# CONVEX ANALYSIS WORKSHOP

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# 1. Convex functions

# 1.1. Definitions and basic properties.

**Definition 1.1.1**: Let  $S \subseteq \mathbb{R}^n$ . A function  $f: S \to \mathbb{R} \cup \{\infty\}$  is convex if **dom** f is a convex set and

$$\forall \boldsymbol{x}, \boldsymbol{y} \in \operatorname{\mathbf{dom}} f, \forall t \in \{0, 1\}, f(t\boldsymbol{x} + (1 - t)\boldsymbol{y} \le tf(\boldsymbol{x}) + (1 - t)f(\boldsymbol{y}))$$

where  $\operatorname{\mathbf{dom}} f$  is the effective domain of f:

$$\operatorname{dom} f \coloneqq \{ \boldsymbol{x} \in S \mid f(\boldsymbol{x}) < \infty \}.$$

[1]

### References

1. Boyd, S., Vandenberghe, L.: Convex Optimization. Cambridge University Press (2004)

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