Изпит-Зад 2-Георги Г.-2001261019

$$A = \begin{pmatrix} 1+\frac{2}{12} & 0.05 & -0.1 \\ 0.2 & 1-\frac{1}{3} & 0 \\ 0.5 & 0 & 1+\frac{3}{5} \end{pmatrix}, b = (1,-8,10)$$

In[∘]:=
```
A = {{1 + 2/12, 0.05, -0.1}, {0.2, 1 - 1/3, 0}, {0.5, 0, 1 + 3/5}}; b = {1, -8, 10};
```

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1. Да се избере итерационен метод за решаването й. (в случая избираме метода на последователните приближения)

In[∘]:=
```
n = Length[A];
```

In[∘]:=
```
IM = IdentityMatrix[n];
```

In[∘]:=
```
B = IM - A;
```

In[∘]:=
```
c = b;
```

In[∘]:=
```
Print["Итерационният процес е x^(k+1) = ",
 N[B // MatrixForm], ". x^(k) + ", N[c // MatrixForm]]
```

Итерационният процес е $x^{(k+1)} = \begin{pmatrix} -0.166667 & -0.05 & 0.1 \\ -0.2 & 0.333333 & 0. \\ -0.5 & 0. & -0.6 \end{pmatrix} . x^{(k)} + \begin{pmatrix} 1. \\ -8. \\ 10. \end{pmatrix}$

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2. Проверка за сходимост ||B|| < 1

In[∘]:=
```
B // MatrixForm
```

Out[∘]//MatrixForm=

$$\begin{pmatrix} -\frac{1}{6} & -0.05 & 0.1 \\ -0.2 & \frac{1}{3} & 0 \\ -0.5 & 0 & -\frac{3}{5} \end{pmatrix}$$

първа норма

In[ ]:= $\text{Max}\left[\text{Table}\left[\sum_{j=1}^{n} \text{Abs}[B[[i, j]]], \{i, n\}\right]\right]$

Out[ ]=

1.1

втора норма

In[ ]:= $\text{Max}\left[\text{Table}\left[\sum_{i=1}^{n} \text{Abs}[B[[i, j]]], \{j, n\}\right]\right]$

Out[ ]=

0.866667

трета норма

In[ ]:= $\sqrt{\sum_{i=1}^{n} \sum_{j=1}^{n} B[[i, j]]^2}$

Out[ ]=

0.895203

Итерираме

In[ ]:=

$$A = \begin{pmatrix} 1+\frac{2}{12} & 0.05 & -0.1 \\ 0.2 & 1-\frac{1}{3} & 0 \\ 0.5 & 0 & 1+\frac{3}{5} \end{pmatrix}; \ b = \{1, -8, 10\};$$

```
n = Length[A];
IM = IdentityMatrix[n];
B = IM - A;
c = b;
Print["Итерационният процес е x^(k+1) = ",
 N[B // MatrixForm], ". x^(k) + ", N[c // MatrixForm]]
x = {4, 6, 0.5}
```

$$normB = Max\left[Table\left[\sum_{i=1}^{n} Abs[B[[i, j]]], \{j, n\}\right]\right];$$

```
Print["Нормата на B е ", N[normB]]
normx0 = Max[Abs[x]];
normc = Max[Abs[c]];
For[k = 0, k ≤ 3, k++,
```

$$Print\left["k = ", N[k], " x^{(k)} = ", N[x], " \varepsilon_k = ", N\left[eps = normB^k \left(normx0 + \frac{normc}{1 - normB}\right)\right]\right];$$

```
 x = B.x + c
]
Print["За сравнение, точното решение е ", N[LinearSolve[A, b]]]
```

Итерационният процес е $x^{(k+1)} = \begin{pmatrix} -0.166667 & -0.05 & 0.1 \\ -0.2 & 0.333333 & 0. \\ -0.5 & 0. & -0.6 \end{pmatrix} . x^{(k)} + \begin{pmatrix} 1. \\ -8. \\ 10. \end{pmatrix}$

Out[ ]=

{4, 6, 0.5}

Нормата на B е 0.866667

k = 0. $x^{(k)}$ = {4., 6., 0.5} ε_k = 81.

k = 1. $x^{(k)}$ = {0.0833333, -6.8, 7.7} ε_k = 70.2

k = 2. $x^{(k)}$ = {2.09611, -10.2833, 5.33833} ε_k = 60.84

k = 3. $x^{(k)}$ = {1.69865, -11.847, 5.74894} ε_k = 52.728

За сравнение, точното решение е {1.88094, -12.5643, 5.66221}

In[ ]:=

$$N\left[\frac{Log\left[\frac{10^{-4}}{normx0 + \frac{normc}{1-normB}}\right]}{Log[normB]}\right]$$

Out[ ]=

95.0713

НУЖНИ СА НИ 96 ИТЕРАЦИИ

In[∘]:=

```
A = (1 + 2/12   0.05   -0.1
     0.2        1 - 1/3  0
     0.5        0        1 + 3/5); b = {1, -8, 10};
n = Length[A];
IM = IdentityMatrix[n];
B = IM - A;
c = b;
Print["Итерационният процес е x^(k+1) = ",
 N[B // MatrixForm], ". x^(k) + ", N[c // MatrixForm]]
x = {4, 6, 0.5}
normB = Max[Table[Sum[Abs[B[[i, j]]], {i, 1, n}], {j, n}]];
Print["Нормата на B е ", N[normB]]
normx0 = Max[Abs[x]];
normc = Max[Abs[c]];
For[k = 0, k ≤ 96, k++,
 Print["k = ", N[k], " x^(k) = ", N[x], " ε_k = ", N[eps = normB^k (normx0 + normc/(1 - normB))]];
 x = B.x + c
]
Print["За сравнение, точното решение е ", N[LinearSolve[A, b]]]
```

Итерационният процес е $x^{(k+1)} = \begin{pmatrix} -0.166667 & -0.05 & 0.1 \\ -0.2 & 0.333333 & 0. \\ -0.5 & 0. & -0.6 \end{pmatrix} . x^{(k)} + \begin{pmatrix} 1. \\ -8. \\ 10. \end{pmatrix}$

Out[∘]=

{4, 6, 0.5}

Нормата на B е 0.866667

k = 0. $x^{(k)}$ = {4., 6., 0.5} ε_k = 81.

k = 1. $x^{(k)}$ = {0.0833333, -6.8, 7.7} ε_k = 70.2

k = 2. $x^{(k)}$ = {2.09611, -10.2833, 5.33833} ε_k = 60.84

k = 3. $x^{(k)}$ = {1.69865, -11.847, 5.74894} ε_k = 52.728

k = 4. $x^{(k)}$ = {1.88414, -12.2887, 5.70131} ε_k = 45.6976

k = 5. $x^{(k)}$ = {1.87054, -12.4731, 5.63715} ε_k = 39.6046

k = 6. $x^{(k)}$ = {1.87561, -12.5318, 5.68244} ε_k = 34.324

k = 7. $x^{(k)}$ = {1.88223, -12.5524, 5.65273} ε_k = 29.7474

k = 8. $x^{(k)}$ = {1.87919, -12.5606, 5.66725} ε_k = 25.7811

k = 9. $x^{(k)}$ = {1.88156, -12.5627, 5.66006} ε_k = 22.3436

k = 10. $x^{(k)}$ = {1.88055, -12.5639, 5.66319} ε_k = 19.3645

k = 11. $x^{(k)}$ = {1.88109, -12.5641, 5.66181} ε_k = 16.7826

k = 12. $x^{(k)}$ = {1.88087, −12.5642, 5.66237} ε_k = 14.5449

k = 13. $x^{(k)}$ = {1.88097, −12.5643, 5.66214} ε_k = 12.6056

k = 14. $x^{(k)}$ = {1.88093, −12.5643, 5.66223} ε_k = 10.9248

k = 15. $x^{(k)}$ = {1.88095, −12.5643, 5.6622} ε_k = 9.46818

k = 16. $x^{(k)}$ = {1.88094, −12.5643, 5.66221} ε_k = 8.20575

k = 17. $x^{(k)}$ = {1.88094, −12.5643, 5.6622} ε_k = 7.11165

k = 18. $x^{(k)}$ = {1.88094, −12.5643, 5.66221} ε_k = 6.16343

k = 19. $x^{(k)}$ = {1.88094, −12.5643, 5.6622} ε_k = 5.34164

k = 20. $x^{(k)}$ = {1.88094, −12.5643, 5.6622} ε_k = 4.62942

k = 21. $x^{(k)}$ = {1.88094, −12.5643, 5.66221} ε_k = 4.01217

k = 22. $x^{(k)}$ = {1.88094, −12.5643, 5.6622} ε_k = 3.47721

k = 23. $x^{(k)}$ = {1.88094, −12.5643, 5.66221} ε_k = 3.01358

k = 24. $x^{(k)}$ = {1.88094, −12.5643, 5.6622} ε_k = 2.61177

k = 25. $x^{(k)}$ = {1.88094, −12.5643, 5.66221} ε_k = 2.26354

k = 26. $x^{(k)}$ = {1.88094, −12.5643, 5.66221} ε_k = 1.96173

k = 27. $x^{(k)}$ = {1.88094, −12.5643, 5.66221} ε_k = 1.70017

k = 28. $x^{(k)}$ = {1.88094, −12.5643, 5.66221} ε_k = 1.47348

k = 29. $x^{(k)}$ = {1.88094, −12.5643, 5.66221} ε_k = 1.27701

k = 30. $x^{(k)}$ = {1.88094, −12.5643, 5.66221} ε_k = 1.10675

k = 31. $x^{(k)}$ = {1.88094, −12.5643, 5.66221} ε_k = 0.95918

k = 32. $x^{(k)}$ = {1.88094, −12.5643, 5.66221} ε_k = 0.831289

k = 33. $x^{(k)}$ = {1.88094, −12.5643, 5.66221} ε_k = 0.72045

k = 34. $x^{(k)}$ = {1.88094, −12.5643, 5.66221} ε_k = 0.62439

k = 35. $x^{(k)}$ = {1.88094, −12.5643, 5.66221} ε_k = 0.541138

k = 36. $x^{(k)}$ = {1.88094, −12.5643, 5.66221} ε_k = 0.468987

k = 37. $x^{(k)}$ = {1.88094, −12.5643, 5.66221} ε_k = 0.406455

k = 38. $x^{(k)}$ = {1.88094, −12.5643, 5.66221} ε_k = 0.352261

k = 39. $x^{(k)}$ = {1.88094, −12.5643, 5.66221} ε_k = 0.305293

k = 40. $x^{(k)}$ = {1.88094, −12.5643, 5.66221} ε_k = 0.264587

k = 41. $x^{(k)}$ = {1.88094, −12.5643, 5.66221} ε_k = 0.229309

k = 42. $x^{(k)}$ = {1.88094, −12.5643, 5.66221} ε_k = 0.198734

k = 43. $x^{(k)}$ = {1.88094, −12.5643, 5.66221} ε_k = 0.172236

k = 44. $x^{(k)}$ = {1.88094, −12.5643, 5.66221} ε_k = 0.149272

k = 45. $x^{(k)}$ = {1.88094, −12.5643, 5.66221} ε_k = 0.129369

k = 46. $x^{(k)}$ = {1.88094, −12.5643, 5.66221} ε_k = 0.11212

k = 47. $x^{(k)}$ = {1.88094, −12.5643, 5.66221} ε_k = 0.0971703

$k = 48.\ x^{(k)} = \{1.88094, -12.5643, 5.66221\}\ \varepsilon_k = 0.0842142$

$k = 49.\ x^{(k)} = \{1.88094, -12.5643, 5.66221\}\ \varepsilon_k = 0.0729857$

$k = 50.\ x^{(k)} = \{1.88094, -12.5643, 5.66221\}\ \varepsilon_k = 0.0632542$

$k = 51.\ x^{(k)} = \{1.88094, -12.5643, 5.66221\}\ \varepsilon_k = 0.0548203$

$k = 52.\ x^{(k)} = \{1.88094, -12.5643, 5.66221\}\ \varepsilon_k = 0.047511$

$k = 53.\ x^{(k)} = \{1.88094, -12.5643, 5.66221\}\ \varepsilon_k = 0.0411762$

$k = 54.\ x^{(k)} = \{1.88094, -12.5643, 5.66221\}\ \varepsilon_k = 0.035686$

$k = 55.\ x^{(k)} = \{1.88094, -12.5643, 5.66221\}\ \varepsilon_k = 0.0309279$

$k = 56.\ x^{(k)} = \{1.88094, -12.5643, 5.66221\}\ \varepsilon_k = 0.0268042$

$k = 57.\ x^{(k)} = \{1.88094, -12.5643, 5.66221\}\ \varepsilon_k = 0.0232303$

$k = 58.\ x^{(k)} = \{1.88094, -12.5643, 5.66221\}\ \varepsilon_k = 0.0201329$

$k = 59.\ x^{(k)} = \{1.88094, -12.5643, 5.66221\}\ \varepsilon_k = 0.0174485$

$k = 60.\ x^{(k)} = \{1.88094, -12.5643, 5.66221\}\ \varepsilon_k = 0.015122$

$k = 61.\ x^{(k)} = \{1.88094, -12.5643, 5.66221\}\ \varepsilon_k = 0.0131058$

$k = 62.\ x^{(k)} = \{1.88094, -12.5643, 5.66221\}\ \varepsilon_k = 0.0113583$

$k = 63.\ x^{(k)} = \{1.88094, -12.5643, 5.66221\}\ \varepsilon_k = 0.00984389$

$k = 64.\ x^{(k)} = \{1.88094, -12.5643, 5.66221\}\ \varepsilon_k = 0.00853137$

$k = 65.\ x^{(k)} = \{1.88094, -12.5643, 5.66221\}\ \varepsilon_k = 0.00739386$

$k = 66.\ x^{(k)} = \{1.88094, -12.5643, 5.66221\}\ \varepsilon_k = 0.00640801$

$k = 67.\ x^{(k)} = \{1.88094, -12.5643, 5.66221\}\ \varepsilon_k = 0.00555361$

$k = 68.\ x^{(k)} = \{1.88094, -12.5643, 5.66221\}\ \varepsilon_k = 0.00481313$

$k = 69.\ x^{(k)} = \{1.88094, -12.5643, 5.66221\}\ \varepsilon_k = 0.00417138$

$k = 70.\ x^{(k)} = \{1.88094, -12.5643, 5.66221\}\ \varepsilon_k = 0.00361519$

$k = 71.\ x^{(k)} = \{1.88094, -12.5643, 5.66221\}\ \varepsilon_k = 0.00313317$

$k = 72.\ x^{(k)} = \{1.88094, -12.5643, 5.66221\}\ \varepsilon_k = 0.00271541$

$k = 73.\ x^{(k)} = \{1.88094, -12.5643, 5.66221\}\ \varepsilon_k = 0.00235336$

$k = 74.\ x^{(k)} = \{1.88094, -12.5643, 5.66221\}\ \varepsilon_k = 0.00203958$

$k = 75.\ x^{(k)} = \{1.88094, -12.5643, 5.66221\}\ \varepsilon_k = 0.00176763$

$k = 76.\ x^{(k)} = \{1.88094, -12.5643, 5.66221\}\ \varepsilon_k = 0.00153195$

$k = 77.\ x^{(k)} = \{1.88094, -12.5643, 5.66221\}\ \varepsilon_k = 0.00132769$

$k = 78.\ x^{(k)} = \{1.88094, -12.5643, 5.66221\}\ \varepsilon_k = 0.00115066$

$k = 79.\ x^{(k)} = \{1.88094, -12.5643, 5.66221\}\ \varepsilon_k = 0.000997242$

$k = 80.\ x^{(k)} = \{1.88094, -12.5643, 5.66221\}\ \varepsilon_k = 0.000864276$

$k = 81.\ x^{(k)} = \{1.88094, -12.5643, 5.66221\}\ \varepsilon_k = 0.000749039$

$k = 82.\ x^{(k)} = \{1.88094, -12.5643, 5.66221\}\ \varepsilon_k = 0.000649167$

$k = 83.\ x^{(k)} = \{1.88094, -12.5643, 5.66221\}\ \varepsilon_k = 0.000562612$

k = 84. $x^{(k)}$ = {1.88094, -12.5643, 5.66221} ε_k = 0.000487597

k = 85. $x^{(k)}$ = {1.88094, -12.5643, 5.66221} ε_k = 0.000422584

k = 86. $x^{(k)}$ = {1.88094, -12.5643, 5.66221} ε_k = 0.000366239

k = 87. $x^{(k)}$ = {1.88094, -12.5643, 5.66221} ε_k = 0.000317407

k = 88. $x^{(k)}$ = {1.88094, -12.5643, 5.66221} ε_k = 0.000275086

k = 89. $x^{(k)}$ = {1.88094, -12.5643, 5.66221} ε_k = 0.000238408

k = 90. $x^{(k)}$ = {1.88094, -12.5643, 5.66221} ε_k = 0.000206621

k = 91. $x^{(k)}$ = {1.88094, -12.5643, 5.66221} ε_k = 0.000179071

k = 92. $x^{(k)}$ = {1.88094, -12.5643, 5.66221} ε_k = 0.000155195

k = 93. $x^{(k)}$ = {1.88094, -12.5643, 5.66221} ε_k = 0.000134502

k = 94. $x^{(k)}$ = {1.88094, -12.5643, 5.66221} ε_k = 0.000116569

k = 95. $x^{(k)}$ = {1.88094, -12.5643, 5.66221} ε_k = 0.000101026

k = 96. $x^{(k)}$ = {1.88094, -12.5643, 5.66221} ε_k = 0.000087556

За сравнение, точното решение е {1.88094, -12.5643, 5.66221}

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4. Какъв е минималния брой итерации, които за нужни за достигане на точност 10^{-7}, работейки по избрания метод при избор на начално приближение x(0) = c?

In[ ]:=

$$N\left[\frac{\text{Log}\left[\frac{10^{-7}}{\text{normx0}+\frac{\text{normc}}{1-\text{normB}}}\right]}{\text{Log}[\text{normB}]}\right]$$

Out[ ]=

143.343