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TuTorial - 2

1) A multiple choice test contains 10 questions there are 4 possible answers for each que
(a) How many ways can a student answer the que. on the test if student answer every que?

(b) How many ways can a student answer the que on the test if the student can leave answers blank?

Ans => (a) possible ways $= 4^{10}$

(b) possible ways $= 5^{10}$ ∵ (4 options: 4^{10} + blanks 1^{10})

2) How many positive integers between 50 & 100

(a) are divisible by 7? which integer are these?
(b) are divisible by 11? which integers are these?
(c) are divisible by both 7 & 11? which integers are these?

Ans:- (a) positive integers between 50 and 100
56, 63, 70, 77, 84, 91, 98 : 7 Integers
(b) 55, 66, 77, 88, 99 : 5 integers
(c) 77 : 1 integers

3) How many Positive integers between 100 & 999 inclusive.

(a) are divisible by 7?
(b) are odd?
(c) Have the same 3 decimal digits?
(d) are not divisible by 4?
(e) are divisible by 3 or 4?
(f) are not divisible by either 3 or 4?
(g) are divisible by 3 but not by 4?
(h) are divisible by 3 and 4?

Ans. Total Integers between 100 & 999 inclusive, are = 900

(a) $\frac{900}{7} = 128.5 \approx 128$ integers

(b) total = 900
$\therefore$ odd = 450 & even = 450
$\therefore$ odd Integers are 450

(c) 111, 222, 333, 444, 555, 666, 777, 888, 999

Total digits : 10

(d) divisible by $4 = \frac{900}{4} = 225$

Not divisible by $4 = 900 - \frac{900}{4} = 900 - 225$

$= 675$

(e) by 3 or 4

$|A| = \frac{900}{3}$ $\quad |B| = \frac{900}{4}$ $\quad |A \cap B| = \frac{900}{3 \times 4}$.

$$|A \cup B| = |A| + |B| - |A \cap B|$$
$$= \frac{900}{3} + \frac{900}{4} - \frac{900}{12} = 450$$

(f) either not divisible by 3 or 4

$$= 900 - |A \cup B|$$
$$= 900 - 450$$
$$= 450.$$

(g) div. by 3 but not by 4

$$|A \cap B'| = |A| - |A \cap B|$$
$$= \frac{900}{3} - \frac{900}{3 \times 4}$$
$$= 300 - 75$$
$$= \underline{225}$$

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(b) div by 3 and 4

$$|A \cap B| = \frac{900}{3 \times 4} = 75$$

4) 7 A drawer contains a dozen brown socks and a dozen black socks, all unmatched. A man takes socks out at random in the dark.

- (a) How many socks must he takes out to be sure that he has atleast two socks of the same colour?
- (b) How many socks he take out to be sure that he has at least two black socks.

Ans (a) By generalized pigeonhole principle
$m=2 \quad n=? \quad k=2$ colors.

$m = \lceil n/k \rceil$

$2 = \lceil n/2 \rceil$

$n = 3$ socks.

$\left[\begin{array}{l} \text{for } n=1 \; \lceil 1/2 \rceil = \lceil 0.5 \rceil = 1 \\ \text{for } n=2 \; \lceil 2/2 \rceil = 1 \end{array} \right.$

for $n=3$ $\lceil 3/2 \rceil = \lceil 1.5 \rceil = 2 \rceil$

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(b) we cannot use Pigonhole principle because we want to make sure that atleast two black socks not just two socks of same colour.

→ In worst case we can select all the brown socks before black, so its 12 brown socks & then next 2 black socks

Hence, we need to take out 14 socks

5) let d be a positive integer show that among any group of d+1 (not necessarily consecutive) integers there are two with exactly the same remainder when they divided by d.

Ans → there are d possible remainders when an d+1 integers are divided by d.

$n = d+1 > n/k = d$

By Pigonhole principle given d+1 integers atleast 2 have same reminders

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6) What is the min. numbers of students each of them comes from one of the 50 state who must be enrolled in a university of that there are atleast 100 who came from the same state?

→ states $= 50 = k$, $m = 100$.

By generalized pigonhole principle

$m = \left\lceil \frac{n}{k} \right\rceil$.

$100 = \left\lceil \frac{n}{50} \right\rceil$

for $n = 4951$.

we get $\left\lceil \frac{4951}{50} \right\rceil = \lceil 99.02 \rceil = 100$.

Hence, min no. of students $= n = 4951$

(7) (a) Show that if 5 integers are selected from the first eight positive integers with a sum equal to 9.

(b) Is the conclusion in part (a) true if four integers are selected rather than five?

→ (a) group the first eight positive integers into subsets of two integers Such that each subset. Hence, two integers have sum 9.

b) No, for ex. $\{1,2,3,4\}$

8) How many permutations of letters ABCDEFGH contain the string ABC?

→ the letters ABC must occur as a block & Individual letters are D, E, F, G, & H.

∴ 6 letters can be arranged in,

$6! = 7\times5\times4\times3\times2\times1$

$= 720$ permutations

9) find the numbers of different one-one functions from the set $\{1,2,3,4\}$ into $\{A, B, C, D, E, F\}$

→ image of $1 \to 6$ ways

$2 \to 5$

$3 \to 4$

$4 \to 3$

Product rule;

this can be completed in

$6\times5\times4\times3$ ways

$= 360$ ways

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10) find the positive integer n such that $C(20, 2n) = C(20, 2n+4)$

$\longrightarrow$ $2n = 2n+4$ $2n+2n+4=20$
but $2n \neq 2n+4$ $\therefore 4n+4=20$
$\therefore n+1=5$
$\therefore n=4$ ($\because {}^nC_r = {}^nC_p$
$r+p,\ r+p=n$)

11) In an athletic club there are 16 male & 10 female members. How many ways can we form a commitee of 7 members subject to the following conditions.

$\longrightarrow$ (a) ${}^{16}C_3 \times {}^{10}C_4$

(b) $({}^{10}C_2 \times {}^{16}C_3) + ({}^{10}C_3 \times {}^{16}C_4) + ({}^{16}C_4 \times {}^{16}C_3) + ({}^{10}C_3 \times {}^{16}C_2) + ({}^{10}C_6 \times {}^{10}C_1) + ({}^{10}C_7 \times {}^{16}C_0)$

(c) $({}^{10}C_3 \times {}^{16}C_2) + ({}^{10}C_4 \times {}^{16}C_3) + ({}^{16}C_3 \times {}^{16}C_4) + ({}^{10}C_2 \times {}^{16}C_5) + ({}^{10}C_2 \times {}^{16}C_6) + ({}^{10}C_0 \times {}^{16}C_7)$

Tutorial - 3

Q-1 what is the coeff of x^9 in $(2-x)^{19}$?

Ans → $(x+y)^n = \sum_{j=0}^{n} \binom{n}{j} x^{n-j} y^j$

$(2-x)^{19} = \sum_{j=0}^{19} \binom{19}{j} (2)^{19-j} (-x)^j$

$= \sum_{j=0}^{19} \binom{19}{j} (-1)^j \cdot 2^{19-j} (x)^j$

now $j=9$

Coeff of x^9 is $\binom{19}{9} (-1)^9 (2)^{10}$

Ans

Q-2 what is the Coeff. of x^8y^9 in $(3x+2y)^{17}$?

$n = 17$, $x = 3x$, $y = 2y$

$(3x+2y)^{17} = \sum_{j=0}^{17} \binom{17}{j} (3x)^{17-j} (2y)^j$

now $j=9$

$= \binom{17}{9} (3)^8 x^8 (2)^9 y^9$

$\therefore$ Coeff. of $x^8y^9 = \binom{17}{9} (3)^8 (2)^9$

Tutorial -3

Q-1 what is the coeff of x^9 in $(2-x)^{19}$?

Ans → $(x+y)^n = \sum_{j=0}^{n} \binom{n}{j} x^{n-j} y^j$

$(2-x)^{19} = \sum_{j=0}^{19} \binom{19}{j} (2)^{19-j} (-x)^j$

$= \sum_{j=0}^{19} \binom{19}{j} (-1)^j \cdot 2^{19-j} (x)^j$

now $j=9$

Coeff of x^9 is $\binom{19}{9} (-1)^9 (2)^{10}$

Ans

Q-2 what is the coeff. of x^8y^9 in $(3x+2y)^{17}$?

$n=17$, $x = 3x$, $y = 2y$

$(3x+2y)^{17} = \sum_{j=0}^{17} \binom{17}{j} (3x)^{17-j} (2y)^j$

now $j=9$

$= \binom{17}{9} (3)^8 x^8 (2)^9 y^9$

$\therefore$ coeff. of $x^8y^9 = \binom{17}{9} (3)^8 (2)^9$

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Q-3 give a formula for coeff. of x^k in the expansion of $(x+1/x)^{100}$

Ans- $(x+1/x)^{100} = \sum_{j=0}^{100} \binom{100}{j} (x)^{100-j} (1/x)^{j}$

$= \sum_{j=0}^{100} \binom{100}{j} (x)^{100-2j}$

we want to determine the coeff of x^k.

$\therefore$ $100-2j=k$.

$\therefore$ $100-k=2j$

$\therefore$ $j = \frac{100-k}{2}$.

$\therefore$ coeff of $x^k = \binom{100}{\frac{100-k}{2}}$

Q-5 How many solutions does the equation $x_1+x_2+x_3=11$ have where x_1, x_2 & x_3 are non negative integers

Ans $n=11$ & $r=3$

$\therefore (n+r-1)=13$

& $r-1=2$

$\therefore$ No. of solutions $\binom{13}{2} = 78$

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Q-4/7 How Suppose that a Cookie shop has four different kinds of cookies. How many different ways can six cookies be chosen.

Ans-> $n=4, r=6$.
Combination with repitition
$C(n+r-1, r)$
$C(4+6-1, 6) = C(9,6)$
$= {}^9C_6 = \frac{9\times8\times7\times6\times5\times4\times3\times2}{6!\times3!}$
$= \frac{504}{6} = \frac{84}{}$ Ans

Q-6/7 How many solutions are there to the equation $x_1+x_2+x_3+x_4+x_5=21$ where x_i, $i=1,2,3,4,5$, is a nonnegative integer such that
(a) $x_1 \geq 1$?
(b) $x_i \geq 1$?
(c) $0 \leq x_1 \leq 10$?
(d) $0 \leq x_1 \leq 3$, $1 \leq x_2 < 4$, and $x_3 \geq 15$?

Ans -> (a) $n=21-1=20$
no of objects $K=5$
$C(n+r-1, r-1) = C(20+5-1, 5-1)$
$= C(24, 4)$
$= {}^{24}C_4 =$
$= \frac{24}{[20! \; 4!]} = 10,626$

(b) → $x_i \geq 2$, $i = 1, 2, 3, 4, 5$
$n = 21 - 10 = 11$
$k = 5$
$C(n+r-1, r-1) = C(11+5-1, 4)$
$= C(15, 4)$
$= {}^{15}C_4$

(c) → $0 \leq x_1 \leq 10$
upper limit $x_1 \geq 0$, $n = 21$, $r = 5$
$C(n+r-1, r-1) = C((21+5), 5-1)$
$= C(25, 5)$
$= {}^{25}C_5$

lower limit $x_1 < 10$
$n = 21 - 11 = 10$
$C(n+r-1, r-1) = C(10+5-1, 5-1)$
$= C(14, 4)$
$=$

(d) $0 < x_1 < 3$, $1 \leq x_2 < 4$, $x_3 \geq 15$

$x_1 \geq 0$, $x_2 \geq 1$ $x_3 \geq 15$
0 not involved   0 to 14 not involve
$n = 21 - 1 - 15 = 5$

$C(n+r-1, r-1) = C(5+5-1, 5-1)$
$= C(9, 4)$

$x_1 \leq 3$, $x_2 \leq 4$
$n = 5-4$ $n = 5-3 = 2.$
$= 1$

$C(n+r-1, r-1) + C(n+r-1, r-1)$
$= {}^5C_4 + {}^6C_4$
$\therefore {}^9C_4 - {}^5C_4 - {}^6C_4 = 106.$

Q-7

Ans-7 $n = 3000$
$x_1 + x_2 + x_3 = 3000$
$\therefore r = 3$

$C(n+r-1, r-1) = C(3000+3-1, 3-1)$
$= C(3002, 2)$
$= {}^{3002}C_2$
$=$

Q-8 How many strings of 10 ternary digits (0, 1, or 2) are there that contain exactly two 0s three 1s, and five 2s?

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Ans-7) $n = 10$
two 0s $\Rightarrow n_1 = 2$
three 1s $\Rightarrow n_2 = 3$
five 2s $\Rightarrow n_3 = 5$

$\therefore$ formula : $\dfrac{n!}{n_1! \times n_2! \times n_3!} = \dfrac{10!}{2! \times 3! \times 5!}$

$= 2520.$

Q-9 $n = 100$ each eqestion - 5 marks $x_i \geq 5$ i=1
no of equation - 10. 0, 1, 2, 3, 4
not involve.

Ans:- $n = 100 - 50 = 50$
$\therefore C(n+r-1, r-1) = C(50+10-1, 10-1)$
$= C(59, 9).$

Q-10 lexicographic order meens increasing order

156423 < 165432 < 231456 < 231465 < 234561 < 314562 < 432656 1 < 435612 < 541236 < 543216 < 654312 < 654321.

Q-11. Next largest permutation in lexicographic order

A : 1432 Ans 2134
B : 54123 Asn 54132.
C : 12453 Ans < 12534
D : 45231 < 45312
E : 6714235 < 6714253

F = 31528764 < 31542678

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Q-12 $X = \{1,2,3\}$, $f, g, h, s : X \to X$ be the function $X \to X$ given

$f = \{(1,3), (2,3), (3,1)\}$
$g = \{(1,2), (2,1), (3,3)\}$
$h = \{(1,1), (2,2), (3,1)\}$
$s = \{(1,1), (2,2), (3,3)\}$

Ans:- $fog : \{(1,1), (2,3), (3,2)\}$
$gof : \{(1,3), (2,2), (3,1)\}$
$fohog : \{(1,1), (2,2), (3,2)\}$
$gog : \{(1,2), (2,1), (3,3)\} = g$
$gos : \{(1,1)(2,2), (3,3)\} = s$ (Identity)
$fos : \{(1,2)(2,3), (3,1)\} = f$

Q-13 let $f(x) = x+2$, $g(x) = x-2$, $h(x) = 3x$, $x \in R$ where R is the set of real numbers, find

Ans
$gof = g(f(x)) = g(x+2) = x+2-2 = x$
$fog = f(g(x)) = f(x-2) = x-2+2 = x$
$fof = f(f(x)) = f(x+2) = x+2+2 = x+4$
$gog = g(g(x)) = g(x-2) = x-2-2 = x-4$
$foh = f(h(x)) = f(3x) = 3x+2$
$hog = h(g(x)) = h(x-2) = 3(x-2) = 3x-6$
$hof = h(f(x)) = h(x+2) = 3(x+2) = 3x+6$
$fohog = f(h(g(x))) = f(h(x-2)) = f(3x-6)$
$= (3x-6)+2$
$= 3x-4$

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Q-14) (i) $f(x) = e^x$

let $x_1, x_2, \in R$; $f(x_1) = f(x_2)$

$$e^{x_1} = e^{x_2}$$

$$\Rightarrow x_1 = x_2$$

$\therefore$ fun is one-one.

$\therefore e^x = \alpha$, $\alpha \in R$

$\therefore \log(e^x) = \log \alpha \in R$

$\therefore x = \log \alpha$

$\alpha = 0$, $\log 0$ is not defined

$\therefore$ fun is not ~~onot~~ onto

tutorial - 4

1. what is the negation of each of these statements?

(a) Today is thursday.

$\neg p$: today is not thursday

(b) p : there is no pollution in New Jersey.

$\neg p$: there is pollution in New Jersey

(c) $p : 2+1=3$

$\neg p : 2+1 \neq 3$

(d) the summer in maine is hot and sunny.

$\neg p$: the summer in maine is not hot and sunny

2. Let p and q be the statements "Swimming at the New Jersey shore is allowed" and "sharks have been spotted near the shore." respectively. Express of these compound statement as an English sentence.

→ Ans :-

here, we have two Sentence

p: "swimming at the New Jersey shore is allowed
q: "sharks have been spotted near the shore".

(a) $\neg p$

→ Swimming at the New Jersey shore is Not allowed

(b) $p \wedge Q$

- Swimming at the new jersey shore are allowed and shark have been spotted near the shore.

(c) $\neg p \vee Q$

- Swimming at the New Jersey shore are allowed or sharks have been spotted near the shore.

(d) $p \rightarrow \neg Q$

- Swimming at the New jersey shore are allowed implise that sharks not spotted near the shore.

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(e) $\neg q \rightarrow p$

- Swimming at the New jersey shore is not allowed implise that sharks have been spotted near the shore.

(f) $\neg p \rightarrow \neg q$

- Swimming at the New jersey shore is not allowed then sharks have not spotted near the shore.

Q.(3) Let P and Q be the Statement

P: It is below freezing.
Q: It is snowing.

Write these propositions using p and q and logical connectivies

(a) It is below freezing and Snowing.
$\rightarrow$ $p \wedge q$

(b) It is below freezing but not snowing
- $p \wedge \neg q$

(c) It is not below freezing and it is not snowing
- $\neg p \wedge \neg q$

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(d) It is either snowing or below freezing (or both)

- $p \vee q$

(e) If it is below freezing, it is also snowing.

- $p \rightarrow q$

(f) It is either below freezing or it is snowing, but it is not snowing if it is below freezing.

- $(p \vee q) \wedge (p \rightarrow q)$

(g) that it is below freezing is necessary and sufficient for it to snowing

- $p \longleftrightarrow q$

Q-(4) Construct truth table.

(a) $p \wedge \neg q$

p	q	$\neg q$	$p \wedge \neg q$
T	T	F	F
T	F	T	T
F	T	F	F
F	F	T	F

(b) $P \vee \neg P$

P	$\neg P$	$P \vee \neg P$
T	F	T
T	F	T
F	T	T
F	T	T

(c) $(P \vee \neg Q) \longrightarrow Q$

P	Q	$\neg Q$	$P \vee \neg Q$	$(P \vee Q) \rightarrow Q$
T	T	F	T	T
T	F	T	T	F
F	T	F	F	T
F	F	T	T	F

(d) $(P \vee Q) \longrightarrow (P \wedge Q)$

P	Q	$P \vee Q$	$P \wedge Q$	$(P \vee Q) \rightarrow (P \wedge Q)$
T	T	T	T	T
T	F	T	F	F
F	T	T	F	F
F	F	F	F	T

(e) $(P \rightarrow Q) \rightleftarrows (\neg Q \rightarrow \neg P) = E$

P	Q	$\neg P$	$\neg Q$	$P \rightarrow Q$	$\neg Q \rightarrow \neg P$	E
T	T	F	F	T	T	T
T	F	F	T	F	F	T
F	T	T	F	T	T	T
F	F	T	T	T	T	T

(f) $(P \to Q) \to (Q \to P) = E$

P	Q	$P \to Q$	$Q \to P$	E
T	T	T	T	T
T	F	F	T	T
F	T	T	F	F
F	F	T	T	T

(g) $\neg P \to (Q \to R) = E$

P	Q	R	$\neg P$	$Q \to R$	E
T	T	T	F	T	F
T	T	F	F	F	T
T	F	T	F	T	F
T	F	F	F	T	F
F	T	T	T	T	T
F	T	F	T	F	T
F	F	T	T	T	T
F	F	F	T	T	T

(h) $(P \to Q) \wedge (\neg P \to R) = E$

P	Q	R	$P \to Q$	$\neg P$	$\neg P \to R$	E
T	T	T	T	F	T	T
T	T	F	T	F	T	T
T	F	T	F	F	T	F
T	F	F	F	F	T	F
F	T	T	T	T	T	T
F	T	F	T	T	T	T
F	F	T	T	T	F	F
F	F	F	T	T	T	T

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Q-5 Show that $P \rightarrow Q$ and $\neg P \vee Q$ are logically equivalant.

P	Q	$\neg P$	$P \rightarrow Q$	$\neg P \vee Q$
T	T	F	T	T
T	F	F	F	F
F	T	T	T	T
F	F	T	T	T

logically equivalance.

Q-6 Show that $\neg(P \vee (\neg P \wedge Q))$ and $\neg P \wedge \neg Q$ are logically by developing a series of logical equivalance.

Ans → $\neg(P \vee (\neg P \wedge Q)) = A$
$\neg P \wedge \neg Q = B.$

P	Q	$\neg P$	$\neg Q$	B	$\neg P \wedge Q$	$P \vee (\neg P \wedge Q)$	A
T	T	F	F	F	F	T	F
T	F	F	T	F	F	T	F
F	T	T	F	F	T	T	F
F	F	T	T	T	F	F	T

logically equivalant

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Q-7 Show that $(P \wedge Q) \rightarrow (P \vee Q)$ is a tautology.

P	Q	$P \wedge Q$	$P \vee Q$	$(P \wedge Q) \rightarrow (P \vee Q)$
T	T	T	T	T
T	F	F	T	T
F	T	F	T	T
F	F	F	F	T

all value true.

Q-8 Show the following equivalences.

(a) $P \rightarrow (Q \vee R) \Leftrightarrow (P \rightarrow Q) \vee (P \rightarrow R)$

$\quad\;\; A \Leftrightarrow B$

P	Q	R	$Q \vee R$	$P \rightarrow Q$	$P \rightarrow R$	A	B
T	T	T	T	T	T	T	T
T	T	F	T	T	F	T	T
T	F	T	T	F	T	T	T
T	F	F	F	F	F	F	F
F	T	T	T	T	F	T	T
F	T	F	T	T	T	T	T
F	F	T	T	T	T	T	T
F	F	F	F	T	T	T	T

$A \Leftrightarrow B$

(b) $(P \to Q) \wedge (R \to Q) \iff (P \vee R) \to Q$

$A \iff B$

P	Q	R	$P\to Q$	$R \to Q$	$P \vee R$	A	B
T	T	T	T	T	T	T	T
T	T	F	T	T	T	T	T
T	F	T	F	F	T	F	F
T	F	F	F	T	T	F	F
F	T	T	T	T	T	T	T
F	T	F	T	T	F	T	T
F	F	T	T	F	T	F	F
F	F	F	T	T	F	T	F

A AND B are logically Equivalent

(c) $\neg(P \rightleftarrows Q) \iff (P \vee Q) \wedge \neg (P \wedge Q)$

$\underbrace{\neg(P \rightleftarrows Q)}_{A} \iff \underbrace{(P \vee Q) \wedge \neg (P \wedge Q)}_{B}$

P	Q	$P \rightleftarrows Q$	$\neg A$	$P \vee Q$	$P \wedge Q$	$\neg P \wedge Q$	B
T	T	T	F	T	T	F	F
T	F	F	T	T	F	T	T
F	T	T	F	T	F	T	T
F	F	T	F	F	F	T	F

Q-9 Show that the following are tautologies

(a) $(P \to Q) \wedge (Q \to R) \to (R \to R) = A$

here $\Rightarrow (P \to Q) \wedge (Q \to R) = B$

P	Q	R	$P \to Q$	$Q \to R$	$R \to R$	B	A
T	T	T	T	T	T	T	T
T	T	F	T	F	F	F	T
T	F	T	F	T	T	F	T
T	F	F	F	T	F	F	T
F	T	T	T	T	T	T	T
F	T	F	T	F	T	F	T
F	F	T	T	T	T	T	T
F	F	F	T	T	T	T	T

$\to$ here is A is tautology.

(b) $(P \vee Q) \wedge (\neg P \vee R) \to (Q \vee R)$.

Ans $\to$ here $\Rightarrow (P \vee Q) \wedge (\neg P \vee R) = A$

$(P \vee Q) \wedge (\neg P \vee R) \to (Q \vee R) = B$

P	Q	R	$P \vee Q$	$\neg P \vee R$	$Q \vee R$	A	B
T	T	T	T	T	T	T	T
T	T	F	T	F	T	F	T
T	F	T	T	T	T	T	T
T	F	F	T	F	F	F	T
F	T	T	T	T	T	T	T
F	T	F	T	T	T	T	T
F	F	T	F	T	T	F	T
F	F	F	F	T	F	F	T

→ here A is tautology

Q-10) <u>Show the Following Implications</u>

(c) $(P \wedge Q) \Rightarrow (P \rightarrow Q)$

P	Q	$P \wedge Q$	$P \rightarrow Q$	$(P \wedge Q) \Rightarrow (P \rightarrow Q)$
T	T	T	T	
T	F	F	F	
F	T	F	T	
F	F	F	T	

(d) $P \Rightarrow (Q \rightarrow P)$

P	Q	$Q \rightarrow P$	$P \Rightarrow (Q \rightarrow P)$
T	T	T	T
T	F	F	F
F	T	T	T
F	F	T	T

Q-II Show the following implications without Constructing (i) the truth tables.

(a) $(P \to (Q \to R)) \Rightarrow (P \to Q) \to (P \to R)$

→ here

$P \to (Q \to R) = A$

$(P \to Q) \to (P \to R) = B$

P	Q	R	$Q \to R$	$P \to Q$	$P \to R$	A	B
T	T	T	T	T	T	T	T
T	T	F	F	T	F	F	F
T	F	T	T	F	T	T	T
T	F	F	T	F	F	T	T
F	T	T	T	T	T	T	T
F	T	F	F	T	T	T	T
F	F	T	T	T	T	T	T
F	F	F	T	T	T	T	T

(b) $(P \to Q) \Rightarrow P \to (P \wedge Q) = A$

P	Q	$P \to Q$	$P \wedge Q$	$P \to (P \wedge Q)$	A
T	T	T	T	T	T
T	F	F	F	F	F
F	T	T	T	T	T
F	F	T	T	T	T

(a) $(P \rightarrow Q) \rightarrow Q \Rightarrow (P \vee Q) = A$

P	Q	$P \vee Q$	$P \rightarrow Q$	$Q \Rightarrow (P \vee Q)$	A
T	T	T	T	T	T
T	F	T	F	T	T
F	T	T	T	T	T
F	F	F	T	T	T

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tutorial-5

Q-1 Express the ~~student~~ statement "Every student in this class has studied calculus" using predicates and quantifier

Ans → $\forall x P(x)$

Q-2 Express the statement "some student in this class have visited mexico" and Every student in this class has visited either Canada or mexico" using Predicates and quantifier

Ans

$\exists x P(x)$

and

$\forall x P(x) \vee \forall x Q(x)$

$\hookrightarrow \forall x (P(x) \vee Q(x))$

Q-3 Consider the statement "Given any positive integer, there is a greater positive integer". Symbolize this statement with and without using the set of positive integers as the universe of discourse.

Ans- $\forall x \; (x+1 > x)$

$x \in Z^+$

or

$\forall x \; (x+1 > x)$

$x > 0, \; x \in Z$

Q-4 What are the negations of the statements $(\forall x)(x^2 > x)$ and $(\exists x)(x^2 = 2)$?

Ans (i) here $\neg(\forall x \; (x^2 > x))$

$= \forall x \; \neg(x^2 > x)$

(ii) $\exists x \; (x^2 = 2)$

$\therefore \neg \exists x \; (x^2 = 2)$

$\therefore \exists x \; (x^2 \neq 2)$

Q-5 Show that $\neg(\forall x)(P(x) \rightarrow Q(x))$ and $(\exists x)(P(x) \wedge \neg Q(x))$ are logically equivalent.

$\neg(\forall x)(P(x) \rightarrow Q(x)) = A$

$(\exists x)(P(x) \wedge \neg Q(x)) = B$

P	Q	$\neg Q$	$P \rightarrow Q$	A	B
T	T	F	T	F	F
T	F	T	F	T	T
F	T	F	T	F	F
F	F	T	T	F	F

A and B are logically equ-

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Q-6 Let L denote the relation "less than or equal to" and D denote the relation "divides" where xDy means "x divides y". Both L and D are defined on the set {1, 2, 3, 4}, write L and D as sets, and find $L \cap D$.

Ans → here two relations
L : "less than or equal to".
D : "divides" where x divides y"

Let's write Set's for L and D

- L = {(1,1) (1,2) (1,3) (1,6) (2,2) (2,3) (2,6) (3,3) (3,6) (6,6}

- D = {(1,1) (1,2) (1,3) (1,6) (2,2) (2,6 (3,3) (3,6) (6,6)}

→ then we find $L \cap D$:

$L \cap D$ = {(1,1) (1,2) (1,3) (1,6) (2,2) (2,6) (3,3) (6,6)}

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Q-7
7

Given $S = \{1, 2, 3, 4\}$ and a relation R on S defined by $R = \{(1,2) (4,3) (2,2) (2,1) (3,1)\}$, show that R is not transitive find a relation $R_1 \supseteq R$ such that R_1 is transitive.

Ans-> here, $R = \{(1,2), (4,3), (2,2), (2,1), (3,1)\}$

So here according to transitive rule.

xRy ~~&~~; $yRz \longrightarrow xRz$.

→ there $(1,2), (2,1) \longrightarrow (1,1)$

$(1,2) \notin R$ set.

→ so R is not transitive.

~> $R_1 = \{(1,2), (2,1), (1,1), (4,3), (3,1) (4,1)$
$(2,2) (2,1), (3,1) (1,2) (3,2)\}$

~> ~~R~~ Here $R_1 \supseteq R$, and R_1 is transitive.

Q-8
4

Given ~~S = {1,2,3,4} and~~ $S = \{1, 2 \dots 10\}$ and a relation R on S where $R = \{(x,y) \mid x+y = 10)$ what are the properties of the relation R?

Ans :- $S = \{1, 2, \ldots\ldots 10\}$

→ here relation $R = \{(x, y) \mid x + y = 10\}$

$R = \{(1,9), (9,1), (2,8), (8,2), (3,7), (7,3), (4,6)$
$(6,4), (5,5)\}$

→ R is a only Symetric

Q-9 Represent each of these relations on $\{1, 2, 3, 4\}$ with a matrix.

(a) $\{(1,2) (1,3) (1,4) (2,3) (2,4) (3,4)\}$

$$A = \begin{vmatrix} 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{vmatrix}$$

(b) $\{(1,1) (1,4) (2,2) (3,3) (4,1)\}$

$$A = \begin{vmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \end{vmatrix}$$

(c) {(1,2) (1,3) (1,4) (2,1) (2,3) (2,4) (3,1) (3,2) (3,4) (4,1) (4,2) (4,3)}

$$A = \begin{vmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{vmatrix}$$

(d) {(2,4) (3,1) (3,2) (3,4)}

$$d = \begin{vmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 1 & 0 & 1 \end{vmatrix}$$

Q-10 List the ordered Pairs in the relations on {1,2,3} corresponding to these matrices.

(a) $\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$ = {(1,1) (1,3) (2,2) (3,1) (3,3)}

(b) $\begin{bmatrix} 0 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 0 \end{bmatrix}$ = {(1,2) (2,2) (3,2)}

(c) $\begin{bmatrix} 1 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 1 \end{bmatrix}$ = {(1,1) (1,2) (1,3) (2,1) (2,3) (3,1) (3,2) (3,3)}

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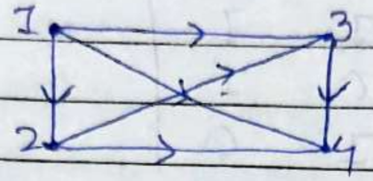
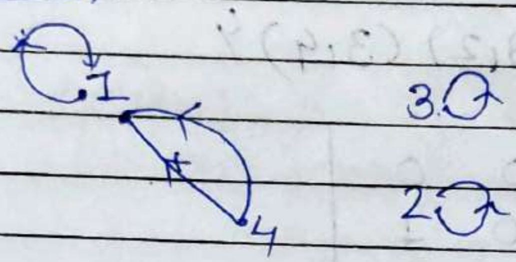
Q-11) Draw the graph of the relations from Quetion 9 and Quetion 10

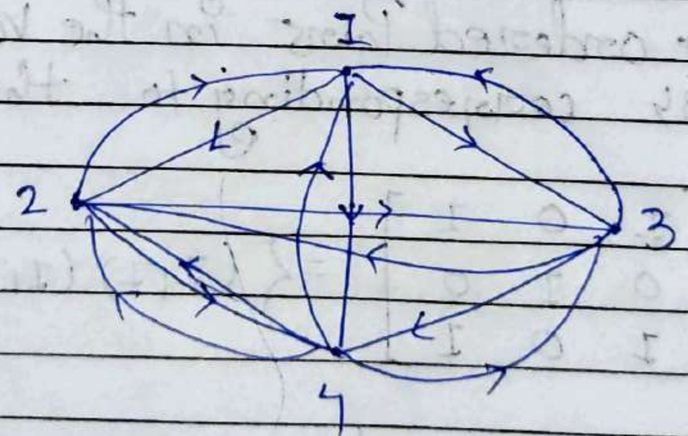
Ans-) $\boxed{9-(a)}$



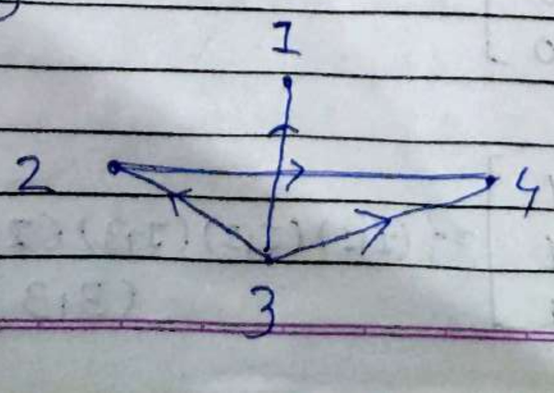
$\boxed{9-(b)}$



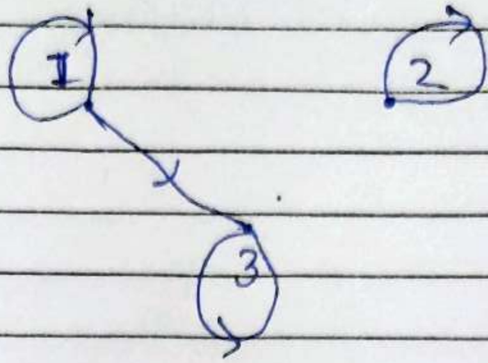
$\boxed{9-(c)}$



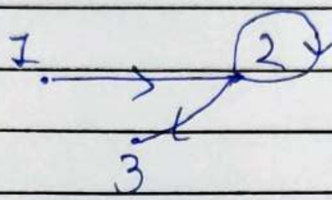
$\boxed{9-(d)}$



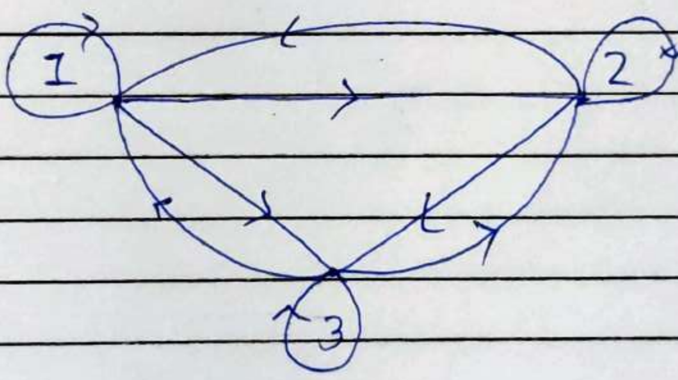
10-(a)



10-(b)



10-(c)



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