$\int_{-\infty}^{\infty} |x(t)|^2 \, dt \geq 0 \to E_T$

$\frac{1}{2T_1} \int_{-T}^{T} |x(t)|^2 \to P_{av}$

$\int_0^1 1^2 \, dt = 1 \to E_T$

$\frac{1}{1} \int_0^1 1^2 \, dt$

$y(t) = \frac{1}{\tau} \int_{-\infty}^{t} x(\tau) \, d\tau$

① $y(t-t_0) = \frac{1}{\tau} \int_{-\infty}^{t-t_0} x(\tau) \, d\tau$

② $y(t) = \frac{1}{\tau} \int_{-\infty}^{t} x(\tau - t_0) \, d\tau$

let $\tau' = \tau - t_0$

$\frac{d\tau'}{d\tau} = 1$

$\therefore$ ② $= \frac{1}{\tau} \int_{-\infty}^{t-t_0} x(\tau') \, d\tau'$

$\therefore$ 非時變

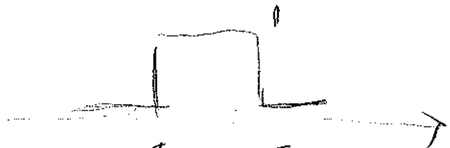
2.4

$\int_{-\infty}^{\infty} e^{-at} \cdot e^{-j\omega t} \, dt \; u(t)$

$\int_0^{\infty} e^{-t(a+j\omega)} \, dt$

$\frac{1}{a+j\omega} \int e \Big|_0^{\infty} \to \frac{-1}{(a+j\omega)}(0-1) = \frac{1}{a+j\omega}$

2.5



$-T_1 \quad T_1$

$\int_{-T_1}^{T_1} 1 \cdot e^{-j\omega t} \, dt$

$\to \frac{1}{-j\omega}\left(e^{-j\omega T_1} - e^{j\omega T_1}\right)$

$\to \frac{2}{\omega} \cdot \frac{e^{j\omega T_1} - e^{-j\omega T_1}}{2j} \to \frac{2}{\omega} \sin(\omega T_1)$

又 $\operatorname{sinc} x = \frac{\sin \pi x}{\pi x}$

$\pi x = \omega T_1$

$x = \frac{\omega T_1}{\pi}$

$\operatorname{sinc} \frac{\omega T_1}{\pi} = \frac{\sin \omega T_1}{\omega T_1} \to \frac{2}{\omega} \sin(\omega T_1)$

$\to T_1 \cdot \operatorname{sinc}\left(\frac{\omega T_1}{\pi}\right)$

2.6

$-W \quad W$

I.F.T

$\frac{1}{2\pi} \int_{-W}^{W} 1 \cdot e^{j\omega t} \, d\omega$

$\to \frac{1}{2\pi \cdot jt} \, e^{j\omega t} \Big|_{-W}^{W}$

$\frac{e^{jWt} - e^{-jWt}}{2\pi jt}$

$\frac{1}{\pi t} \quad \frac{e^{jWt} - e^{-jWt}}{2j}$

the consider x and ω

$\frac{1}{\pi t} \sin(Wt) \to \frac{\sin Wt}{\pi t}$

2.7

2.5.

$h_1(t) = u(t+1)$

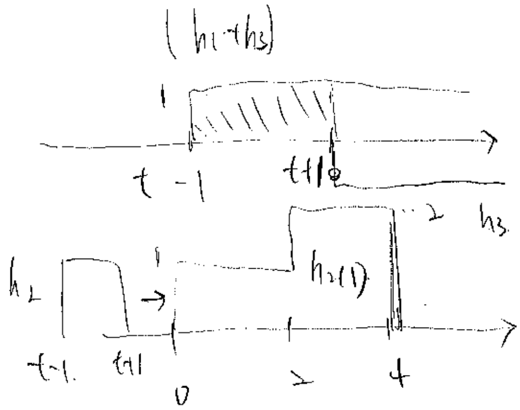
$h_2(t) = u(t) + u(t-2) - 2u(t-4)$

$h_3(t) = -u(t-1)$

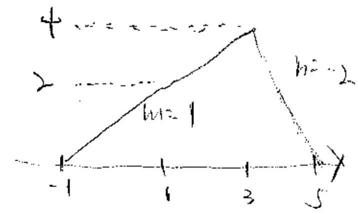
$h_T(t) = [h_2(t) * h_3(t)] + [h_2(t) * h_1(t)]$

$\Rightarrow h_2(t) * [h_3 + h_1](t)$

$(h_1 + h_3)$



$h_T(t)$



$t+1=0,\ t=-1 \uparrow$

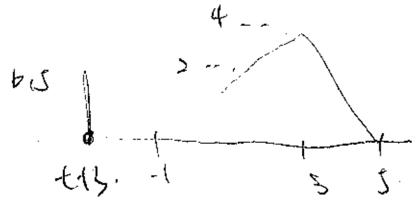
$t+1=2,\ t=1.$ Max Area $= 2$

$t+1>2,\ t>1,\ \uparrow\uparrow$

$t+1=4,\ t=3,$ Max

$t-1=4,\ t=5.\ \downarrow.$ Area $=0$

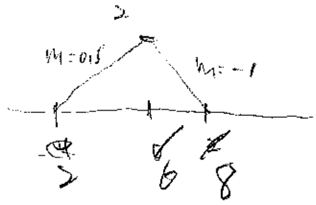
$x(t) * h_T(t) = y(t)$



$t+3=-1,\ t=-4$, Area $\uparrow$

$t+3=3,\ t=0$, Area $\uparrow$ Max $= 2$.

$t+3=5,\ t=2$, Area $=0$



not mem

stable.

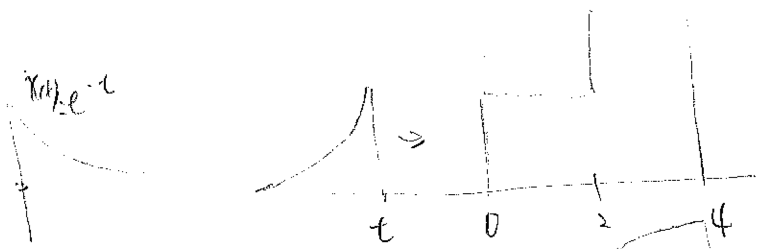
$x=y$

$\frac{1}{2}\sin(x-1)$

2.16 (b)

$y_2(t) = x(t) * h_2(t)$

$Y_2(j\omega)$



$\int_0^t e^{-t}\,dt + \int_0^2 e^{-t}\,dt + \int_2^t e^{-t}\,dt + \int_2^4 e^{-t}\,dt$

$\frac{1}{-1} e^{-t}\Big|_0^t \quad \int_0^2 1 \cdot e^{-(t-\tau)}\,d\tau$

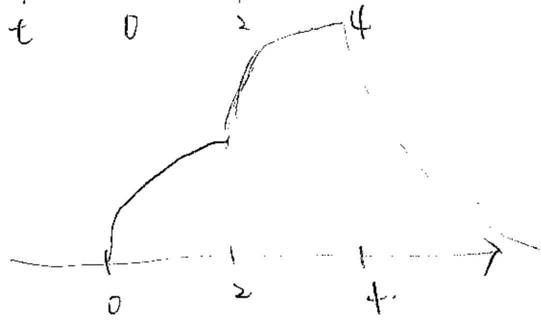
$-1(e^{-t} - 1) \cdot -(e^{-2} - e^0)t$

$\int_{-\infty}^0 e^t \cdot e^{-j\omega t}\,dt + \int_0^\infty e^{-t} e^{-j\omega t}\,dt$

$\int_{-\infty}^0 e^{t(1-j\omega)}\,dt + \int_0^\infty e^{t(-1-j\omega)}\,dt$

$= \frac{1}{1-j\omega}(1-0) + \frac{1}{-1-j\omega}(0-1)$

$= \frac{1}{a-j\omega} + \frac{1}{a+j\omega}$



when $t=0,\ \uparrow\uparrow$

$t=2$, Area $= \int_0^2 e^{-t}\,dt$

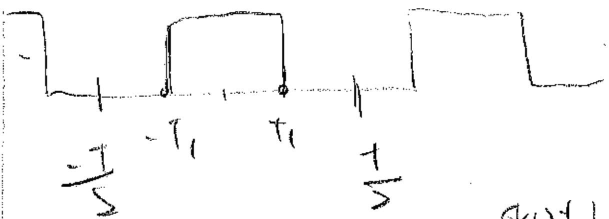
$t>2,\ \uparrow\uparrow$

$t=4$, Area $\to$ Max

$> \int_0^2 e^{-t}\,dt$

$t>4\ \downarrow\downarrow$

3.3



$$X(k) = \frac{1}{T}\int_{-T_1}^{T_1} 1 \cdot e^{-jk\omega t}\, dt$$

$$\frac{1}{T \cdot jk\omega}\left(e^{jk\omega t}\Big|_{-T_1}^{T_1}\right)$$

$$e^{jk\omega t}\Big|_{k=0}$$

$$\frac{1}{Tjk\omega}\left(e^{jk\omega T_1} - e^{-jk\omega T_1}\right).$$

$$k(0) = \frac{2T_1}{T} \rightarrow \frac{1}{T}\int_{-T_1}^{T_1} 1\, dt$$

$$\omega_0 = \frac{2\pi}{T}$$

$$\rightarrow \frac{2\sin(k\omega T_1)}{T\, k\omega.} = \frac{2\sin\left(k\frac{2\pi}{T}T_1\right)}{\cancel{T}\frac{2\pi}{T}\, k} = \frac{\sin\left(k2\pi\frac{T_1}{T}\right)}{\pi k}$$

if $k \neq 0$

3.4

$$x(t) = \sum_{t=-\infty}^{\infty} \delta(t-T)$$

$$\omega = \frac{2\pi}{T}. \quad X(k) = \frac{1}{T}\int_{-\frac{T}{2}}^{\frac{T}{2}} \delta(t)\, e^{jk\omega t}\, dt$$

$$X(k) = \frac{1}{T}$$

$x(t)$

$$q(t) = x(t - T_1) - x(t + T_1)$$

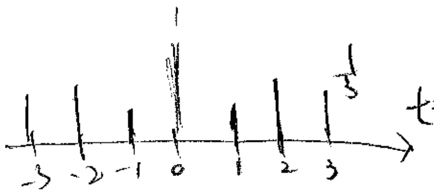
$$Q(k) = X(k)e^{+jk T_1 \omega} - X(k)e^{-jk\omega T_1}$$

$$\frac{2j}{T}\left(\frac{e^{+jk\omega T_1} - e^{-jk\omega T_1}}{2j}\right) = \frac{2j}{T}\sin(k\omega T_1)$$

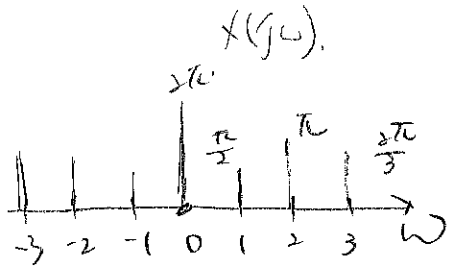
$$k \xrightarrow{2\pi} \omega$$

3.5

$$\int_{-\infty}^{\infty} e^{t} \cdot e^{-j\omega t}\, dt$$



$\rightarrow$



$$= \int_0^{\infty} e^{-t(1+j\omega)}\, dt \qquad X(k) \cdot e^{jk\omega t}$$

$$y(t) = x(t) * h(t).$$

$$\frac{-1}{(1+j\omega)}(-1) = \frac{1}{1+j\omega}$$

$$\omega = 2\pi \qquad f = 1$$

$$\rightarrow X(j\omega) \cdot H(j\omega)$$

$$0: \frac{2\pi}{(1+j\omega)} \times \frac{1}{2\pi}$$

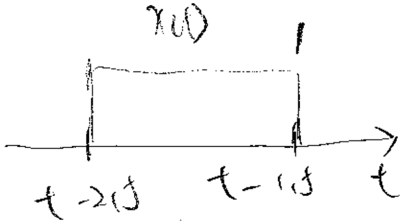
$$Y(1) = \frac{\frac{\pi}{2}}{(1+j \cdot 2\pi)} \times \frac{1}{2\pi} \cdot (1+j2\pi)\, 4$$

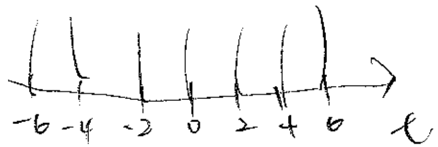
$$\int_2 1 \cdot 2$$

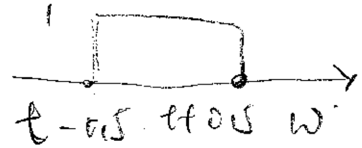
$$\int_2^4 1 \cdot 2\, dt$$

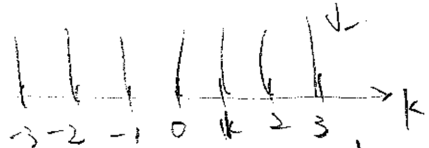
$$= 2(4-2) = 4$$

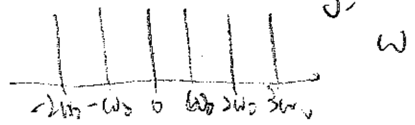
$x(t)$ $h(t)$ $T=2$, $\omega=\frac{2\pi}{T}=\pi=\omega_0$ $h_2(t)$ $\frac{\sin(0.5\omega)}{0.5\omega}$





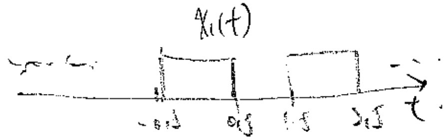






① $x_1(t) = x(t) * h_1(t)$

$X_1(j\omega)$



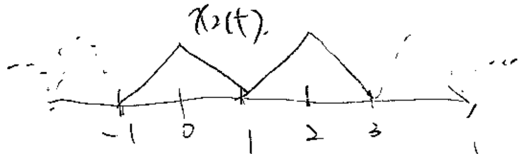
$t-1.5=-2$, $\sqcap$ until $t-2.5=-2$

$t=-0.5$ $\quad t=0.5$

$t-1.5=0$, $\sqcap$, until $t-2.5=0$

$t=1.5$ $\quad t=2.5$

② $x_2(t) = x_1(t) * h_2(t)$



$t+0.5=-0.5$, $t=-1$ $\sqcap$

$t+0.5=0.5$, $t=0$, Max

$t-0.5=0.5$, $t=1$, $\downarrow=0$

$t+0.5=1.5$, $t=1$ $\sqcap$

$t+0.5=2.5$, $t=2$ Max

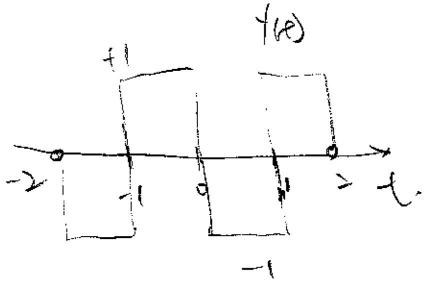
$t-0.5=2.5$ $\quad t=3$ $=0$

$y(t) = x_2(t) * h_3(t)$

$X_2(j\omega)\cdot H_3(j\omega)$

$Y(j\omega) = j\omega\, X_2(j\omega)$

$\frac{dx_2(t)}{dt} = y(t)$



③ $T=2$, 求 $Y[k]$.

$y(t)=\sum_{k=-\infty}^{\infty} Y[k]\cdot e^{-jk\omega t}$, $Y[k]=\frac{1}{T}\int_T y\, e^{jk\omega t}dt$

$\frac{1}{2}\int_{-1}^{0} 1\cdot e^{jk\omega t}dt + \frac{-1}{2}\int_0^1 1\cdot e^{jk\omega t}dt$

$\frac{1}{2jk\omega}\left(e^0 - e^{-jk\omega}\right) + \frac{-1}{2jk\omega}\left[-e^{jk\omega} + e^0\right]$

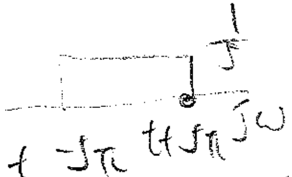
$\frac{2}{2jk\omega}$ $\quad \frac{-e^{-jk\omega}-e^{jk\omega}}{2jk\omega}$

$\frac{1}{jk\omega}\left(2 - \frac{e^{jk\omega}+e^{-jk\omega}}{2}\right)$

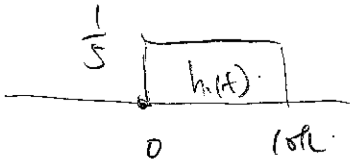
$\frac{1}{jk\omega}\left[2+\cos(k\omega)\right]$

$\frac{1}{j\pi k}\left[1+\cos(k\pi)\right]$

$h(t) \rightarrow H(j\omega) =$



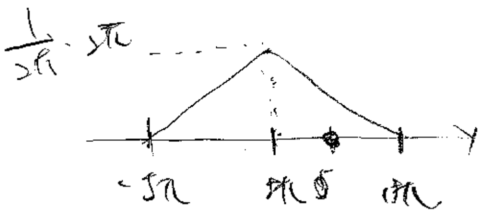
$H(j(\omega - 5\pi)) = h(t) \cdot e^{j5\pi t}$



$t+5\pi = 0, \ t = -5\pi, \uparrow$

$t+5\pi = 10\pi, \ t = 5\pi \ max$

$t-5\pi = 10\pi, \ t = 15\pi, \downarrow 0$



$\frac{1}{5} \cdot \int_0^{10\pi} \frac{1}{25} dx$

$\cancel{\frac{1}{5}}(10\pi - 0)$

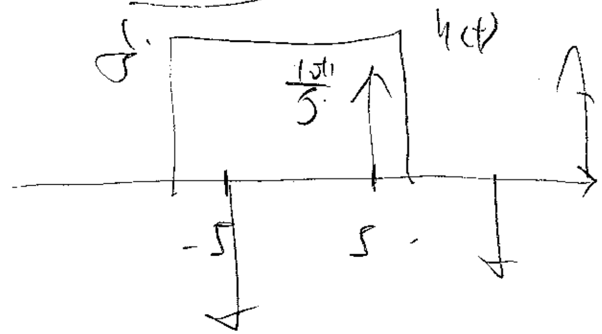
$\int_{-\infty}^{\infty} |h(t)|^2 dt = \frac{1}{2\pi} \int_{-\infty}^{\infty} |H(j\omega)|^2 dt$

$\frac{1}{2\pi} \int_{-5\pi}^{5\pi} \left(\frac{1}{5}\right)^2$

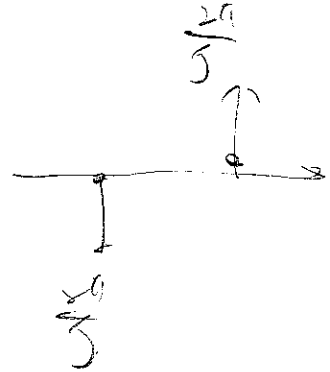
$x = \frac{10\pi}{25} \cdot \frac{1}{10\pi} = \frac{5}{t}$

$z(t) = x(t) * s(t)$

$Z(j\omega) = \cancel{X(j\omega)} \cdot S(j\omega) \cdot Y(j\omega)$



$\Rightarrow$



行きます
来ます
帰ります
がっこう
スーパー
駅
て
ふね
でんしゃ
ちかてつ
しんかんせん
バス
自転車
じてんしゃ
あるいて
ひと
ともだち 友達
かれ 彼
かのじょ 彼女
かぞく

3.2

$x(t)=\sin(5t)$

① $\int_{-\infty}^{\infty} \sin(5t)\,dt = 0$

② $x_1(t) \xrightarrow{F.T} j\omega X(j\omega)$

$\frac{dx}{dt} \Rightarrow j\omega X(j\omega)$

$x_1 = \frac{dx}{dt} \Rightarrow 5\cos(5t)$ #

③ $x\, e^{j\omega t} \rightarrow X(j(\omega - 0.5\pi))$

$x\, e^{j0.5\pi t}$

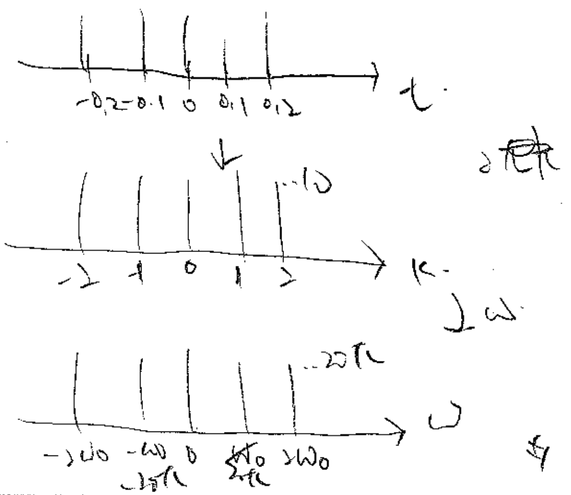
$\sin(5t)\cdot e^{j0.5\pi t}$

$1 = \frac{1}{T}\int_{-\infty}^{\infty} \delta(t) e^{-jk\omega t}\,dt$ #

④ $s(t) \rightarrow$ $s[k] = \frac{1}{T}\int_{-\infty}^{\infty} s(t)\,dt$ $T = \frac{1}{10}$

$\omega_0 = 20\pi$

$T = 0.1$



#

$\sum_{k=-\infty}^{\infty}$

⑤

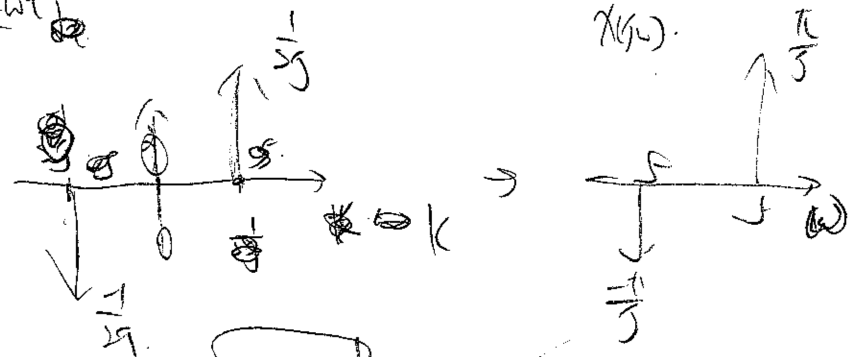
#

$y(t) = x(t)\cdot s(t) \rightarrow Y(j\omega) = X(j\omega) * S(j\omega) \times \frac{1}{2\pi}$

⑥ $x(t) = \sin(5t)$

$x(t) = \sum_{k=-1}^{1} X[k]\cdot e^{-jk\omega t}$

$\frac{e^{j5t} - e^{-j5t}}{2j}$



$k+5=0,\ k=-5$

$k-5=-5,\ k=5$

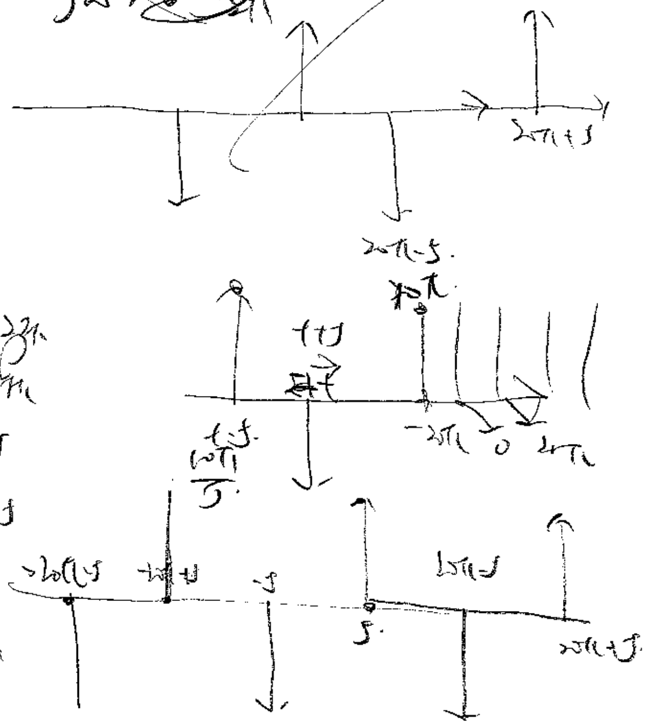
$k+5=1$

$k+5=20\pi$, $k = 20\pi$

$k-5=20\pi$ $k = 15\pi$

$20\pi - 5$

$20\pi + 5$



$20\pi \times \frac{\pi}{j} \times \frac{1}{2\pi}$

$= \frac{10\pi}{j}$

1(a).

$$\int_{-\infty}^{\infty} |x(t)^2| dt = \frac{1}{2\pi}\int_{-\infty}^{\infty} |X(j\omega)|^2 d\omega$$

$$= \frac{1}{2\pi}\int_{-1}^{-0.5} \pi \, d\omega + \int_{-0.5}^{0.5} \frac{\pi}{4} d\omega + \int_{0.5}^{1} \pi d\omega$$

$$= \frac{1}{2\pi}\left(\pi(0.5) + \frac{\pi}{4}(1) + 0.5\pi\right) = \frac{5}{8}$$

(b)

$$\frac{1}{2\pi}\int_{-1}^{0} \pi \, dt + \int_{0}^{1} \pi \, dt$$

$$\frac{1}{2\pi}(\pi + \pi) = 1$$

2. $\frac{d}{dt}x(t)\Big|_{t=0} \to j\omega X(j\omega)$.

(a)

$$\int x(t) e^{-j\omega t} dt \Big|_{t=0}$$

$$\int_{-1}^{-0.5} j\omega \cdot \sqrt{\pi} \, d\omega + \int_{-0.5}^{0.5} j\omega \cdot \frac{\sqrt{\pi}}{2} d\omega + \int_{0.5}^{1} j\omega\sqrt{\pi} \, d\omega$$

$$= \frac{j\sqrt{\pi}\omega^2}{2}\Big|_{-1}^{-0.5} + \frac{\sqrt{\pi}}{4} j\omega^2\Big|_{-0.5}^{0.5} + \frac{j\sqrt{\pi}}{2}\omega^2\Big|_{0.5}^{1}$$

$$\frac{j\sqrt{\pi}}{2}(0.25 - 1 + 1 - 0.25) + \frac{\sqrt{\pi}}{4} j(0.5^2 - 0.5^2) = 0$$

(b)

$$\int_{-1}^{0} j\omega \cdot (-j\sqrt{\pi}) d\omega + \int_{0}^{1} j\omega (j\sqrt{\pi}) d\omega$$

$$\int_{-1}^{0} \omega\sqrt{\pi} \, d\omega - \int_{0}^{1} \omega\sqrt{\pi} \, d\omega$$

$$\frac{\sqrt{\pi}}{2}\omega^2\Big|_{-1}^{0} - \frac{\sqrt{\pi}}{2}\omega^2\Big|_{0}^{1}$$

$$\frac{\sqrt{\pi}}{2}(0-1) - \frac{\sqrt{\pi}}{2}(1-0) = -\sqrt{\pi}$$

2.13.

$X(j\omega) = \frac{1}{a+j\omega}$, $H(j\omega) = \frac{1}{a+j\omega}$ $Y(t) = H(j\omega)\cdot X(j\omega) \to \frac{1}{(a+j\omega)^2}$

let $\frac{dz(t)}{dt} =$ $\frac{dY(j\omega)}{dj\omega} = Z(j\omega)$

$Z(j\omega) = \frac{1}{a+j\omega}$ $\to$

$Y(t)\cdot(-jt)$

$z(t) \to e^{-at}u(t)\cdot(-jt)\, j.$

$\to$

2.15

$Y(t) = \int_{-\infty}^{\infty} x(\tau)\, h(\tau - t)\, d\tau$ $sgn(t)$

$u(\tau - t)$ $\frac{d\,sgn(t)}{dt} = 2\delta(t)$ $\to j\omega S(j\omega) = 2$

$\to \frac{2}{j\omega}$

$\to X(j\omega)\cdot \underline{U(j\omega)}$

$\frac{1}{2} + \frac{1}{2}sgn(t)$. $1 \to 2\pi\delta(j\omega)$

$X(j\omega)\cdot\left(\pi\delta(j\omega) + \frac{1}{j\omega}\right)$ $\pi\delta(j\omega)\cdot \underset{0}{X(j\omega)} + \frac{X(j\omega)}{j\omega}$

2.4

$j\omega V_o(j\omega) = \frac{V_i(j\omega)}{RC} - \frac{V_o}{RC}$

$V_o\left(j\omega + \frac{1}{RC}\right) = \frac{V_i(j\omega)}{RC}$

$V_o = \frac{V_i(j\omega)}{RC\left(j\omega + \frac{1}{RC}\right)} \to \frac{V_i}{RC}\, e^{-\frac{1}{RC}t} = V_i(t)$

$H(j\omega)$ $h(t)$.

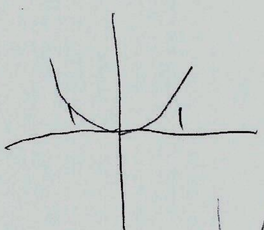
$V_o = u(t) * \frac{e^{-\frac{1}{RC}t}}{RC}\, u(t)$

$\frac{1}{RC}\int_0^{\infty} e^{-\frac{1}{RC}(t-\tau)}\, u(t-\tau)\, d\tau$

$\frac{1}{RC}\int_0^{t} e^{-\frac{1}{RC}(t-\tau)}\, d\tau$

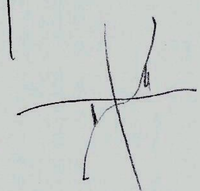
$\to \frac{RC}{RC}\left(e^{-\frac{1}{RC}(0)} - e^{-\frac{1}{RC}(t-0)}\right)$

$\to 1 - e^{-\frac{t}{RC}}$



$x(t)$

$y(t) = y(-t) \to$ 偶



$y(t) = -y(t)$

$y(t) = -y(-t)$

3.3.

① $\int_{-\infty}^{\infty} \sin(5x)\,dx = 0$

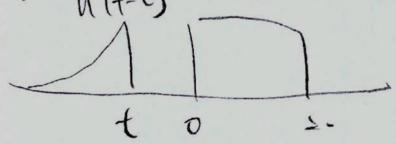
②.

$x_1(t) \xrightarrow{F.T.} j\omega X(j\omega)$

$\frac{d\,x(t)}{dt} \to j\omega X(j\omega)$

$x(t) = \sin(5t)$. $x_1(t) = 5\cos(5t)$

2.2 $h(t-\tau)$ $x(t)$

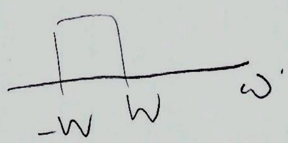


$\int_0^t 1 \cdot e^{-(t-\tau)}\,d\tau$

$e^{-0} - e^{-t}$

$\to 1 - e^{-t}$

3.6.



$\frac{1}{2\pi}\int_{-W}^{W} 1 \cdot e^{j\omega t}\,d\omega$

$\frac{1}{2\pi j t}\left(e^{jWt} - e^{-jWt}\right)$

$\frac{\sin(Wt)}{\pi t}$

$\int_0^{\infty} e^{-bt} \cdot e^{-j\omega t}\,dt$

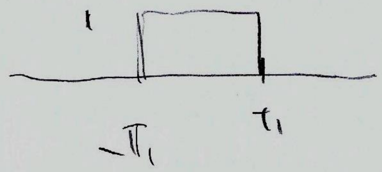
$e^{-t(b+j\omega)} \to \frac{1}{b+j\omega}$

③ $x_2(t) \xrightarrow{F.T.} X[j(\omega - 0.5\pi)]$. $x(t-t_0) = X(j\omega)\cdot e^{-j\omega t_0}$

$x(t)\cdot e^{j0.5\pi t}$

$x(t)\cdot e^{j\omega t} \overset{I.F.T.}{\leftarrow} X[j(\omega - 0.5\pi)]$

3.5.



$\int_{-T_1}^{T_1} 1 \cdot e^{-j\omega t}\,dt$

$\frac{1}{j\omega}\left(e^{j\omega T_1} - e^{-j\omega T_1}\right)$

$\frac{2}{\omega} \quad \frac{e^{j\omega T_1} - e^{-j\omega T_1}}{2j}$

$\frac{2}{\omega}\sin(\omega T_1)$.

$x \quad \mathrm{sinc}(x) = \frac{\sin \pi x}{\pi x}$

$\pi x = \omega T_1$

$\omega = \frac{\pi x}{T_1}$

$\frac{2}{\frac{\pi x}{T_1}} \sin(\omega T_1)$

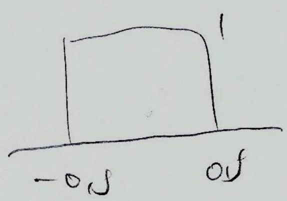
$\frac{2T_1 \sin(\omega T_1)}{\pi x}$

sinc

3.8

$X(j\omega)e^{-j\omega t_0}$

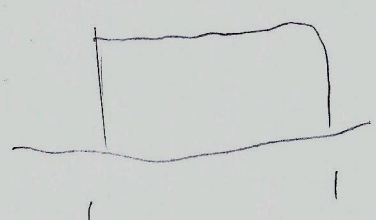
3.9. rect.



$\to \frac{\sin(0.5t)}{\pi t}$

$e^{-bt}u(t)\cdot\frac{1}{a-b} + e^{-at}u(t)\cdot\frac{1}{b-a}$

$\frac{u(t)}{a-b}\left(e^{-bt} - e^{-at}\right)$



$Y = \frac{1}{(b+j\omega)(a+j\omega)}$

$\to \frac{A}{b+j\omega} + \frac{B}{a+j\omega} = \frac{1}{\ldots}$

$X(j\omega) = \frac{1}{b+j\omega}$ $H(j\omega) = \frac{1}{a+j\omega}$

$A(a+j\omega) + B(b+j\omega) = 1$

$\frac{1}{b-a} = B$. $\frac{1}{a-b} = A$

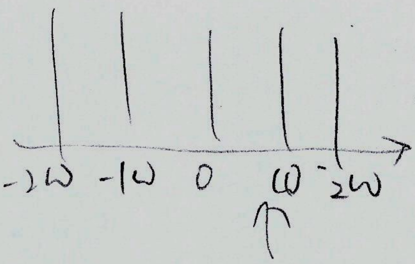
3.3

$\int_{-\infty}^{\infty} \sin(5t)\,dt = 0$

$x_1(t) = \sin(5t)$.

$5\cos(5t)$

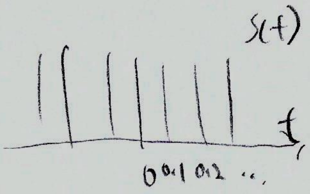
$5\omega t$



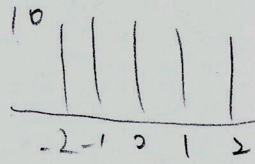
$x_1(t) \Rightarrow j\omega X(j\omega)$

$\frac{dx}{dt} \Rightarrow j\omega Y(j\omega)$.

$e^{0.5\pi t}$ $\cdot \sin(5t)$



$\frac{1}{T}\int \delta(t-\frac{n}{10}) e^{-jk\omega t} dt$ $\rightarrow$

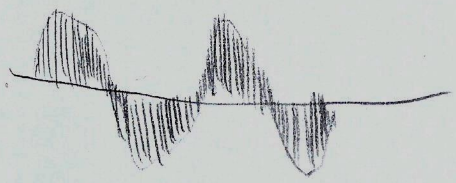


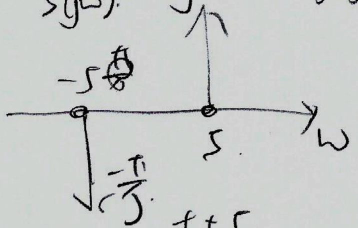
$\rightarrow T = 0.1,\ S(k) = 10$

$y(t)$

$\omega = 2\pi$

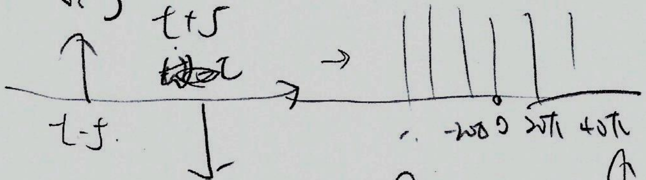
$S(j\omega) = \sum_{k=-\infty}^{\infty} 20\pi\,\delta(\omega - k\omega)$



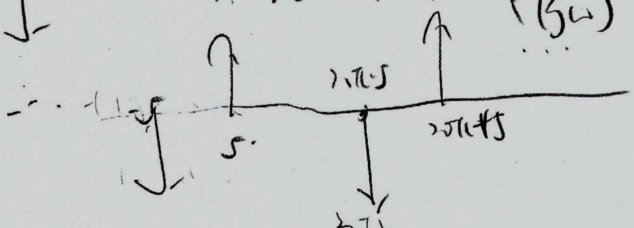


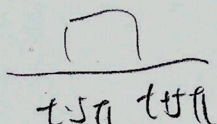
$y(t) = x(t)\cdot h(t)$

$\Rightarrow \frac{1}{2\pi} X(j\omega) * h(j\omega)$



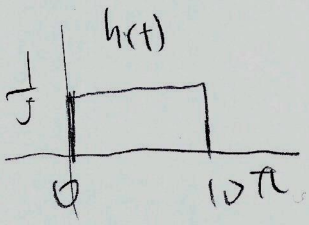
$\frac{20\pi\cdot\pi}{2\pi j}$.





$\cancel{\frac{20\pi}{j\,2}}$ $\frac{10\pi}{j}\,\delta(\omega - k\omega_0|_{20\pi}$

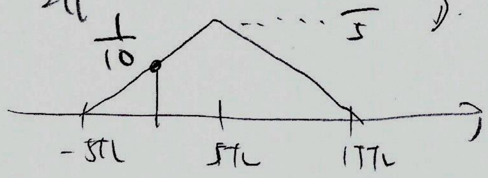
$t \quad t = 0,\ t = 5$



$h(t)\cdot h_1(t) = h(t) * h_1(t) \cdot \frac{1}{2\pi}$

$\frac{1}{5\pi}\int_0^{10\pi} \frac{1}{5}\cdot\frac{1}{5}\,dt$

$\cancel{\frac{10\pi}{25}}$ $\frac{10\pi}{50\pi} = \frac{1}{5}$



$\int_{-\infty}^{\infty} h(t) h_1(t) \cdot e^{-5j\omega t}\, dt$ $\quad \omega = 5\pi$

$\omega = 0$

$\int_{-\infty}^{\infty} h_1$

$\int_{-\infty}^{\infty} |h(t)|^2 = \frac{1}{2\pi}\int |H(j\omega)|^2 d\omega$

$10\pi \cdot \frac{1}{5}\cdot\frac{1}{2\pi} = 1$

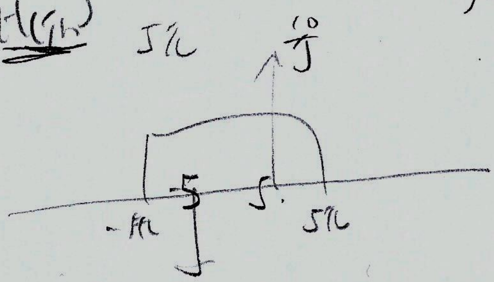
$z(t) = y(t) * h(t)$.

$= Y(j\omega) H(j\omega)$

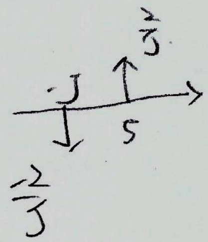
± 5.

$y(t)\ \frac{x_1(t)}{R(t)}$

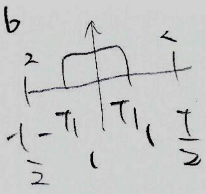
$y(t) = \frac{x(t-1)}{R(t-1)}$



3.



3.6



$\frac{1}{T}\int_{-\frac{T}{2}}^{\frac{T}{2}} 1 \cdot e^{-jk\omega t}\,dt$

$$\frac{e^{jkT_1} - e^{-jkT_1}}{T \cdot jk\omega} \Rightarrow \frac{e^{jk\frac{\pi}{2}} - e^{jk\frac{\pi}{2}}}{\cancel{\frac{\pi}{2}} \cdot 4\, j\, 2\pi k} \to \frac{e^{jk\frac{\pi}{2}} - e^{jk\frac{\pi}{2}}}{j2\pi k}$$

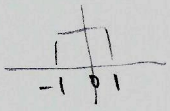
$T = 4$

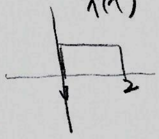
$\omega = \frac{\pi}{2}$

$x(t)$

$x(t-1)$

$x(t)$





$\to x(t) \cdot e^{-jk\frac{\pi}{2}}$

$$\left(\frac{\sin(k\frac{\pi}{2})}{\pi k} \cdot e^{-jk\frac{\pi}{2}}\right) \quad \frac{1}{2} \qquad \frac{\sin(k\frac{\pi}{2})}{\pi k}$$

$$x(t).\quad g(k) = \frac{-1}{2} + \frac{\sin(k\frac{\pi}{2})}{\pi k} e^{-jk\frac{\pi}{2}},\quad k \neq 0.\quad k = 0,\ g(k) = 0$$

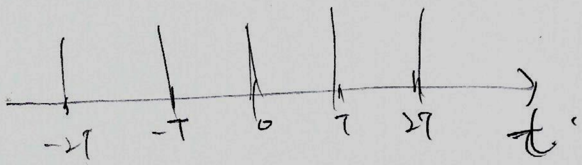
$$g(t) = \frac{dx}{dt} \quad G(j\omega) \cdot j\omega \Leftarrow \frac{dx}{dt} \qquad g(j\omega) = X(j\omega) \cdot j\omega \qquad X(j\omega) = \frac{g(j\omega)}{j\omega}$$

$\omega = \frac{\pi}{2}$

$$\frac{\sin(k\frac{\pi}{2})}{\pi k \cdot j\omega}$$

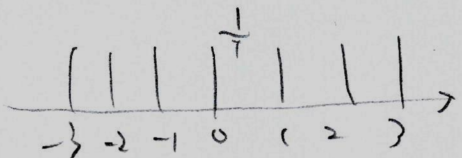
$$= \frac{1}{j \cdot (\frac{\pi^2}{2} \cdot k)}$$

3.1 $x(t)$

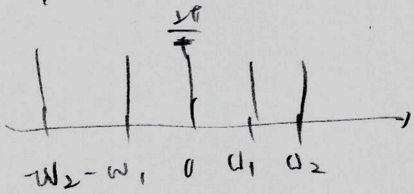


$$\to \frac{1}{T}\int_{-\frac{T}{2}}^{\frac{T}{2}} \delta(t)\, e^{jk\omega t}\,dt = \frac{1}{T}$$

$\omega = \frac{2\pi}{T}$



ωX



$\to \sum$

$$X(j\omega) = \sum_{k=-\infty}^{\infty} \cdot \frac{2\pi}{T} \cdot \delta(t - k\omega)$$

$\delta(t-k\omega)$

$$\frac{\sin(\omega T_1)}{\omega T_1}$$

$2T_1 \operatorname{sinc}(\frac{\omega T_1}{\pi})$

$\omega T_1 = \pi x$

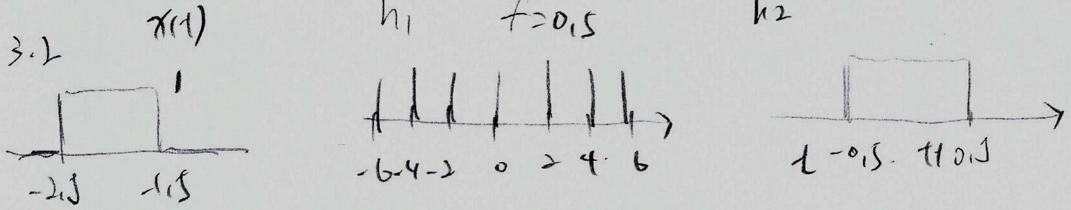
$$\operatorname{sinc}(?) = \frac{2T_1}{\cancel{\omega}} \sin(\omega T_1)$$

$x = \frac{\omega T_1}{\pi}$

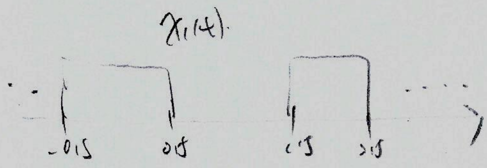
$$\operatorname{sinc}(x) = \frac{\sin \pi x}{\pi x}$$

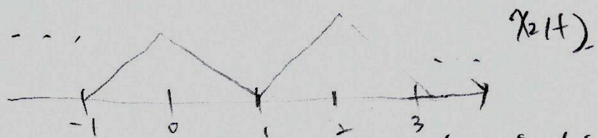
$$\operatorname{sinc}\left(\frac{\omega T_1}{\pi}\right) = \frac{\sin \omega T_1}{\omega T_1}$$

3.2



$x_1(t) = x(t) * h_1(t)$ $\qquad$ $x_2(t) = x_1(t) * h_2(t)$.





$t-1.5=-2,\ t=-0.5$ $\qquad$ $t-1.5=0,\ t=1.5$

$t-2.5=-2,\ t=0.5$ $\qquad$ $t-2.5=0,\ t=2.5$

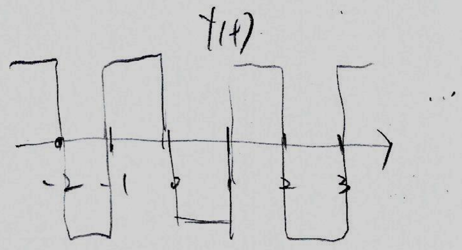
$t+0.5=-0.5,\ t=-1$ $\qquad$ $t+0.5=1.5,\ t=1.1?$

$t+0.5=0.5,\ t=0$, Max $\qquad$ $t+0.5=2.5,\ t=2$, Max

$t-0.5=0.5,\ t=1$, 0 $\qquad$ $t-0.5=2.5,\ t=3$. 0

$Y(j\omega) = j\omega X_2(j\omega)$.

$\frac{d x_2(t)}{dt} \leftrightarrow j\omega X_2(j\omega)$



3.1

$\sin(\omega t) \to \frac{e^{j\omega t} - e^{-j\omega t}}{j2}$. $\qquad$ $x(t) = \sum_{-\infty}^{\infty} X[k] e^{jk\omega t}$ $\qquad$ $X[-1] = \frac{-1}{j2}$, $X[1] = \frac{1}{j2}$, other, 0

3.2

$1 + \frac{e^{j\omega t} - e^{-j\omega t}}{2j} + \frac{e^{j\omega t} + e^{-j\omega t}}{1} + \frac{e^{j(2\omega t + \frac{\pi}{4})} + e^{-j(2\omega t + \frac{\pi}{4})}}{2}$

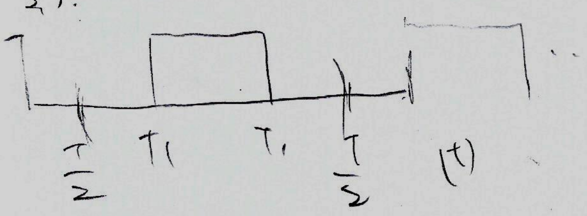
$\frac{e^{\frac{\pi}{4}}}{2}$ $\qquad$ $\frac{e^{\frac{-\pi}{4}}}{2}$

$X[0] = 1$

$X[1] = 1 + \frac{1}{2j}$, $X[-1]\ 1 - \frac{1}{2j}$.

$X[2] = \frac{1}{2}\left(\frac{1}{\sqrt{2}} + j\frac{1}{\sqrt{2}}\right)$ $\quad$ $X[-2] = \frac{1}{2}\left(\frac{1}{\sqrt{2}} - j\frac{1}{\sqrt{2}}\right)$.

3.3.



$\frac{1}{T}\int_{-T_1}^{T_1} 1 e^{0} dt = \frac{2T_1}{T}$

$x(t) = \sum_{-\infty}^{\infty} X[k] e^{jk\omega t}$

$X[k] = \frac{1}{T}\int_{-T_1}^{T_1} 1 \cdot e^{-jk\omega t} dt$

$\frac{1}{-jk\omega T}\left(e^{-jk\omega T_1} - e^{jk\omega T_1}\right)$

$\frac{2}{k\omega T} \cdot \frac{e^{+jk\ldots} - e^{-}}{j2}$ $\to \frac{2}{k\omega T}\sin(k\omega T_1)\Big|_{k \neq 0,\ \omega = \frac{2\pi}{T}}$

離散

分佈名稱	Pmf	var	Mean
白努力 Bernoulli(p)	$P_x(x) = p, x = 1$ $P_x(x) = 1 - p, x = 0$	$Var = p(1-p)$	$E(x) = p$
二項式 Binomial(n,p)	$C_k^n p^k (1-p)^{n-k}$ $0, other$	$Var = np(1-p)$	$E(x) = np$
幾何 Geometric(p)	$P_x(k) = (1-p)^{k-1}p$ $0, other$	$Var = \frac{1-p}{p^2}$	$E(x) = \frac{1}{p}$
Poisson(λ)	$P_x(k) = e^{-\lambda}\frac{\lambda^k}{k!}$ $0, other$	$Var = \lambda$	$E(x) = \lambda$
Hypergeometric(R,W,n)	$P_x(k) = \frac{C_k^R C_{n-k}^W}{C_n^{R+W}}$	$n\frac{R}{R+W}\frac{W}{R+W}\frac{R+W-n}{R+W-1}$	$E(x) = \frac{R}{R+W}n$
負二項(p,r)	$C_{r-1}^{k-1}p^{r-1}(1 - p)^{k-r}p$	$Var = r\frac{1-p}{p^2}$	$E(x) = r\frac{1}{p}$

連續

$$\boldsymbol{E(x) = \int_{-\infty}^{\infty} xf(x)dx \, , E\big(h(x)\big) = \int_{-\infty}^{\infty} h(x)f(x)dx}$$

Moments

$$\boldsymbol{E(x^n) = \int_{-\infty}^{\infty} x^n f(x)dx \, , E\big(h(x)\big)}$$

標準差$\boldsymbol{\sigma}$

$$\boldsymbol{\sqrt{Var(x)} = \sigma_X = \sqrt{\sigma_X^2}}$$

分佈名稱	PDF	var	Mean
Uniform(a,b)	$\boldsymbol{f(x) = \frac{1}{b-a}}$ $\boldsymbol{F(x) = \begin{cases} 0, x < a \\ \frac{x-a}{x-b}, a < x < b \\ 1, x > b \end{cases}}$	$\boldsymbol{\frac{1}{12}(b-a)^2}$	$\boldsymbol{E(x) = \frac{b-a}{1}}$
Exponentia($\boldsymbol{\lambda}$)	$\boldsymbol{f(x) = u(x)\lambda e^{-\lambda x}}$ $\boldsymbol{F(x) = u(x)\big(1 - e^{-\lambda x}\big)}$ $\boldsymbol{P(x > t) = 1 - F(x) = e^{-\lambda t}}$	$\boldsymbol{Var(x) = \frac{1}{\lambda^2}}$	$\boldsymbol{E(x) = \frac{1}{\lambda}, E(x^2) = \frac{2}{\lambda^2}}$
Gamma($\boldsymbol{\alpha, \lambda}$)	$\boldsymbol{u(x)\lambda^\alpha \frac{x^{\alpha-1}}{\Gamma(\alpha)} e^{-\lambda x}}$	$\boldsymbol{Var(x) = \frac{\alpha}{\lambda^2}}$	$\boldsymbol{E(x) = \frac{\alpha}{\lambda}}$
Weibull($\boldsymbol{\alpha, \lambda}$)	$\boldsymbol{u(x)\alpha\lambda(\lambda x)^{\alpha-1} e^{-(\lambda x)^\alpha}}$ $\boldsymbol{F(x) = (1 - e^{-(\lambda x)^\alpha})}$	$\frac{1}{\lambda^2}(\Gamma\left(1+\frac{2}{\alpha}\right) - \Gamma\left(1+\frac{1}{\alpha}\right)^2)$	$\boldsymbol{E = \frac{1}{\lambda}\Gamma(1 + \frac{1}{\alpha})}$
Gaussian($\boldsymbol{\mu, \sigma^2}$)	$\boldsymbol{f(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{\frac{-(x-\mu)^2}{2\sigma^2}}}$ $\boldsymbol{F(x) = \phi(\frac{x-\mu}{\sigma})}$	$\boldsymbol{Var(x) = \sigma^2}$	$\boldsymbol{E = \mu}$

定理:

$$\mathrm{T}(\alpha+1)=\alpha\mathrm{T}(\alpha)$$

範例:

$$\frac{5}{2}!=\mathrm{T}\left(\frac{5}{2}+1\right),=\frac{5}{2}\times\mathrm{T}\left(\frac{5}{2}\right),=\frac{5}{2}\times\mathrm{T}\left(\frac{3}{2}+1\right)=\frac{5}{2}\times\frac{3}{2}\mathrm{T}\left(\frac{3}{2}\right)=\frac{5}{2}\times\frac{3}{2}\mathrm{T}\left(\frac{1}{2}+1\right)=\frac{5}{2}\times\frac{3}{2}\times\frac{1}{2}\mathrm{T}(\frac{1}{2})$$

又$\mathrm{T}\left(\frac{1}{2}\right)=\sqrt{\pi}$,所以$\frac{5}{2}!=\frac{5}{2}\times\frac{3}{2}\times\frac{1}{2}\times\sqrt{\pi}=\frac{15}{8}\sqrt{\pi}$

定理:

$$f_x|dx|=f_y|dy|,f_y=f_x|\frac{dy}{dx}|$$

範例:

假設有一分佈

$$f_X(x)=\begin{cases}\frac{3}{2}(2x-x^2), if 0<x<1\\ 0 \quad\quad ,other\end{cases}$$

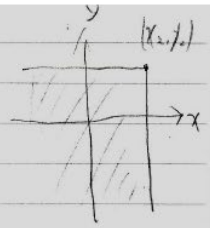
有另外一個隨機變數$Y=ln(X)$ ，求$f_Y(y)$

可知$x=e^y$。$\frac{dx}{dy}=(e^y)'=e^y$ 所以根據$f_y=f_x|\frac{dy}{dx}|$可得:

$$\frac{3}{2}(2e^y-e^{2y})e^y$$

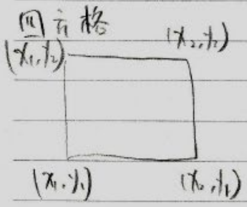
Joint CDF

有兩個 r.m X,Y. 可寫成 $F_{X,Y}(x,y)$



- $0\le F_{X,Y}(x,y)\le 1$.
- $x_1<y_1$ 且 $x_2<y_2$ $\to F_{X,Y}(x_1,y_1)<F_{X,Y}(x_2,y_2)$
- $F_{X,Y}(x,\infty)=P(X\le x,Y\le\infty)\to F_X(x)$
- $F_{X,Y}(\infty,y)=F_Y(y)$
- $F_{X,Y}(\infty,\infty)=P(X\le\infty,Y\le\infty)=1$
- $F_{X,Y}(x,-\infty)=P(X\le x,Y\le-\infty)\le P(Y\le\infty)=0$

四方格



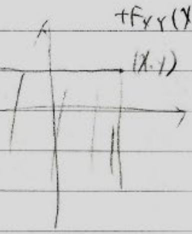
方框內的機率

$P(x_1\le X\le x_2,y_1\le y\le y_2)$

$=F_{X,Y}(x_2,y_2)-F_{X,Y}(x_2,y_1)-F_{X,Y}(x_1,y_2)$

$+F_{X,Y}(x_1,y_1)$

Joint PMF

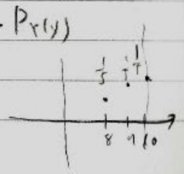


$P_{X,Y}(x,y)=P(X=x,Y=y)$

且 $F_{X,Y}(x,y)=\sum_{v=-\infty}^{y}\sum_{u=-\infty}^{x}P_{X,Y}(u,v)$

- $P_{X,Y}(x,y)$ 介於 $0\sim1$
- $\sum_{x=-\infty}^{\infty}\sum_{y=-\infty}^{\infty}P_{X,Y}(x,y)=1$
- $X\ Y$ 獨立 . $P_{X,Y}(x,y)=P(X=x,Y=y)=P_X(x)\cdot P_Y(y)$

若有一事件B. ≤ 10 . 可由



$\sum_{(x,y)\in B}P_{X,Y}(x,y)=P(B)$.

Joint PDF

$f_{X,Y}(x,y)=\lim_{\substack{\Delta x\to0\\ \Delta y\to0}}\frac{P\left(\boxed{\quad}\,\Delta y\right)}{\Delta x\cdot\Delta y}$

↳ 透過四方格定理 + 整理 $\to f_{X,Y}(x,y)=\frac{\partial^2F_{X,Y}(x,y)}{\partial x\,\partial y}$

$F_{X,Y}(x,y)=\int_{-\infty}^{x}\int_{-\infty}^{y}f_{X,Y}(u,v)\,dv\,du$

且若 $\Delta x,\Delta y$ 很小. $P\left((X,Y)\in\boxed{\quad}\,\Delta y\right)$

↳ $\Delta x\cdot\Delta y\,f_{X,Y}(x,y)$

Joint PDF 可以決定他的機率分佈

- $f_{X,Y}(x,y)\ge1$
- $\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}f_{X,Y}(x,y)=1$
- 如果 X.Y 獨立. $f_{X,Y}(x,y)=f_X(x)\times f_Y(y)$
- $P(B)=\int\int_{(x,y)\in B}f_{X,Y}(x,y)\,dx\,dy$。

MPMF Marginal PMF 算了 找 MPMF

	X=8	X=9	X=10	
Y=8	0.2	0.1	0.05	$P_Y(8)=0.35$
Y=9	0.05	0.2	0.1	$P_Y(9)=0.35$
Y=10	0.05	0.1	0.15.	$P_Y(10)=0.3$
$P_X(8)$	0.3	0.4	0.3.	

$P_X(X=x)=\sum_{y=-\infty}^{\infty}P_{X,Y}(x,y)$

$P_Y(Y=y)=\sum_{x=-\infty}^{\infty}P_{X,Y}(x,y)$

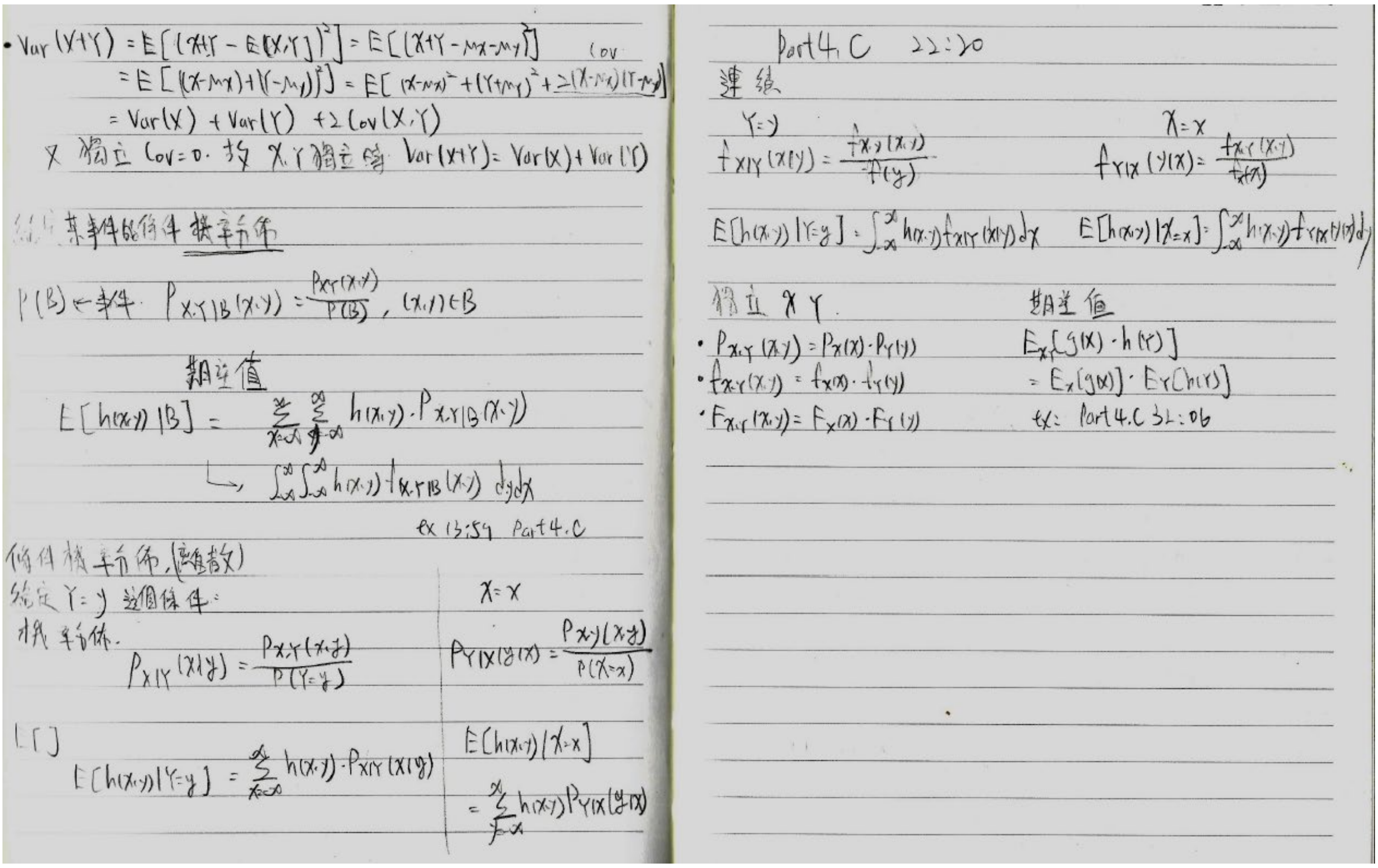
找 MPDF

$f_X(x)=\int_{-\infty}^{\infty}f_{X,Y}(x,y)\,dy$

$f_Y(y)=\int_{-\infty}^{\infty}f_{X,Y}(x,y)\,dx$

Joint 系列特性

- $0 \le F_{XY}(x,y) \le 1$
- $f_X(x) = \int_0^x f_{XY}(x,y)dy\,, 0 \le y \le x \le 1$
- $f_Y(y) = \int_y^1 f_{XY}(x,y)dx\,, 0 \le y \le x \le 1$
- $f_{X|Y}(X|Y) = \frac{f_{xy}}{f_y}$, $f_{XY} = f_X \times f_Y$當 XY獨立。
- $E\big(f_X(x)\big) = \int_0^1 x \int_0^x f_{xy}(x,y)dydx\,, 0 \le y \le x \le 1$



$\bullet\ Var(X+Y) = E[(X+Y - E[X+Y])^2] = E[(X+Y-\mu_x-\mu_y)^2]$ Cov

$= E[((X-\mu_x)+(Y-\mu_y))^2] = E[(X-\mu_x)^2+(Y-\mu_y)^2+\underline{2(X-\mu_x)(Y-\mu_y)}]$

$= Var(X) + Var(Y) + 2Cov(X,Y)$

X 獨立 Cov=0. 故 X,Y 獨立時 $Var(X+Y) = Var(X) + Var(Y)$

給定某事件的條件 機率分佈

$P(B) \leftarrow$ 事件. $P_{X,Y|B}(x,y) = \frac{P_{XY}(x,y)}{P(B)}$, $(x,y) \in B$

期望值

$E[h(x,y)|B] = \sum_{x=-\infty}^{\infty}\sum_{y=-\infty}^{\infty} h(x,y)\cdot P_{X,Y|B}(x,y)$

$\hookrightarrow \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} h(x,y) f_{X,Y|B}(x,y)\,dydx$

ex 13:59 Part4.C

條件機率分佈 (離散)

給定 $Y=y$ 這個條件：

機率分佈.

$P_{X|Y}(x|y) = \frac{P_{X,Y}(x,y)}{P(Y=y)}$

$X=x$

$P_{Y|X}(y|x) = \frac{P_{X,Y}(x,y)}{P(X=x)}$

E[ ]

$E[h(x,y)|Y=y] = \sum_{x=-\infty}^{\infty} h(x,y)\cdot P_{X|Y}(x|y)$

$E[h(x,y)|X=x] = \sum_{y=-\infty}^{\infty} h(x,y) P_{Y|X}(y|x)$

Part4.C 22:20

連續

$Y=y$

$f_{X|Y}(x|y) = \frac{f_{X,Y}(x,y)}{f(y)}$

$X=x$

$f_{Y|X}(y|x) = \frac{f_{X,Y}(x,y)}{f_X(x)}$

$E[h(x,y)|Y=y] = \int_{-\infty}^{\infty} h(x,y) f_{X|Y}(x|y)\,dx$ $E[h(x,y)|X=x] = \int_{-\infty}^{\infty} h(x,y) f_{Y|X}(y|x)\,dy$

獨立 X Y.

- $P_{X,Y}(x,y) = P_X(x)\cdot P_Y(y)$
- $f_{X,Y}(x,y) = f_X(x)\cdot f_Y(y)$
- $F_{X,Y}(x,y) = F_X(x)\cdot F_Y(y)$

期望值

$E_{X,Y}[g(X)\cdot h(Y)]$

$= E_X[g(X)]\cdot E_Y[h(Y)]$

ex: Part4.C 32:06

相關 Cov

- Correlation coefficient $\rho = \frac{Cov(X,Y)}{\sigma_x\sigma_y}$
- Covariance $Cov(X,Y) = E\big[E(X-\mu_x)(Y-\mu_y)\big] = E(XY) - E(X)E(Y)$
- 標準差 ： $\sigma_x = \sqrt{Var(x)}$
- $Var(x) = E(x^2) - E(x)^2$
- $E(x) = \sum_{-\infty}^{\infty} xP(x)$ or $\int_{-\infty}^{\infty} xf(x)dx$
- $E(x^n) = \sum_{-\infty}^{\infty} x^nP(x)$ or $\int_{-\infty}^{\infty} x^nf(x)dx$

Bivariate Normal Distribution

$$f(x) = \frac{1}{2\pi\sigma_x\sigma_y\sqrt{1-\rho^2}} e^{\frac{-1}{2}\left[\frac{1}{1-\rho^2}\left(\frac{(x-\mu_x)^2}{\sigma_x^2} - 2\rho\frac{x-\mu_x}{\sigma_x}\frac{y-\mu_y}{\sigma_y} + \frac{(y-\mu_y)^2}{\sigma_y^2}\right)\right]}$$

MGF:

MGF:

離散 ： $M_X(t) = E(e^{tX}) = \sum_x e^{tX} P_X(x)$

連續 ： $M_X(t) = E(e^{tX}) = \int_x e^{tX} f_X(x) dx$

分布	mgf
Poisson(λ)	$M_X(t)e^{-\lambda(1-e^{-t})}$
白努力**(p)**	$M_X(t) = 1 - p + pe^t$
二項式**(n,p)**	$M_X(t) = (1 - p + pe^t)^n$
Exponential(λ)	$M_X(t) = \frac{\lambda}{\lambda - t}$
高斯 **G(0,1)**	$M_X(t) = e^{\frac{t^2}{2}}$
高斯 **G(μ, σ^2)**	$M_X(t) = e^{\mu t + \frac{\sigma^2 t^2}{2}}$

定理:

$$E(x^n) = M_X(0)^n$$
$$Z = X_1 + \cdots + X_n, M_Z(t) = M_{X_1}(t) \times \ldots \times M_{X_n}(t)$$

若高斯分布的隨機變數$Y = 3X - 7$，可得$\sigma = 3, \mu = 7 (X = \frac{Y+7}{3})$

中央極限定理:

$X_1, X_2, \ldots, X_n$ ，若獨立，且有相同機率分布，當 n 接近∞，

$$X = X_1 + X_2 + \cdots + X_n \sim N\left(\mu_{X_1+X_2+\cdots+X_n}, \sigma^2_{X_1+X_2+\cdots+X_n}\right) = G(n\mu_{X_1}, n\sigma^2_{X1})$$

範例:

5.(10%) A fair die is rolled 200 times. What is the approximate probability that the sum of the outcomes is between 700 and 725? $\sim(\mu+6\sim\mu)$
<Hint> Find the mean and variance of rolling a die. Then, apply the central limit theorem.
<P.S.> You can write down your answer as a numerical value. Or, if you do not have the relevant table for carrying out the numerical computation, you can express your answer in terms of $\Phi(\cdot)$: the cummulative distribution function of the standard Gaussian random variable.

投骰子$E(X) = \frac{1+\cdots+6}{6} = 3.5, E(X^2) = \frac{1+2^2+\cdots+6^2}{6} = 15.17, Var(x) = E(X^2) - E(X)^2 = 2.92$

投很多次變常態分布，又$Var(x) = \sigma^2 = 2.92, \mu = 3.5$，大數法則:

$$G\left(n\mu_{X_1}, n\sigma^2_{X1}\right) = G(200 \times 3.5, 200 \times 2.92) = G(700, 584)$$

且找$P(700 < X < 725) = P(X = 725) - P(X = 700)$又高斯分布 cdf $F_X(X) = \phi(\frac{x-\mu}{\sigma})$所以可得:

$$P(700 < X < 725) = \phi\left(\frac{725 - 700}{\sqrt{584}}\right) - \phi\left(\frac{700 - 700}{\sqrt{584}}\right) = \phi(1.03) - \phi(0)$$

$E[X^n]$ 叫 moment

- MGF 和 moment 的關係，

$$\phi_X(s) = E[e^{sX}] = \int_{-\infty}^{\infty} e^{sx} f(x)\, dx \quad x \cdot e^{sx}$$

$$\phi_X'(s) = \frac{d}{ds}\int_{-\infty}^{\infty} e^{sx} f_X(x)\, dx = \int_{-\infty}^{\infty} \frac{de^{sx}}{ds} f_X(x)\, dx$$

$$= \int_{-\infty}^{\infty} x e^{sx} f_X(x)\, dx$$

$$\phi_X'(0) = \int_{-\infty}^{\infty} x \cdot 1 \cdot f_X(x)\, dx = E[X]$$

$$\phi_X^{(n)}(s)\big|_{s=0} = \int_{-\infty}^{\infty} x^n \cdot e^{sx} \cdot f_X(x)\, dx \Big|_{s=0} = \int_{-\infty}^{\infty} x^n f_X(x)\, dx = E[X^n]$$

離散分佈的 MGF．

- $X \sim$ Bernoulli(p): 白努力 $P_X(0) = 1-p$ $P_X(1) = p$

$$\phi_X(s) = E[e^{sX}] = e^{s\cdot 0} \cdot P(0) + e^{s\cdot 1} \cdot P(1)$$

$$= 1-p + e^s \cdot p = pe^s + 1 - p$$

- $X \sim$ Bin：二項 $X = X_1 + X_2 + \cdots$ 又獨立，都是白努力的和．

$$\phi_X(s) = (1 - p + pe^s)^n$$

$$\phi_X(s) = E[e^{sX}]$$

- Geometric(p) → 一次成功要幾次
- Pascal(k, p) → 成功 k 次要幾次
- Poisson(α):
- UNIF(a, b):

■ In the special case: $(X, Y) \sim \text{bi-Gauss}(\mu_1, \mu_2, \sigma_1^2, \sigma_2^2, \rho)$, the conditional mean turns out to be in a linear form:

$$\mu_2 + \rho \cdot \frac{\sigma_2}{\sigma_1} \cdot (x - \mu_1) \quad (\#17)$$

Bivariate Normal Distribution

$$f(x) = \frac{1}{2\pi\sigma_x\sigma_y\sqrt{1-\rho^2}} e^{\frac{-1}{2}\left[\frac{1}{1-\rho^2}\left(\frac{(x-\mu_x)^2}{\sigma_x^2} - 2\rho\frac{x-\mu_x}{\sigma_x}\frac{y-\mu_y}{\sigma_y} + \frac{(y-\mu_y)^2}{\sigma_y^2}\right)\right]}$$

Based on an observation on X, we try to estimate Y (i.e. produce a value that Y is likely to assume).

The estimator is expressed as $Y = g(X)$, where $g(\cdot)$ is some function.

According to the criterion of minimum mean squared error (MMSE), the goal is to minimize

$E\left(\left(Y - g(X)\right)^2 | X = x\right)$ (#15)

The solution to the minimization of Eq.(#15) is:

$g(X) = E(Y|X = x)$ (#16) #結論

Prf: $\text{LHS of (\#15)} = \int_{-\infty}^{\infty} \left(y - g(x)\right)^2 \cdot f_{Y|X}(y|x)\, dy$