20240914
Condensed matter physics: from magnetism to quantum spin liquid II

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https://github.com/ryuikaneko/lecturenote_2024_condmat

Rough outline

Hamiltonian for electrons in atoms

$$\mathcal{H} = -\sum_{j}^{N_e} \frac{\hbar^2}{2m}\nabla_j^2 - \sum_{\alpha}^{N_i} \frac{\hbar^2}{2M_\alpha}\nabla_\alpha^2 - \sum_{j}^{N_e}\sum_{\alpha}^{N_i} \frac{Z_\alpha e^2}{|\vec{r}_j - \vec{R}_\alpha|} + \sum_{j \ll k}^{N_e} \frac{e^2}{|\vec{r}_j - \vec{r}_k|} + \sum_{\alpha \ll \beta}^{N_j} \frac{Z_\alpha Z_\beta e^2}{|\vec{R}_\alpha - \vec{r}_\beta|}$$

Hubbard model

$$\hat{H} = -\sum_{ij\sigma} t_{ij}\, \hat{c}_{i\sigma}^{+}\, \hat{c}_{j\sigma} + \sum_i U_i\, \hat{n}_{i\uparrow}\, \hat{n}_{i\downarrow}$$

Heisenberg model

$$\hat{H} = \sum_{ij} J_{ij}\, \hat{S}_i \cdot \hat{S}_j$$

magnetism, quantum spin liquid, ...

Outline

- Brief review of itinerant electron systems
  - Electrons in crystals
  - Second quantization
  - Hubbard models
  - Solving models for simple cases
    - Noninteracting and atomic limits
    - Mean-field approximation
    - Exact diagonalization (2 sites)

      → Strong coupling limit: Heisenberg model (spin system)
- Magnetism and quantum spin liquid
  - Spin models
  - Frustrated magnetism and quantum spin liquid
  - Kitaev honeycomb spin liquid

Spin models

- Heisenberg model

$$H = \sum_{ij} J_{ij} \boldsymbol{S}_i \cdot \boldsymbol{S}_j = \sum_{ij} J_{ij}(S_i^x S_j^x + S_i^y S_j^y + S_i^z S_j^z)$$

- XXZ model

$$H = \sum_{ij} [J_{ij}^{xy}(S_i^x S_j^x + S_i^y S_j^y) + J_{ij}^z S_i^z S_j^z)]$$

- XY model (J_{ij}^z=0)

$$H = \sum_{ij} J_{ij}^{xy}(S_i^x S_j^x + S_i^y S_j^y)$$

- Classical Ising model (J_{ij}^{xy}=0)

$$H = \sum_{ij} J_{ij}^z S_i^z S_j^z$$

- Transverse-field Ising model

$$H = \sum_{ij} J_{ij} S_i^z S_j^z - \sum_i \Gamma_i S_i^x$$

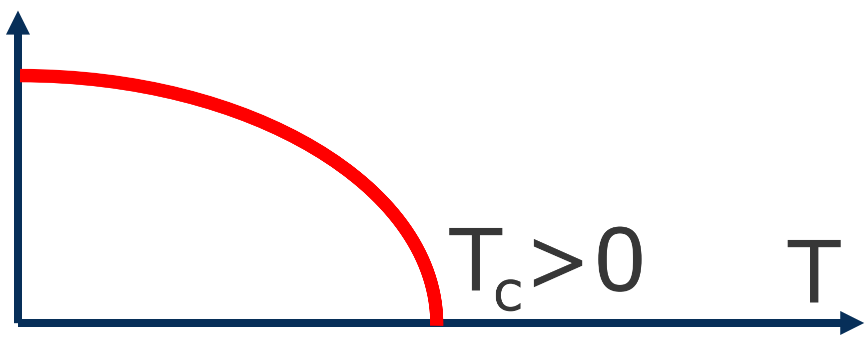
Mermin-Wagner theorem

- Continuous symmetries cannot be spontaneously broken at finite temperature in systems with sufficiently short-range interactions in dimensions D<=2.

N. D. Mermin and H. Wagner, PRL.17.1133 ('66)

Spatial dimension	Tc>0?
1D	x
2D	x
3D	✓
4D	✓
…	…

Magnetic order



Mermin-Wagner theorem

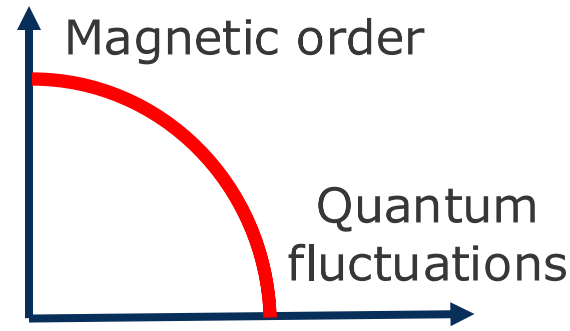
- Continuous symmetries cannot be spontaneously broken at finite temperature in systems with sufficiently short-range interactions in dimensions D<=2. N. D. Mermin and H. Wagner, PRL.17.1133 ('66)

- Note 1: Of course, discrete symmetries can be spontaneously broken even in D=2 (e.g., Tc>0 in 2D Ising models).

- Note 2: MW theorem does not state anything about the topological phase transition (e.g., Berezinskii-Kosterlitz-Thouless (BKT) transition occurs in systems with U(1) symmetry in 2D).

“Mermin-Wagner theorem” at T=0

- Quantum fluctuations in low spatial dimensions sometimes behave like thermal fluctuations
- Consider
  - Continuous spin systems on lattices
  - Interaction J_{ij} is short-range
  - $[H,O]\neq 0$ (O: operator of order parameter)



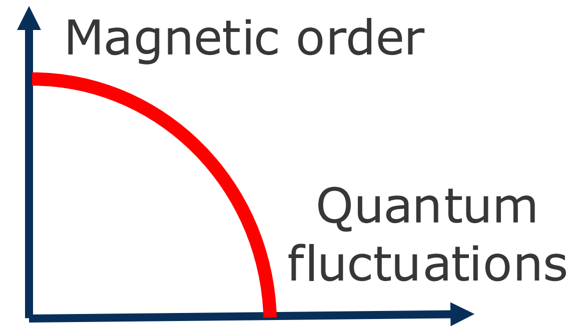
- Then,

Spatial dimension	Magnetic Long-range order at T=0?
1D	x
2D	✓
3D	✓
…	…

L. Pitaevskii and S. Stringari, JLowTempPhys.85.377(’91)

“Mermin-Wagner theorem” at T=0

- Quantum fluctuations in low spatial dimensions sometimes behave like thermal fluctuations
- Consider
  - Continuous spin systems on lattices
  - Interaction J_{ij} is short-range
  - Hamiltonian is not frustration-free



(c.f. For more rigorous argument, see H. Watanabe et al., arXiv:2310.16881)

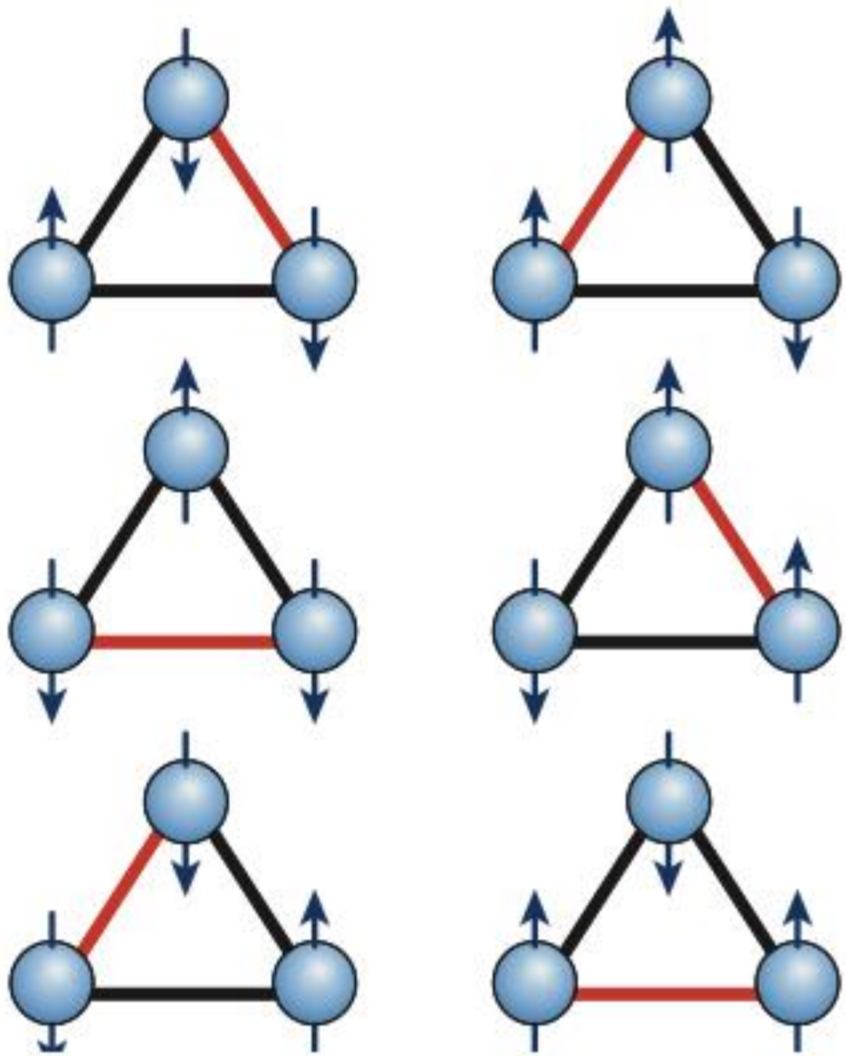
- Then,

Spatial dimension	Magnetic Long-range order at T=0?
1D	x
2D	✓
3D	✓
…	…

L. Pitaevskii and S. Stringari, JLowTempPhys.85.377(’91)

No long-range order even in 2D, 3D at T=0?

- Example 1: Classical triangular Ising model



- Consider antiferromagnetic (AF) interaction J>0
- For 3 sites, degenerate ground states

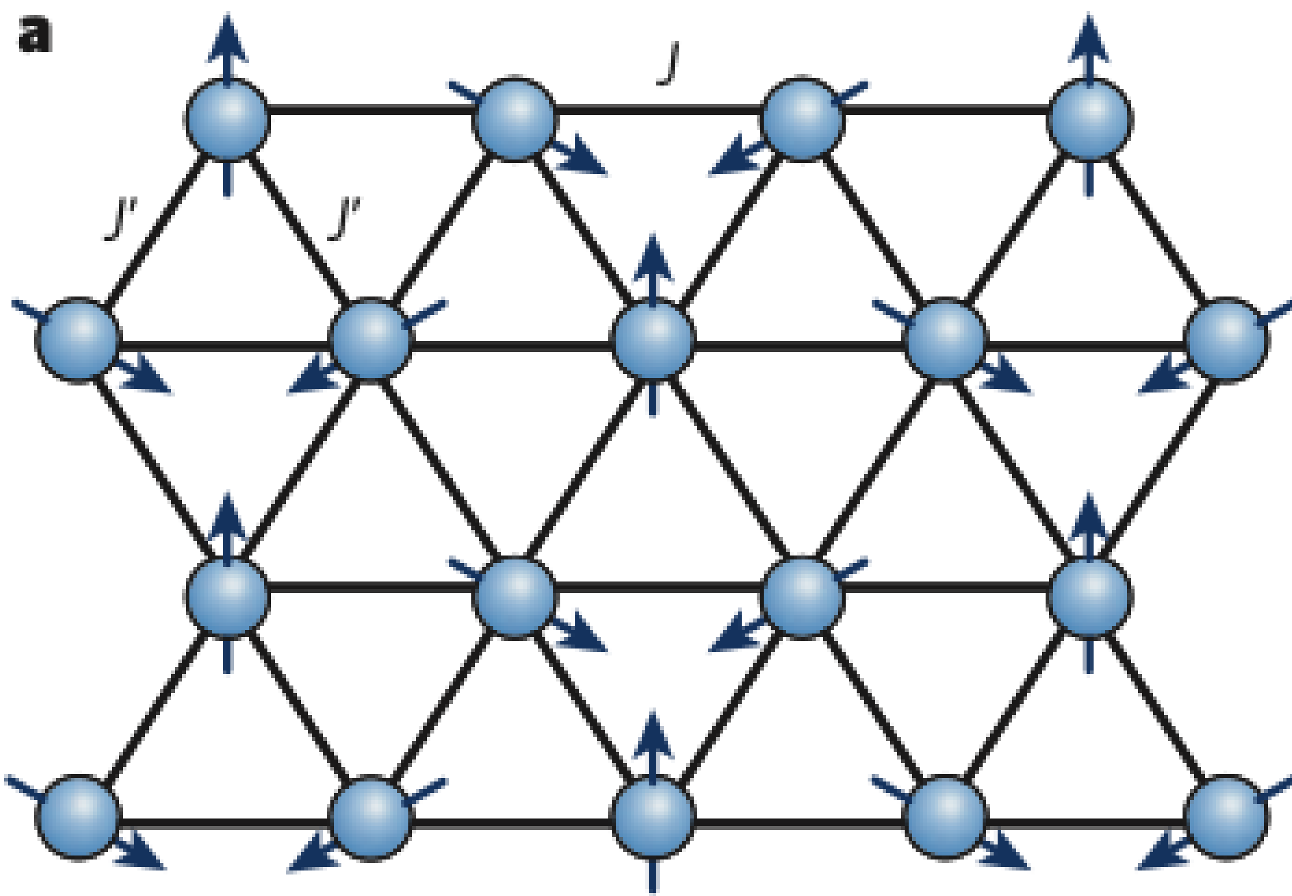
- For infinite sites, spin correlation shows power-law decay at T=0 $\langle S_0 S_r \rangle \sim r^{-\eta}$ ($\eta=1/2$)
- But no long-range order $\Sigma_i \exp(iQr_i)\langle S_i \rangle=0$ ($Q=(2\pi/3,2\pi/3)$)

J. Stephenson, J.Math.Phys.11.413 ('70);
S. Alexander and P. Pincus, J.Phys.A:Math.Gen.13.263 ('80)

Geometrical frustration

No long-range order even in 2D, 3D at T=0?

- Example 2: Classical triangular Heisenberg model (ordered case)



120 Neel order at T=0

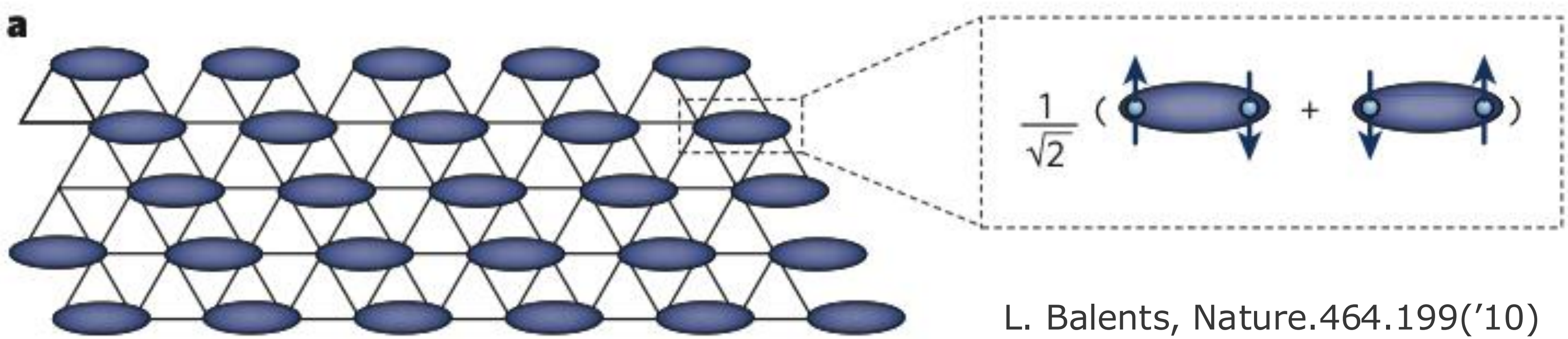
No long-range order even in 2D, 3D at T=0?

- In quantum spin 1/2 triangular Heisenberg model
  - Ground state has no magnetic order?!

    P. W. Anderson

    “Resonating valence bonds: a new kind of insulator”

    Mater. Res. Bull. 8, 153–160 (1973)

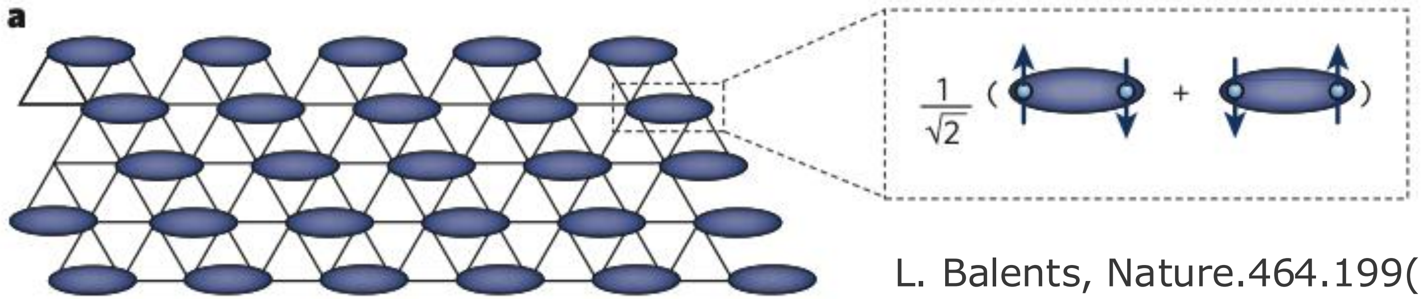


L. Balents, Nature.464.199(’10)

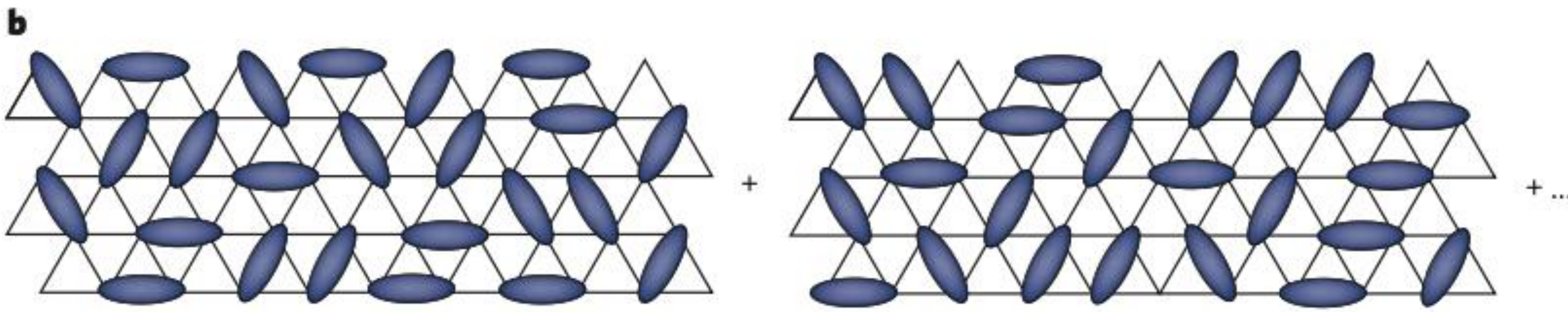
Example of a singlet dimerized state

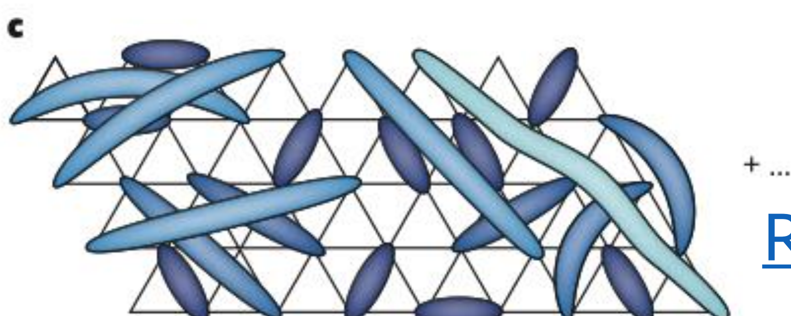
No long-range order even in 2D, 3D at T=0?

- In quantum spin 1/2 triangular Heisenberg model



L. Balents, Nature.464.199('10)





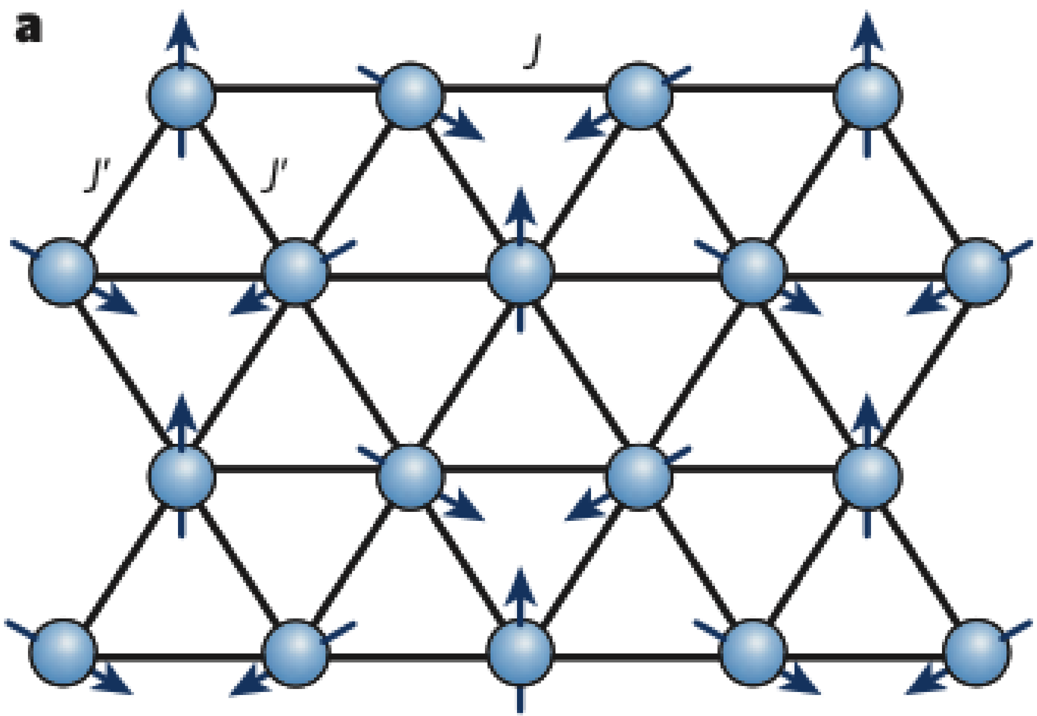
Dimer order is also absent?
Dimer dynamically moves?

Resonating valence bond state
(RVB state)

No long-range order even in 2D, 3D at T=0?

- In quantum spin 1/2 triangular Heisenberg model

- Unfortunately, it is now widely believed that the true ground state has long-range magnetic order

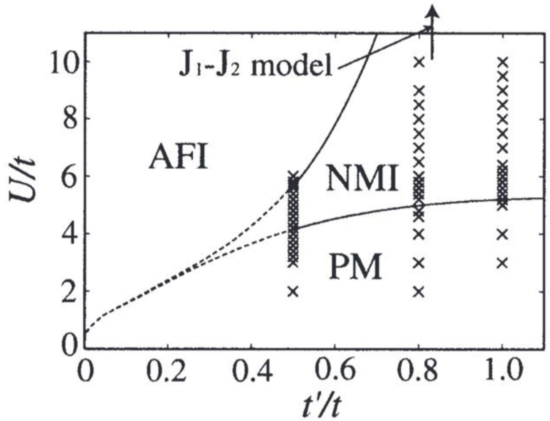


- Exact diagonalization
  - B. Bernu et al., PRB.50.10048('94)
- Variational Monte Carlo (VMC)
  - L. Capriotti et al., PRL.82.3899 ('99)
- Density matrix renormalization group (DMRG)
  - S. White and A. L. Chernyshev, PRL.99.127004('07)

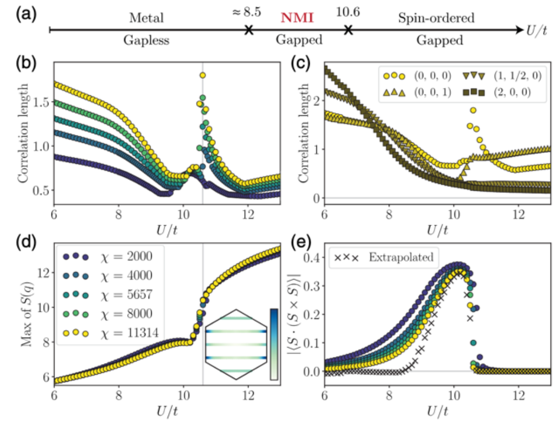
- $\Sigma_i \exp(iQr_i)\langle S_i\rangle/S_{max}=0.41(3)$ $(Q=(2\pi/3,2\pi/3),\ S_{max}=1/2)$

To realize quantum spin liquid…

- Effects of charge fluctuations

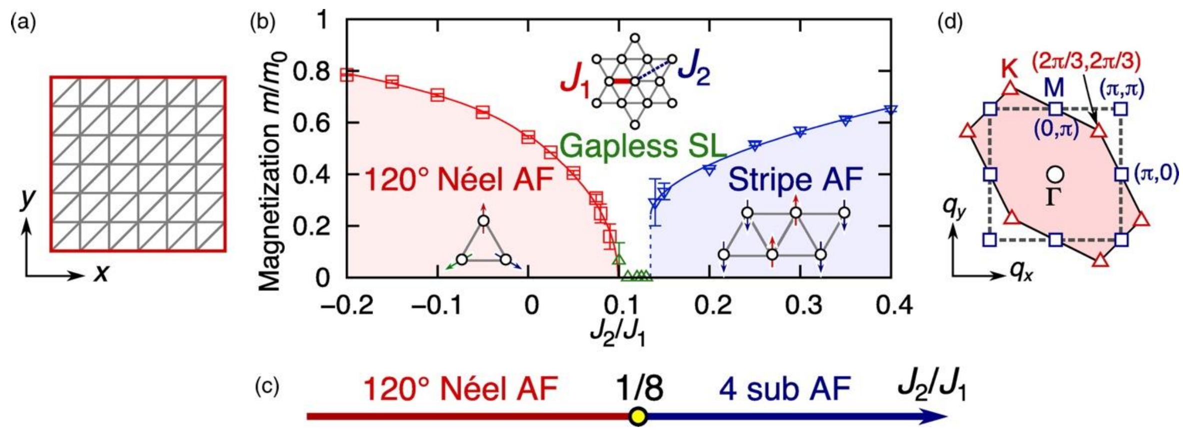


H. Morita et al., JPSJ.71.2109 (’02)



A. Szasz et al., PRX.10.021042 (’20)

- Effects of next-nearest-neighbor interaction



R. Kaneko et al., JPSJ.83.093707 (’14)

- And so on…

Quantum spin liquid (QSL)

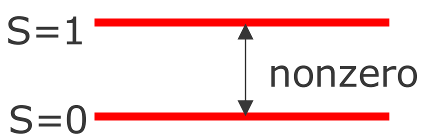
- No long-range order of spin, dimer, …

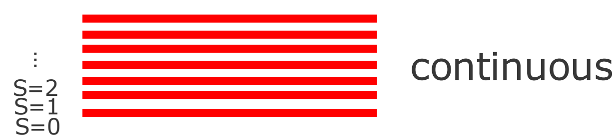
- Insulating (charge excitation: open)
  - Metal shows no order, but it is not quantum spin liquid

- Characterized by spin excitation (for $N_s \rightarrow \infty$)
  - Open: Gapped spin liquid
    - Often characterized by topological entanglement entropy

    A. kitaev and J. Preskill, PRL.96.110404 ('06)
    M. Levin and X.-G. Wen, PRL.96.110405 ('06)
  - Closed: Gapless spin liquid
    - Sometimes characterized by spinon pairing symmetry (just as in the case of pairing symmetry in superconductors)

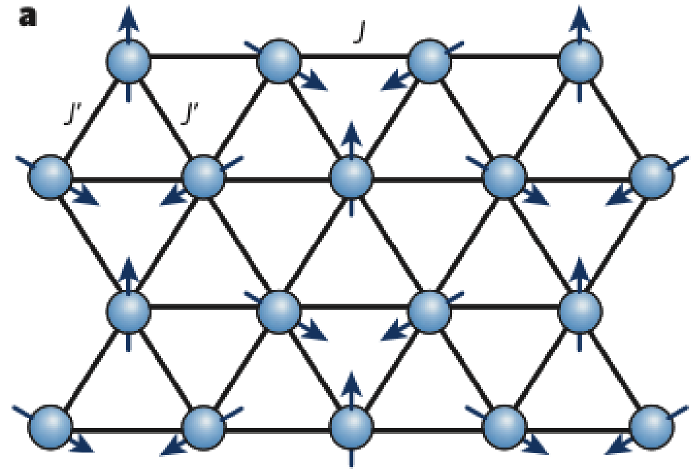
    Y. Zhang et al., PRL.107.067202('11)



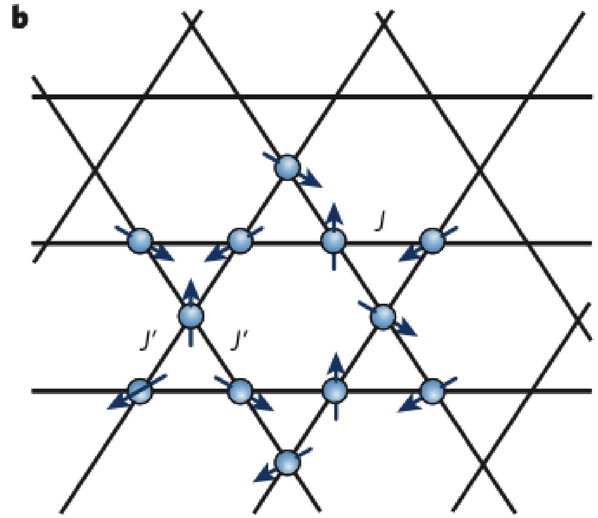


Example? Candidate materials?

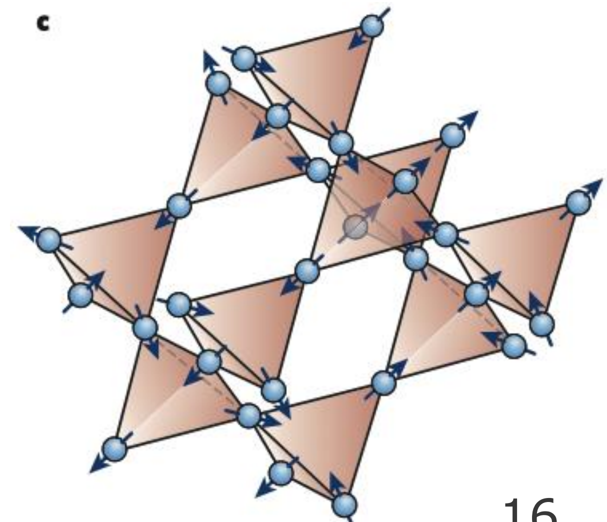
- Triangular lattice
  - Organic compounds
    - κ-$(BEDTTTF)_2Cu_2(CN)_3$
    - $EtMe_3Sb[Pd(dmit)_2]_2$
  - Antiferromagnets
    - $YbMgGaO_4$



- Kagome lattice
  - Antiferromagnets
    - Herbertsmithite
    - Volborthite
    - Vesignieite
    - Kapellasite



- Pyrochlore lattice
  - Antiferromagnets
    - $Tb_2Hf_2O_7$, $Tb_2Hf_2O_7$, $Ce_2Zr_2O_7$, …



Why do we study QSL?

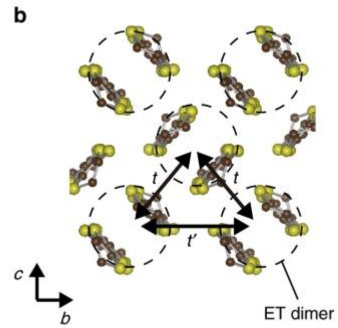
- Related to superconductivity (SC)
  - Doping QSL → High T_c SC?

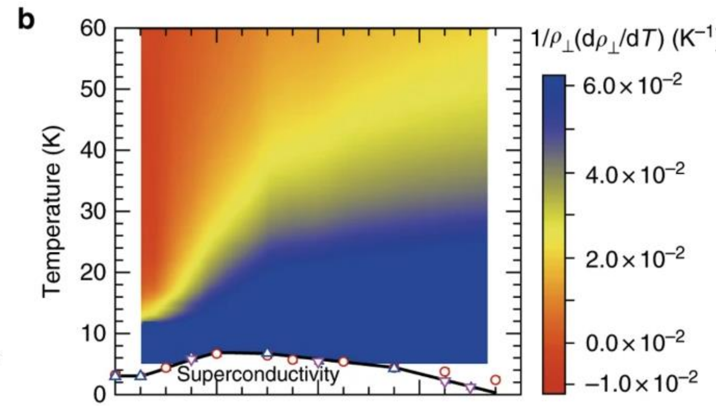
Article | Open access | Published: 02 October 2017

Anomalous metallic behaviour in the doped spin liquid candidate κ-$(ET)_4Hg_{2.89}Br_8$

Hiroshi Oike, Yuji Suzuki, Hiromi Taniguchi, Yasuhide Seki, Kazuya Miyagawa & Kazushi Kanoda

Nature Communications **8**, Article number: 756 (2017) | Cite this article





- Quantum computation
  - Using Majorana fermion in Kitaev spin liquid

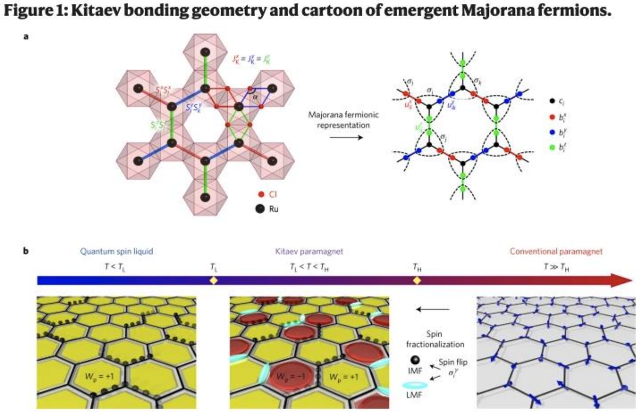
Letter | Published: 18 September 2017

Majorana fermions in the Kitaev quantum spin system α-$RuCl_3$

Seung-Hwan Do, Sang-Youn Park, Junki Yoshitake, Joji Nasu, Yukitoshi Motome, Yong Seung Kwon, D. T. Adroja, D. J. Voneshen, Kyoo Kim, T.-H. Jang, J.-H. Park, Kwang-Yong Choi & Sungdae Ji

Nature Physics **13**, 1079–1084 (2017) | Cite this article

Figure 1: Kitaev bonding geometry and cartoon of emergent Majorana fermions.



- (Just for fun)
  - Unconventional quasiparticles, statistics, …

Difficulty in searching for QSL theoretically

- However, no analytical solution
  - Exception 1: “QSL” in 1D (Tomonaga-Luttinger liquid)
  - Exception 2: Kitaev QSL in 2D

- Mean-field approximation is not applicable
  - Any order parameter is 0

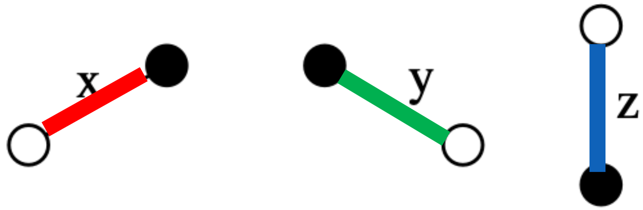
- No universal numerical methods
  - Exact diagonalization: Size too small
  - Quantum Monte Carlo method: Negative sign problem

- We often use variational Monte Carlo and Tensor-network methods

Kitaev QSL (rare example of solvable QSL)

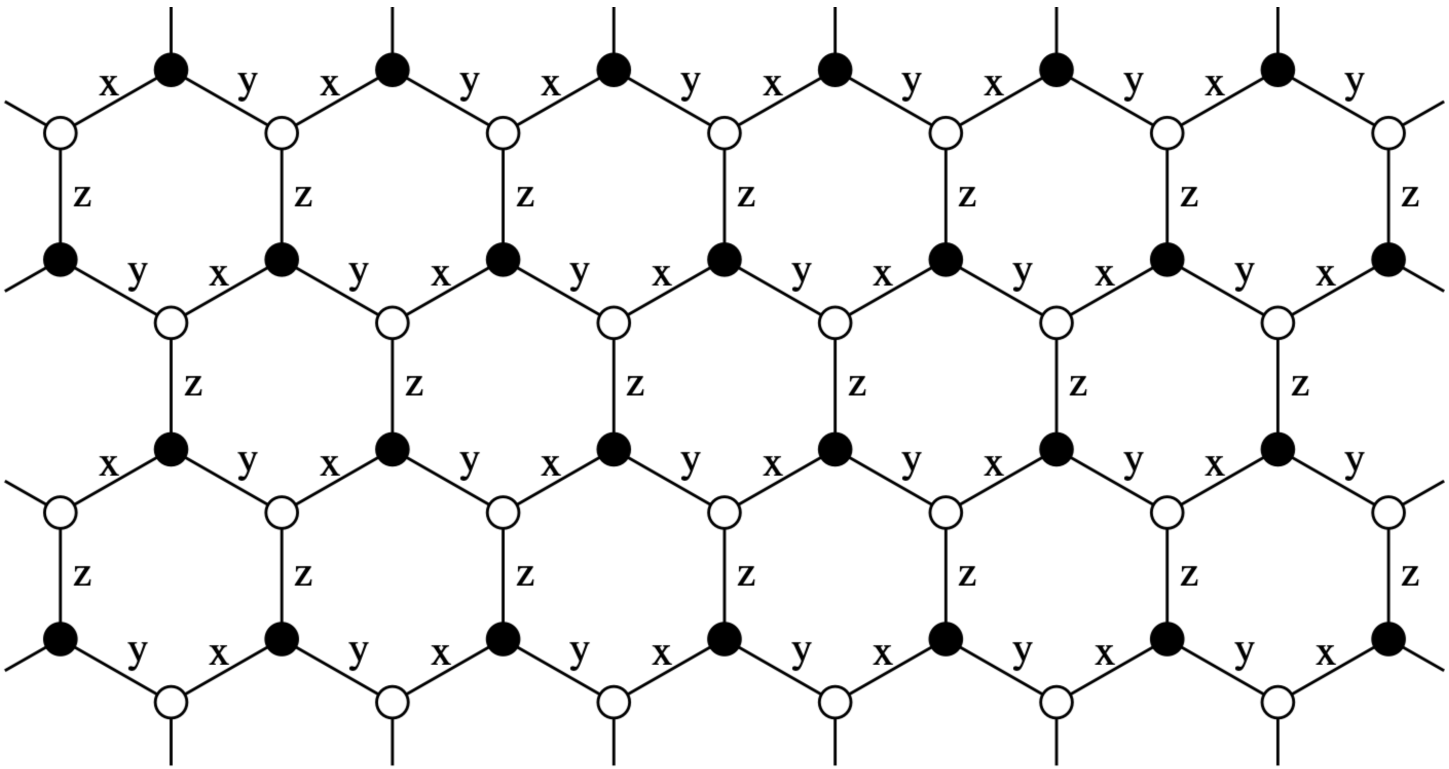
- Model

A. kitaev,
AnnPhys.321.2 ('06)



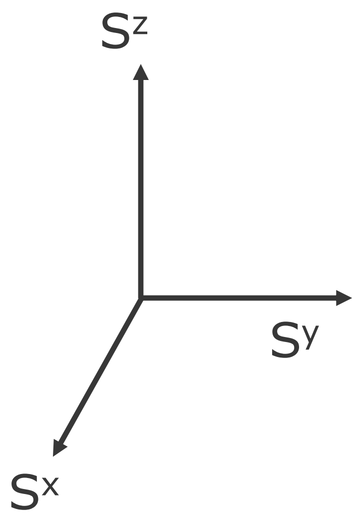
Bond-dependent
Ising interaction

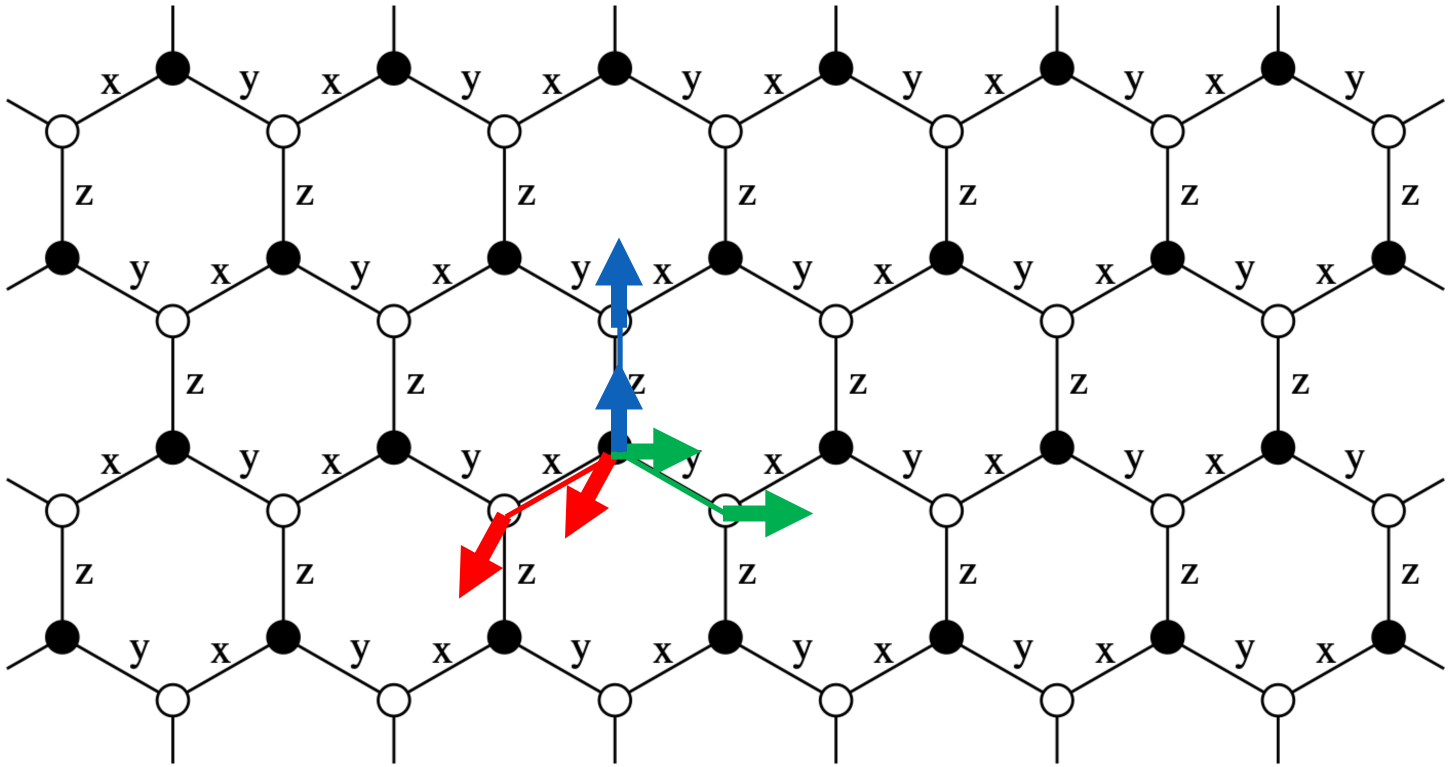
$$H = -\underline{J_x} \sum_{x\text{-links}} \sigma_j^x \sigma_k^x - \underline{J_y} \sum_{y\text{-links}} \sigma_j^y \sigma_k^y - \underline{J_z} \sum_{z\text{-links}} \sigma_j^z \sigma_k^z$$



Kitaev QSL (rare example of solvable QSL)

- Intuitive understanding (bond frustration)



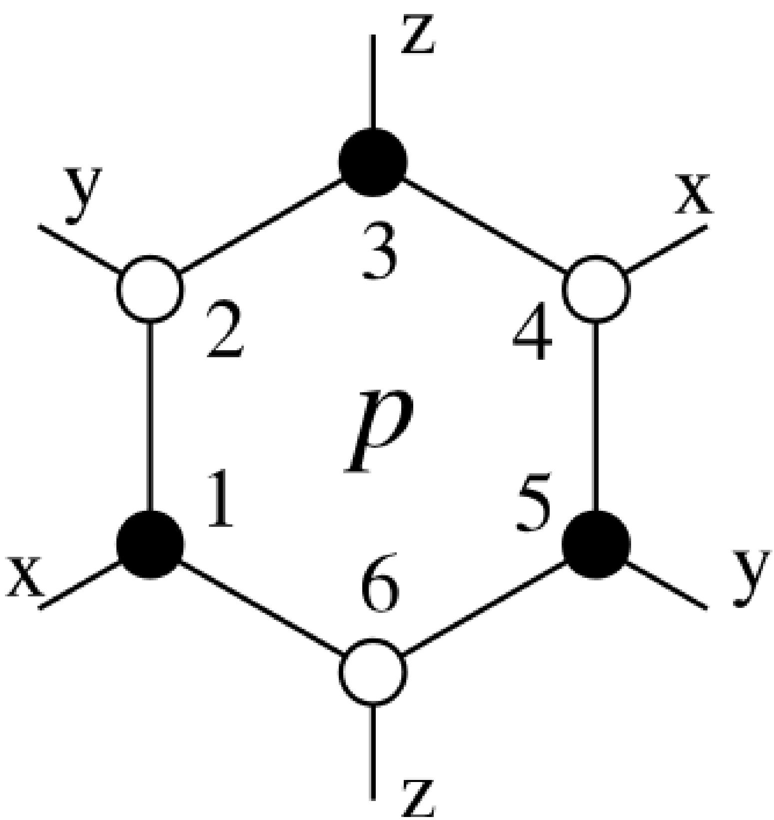


Kitaev QSL (rare example of solvable QSL)

- Common way of solving the problem
  - Using conserved quantities,
    extending the Hilbert space (original way)
    [A. Kitaev, AnnPhys.321.2('06)]
  - Apply Jordan-Wigner transformation
    [X.-Y. Feng et al., PRL.98.087204('07); H.-D. Chen and J. Hu, PRB.76.193101('07); H.-D. Chen and Z. Nussinov, JPhysA:MathTheor.41.075001('08); S. Mandal et al., JPhysA:MathTheor.45.335304('12)]
  - Similar to the original way,
    but without extending the Hilbert space
    [J. Fu et al., PRB.97.115142 ('18)]
- Focus on the original way

Kitaev QSL (rare example of solvable QSL)

- Conserved quantities



$$W_p = \sigma_1^x \sigma_2^y \sigma_3^z \sigma_4^x \sigma_5^y \sigma_6^z$$

After some tedious calculations, we obtain

- $[H, W_p] = 0$ for all p
- $[W_p, W_{p'}] = 0$ for all p, p'
- $W_p^2 = 1 \rightarrow W_p = +1$ or -1

Describe eigenstates of H using eigenstates of W_p

Kitaev QSL (rare example of solvable QSL)

- Enlarge Hilbert space using Majorana fermions ($c=c^\dagger$)

Original spin basis → Enlarged Hilbert space → After projection

$D=(2c_1^\dagger c_1-1)(2c_2^\dagger c_2-1)$
$P=(1+D)/2$

Original spin basis	Enlarged Hilbert space		After projection
$\|\uparrow\rangle$	$\|00\rangle$		$\|00\rangle$
	$\|01\rangle$	Unphysical	$D=(2*0-1)(2*1-1)=-1$ $P=0$ Disappear
	$\|10\rangle$	Unphysical	$D=(2*1-1)(2*0-1)=-1$ $P=0$ Disappear
$\|\downarrow\rangle$	$\|11\rangle$		$\|11\rangle$

$$\tilde{\sigma}^x = ib^x c, \quad \tilde{\sigma}^y = ib^y c, \quad \tilde{\sigma}^z = ib^z c$$

$\sigma^x, \sigma^y, \sigma^z$ $\qquad c=c_1^\dagger+c_1,\ b^x=i(c_1^\dagger-c_1),\ b^y=c_2^\dagger+c_2,\ b^z=i(c_2^\dagger-c_2)$

Solve the problem in the enlarged Hilbert space
and finally apply the projection to obtain the physical state

Kitaev QSL (rare example of solvable QSL)

- Enlarge Hilbert space using Majorana fermions

$$H = -J_x \sum_{x\text{-links}} \sigma_j^x \sigma_k^x - J_y \sum_{y\text{-links}} \sigma_j^y \sigma_k^y - J_z \sum_{z\text{-links}} \sigma_j^z \sigma_k^z$$

$$\sigma_j^x \mapsto -\mathrm{i} b_j^y b_j^z, \quad \sigma_j^y \mapsto -\mathrm{i} b_j^z b_j^x, \quad \sigma_j^z \mapsto -\mathrm{i} b_j^x b_j^y$$

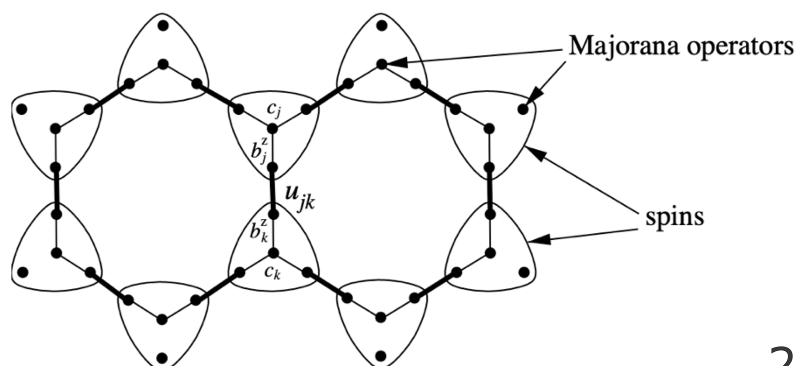
$$(\mathrm{i} b_j^\alpha c_j)(\mathrm{i} b_k^\alpha c_k) = -\mathrm{i}(\mathrm{i} b_j^\alpha b_k^\alpha) c_j c_k$$

$$\widetilde{H} = \frac{\mathrm{i}}{4} \sum_{j,k} \hat{A}_{jk} c_j c_k, \qquad \hat{A}_{jk} = \begin{cases} 2J_{\alpha_{jk}} \hat{u}_{jk} & \text{if } j \text{ and } k \text{ are connected,} \\ 0 & \text{otherwise,} \end{cases}$$

Quadratic-“like” Hamiltonian

$$\hat{u}_{jk} = \mathrm{i} b_j^{\alpha_{jk}} b_k^{\alpha_{jk}}.$$

Looks not free because of u_{jk}



Kitaev QSL (rare example of solvable QSL)

- It IS actually a quadratic Hamiltonian

After some tedious calculations, we obtain

- $u_{jk} = -u_{kj}$
- $[H, u_{jk}] = 0$ for all j, k
- $[u_{jk}, u_{j'k'}] = 0$ for all j, k, j', k'
- $u_{jk}^2 = 1 \rightarrow u_{jk} = +1$ or -1

$$w_p = \prod_{(j,k)\in \mathrm{boundary}(p)} u_{jk}$$

$(j \in \text{even sublattice}, k \in \text{odd sublattice})$

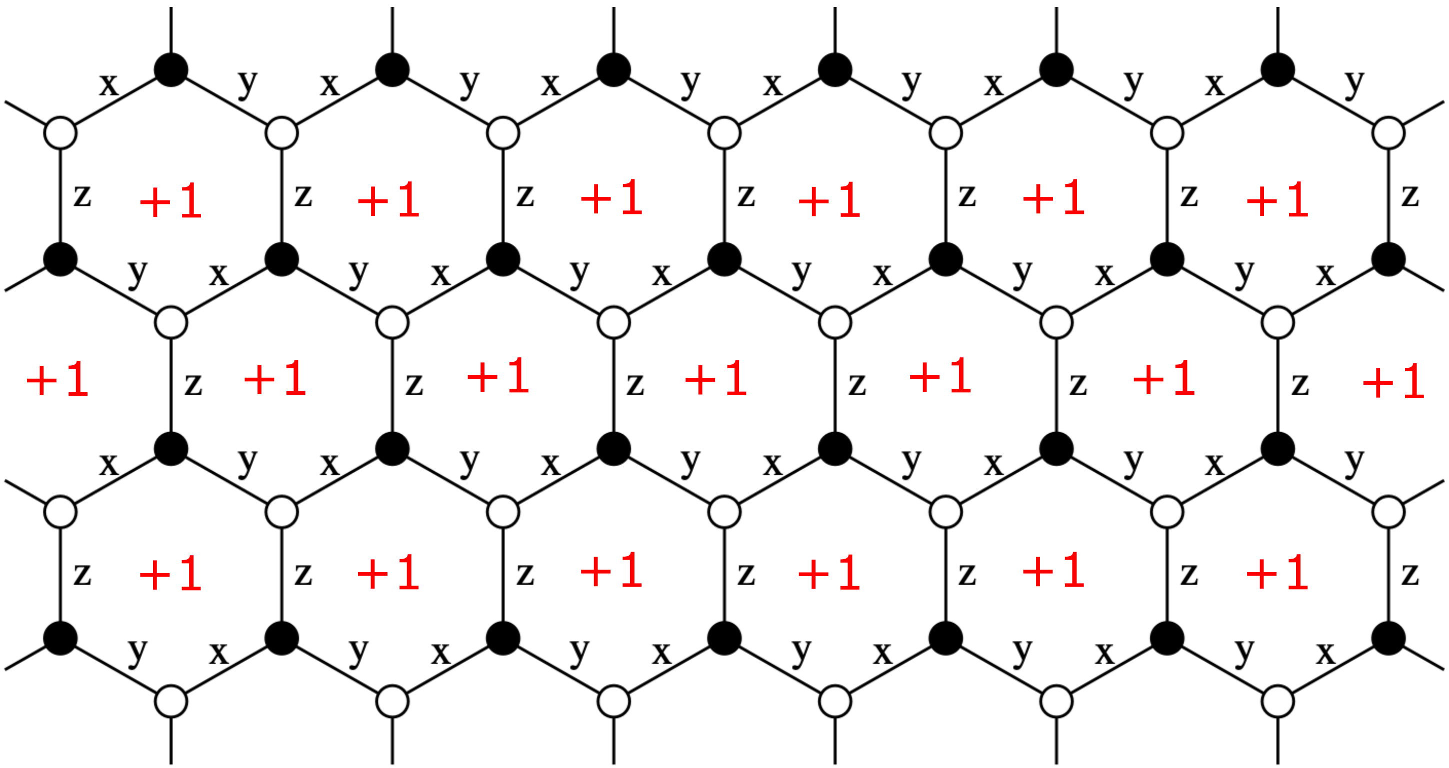
The role of u_{jk} and W_p is the same

$$\widetilde{H} = \frac{i}{4} \sum_{j,k} \hat{A}_{jk} c_j c_k$$

Quadratic Hamiltonian because u_{jk} is just a number

Kitaev QSL (rare example of solvable QSL)

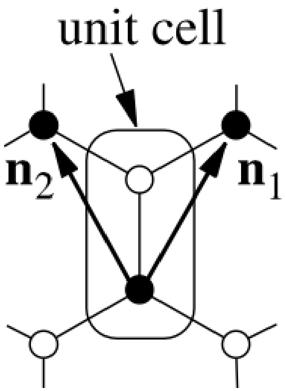
- Configuration of u_{jk}?
  - According to Lieb's theorem, [E. H. Lieb, PRL.73.2158 ('94)]
    $u_{jk}=+1$ ($W_p=+1$) for all j, k (p) in the ground state
  - Often called "flux free" or "Vortex free"



Kitaev QSL (rare example of solvable QSL)

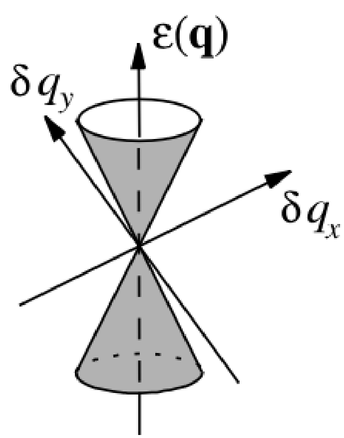
- Spectrum of the quadratic Hamiltonian

$$H_{\text{vortex-free}} = \frac{i}{4}\sum_{j,k} A_{jk} c_j c_k, \quad A_{jk} = 2J_{\alpha_{jk}} u_{jk}^{\text{std}}$$



$$i\widetilde{A}(\mathbf{q}) = \begin{pmatrix} 0 & if(\mathbf{q}) \\ -if(\mathbf{q})^* & 0 \end{pmatrix}, \quad \varepsilon(\mathbf{q}) = \pm|f(\mathbf{q})|,$$

$$f(\mathbf{q}) = 2\left(J_x e^{i(\mathbf{q},\mathbf{n}_1)} + J_y e^{i(\mathbf{q},\mathbf{n}_2)} + J_z\right), \quad \mathbf{n}_1 = \left(\tfrac{1}{2}, \tfrac{\sqrt{3}}{2}\right),\ \mathbf{n}_2 = \left(-\tfrac{1}{2}, \tfrac{\sqrt{3}}{2}\right)$$



When $J_x \sim J_y \sim J_z$,
Dirac cone dispersion appears

Kitaev QSL (rare example of solvable QSL)

- Reduce enlarged Hilbert space:
  Project out unphysical states

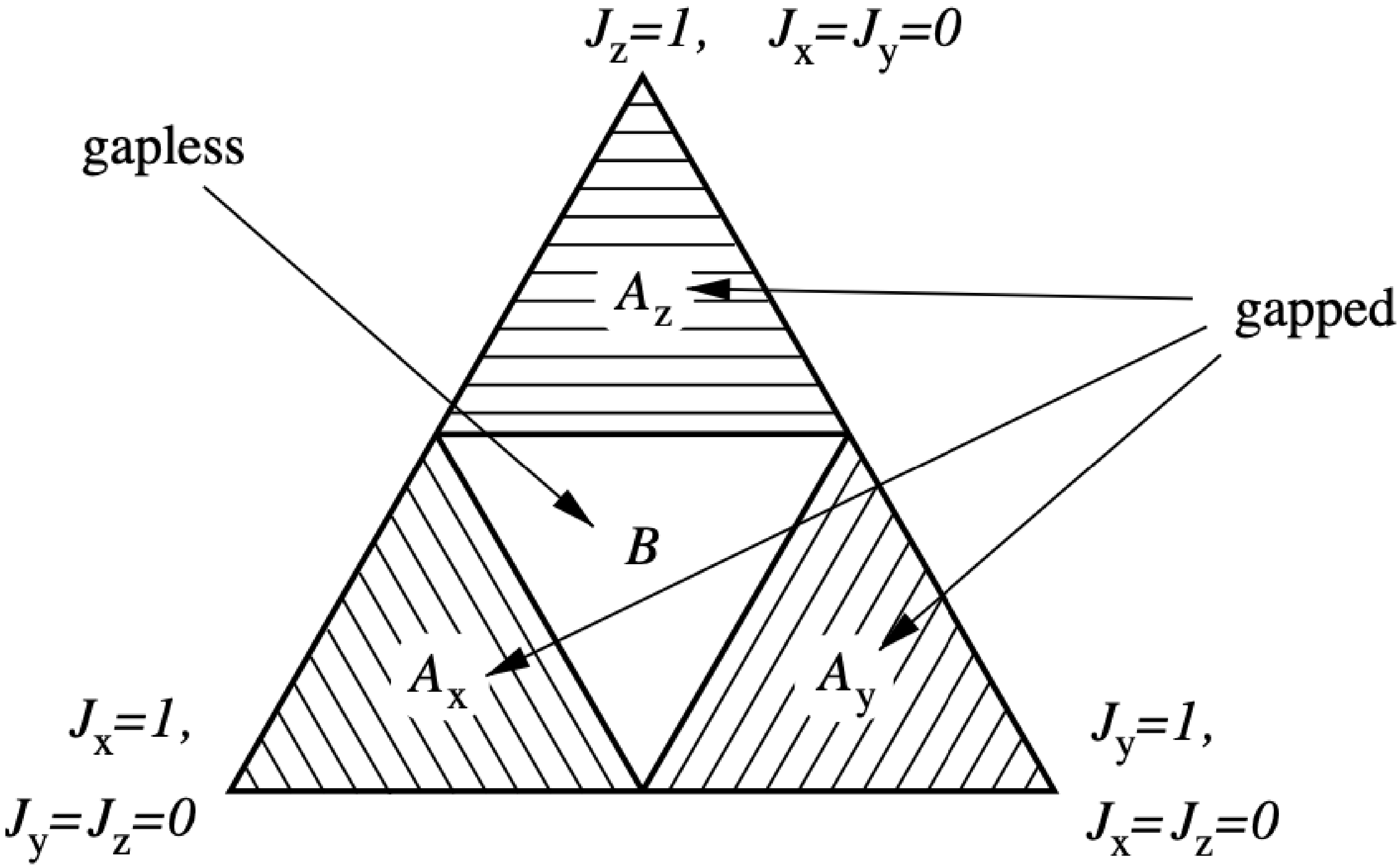
- Projection operator commutes with Hamiltonian!!
  - P=(1+D)/2
  - [H,P]=0

- As for the energy spectrum, we do not need to care about the projection

- As for the eigenstate, $|\Psi_w\rangle = \prod_j \left(\frac{1+D_j}{2}\right)|\widetilde{\Psi}_u\rangle$

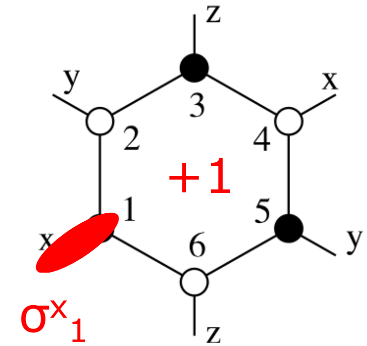
Kitaev QSL (rare example of solvable QSL)

- Ground state phase diagram

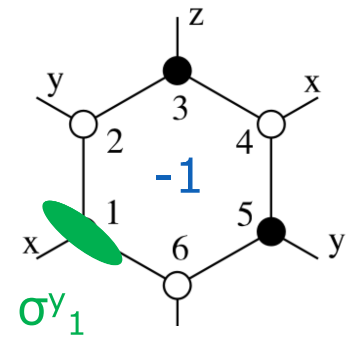


Kitaev QSL (rare example of solvable QSL)

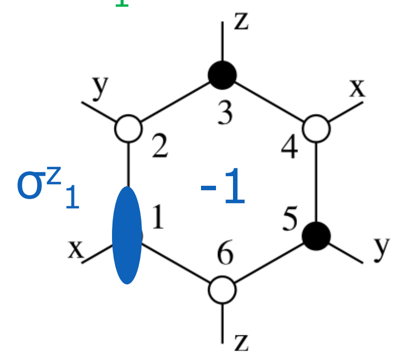
- Short-range spin correlation
- Applying σ^a flips the flux



  - $W_p \sigma^x{}_1 = +\sigma^x{}_1 W_p$ (commute)



  - $W_p \sigma^y{}_1 = -\sigma^y{}_1 W_p$ (anticommute)

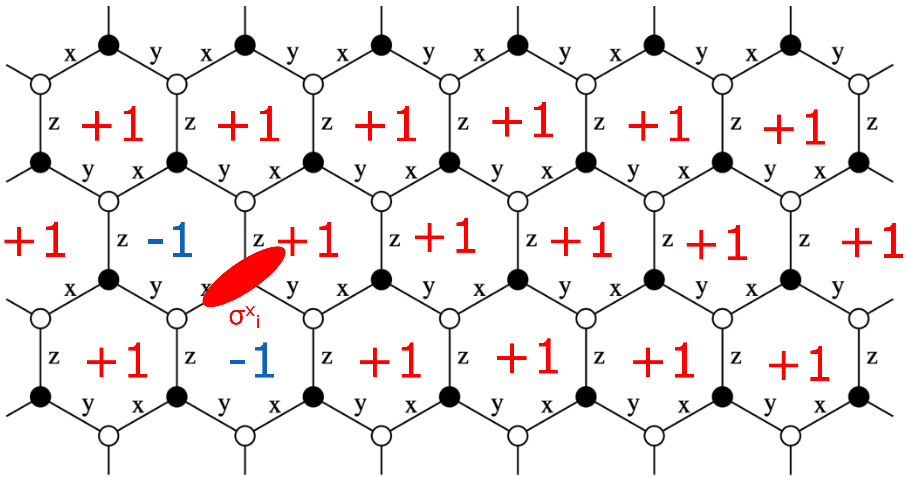


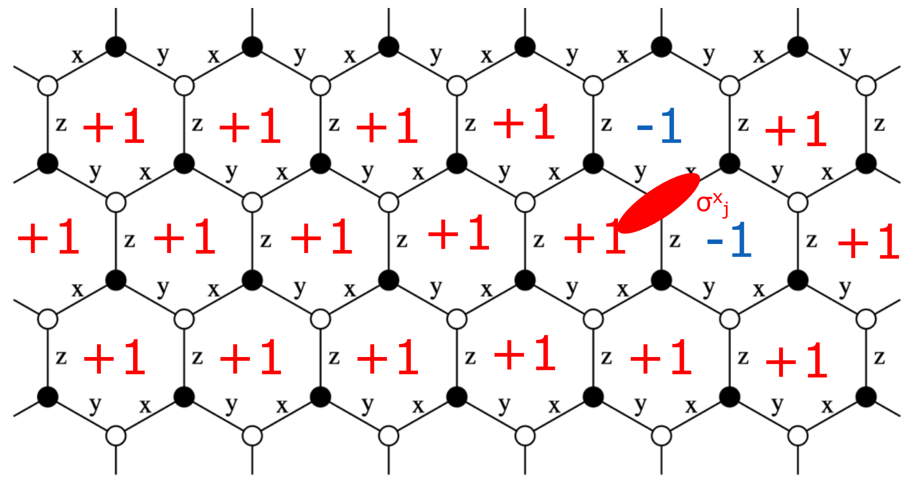
  - $W_p \sigma^z{}_1 = -\sigma^z{}_1 W_p$ (anticommute)
  - $W_p \sigma^y{}_1 |\text{eigenstate}(+1)\rangle = -\sigma^y{}_1 W_p |\text{eigenstate}(+1)\rangle = -\sigma^y{}_1 |\text{eigenstate}(+1)\rangle$

Kitaev QSL (rare example of solvable QSL)

- Short-range spin correlation

$|\psi_i\rangle = \sigma^x_i|\psi\rangle$ ← Different flux sector
Orthogonal each other → $|\psi_j\rangle = \sigma^x_j|\psi\rangle$





$$\langle\psi_i|\psi_j\rangle = 0 \rightarrow \langle\psi|\sigma^x_i\,\sigma^x_j|\psi\rangle = 0$$

Spin correlation is 0 except i and j on the same bond
for all eigenstates $|\psi\rangle$

Short-range QSL for any eigenstates

[G. Baskaran et al., PRL.98.247201 ('07)]

Kitaev QSL (rare example of solvable QSL)

- Relation to superconductivity

[F. J. Burnell and C. Nayak, PRB.84.125125('11); S. Mandal et al., JPhysA:MathTheor.45.335304('12)]

- Solution by Jordan-Wigner transformation on a torus
- p-wave superconductor (but with projection)

$$H = \sum_k (\epsilon_k \psi_k^\dagger \psi_k - \epsilon_k \psi_{-k} \psi_{-k}^\dagger + \mathrm{i}\delta_k \psi_k^\dagger \psi_{-k}^\dagger - \mathrm{i}\delta_k \psi_{-k}\psi_k)$$
$$+ \epsilon_{0,0}\psi_{0,0}^\dagger \psi_{0,0} + \epsilon_{\pi,0}\psi_{\pi,0}^\dagger \psi_{\pi,0} + \epsilon_{0,\pi}\psi_{0,\pi}^\dagger \psi_{0,\pi} + \sum_k \epsilon_k - MNJ_z$$

$$\epsilon_k = 2(J_x \cos k_x + J_y \cos k_y + J_z)$$
$$\boxed{\delta_k = 2(J_x \sin k_x + J_y \sin k_y)}$$
$$E_k = \sqrt{\epsilon_k^2 + \delta_k^2} \quad \cos 2\theta_k = \epsilon_k / E_k$$

$$H = \sum E_k (\alpha_k^\dagger \alpha_k - \beta_k^\dagger \beta_k) + \epsilon_{0,0}\psi_{0,0}^\dagger \psi_{0,0} + \epsilon_{\pi,0}\psi_{\pi,0}^\dagger \psi_{\pi,0} + \epsilon_{0,\pi}\psi_{0,\pi}^\dagger \psi_{0,\pi}$$
$$- \frac{1}{2}(\epsilon_{0,0} + \epsilon_{0,\pi} + \epsilon_{\pi,0}) + \left(\sum_{k'} \frac{1}{2}\epsilon_{k'} - N_z J_z\right).$$

$$\begin{pmatrix} \alpha_k \\ \beta_k \end{pmatrix} = \begin{pmatrix} \cos\theta_k & -\mathrm{i}\,\sin\theta_k \\ -\mathrm{i}\,\sin\theta_k & \cos\theta_k \end{pmatrix} \begin{pmatrix} \psi_k \\ \psi_{-k}^\dagger \end{pmatrix}$$

- This is also the case for SU(2) slave fermion formalism

(although SU(2) slave fermion formalism is an approximation)

$$\hat{S}_i^\alpha = \frac{1}{2} f_{i\alpha}^\dagger \sigma_{\alpha\beta}^\alpha f_{i\beta} \qquad n_{i\uparrow} + n_{i\downarrow} = 1, \quad f_{i\uparrow}^\dagger f_{i\downarrow}^\dagger = 0, \quad f_{i\uparrow} f_{i\downarrow} = 0$$

Kitaev QSL (rare example of solvable QSL)

- Relation to superconductivity

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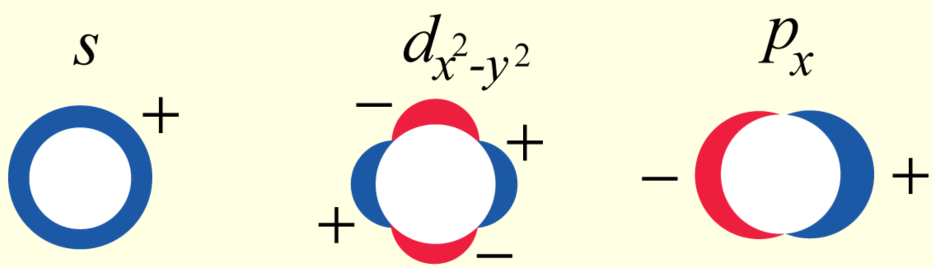


+exp(+ikx)-exp(-ikx) = 2 i sin(kx)

Taken from https://zvine-ap.eng.hokudai.ac.jp/~asano/research.html

$$\epsilon_k = 2(J_x \cos k_x + J_y \cos k_y + J_z)$$

$$\boxed{\delta_k = 2(J_x \sin k_x + J_y \sin k_y)}$$

$$E_k = \sqrt{\epsilon_k^2 + \delta_k^2} \quad \cos 2\theta_k = \epsilon_k / E_k$$

$$\begin{pmatrix} \alpha_k \\ \beta_k \end{pmatrix} = \begin{pmatrix} \cos\theta_k & -\mathrm{i}\sin\theta_k \\ -\mathrm{i}\sin\theta_k & \cos\theta_k \end{pmatrix} \begin{pmatrix} \psi_k \\ \psi^\dagger_{-k} \end{pmatrix}$$

- This is also the case for SU(2) slave fermion formalism

(although SU(2) slave fermion formalism is an approximation)

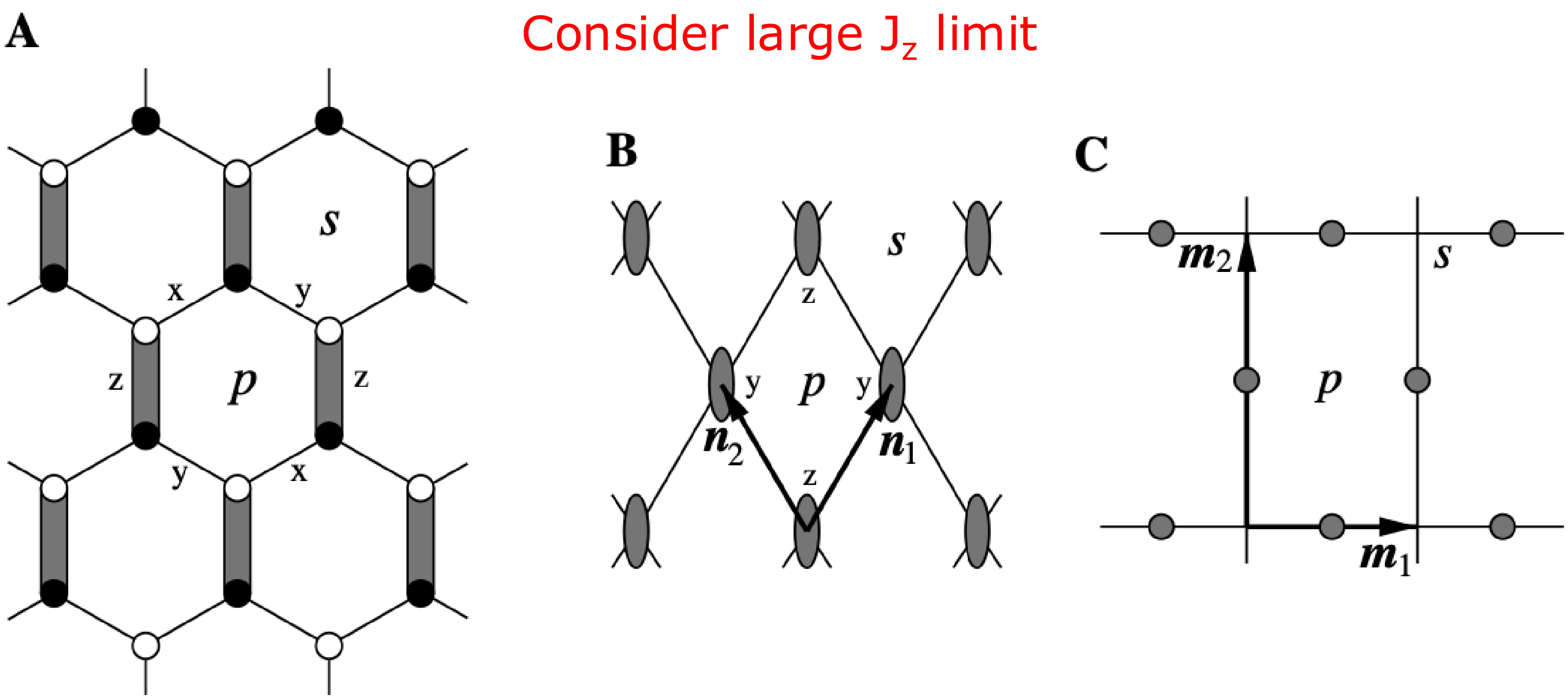
$$\hat{S}_i^\alpha = \frac{1}{2} f^\dagger_{i\alpha} \sigma^\alpha_{\alpha\beta} f_{i\beta} \qquad n_{i\uparrow} + n_{i\downarrow} = 1, \quad f^\dagger_{i\uparrow} f^\dagger_{i\downarrow} = 0, \quad f_{i\uparrow} f_{i\downarrow} = 0$$

Kitaev QSL (rare example of solvable QSL)

- Relation to Toric code model

$$H_0 = -J_z \sum \sigma_j^z \sigma_k^z, \quad V = -J_x \sum \sigma_j^x \sigma_k^x - J_y \sum \sigma_j^y \sigma_k^y$$

Consider large J_z limit



$$H_{\text{eff}} = -\frac{J_x^2 J_y^2}{16|J_z|^3} \sum_p Q_p, \quad Q_p = \sigma^y_{\text{left}(p)} \sigma^y_{\text{right}(p)} \sigma^z_{\text{up}(p)} \sigma^z_{\text{down}(p)}$$

Kitaev QSL (rare example of solvable QSL)

- Relation to Toric code model

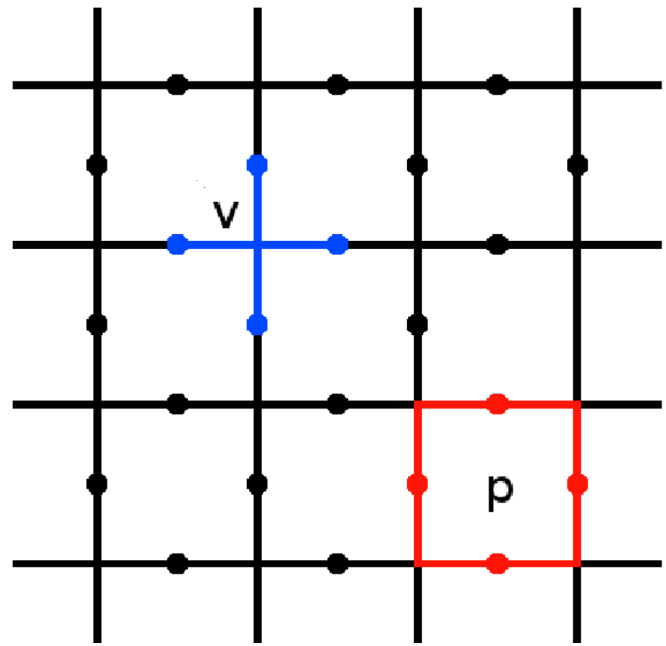
$$H_{\text{eff}} = -J_{\text{eff}} \left(\sum_{\text{vertices}} Q_s + \sum_{\text{plaquettes}} Q_p \right) \qquad J_{\text{eff}} = J_x^2 J_y^2 / (16 |J_z|^3)$$

Unitary transformation

$$U = \prod_{\text{zontal links}} X_j \prod_{\text{vertical links}} Y_k$$

$$H'_{\text{eff}} = U H_{\text{eff}} U^\dagger$$

$$= -J_{\text{eff}} \left(\sum_{\text{vertices}} A_s + \sum_{\text{plaquettes}} B_p \right)$$



Kitaev QSL (rare example of solvable QSL)

- Relation to Toric code model

Toric code

2 languages

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From Wikipedia, the free encyclopedia

The **toric code** is a topological quantum error correcting code, and an example of a stabilizer code, defined on a two-dimensional spin lattice.[1] It is the simplest and most well studied of the quantum double models.[2] It is also the simplest example of topological order—Z_2 topological order (first studied in the context of Z_2 spin liquid in 1991).[3][4] The toric code can also be considered to be a Z_2 lattice gauge theory in a particular limit.[5] It was introduced by Alexei Kitaev.

The toric code gets its name from its periodic boundary conditions, giving it the shape of a torus. These conditions give the model translational invariance, which is useful for analytic study. However, some experimental realizations require open boundary conditions, allowing the system to be embedded on a 2D surface. The resulting code is typically known as the planar code. This has identical behaviour to the toric code in most, but not all, cases.

Original motivation seems to be quantum computation

Strongly influences the field of condensed matter physics

1997:
Toric code model

↓

2005:
Kitaev honeycomb model

(Toric code model and RVB states are mentioned in introduction)

arXiv > quant-ph > arXiv:quant-ph/9707021

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Quantum Physics

[Submitted on 9 Jul 1997]

Fault-tolerant quantum computation by anyons

A. Yu. Kitaev

A two-dimensional quantum system with anyonic excitations can be considered as a quantum computer. Unitary transformations can be performed by moving the excitations around each other. Measurements can be performed by joining excitations in pairs and observing the result of fusion. Such computation is fault-tolerant by its physical nature.

Comments: 27 pages, Latex2e, uses amssymb.sty, 13 Postscript figures

Subjects: **Quantum Physics (quant-ph)**; Mesoscale and Nanoscale Physics (cond-mat.mes-hall); High Energy Physics - Theory (hep-th)

Cite as: arXiv:quant-ph/9707021
(or arXiv:quant-ph/9707021v1 for this version)
https://doi.org/10.48550/arXiv.quant-ph/9707021

Journal reference: Annals Phys. 303 (2003) 2-30

Related DOI: https://doi.org/10.1016/S0003-4916%2802%2900018-0

Submission history

From: Alexei Kitaev [view email]
[v1] Wed, 9 Jul 1997 18:28:27 UTC (40 KB)

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17 blog links (what is this?)

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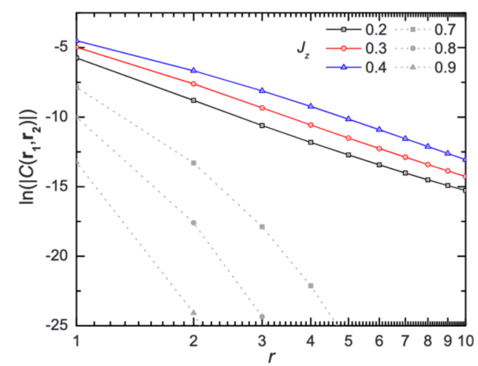
What is unconventional about Kitaev QSL?

(in my opinion)

- It is often the case that
  - Gapped QSL → Short-range magnetic order
    $\langle S_0 . S_r \rangle \sim \exp(-r/\xi)$
  - Gapless QSL → Quasi-long-range magnetic order
    $\langle S_0 . S_r \rangle \sim r^{-\eta}$
- In Kitaev QSL
  - Gapless QSL → Short-range magnetic order
    $\langle S_0 . S_r \rangle \sim \exp(-r/\xi)$

  (Because spin rotational symmetry is explicitly broken and total spin is not a good quantum number in Kitaev model)
  - In contrast, dimer-dimer correlation shows quasi-long-range order

    [S. Yang et al., PRA.78.012304 ('08)]



What is unconventional about Kitaev QSL?

(in my opinion)

- It is often the case that
  - Only low-energy states exhibit QSL behavior
  - At high temperatures, we cannot see QSL behavior

- In Kitaev QSL
  - All the eigenstates are QSL
  - QSL behavior (short-range spin correlations) even at high temperatures
    [G. Baskaran et al., PRL.98.247201 ('07)]
  - QSL behavior in real-time dynamics as well
    [A. Lavasani et al., PRB.108.115135 ('23)]
  - It can also be extended to general spin S
    [G. Baskaran et al., PRB.78.115116 ('08)]

Candidate material

PRL **102,** 017205 (2009) PHYSICAL REVIEW LETTERS week ending 9 JANUARY 2009

Mott Insulators in the Strong Spin-Orbit Coupling Limit: From Heisenberg to a Quantum Compass and Kitaev Models

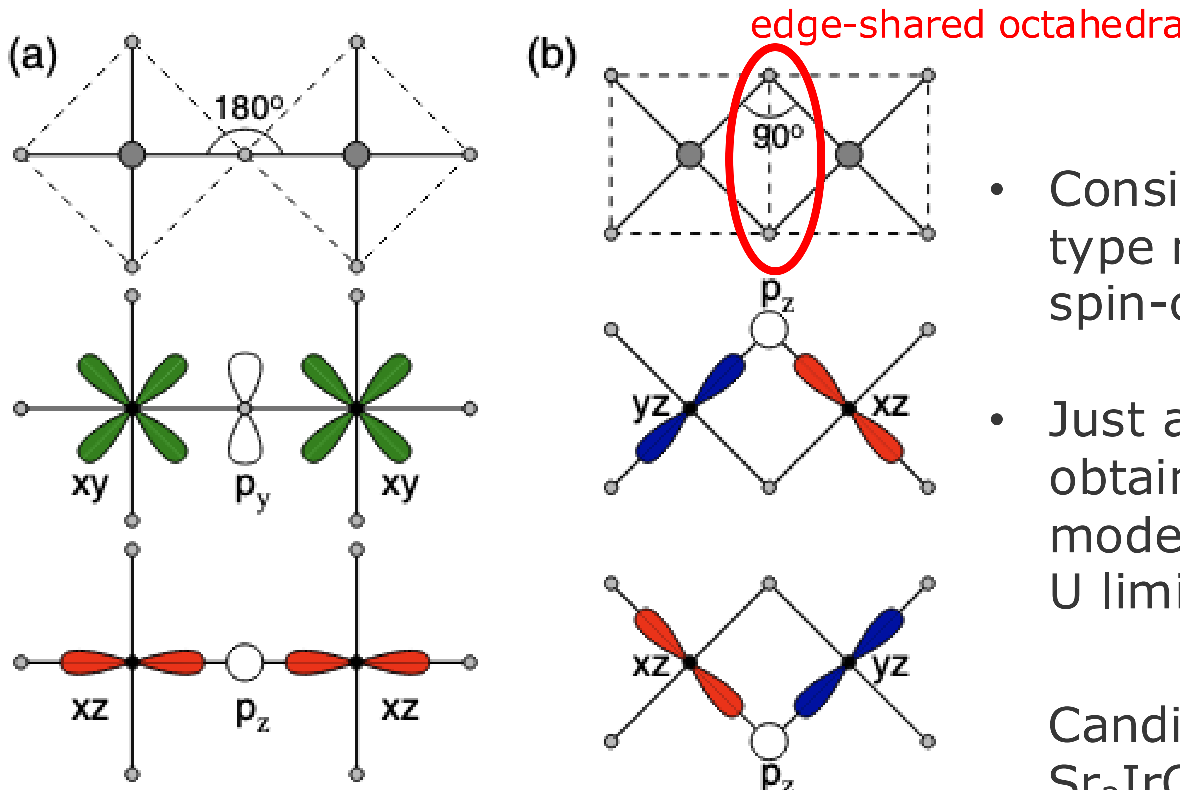
G. Jackeli[1,*] and G. Khaliullin[1]

[1] *Max-Planck-Institut für Festkörperforschung, Heisenbergstrasse 1, D-70569 Stuttgart, Germany*

(Received 21 August 2008; published 6 January 2009)

We study the magnetic interactions in Mott-Hubbard systems with partially filled t_{2g} levels and with strong spin-orbit coupling. The latter entangles the spin and orbital spaces, and leads to a rich variety of the low energy Hamiltonians that extrapolate from the Heisenberg to a quantum compass model depending on the lattice geometry. This gives way to "engineer" in such Mott insulators an exactly solvable spin model by Kitaev relevant for quantum computation. We, finally, explain "weak" ferromagnetism, with an anomalously large ferromagnetic moment, in Sr_2IrO_4.

DOI: 10.1103/PhysRevLett.102.017205 PACS numbers: 75.30.Et, 71.70.Ej, 75.10.Jm



$$\mathcal{H}_{ij} = J_1 \vec{S}_i \cdot \vec{S}_j + J_2 (\vec{S}_i \cdot \vec{r}_{ij})(\vec{r}_{ij} \cdot \vec{S}_j)$$

$$\mathcal{H}_{ij}^{(\gamma)} = -J S_i^{\gamma} S_j^{\gamma}$$

- Consider Hubbard-type model with spin-orbit interaction
- Just as in the case of obtaining Heisenberg model, consider large U limit

Candidates: Sr_2IrO_4, Na_2IrO_3, α-$RuCl_3$, …

Candidate material

Article | Published: 04 April 2016

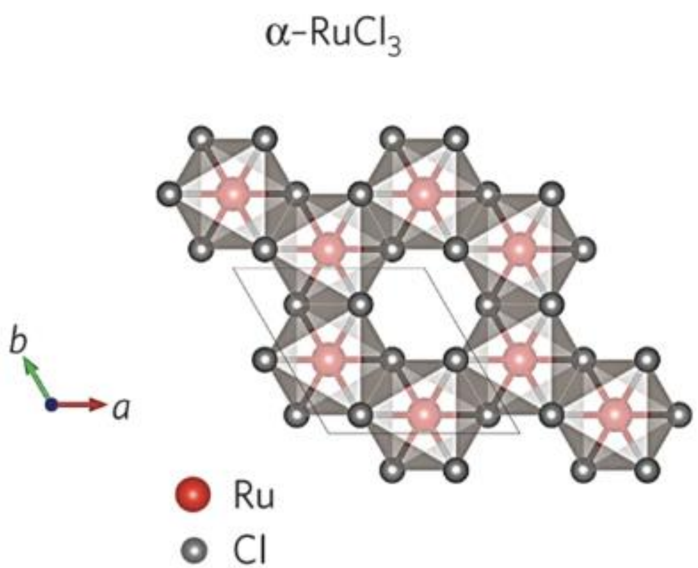
Proximate Kitaev quantum spin liquid behaviour in a honeycomb magnet

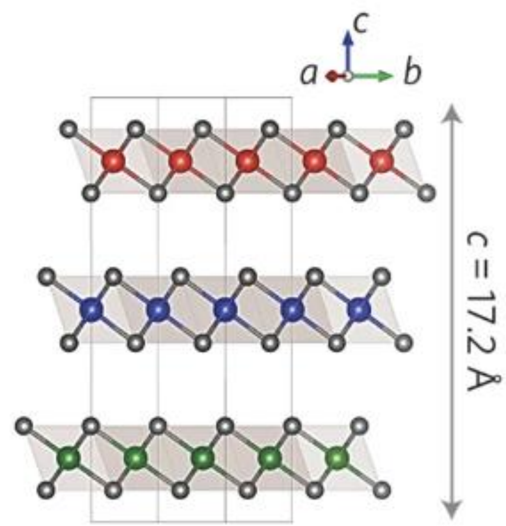
A. Banerjee, C. A. Bridges, J.-Q. Yan, A. A. Aczel, L. Li, M. B. Stone, G. E. Granroth, M. D. Lumsden, Y. Yiu, J. Knolle, S. Bhattacharjee, D. L. Kovrizhin, R. Moessner, D. A. Tennant, D. G. Mandrus & S. E. Nagler

Nature Materials **15**, 733–740 (2016) | Cite this article

- Ground state actually shows magnetic order
- At low temperatures, it exhibits QSL-like behavior

a





Candidate material

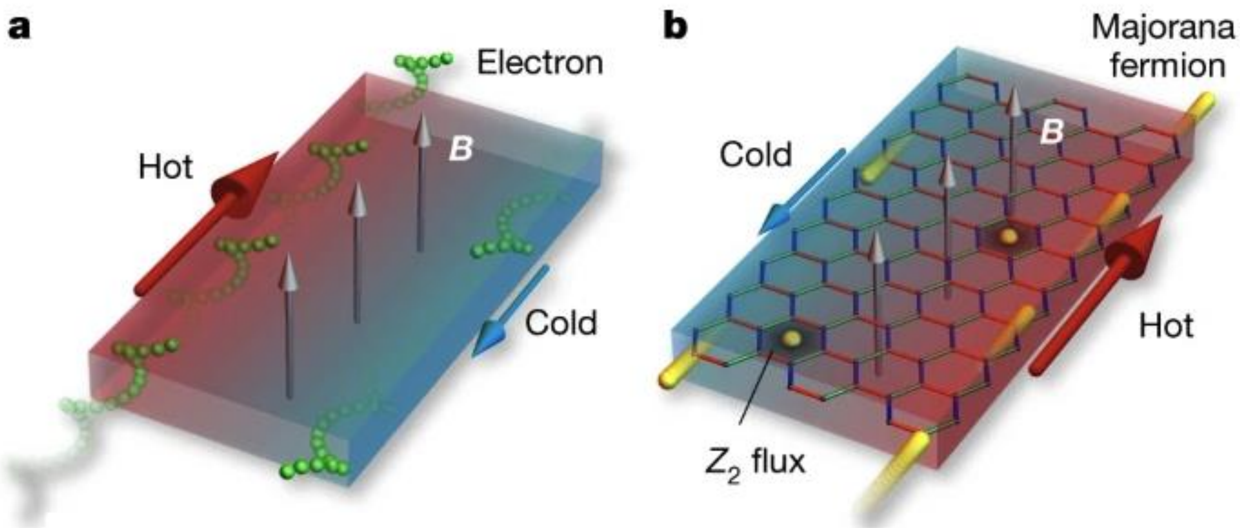
Letter | Published: 11 July 2018

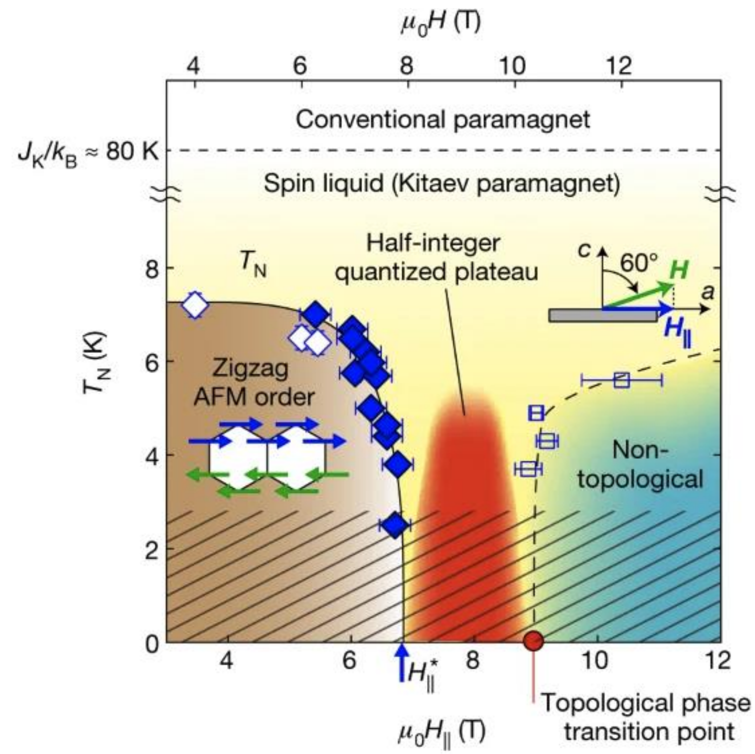
Majorana quantization and half-integer thermal quantum Hall effect in a Kitaev spin liquid

Y. Kasahara, T. Ohnishi, Y. Mizukami, O. Tanaka, Sixiao Ma, K. Sugii, N. Kurita, H. Tanaka, J. Nasu, Y. Motome, T. Shibauchi & Y. Matsuda

Nature **559**, 227–231 (2018) | Cite this article

Find QSL under magnetic field





Candidate material

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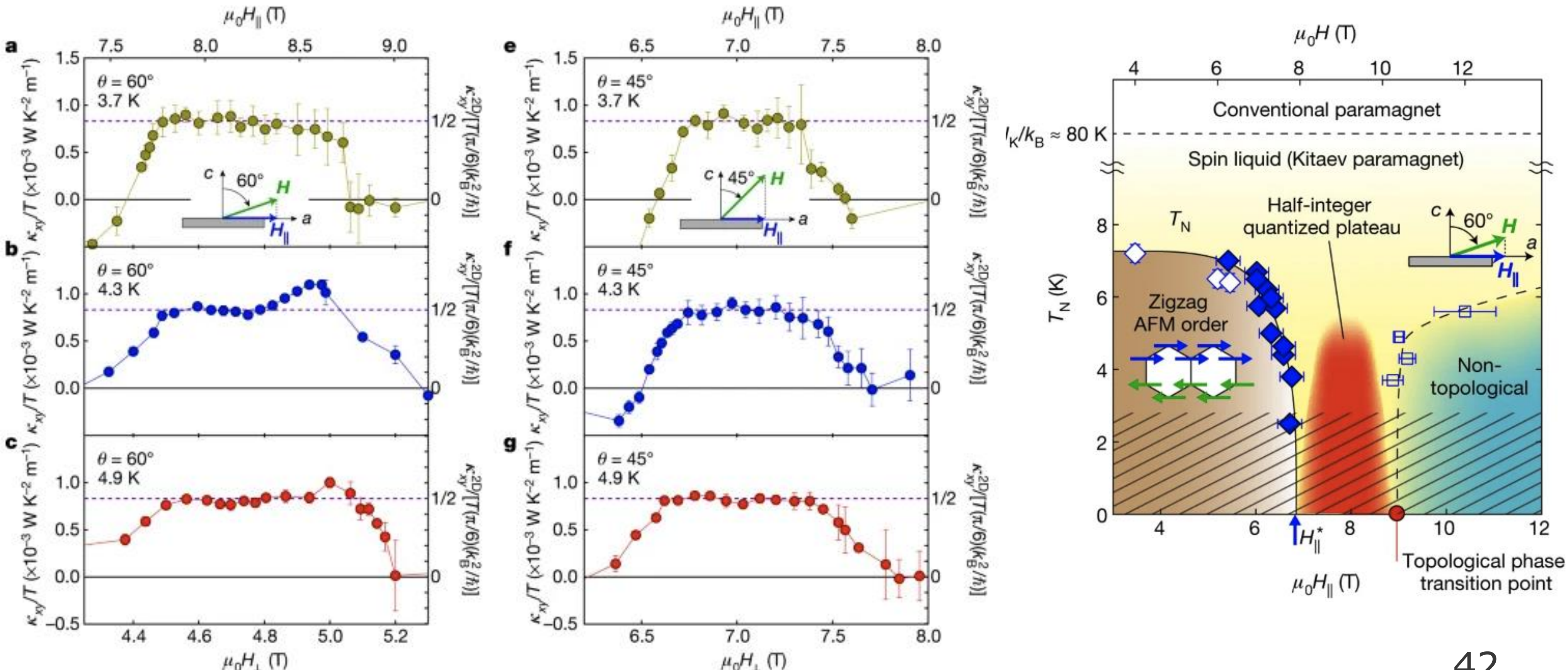
Find QSL under magnetic field



Candidate "material"

- Ultracold atoms in optical lattices, Rydberg atom arrays

Open Access

Engineering and Probing Non-Abelian Chiral Spin Liquids Using Periodically Driven Ultracold Atoms

Bo-Ye Sun, Nathan Goldman, Monika Aidelsburger, and Marin Bukov
PRX Quantum **4**, 020329 – Published 19 May 2023

Open Access

Non-Abelian Floquet Spin Liquids in a Digital Rydberg Simulator

Marcin Kalinowski, Nishad Maskara, and Mikhail D. Lukin
Phys. Rev. X **13**, 031008 – Published 21 July 2023

Feasibility of Kitaev quantum spin liquids in ultracold polar molecules

Kiyu Fukui, Yasuyuki Kato, Joji Nasu, and Yukitoshi Motome
Phys. Rev. B **106**, 014419 – Published 25 July 2022

Recent topics (my research)

- How to efficiently represent Kitaev gapless QSL using tensor network?
- Gapless state
  - Often logarithmic correction in entanglement entropy
  - 1D tensor network (matrix product state) is not good at representing gapless states
- 2D tensor network (projected entangled pair state (PEPS) / tensor product state)
  - Do not prohibit efficient description of gapless states (e.g. criticality of 2D classical Ising model by D_{bond}=2)
    [see R. Orus, AnnPhys.349.117('14)]
- Honeycomb gapless spin liquid
  - Spinon Dirac dispersion → no logarithmic correction
    [see Y. Zhang et al., PRL.107.067202('11)]

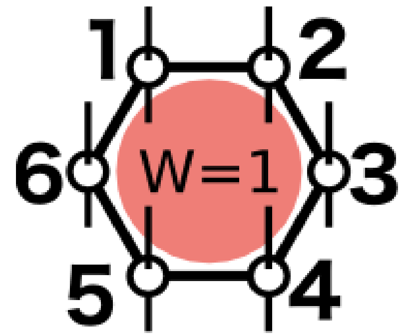
Recent topics (my research)

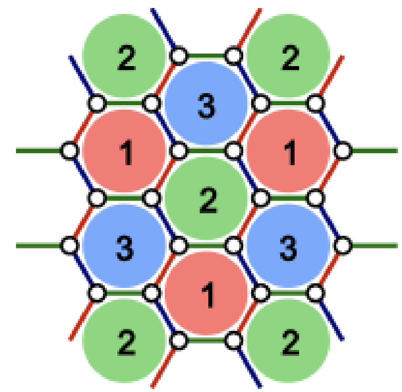
- PEPS
  - Efficiently describe “2D classical Ising”-like gapless state
- Kitaev gapless QSL
  - “2D classical Ising”-like in a sense that solvable by Jordan-Wigner transformation (quantum 1D Ising-like)
  - Gapless but no log correction in entanglement
- Is there any efficient PEPS representation of Kitaev gapless QSL?
  - Yes!
  - [H. Y. Lee et al., PRL.123.087203(’19)]

- c.f. Representation in fermion basis: [P. Schmoll and R. Orus, PRB.95.045112(’17)]
- We are interested in representation in spin basis

Recent topics (my research)

- Prepare eigenstates of flux: $|\mathbf{trial}\rangle = \mathcal{P}_{\boldsymbol{W=1}}|\psi\rangle$
  - $\mathcal{P}_{\boldsymbol{W=1}}$: written by $\mathbf{3}$ tensor product operators
  - $|\psi\rangle$: can be anything
  - Magnetization is $\mathbf{0}$ by Elitzur's theorem
- Apply simple update and see how symmetry evolves
- First, focus on the isotropic FM Kitaev model
- Then, add perturbations



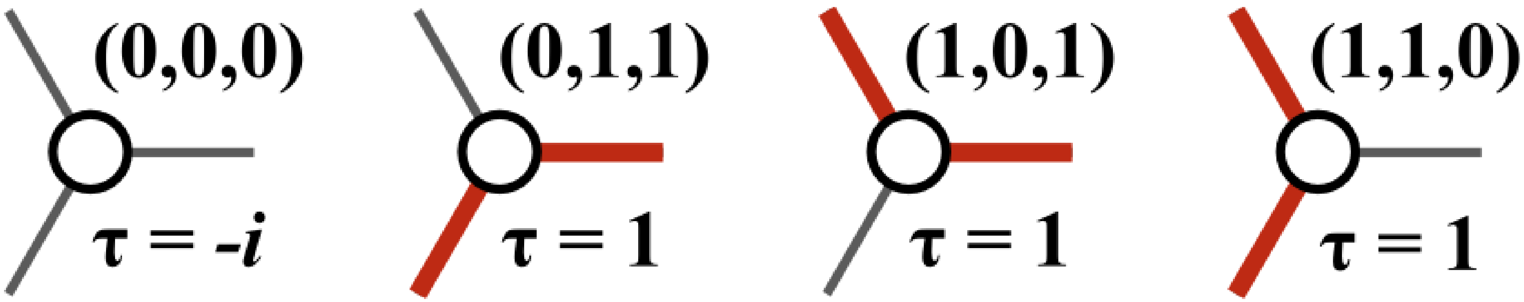


Recent topics (my research)

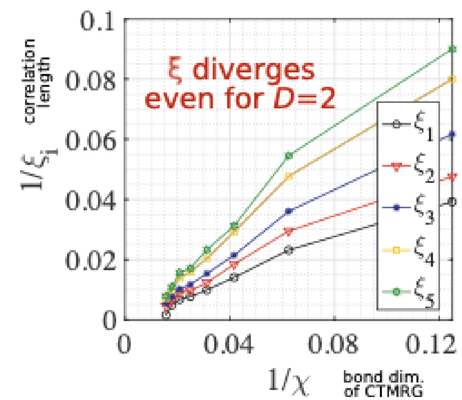
- Choose $|D=4\rangle = \mathcal{P}_{W=1}|\vec{S} \propto (1,1,1)\rangle$
- Written in $D=2$ by the Loop Gas operator:

$$|T_{xyz}\rangle = \hat{Q}_{\mathrm{LG}}|\vec{S} \propto (1,1,1)\rangle \qquad \hat{Q}_{\mathrm{LG}} = \mathrm{Tr} \prod_{\{\mathrm{site}\}} Q^{(\mathrm{site})}_{xyzm_1m_2}|m_1\rangle\langle m_2|$$

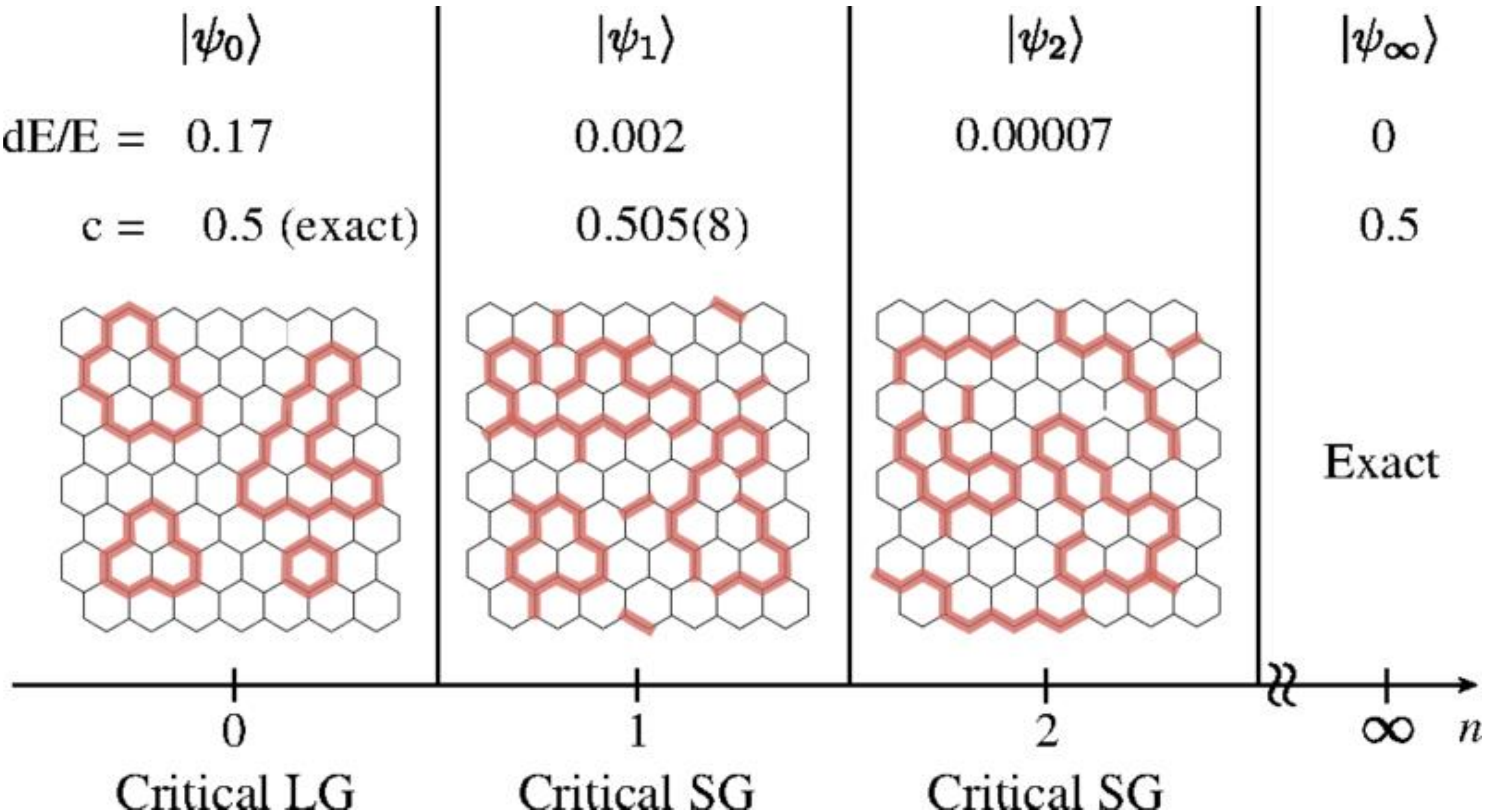
$$Q_{xyzm_1m_2} = \tau_{xyz}[(\sigma^x)^{1-x}(\sigma^y)^{1-y}(\sigma^z)^{1-z}]_{m_1m_2}$$



- Satisfy the following conditions even before optimization:
  - Nonmagnetic
  - Rotational symmetry: $E_x = E_y = E_z$
  - Energy better than classical: $E = -0.16349$
    cf. $E_{\mathrm{classical}} = -0.125$, $E_{\mathrm{exact}} = -0.19682$
  - Criticality of KSL: $c = 1/2$ (2D Ising)



Recent topics (my research)



Recent topics (my research)

Article | Open access | Published: 02 April 2020

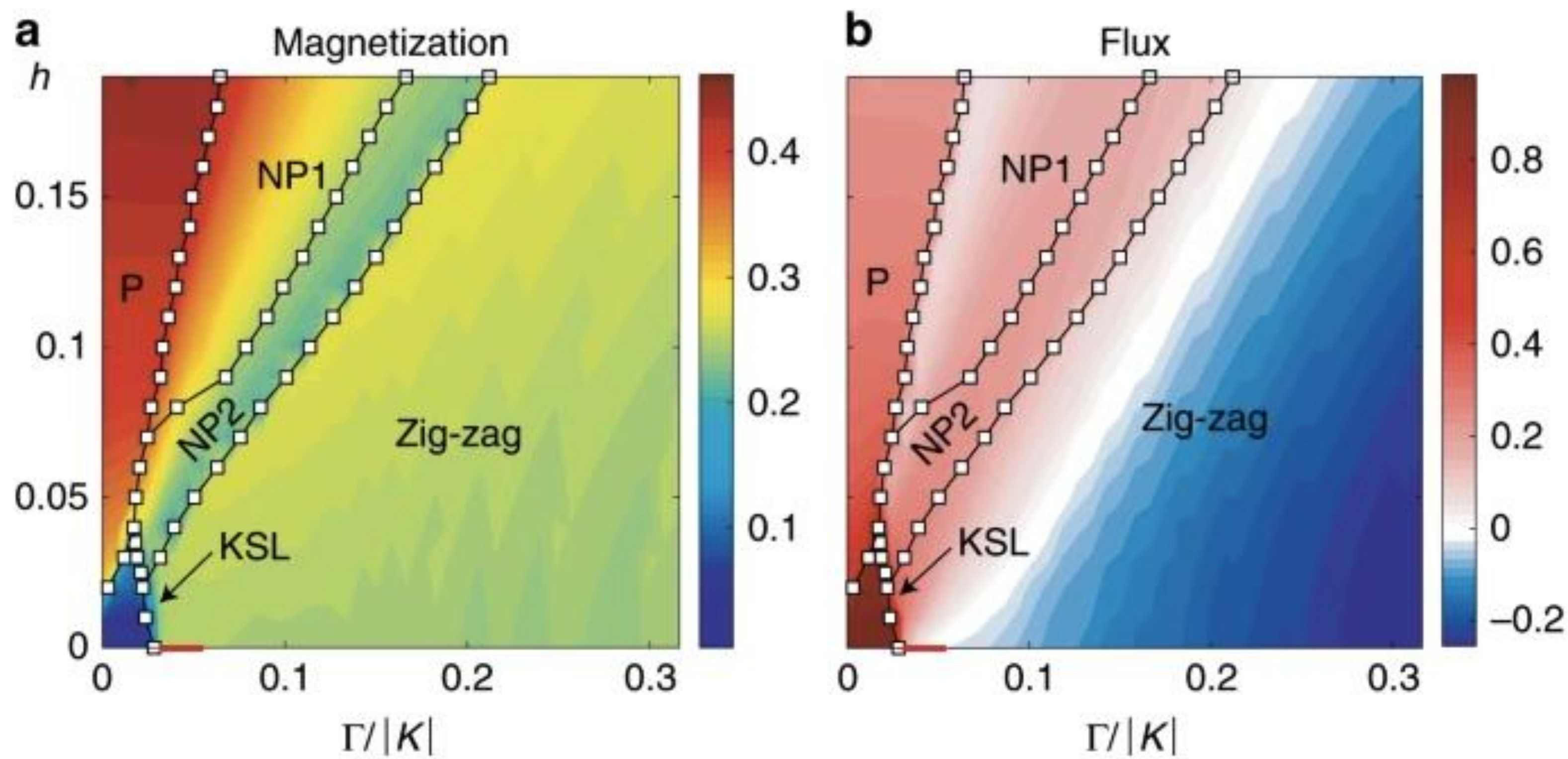
Magnetic field induced quantum phases in a tensor network study of Kitaev magnets

Hyun-Yong Lee, Ryui Kaneko, Li Ern Chern, Tsuyoshi Okubo, Youhei Yamaji, Naoki Kawashima & Yong Baek Kim

Nature Communications 11, Article number: 1639 (2020) | Cite this article

$$\hat{H}_{ij}^{\gamma} = -\frac{\mathbf{h}}{3}\cdot(\mathbf{S}_i + \mathbf{S}_j) + K S_i^{\gamma} S_j^{\gamma} + \Gamma(S_i^{\mu} S_j^{\nu} + S_i^{\nu} S_j^{\mu})$$
$$+ \Gamma'(S_i^{\mu} S_j^{\gamma} + S_i^{\gamma} S_j^{\mu} + S_i^{\nu} S_j^{\gamma} + S_i^{\nu} S_j^{\gamma}),$$

Fig. 1: Ground-state phase diagram.



Today's summary

- Brief review of itinerant electron systems
  - Electrons in crystals
  - Second quantization
  - Hubbard models
  - Solving models for simple cases
    - Noninteracting and atomic limits
    - Mean-field approximation
    - Exact diagonalization (2 sites)

      → Strong coupling limit: Heisenberg model (spin system)
- Magnetism and quantum spin liquid
  - Spin models
  - Frustrated magnetism and quantum spin liquid
  - Kitaev honeycomb spin liquid

https://github.com/ryuikaneko/lecturenote_2024_condmat