Department of Mathematics, School of advanced sciences

Winter Semester(2022-23) Instructor: Dr.Raghavendar K

Applications of Differential and Difference Equations (MAT2002)

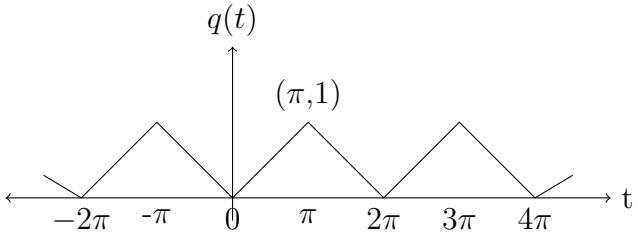
Worksheet-II

---

1. Find the Fourier series expansion for the following functions.

   (a) The clipped response of a half-wave rectifier is the periodic function $f(t)$ of period 2π defined over the period $0 \le t \le 2\pi$ by $f(t)=\begin{cases} 5\sin t, & 0 \le t \le \pi, \\ 0, & \pi \le t \le 2\pi. \end{cases}$

   (b) The charge $q(t)$ with periodicity 2π on the plates of a capacitor at time t is shown in the following figure. Express $q(t)$ as a Fourier series expansion in the interval $[0, 2\pi]$



   (c) $f(t)=\begin{cases} \pi^2, & -\pi < t < 0, \\ (t-\pi)^2, & 0 < t < \pi. \end{cases}$ Using this result evaluate the following sums.

   i. $\sum_{n=1}^{\infty} \frac{1}{n^2}$

   ii. $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^2}$

   (d) $f(t) = \pi + t, -\pi < t < \pi$, Hence show that $\frac{\pi}{4} = 1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \ldots$

2. Find half range Fourier Cosine and Sine expansion.

   (a) $f(t) = \sin 3t, 0 < t < \pi$

   (b) $f(t) = e^{-t}, 0 < t < 2$

3. Using the Parseval's idenitity prove the following

   (a) $\int_{-\pi}^{\pi} \cos^4 x dx = \frac{3\pi}{4}$

   (b) $f(t)=\begin{cases} 1, & -\pi/2 < t < \pi/2, \\ -1, & \pi/2 < t < 3\pi/2. \end{cases}$ Deduce that $1 + \frac{1}{9} + \frac{1}{25} + \ldots = \frac{\pi^2}{8}$

4. Find the first two harmonics for the following data

(a)

t	0	$\pi/2$	π	$3\pi/2$
f(t)	1	2	3	4

(b)

t	0	$\pi/3$	$2\pi/3$	π	$4\pi/3$	$5\pi/3$
f(t)	3	4	5	3	-4	-12