Задача. Нека са дадени две криви на Безие $C_1(u)$ и $C_2(u)$, където първата се дефинира чрез контролните точки: $P_0(0,-2)$, $P_1(-2,-2)$, $P_2(-2,0)$, $P_3(0,0)$, а втората: $Q_0 = P_3(0,0)$, $Q_1(2,0)$, $Q_2(2,3)$, $Q_3(0,3)$; u, v са в $[0,1]$.
Проверете непрекъснатостта до втора степен в точка $(0,0)$ (т.е. C^1-, C^2-, G^1-, G^2- непр. и кривинна непрек.).

Решение:

$C_1(u)$: $P_0(0,-2)$, $P_1(-2,-2)$, $P_2(-2,0)$, $\underline{P_3(0,0)}$ $\quad n=3$

$C_2(u)$: $\underline{Q_0(0,0)}$, $Q_1(2,0)$, $Q_2(2,3)$ $Q_3(0,3)$ $\quad n=3$

$C_1(1) = C_2(0) = P_3 = Q_0(0,0)$. - точка на съединяване.

1. ? $\exists$ C^1-непр. $\iff$? $\dot{C}_1(1) = \dot{C}_2(0)$

$\dot{C}_1(1) = 3[P_3 - P_2] = 3[(0,0)-(-2,0)] = 3(2,0) = \underline{(6,0)}$

$\dot{C}_2(0) = 3[Q_1 - Q_0] = 3[(2,0)-(0,0)] = 3(2,0) = (6,0)$

$\Rightarrow \dot{C}_1(1) = \dot{C}(0) \Rightarrow \exists$ C^1-непр.

$\dot{C}_1(1) \uparrow\uparrow \dot{C}(0) \Rightarrow \exists$ G^1-непр.

2. ? $\exists$ C^2-непр. $\iff$ $\ddot{C}_1(1) = \ddot{C}_2(0)$

$\ddot{C}_1(1) = 3\cdot 2\,[P_3 - 2P_2 + P\ ] = 6[(0,0) - 2(-2,0) + (-2,-2)]$
$= 6[(4,0) + (-2,-2)] = 6(2,-2) = \underline{(12,-12)}$

$\ddot{C}_2(0) = 3\cdot 2\,[Q_2 - 2Q_1 + Q_0] = 6[(2,3) - 2(2,0) + (0,0)]$
$= 6[(2,3) - (4,0)] = 6(-2,3) = \underline{(-12,18)}$

$\Rightarrow \ddot{C}_1(1) \neq \ddot{C}_2(0) \Rightarrow \nexists$ C^2-непр.

3\. ? $\exists\, G^2$-непр. $\Leftrightarrow$? $\ddot{c}_1(1)-\ddot{c}_2(0) \uparrow\uparrow \dot{c}_1(1)=\dot{c}_2(0)$

$\ddot{c}_1(1)=(12,-12)$

$\ddot{c}_2(0)=(-12,18)$

$\ddot{c}_1(1)-\ddot{c}_2(0)=(12,-12)-(-12,18)=(24,-30)$

? $\ddot{c}_1(1)-\ddot{c}_2(0)=\lambda\dot{c}_1(1)$

$(24,-30)\neq\lambda(6,0)$

$\Rightarrow \not\exists\, G^2$-непр.

4\. ? $\varkappa$-непр $\Leftrightarrow$? $\varkappa_{c_1}(1)=\varkappa_{c_2}(0)$

$$\varkappa_{c_1}(1)=\frac{|\dot{c}_1(1)\times\ddot{c}_1(1)|}{|\dot{c}_1(1)|^3} = \varkappa_{c_2}(0)=\frac{|\dot{c}_2(0)\times\ddot{c}_2(0)|}{|\dot{c}_2(0)|^3}$$

$\dot{c}_1(1)=(6,0)\to\dot{c}_1(1)=(6,0,0)$

$\ddot{c}_1(1)=(12,-12)\to\ddot{c}_1(1)=(12,-12,0)$

$|\dot{c}_1(1)|=\sqrt{6^2+0^2+0^2}=6$

$\dot{c}_1(1)\times\ddot{c}_1(1)=(0,0,-72)$

$|\dot{c}_1(1)\times\ddot{c}_1(1)|=\sqrt{0^2+0^2+(-72)^2}=72$

$$\varkappa_{c_1}(1)=\frac{|\dot{c}_1(1)\times\ddot{c}_1(1)|}{|\dot{c}_1(1)|^3}=\frac{72}{6}=12$$

$\dot{c}_2(0)=(6,0)\to\dot{c}_2(0)=(6,0,0)$

$\ddot{c}_2(0)=(-12,18)\to\ddot{c}_2(0)=(-12,18,0)$

$|\dot{c}_2(0)|=\sqrt{6^2+0^2+0^2}=6$

$\dot{c}_2(0)\times\ddot{c}_2(0)=(0,0,108)$

$|\dot{c}_2(0)\times\ddot{c}_2(0)|=\sqrt{0^2+0^2+108^2}=108$

$$\varkappa_{c_2}(0)=\frac{|\dot{c}_2(0)\times\ddot{c}_2(0)|}{|\dot{c}_2(0)|^3}=\frac{108}{6}=18$$

$\Rightarrow \varkappa_{c_1}(1)\neq\varkappa_{c_2}(0)\to\not\exists\,\varkappa$-непр