Combinactories and Propositional Logic

Introdution :- Recursive Equations.

Master's Theorem is applicable to Recurssive equation for Devide and Conquer approach.

$$T(n) = a(T(n/b)) + k$$

→ we get closed approximation.

But in 2nd type of equations.

$$\begin{cases} T(n) = T(n-1) + T(n-2) + k \\ T(1) = 1 \end{cases}$$

~~Imp~~

→ Sometimes you will get the answer just by putting values.

Sequence → Order of any object.

Ex → 0, 2, 4, 6, 8, 10, ----) T(n)

n = 0, 1, 2, 3, 4, 5, --

$T(0) \to 0$, $T(1) \to 2$, $T(2) \to 4$, ---

Recurrence relation

$$T(n) = T(n-1) + 2 \; : n \geq 1$$

Initial Condⁿ → [$T(0) = 0$, $T(1) = 2$

Solⁿ

Suppose solⁿ is $T(n) = 2n$ then it must satisfy the recurrence relation.

$LHS = T(n) = 2n$

$RHS = T(n-1) + 2 = 2(n-1) + 2$
$= 2n$

LHS = RHS.

So, $T(n) = 2n$ will be the solution.

Q → The recurrence eqn

$T(1) = 1$

$T(n) = 2T(n-1) + n \quad n \geq 2$

evaluates to.

a) $2^{n+1} - n - 2$ b) $2^n - n$

c) $2^{n+1} - 2n - 2$ d) $2^n + n$

Put $n=1$ then found that only a and b are valid.
because $T(1) = 1$.

Then found $T(2) = 2T(2-1) + 2$
$= 2T(1) + 2 \rightarrow 4$

Only option a satisfies $T(2) \rightarrow 4$

So, Ans $\rightarrow$ (a).

Q $T(2^k) = 3T(2^{k-1}) + 1$
$T(1) = 1$

Sol^n put $2^k = n$

then $T(n) = 3T(n/2) + 1$

$a = 3$, $b = 2$ $k = 0$

$a < b^k \rightarrow O(n^{\log_2 3})$

$T(n) \rightarrow O(n^{\log_2 3})$

$T(2^k) \rightarrow O(2^{k\log_2 3}) \rightarrow O(2^{\log_2 3^k}) = O(3^k)$

Using Elimination method

a) $T(2^k) = 2^k$ not $\boxed{T(k) = 2^k}$ ** Remember

b) $T(3^k) = (3^{k+1} - 1)/2$

c) $T(2^k) - 3^{\log_2 k}$

d) $T(2^k) = 2^{\log_3 k}$

Option c and d are invalid because weget $T(1)$
at $k=0$ because $T(2^0) = T(1)$

and in $3^{\log_2 k}$ and $2^{\log_3 k}$ at $k=0$ log value
is not defined.

Q Let x_n denotes no. of binary string of length n contain no consecutive zero. which of the following recurrence does x_n satisfy

gm

a) $x_n = 2x_{n-1}$ b) $x_n = x_{n/2} + 1$

c) $x_n = x_{n/2} + n$ d) $x_n = x_{n-1} + x_{n-2}$

Soln $x_1 = 2 \{0, 1\}$ $x_3 = 5 \{010, 011, 101, 110, 111\}$

$x_2 = 3 \{01, 10, 11\}$

So the ans is → (d)

because $x_3 = x_{3/2} + 1$ in option b

and $x_{3/2}$ can't be evaluated.

Generating functions

→ Not useful in Computer science directly but useful in computation.

Series → a_1, a_2, a_3, a_4, a_5

Task → Generate a generating fn of the series.

Approach → Give some weight to each term in series;

$$\underbrace{x^0 a_1 + a_2 x^1 + a_3 x^2 + a_4 x^3 + a_5 x^4}_{\text{Generating function for series}}$$

→ If sequence is not finite then generating fn will also infinite.

Ex → Sequence → 1, 1, 1, 1, ...

Generating fn → $1 + x + x^2 + x^3 + \dots$

Compact form of generating fn

1) $(x+1) = \frac{(x^2-1)}{(x-1)} = \frac{(1-x^2)}{(1-x)}$

2) $(1+x+x^2) = \frac{(x^3-1)}{(x-1)} = \frac{(1-x^3)}{(1-x)}$

3) $1 + x + x^2 + \dots x^n$

$= \left(\frac{x^n - 1}{x-1}\right) = \left(\frac{1-x^n}{1-x}\right)$

Pigeonhole Principle

If 'k' is a positive integer and 'k+1' or more objects are placed into 'k' boxes, then there is at least one box containing two or more objects.

Ex → If $k = 3$ and $k+1 = 4$ then there are many ways to put the object in boxes such that atleast 1 box contain 2 or more objects

Ex → Among any group of 367 people there must be atleast two same birthday.

Sol^n Max. days in a year can be possible → 366 (leap yr)
No. of people = 366+1

So allot each day to 366 people and allot any day to remaining 1.
In every case it is going to follow pigeonhole principles

Generalised Pigeonhole Principle

If 'N' objects are placed in 'k' boxes then atleas one box containing atleast $\lceil N/k \rceil$ objects. $[K < N]$

Propositional Calculus Prepo with logic

Proposition: Any declarative statement with truth values.

Certainity [My name is Aryan. → Proposition (T)
I am 12 yr. old. → Proposition (false)

Uncertain [You are right or wrong → Not a proposition.

Propositional variable → variable / symbol assign to a particular statement

Relation Ex → I am right : 'p'

Connectives → Things that connects more than one prepositional statement

① $\sim p$: I am not right. [Negation]
(or) $\sim p$: It is not the case that I am not right

② $\wedge$: p: I got 80% marks.
q: I got A grade.] conjunction

$(p \wedge q)$: I got 80% marks and A grade.

③ $\vee$: Disjunction
$(p \vee q)$: I got 80% marks or A grade.

p	$\sim p$
T	F
F	T

p	q	$p \wedge q$	$p \vee q$	$p \oplus q$ (XOR)
T	T	T	T	False
T	F	F	T	T
F	T	F	T	T
F	F	F	F	False

Imp.
Implications

p ⟶ q
(hypothesis) (conclusion)

p: You get 60 marks in Gate
q: You will go to IIT's

$p \to q$: If you get 60 marks in Gate then you will go to IIT's.

p	q	$p \Rightarrow q$
T	T	T
F	F	T
T	F	F
F	T	T

→ Implication have no relation with if-else in programming and general english.

→ If $(p \to q)$ is always true called 'Tautology'.

① If p then q
② If p, q
③ p is sufficient for q
④ q if p
⑤ q when p

→ A necessary condition for p is q.
→ q unless $\sim p$
→ p implies q
→ p only if q
→ A sufficient condition for q is p

→ q whenever p
→ q is necessary for p
→ q follows from p

These 3 and all previous are ways to respresent $p \to q$.

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for $(p \to q)$

Converse: $q \to p$

Contrapositive: $\sim q \to \sim p$

Inverse: $\sim p \to \sim q$

Bi-conditional : $p (\leftrightarrow) q$: $(p \to q) \wedge (q \to p)$

if and only if (iff)

X-Nor $\odot$

V

p	q	$p \leftrightarrow q$
T	T	T
T	F	F
F	T	F
F	F	T

Consistent System: 3

The system of statement is consistent if all the statements becomes 'True' simultaneously at a time.

Ex.

→ The diagonastic message is stored in buffer (p) or (V) it is transmitted (q).

→ The diagnostic message is not stored in buffer $(\sim p)$

→ If the diagnostic message stored in buffer then it is transmitted. $(p \to q)$ or $(\sim p \vee q)$

p	q	$\sim p$	$p \to q$	$p \vee q$
T	T	F	T	T
T	F	F	F	T
F	T	T	T	T
F	F	T	T	F

All three are true if $p \to$ false, $q \to$ True

2nd method → To make all the statement true at a time do analysis like.

1. $\sim p$ will true if $p \rightarrow$ false
2. $(p \vee q)$ will true if $q \rightarrow$ true bcoz p is already false above

So $p \rightarrow$ false, $q \rightarrow$ true.

Equivalences → You can by-heart it, but don't need to do this because its easy to evaluate.

1. $p \rightarrow q = \sim p \vee q = \sim q \rightarrow \sim p$
2. $p \vee q = \sim p \rightarrow q$
3. $p \wedge q = \sim(q \rightarrow \sim p)$
4. $\sim(p \rightarrow q) \equiv p \wedge \sim q$
5. $Q \wedge \sim Q \rightarrow \phi$

Precedence Table

$\sim$	1
$\wedge$	2
$\vee$	3
$\rightarrow$	4
$\leftrightarrow$	5

Priority decreases

De-Morgan's Law

$\sim(p \vee q) \cong \sim p \wedge \sim q$

$\sim(p \wedge q) \cong \sim p \vee \sim q$

Ex → Ravi has a computer, and, a phone $(p \wedge q)$

computer = p, and = $\wedge$, phone = q

$\sim(p \wedge q)$ = Ravi doesn't have a computer
or
Ravi doesn't have a phone

Argument / Inference: If a set of premises $\{P_1, P_2 .. P_n\}$ yield another proposition Q (conclusion) the whole process is called argument.

Note: An argument is said to be valid if the conclusion Q can be derived from the premises by applying the rule of inferences.

An argument is denoted by $\{\underbrace{P_1, P_2, P_3 \cdots P_n}\} \vdash Q$

Proposition which are 'true' called as $\boxed{\text{Premises}}$

Rules of Inference

Representation of a valid argument.

1. The argument $\{P_1, P_2, \cdots P_n\} \vdash Q$ is valid.
2. The statement $\{P_1, P_2, \cdots P_n\} \Rightarrow Q$ is true.
3. The statement formula

$$(P_1 \wedge P_2 \wedge \cdots P_n) \longrightarrow Q \text{ is a tautology.}$$

1. Simplification Rule

a) $\dfrac{P \wedge Q}{P/Q}$ valid or $(P \wedge Q) \Rightarrow P$ is true / Q is true

$(P \wedge Q) \rightarrow P$ is tautology. / Q is tautology

2) Addition Rule

a) $\dfrac{P}{P \vee Q}$ valid If $P \rightarrow$ true then $(P \vee Q) \rightarrow$ true

b) $\dfrac{Q}{P \vee Q}$ same as above $Q \rightarrow$ true then $(P \vee Q) \rightarrow$ true

or $P \rightarrow (P \vee Q)$ is true $Q \rightarrow (P \vee Q)$ is true

(3) $\dfrac{\sim P}{P \rightarrow Q}$ valid

(4) $\dfrac{Q}{P \rightarrow Q}$: valid $[P \rightarrow q = \sim p \vee q]$

(5) $\dfrac{\sim(P \rightarrow Q)}{P}$ valid

(6) $\dfrac{\sim(P \rightarrow Q)}{\sim Q}$ valid

$\sim(P \rightarrow Q) = P \wedge \sim Q$ is true if

P is true and $\sim Q$ is true.

7. Conjuction

$$\begin{array}{l} Q \\ P \\ \hline P \wedge Q \end{array}$$ valid

(4)

8. Disjunctive Syllogism

$$\begin{array}{l} P \vee Q \\ \sim P \\ \hline Q \end{array}$$ valid

9. Conjunctive syllogism

$$\begin{array}{l} \sim(P \wedge Q) \\ P \\ \hline \sim Q \end{array}$$ valid

10. Modus Ponen / Rule of detatchment

$$\begin{array}{l} P \to Q \\ P \\ \hline Q \end{array}$$ valid

11. Modus Tollen or Rule of contrapositive

$$\begin{array}{l} P \to Q \\ \sim Q \\ \hline \sim P \end{array}$$ valid

12. Transitivity

$$\begin{array}{l} P \to Q \\ Q \to R \\ \hline P \to R \end{array}$$ valid

13. Dilemma

$$\begin{array}{l} P \vee Q \\ P \to R \\ Q \to R \\ \hline R \end{array}$$ valid

14. Constructive Dilemma

$$\begin{array}{l} P \vee Q \\ P \to R \\ Q \to S \\ \hline R \vee S \end{array}$$ valid

15. Distructive Dilemma

$$\begin{array}{l} P \to R \\ Q \to S \\ \sim R \vee \sim S \\ \hline \sim P \vee \sim Q \end{array}$$ valid

Invalid arguments (fallacies)

① fallacy of assuming converse

$$\begin{array}{l} P \to Q \\ Q \\ \hline P \end{array}$$ Not Valid

$\sim P \vee Q$ if Q is true then P can be T/F any thing but here it Consider P as true only so, fallacy.

But if you assume the converse as

$$\begin{array}{l} Q \to P \\ Q \\ \hline P \end{array}$$

then it is valid.

(2) fallacy of assuming inverse

$$\begin{array}{l} P \to Q \\ \sim P \\ \hline \sim Q \end{array}$$

(3) Non Sequitur

(5)

$$\begin{array}{l} p \\ q \\ \hline r \end{array}$$

Not valid

Here p, q are not related.

Q which of following are not valid?

a) The conclusion $\sim p$ follows from premises $\{P \to Q, Q \to R, \sim R\}$

$$\begin{array}{l} P \to Q \\ Q \to R \\ \hline P \to R \quad \text{(transitivity)} \\ \sim R \\ \hline \sim P \end{array}$$

| Modus tollens.

so valid.

b) $(p \to q)$ follows $\{\sim P, P \vee q\}$

$$\begin{array}{l} \sim P \\ P \vee q \\ \hline q \end{array}$$

then $q \vee \sim p$ also true

$(p \to q)$

$$\begin{array}{l} A \cup B \\ B \to C \\ A \to D \\ \sim D \\ \hline C \end{array}$$

$(\sim A \cup D) \cap \sim D \Rightarrow \sim A$

$A \cup B$

B

$B \cap (\sim B \cup C) = C$

Q Consider the following set of premises which of the following is true

$S_1 : A \to B,\ A \to C,\ B \to \sim C,\ A$

$S_2 : \sim A \vee B,\ \sim B,\ A$

a) S_1 is inconsistent

b) S_2 ————

c) both S_1 and S_2 are inconsistent

d) ———— are consistent

$A \to B$

$A \to C$ $\sim A$ B

$B \to \sim C$ $\sim C$

A C

Both C and $\sim C$ can't be true at a time

$\sim A \vee B$ $\sim A$

$\sim B$

$A \to A$

Both can't be true at a time

Conditional proof (6)

If a set of premises $[\{P_1, P_2, P_3 \ldots P_n\} \wedge Q] \Rightarrow R$ then

$\{P_1, P_2 \ldots P_n\} \Rightarrow (Q \to R)$

Ex → Check whether the argument is valid or not

$\{\sim(p \wedge q),\ (q \vee r),\ (r \to s)\} \Rightarrow (P \to S)$

$\{\sim(p \wedge q),\ (q \vee r),\ (r \to s),\ P\} \Rightarrow S$

$\sim q$

r

$\underline{\underline{S}}$

So valid.

Proposition function

Proposition statement: 2 is greater than 10.] false

———— function : $P(x) = x$ is greater than 10] True/false

Variable Predicate

Ex $P(2) \to$ false

$P(12) \to$ True

(2) $R(x)$ = Ravi has a x phone

R(samsung), R(Apple) ---

Computer 'x' is connected to network 'y' $\rightarrow P(x, y)$

Quantifiers: [Do atleast 20 question on this topic]

$P(x): x > 2$

Domain of Disclosure: $\{1, 2, 3, 4\}$ [Value that x can take]

$P(1) \rightarrow F \quad P(2) \rightarrow F \quad P(3) \rightarrow T \quad P(4) \rightarrow T$

(1) $\forall x \, P(x) \rightarrow$ If the proposition function $P(x)$ is True for all x then it returns True.

$\forall$: for each x, for all x, for every x

So, for $P(x)$ $x > 2$

$\forall x \, P(x) \rightarrow$ False

If Domain = $\{3, 4\}$ then $P(x) = x > 2$ is following $\forall x \, P(x)$

Here, $\forall x$ quantifier.

(2) $\exists x \, P(x) \rightarrow$ If atleast one value in domain for which $P(x)$ is True.

$\exists$: There exist, Some

$\forall x \, P(x) = P(3) \wedge P(4)$ ($\wedge$: Intersection)

$= T \wedge T$

$= T$

$\exists x \, P(x) = P(3) \vee P(4)$ ($\vee$: Union)

$= T \vee T$

$= T$

(7)

Examples $P(x): x > 3$

Domain: $\{4, 5, 6\}$

$\forall x \, P(x) \rightarrow P(4) \wedge P(5) \wedge P(6)$

$\rightarrow T \wedge T \wedge T$

$\rightarrow T$

$\exists x \, P(x) \rightarrow P(4) \vee P(5) \vee P(6)$

$\rightarrow$ True.

Imp

$\Rightarrow$ If $\forall x \, P(x)$ is true for same domain and propositional function $\exists x \, P(x)$ will always True, but its converse is not always True.

(8)

$\Rightarrow$ $\forall$ and $\exists$ have highprecedence over other connectives

Ex. $\underbrace{\forall x P(x)}_{\text{Result}} \vee \underbrace{Q(x)}_{\text{Result}} \longrightarrow ((\forall x P(x)) \vee Q(x))$

Free and Bounded Variable

$\forall x P(x) \vee Q(x)$

$P(x)$ → Bounded with quantifier.

$Q(x)$ → free

Scope

$\underrightarrow{\forall x P(x)} \vee Q(x)$

$(\forall x)$ have scope upto $P(x)$ only

Distribution Quantifiers

$\forall x (x+y+z=10) \cong (\forall x(x)) + y + z = 10$

→ Only x is bounded here.

$\exists x P(x) \vee \forall x Q(x) \cong \exists x P(x) \vee \forall y Q(y)$

x is bounded to $\exists$

x is bounded to $\forall$

Similar to LHS but easy to understand.

Ex: $\forall x P(x) = P(x_1) \wedge P(x_2) \wedge P(x_3)$

$D: \{x_1, x_2, x_3\}$

$\forall x (P(x) \wedge Q(x)) \Rightarrow (P(x_1) \wedge Q(x_1)) \wedge (P(x_2) \wedge Q(x_2))$

$D: \{x_1, x_2\}$

same

$\forall x P(x) \wedge \forall x Q(x) = P(x_1) \wedge P(x_2) \wedge Q(x_1) \wedge Q(x_2)$

$D: \{x_1, x_2\}$

a) So, $\forall$ quantifier is distributive over '$\wedge$'

$\forall (x) (P(x) \vee Q(x)) = (P(x_1) \vee Q(x_1)) \wedge (P(x_2) \vee Q(x_2))$

$\forall (x) P(x) \vee \forall x Q(x) = (P(x_1) \wedge P(x_2)) \vee (Q(x_1) \wedge Q(x_2))$

So, $\forall$ is not distributive over '$\vee$'.

x (5) DBMS

	Conjuction($\wedge$)	Disjuction($\vee$)
$\forall$	Distributed	Non-Distributed
E	~~Non-distributed~~ Distributed	Distributed

10

Quantifiers with negation

$\sim(\forall x\, P(x))$

$\sim(\exists x\, P(x))$

Ex: Every student in class has taken a Course in CS.

or

P(x): x has taken a course in CS.

$\forall x\, P(x)$

Domain : student of class.

$\sim(\forall x\, P(x)) \rightarrow$ No student in class has taken a Course in CS.

$$\boxed{\sim\forall x\, P(x) \cong \exists x \sim P(x)}$$

$\rightarrow$ There exist atleast one student who has not taken a Course in CS.

$$\boxed{\sim\exists x\, P(x) \cong \forall x \sim P(x)}$$

Important examples

$\sim\forall x\, (\underbrace{P(x) \wedge Q(x)}_{R(x)}) \cong \exists x \sim R(x)$

Imp.

$\cong \exists x\, (\sim P(x) \vee \sim Q(x))$

$\sim\forall x\, (\underbrace{P(x) \rightarrow Q(x)}_{(\sim P(x) \vee Q(x))}) \cong \exists x\, (P(x) \wedge \sim Q(x))$

Imp.

$\sim\exists x\, (P(x) \wedge Q(x)) \cong \forall x\, (\sim P(x) \vee \sim Q(x))$

$\cong \forall x\, (P(x) \rightarrow \sim Q(x))$

$\sim\exists x\, ((P(x) \rightarrow Q(x)) \cong \forall x\, (P(x) \wedge \sim Q(x))$

Ex : Not all that glitter is gold
$\sim$ p

a) $\forall x$: glitter(x) $\rightarrow \sim$ gold(x)

b) $\forall x$: gold(x) $\wedge \sim$ glitter(x)

c) $\exists x$: gold(x) $\wedge \sim$ glitters(x)

d) $\exists x$: glitters(x) $\wedge \sim$ gold(x)

p(x) :- $\forall x$: glitter $\rightarrow$ gold(x) $(P \rightarrow Q)$

$\sim$ p(x) : $\exists x$: glitter(x) $\wedge \sim$ gold(x).

Using $\sim \forall x (P \rightarrow Q) = \exists x (P \wedge \sim Q)$

Important points to note :-

→ $\forall x (P(x) \Rightarrow Q(x)) \Rightarrow (\forall(x) P(x) \Rightarrow \forall x Q(x))$

→ whenever a predicate is satisfiable then its negation is also satisfiable.

→ In substrings select the continuous elements.

→ No. of ways of distributing 'n' similar things among 'r' different things equals to $(n+r-1) C_r$

→ Ex : Distributing 10 flowers among 2 girls = $^{11}C_{10} = 11$

→ No. of derangement (Q. 3.16 : Combinatorics).

→ Tautology (Always 1), Contradiction (Always 0), Contingency (0/1).

→ $p \equiv q$ denotes p, q are logically equivalent

→ $\forall x (P(x) \wedge Q(x)) \leftrightarrow \forall x P(x) \wedge \forall x Q(x)$

$\forall(x) P(x) \vee \forall x Q(x)) \rightarrow \forall(x) (P(x) \vee Q(x))$

$\exists x (P(x) \vee Q(x)) \leftrightarrow \exists x P(x) \vee \exists x Q(x)$

$\exists x (P(x) \wedge Q(x)) \longrightarrow \exists x P(x) \wedge \exists x Q(x)$

Day 3

Discrete Maths (Part-I : P2)

(1) $p \to q$ can be write as: p is sufficient for q, q is necessary for p, p only if q, q unless $\sim p$, p implies q, q if p, q when p, if p, q etc.

(2) $p \leftrightarrow q$ is equivalent to X-NOR, and $(p \to q) \wedge (q \to p)$.

(3) System is consistent if all the logics of system is true.

(4) $p \to q \cong \sim p \vee q$.

(5) Inference Rules

(1) Disjunctive Syllogism: $\dfrac{P \vee Q \quad \sim P}{Q}$

(2) Conjunctive Syllogism: $\dfrac{\sim (P \wedge Q) \quad P}{\sim Q}$

(3) Modus Ponen / Rule of detachment: $\dfrac{P \to Q \quad P}{Q}$

(4) Modus Tollen / Rule of contrapositive: $\dfrac{P \to Q \quad \sim Q}{\sim P}$

(5) Transitivity: $\dfrac{P \to Q \quad Q \to R}{P \to R}$

(6) Dilemma: $\dfrac{P \vee Q \quad P \to R \quad Q \to R}{R}$

(6) $[\{P_1, P_2, P_3 \ldots P_n\} \wedge Q] \Rightarrow R$ can be written as
$\{P_1, P_2, P_3 \ldots P_n\} \Rightarrow (Q \to R)$.

(7) Quantifiers: $\forall(x)\, P(x) = P(x_1) \wedge P(x_2) \wedge \ldots$
$\exists x\, P(x) = P(x_1) \vee P(x_2) \vee \ldots$

If $\forall x\, P(x)$ is true then in same domain $\exists x\, P(x)$ is true but converse is not true.

(8)

	$\wedge$	$\vee$
$\forall(x)$	Distributed	Non distributed
$\exists(x)$	Non-Distributed	Distributed

$\sim \forall(x)\, P(x) \cong \exists(x) \sim P(x)$

→ $P \equiv q$ denotes p and q are logical equivalent.

Imp Question: $\cancel{\forall(x)}\, P(x) \vee \forall(x)\, Q(x) \longrightarrow \forall(x)\, [P(x) \vee Q(x)]$

not both side.

→ $\forall(x) [P \Rightarrow q] \Rightarrow [\forall(x) P(x) \Rightarrow \forall(x) Q(x)]$

→ Tautology → All true   Contingency → True / false mixed

Contradiction → All false

→ Satisfiable : Either tautology or contingency.

Unsatisfiable : Contradiction.

→ If predicate logic is satisfiable then its negation is also satisfiable.

→ In CNF, for any formula, there is a truth assignment for which atleast half the clause evaluates to true.

→ $\exists x[(P(x)] \vee [Q(x)) \leftrightarrow \exists x\,(p(x) \vee q(x))$

and

$\exists x\,(p(x) \vee q(x)) \longleftrightarrow \exists x\, P(x) \vee \exists x\, Q(x)$

<u>Both side distributed</u>

→ Incase of every use $\forall(x)\, a(x) \rightarrow$, and with some use $\exists(x)\, a(x) \wedge$.

→ $\exists x\,(A(x) \wedge B(x)) \rightarrow \exists x\, A(x) \wedge \exists x\, B(x)$

→ $\exists! x$ means only or exactly 1.

→ All the sets over rational no. and integers are countable.