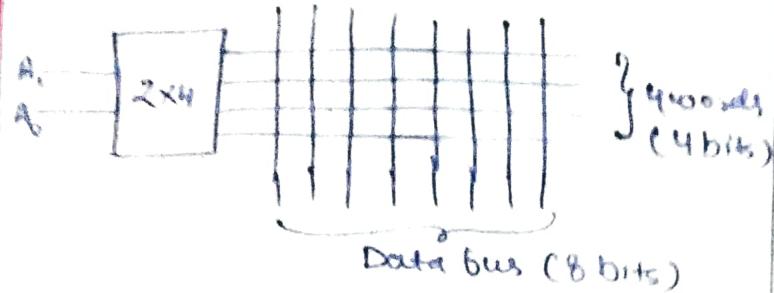
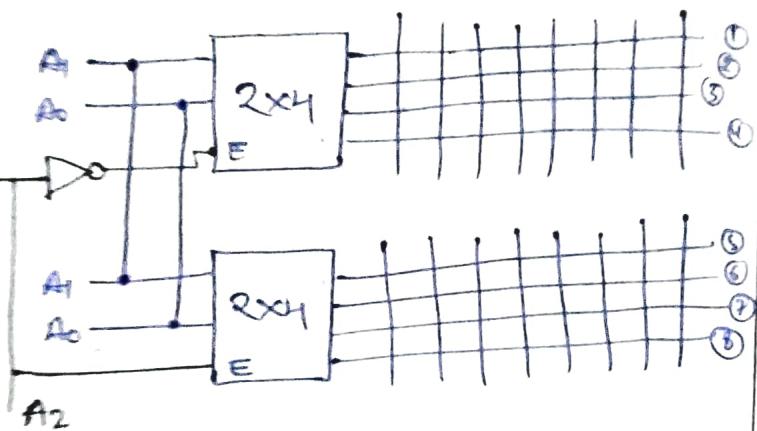


## Address Expansion of ROM



$$M/m \text{ size} = 8 \times 4 = 2^5 \text{ bits}$$

Use above ROM to construct a ROM of size  $(8 \times 8) = 2^6$  bits



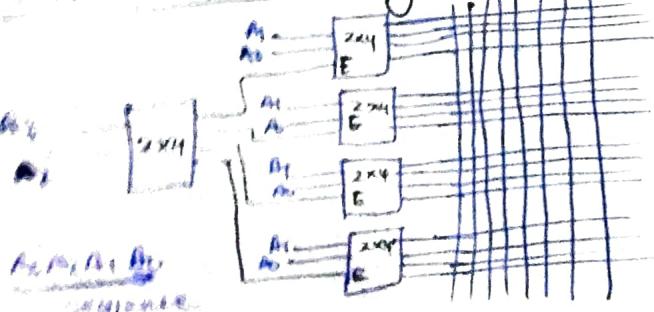
<u>dp line</u>		
$A_2$	$A_1$	$A_0$
0	0	0
1	1	1
1	1	1
1	1	1

Same as  
no. of  
address

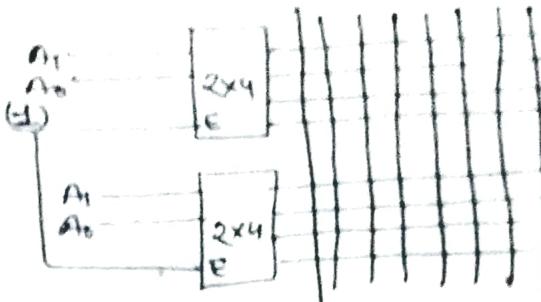
→ we are increasing no. of words without increasing size of word.

→ So, we can say that 3x8 decoder works as 8x8 ROM.

## Word Expansion of ROM



→ It is a  $16 \times 8$  ROM



→ If you set the enable as 1 then only four combinations are possible.

$A_1$	$A_0$	And at each combination 8 buses from each decoder get selected and total 16 buses get selected.
0	0	8
0	1	buses from each decoder get selected
1	0	and total 16 buses
1	1	get selected.

→ So, ROM side becomes  $(4 \text{ words} \times 16 \text{ bits})$

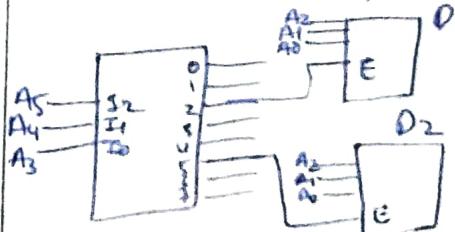
This way we can expand the word size too.

→ This method can apply to RAM also.

→ we can extend this concept upto larger memory.

Finding address range of devices

If  $A_5 A_4 A_3 A_2 A_1 A_0$  are addresses, then ranges does  $D_1$  and  $D_2$  get?



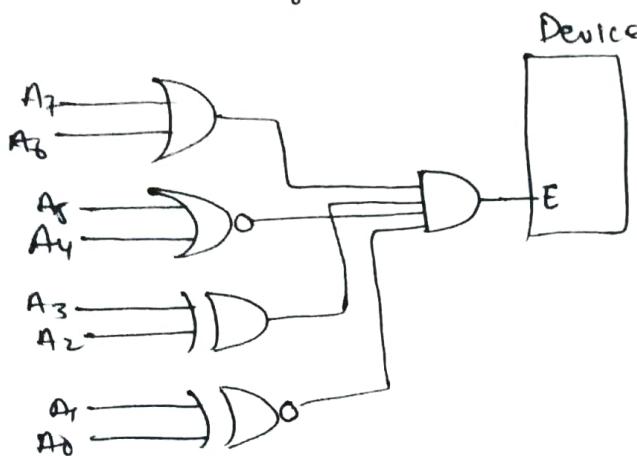
$A_5 A_4 A_3 A_2 A_1 A_0$   
 $D_1 \rightarrow 0 \quad 1 \quad 0 \quad 0 \quad 0 \quad 0 \quad \text{start}$   
 $D_2 \rightarrow 1 \quad 0 \quad 1 \quad 0 \quad 0 \quad 0 \quad \text{start}$

$D_1 \rightarrow (16 \text{ to } 23)$

$D_2 \rightarrow (40 \text{ to } 47)$

### Example on enabling a device

Consider a device that is enabled using 8 address bits as shown below. For how many address the device is enabled?



- a) 1   b) 8   c) 12   d) None

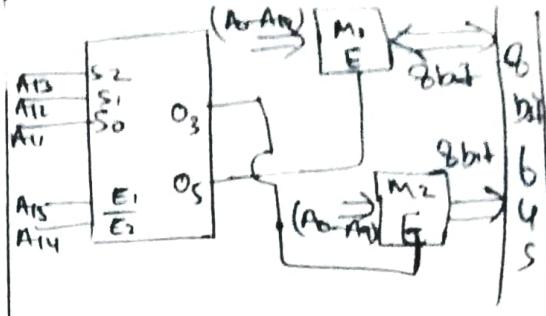
$$\begin{array}{r}
 \begin{array}{cc} A_6 & A_7 \\ 0 & | \\ 1 & 0 \\ 1 & 1 \end{array} \quad
 \begin{array}{cc} A_5 & A_4 \\ 0 & 0 \\ | & | \\ 0 & 1 \\ 1 & 0 \end{array} \quad
 \begin{array}{cc} A_3 & A_2 \\ 0 & | \\ 1 & 0 \\ | & | \\ 0 & 1 \\ 1 & 0 \end{array} \\
 \hline
 \begin{array}{c} (3) \\ \hline A_0 & A_1 \\ 0 & 0 \\ 1 & 1 \end{array} \quad
 \begin{array}{c} (2) \\ \hline \end{array}
 \end{array}$$

$\frac{3 \times 2 \times 2 \times 1}{(2)}$

Total 12 address make device enabled.

### Finding the address range of memory devices

Consider the following interface of two m/m devices with 3x8 decoder. Compute the nature and address range of each m/m device.



Sol<sup>n</sup> M<sub>1</sub> connects with a dual direction bus means we can perform both read and write on it.

M<sub>2</sub> connects with one directional bus so we can only read from it.

M<sub>2</sub>  $\rightarrow$  ROM      M<sub>1</sub>  $\rightarrow$  RAM.

M<sub>1</sub>  $\rightarrow$  (A<sub>0</sub>-A<sub>10</sub>) 11 address bit means  $2^{11}$  words can store

M<sub>2</sub>  $\rightarrow$  (A<sub>0</sub>-A<sub>9</sub>) 10 bit address means  $2^{10}$  words can store

Size of RAM  $\rightarrow 2^{11} \times 8 = 2^{14}$  bits

Size of ROM  $\rightarrow 2^{10} \times 8 = 2^{13}$  bits

Address range for M<sub>1</sub>

A <sub>15</sub>	A <sub>14</sub>	A <sub>13</sub>	A <sub>12</sub>	A <sub>11</sub>	A <sub>10</sub>	A <sub>9</sub>	A <sub>8</sub>	A <sub>7</sub>
1	0	1	0	1	0	—	0	—
1	1	.	.	.	1	—	1	—
1	0	1	0	1	1	—	1	—

E<sub>1</sub>=1, E<sub>2</sub>=0      O<sub>5</sub>

Address range for M<sub>2</sub>

A <sub>15</sub>	A <sub>14</sub>	A <sub>13</sub>	A <sub>12</sub>	A <sub>11</sub>	A <sub>10</sub>	A <sub>9</sub>	A <sub>8</sub>	A <sub>7</sub>
1	0	0	1	1	0	0	—	0
1	—	—	—	—	1	—	1	—

[9800H - 9FFFH]

Here we choose  $\phi$  that comes first as '0' bcoz we are seeking towards getting starting add. and  $\phi$  that comes at last equals to 1 to get final address.

→ But we can give any value to  $\phi$  don't care so each m/m location can get two address.

→ In ROM  $2^n \times 8 \rightarrow$  m/m locations were there, and each m/m location will get 2 address.

Total no. of address in ROM =  $2^{14}$   
as  $2 \times 2^n \times 8$ .

### Introduction to Encoders

→ Encoders are of two types-

- Non-priority
- Priority

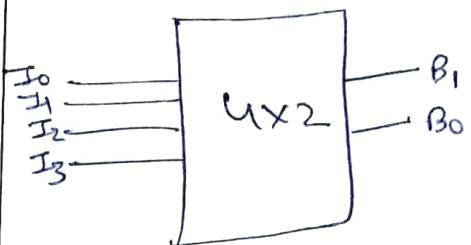
Ex - 4x2 Encoder (Non-Priority)

$I_0$	$I_1$	$I_2$	$I_3$	$B_1$	$B_0$
0	0	0	1	1	1
0	0	1	0	1	0
0	1	0	0	0	1
1	0	0	0	0	0

$$B_1(I_0, I_1, I_2, I_3) = I_0'I_1'I_2'I_3 + I_0'I_1'I_2I_3 \\ = I_0'I_1'(I_2 \oplus I_3).$$

$$B_2(I_0, I_1, I_2, I_3) = I_0'I_1'I_2'I_3 \\ + I_0'I_1I_2'I_3$$

$$\Rightarrow I_0'I_2'(I_1 \oplus I_3)$$



### Encoders

→ They convert one code to another

→ They perform lossless compression.

→ They are of two types

a) Non-priority (Doesn't support simultaneous IP activation).

b) Priority (Support simultaneous IP activation and used for interrupt servicing).

→ Due to static priorities, the lower priorities IP is exposed to starvation.

### Priority Encoding:

Say, priority is  $I_3 > I_2 > I_1 > I_0$   
Highest suffix input gives the highest priority.

$I_0$	$I_1$	$I_2$	$I_3$	$B_1$	$B_0$
$\phi$	$\phi$	$\phi$	1	1	1
$\phi$	$\phi$	1	0	1	0
$\phi$	1	0	0	0	1
1	0	0	0	0	0

$$B_1 = I_3 + I_2 I_3' = \underline{I_2 + I_3}$$

$$B_0 = I_3 + I_1 I_3' I_2' \\ = \underline{(I_3 + I_1) \cdot (I_3 + I_2')}$$

→ We can extend it to any no. of inputs, logic will be same.

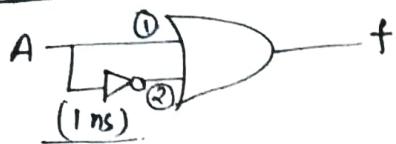
## Introduction to Hazards

Hazards are of two type

- ① Temporary
- ② Permanent

### ① Temporary Hazard

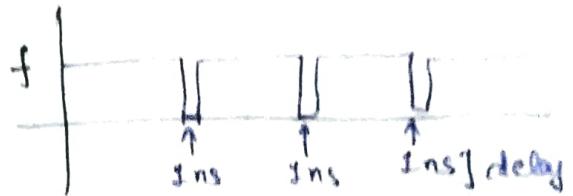
Ex-



Say we have a 'not gate' that takes 1ns to pass a signal then,

assume initially  $A=1$  then value at ① = 1 and value at ② = 0. It might take 1ns to provide value at ② as 0. After setting values we get output at  $f=1$ .

And then suddenly we switch the value at  $A=0$  then value at ① becomes '0' at the same instant but value at ② take 1ns to become '1'. For that one second both location ① and ② will show '0' and  $f$  becomes 0 for 1ns, called as hazards.



It is a temporary hazard, and it is going to correct itself.

### Permanent Hazard

(Short circuit or open  
(always 1)      (always 0))



Say link get broken at B, so it will always show '0' whatever input you give, also called stuck at '0'.

These errors needs to remove by self.

### Hazards in digital circuit

→ The oral function of a digital circuit is called hazard. They can be permanent or temporary. The temporary hazard is due to uneven delays of 1ps.

→ The permanent hazards are resulted due to open circuit or short circuits of the connecting leads (terminals).

→ The hazards can be stuck at 0 (S-a-0) or stuck at 1 (S-a-1)

→ The single-fault analysis is made using "path sensitization technique". This techniques provides test vector.

### Procedure to find Test Vector

Cont

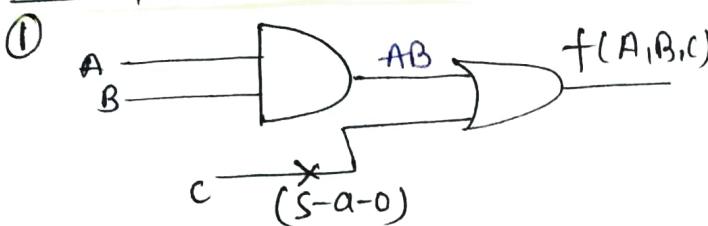
① Inactivate all the other paths except the tested one. For 'AND' or 'NAND' apply the logic 1 for inactivation.

for 'OR' or 'NOR' apply the logic 0 from inactivation.

② Apply the opposite logic value at the tested places

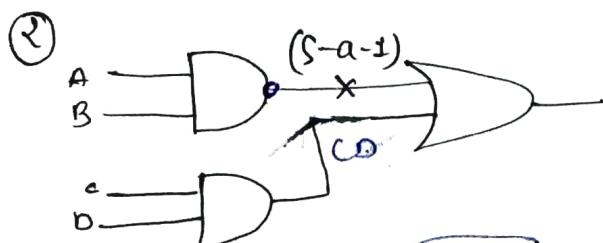
③ The input combinations satisfying the above requirements form the test vectors

### Examples on test vector



Make  $AB = 0$  so that 0/b become depend completely on C values

A	B	C	Choose value of C as opposite to other values
0	0	1	
0	1	1	
1	0	1	



Put $CO = 0$		A	B
C	D	(S-a-1)	1 0
0	0	0	0 1
0	1	0	0 0
1	0	0	

So, for fig ②

A	B	C	D
0	0	0	0
0	1	0	0
1	0	0	0
0	0	0	1
0	1	0	1
1	0	0	1
0	0	1	0
0	1	1	0
1	0	1	0

9  
Test  
Vectors

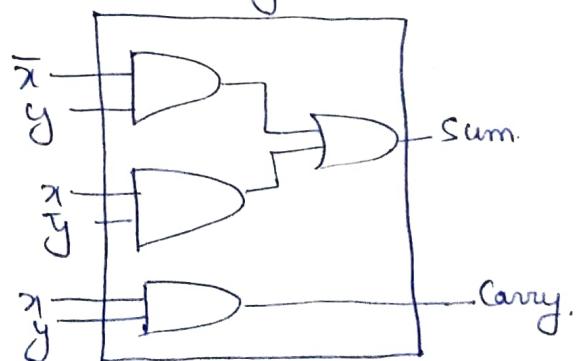
### Half Adder

It can do sum of 2 bits only.

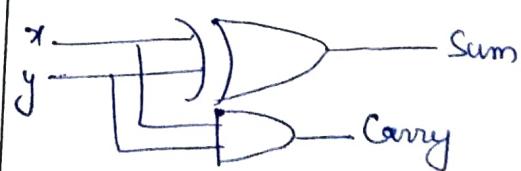
Inputs		Outputs	
x	y	sum	carry
0	0	0	0
0	1	1	0
1	0	1	0
1	1	0	1

$$S = \bar{x}y + x\bar{y} = x \oplus y$$

$$\text{Carry} = xy$$



or



## full Adder.

- It can perform the sum of 3 bits.
- It can be constructed by using two half adders
- It is 3*1/b* too o/b device

x	y	z	S	C
0	0	0	0	0
0	0	1	1	0
0	1	0	1	0
0	1	1	0	1
1	0	0	1	0
1	0	1	0	1
1	1	0	0	1
1	1	1	1	1

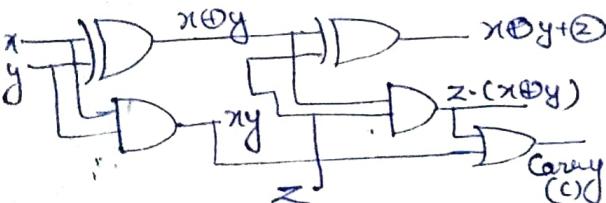
K-map for S

	xy	00	01	11	10
z	0	0	1	0	1
	1	1	0	1	0

$$S = \bar{x}y\bar{z} + \bar{x}\bar{y}z + xy^2 \\ + x\bar{y}\bar{z}$$

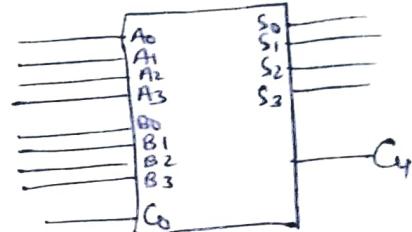
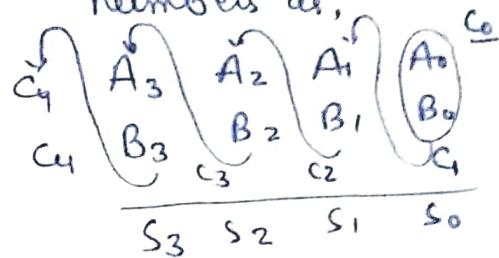
$$S = x \oplus y \oplus z$$

and, C =  $\bar{x}y + yz + zx$   
 $= z(x \oplus y) + xy$

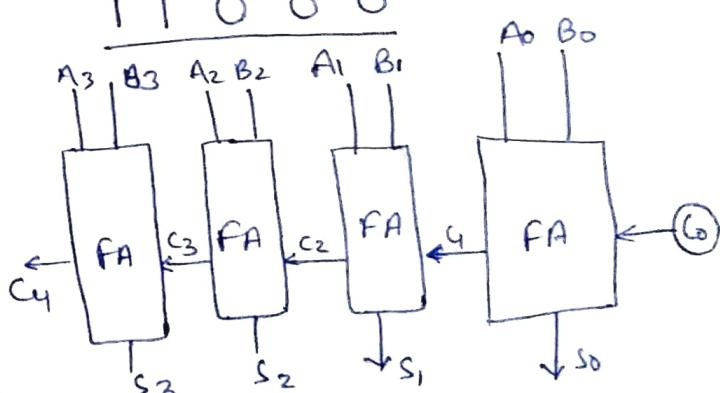
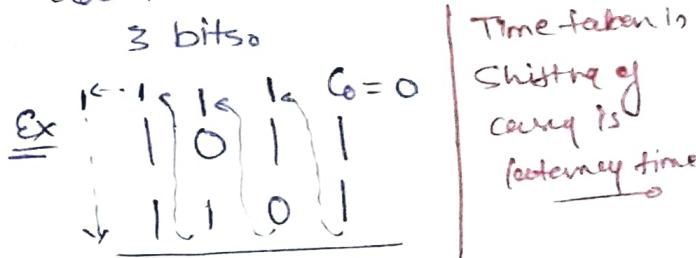


## Ripple Carry Adder

Conventions: we are taking 2, 4 bit numbers as,



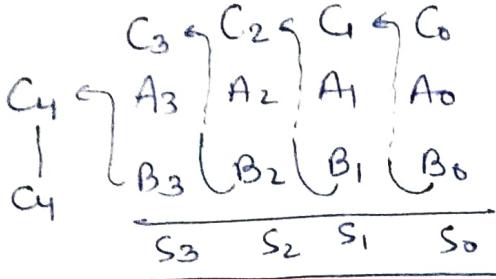
If you take above machine it will take 2<sup>9</sup> inputs means 2<sup>9</sup> combination so, truth table become very large. So, we will use concept of full adder bcz it performs add of 3 bits.



→ Time taken by complete system to add the numbers is high because all the FA works sequentially.

→ To solve this problem of time we have carry look ahead adders.

## Carry Look Ahead Adder



$$G_i = C_0 (A_0 \oplus B_0) + A_0 B_0$$

$$C_2 = G_1 (A_1 \oplus B_1) + A_1 B_1$$

$$C_3 = C_2 (A_2 \oplus B_2) + A_2 B_2$$

$$C_4 = C_3 (A_3 \oplus B_3) + A_3 B_3$$

say,  $G_i^o = A_i B_i$ ,  $P_i^o = A_i \oplus B_i$   
 ↪ Generate fn      ↪ Propagation fn

$$C_1 = C_0 P_0 + G_0$$

$$C_2 = C_1 P_1 + G_1$$

$$C_3 = C_2 P_2 + G_2 \quad C_4 = C_3 P_3 + G_3$$

In terms of  $C_0$ ,

$$C_1 = C_0 P_0 + G_0$$

$$C_2 = (C_0 P_0 + G_0) P_1 + G_1$$

$$= C_0 P_0 P_1 + G_0 P_1 + G_1$$

$$C_3 = (C_0 P_0 P_1 + G_0 P_1 + G_1) P_2 + G_2$$

$$= C_0 P_0 P_1 P_2 + G_0 P_1 P_2 + G_1 P_2 + G_2$$

$$C_4 = C_0 P_0 P_1 P_2 P_3 + G_0 P_1 P_2 P_3 + G_1 P_2 P_3 + G_2 P_3 + G_3$$

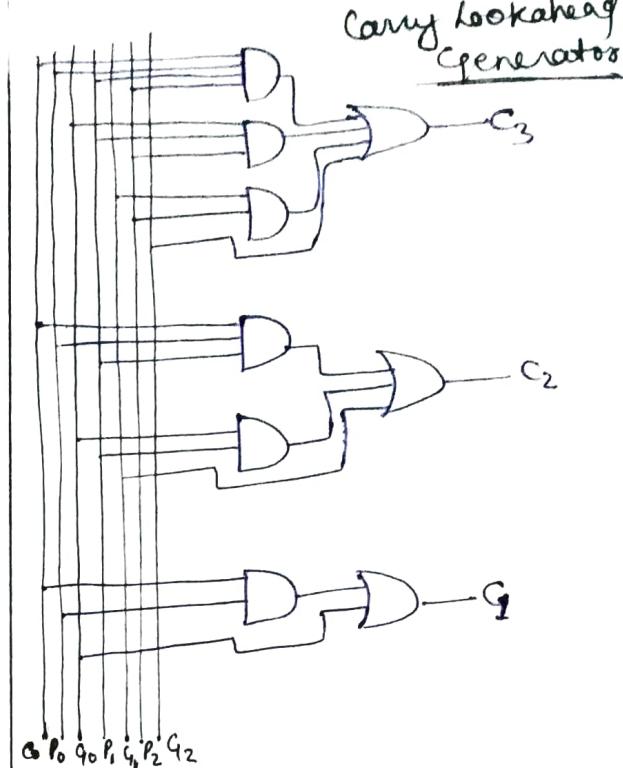
## Implementation



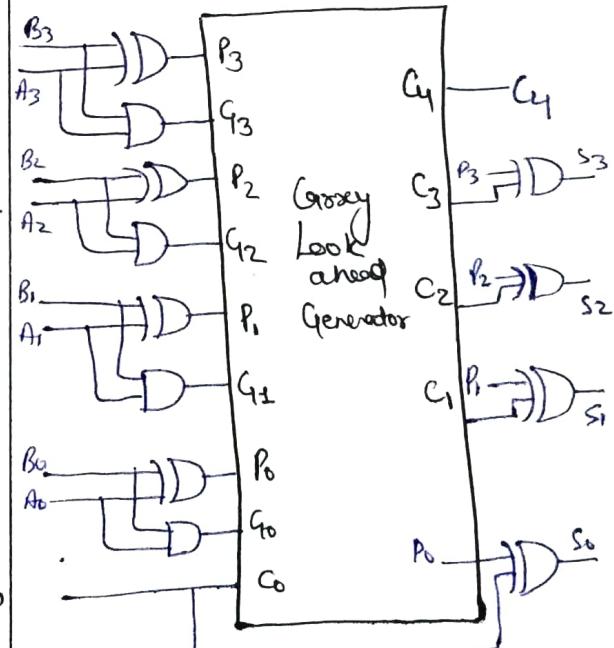
for n-bit  
addition

AND Gate  
 $= n \times \frac{n+1}{2}$

OR Gate  $\approx n$



It is better than ripple carry adder because it doesn't work in a sequential manner. It depends only on starting input parameters.



Advantages :- ① There is no rippling bcoz no one is waiting for the response from previous stages

## Hybrid Adder.

- Ripple carry adder is simple and cheap but slower.
- Carry look ahead is complex and costly but fast.
- So, both have its own pros and cons.

Suppose, we are adding two 8 bit numbers, in ripple adder.

$$\begin{array}{ccccccccccccc}
 & & & & & & & & & & & & & & \\
 C_8 & C_7 & C_6 & C_5 & C_4 & C_3 & C_2 & C_1 & & & & & & & \\
 \downarrow & & & & & & & \\
 A_8 & A_7 & A_6 & A_5 & A_4 & A_3 & A_2 & A_1 & & & & & & & \\
 B_8 & B_7 & B_6 & B_5 & B_4 & B_3 & B_2 & B_1 & & & & & & & \\
 \hline
 S_8 & S_7 & S_6 & S_5 & S_4 & S_3 & S_2 & S_1 & & & & & & & \\
 \end{array}$$

In carry look ahead generator all the carries generated directly with no dependency on previous phase.

$$\begin{array}{ccccccccccccc}
 & & & & & & & & & & & & & & \\
 C_7 & C_6 & C_5 & C_4 & C_3 & C_2 & C_1 & & & & & & & \\
 \downarrow & & & & & & & \\
 A_8 & A_7 & A_6 & A_5 & A_4 & A_3 & A_2 & A_1 & & & & & & & \\
 B_8 & B_7 & B_6 & B_5 & B_4 & B_3 & B_2 & B_1 & & & & & & & \\
 \hline
 S_8 & S_7 & S_6 & S_5 & S_4 & S_3 & S_2 & S_1 & & & & & & & \\
 \end{array}$$

In hybrid we combine both of them,

$$\begin{array}{ccccc}
 & & & & \\
 & & C_3 & C_2 & C_1 \\
 & & \downarrow & \downarrow & \downarrow \\
 \boxed{A_8 A_7 A_6 A_5} & & A_4 & A_3 & A_2 A_1 \\
 & & \boxed{B_8 B_7 B_6 B_5} & & \\
 & & & & \\
 & & & & \\
 & & & & \\
 \text{Phase 2} & & \text{Phase 1} & & \\
 \end{array}$$

So here we compute the phase 1 first and then whatever the final carry we get ( $C_4$ ) we provide that value as  $C_0$  to another phase.

Since, 8 bit adder must have complex ckt but 4 bit adder have less complex ckts.

## Serial Adder.

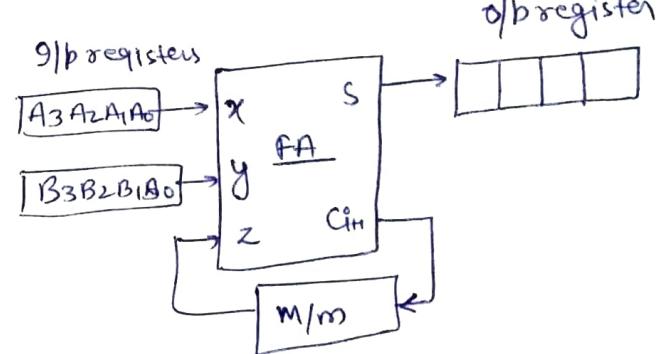
$$\begin{array}{cccc}
 0 & 0 & 0 & 0 \\
 0 & 0 & 0 & A_4 \\
 0 & 0 & A_4 & A_3 \\
 0 & A_4 & A_3 & A_2 \\
 \hline
 A_1 & A_3 & A_2 & A_1
 \end{array}$$

$$\boxed{C_1} \quad S_2 \quad S_3 \quad C_4$$

$$\begin{array}{cccc}
 S_0 & & & \\
 S_1 & S_2 & S_3 & \\
 S_2 & S_3 & S_1 & \\
 S_3 & S_2 & S_1 & \\
 \hline
 B_4 & B_3 & B_2 & B_1 \\
 0 & B_4 & B_3 & B_2 \\
 0 & 0 & B_4 & B_3 \\
 0 & 0 & 0 & B_4 \\
 0 & 0 & 0 & 0
 \end{array}$$

Everything will perform in shift registers.

It comes under sequential ckts



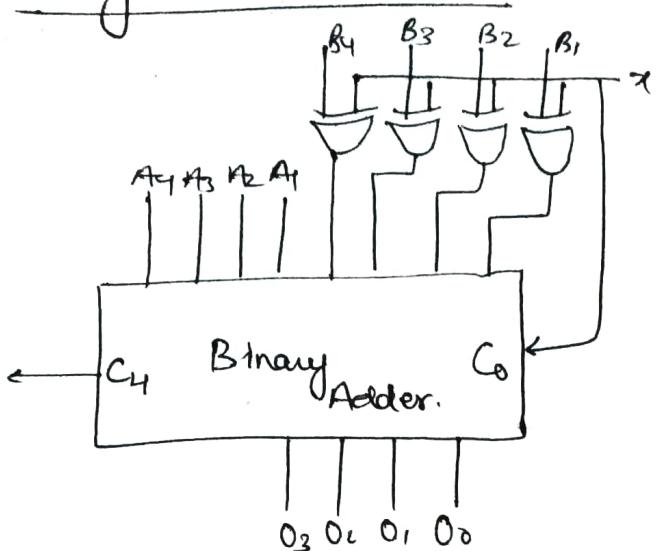
Output is depends on inputs and carry of previous inputs.

→ It is a serial adder and very slow as compare to all the others.

→ It is cheapest one bcoz we need only one FA.

→ We will read about in sequential ckts.

## Binary Adder / Subtractor



So, this ckt can work as both adder and subtractor as, we know -

$$B \oplus 0 = B\bar{0} + \bar{B}0 = B$$

$$B \oplus 1 = B\bar{1} + \bar{B}1 = \bar{B}$$

If  $x=0$  we receive all the bits as it is, and adder will perform  $(A+B)$ .

If  $x=1$  then it will perform  $(A+\bar{B}+1)$

$$\begin{array}{c} \uparrow \\ C_0 \\ \Rightarrow A + 2's \text{ complement of } B \\ = \underline{\underline{A-B}}. \end{array}$$

→ So, we can implement various operation using binary adder.

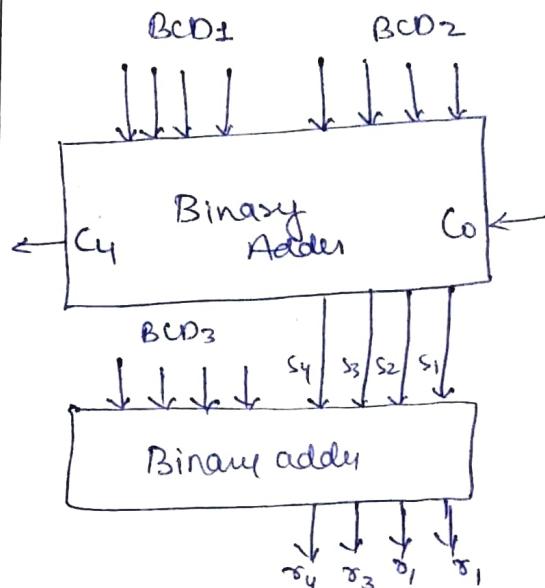
Ex

A	B	X	function
a) BCD 0011	0	0	$\rightarrow$ (BCD to XS-3) convertor
b) XS-3 0011	1	1	$\rightarrow$ (XS-3 to BCD) convertor
c) 1001 BCD	1	1	$\rightarrow$ (9's complement of BCD)
d) 1010 BCD	1	1	$\rightarrow$ (10's complement of BCD)

## BCD Adder

If output is greater than 9 or we are generating a carry then output is incorrect or not in BCD.

→ for correction we add 6 in outputs



If there is no need of correction then give BCD3 as '0' (zero).

Correction is required if

$$\underline{s_4 s_3 s_2 s_1 > 9} \text{ or } \underline{C_4 = 1}.$$

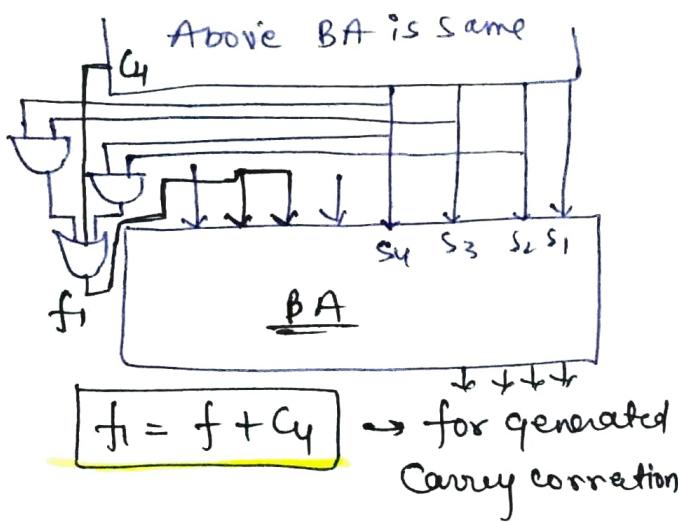
To detect  $s_4 s_3 s_2 s_1 > 9$ ,

S <sub>4</sub>	S <sub>3</sub>	S <sub>2</sub>	S <sub>1</sub>	f
1	0	1	0	1
1	0	1	1	1
1	1	0	0	1
1	1	0	1	1
1	1	1	0	1
1	1	1	1	1

$f$  is a function that gives '1' for  $S_4 S_3 S_2 S_1 > 9$  otherwise gives '0'.

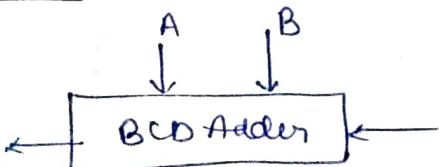
$S_4 S_3$	00	01	11	10
$S_2 S_1$	00		1	
00			1	
01				1
11			1	1
10			1	1

$f = S_4 S_2 + S_4 S_3$  will act as alarm for output  $> 9$ .  
→ so we do following changes in previous ckts



### Invalid Combinations for BCD

#### Adder



How many invalid combinations at A, B can occur to give incorrect output?

Invalid combn: Total Combination - Valid combn

Ex: 4 bits numbers A and B

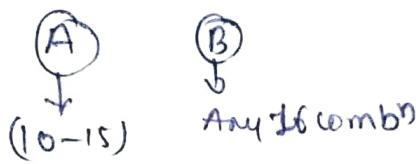
$$\text{Total Comb}^n = 16 \times 16 = 256$$

$$\begin{aligned}\text{Valid Comb}^n &= 10 \times 10 (0-9) \\ &= 100\end{aligned}$$

$$\begin{aligned}\text{Invalid Comb}^n &= 256 - 100 \\ &= 156\end{aligned}$$

#### ② Approach

(A)  $4 \text{ bit} = 2^4 = 16 \text{ combinat}$   
(0-9) valid ( $10-15$ ) invalid  
for BCD



$$\begin{aligned}(4 \times 6) + (6 \times 16) \\ \downarrow \\ \text{if } B \text{ belongs to } (10-15)\end{aligned}$$

Total invalid =

$$\begin{aligned}(16 \times 6) + (16 \times 6) - (6 \times 6) \\ = 156\end{aligned}$$

[for common in both]

### 2-Bit Comparator

→ for unsigned number

$A_2 A_1$	$B_2 B_1$	$A > B$	$A = B$	$A < B$
0	0			
0	1			
1	0			
1	1			

→ if we go like this it will take a lot of time to analyse if we have input size is more than 2 bit. So we go for another approach

$X-NOR$  can be used as two bit comparators as.

a	b	a $\oplus b \oplus 1$	
0	0	1	→ Same bits
0	1	0	→ Different bits
1	0	0	
1	1	1	→ Same bits

$\frac{A}{A_2 A_1} \quad \frac{B}{B_2 B_1}$  ] 2 bits numbers.

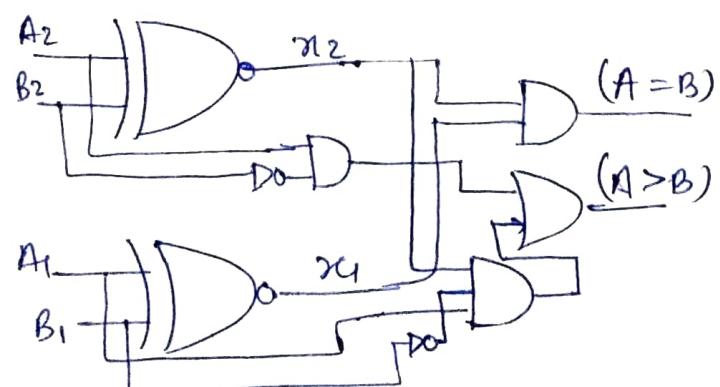
Case(i)  $A = B$  if  $A_2 = B_2$  and  $A_1 = B_1$   
 $\therefore x_1 = x_2 = 1$  or  $x_1 = x_2 = 0$

Case(ii)  $(A > B)$  if  $(A_2 > B_2)$  or  
 $(1) \quad (0)$   
 $(A_2 = B_2 \text{ and } A_1 > B_1)$   
 $(1) \quad 0$

$$\Rightarrow (A_2 \overline{B}_2 + x_2 \cdot A_1 \overline{B}_1) = 1$$

Case 3  $(A < B)$  if  $(A_2 < B_2)$  or  
 $(0) \quad 1$   
 $(A_2 = B_2 \text{ and } A_1 < B_1)$   
 $(0) \quad 1$

$$\Rightarrow \overline{A}_2 B_1 + (x_2 \cdot \overline{A}_1 B_1) = 1$$



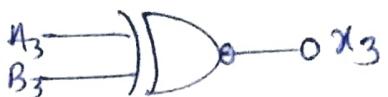
Similarly we can design for  $(A < B)$ .

### 3,4 bit Comparator

$$A = A_3 A_2 A_1$$

$$B = B_3 B_2 B_1$$

We assume that we already have 1 bit comparator as



Case 1:  $A = B$  if

$$x_1 \cdot x_2 \cdot x_3 = 1$$

Case 2 :  $(A > B)$  if

$$A_3 > B_3 = A_3 \overline{B}_3$$

$$(1) \quad (0)$$

$$A_3 = B_3 \text{ and } A_2 > B_2 \\ = x_3 \cdot A_2 \overline{B}_2$$

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$$A_3 = B_3 \text{ and } A_2 = B_2 \text{ and }$$

$$A_2 > B_1$$

$$= x_3 \cdot x_2 \cdot \overline{B}_1 A_2$$

Case 3  $(A < B)$  if

$$A_3 B_3 + x_2 \overline{A}_2 B_2 + x_2 x_3 \overline{B}_1 \overline{A}_2 = 1$$

Similarly we can designed for 4 bit comparators

Analyzing all the cases of  
Comparator

## 2-bit Comparator

$A_2$	$A_1$	$B_2$	$B_1$	
0	0	0	0	(0)
0	0	0	1	(1)
0	0	1	0	(2)
0	0	1	1	(3)
<hr/>		0	0	0 - 0
<hr/>		0	1	- 1
<hr/>		1	0	0 - 2
<hr/>		1	1	- 3
<hr/>		1	0	0 0 - 0
<hr/>		1	0	0 1 - 1
<hr/>		1	0	1 0 - 2
<hr/>		1	0	1 1 - 3
<hr/>		1	1	0 0 - 0
<hr/>		1	1	0 1 - 1
<hr/>		1	1	1 0 - 2
<hr/>		1	1	1 1 - 3

$A = B$  have 4 combinations.

$$16 - 4 = 12$$

$$\frac{12}{2} = (A > B \text{ and } A < B)$$

bcz both have symmetric combinations.

for  $n$  bit comparison

$A = B$  for  $2^n$  combination.

Remaining combinations

$$2^{2n} - 2^n$$

$$A > B = A < B = 2^{2n-1} - 2^{n-1}$$

Time Complexity of ripple carry Adder =  $\Theta(n)$   
( $n \rightarrow$  no. of Full Adders)

Time complexity of look ahead adder =  $\underline{\underline{\Theta(\log_k n)}}$

$k = \text{fan-in}$

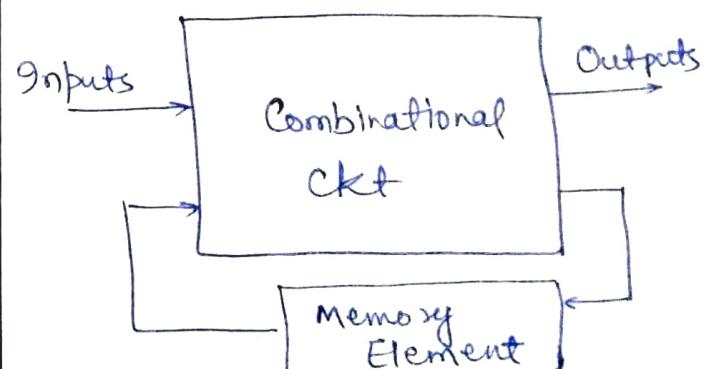
$n \rightarrow \text{no. of inputs}$

no. of levels  $l = \boxed{\log_k(n+1)}$

## Sequential Circuits

→ The external o/p of a sequential circuit depends on external i/p's and on present content of m/m elements".

→ The present contents of the m/m elements are called as present state and new content of m/m elements are obtained by taking external i/p's and present state is called as next state.



## Latch and flip flop

1 bit memory cell: A m/m element is some medium in which one bit of information

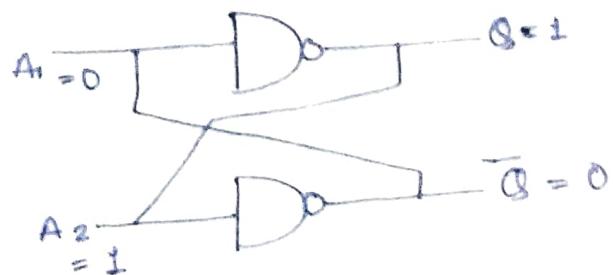
Can be stored or retained until necessary, and thereafter its content can be replaced by another value. It means flip flop can store a bit for long time and also modify it if required.

→ The binary or digital memory circuit is known as flipflop.

→ It has 2 stable states which are known as '1' or '0'. So it is also called bistable multivibrator.

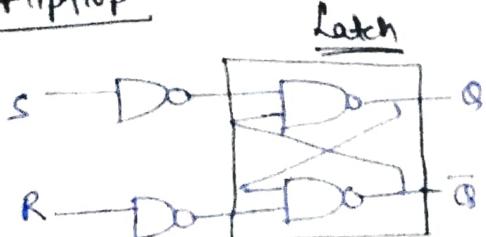
→ flip-flop is basic block of RAMs.

→ We can use any functionally complete gate to develop a flipflop, but generally we use NAND gates.



In flipflop we can change the inputs but in latches we can't change the inputs so each value get locked after setting an input.

### SR- flipflop



$Q_n$  → present state  
 $Q_{n+1}$  → next state

$Q_{n+1} = f(S, R, Q_n)$   
Characteristics table

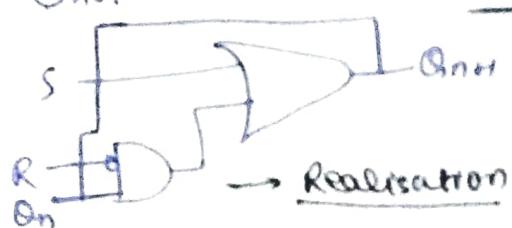
S	R	$Q_n$	$Q_{n+1}$
0	0	0	0
0	0	1	1
0	1	0	0
0	1	1	0
1	0	0	1
1	0	1	1
1	1	0	∅
1	1	1	∅

Set {  $0 \rightarrow 1$  } Reset {  $0 \rightarrow 0$  } Invalid {  $1 \rightarrow 0$  }

$\emptyset \rightarrow S=R=1$  set the value of  $Q, \bar{Q}$  both as 1 and that is invalid. so we don't care.

$Q_n$	00	01	11	10
0	∅	1	1	1
1	1	∅	1	1
1	1	1	∅	1
1	1	1	1	∅

$Q_{n+1} = S + R Q_n$  → Characteristics equation



$Q_n$	$Q_{n+1}$	S	R
0 → 0	0	∅	∅
0 → 1	1	0	0
1 → 0	0	0	1
1 → 1	1	1	0

Exitation table

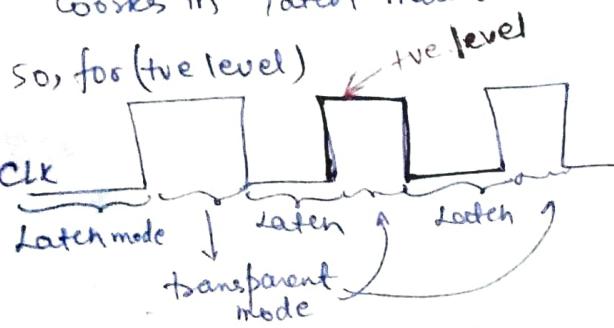
## function table

S	R	Out
0	0	On
0	1	0
1	0	1
1	1	∅

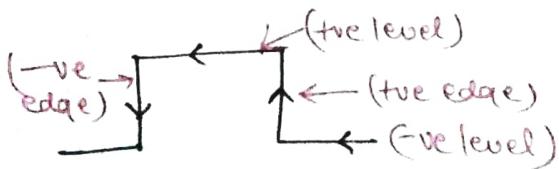
## Clocked flip flops

Sometimes it might be possible that S and R both have need of a constant value for long so both sets its value as required but due to noise value get changed and output get fluctuated.

To resolve above issue we use clock signal as CLK. In the initial CLK=0, so SR flip flop is in latch mode. Similarly at any time if CLK=0 irrespective of values of S & R, flip flop works in latch mode.



In transparent mode value of Out and o/p depends on S/R.



## +ve level triggered



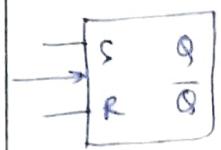
### +ve level



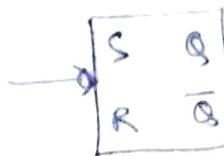
### -ve Level



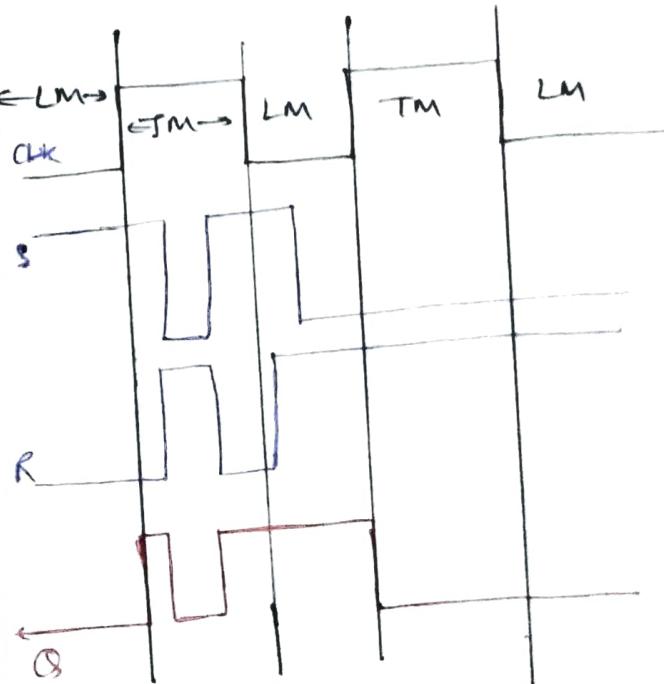
### +ve edge



### -ve edge



## Positive Level Triggered



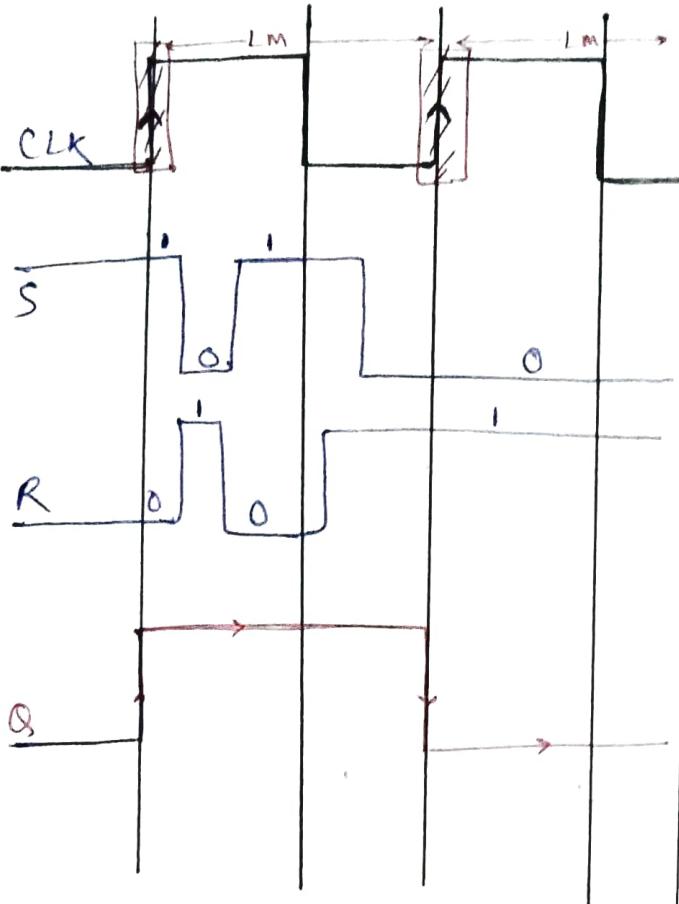
Lm → Latch mode TM → Transparent mode

CLK, S/R are input signals and Q follows the all 3 to decide o/p.

→ Q can change its value in TM only and Lm it will continue the present state independent of inputs.

→ Initial value Q assumes as '0'.

## Edge Triggered (Tve)



Characteristic table

J	K	Q <sub>n</sub>	Q <sub>n+1</sub>
0	0	0	0 } 0 <sub>n</sub>
0	0	1	1 }
0	1	0	0 } 0
0	1	1	0 }
1	0	0	1 } 1
1	0	1	1 }
1	1	0	1 } 0 <sub>n</sub>
1	1	1	0 }

Characteristic Eqn.

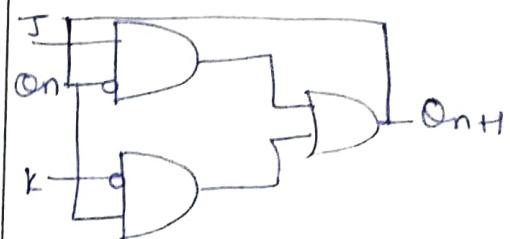
$$Q_{n+1} = \bar{J}Q_n + \bar{K}Q_n \quad [ \text{essential prime implicant} ]$$

JK	00	01	11	10
Q <sub>n</sub>	0		1	1
1	1	1	1	0

So we can say that Latch and transparent mode changes according to mode of flip flop

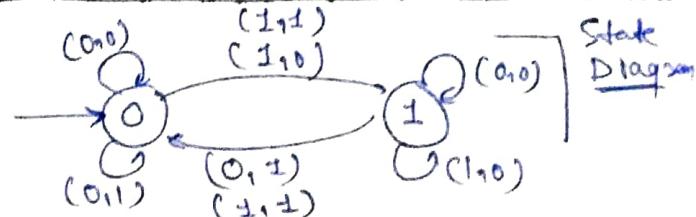
## JK flip flop

JK is same as SR flip flop ( $J=S$ ,  $K=R$ ). It is also defined  $J=K=1$  and the flip-flop perform complementation.



Excitation Table

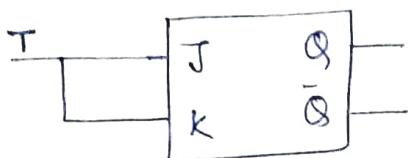
Q <sub>n</sub>	On	On+1	J	K	On the basis of Characteristic table
0	$0 \rightarrow 0$	0	0	$\emptyset$	
0	$0 \rightarrow 1$	1	$\emptyset$	$\emptyset$	
1	$1 \rightarrow 0$	$\emptyset$	$\emptyset$	1	
1	$1 \rightarrow 1$	$\emptyset$	$\emptyset$	0	



(0) → denotes reset state

(1) → denotes set state

### T- flipflop . (Toggle FF)



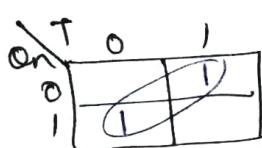
Combine both input

### function Table

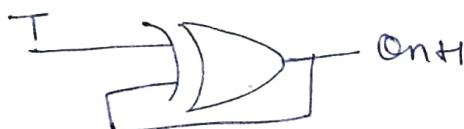
T	On	On+H
0	0	0 ] Latching
1	1	0 ] J Complementing

### Characteristic table

T	On	On+H
0	0	0 ] on
0	1	1 ] on
1	0	1 ] $\bar{on}$
1	1	0 ] $\bar{on}$



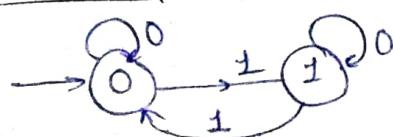
$$On+H = T \oplus On$$



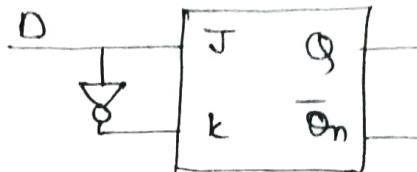
### Excitation table

Q	On	On+H	T
0 → 0	0	0	0
0 → 1	0	1	1
1 → 0	1	1	1
1 → 1	1	0	0

### State diagram



### D flipflop (Delay FF)



D	On	On+H
0	0	0 (J=0, K=1)
1	1	1 (J=1, K=0)

### Characteristic Table

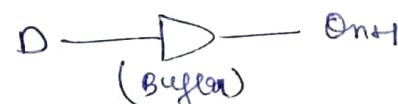
D	On	On+H
0	0	0 } D=0
0	1	0 }
1	0	1 }
1	1	1 } D=1

$$On+H = D \cdot On + D \cdot \bar{On}$$

$$= D$$

→ Output is directly depends on input. So no need of m/m element or feedback.

→ It use to provide delay only.

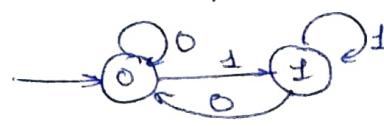


→ It use to remove temporary hazards due to delays.

### Excitation table

On	On+H	D
0 → 0	0	0 } Depends on On
0 → 1	1	1 }
1 → 0	0	0 }
1 → 1	1	1 }

### State diagram



## Example on flip flop. ①

Characteristic eqn of a flip flop  
is  $Q_n = \bar{x}_1 \bar{Q} + \bar{x}_2 Q$ . Define  
the behaviour of this.

### function table

$x_1$	$x_2$	$Q_n$	Behaviour
0	0	1	→ Set
0	1	0	→ toggle
1	0	0	→ Latch
1	1	0	→ Reset

### Characteristic table

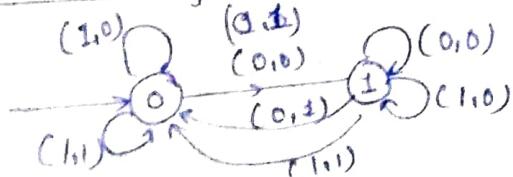
$x_1$	$x_2$	$Q$	$Q_n$
0	0	0	1 } set
0	0	1	1
0	1	0	1 } toggle
0	1	1	0
1	0	0	0 } latch
1	0	1	1
1	1	0	0 } Reset
1	1	1	0

$$Q_n = \bar{x}_1 \bar{Q} + \bar{x}_2 Q \rightarrow \text{satisfying the characteristic table.}$$

### Excitation table

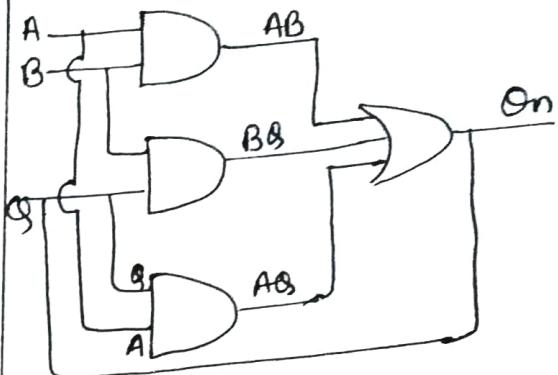
$Q$	$On$	$x_1$	$x_2$
0 → 0	1	∅	
0 → 1	0	∅	
1 → 0	∅	1	
1 → 1	∅	0	

### State diagram



## Example ②

Consider the following realisation. construct the excitation tables



Sol'n

### characteristic eqn

$$Q_n = AB + BQ + AQ$$

### function table

A	B	$Q_n$
0	0	0 → Reset
0	1	0 } latch
1	0	0 }
1	1	1 → Set

### Characteristic table

A	B	$Q$	$Q_n$
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	1
1	1	1	1

### Excitation table

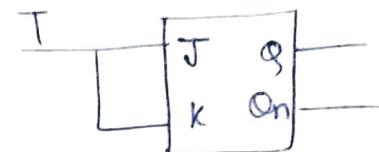
$Q$	$On$	$AB$	→ Wrong table
0 → 0	1	∅∅	
0 → 1	1	11	
1 → 0	0	00	
1 → 1	∅	∅∅	

We can write  $\phi\phi$  bcoz it also includes (1,1) but have only 3 combination as  $(0,0), (0,1), (1,0)$ . So we write it separately.

$Q$	$On$	$A$	$B$	
0	0	0	$\phi$	$\neq (\phi\phi)$
0	0	1	0	
1	0	0	0	
0	1	1	1	
1	1	1	$\phi$	$\neq [\phi,\phi]$
1	1	0	1	

$On$	$T$	$Q$	
0	0	$\phi$	$\phi$
0	1	0	1
1	0	1	0

So,  $J = K = T$

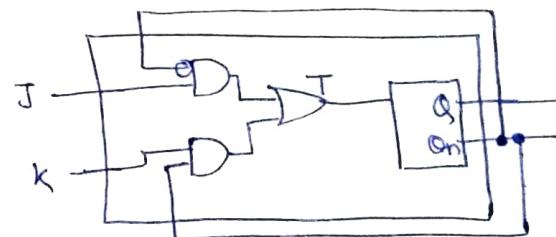


Convert T to JK

$J$	$K$	$On$	$On+1$	$T$
0	0	0	0	0
0	0	1	1	0
0	1	0	0	0
0	1	1	0	1
1	0	0	1	1
1	0	1	1	0
1	1	0	1	1
1	1	1	0	1

$JK$	$On$	00	01	11	10
0	0	$\phi$	0	1	1
1	1	1	1	$\phi$	0

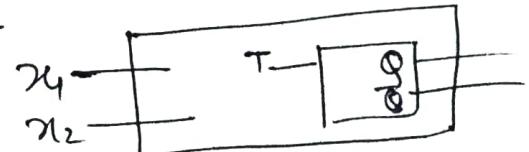
$$T = On K + \bar{J} On$$



Example on conversion

A new flip-flop  $x_1x_2$  has characteristic expression  $On = \bar{x}_1\bar{Q} + \bar{x}_2Q$ . Realise it using T flip-flop.

Soln



Ex → JK to T FF

$T$	$On$	$On+1$	$J$	$K$
0	0	0	0	$\phi$
0	1	1	$\phi$	0
1	0	1	$\frac{1}{2}$	$\phi$
1	1	0	$\phi$	$\frac{1}{2}$

$$J = T$$

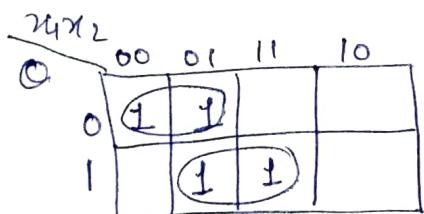
$On$	$T$	$Q$
0	0	$\phi$
1	$\phi$	0

## function Table

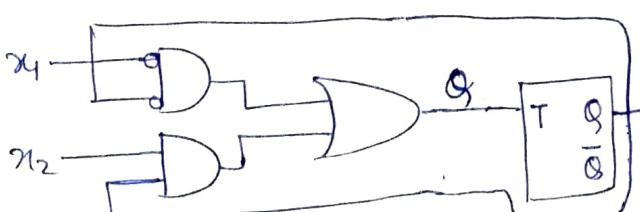
$x_1$	$x_2$	$Q_n$
0	0	1
0	1	0
1	0	0
1	1	0

## Characteristic table

$x_4$	$x_2$	$Q$	$Q_n$	T
0	0	0	1	1
0	0	1	1	0
0	1	0	1	1
0	1	1	0	1
1	0	0	0	0
1	0	1	1	0
1	1	0	0	0
1	1	1	0	1

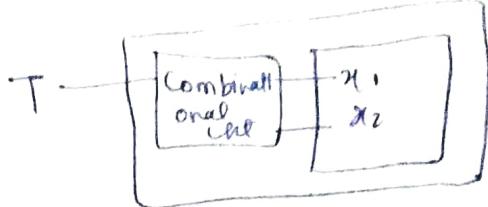


$$T = \overline{x_4}Q + x_2Q$$



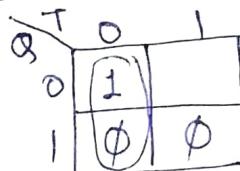
→ follow the convention given in question for On and On+1

Ex-2 Convert  $x_1x_2 \rightarrow T$   
given that  $Q_n = \overline{x_1}Q + x_2Q$



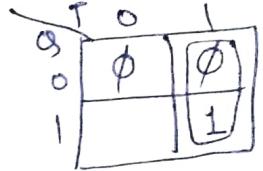
T	Q	On	$x_1$	$x_2$
0	0	0	1	$\emptyset$
0	1	1	$\emptyset$	0
1	0	1	0	$\emptyset$
1	1	0	$\emptyset$	1

for  $x_1$

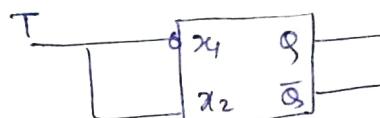


$$x_1 = T^1$$

for  $x_2$

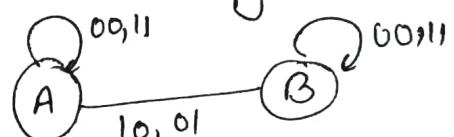


$$x_2 = T$$



QMB

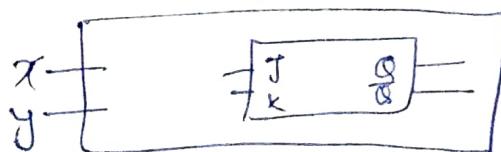
An X-Y flipflop is represented by the following state diagram



To realise it using JK FF.

Find J & K.

→ Either  $A, B = \{0, 1\}$  or there is no direction on edge joining (A) and (B) and it's not a typing mistake



$x$	$y$	$Q_{n+1}$	$J$	$K$
0	0	0	0	$\emptyset$
0	0	1	$\emptyset$	0
0	1	1	$\perp$	$\emptyset$
0	1	0	$\emptyset$	$\perp$
1	0	1	$\perp$	$\emptyset$
1	0	0	$\emptyset$	$\perp$
1	1	0	0	$\emptyset$
1	1	1	$\emptyset$	0

functional table

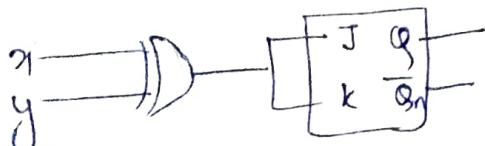
$x$	$y$	$Q_{n+1}$	from given state diagram
0	0	$Q_n$	
0	1	$\overline{Q_n}$	
1	0	$\overline{Q_n}$	
1	1	$Q_n$	

Using minimisation from k-map we get

$$J = \bar{x}y + xy' = x \oplus y$$

$$K = \bar{x}y + x\bar{y}' = x \oplus y$$

$$\text{So, } J = K = x \oplus y$$



what is the behaviour of following one-input flip flop  $x \oplus Q$



a) D-FF      b) T-FF

c) Inverted D-FF    d) Inverted T-FF

<u>Soln</u>	$x$	$Q_n$	$Q_{n+1}$	T
	0	0	0	0
	0	1	0	1
	1	0	1	1
	1	1	1	0

$$Q_{n+1} = x$$

Delay provided only so one in D-FF.

### Counters

The counters are used to provide accurate timing and control signals.

→ These are of two types - :

a) Synchronous    b) Asynchronous

→ In synchronous counters all the flipflops respond to the same clock instances.

→ In asynchronous counters, the output of one flipflop drives the clock of another flipflop.

→ Synchronous counters are faster than asynchronous counters.

→ Due to simplicity of design, asynchronous counters are used in IC fabrication.

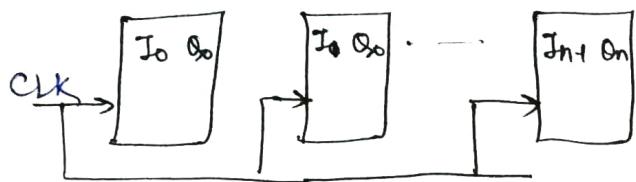
→ The simplified version of synchronous counter is called shift counters.

→ D-FF is basic element of shift counters.

The ring and Johnson counters are further simplified version of shift counters.

### Asynchronous and Synchronous Counters

#### ① Synchronous



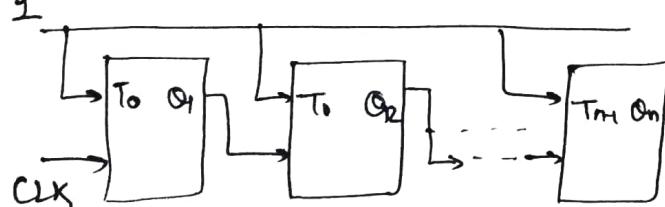
Input can be function of o/p but we are just explaining the concept of counter here.

Propagation delay in synchronous counter is,

$$T_{bdyn} = T_{FF} + T_{combinational}$$

$$T_{CLK} \geq T_{bdyn}$$

#### ② Asynchronous



Propagation delay in asynchronous counter (rippling effect) is,

$$T_{bdasyn} = N \times T_{FF} + T_{combinational}$$

$$T_{CLK} \geq T_{bdasyn}$$

T<sub>CLK</sub> → After how much time you will give 1<sup>st</sup> input.

### Shift Counters

In synchronous counters,

$$\text{Input } (I_i) = f(Q_0, Q_1, \dots, Q_{N-1})$$

$$0 \leq i \leq N-1$$

→ It's design is more complex  
So, simplification are done as follows.

$$1) I_i = Q_{i-1}$$

$$\therefore I_0 = Q_0$$

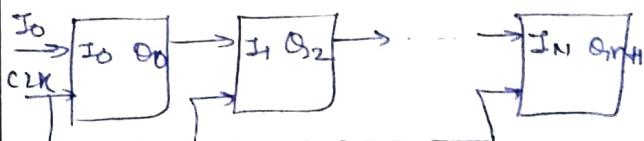
$$I_1 = Q_1$$

⋮

$$I_N = Q_{N-1}$$

Here data is shifted from one counter to another next FF. So it is called as Shift Counter.

→ All are D-FF in shift register



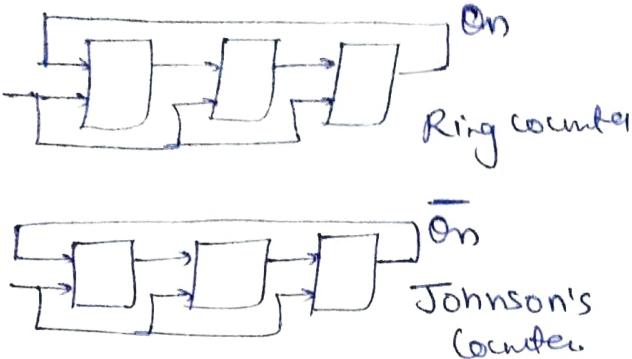
### In shift counter

$$I_0 = f(Q_{N-1})$$

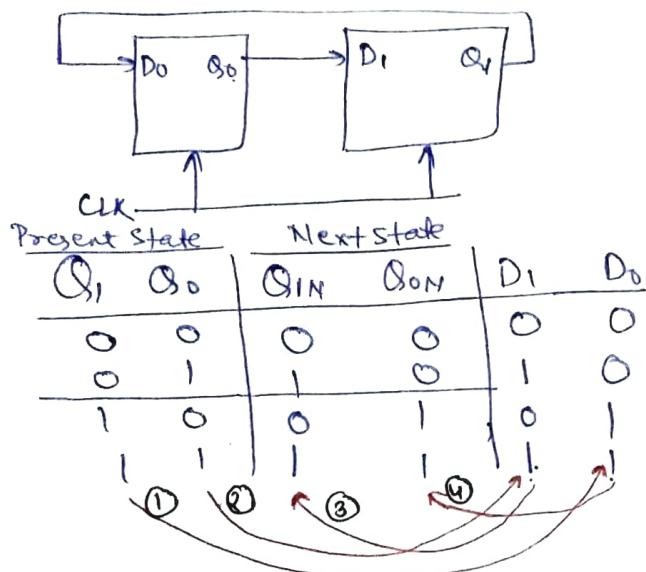
$$I_0 = Q_N \\ (\text{Ring Counter})$$

$$I_0 = \bar{Q}_{N-1} \\ (\text{Johnson Counter})$$

→ If we have 'n' flip flops then  $2^n$  states are possible then counter size should be  $\text{mod}(2^n)$ .

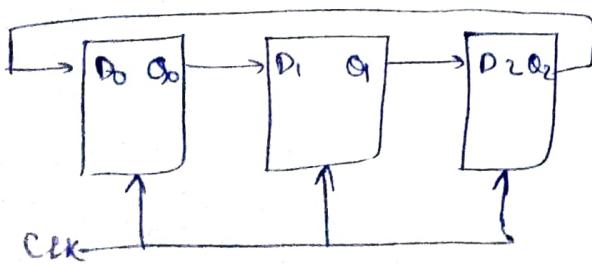


### Mod 2 ring counter



~~Off~~ So, even we have  $2^N$  states for (N-FF) but we get only N useful state so, it is (mod N) counter. and here N is 2. (Ring Counter)

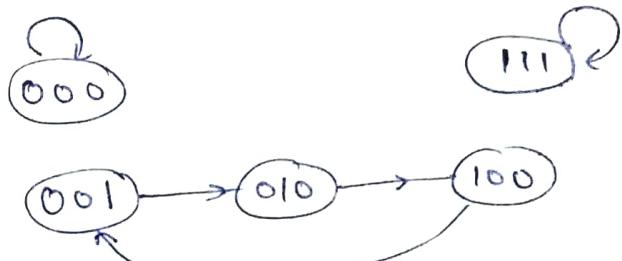
### Mod 3 Counter



Q <sub>2</sub>	Q <sub>1</sub>	Q <sub>0</sub>	Q <sub>2N</sub>	Q <sub>1N</sub>	Q <sub>0N</sub>	D <sub>2</sub>	D <sub>1</sub>	D <sub>0</sub>
0	0	0	0	0	0	0	0	0
0	0	1	0	1	0	0	1	0
0	1	0	1	0	0	1	0	0
0	1	1	1	1	0	1	1	0
1	0	0	0	0	1	0	0	1
1	0	1	0	1	1	0	1	1
1	1	0	1	0	1	1	0	1
1	1	1	1	1	1	1	1	1

Present                  Next

$$D_0 = Q_2, D_1 = Q_0, D_2 = Q_1$$



So, in mod 3 counter only ③ states are participating.

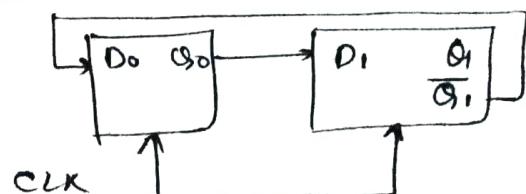


→ Transitions are from present state to next state.

→ At each clock bits get shift to next flip-flop and we reach the next state of DFA.

→ we can use this counter in Round-Robin Scheduling bcoz each time only one device gets active.

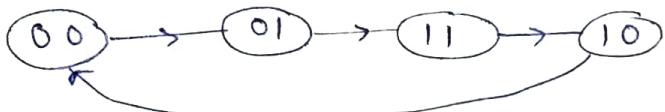
### Mod 4 Johnson Counter



→ It is also called twisted ring counter or Johnson's Counter.

$Q_1$	$Q_0$	$Q_{IN}$	$Q_{ON}$	$D_0$	$D_1$
0	0	0	1	1	0
0	1	1	1	1	1
1	0	0	0	0	0
1	1	1	0	0	1

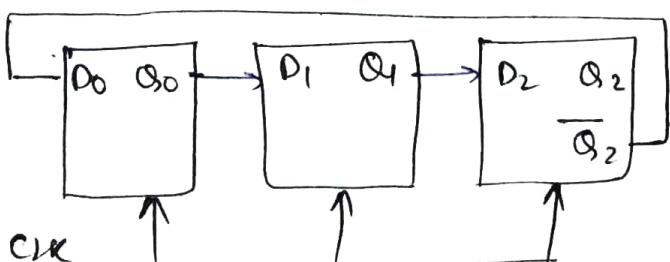
$$D_0 = \bar{Q}_1, \quad D_1 = Q_0$$



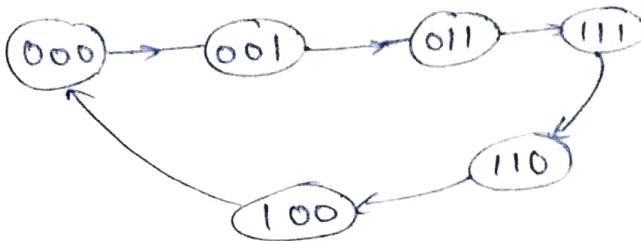
→ 4 states are involved here  
So it is mod 4 counter.  
It means after each trip of 4 shift we reach to initial state.

Ques: In Johnson Counter if  $N$  flipflops are present then it is  $(mod 2^N)$  counter.

### Mod 6 Johnson Counter



$Q_2$	$Q_1$	$Q_0$	$Q_{2N}$	$Q_{IN}$	$Q_{ON}$	$D_2$	$D_1$	$D_0$
0	0	0	0	0	1	0	0	1
0	0	1	0	1	1	0	1	1
0	1	0	1	0	1	1	0	1
0	1	1	1	1	1	1	1	1
1	0	0	0	0	0	0	0	0
1	0	1	0	1	0	0	1	0
1	1	0	1	0	0	1	0	0
1	1	1	1	0	0	1	1	0



→ So after 6 shifts we get the first input again.

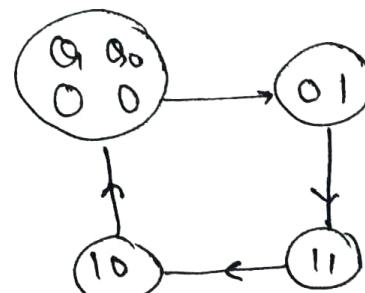
→ So it is mod 6\*1 counter.  
→ We will get reverse of b on reversing  $\frac{Q_2}{Q_1}$  &  $\frac{Q_1}{Q_0}$   
Mod 4 gray counter using T-FF

### T-FF

#### Designing a counter from functions

- ① Get the state table from State diagram.
- ② Identify flipflops to be used and replace the concerned next state using excitation table of associate flip flop.
- ③ Get the expression for ips and realize them.

Q: Design a synchronous counter for the following using T-FFs.



Soln

Cont

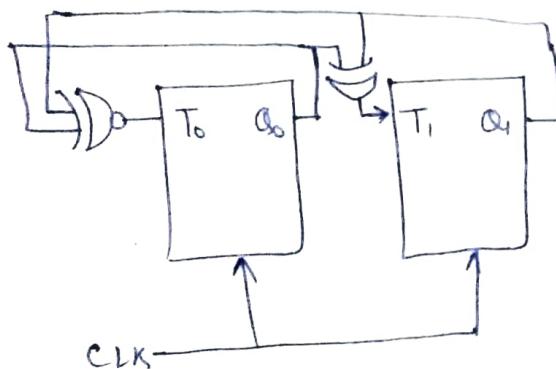
## State table

$Q_1\ Q_0$	$Q_{IN}\ Q_{ON}$	$T_1\ T_0$
0 0	0 1	0 1
0 1	1 1	1 0
1 1	1 0	0 1
1 0	0 0	1 0

for  $T_1$  entries Consider  $Q_1 \rightarrow Q_{IN}$   
for  $T_0$  entries —————  $Q_0 \rightarrow Q_{ON}$

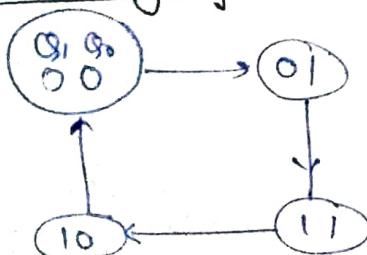
$$T_1 = \overline{Q_1}Q_0 + Q_1\overline{Q_0} = Q_1 \oplus Q_0$$

$$T_0 = \overline{Q_1}Q_0 + Q_1Q_0 = Q_1 \odot Q_0$$



→ Since in each transition in DFA only 1 bit is changing and it is called gray codes  $[00 \rightarrow 01 \rightarrow 11 \rightarrow 10]^{(00)}$   
 → So it is mod 4 gray counter.

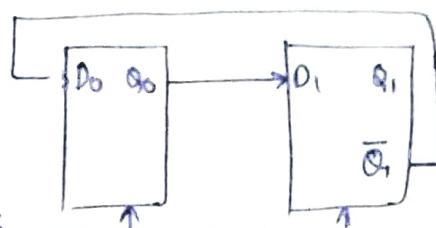
## Mod 4 gray counter using D-FF



$Q_1\ Q_0$	$Q_{IN}\ Q_{ON}$	$D_1\ D_0$
0 0	0 1	0 1
0 1	1 1	1 1
1 1	1 0	1 0
1 0	0 0	0 0

$$D_1 = Q_0\overline{Q}_1 + Q_1Q_0 = Q_0$$

$$D_0 = \overline{Q}_1\overline{Q}_0 + \overline{Q}_1Q_0 = \overline{Q}_1$$



→ Johnson's counter for mod 4 gray code.

→ So, DFF is better than T-FF.

## Mod 4 gray counter using J-T and J-D flip flop

$Q_1\ Q_0$	$Q_{IN}\ Q_{ON}$	$D_1\ T_0$
0 0	0 1	0 1
0 1	1 1	1 0
1 0	0 0	0 0
1 1	1 0	1 1

$$D_1 = Q_0 \quad T_0 = \overline{Q}_1\overline{Q}_0 + Q_1Q_0 \\ = Q_1 \odot Q_0$$

