Изпит по КЧМ, И4, РБ, Име: Мкртич Чивиджян, Фак. № 2001261044

Задача 2:

Условие: $A = \begin{pmatrix} 1+\frac{2}{b+3} & 0.05 & -0.1 \\ 0.2 & 1-\frac{1}{a+2} & 0 \\ 0.5 & 0 & 1+\frac{3}{a+4} \end{pmatrix}$, $c = \begin{pmatrix} a \\ a-b \\ b+1 \end{pmatrix}$

In[∘]:=
```
A = (1 + 2/7   0.05   -0.1
     0.2       1 - 1/6   0
     0.5       0      1 + 3/8); b = {4, 0, 5};

Print["За сравнение точното решение е ", N[LinearSolve[A, b]]]
```

За сравнение точното решение е {3.33082, -0.799397, 2.42516}

а)

In[∘]:=
```
n = Length[A];
IM = IdentityMatrix[n];
B = IM - A;
c = b;
Print["Итерационният процес е x^(k+1) = ", B // MatrixForm, ". x^(k) + ", c // MatrixForm]
```

Итерационният процес е $x^{(k+1)} = \begin{pmatrix} -\frac{2}{7} & -0.05 & 0.1 \\ -0.2 & \frac{1}{6} & 0 \\ -0.5 & 0 & -\frac{3}{8} \end{pmatrix} . x^{(k)} + \begin{pmatrix} 4 \\ 0 \\ 5 \end{pmatrix}$

б)

първа норма

In[∘]:=
```
Max[Table[Sum[Abs[B[[i, j]]], {j, 1, n}], {i, n}]]
```

Out[∘]=

0.875

втора норма

In[ ]:= $\text{Max}\left[\text{Table}\left[\sum_{i=1}^{n} \text{Abs}[B[[i, j]]], \{j, n\}\right]\right]$

Out[ ]=

0.985714

трета норма

In[ ]:= $\sqrt{\sum_{i=1}^{n}\sum_{j=1}^{n} B[[i, j]]^2}$

Out[ ]=

0.743327

Избираме най-малката възможна норма, която в случая е трета.

Нормата на матрицата B е по-малка от 1, следователно процесът ще е сходящ при всеки избор на начално приближение.

в)

In[ ]:= x = {5, 12, -2}; (*изборът на начално приближение е произволен*)

In[ ]:= $\text{normB} = \sqrt{\sum_{i=1}^{n}\sum_{j=1}^{n} B[[i, j]]^2};$

In[ ]:= normx0 = Norm[x, 1];
normc = Norm[c, 1];

In[ ]:= For[k = 0, k ≤ 3, k++,

$\text{Print}\left["k = ", k, " x^{(k)} = ", N[x], " \varepsilon_k = ", \text{eps} = \text{normB}^k \left(\text{normx0} + \frac{\text{normc}}{1 - \text{normB}}\right)\right];$

x = B.x + c

]

k = 0 $x^{(k)}$ = {5., 12., -2.} ε_k = 54.0641

k = 1 $x^{(k)}$ = {1.77143, 1., 3.25} ε_k = 40.1873

k = 2 $x^{(k)}$ = {3.76888, -0.187619, 2.89554} ε_k = 29.8723

k = 3 $x^{(k)}$ = {3.22211, -0.785045, 2.02974} ε_k = 22.2049

г)

In[ ]:= $N[10^{-4}]$

Out[ ]=

0.0001

In[ ]:= epszad = 0.0001;

In[ ]:= eps = 1;

In[ ]:=
```
For[k = 0, eps ≥ epszad, k++,
 Print["k = ", k, " x^(k) = ", N[x], " ε_k = ", eps = normB^k (normx0 + normc/(1 - normB))];
 x = B.x + c
]
```

k = 0 $x^{(k)}$ = {3.33082, -0.799397, 2.42516} ε_k = 54.0641
k = 1 $x^{(k)}$ = {3.33082, -0.799397, 2.42516} ε_k = 40.1873
k = 2 $x^{(k)}$ = {3.33082, -0.799397, 2.42516} ε_k = 29.8723
k = 3 $x^{(k)}$ = {3.33082, -0.799397, 2.42516} ε_k = 22.2049
k = 4 $x^{(k)}$ = {3.33082, -0.799397, 2.42516} ε_k = 16.5055
k = 5 $x^{(k)}$ = {3.33082, -0.799397, 2.42516} ε_k = 12.269
k = 6 $x^{(k)}$ = {3.33082, -0.799397, 2.42516} ε_k = 9.11989
k = 7 $x^{(k)}$ = {3.33082, -0.799397, 2.42516} ε_k = 6.77906
k = 8 $x^{(k)}$ = {3.33082, -0.799397, 2.42516} ε_k = 5.03906
k = 9 $x^{(k)}$ = {3.33082, -0.799397, 2.42516} ε_k = 3.74567
k = 10 $x^{(k)}$ = {3.33082, -0.799397, 2.42516} ε_k = 2.78426
k = 11 $x^{(k)}$ = {3.33082, -0.799397, 2.42516} ε_k = 2.06962
k = 12 $x^{(k)}$ = {3.33082, -0.799397, 2.42516} ε_k = 1.5384
k = 13 $x^{(k)}$ = {3.33082, -0.799397, 2.42516} ε_k = 1.14354
k = 14 $x^{(k)}$ = {3.33082, -0.799397, 2.42516} ε_k = 0.850022
k = 15 $x^{(k)}$ = {3.33082, -0.799397, 2.42516} ε_k = 0.631844
k = 16 $x^{(k)}$ = {3.33082, -0.799397, 2.42516} ε_k = 0.469667
k = 17 $x^{(k)}$ = {3.33082, -0.799397, 2.42516} ε_k = 0.349116
k = 18 $x^{(k)}$ = {3.33082, -0.799397, 2.42516} ε_k = 0.259508
k = 19 $x^{(k)}$ = {3.33082, -0.799397, 2.42516} ε_k = 0.192899
k = 20 $x^{(k)}$ = {3.33082, -0.799397, 2.42516} ε_k = 0.143387
k = 21 $x^{(k)}$ = {3.33082, -0.799397, 2.42516} ε_k = 0.106584
k = 22 $x^{(k)}$ = {3.33082, -0.799397, 2.42516} ε_k = 0.0792265
k = 23 $x^{(k)}$ = {3.33082, -0.799397, 2.42516} ε_k = 0.0588912
k = 24 $x^{(k)}$ = {3.33082, -0.799397, 2.42516} ε_k = 0.0437754
k = 25 $x^{(k)}$ = {3.33082, -0.799397, 2.42516} ε_k = 0.0325395
k = 26 $x^{(k)}$ = {3.33082, -0.799397, 2.42516} ε_k = 0.0241875
k = 27 $x^{(k)}$ = {3.33082, -0.799397, 2.42516} ε_k = 0.0179792
k = 28 $x^{(k)}$ = {3.33082, -0.799397, 2.42516} ε_k = 0.0133644
k = 29 $x^{(k)}$ = {3.33082, -0.799397, 2.42516} ε_k = 0.00993416
k = 30 $x^{(k)}$ = {3.33082, -0.799397, 2.42516} ε_k = 0.00738433
k = 31 $x^{(k)}$ = {3.33082, -0.799397, 2.42516} ε_k = 0.00548897
k = 32 $x^{(k)}$ = {3.33082, -0.799397, 2.42516} ε_k = 0.0040801
k = 33 $x^{(k)}$ = {3.33082, -0.799397, 2.42516} ε_k = 0.00303285

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k = 34 x(k) = {3.33082, -0.799397, 2.42516} εk = 0.0022544
k = 35 x(k) = {3.33082, -0.799397, 2.42516} εk = 0.00167576
k = 36 x(k) = {3.33082, -0.799397, 2.42516} εk = 0.00124564
k = 37 x(k) = {3.33082, -0.799397, 2.42516} εk = 0.000925916
k = 38 x(k) = {3.33082, -0.799397, 2.42516} εk = 0.000688258
k = 39 x(k) = {3.33082, -0.799397, 2.42516} εk = 0.000511601
k = 40 x(k) = {3.33082, -0.799397, 2.42516} εk = 0.000380287
k = 41 x(k) = {3.33082, -0.799397, 2.42516} εk = 0.000282678
k = 42 x(k) = {3.33082, -0.799397, 2.42516} εk = 0.000210122
k = 43 x(k) = {3.33082, -0.799397, 2.42516} εk = 0.00015619
k = 44 x(k) = {3.33082, -0.799397, 2.42516} εk = 0.0001161
k = 45 x(k) = {3.33082, -0.799397, 2.42516} εk = 0.0000863002
```

In[∘]:=
```
Print["За сравнение точното решение е ", N[LinearSolve[A, b]]]
```

```
За сравнение точното решение е {3.33082, -0.799397, 2.42516}
```

Извод: Необходими са ни 45 итерации за достигане на исканата точност

д)

```
k = 45 x(k) = {3.33082, -0.799397, 2.42516} εk = 0.0000863002
```

Извод: крайният резултат се представя с 10 знака и за нужни 5 за междинните изчисления

е)

In[∘]:=
```
N[10^-7]
```

Out[∘]=
$1. \times 10^{-7}$

In[∘]:=
```
epszad = 1. * 10^-7;
```

In[∘]:=
```
eps = 1;
```

In[∘]:=
```
For[k = 0, eps ≥ epszad, k++,
 Print["k = ", k, " x(k) = ", N[x], " εk = ", eps = normB^k (normx0 + normc/(1 - normB))];
 x = B.x + c
]
```

```
k = 0 x(k) = {3.33082, -0.799397, 2.42516} εk = 54.0641
k = 1 x(k) = {3.33082, -0.799397, 2.42516} εk = 40.1873
k = 2 x(k) = {3.33082, -0.799397, 2.42516} εk = 29.8723
k = 3 x(k) = {3.33082, -0.799397, 2.42516} εk = 22.2049
k = 4 x(k) = {3.33082, -0.799397, 2.42516} εk = 16.5055
k = 5 x(k) = {3.33082, -0.799397, 2.42516} εk = 12.269
k = 6 x(k) = {3.33082, -0.799397, 2.42516} εk = 9.11989
```

$k = 7 \; x^{(k)} = \{3.33082, -0.799397, 2.42516\} \; \varepsilon_k = 6.77906$

$k = 8 \; x^{(k)} = \{3.33082, -0.799397, 2.42516\} \; \varepsilon_k = 5.03906$

$k = 9 \; x^{(k)} = \{3.33082, -0.799397, 2.42516\} \; \varepsilon_k = 3.74567$

$k = 10 \; x^{(k)} = \{3.33082, -0.799397, 2.42516\} \; \varepsilon_k = 2.78426$

$k = 11 \; x^{(k)} = \{3.33082, -0.799397, 2.42516\} \; \varepsilon_k = 2.06962$

$k = 12 \; x^{(k)} = \{3.33082, -0.799397, 2.42516\} \; \varepsilon_k = 1.5384$

$k = 13 \; x^{(k)} = \{3.33082, -0.799397, 2.42516\} \; \varepsilon_k = 1.14354$

$k = 14 \; x^{(k)} = \{3.33082, -0.799397, 2.42516\} \; \varepsilon_k = 0.850022$

$k = 15 \; x^{(k)} = \{3.33082, -0.799397, 2.42516\} \; \varepsilon_k = 0.631844$

$k = 16 \; x^{(k)} = \{3.33082, -0.799397, 2.42516\} \; \varepsilon_k = 0.469667$

$k = 17 \; x^{(k)} = \{3.33082, -0.799397, 2.42516\} \; \varepsilon_k = 0.349116$

$k = 18 \; x^{(k)} = \{3.33082, -0.799397, 2.42516\} \; \varepsilon_k = 0.259508$

$k = 19 \; x^{(k)} = \{3.33082, -0.799397, 2.42516\} \; \varepsilon_k = 0.192899$

$k = 20 \; x^{(k)} = \{3.33082, -0.799397, 2.42516\} \; \varepsilon_k = 0.143387$

$k = 21 \; x^{(k)} = \{3.33082, -0.799397, 2.42516\} \; \varepsilon_k = 0.106584$

$k = 22 \; x^{(k)} = \{3.33082, -0.799397, 2.42516\} \; \varepsilon_k = 0.0792265$

$k = 23 \; x^{(k)} = \{3.33082, -0.799397, 2.42516\} \; \varepsilon_k = 0.0588912$

$k = 24 \; x^{(k)} = \{3.33082, -0.799397, 2.42516\} \; \varepsilon_k = 0.0437754$

$k = 25 \; x^{(k)} = \{3.33082, -0.799397, 2.42516\} \; \varepsilon_k = 0.0325395$

$k = 26 \; x^{(k)} = \{3.33082, -0.799397, 2.42516\} \; \varepsilon_k = 0.0241875$

$k = 27 \; x^{(k)} = \{3.33082, -0.799397, 2.42516\} \; \varepsilon_k = 0.0179792$

$k = 28 \; x^{(k)} = \{3.33082, -0.799397, 2.42516\} \; \varepsilon_k = 0.0133644$

$k = 29 \; x^{(k)} = \{3.33082, -0.799397, 2.42516\} \; \varepsilon_k = 0.00993416$

$k = 30 \; x^{(k)} = \{3.33082, -0.799397, 2.42516\} \; \varepsilon_k = 0.00738433$

$k = 31 \; x^{(k)} = \{3.33082, -0.799397, 2.42516\} \; \varepsilon_k = 0.00548897$

$k = 32 \; x^{(k)} = \{3.33082, -0.799397, 2.42516\} \; \varepsilon_k = 0.0040801$

$k = 33 \; x^{(k)} = \{3.33082, -0.799397, 2.42516\} \; \varepsilon_k = 0.00303285$

$k = 34 \; x^{(k)} = \{3.33082, -0.799397, 2.42516\} \; \varepsilon_k = 0.0022544$

$k = 35 \; x^{(k)} = \{3.33082, -0.799397, 2.42516\} \; \varepsilon_k = 0.00167576$

$k = 36 \; x^{(k)} = \{3.33082, -0.799397, 2.42516\} \; \varepsilon_k = 0.00124564$

$k = 37 \; x^{(k)} = \{3.33082, -0.799397, 2.42516\} \; \varepsilon_k = 0.000925916$

$k = 38 \; x^{(k)} = \{3.33082, -0.799397, 2.42516\} \; \varepsilon_k = 0.000688258$

$k = 39 \; x^{(k)} = \{3.33082, -0.799397, 2.42516\} \; \varepsilon_k = 0.000511601$

$k = 40 \; x^{(k)} = \{3.33082, -0.799397, 2.42516\} \; \varepsilon_k = 0.000380287$

$k = 41 \; x^{(k)} = \{3.33082, -0.799397, 2.42516\} \; \varepsilon_k = 0.000282678$

$k = 42 \; x^{(k)} = \{3.33082, -0.799397, 2.42516\} \; \varepsilon_k = 0.000210122$

$k = 43 \; x^{(k)} = \{3.33082, -0.799397, 2.42516\} \; \varepsilon_k = 0.00015619$

$k = 44 \; x^{(k)} = \{3.33082, -0.799397, 2.42516\} \; \varepsilon_k = 0.0001161$

$k = 45 \; x^{(k)} = \{3.33082, -0.799397, 2.42516\} \; \varepsilon_k = 0.0000863002$

k = 46 $x^{(k)}$ = {3.33082, -0.799397, 2.42516} ε_k = 0.0000641493

k = 47 $x^{(k)}$ = {3.33082, -0.799397, 2.42516} ε_k = 0.0000476839

k = 48 $x^{(k)}$ = {3.33082, -0.799397, 2.42516} ε_k = 0.0000354448

k = 49 $x^{(k)}$ = {3.33082, -0.799397, 2.42516} ε_k = 0.0000263471

k = 50 $x^{(k)}$ = {3.33082, -0.799397, 2.42516} ε_k = 0.0000195845

k = 51 $x^{(k)}$ = {3.33082, -0.799397, 2.42516} ε_k = 0.0000145577

k = 52 $x^{(k)}$ = {3.33082, -0.799397, 2.42516} ε_k = 0.0000108211

k = 53 $x^{(k)}$ = {3.33082, -0.799397, 2.42516} ε_k = 8.04364×10^{-6}

k = 54 $x^{(k)}$ = {3.33082, -0.799397, 2.42516} ε_k = 5.97906×10^{-6}

k = 55 $x^{(k)}$ = {3.33082, -0.799397, 2.42516} ε_k = 4.4444×10^{-6}

k = 56 $x^{(k)}$ = {3.33082, -0.799397, 2.42516} ε_k = 3.30364×10^{-6}

k = 57 $x^{(k)}$ = {3.33082, -0.799397, 2.42516} ε_k = 2.45569×10^{-6}

k = 58 $x^{(k)}$ = {3.33082, -0.799397, 2.42516} ε_k = 1.82538×10^{-6}

k = 59 $x^{(k)}$ = {3.33082, -0.799397, 2.42516} ε_k = 1.35685×10^{-6}

k = 60 $x^{(k)}$ = {3.33082, -0.799397, 2.42516} ε_k = 1.00859×10^{-6}

k = 61 $x^{(k)}$ = {3.33082, -0.799397, 2.42516} ε_k = 7.4971×10^{-7}

k = 62 $x^{(k)}$ = {3.33082, -0.799397, 2.42516} ε_k = 5.5728×10^{-7}

k = 63 $x^{(k)}$ = {3.33082, -0.799397, 2.42516} ε_k = 4.14241×10^{-7}

k = 64 $x^{(k)}$ = {3.33082, -0.799397, 2.42516} ε_k = 3.07917×10^{-7}

k = 65 $x^{(k)}$ = {3.33082, -0.799397, 2.42516} ε_k = 2.28883×10^{-7}

k = 66 $x^{(k)}$ = {3.33082, -0.799397, 2.42516} ε_k = 1.70135×10^{-7}

k = 67 $x^{(k)}$ = {3.33082, -0.799397, 2.42516} ε_k = 1.26466×10^{-7}

k = 68 $x^{(k)}$ = {3.33082, -0.799397, 2.42516} ε_k = 9.40056×10^{-8}

In[ ]:=
```
Print["За сравнение точното решение е ", N[LinearSolve[A, b]]]
```

За сравнение точното решение е {3.33082, -0.799397, 2.42516}

Извод: Необходими са ни 67 итерации за достигане на исканата точност