, 0.594752, 0.543075, 0.469747,
0.377692, 0.270579, 0.152679, 0.0286925, 0.0964381, 0.13755}

Визуализация на резултатите

In[90]:= **points = Table[{xt〚i〛, z〚i〛}, {i, 1, n + 1}];**
grp = ListPlot[points, PlotStyle → Black];
grf = Plot[f'[x], {x, a, b}, PlotStyle → Red];
Show[grf, grp]

Out[93]=

Намиране на производните с точност $O(h^2)$ - втори порядък на грешката

Първа производна

попълваме вътрешните точки

In[94]:= `z = Table[` $\dfrac{yt[[i+1]] - yt[[i-1]]}{2\,h}$ `, {i, 2, n}]`

Out[94]=

{0.364335, 1.59493, 2.76194, 3.81884, 4.72349, 5.43984, 5.93931, 6.202, 6.21744}

добавяме последния елемент

In[95]:= `AppendTo[z,` $\dfrac{yt[[n-1]] - 4\,yt[[n]] + 3\,yt[[n+1]]}{2\,h}$ `]`

Out[95]=

{0.364335, 1.59493, 2.76194, 3.81884, 4.72349, 5.43984, 5.93931, 6.202, 6.21744, 6.10785}

добавяме първия елемент

In[96]:= `PrependTo[z,` $\dfrac{-3\,yt[[1]] + 4\,yt[[2]] - yt[[3]]}{2\,h}$ `]`

Out[96]=

{−0.885983, 0.364335, 1.59493, 2.76194,
 3.81884, 4.72349, 5.43984, 5.93931, 6.202, 6.21744, 6.10785}

Оценка на грешката

теоретична грешка

In[97]:= `h`^{2}

Out[97]=

0.01

истинската грешка

In[98]:= `z (*приближени стойност*)`

Out[98]=

{−0.885983, 0.364335, 1.59493, 2.76194,
 3.81884, 4.72349, 5.43984, 5.93931, 6.202, 6.21744, 6.10785}

In[99]:= `f'[xt] (*точни стойност*)`

Out[99]=

{−0.886683, 0.366776, 1.60561, 2.78044, 3.84442,
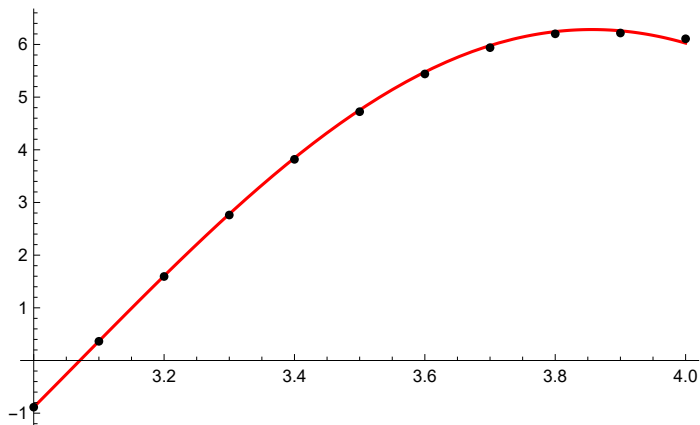 4.75513, 5.47627, 5.97909, 6.24354, 6.25909, 6.0251}

In[100]:=

`Abs[z − f'[xt]]`

Out[100]=

{0.000700299, 0.00244028, 0.0106827, 0.0184992, 0.0255782,
 0.0316375, 0.0364355, 0.039781, 0.0415405, 0.0416439, 0.0827556}

Визуализация на резултатите

In[101]:=
```
points = Table[{xt[[i]], z[[i]]}, {i, 1, n + 1}];
grp = ListPlot[points, PlotStyle → Black];
grf = Plot[f'[x], {x, a, b}, PlotStyle → Red];
Show[grf, grp]
```

Out[104]=

Втора производна

попълваме вътрешните точки

In[105]:=
$$z = \text{Table}\left[\frac{yt[[i-1]] - 2\,yt[[i]] + yt[[i+1]]}{h^2}, \{i, 2, n\}\right]$$

Out[105]=
$$\{12.5032, 12.1087, 11.2315, 9.90651, 8.18658, 6.14029, 3.8492, 1.40466, -1.09588\}$$

добавяме последния елемент - SKIP

добавяме първия елемент - SKIP

Оценка на грешката

теоретична грешка

In[106]:=
$$h^2$$

Out[106]=
0.01

истинската грешка

In[107]:=
```
z (*приближени стойност*)
```

Out[107]=
$$\{12.5032, 12.1087, 11.2315, 9.90651, 8.18658, 6.14029, 3.8492, 1.40466, -1.09588\}$$

In[108]:=
```
f''[xt] (*точни стойност*)
```
Out[108]=
$\{12.4406, 12.5449, 12.1491, 11.269, 9.93959,$
$8.21393, 6.1608, 3.86206, 1.40935, -1.09954, -3.5646\}$

In[109]:=
```
Table[Abs[z[[i]] - f''[xt[[i + 1]]]], {i, 1, n - 1}]
```
Out[109]=
$\{0.0417608, 0.0404432, 0.0375133, 0.0330878,$
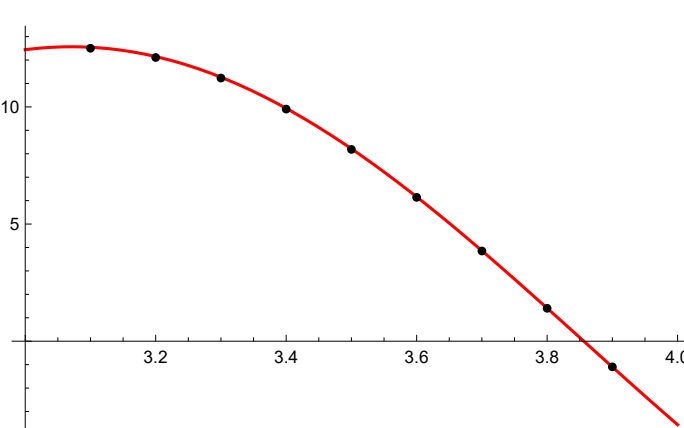$0.0273433, 0.0205086, 0.0128564, 0.00469157, 0.00366027\}$

Визуализация на резултатите

In[110]:=
```
points = Table[{xt[[i + 1]], z[[i]]}, {i, 1, n - 1}];
grp = ListPlot[points, PlotStyle → Black];
grf = Plot[f''[x], {x, a, b}, PlotStyle → Red];
Show[grf, grp]
```
Out[113]=

Числено диференциране - увеличаваме точността чрез сгъстяване на мрежата

Табличната функция

съставяне на равномерна мрежа

In[114]:=
```
a = 3.; b = 4;
n = 100;
h = (b - a)/n;
xt = Table[a + i * h, {i, 0, n}]
```

Out[117]=
```
{3., 3.01, 3.02, 3.03, 3.04, 3.05, 3.06, 3.07, 3.08, 3.09, 3.1, 3.11, 3.12, 3.13, 3.14,
 3.15, 3.16, 3.17, 3.18, 3.19, 3.2, 3.21, 3.22, 3.23, 3.24, 3.25, 3.26, 3.27, 3.28,
 3.29, 3.3, 3.31, 3.32, 3.33, 3.34, 3.35, 3.36, 3.37, 3.38, 3.39, 3.4, 3.41, 3.42, 3.43,
 3.44, 3.45, 3.46, 3.47, 3.48, 3.49, 3.5, 3.51, 3.52, 3.53, 3.54, 3.55, 3.56, 3.57,
 3.58, 3.59, 3.6, 3.61, 3.62, 3.63, 3.64, 3.65, 3.66, 3.67, 3.68, 3.69, 3.7, 3.71,
 3.72, 3.73, 3.74, 3.75, 3.76, 3.77, 3.78, 3.79, 3.8, 3.81, 3.82, 3.83, 3.84, 3.85,
 3.86, 3.87, 3.88, 3.89, 3.9, 3.91, 3.92, 3.93, 3.94, 3.95, 3.96, 3.97, 3.98, 3.99, 4.}
```

In[118]:=
```
f[x_] := π Cos[2 x - 3]
yt = f[xt]
```

Out[119]=
```
{-3.11015, -3.1184, -3.12539, -3.13114, -3.13564, -3.13888, -3.14086, -3.14159,
 -3.14106, -3.13928, -3.13624, -3.13194, -3.12639, -3.1196, -3.11155, -3.10226,
 -3.09173, -3.07996, -3.06696, -3.05273, -3.03729, -3.02062, -3.00275, -2.98368,
 -2.96342, -2.94197, -2.91934, -2.89554, -2.87059, -2.84449, -2.81725, -2.78888,
 -2.7594, -2.72882, -2.69714, -2.66438, -2.63056, -2.59569, -2.55978, -2.52284,
 -2.4849, -2.44596, -2.40604, -2.36516, -2.32334, -2.28058, -2.23692, -2.19236,
 -2.14692, -2.10062, -2.05348, -2.00552, -1.95676, -1.90722, -1.85691, -1.80586,
 -1.75409, -1.70162, -1.64846, -1.59465, -1.5402, -1.48513, -1.42947, -1.37324,
 -1.31646, -1.25915, -1.20134, -1.14304, -1.08429, -1.02511, -0.965515,
 -0.905535, -0.845192, -0.784512, -0.723518, -0.662235, -0.600686, -0.538898,
 -0.476893, -0.414698, -0.352338, -0.289836, -0.227218, -0.16451, -0.101735,
 -0.0389201, 0.0239105, 0.0867316, 0.149518, 0.212244, 0.274886, 0.337418, 0.399815,
 0.462051, 0.524104, 0.585946, 0.647554, 0.708903, 0.769968, 0.830726, 0.891151}
```

In[120]:=
```
Length[xt]
```

Out[120]=
```
101
```

Намиране на производните с точност O(h) - първи порядък на грешката

Първа производна

In[121]:=

$$z = Table\left[\frac{yt[\![i+1]\!] - yt[\![i]\!]}{h}, \{i, 1, n\}\right]$$

Out[121]=

{-0.824423, -0.699691, -0.57468, -0.449438, -0.324017, -0.198466, -0.0728359,
 0.0528234, 0.178462, 0.304028, 0.429474, 0.554747, 0.679799, 0.804578, 0.929036,
 1.05312, 1.17679, 1.29998, 1.42266, 1.54476, 1.66625, 1.78707, 1.90718, 2.02652,
 2.14505, 2.26273, 2.3795, 2.49531, 2.61013, 2.72391, 2.8366, 2.94815, 3.05852, 3.16767,
 3.27555, 3.38212, 3.48734, 3.59117, 3.69355, 3.79446, 3.89386, 3.99169, 4.08793,
 4.18253, 4.27546, 4.36668, 4.45616, 4.54385, 4.62972, 4.71375, 4.79588, 4.8761, 4.95437,
 5.03065, 5.10493, 5.17716, 5.24732, 5.31538, 5.38132, 5.4451, 5.50671, 5.56611,
 5.62329, 5.67822, 5.73087, 5.78124, 5.82929, 5.87501, 5.91838, 5.95938, 5.998, 6.03422,
 6.06803, 6.09941, 6.12835, 6.15484, 6.17886, 6.20042, 6.21949, 6.23608, 6.25018,
 6.26177, 6.27086, 6.27744, 6.28151, 6.28306, 6.28211, 6.27864, 6.27266, 6.26417,
 6.25317, 6.23968, 6.22368, 6.2052, 6.18424, 6.1608, 6.1349, 6.10655, 6.07575, 6.04252}

добавяме последния елемент

In[122]:=

$$AppendTo\left[z, \frac{yt[\![n+1]\!] - yt[\![n]\!]}{h}\right]$$

Out[122]=

{-0.824423, -0.699691, -0.57468, -0.449438, -0.324017, -0.198466, -0.0728359,
 0.0528234, 0.178462, 0.304028, 0.429474, 0.554747, 0.679799, 0.804578,
 0.929036, 1.05312, 1.17679, 1.29998, 1.42266, 1.54476, 1.66625, 1.78707,
 1.90718, 2.02652, 2.14505, 2.26273, 2.3795, 2.49531, 2.61013, 2.72391, 2.8366,
 2.94815, 3.05852, 3.16767, 3.27555, 3.38212, 3.48734, 3.59117, 3.69355,
 3.79446, 3.89386, 3.99169, 4.08793, 4.18253, 4.27546, 4.36668, 4.45616,
 4.54385, 4.62972, 4.71375, 4.79588, 4.8761, 4.95437, 5.03065, 5.10493, 5.17716,
 5.24732, 5.31538, 5.38132, 5.4451, 5.50671, 5.56611, 5.62329, 5.67822, 5.73087,
 5.78124, 5.82929, 5.87501, 5.91838, 5.95938, 5.998, 6.03422, 6.06803, 6.09941,
 6.12835, 6.15484, 6.17886, 6.20042, 6.21949, 6.23608, 6.25018, 6.26177, 6.27086,
 6.27744, 6.28151, 6.28306, 6.28211, 6.27864, 6.27266, 6.26417, 6.25317, 6.23968,
 6.22368, 6.2052, 6.18424, 6.1608, 6.1349, 6.10655, 6.07575, 6.04252, 6.04252}

Оценка на грешката

теоретична грешка

In[123]:=

h

Out[123]=

0.01

истинската грешка

In[124]:=

z (*приближени стойност*)

Out[124]=

{−0.824423, −0.699691, −0.57468, −0.449438, −0.324017, −0.198466, −0.0728359, 0.0528234, 0.178462, 0.304028, 0.429474, 0.554747, 0.679799, 0.804578, 0.929036, 1.05312, 1.17679, 1.29998, 1.42266, 1.54476, 1.66625, 1.78707, 1.90718, 2.02652, 2.14505, 2.26273, 2.3795, 2.49531, 2.61013, 2.72391, 2.8366, 2.94815, 3.05852, 3.16767, 3.27555, 3.38212, 3.48734, 3.59117, 3.69355, 3.79446, 3.89386, 3.99169, 4.08793, 4.18253, 4.27546, 4.36668, 4.45616, 4.54385, 4.62972, 4.71375, 4.79588, 4.8761, 4.95437, 5.03065, 5.10493, 5.17716, 5.24732, 5.31538, 5.38132, 5.4451, 5.50671, 5.56611, 5.62329, 5.67822, 5.73087, 5.78124, 5.82929, 5.87501, 5.91838, 5.95938, 5.998, 6.03422, 6.06803, 6.09941, 6.12835, 6.15484, 6.17886, 6.20042, 6.21949, 6.23608, 6.25018, 6.26177, 6.27086, 6.27744, 6.28151, 6.28306, 6.28211, 6.27864, 6.27266, 6.26417, 6.25317, 6.23968, 6.22368, 6.2052, 6.18424, 6.1608, 6.1349, 6.10655, 6.07575, 6.04252, 6.04252}

In[125]:=

f'[xt] (*точни стойност*)

Out[125]=

{−0.886683, −0.762108, −0.637228, −0.512093, −0.386753, −0.261259, −0.13566, −0.0100069, 0.11565, 0.241261, 0.366776, 0.492143, 0.617314, 0.742238, 0.866865, 0.991145, 1.11503, 1.23847, 1.36141, 1.48381, 1.60561, 1.72677, 1.84725, 1.96698, 2.08592, 2.20404, 2.32127, 2.43757, 2.55289, 2.6672, 2.78044, 2.89256, 3.00353, 3.1133, 3.22182, 3.32906, 3.43496, 3.53949, 3.6426, 3.74426, 3.84442, 3.94304, 4.04008, 4.13551, 4.22928, 4.32136, 4.41172, 4.5003, 4.58709, 4.67205, 4.75513, 4.83631, 4.91556, 4.99284, 5.06813, 5.14139, 5.21259, 5.2817, 5.34871, 5.41357, 5.47627, 5.53678, 5.59507, 5.65113, 5.70493, 5.75644, 5.80565, 5.85254, 5.89709, 5.93928, 5.97909, 6.01651, 6.05153, 6.08413, 6.11429, 6.142, 6.16726, 6.19005, 6.21037, 6.2282, 6.24354, 6.25639, 6.26673, 6.27456, 6.27989, 6.2827, 6.283, 6.28079, 6.27607, 6.26883, 6.25909, 6.24684, 6.2321, 6.21486, 6.19513, 6.17293, 6.14826, 6.12113, 6.09155, 6.05954, 6.0251}

In[126]:=

Abs[z − f'[xt]]

Out[126]=

{0.0622601, 0.0624167, 0.0625483, 0.0626549, 0.0627364, 0.0627928, 0.0628242, 0.0628303, 0.0628114, 0.0627673, 0.0626982, 0.0626039, 0.0624846, 0.0623403, 0.0621711, 0.061977, 0.0617582, 0.0615146, 0.0612464, 0.0609537, 0.0606367, 0.0602953, 0.0599299, 0.0595405, 0.0591273, 0.0586904, 0.0582301, 0.0577464, 0.0572397, 0.0567101, 0.0561578, 0.055583, 0.054986, 0.054367, 0.0537262, 0.053064, 0.0523805, 0.0516761, 0.050951, 0.0502056, 0.04944, 0.0486547, 0.0478499, 0.047026, 0.0461833, 0.0453221, 0.0444427, 0.0435456, 0.0426311, 0.0416995, 0.0407513, 0.0397867, 0.0388062, 0.0378102, 0.0367991, 0.0357733, 0.0347332, 0.0336791, 0.0326116, 0.0315311, 0.0304379, 0.0293326, 0.0282155, 0.0270871, 0.0259479, 0.0247984, 0.0236389, 0.0224699, 0.021292, 0.0201055, 0.0189111, 0.017709, 0.0164999, 0.0152841, 0.0140623, 0.0128348, 0.0116022, 0.0103649, 0.00912353, 0.00787849, 0.00663029, 0.00537944, 0.00412644, 0.00287179, 0.00161599, 0.000359538, 0.000897052, 0.00215328, 0.00340865, 0.00466266, 0.0059148, 0.00716458, 0.00841149, 0.00965504, 0.0108947, 0.01213, 0.0133605, 0.0145857, 0.0158049, 0.0170179, 0.0174208}

Визуализация на резултатите

In[127]:=
```
points = Table[{xt〚i〛, z〚i〛}, {i, 1, n + 1}];
grp = ListPlot[points, PlotStyle → Black];
grf = Plot[f'[x], {x, a, b}, PlotStyle → Red];
Show[grf, grp]
```
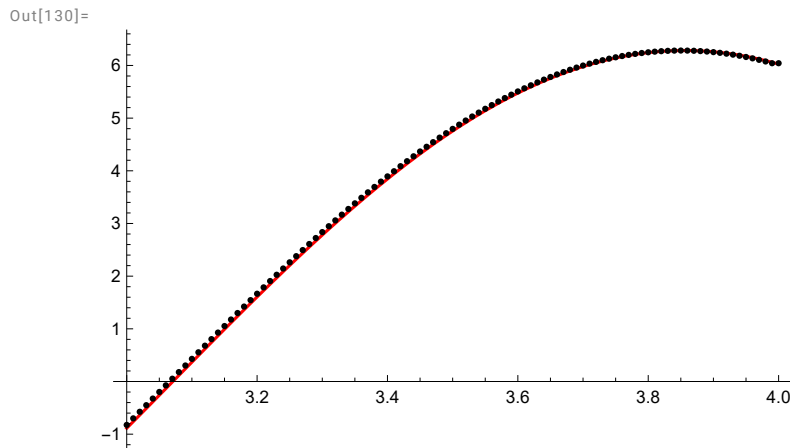
Out[130]=

Намиране на производните с точност $O(h^2)$ - втори порядък на грешката

Първа производна

попълваме вътрешните точки

In[131]:=
$$z = \text{Table}\left[\frac{yt〚i + 1〛 - yt〚i - 1〛}{2\,h}, \{i, 2, n\}\right]$$

Out[131]=
{−0.762057, −0.637186, −0.512059, −0.386728, −0.261242, −0.135651, −0.0100063,
 0.115643, 0.241245, 0.366751, 0.49211, 0.617273, 0.742189, 0.866807, 0.991079, 1.11495,
 1.23838, 1.36132, 1.48371, 1.60551, 1.72666, 1.84712, 1.96685, 2.08579, 2.20389,
 2.32111, 2.43741, 2.55272, 2.66702, 2.78025, 2.89237, 3.00333, 3.11309, 3.22161,
 3.32884, 3.43473, 3.53925, 3.64236, 3.74401, 3.84416, 3.94277, 4.03981, 4.13523, 4.229,
 4.32107, 4.41142, 4.5, 4.58679, 4.67173, 4.75481, 4.83599, 4.91523, 4.99251, 5.06779,
 5.14104, 5.21224, 5.28135, 5.34835, 5.41321, 5.47591, 5.53641, 5.5947, 5.65075,
 5.70455, 5.75606, 5.80526, 5.85215, 5.8967, 5.93888, 5.97869, 6.01611, 6.05113,
 6.08372, 6.11388, 6.14159, 6.16685, 6.18964, 6.20996, 6.22779, 6.24313, 6.25597,
 6.26631, 6.27415, 6.27947, 6.28228, 6.28258, 6.28037, 6.27565, 6.26841, 6.25867,
 6.24642, 6.23168, 6.21444, 6.19472, 6.17252, 6.14785, 6.12072, 6.09115, 6.05913}

добавяме последния елемент

In[132]:=

$$\text{AppendTo}\left[z, \frac{yt[\![n-1]\!] - 4\,yt[\![n]\!] + 3\,yt[\![n+1]\!]}{2\,h}\right]$$

Out[132]=

{−0.762057, −0.637186, −0.512059, −0.386728, −0.261242, −0.135651, −0.0100063,
0.115643, 0.241245, 0.366751, 0.49211, 0.617273, 0.742189, 0.866807, 0.991079, 1.11495,
1.23838, 1.36132, 1.48371, 1.60551, 1.72666, 1.84712, 1.96685, 2.08579, 2.20389,
2.32111, 2.43741, 2.55272, 2.66702, 2.78025, 2.89237, 3.00333, 3.11309, 3.22161,
3.32884, 3.43473, 3.53925, 3.64236, 3.74401, 3.84416, 3.94277, 4.03981, 4.13523, 4.229,
4.32107, 4.41142, 4.5, 4.58679, 4.67173, 4.75481, 4.83599, 4.91523, 4.99251, 5.06779,
5.14104, 5.21224, 5.28135, 5.34835, 5.41321, 5.47591, 5.53641, 5.5947, 5.65075, 5.70455,
5.75606, 5.80526, 5.85215, 5.8967, 5.93888, 5.97869, 6.01611, 6.05113, 6.08372,
6.11388, 6.14159, 6.16685, 6.18964, 6.20996, 6.22779, 6.24313, 6.25597, 6.26631,
6.27415, 6.27947, 6.28228, 6.28258, 6.28037, 6.27565, 6.26841, 6.25867, 6.24642,
6.23168, 6.21444, 6.19472, 6.17252, 6.14785, 6.12072, 6.09115, 6.05913, 6.02591}

добавяме първия елемент

In[133]:=

$$\text{PrependTo}\left[z, \frac{-3\,yt[\![1]\!] + 4\,yt[\![2]\!] - yt[\![3]\!]}{2\,h}\right]$$

Out[133]=

{−0.886789, −0.762057, −0.637186, −0.512059, −0.386728, −0.261242, −0.135651,
−0.0100063, 0.115643, 0.241245, 0.366751, 0.49211, 0.617273, 0.742189,
0.866807, 0.991079, 1.11495, 1.23838, 1.36132, 1.48371, 1.60551, 1.72666,
1.84712, 1.96685, 2.08579, 2.20389, 2.32111, 2.43741, 2.55272, 2.66702,
2.78025, 2.89237, 3.00333, 3.11309, 3.22161, 3.32884, 3.43473, 3.53925,
3.64236, 3.74401, 3.84416, 3.94277, 4.03981, 4.13523, 4.229, 4.32107, 4.41142,
4.5, 4.58679, 4.67173, 4.75481, 4.83599, 4.91523, 4.99251, 5.06779, 5.14104,
5.21224, 5.28135, 5.34835, 5.41321, 5.47591, 5.53641, 5.5947, 5.65075, 5.70455,
5.75606, 5.80526, 5.85215, 5.8967, 5.93888, 5.97869, 6.01611, 6.05113, 6.08372,
6.11388, 6.14159, 6.16685, 6.18964, 6.20996, 6.22779, 6.24313, 6.25597, 6.26631,
6.27415, 6.27947, 6.28228, 6.28258, 6.28037, 6.27565, 6.26841, 6.25867, 6.24642,
6.23168, 6.21444, 6.19472, 6.17252, 6.14785, 6.12072, 6.09115, 6.05913, 6.02591}

Оценка на грешката

теоретична грешка

In[134]:=

$$h^2$$

Out[134]=

0.0001

истинската грешка

In[135]:=

z (*приближени стойност*)

Out[135]=

{−0.886789, −0.762057, −0.637186, −0.512059, −0.386728, −0.261242, −0.135651,
  −0.0100063, 0.115643, 0.241245, 0.366751, 0.49211, 0.617273, 0.742189,
  0.866807, 0.991079, 1.11495, 1.23838, 1.36132, 1.48371, 1.60551, 1.72666,
  1.84712, 1.96685, 2.08579, 2.20389, 2.32111, 2.43741, 2.55272, 2.66702,
  2.78025, 2.89237, 3.00333, 3.11309, 3.22161, 3.32884, 3.43473, 3.53925,
  3.64236, 3.74401, 3.84416, 3.94277, 4.03981, 4.13523, 4.229, 4.32107, 4.41142,
  4.5, 4.58679, 4.67173, 4.75481, 4.83599, 4.91523, 4.99251, 5.06779, 5.14104,
  5.21224, 5.28135, 5.34835, 5.41321, 5.47591, 5.53641, 5.5947, 5.65075, 5.70455,
  5.75606, 5.80526, 5.85215, 5.8967, 5.93888, 5.97869, 6.01611, 6.05113, 6.08372,
  6.11388, 6.14159, 6.16685, 6.18964, 6.20996, 6.22779, 6.24313, 6.25597, 6.26631,
  6.27415, 6.27947, 6.28228, 6.28258, 6.28037, 6.27565, 6.26841, 6.25867, 6.24642,
  6.23168, 6.21444, 6.19472, 6.17252, 6.14785, 6.12072, 6.09115, 6.05913, 6.02591}

In[136]:=

f'[xt] (*точни стойност*)

Out[136]=

{−0.886683, −0.762108, −0.637228, −0.512093, −0.386753, −0.261259, −0.13566,
  −0.0100069, 0.11565, 0.241261, 0.366776, 0.492143, 0.617314, 0.742238,
  0.866865, 0.991145, 1.11503, 1.23847, 1.36141, 1.48381, 1.60561, 1.72677,
  1.84725, 1.96698, 2.08592, 2.20404, 2.32127, 2.43757, 2.55289, 2.6672, 2.78044,
  2.89256, 3.00353, 3.1133, 3.22182, 3.32906, 3.43496, 3.53949, 3.6426, 3.74426,
  3.84442, 3.94304, 4.04008, 4.13551, 4.22928, 4.32136, 4.41172, 4.5003, 4.58709,
  4.67205, 4.75513, 4.83631, 4.91556, 4.99284, 5.06813, 5.14139, 5.21259,
  5.2817, 5.34871, 5.41357, 5.47627, 5.53678, 5.59507, 5.65113, 5.70493, 5.75644,
  5.80565, 5.85254, 5.89709, 5.93928, 5.97909, 6.01651, 6.05153, 6.08413,
  6.11429, 6.142, 6.16726, 6.19005, 6.21037, 6.2282, 6.24354, 6.25639, 6.26673,
  6.27456, 6.27989, 6.2827, 6.283, 6.28079, 6.27607, 6.26883, 6.25909, 6.24684,
 6.2321, 6.21486, 6.19513, 6.17293, 6.14826, 6.12113, 6.09155, 6.05954, 6.0251}

In[137]:=

Abs[z − f'[xt]]

Out[137]=

{0.000105768, 0.0000508062, 0.000042481, 0.0000341389, 0.000025783, 0.0000174169,
  9.04383×10^{-6}, 6.67116×10^{-7}, 7.70986×10^{-6}, 0.0000160838, 0.0000244512,
  0.0000328089, 0.0000411535, 0.0000494815, 0.0000577899, 0.000066075, 0.0000743338,
  0.0000825628, 0.0000907588, 0.0000989185, 0.000107039, 0.000115116, 0.000123147,
  0.000131129, 0.000139059, 0.000146933, 0.000154748, 0.000162501, 0.00017019,
  0.00017781, 0.000185359, 0.000192834, 0.000200232, 0.000207549, 0.000214784,
  0.000221933, 0.000228993, 0.000235961, 0.000242835, 0.000249612, 0.000256289,
  0.000262864, 0.000269333, 0.000275695, 0.000281946, 0.000288085, 0.000294108,
  0.000300014, 0.0003058, 0.000311463, 0.000317002, 0.000322414, 0.000327698,
  0.00033285, 0.000337868, 0.000342752, 0.000347499, 0.000352107, 0.000356573,
  0.000360898, 0.000365078, 0.000369111, 0.000372998, 0.000376735, 0.000380321,
  0.000383755, 0.000387036, 0.000390162, 0.000393131, 0.000395944, 0.000398598,
  0.000401093, 0.000403427, 0.0004056, 0.000407611, 0.000409459, 0.000411143,
  0.000412662, 0.000414016, 0.000415205, 0.000416228, 0.000417084, 0.000417774,
  0.000418296, 0.000418651, 0.000418838, 0.000418859, 0.000418711, 0.000418396,
  0.000417914, 0.000417264, 0.000416448, 0.000415465, 0.000414316, 0.000413001,
  0.000411521, 0.000409876, 0.000408067, 0.000406095, 0.000403961, 0.000806798}

Визуализация на резултатите

In[138]:=
```
points = Table[{xt[[i]], z[[i]]}, {i, 1, n + 1}];
grp = ListPlot[points, PlotStyle → Black];
grf = Plot[f'[x], {x, a, b}, PlotStyle → Red];
Show[grf, grp]
```
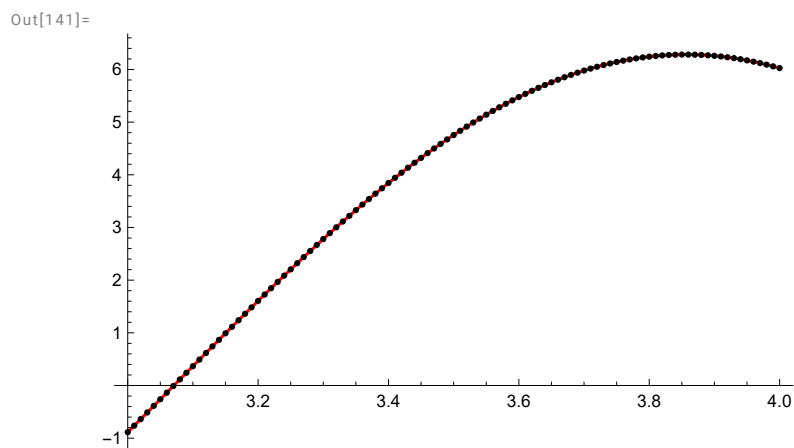
Out[141]=

Втора производна

попълваме вътрешните точки

In[142]:=
$$z = \text{Table}\left[\frac{yt[[i-1]] - 2\,yt[[i]] + yt[[i+1]]}{h^2}, \{i, 2, n\}\right]$$

Out[142]=
{12.4732, 12.5012, 12.5241, 12.5421, 12.5551, 12.563, 12.5659, 12.5638, 12.5567, 12.5445, 12.5273, 12.5052, 12.478, 12.4458, 12.4086, 12.3665, 12.3194, 12.2674, 12.2105, 12.1487, 12.0821, 12.0106, 11.9343, 11.8533, 11.7675, 11.677, 11.5818, 11.482, 11.3776, 11.2686, 11.1552, 11.0372, 10.9149, 10.7882, 10.6572, 10.5219, 10.3824, 10.2388, 10.091, 9.93926, 9.78351, 9.62385, 9.46034, 9.29304, 9.12203, 8.94737, 8.76913, 8.58738, 8.4022, 8.21365, 8.02183, 7.82679, 7.62862, 7.4274, 7.22321, 7.01613, 6.80624, 6.59364, 6.37839, 6.16059, 5.94033, 5.7177, 5.49277, 5.26565, 5.03642, 4.80518, 4.57202, 4.33702, 4.1003, 3.86193, 3.62202, 3.38066, 3.13794, 2.89398, 2.64885, 2.40266, 2.15552, 1.90751, 1.65874, 1.4093, 1.1593, 0.908842, 0.658016, 0.406927, 0.155675, −0.0956388, −0.346915, −0.598052, −0.84895, −1.09951, −1.34963, −1.59921, −1.84814, −2.09634, −2.34371, −2.59013, −2.83552, −3.07977, −3.32279}

добавяме последния елемент - SKIP

добавяме първия елемент - SKIP

Оценка на грешката

теоретична грешка

In[143]:=
$$h^2$$

Out[143]=
0.0001

истинската грешка

In[144]:=

z (*приближени стойност*)

Out[144]=

{12.4732, 12.5012, 12.5241, 12.5421, 12.5551, 12.563, 12.5659, 12.5638, 12.5567, 12.5445, 12.5273, 12.5052, 12.478, 12.4458, 12.4086, 12.3665, 12.3194, 12.2674, 12.2105, 12.1487, 12.0821, 12.0106, 11.9343, 11.8533, 11.7675, 11.677, 11.5818, 11.482, 11.3776, 11.2686, 11.1552, 11.0372, 10.9149, 10.7882, 10.6572, 10.5219, 10.3824, 10.2388, 10.091, 9.93926, 9.78351, 9.62385, 9.46034, 9.29304, 9.12203, 8.94737, 8.76913, 8.58738, 8.4022, 8.21365, 8.02183, 7.82679, 7.62862, 7.4274, 7.22321, 7.01613, 6.80624, 6.59364, 6.37839, 6.16059, 5.94033, 5.7177, 5.49277, 5.26565, 5.03642, 4.80518, 4.57202, 4.33702, 4.1003, 3.86193, 3.62202, 3.38066, 3.13794, 2.89398, 2.64885, 2.40266, 2.15552, 1.90751, 1.65874, 1.4093, 1.1593, 0.908842, 0.658016, 0.406927, 0.155675, −0.0956388, −0.346915, −0.598052, −0.84895, −1.09951, −1.34963, −1.59921, −1.84814, −2.09634, −2.34371, −2.59013, −2.83552, −3.07977, −3.32279}

In[145]:=

f''[xt] (*точни стойност*)

Out[145]=

{12.4406, 12.4736, 12.5016, 12.5246, 12.5425, 12.5555, 12.5634, 12.5664, 12.5642, 12.5571, 12.5449, 12.5278, 12.5056, 12.4784, 12.4462, 12.409, 12.3669, 12.3198, 12.2678, 12.2109, 12.1491, 12.0825, 12.011, 11.9347, 11.8537, 11.7679, 11.6774, 11.5822, 11.4824, 11.378, 11.269, 11.1555, 11.0376, 10.9153, 10.7886, 10.6575, 10.5223, 10.3828, 10.2391, 10.0914, 9.93959, 9.78384, 9.62417, 9.46065, 9.29335, 9.12233, 8.94767, 8.76942, 8.58767, 8.40248, 8.21393, 8.02209, 7.82705, 7.62887, 7.42765, 7.22345, 7.01636, 6.80647, 6.59386, 6.3786, 6.1608, 5.94053, 5.71789, 5.49295, 5.26583, 5.03659, 4.80534, 4.57217, 4.33717, 4.10043, 3.86206, 3.62214, 3.38077, 3.13805, 2.89407, 2.64894, 2.40274, 2.15559, 1.90757, 1.65879, 1.40935, 1.15934, 0.908872, 0.658038, 0.406941, 0.155681, −0.095642, −0.346926, −0.598072, −0.848978, −1.09954, −1.34967, −1.59926, −1.84821, −2.09641, −2.34378, −2.59022, −2.83561, −3.07987, −3.3229, −3.5646}

In[146]:=

```
Table[Abs[z〚i〛 - f''[xt〚i + 1〛]], {i, 1, n - 1}]
```

Out[146]=

$\{0.000415781, 0.000416714, 0.00041748, 0.000418079, 0.000418511, 0.000418776,$
$0.000418873, 0.000418802, 0.000418565, 0.000418159, 0.000417587, 0.000416847,$
$0.00041594, 0.000414868, 0.000413629, 0.000412225, 0.000410656, 0.000408923,$
$0.000407026, 0.000404966, 0.000402744, 0.000400362, 0.000397819, 0.000395117,$
$0.000392257, 0.00038924, 0.000386067, 0.00038274, 0.00037926, 0.000375628,$
$0.000371846, 0.000367915, 0.000363837, 0.000359614, 0.000355247, 0.000350737,$
$0.000346087, 0.000341299, 0.000336375, 0.000331315, 0.000326124, 0.000320801,$
$0.000315351, 0.000309774, 0.000304074, 0.000298252, 0.00029231, 0.000286252,$
$0.000280079, 0.000273794, 0.0002674, 0.000260898, 0.000254292, 0.000247585,$
$0.000240778, 0.000233876, 0.000226879, 0.000219792, 0.000212617, 0.000205357,$
$0.000198015, 0.000190594, 0.000183096, 0.000175525, 0.000167884, 0.000160176,$
$0.000152404, 0.00014457, 0.000136679, 0.000128734, 0.000120736, 0.000112691,$
$0.0001046, 0.0000964678, 0.0000882968, 0.0000800904, 0.000071852, 0.000063585,$
$0.0000552924, 0.0000469777, 0.0000386443, 0.0000302954, 0.0000219343,$
$0.0000135645, 5.18931 \times 10^{-6}, 3.18805 \times 10^{-6}, 0.000011564, 0.0000199355,$
$0.0000282989, 0.000036651, 0.0000449884, 0.0000533079, 0.000061606,$
$0.0000698796, 0.000078125, 0.0000863394, 0.0000945192, 0.000102661, 0.000110762\}$

Визуализация на резултатите

In[147]:=

```
points = Table[{xt〚i + 1〛, z〚i〛}, {i, 1, n - 1}];
grp = ListPlot[points, PlotStyle → Black];
grf = Plot[f''[x], {x, a, b}, PlotStyle → Red];
Show[grf, grp]
```

Out[150]=