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MAT-2002

WORKSHEET-2

Q.1) Find the fourier series Expansion for the following functions

(a) The clipped response of a half-wave rectifier is the periodic function of "t" of period 2π defined over the period $0 \le t \le 2\pi$ by

$$f(t) = \begin{cases} 5\sin t, & 0 \le t \le \pi \\ 0, & \pi \le t \le 2\pi \end{cases}$$

A) Given $f(t+2\pi) = f(t)$,

Hence by fourier-series Euler's formula:

$$\boxed{f(t) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos n\cancel{\omega}t + b_n \sin n\cancel{\omega}t}$$

T = time period.

$$a_0 = \frac{2}{T} \int_0^T f(t)\,dt \qquad [T = 2\pi]$$

$$a_0 = \frac{2}{2\pi}\int_0^{2\pi} f(t)\,dt$$

$$\Rightarrow a_0 = \frac{1}{\pi}\left[\int_0^{\pi} 5\sin t\,dt + \int_{\pi}^{2\pi} 0\,dt\right]$$

$$\Rightarrow a_0 = \frac{1}{\pi}\left[5\cdot(-\cos t)\Big|_0^{\pi} + 0\right]$$

$$\Rightarrow a_0 = \frac{5}{\pi}\left(-[-1-1]\right) \Rightarrow \boxed{a_0 = 10/\pi} \rightarrow (1)$$

$$\Rightarrow a_n = \frac{2}{T}\int_0^{T} f(t)\cos nt\,dt. \qquad \omega = \frac{2\pi}{T} = 1$$

$$\Rightarrow a_n = \frac{2}{2\pi}\left[\int_0^{\pi} 5\sin t\cos nt\,dt + \int_{\pi}^{2\pi} 0\,dt\right]$$

$$\Rightarrow a_n = \frac{1}{\pi}\left[5\int_0^{\pi} \sin t\cos nt\,dt + 0\right]$$

$$\Rightarrow a_n = \frac{5}{2\pi}\int_0^{\pi} 2\sin t\cos nt\,dt$$

$$\Rightarrow a_n = 5/2\pi \int_0^{\pi} \left(\sin(n+1)t - \sin(n-1)t\right)dt$$

$$\Rightarrow a_n = \frac{5}{2\pi}\int_0^{\pi} \sin(n+1)t\,dt - \frac{5}{2\pi}\int_0^{\pi}\sin(n-1)t\,dt$$

$$\Rightarrow a_n = \frac{5}{2\pi}\left\{ -\frac{\cos(n+1)t}{(n+1)}\Big|_0^{\pi} + \frac{\cos(n-1)t}{(n-1)}\Big|_0^{\pi}\right\}$$

$$\Rightarrow a_n = \frac{5}{2\pi}\left\{\frac{-1}{(n+1)}\left((-1)^{n+1}-1\right) + \frac{1}{(n-1)}\left(-1^{n-1}-1\right)\right\}$$

$$\Rightarrow a_n = \frac{5}{2\pi}\left\{-\frac{(-1)^{n+1}}{n+1} + \frac{(-1)^{n-1}}{n-1} - \frac{2}{n^2-1}\right\}$$

$$\Rightarrow a_n = \frac{5}{2\pi}\left\{\frac{-2(-1)^n}{n^2-1} - \frac{2}{n^2-1}\right\} \Rightarrow \boxed{a_n = \frac{-5}{\pi}\left[\frac{(-1)^n+1}{n^2-1}\right]} \quad (2)$$

$\forall\, n \neq 1$

$$\Rightarrow b_n = \frac{2}{T}\int_0^{T} f(t)\sin nt\,dt$$

$$\Rightarrow b_n = \frac{2}{2\pi}\left[\int_0^{\pi} 5\sin t\sin nt\,dt + \int_{\pi}^{2\pi} 0\,\sin nt\,dt\right]$$

$$\Rightarrow b_n = \frac{1}{\pi}\cdot 5\int_0^{\pi}\sin t\sin nt\,dt$$

$$\Rightarrow b_n = \frac{5}{2\pi}\int_0^{\pi} 2\sin t \sin nt \, dt$$

$$\Rightarrow b_n = \frac{5}{2\pi}\left[\int_0^{2\pi} \cos(n-1)t \, dt - \int_0^{2\pi} \cos(n+1)t \, dt\right]$$

$$\Rightarrow b_n = \frac{5}{2\pi}\left\{ \frac{\sin(n-1)t}{(n-1)} - \frac{\sin(n+1)t}{(n+1)} \Big|_0^{\pi} \right\}$$

$$\Rightarrow \boxed{b_n = \frac{5}{2\pi}\{0-0\} = 0.} \rightarrow 3$$

Hence from a_0, a_n, b_n and Euler's formula

$$f(t) = \frac{5}{\pi} + \sum_{n=1}^{\infty} \frac{-5}{\pi}\left[\frac{(-1)^n+1}{n^2-1}\right]\cos nt$$

$$\Rightarrow \boxed{f(t) = \frac{5}{\pi} - \frac{5}{\pi}\sum_{n=1}^{\infty}\left(\frac{(-1)^n+1}{n^2-1}\right)\cos nt} \quad \forall n \neq 1$$

(b) The charge $q(t)$ with periodicity 2π on the plates of a capacitor at time t is shown in the following figure. Express $q(t)$ as a fourier series Expansion in $[0, 2\pi]$

for $n=1$

$$a_1 = \frac{2}{2\pi}\int_0^T f(t)\cos t\,dt \quad \because \omega=1.$$

$$\Rightarrow a_1 = \frac{1}{\pi}\int_0^{\pi} 5\sin t\cos t\,dt$$

$$\Rightarrow a_1 = \frac{5}{2\pi}\int_0^{\pi} 2\sin t\cos t\,dt$$

$$\Rightarrow a_1 = \frac{5}{2\pi}\int_0^{\pi}\sin 2t\,dt \Rightarrow a_1 = \frac{5}{2\pi}\left.\frac{-\cos 2t}{2}\right|_0^{\pi}$$

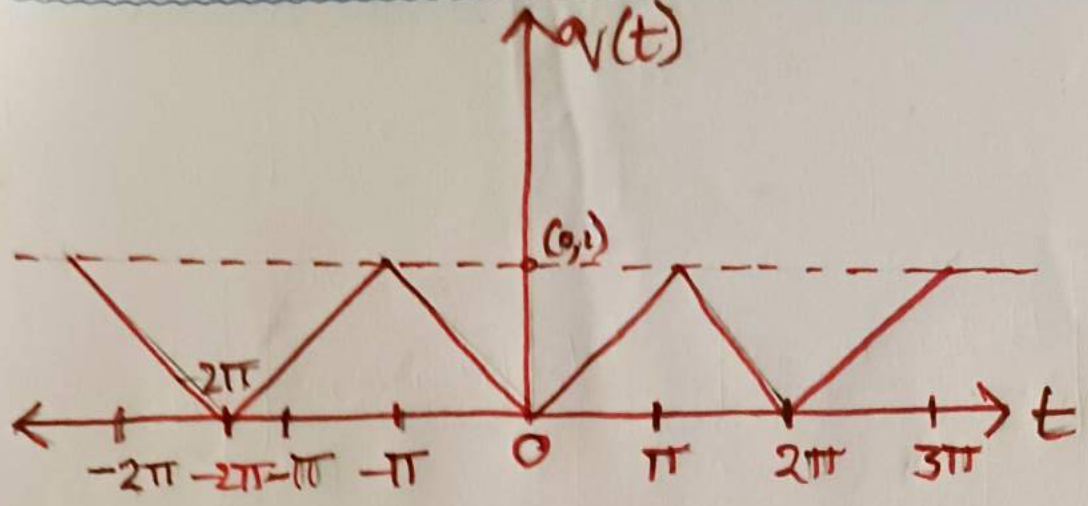
$$\Rightarrow a_1 = \frac{5}{2\pi}\,{-\tfrac{1}{2}}\left[1-1\right] = 0.$$

$$\Rightarrow b_1 = \frac{2}{2\pi}\int_0^T f(t)\sin t\,dt \quad \because \omega=1$$

$$\Rightarrow b_1 = \frac{1}{\pi}\int_0^{\pi} 5\sin^2 t\,dt \Rightarrow b_1 = \frac{5}{\pi}\int_0^{\pi}\frac{1-\cos 2t}{2}\,dt$$

$$\Rightarrow b_1 = \frac{5}{\pi}\left[\int_0^{\pi}\frac{1}{2}\,dt - \frac{1}{2}\int_0^{\pi}\cos 2t\,dt\right]$$

$$\Rightarrow b_1 = \frac{5}{\pi}\times\frac{\pi}{2} \Rightarrow 5/2.$$



A) Given.

$$q(t+2\pi) = q(t) \; ; \; T = 2\pi$$

also

$$q(t) = \begin{cases} t/\pi \, , & 0 \le t \le \pi \\ -t/\pi + 2 \, , & \pi \le t \le 2\pi \end{cases}$$

By Euler's Formula's and Fourier series expansion.

$$q(t) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos n\omega t + b_n \sin n\omega t$$

$$\omega = \frac{2\pi}{T} = 1$$

$$\Rightarrow \boxed{a_0 = \frac{2}{T} \int_0^T q(t)\,dt}$$

$$\Rightarrow a_0 = \frac{2}{2\pi} \left\{ \int_0^{\pi} \frac{t}{\pi} dt + \int_{\pi}^{2\pi} \left(-\frac{t}{\pi} + 2 \right) dt \right\}$$

$$\Rightarrow a_0 = \frac{1}{\pi}\left\{\frac{1}{\pi}\frac{t^2}{2}\Big|_0^{\pi} - \frac{1}{2\pi}t^2\Big|_{\pi}^{2\pi} + 2t\Big|_{\pi}^{2\pi}\right\}$$

$$\Rightarrow a_0 = \frac{1}{\pi}\left\{\frac{1}{2\pi}(\pi^2) - \frac{1}{2\pi}(4\pi^2 - \pi^2) + 2(\pi)\right\}$$

$$\Rightarrow a_0 = \frac{1}{\pi}\left\{\frac{\pi}{2} - \frac{3\pi}{2} + 2\pi\right\} \Rightarrow \boxed{a_0 = 1}$$

$$\Rightarrow a_n = \frac{2}{T}\int_0^T f(t)\cos nt\,dt \quad [\because \omega = 1]$$

$$\Rightarrow a_n = \frac{2}{2\pi}\left[\int_0^{\pi}\frac{t}{\pi}\cos nt\,dt + \int_{\pi}^{2\pi}\left(\frac{-t}{\pi}+2\right)\cos nt\,dt\right]$$

$$\Rightarrow a_n = \frac{1}{\pi}\left\{\frac{1}{\pi}\int_0^{\pi}t\cos nt\,dt - \frac{1}{\pi}\int_{\pi}^{2\pi}t\cos nt\,dt + 2\int_{\pi}^{2\pi}\cos nt\,dt\right\}$$

$$\Rightarrow a_n = \frac{1}{\pi}\left\{\frac{1}{\pi}\left(\frac{t\sin nt}{n}\Big|_0^{\pi} - \frac{1}{n^2}\cos nt\Big|_0^{\pi}\right) - \frac{1}{\pi}\left(\frac{t\sin nt}{n}\Big|_{\pi}^{2\pi} - \frac{1}{n^2}\cos nt\Big|_{\pi}^{2\pi}\right) + 2\cdot\frac{\sin nt}{n}\Big|_{\pi}^{2\pi}\right\}$$

$$\Rightarrow a_n = \frac{1}{\pi}\left\{\frac{1}{\pi}\left(0 - \frac{(-1)^n - 1}{n^2}\right) - \frac{1}{\pi}\left(0 - \frac{1-(-1)^n}{n^2}\right) + 2\frac{(0-0)}{n}\right\}$$

$$\Rightarrow a_n = \frac{1}{\pi}\left\{\frac{1-(-1)^n}{n^2\pi} + \frac{1-(-1)^n}{n^2\pi} + 0\right\}$$

$$\Rightarrow \boxed{a_n = \frac{2}{\pi^2}\left(\frac{1-(-1)^n}{n^2}\right)}$$

$$\Rightarrow b_n = \frac{2}{T}\int_0^T f(t)\sin nt\,dt \quad [\because \omega = 1]$$

$$\Rightarrow b_n = \frac{2}{2\pi}\int_0^\pi \frac{t}{\pi}\sin nt\,dt + \int_\pi^{2\pi}\left(\frac{-t}{\pi}+2\right)\sin nt\,dt$$

$$\Rightarrow b_n = \frac{1}{\pi}\left\{\frac{1}{\pi}\int_0^\pi t\sin nt\,dt - \frac{1}{\pi}\int_\pi^{2\pi} t\sin nt\,dt + 2\int_\pi^{2\pi}\sin nt\,dt\right\}$$

$$\Rightarrow b_n = \frac{1}{\pi}\left\{\frac{1}{\pi}\left(\frac{-t\cos nt}{n}\Big|_0^\pi + \frac{\sin nt}{n^2}\Big|_0^\pi\right) - \frac{1}{\pi}\left[\frac{-t\cos nt}{n}\Big|_\pi^{2\pi} + \frac{\sin nt}{n^2}\Big|_\pi^{2\pi}\right] + \cdots\right.$$

$$\left.\cdots 2\left(-\frac{\cos nt}{n}\Big|_\pi^{2\pi}\right)\right\}$$

$$\Rightarrow b_n = \frac{1}{\pi}\left\{\frac{1}{\pi}\left(\frac{-\pi(-1)^n}{n} + 0\right) - \frac{1}{\pi}\left(\frac{-2\pi + \pi(-1)^n}{n} + 0\right) + 2\left(\frac{-1+(-1)^n}{n}\right)\right\}$$

$$b_n = \frac{1}{\pi}\left\{-\frac{(-1)^n}{n} - \frac{(-1)^n}{n} + \frac{2}{n} - \frac{2}{n} + \frac{2(-1)^n}{n}\right\}$$

$$\boxed{b_n = 0}$$

Hence, from a_0, a_n, b_n and Eq-①.

$$q(t) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos nt + b_n \sin nt \quad [\because \omega = 1]$$

$$\Rightarrow q(t) = \frac{1}{2} + \sum_{n=1}^{\infty} \frac{2}{\pi^2}\frac{1-(-1)^n}{n^2}\cos nt$$

$$\Rightarrow \boxed{q(t) = \frac{1}{2} + \frac{2}{\pi^2}\sum_{n=1}^{\infty}\frac{1-(-1)^n}{n^2}\cos nt}$$

(c) $f(t) = \begin{cases} \pi^2, & -\pi < t < 0 \\ (t-\pi)^2, & 0 < t < \pi \end{cases}$, using this result evaluate the following sums (i) $\sum_{n=1}^{\infty}\frac{1}{n^2}$ (ii) $\sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{n^2}$

A) From fourier-Expansion.

$$f(t) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos n\omega t + b_n \sin n\omega t$$

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From Euler's formulas. where given

$$T = \pi - (-\pi) = 2\pi$$
$$\omega = \frac{2\pi}{T} = 1$$

$$a_0 = \frac{2}{T}\int_0^T f(t)\,dt$$

$$a_n = \frac{2}{T}\int_0^T f(t)\cos n\omega t\,dt \;;\; b_n = \frac{2}{T}\int_0^T f(t)\sin n\omega t\,dt$$

$$\Rightarrow a_0 = \frac{2}{2\pi}\left[\int_{-\pi}^{0} \pi^2\,dt + \int_0^{\pi} (t-\pi)^2\,dt\right]$$

$$\Rightarrow a_0 = \frac{1}{\pi}\left[\pi^2 t\Big|_{-\pi}^{0} + \frac{1}{3}(t-\pi)^3\Big|_{0}^{\pi}\right]$$

$$\Rightarrow a_0 = \frac{1}{\pi}\left[\pi^2(\pi) + \frac{1}{3}\left[+\pi^3 + 0\right]\right]$$

$$\Rightarrow a_0 = \frac{1}{\pi}\left[\pi^3 + \frac{\pi^3}{3}\right] \Rightarrow \boxed{a_0 = \frac{4\pi^2}{3}}$$

$$\Rightarrow a_n = \frac{2}{T}\int_0^T f(t)\cos nt\,dt \quad [\because \omega = 1]$$

$$\Rightarrow a_n = \frac{1}{\pi}\left[\int_{-\pi}^{0} \pi^2 \cos nt + \underbrace{\int_0^{\pi} (t-\pi)^2 \cos nt\,dt}_{I_2}\right] \text{—— (i)}$$

$\Rightarrow$ Solvation of I_2

$$I_2 = \int_0^{\pi} (t-\pi)^2 \cos nt\, dt$$

Let $t-\pi = m$

$t = m+\pi$

$dt = dm.$

$$I_2 = \int_{-\pi}^{0} m^2 \cos n(m+\pi)\, dm.$$

(or)

$$I_2 = \int_0^{\pi} t^2 \cos nt\, dt - 2\pi \int_0^{\pi} t \cos nt\, dt + \pi^2 \int_0^{\pi} \cos nt\, dt$$

$$I_2 = \left. \frac{t^2 \sin nt}{n} + \frac{2t\cos nt}{n^2} - 2\frac{\sin nt}{n^3} \right|_0^{\pi}$$

$$- 2\pi \left[\left. \frac{t \sin nt}{n} \right|_0^{\pi} + \left. \frac{\cos nt}{n^2} \right|_0^{\pi} \right] + \pi^2 \left. \frac{\sin nt}{n} \right|_0^{\pi}$$

$$I_2 = \frac{2\pi(-1)^n}{n^2} - 2\pi \left[\frac{(-1)^n - 1}{n^2} \right] \Rightarrow I_2 = \frac{2\pi}{n^2} \left[(-1)^n - (-1)^n + 1 \right]$$

$$\boxed{\Rightarrow I_2 = \frac{2\pi}{n^2}}$$

from Equation -(i), subs of I_2

$$\Rightarrow a_n = \frac{1}{\pi} \left\{ \left. \pi^2 \frac{\sin nt}{n} \right|_{-\pi}^{0} + \frac{2\pi}{n^2} \right\}$$

$$\Rightarrow a_n = \frac{1}{\pi}\left\{0 + \frac{2\pi}{n^2}\right\}$$

$$\Rightarrow \boxed{a_n = 2/n^2}$$

$$\Rightarrow b_n = \frac{2}{T}\int_0^T f(t)\sin nt\,dt \quad [\because \omega = 1]$$

$$\Rightarrow b_n = \frac{1}{\pi}\left[\int_{-\pi}^{0} \pi^2 \sin nt + \underbrace{\int_0^\pi (t-\pi)^2 \sin nt\,dt}_{I_2}\right] \text{ —— (i)}$$

Solvation of I_2

$$I_2 = \int_0^\pi (t-\pi)^2 \sin nt\,dt$$

$$I_2 = \int_0^\pi t^2 \sin nt\,dt - 2\pi\int_0^\pi t\sin nt\,dt + \pi^2\int_0^\pi \sin nt\,dt$$

$$I_2 = \frac{-t^2\cos nt}{n}\Big|_0^\pi + \frac{2t\sin nt}{n^2}\Big|_0^\pi + \frac{2\cos nt}{n^3}\Big|_0^\pi$$

$$-2\pi\left[\frac{-t\cos nt}{n}\Big|_0^\pi + \frac{\sin nt}{n^2}\Big|_0^\pi\right] + \pi^2\left(\frac{-\cos nt}{n}\Big|_0^\pi\right)$$

$$I_2 = \frac{-\pi^2(-1)^n}{n} + \frac{2(-1)^n - 2}{n^3} - 2\pi\left[\frac{-\pi(-1)^n}{n}\right] - \pi^2\left[\frac{(-1)^n - 1}{n}\right]$$

$$I_2 = \frac{2(-1)^n}{n^3} - \frac{2}{n^3} + \frac{\pi^2}{n}$$

from — (i), on subs of I_2

$$b_n = \frac{1}{\pi}\left[-\left[\frac{\pi^2 \cos nt}{n}\right]_{-\pi}^{0} + \frac{2(-1)^n}{n^3} - \frac{2}{n^3} + \frac{\pi^2}{n}\right]$$

$$\Rightarrow b_n = \frac{1}{\pi}\left[-\pi^2\frac{(1-(-1)^n)}{n} + \frac{2(-1)^n}{n^3} - \frac{2}{n^3} \pm \frac{\pi^2}{n}\right]$$

$$\Rightarrow b_n = \frac{1}{\pi}\left[\frac{-\pi^2}{n} + \frac{\pi^2(-1)^n}{n} + \frac{2(-1)^n}{n^3} - \frac{2}{n^3} + \frac{\pi^2}{n}\right]$$

$$\Rightarrow b_n = \frac{1}{\pi}\left[\frac{n^2\pi^2(-1)^n + 2(-1)^n - 2}{n^3}\right]$$

$$\Rightarrow \boxed{b_n = \pi \cdot \frac{(-1)^n}{n} + 2\frac{((-1)^n - 1)}{n^3}}$$

Hence, from fourier series Expansion.

$$f(t) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos n\omega t + b_n \sin n\omega t$$

$$f(t) = \frac{\pi^2}{3} + \sum_{n=1}^{\infty} \frac{2}{n^2}\cos nt + \left(\frac{\pi(-1)^n}{n} + 2\cdot\frac{((-1)^n - 1)}{n^3}\right)\sin nt$$

If $t=0$.

$$f(0) = \frac{\cancel{2}\pi^2}{3} + \sum_{n=1}^{\infty} \frac{2}{n^2}(1) + 0.$$

Since, 'f' is discontinuous at "0".

$$f(0) = \frac{1}{2}\left(f(0^+) + f(0^-)\right)$$

$$f(0) = \frac{1}{2}\left[\lim_{t\to 0^+} (t-\pi)^2 + \lim_{t \to 0^-} \pi^2\right]$$

$$\Rightarrow f(0) = \frac{1}{2}\left(\pi^2 + \pi^2\right) = \pi^2$$

$$\Rightarrow \pi^2 = \frac{\cancel{2}\pi^2}{3} + 2 \sum_{n=1}^{\infty} \frac{1}{n^2}.$$

$$\Rightarrow + \frac{\cancel{2}\pi^2}{3} = 2 \sum_{n=1}^{\infty} \frac{1}{n^2} \Rightarrow \boxed{\sum_{n=1}^{\infty} \frac{1}{n^2} = \frac{+\pi^2}{6}}$$

Similarly, $t=\pi$

$$f(\pi) = \frac{2\pi^2}{3} + \sum_{n=1}^{\infty} \frac{2}{n^2} \cos n\pi + 0.$$

$$f(\pi) = \frac{2\pi^2}{3} + 2 \sum_{n=1}^{\infty} \frac{(-1)^n}{n^2}$$

$$f(\pi) = \frac{2\pi^2}{3} - 2\sum_{n\geq 1}^{\infty} \frac{(-1)^{n+1}}{n^2}.$$

$$\Rightarrow \sum_{n\geq 1}^{\infty} \frac{(-1)^{n+1}}{n^2} = \frac{2\pi^2/3 - f(\pi)}{2} = \boxed{\pi^2/3}$$

Since, the function is ~~too~~ to be converted into fourier series expansion. for, the convergence of a_0, a_n, b_n i.e., Coefficients, the function ~~is~~ must be continuous over the given interval and also its first & second derivatives must also be continuous.

Hence. $f(\pi) = f(\pi^-) = \lim_{t \to \pi^-} f(t)$

$$\Rightarrow f(\pi) = \lim_{t \to \pi^-} (t-\pi)^2 = 0.$$

(d) $f(t) = \pi + t$, $-\pi < t < \pi$, Hence show that

$$\frac{\pi}{4} = 1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \cdots$$

A) Assuming $f(t+2\pi) = f(t)$; $T = 2\pi$

from fourier-Series Expansion of Euler's formula.

$$f(t) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos n\omega t + b_n \sin n\omega t n.$$

$\Rightarrow \omega = \frac{2\pi}{T} = \frac{2\pi}{2\pi} = 1$

$\Rightarrow a_0 = \frac{2}{T} \int_0^T f(t)\,dt$

$\Rightarrow a_0 = \frac{2}{2\pi} \int_{-\pi}^{\pi} (t+\pi)\,dt \Rightarrow a_0 = \frac{1}{\pi}\left[\frac{t^2}{2}\Big|_{-\pi}^{\pi} + \pi t\Big|_{-\pi}^{\pi} \right]$

$\Rightarrow \boxed{a_0 = \frac{1}{\pi}\left[0 + 2\pi^2\right] = 2\pi}$

$\Rightarrow a_n = \frac{2}{T} \int_0^T f(t) \cos nt\,dt$

$\Rightarrow a_n = \frac{1}{\pi} \int_{-\pi}^{\pi} (t+\pi) \cdot \cos nt\,dt$

$\Rightarrow a_n = \frac{1}{\pi} \left\{ \frac{t \sin nt}{n}\Big|_{-\pi}^{\pi} - \frac{\cos nt}{n^2}\Big|_{-\pi}^{\pi} + \pi \frac{\sin nt}{n}\Big|_{-\pi}^{\pi} \right\}$

$$\Rightarrow \boxed{a_n = \frac{1}{\pi}\left\{0 - \frac{1}{n^2}\left((-1)^n - (-1)^n\right) + 0\right\} = 0}$$

$$\Rightarrow b_n = \frac{2}{T}\int_0^T f(t)\sin nt\,dt$$

$$\Rightarrow b_n = \frac{1}{\pi}\int_{-\pi}^{\pi}(t+\pi)\sin nt\,dt$$

$$\Rightarrow b_n = \frac{1}{\pi}\left\{ -\frac{t\cos nt}{n}\Big|_{-\pi}^{\pi} + \frac{\sin nt}{n^2}\Big|_{-\pi}^{\pi} + \pi\,\frac{-\cos nt}{n}\Big|_{-\pi}^{\pi} \right\}$$

$$\Rightarrow b_n = \frac{1}{\pi}\left\{ \frac{-1}{n}\left[\pi(-1)^n + \pi(-1)^n\right] + 0 + \frac{\pi(-1)}{n}\left[(-1)^n - (-1)^n\right] \right\}$$

$$\Rightarrow b_n = \frac{1}{\pi}\left\{ \frac{-1}{n}\left[2\pi(-1)^n\right] \right\} \Rightarrow \boxed{b_n = \frac{2(-1)^{n+1}}{n}}$$

Hence, on Subs a_0, a_n, b_n.

$$f(t) = \pi + \sum_{n=1}^{\infty} \frac{2(-1)^{n+1}}{n}\sin nt$$

let $t = \pi/2$

$$f(\pi/2) = \pi + 2\sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{n}\sin n\pi/2$$

$$\frac{3\pi}{2} = \pi + 2\left[1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \cdots\right]$$

$$\Rightarrow \boxed{1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \cdots = \boxed{\frac{\pi}{4}}}$$

Q.2) Find half range Fourier Cosine and Sine Expansion.

(i) $f(t) = \sin 3t, \quad 0 < t < \pi$

A) Let $F(t)$ be a new function

(i) FOURIER COSINE SERIES.

For this series the newly defined function is to be in even form.

$$F(t) = \begin{cases} f(-t), & -\pi < t < 0. \\ f(t), & 0 < t < \pi \end{cases}$$

Now fourier series Expansion of $F(t)$ is

$$F(t) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos n\omega t$$

Since,

$$f(t+2\pi) = f(t), \quad T = 2\pi$$

$$\omega = \frac{2\pi}{T} = \frac{2\pi}{2\pi} = 1$$

$$a_0 = \frac{2}{2\pi}\left[\int_{-\pi}^{0} f(t)\,dt + \int_{0}^{\pi} f(t)\,dt\right]$$

$$a_0 = \frac{1}{\pi}\left[\int_{-\pi}^{0} -\sin 3t\,dt + \int_{0}^{\pi} \sin 3t\,dt\right]$$

$$a_0 = \frac{1}{\pi}\left[\frac{\cos 3t}{3}\Big|_{-\pi}^{0} - \frac{\cos 3t}{3}\Big|_{0}^{\pi}\right]$$

$$a_0 = \frac{1}{\pi}\left[\frac{1}{3}[1+1] - \frac{1}{3}[-1-1]\right] \Rightarrow \boxed{a_0 = \frac{4}{3\pi}}$$

$$a_n = \frac{1}{\pi}\left[\int_{-\pi}^{0} f(-t)\cos nt\,dt + \int_{0}^{\pi} f(t)\cos nt\,dt\right]$$

$$\Rightarrow a_n = \frac{1}{\pi}\left[-\int_{-\pi}^{0} \sin 3t \cos nt\,dt + \int_{0}^{\pi} \sin 3t \cos nt\,dt\right]$$

$$\Rightarrow a_n = \frac{1}{2\pi}\left[-\int_{-\pi}^{0} 2\sin 3t \cos nt\,dt + \int_{0}^{\pi} 2\sin 3t \cos nt\,dt\right.$$

$$\Rightarrow a_n = \frac{1}{2\pi}\left[-\int_{-\pi}^{0}(\sin(n+3)t - \sin(n-3)t)\,dt + \int_{0}^{\pi}\sin(n+3)t - \sin(n-3)t\,dt\right]$$

$$\Rightarrow a_n = \frac{1}{2\pi}\left\{\frac{\cos(n+3)t}{n+3}\Big|_{-\pi}^{0} - \frac{\cos(n-3)t}{(n-3)}\Big|_{-\pi}^{0} - \frac{\cos(n+3)t}{(n+3)}\Big|_{0}^{\pi} + \frac{\cos(n-3)t}{(n-3)}\Big|_{0}^{\pi}\right\}$$

$$\Rightarrow a_n = \frac{1}{2\pi}\left\{\frac{1}{n+3} - \frac{(-1)^{n+3}}{n+3} - \frac{1}{(n-3)} + \frac{(-1)^{n-3}}{n-3} - \frac{(-1)^{n+3}}{n+3} + \frac{1}{n+3} + \frac{(-1)^{n-3}}{n-3} - \frac{1}{n-3}\right\}$$

$$\Rightarrow a_n = \frac{1}{\pi}\left\{\frac{-6}{n^2-9} - \frac{(-1)^{n+3}}{n+3} + \frac{(-1)^{n-3}}{n-3}\right\}$$

$$\Rightarrow a_n = \frac{1}{\pi}\left[\frac{-6}{n^2-9} + (-1)^n\left[\frac{-6}{n^2-9}\right]\right]$$

$$\Rightarrow \boxed{a_n = \frac{-6}{\pi}\left[\frac{(-1)^n+1}{n^2-9}\right] \quad \forall\, n \neq 3.}$$

Hence, $F(t) = \frac{2}{3\pi} + \sum_{n\geq 1}^{\infty} \frac{-6}{\pi}\frac{(-1)^n+1}{n^2-9}\cos nt$

$$\Rightarrow \boxed{F(t) = \frac{2}{3\pi} - \frac{6}{\pi}\sum_{n\geq 1}^{\infty}\left(\frac{(-1)^n+1}{n^2-9}\right)\cos nt \quad \forall\, n \neq 3}$$

for $n=3$.

$$a_n = \frac{2}{2\pi}\left[-\int_{-\pi}^{0} \sin 3t \cos 3t\,dt + \int_{0}^{\pi} \sin 3t \cos 3t\,dt\right]$$

$$\Rightarrow a_n = \frac{1}{2\pi}\left[-\int_{-\pi}^{0} \sin 6t\,dt + \int_{0}^{\pi} \sin 6t\,dt\right]$$

$$\Rightarrow a_3 = \frac{1}{2\pi}\left[\frac{\cos 6t}{6}\Big|_{-\pi}^{0} - \frac{\cos 6t}{6}\Big|_{0}^{\pi}\right]$$

$$\Rightarrow a_3 = \frac{1}{2\pi}\left[\frac{1}{6}[1-1] - \frac{1}{6}[1-1]\right] = 0.$$

Hence,

$$F(t) = \frac{2}{3\pi} - \frac{6}{\pi}\sum_{n=1}^{\infty}\frac{(-1)^n+1}{n^2-9}\cos nt \quad \forall\, n \neq 3.$$

$$\text{Eq } F(t) = \frac{2}{3\pi} - \frac{6}{\pi}\sum_{n=1}^{2}\frac{(-1)^n+1}{n^2-9}\cos nt - \frac{6}{\pi}\sum_{n=4}^{\infty}\frac{(-1)^n+1}{n^2-9}\cos nt$$

(ii) FOURIER SINE SERIES.

For, fourier sine series f(t) must be an odd function.

$$F(t) = \begin{cases} -f(t), & -\pi < t < 0 \\ f(t), & 0 < t < \pi \end{cases}$$

Now, fourier expansion of F(t) is.

$$F(t) = \sum_{n=1}^{\infty} b_n \sin n\omega t \qquad \omega = \frac{2\pi}{T} \text{ also}$$

$$F(t+2\pi) = F(t)$$

$$T = 2\pi$$

$$\omega = 1$$

$$\Rightarrow b_n = \frac{2}{T}\int_0^T F(t) \sin n\omega t \, dt$$

$$\Rightarrow b_n = \frac{1}{\pi}\left[\int_{-\pi}^{0} \sin 3t \sin nt \, dt + \int_0^{\pi} \sin 3t \sin nt \, dt\right]$$

$$\Rightarrow b_n = \frac{1}{\pi}\left[\int_{-\pi}^{\pi} \sin 3t \sin nt \, dt\right]$$

$$\Rightarrow b_n = \frac{2}{\pi}\left[\int_0^{\pi} \sin 3t \sin nt \, dt\right]$$

$$\Rightarrow b_n = \frac{2}{2\pi}\left[\int_0^{\pi} 2\sin 3t \sin nt\,dt\right]$$

$$\Rightarrow b_n = \frac{1}{\pi}\left[\int_0^{\pi}[\cos(n-3)t - \cos(n+3)t]\,dt\right]$$

$$\Rightarrow b_n = \frac{1}{\pi}\left\{ \frac{\sin(n-3)t}{n-3}\Big|_0^{\pi} - \frac{\sin(n+3)t}{n+3}\Big|_0^{\pi} \right\}$$

$$\boxed{\Rightarrow b_n = \frac{1}{\pi}(0-0) = 0 \;\forall\; n \neq 3.}$$

for $n=3$

$$\Rightarrow b_3 = \frac{2}{2\pi}\left[\int_{-\pi}^{0} \sin 3t \cdot \sin 3t\,dt + \int_0^{\pi} \sin 3t \cdot \sin 3t\,dt\right]$$

$$\Rightarrow b_3 = \frac{1}{\pi}\left[\int_{-\pi}^{\pi} \sin^2 3t\,dt\right]$$

$$\Rightarrow b_3 = \frac{2}{\pi}\left[\int_0^{\pi} \sin^2 3t\,dt\right]$$

$$\Rightarrow b_3 = \frac{2}{\pi}\left[\int_0^{\pi} \frac{1-\cos 6t}{2}\,dt\right]$$

$$\Rightarrow b_3 = \frac{2}{\pi}\left[\frac{\pi}{2} - \frac{1}{2}\cdot\frac{\sin 6t}{6}\Big|_0^{\pi}\right] \Rightarrow \boxed{b_3 = 1}$$

Hence.

$$F(t) = b_3 \sin 3t$$

$$\boxed{F(t) = \sin 3t \quad \forall n \in \mathbb{N}}$$

(b) $f(t) = e^{-t}$, $0<t<2$.

A) (i) FOURIER COSINE SERIES.

Let $F(t)$ be a new function defined as

$$F(t) = \begin{cases} f(-t) & , \ -2<t<0. \\ f(t) & , \ 0<t<2. \end{cases}$$

and $F(t+4) = F(t)$.

where $T = 4$, $\omega = \frac{2\pi}{T} = \pi/2$

$$a_0 = \frac{2}{T}\int_0^T F(t)\,dt.$$

$$\Rightarrow a_0 = \frac{2}{4}\left[\int_{-2}^{0} f(-t)\,dt + \int_0^2 f(t)\,dt\right]$$

$$\Rightarrow a_0 = \frac{1}{2}\left[\int_{-2}^{0} e^{t}\,dt + \int_0^2 e^{-t}\,dt\right]$$

$$\boxed{\Rightarrow a_0 = \frac{1}{2}\left[1 - e^{-2} - e^{-2} + 1\right] = e^{-2}}$$

$$a_n = \frac{2}{T}\int_0^T F(t)\cos \tfrac{\pi}{2} n t \, dt.$$

$$\Rightarrow a_n = \frac{2}{4}\left[\int_{-2}^{0} f(-t)\cos n\tfrac{\pi}{2} t\, dt + \int_0^2 f(t)\cos \frac{n\pi}{2} t\, dt\right]$$

$$\Rightarrow a_n = \frac{1}{2}\left[\int_{-2}^{0} e^{t}\cos n\tfrac{\pi}{2} t\, dt + \int_0^2 e^{-t}\cos \frac{n\pi}{2} t\, dt\right]$$

$$\Rightarrow a_n = \frac{1}{2}\left[\frac{e^{t}}{1+\frac{n^2\pi^2}{4}}\left[\cos\frac{n\pi}{2}t + \frac{n\pi}{2}\sin\frac{n\pi}{2}t\right]\Bigg|_{-2}^{0} + \cdots\right.$$

$$\left.\cdots \frac{e^{-t}}{1+\frac{n^2\pi^2}{4}}\left[-\cos\frac{n\pi}{2}t + \frac{n\pi}{2}\sin\frac{n\pi t}{2}\right]\Bigg|_0^2\right]$$

$$\Rightarrow a_n = \frac{2}{n^2\pi^2+4}\left[\cos\frac{n\pi}{2} - (-1)^n e^{-2} - (-1)^n e^{-2} + \cos\frac{n\pi}{2}\right]$$

$$\Rightarrow a_n = \frac{4}{n^2\pi^2+4}\left[\cos\frac{n\pi}{2} - (-1)^n e^{-2}\right]$$

$$\boxed{\Rightarrow a_n = \frac{4}{n^2\pi^2+4}\left[\cos\frac{n\pi}{2} + (-1)^{n+1} e^{-2}\right]}$$

Hence.

$$\boxed{F(t) = \frac{e^{-2}}{2} + \sum_{n=1}^{\infty} \frac{4}{n^2\pi^2+4}\left[\cos\frac{n\pi}{2} + (-1)^n e^{-2}\right]\cos\frac{n\pi}{2}t}$$

(ii) FOURIER SINE SERIES

Let $F(t)$ be a new function defined as

$$F(t) = \begin{cases} -f(-t), & -2 < t < 0 \\ f(t), & 0 < t < 2 \end{cases}$$

and $F(t+4) = F(t)$ where $T=4$, $\omega = \frac{2\pi}{4} = \pi/2$

$$\Rightarrow b_n = \frac{2}{4}\left[\int_{-2}^{0} -f(-t)\,dt \sin\frac{n\pi}{2}t + \int_0^2 f(t)\,dt \sin\frac{n\pi}{2}t\right]$$

$$\Rightarrow b_n = \frac{1}{2}\left[\int_{-2}^{0} -e^{t}\,dt \sin\frac{n\pi}{2}t + \int_0^2 e^{-t}\,dt \sin\frac{n\pi}{2}t\right]$$

$$\Rightarrow b_n = \frac{1}{2}\left[-\int_{-2}^{0} e^{t}\sin\frac{n\pi}{2}t\,dt + \int_0^2 e^{-t}\sin\frac{n\pi}{2}t\,dt\right]$$

$$\Rightarrow b_n = \frac{1}{2}\left[-\frac{e^{t}}{1+\frac{n^2\pi^2}{4}}\left[\sin\frac{n\pi}{2}t - \frac{n\pi}{2}\cos\frac{n\pi}{2}t\right]\Bigg|_{-2}^{0} + \ldots\right.$$

$$\cdots \frac{e^{-t}}{1+\frac{n^2\pi^2}{4}}\left[-\sin\frac{n\pi}{2}t - \frac{n\pi}{2}\cos\frac{n\pi}{2}t\right]\Big|_0^2\ \Big]$$

$$b_n = \frac{2}{n^2\pi^2+4}\left[\frac{n\pi}{2}\cancel{\cos\frac{n\pi}{2}} + e^{-2}\left(-\frac{n\pi}{2}(-1)^n\right)\right] - e^{-2}\left(\frac{n\pi}{2}(-1)^n\right) + \frac{n\pi}{2}\Big]$$

$$b_n = \frac{2}{n^2\pi^2+4}\left[n\pi - e^{-2}n\pi(-1)^n\right]$$

$$\boxed{b_n = \frac{2n\pi}{n^2\pi^2+4}\left[1-e^{-2}(-1)^n\right]}$$

$$F(t) = \sum_{n=1}^{\infty}\frac{2n\pi}{n^2\pi^2+4}\left[1+e^{-2}(-1)^{n+1}\right]\sin\frac{n\pi}{2}t$$

$$\boxed{F(t) = 2\pi\sum_{n=1}^{\infty}\frac{n}{n^2\pi^2+4}\left(1+e^{-2}(-1)^{n+1}\right)\sin\frac{n\pi}{2}t}$$

Q.3) Using the Parseval's Identity prove the following.

(a) $\int_{-\pi}^{\pi}\cos^4 x\,dx = \frac{3\pi}{4}$

To prove $\int_{-\pi}^{\pi} \cos^4 x \, dx = \frac{3\pi}{4}$.

Parseval's Identity

$$\boxed{\frac{1}{T}\int_{0}^{T} [f(t)]^2 \, dt = \frac{a_0^2}{2} + \sum_{n=1}^{\infty} a_n^2 + b_n^2}$$

Let $f(x) = \cos^2 x$. and. $f(x)$ is defined over. $[-\pi, \pi]$ and $f(x+2\pi) = f(x)$. , $T = 2\pi$, $\omega = 1$

$\Rightarrow$ where a_0, a_n, b_n are Euler's formula - coefficients of fourier Expansion.

$$a_0 = \frac{2}{T}\int_{0}^{T} f(t)\, dt$$

$$\Rightarrow a_0 = \frac{2}{2\pi}\int_{-\pi}^{\pi} \cos^2 t \, dt \Rightarrow a_0 = \frac{1}{\pi}\int_{-\pi}^{\pi} \cos^2 x \, dx.$$

$$\Rightarrow a_0 = \frac{1}{\pi}\int_{-\pi}^{\pi} \frac{1+\cos 2x}{2} \, dx.$$

$$\Rightarrow a_0 = \frac{1}{\pi}\left[\frac{x}{2}\Big|_{-\pi}^{\pi} + \frac{1}{4}\sin 2x\Big|_{-\pi}^{\pi}\right] \Rightarrow a_0 = \frac{1}{\pi}\cdot\frac{1}{2}(2\pi)$$

$\Rightarrow \boxed{a_0 = 1}$

$$\Rightarrow a_n = \frac{2}{2\pi}\int_{-\pi}^{\pi} f(t)\cos n\omega t\,dt$$

$$\Rightarrow a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} \cos^2 t \cdot \cos nt\,dt$$

$$\Rightarrow a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} \left[\frac{1+\cos 2t}{2}\right]\cos nt\,dt$$

$$\Rightarrow a_n = \frac{1}{\pi}\left[\int_{-\pi}^{\pi} \frac{\cos nt}{2}\,dt + \frac{1}{2}\int_{-\pi}^{\pi} \cos 2t\cos nt\,dt\right]$$

$$\Rightarrow a_n = \frac{1}{\pi}\left[\frac{1}{2}\left.\frac{\sin nt}{n}\right|_{-\pi}^{\pi} + \frac{1}{4}\int_{-\pi}^{\pi}\left[\cos(n+2)t - \cos(n-2)t\right]dt\right]$$

$$\Rightarrow a_n = \frac{1}{\pi}\left[\frac{1}{2}(0) + \frac{1}{4}\left[\left.\frac{\sin(n+2)t}{(n+2)}\right|_{-\pi}^{\pi} - \left.\frac{\sin(n-2)t}{(n-2)}\right|_{-\pi}^{\pi}\right]\right]$$

$\forall n \neq 2$

$\Rightarrow \boxed{a_n = 0 \quad \forall n \neq 2,\ n \in R}$

for $n = 2$

$$a_2 = \frac{2}{T}\int_{0}^{T} f(t)\cos 2\omega t\,dt$$

$\Rightarrow a_2 = \frac{2}{2\pi}\int_{-\pi}^{\pi} \cos^2 t \cdot \cos 2t\, dt$

$\Rightarrow a_2 = \frac{1}{\pi}\int_{-\pi}^{\pi}\left[\frac{1+\cos 2t}{2}\right]\cos 2t\, dt$

$\Rightarrow a_2 = \frac{1}{\pi}\left[\int_{-\pi}^{\pi}\frac{\cos 2t}{2}dt + \frac{1}{2}\int_{-\pi}^{\pi}\cos^2 2t\, dt\right]$

$\Rightarrow a_2 = \frac{1}{\pi}\left[\frac{1}{2}\frac{\sin 2t}{2}\Big|_{-\pi}^{\pi} + \frac{1}{4}(1+\cos 4t)dt\Big|_{-\pi}^{\pi}\right]$

$\Rightarrow a_2 = \frac{1}{\pi}\left[\frac{1}{2}(0) + \frac{1}{4}\left[t\Big|_{-\pi}^{\pi} + \frac{\sin 4t}{4}\Big|_{-\pi}^{\pi}\right]\right]$

$\Rightarrow a_2 = \frac{1}{\pi}\left[0 + \frac{1}{4}[\pi+\pi] + \frac{1}{16}(0)\right]$

$\Rightarrow \boxed{a_2 = 1/2}$

$\Rightarrow b_n = \frac{2}{T}\int_0^T f(t)\sin n\omega t\, dt$

$\Rightarrow b_n = \frac{2}{2\pi}\int_{-\pi}^{\pi}\cos^2 t \sin nt\, dt$

$\Rightarrow b_n = 1/\pi \int_{-\pi}^{\pi}\left(\frac{1+\cos 2t}{2}\right)\sin nt\, dt$

$$\Rightarrow b_n = \frac{1}{\pi}\int_{-\pi}^{\pi}\left(\frac{1+\cos 2t}{2}\right)\sin nt\,dt$$

$$\Rightarrow b_n = \frac{1}{\pi}\left\{\int_{-\pi}^{\pi}\frac{\sin nt}{2}\,dt + \frac{1}{2}\int_{-\pi}^{\pi}\cos 2t\sin nt\,dt\right\}$$

$$\Rightarrow b_n = \frac{1}{\pi}\left\{-\frac{1}{2}\frac{\cos nt}{n}\Big|_{-\pi}^{\pi} + \frac{1}{4}\int_{-\pi}^{\pi}2\cos 2t\sin nt\,dt\right\}$$

$$\Rightarrow b_n = \frac{1}{\pi}\left\{-\frac{1}{2n}\left[(-1)^n-(-1)^n\right] + \frac{1}{4}\int_{-\pi}^{\pi}\left[\sin(n+2)t+\sin(n-2)t\right]dt\right\}$$

$$\Rightarrow b_n = \frac{1}{\pi}\left\{\frac{1}{4}\left(\frac{-\cos(n+2)t}{(n+2)}\Big|_{-\pi}^{\pi} + \frac{-\cos(n-2)t}{(n-2)}\Big|_{-\pi}^{\pi}\right)\right\}$$

$$\Rightarrow b_n = \frac{1}{4\pi}\left\{-\frac{1}{(n-2)}\left[(-1)^{n-2}-(-1)^{n-2}\right] - \frac{1}{(n+2)}\left[(-1)^{n+2}-(-1)^{n+2}\right]\right\}$$

$$\Rightarrow \boxed{b_n = 0 \ \forall\ n \neq 2}$$

for $n=2$

$$\Rightarrow b_2 = \frac{2}{T}\int_0^T f(t)\sin 2\omega t\,dt$$

$$\Rightarrow b_2 = \frac{2}{2\pi}\int_{-\pi}^{\pi}\cos^2 t\sin 2t\,dt$$

$$\Rightarrow b_2 = \frac{1}{\pi}\int_{-\pi}^{\pi} \frac{1+\cos 2t}{2}(\sin 2t)\,dt$$

$$\Rightarrow b_2 = \frac{1}{2\pi}\int_{-\pi}^{\pi} \sin 2t + \sin 2t\cos 2t\,dt$$

$$\Rightarrow b_2 = \frac{1}{2\pi}\left.\frac{-\cos 2t}{2}\right|_{-\pi}^{\pi} + \frac{1}{4\pi}\int_{-\pi}^{\pi}\sin 4t\,dt$$

$$\Rightarrow b_2 = \frac{-1}{4\pi}\left[1-1\right] + \frac{1}{4\pi}\left.\frac{-\cos 4t}{4}\right|_{-\pi}^{\pi}$$

$$\Rightarrow b_2 = \frac{-1}{4\pi}(0) + -\frac{1}{16\pi}\left[1-1\right] = 0.$$

$$\Rightarrow \boxed{b_2 = 0}$$

Hence, from Parseval's Identity

$$\Rightarrow \frac{1}{T}\int_{0}^{T}|f(t)|^2\,dt = \frac{a_0^2}{2} + \sum_{n=1}^{\infty} a_n^2 + b_n^2$$

$$\Rightarrow \frac{1}{\pi}\int_{-\pi}^{\pi}\cos^4 x\,dx = \frac{1}{2} + a_2^2$$

$$\Rightarrow \boxed{\int_{-\pi}^{\pi}\cos^4 x\,dx = \pi\left[\frac{1}{2}+\frac{1}{4}\right] = \frac{3\pi}{4}.}$$

(ii) $f(t) = \begin{cases} +1, & -\pi/2 < t < \pi/2 \\ -1, & \pi/2 < t < 3\pi/2 \end{cases}$, Deduce that

$$1 + \frac{1}{9} + \frac{1}{25} + \cdots = \frac{\pi^2}{8}.$$

A) Given $f(t) \rightarrow$ defined over $\left(\frac{-\pi}{2}, \frac{3\pi}{2}\right)$,

Let us assume that

$$f(t+2\pi) = f(t), \quad T = 2\pi$$

$$\omega = \frac{2\pi}{T} = 1$$

by Parseval's Identity

$$\boxed{T \int_0^T |f(t)|^2 dt = \frac{a_0^2}{2} + \sum_{n=1}^{\infty} a_n^2 + b_n^2}$$

where a_0, a_n, b_n are Euler's [Expansion] coefficients

$$\Rightarrow a_0 = \frac{2}{T} \int_0^T f(t)\,dt$$

$$\Rightarrow a_0 = \frac{2}{2\pi}\left[\int_{-\pi/2}^{\pi/2} dt + \int_{\pi/2}^{3\pi/2} -dt\right] \Rightarrow \boxed{a_0 = 0.}$$

$$\Rightarrow a_n = \frac{2}{T}\int_0^T f(t)\cos n\omega t\,dt$$

$$\Rightarrow a_n = \frac{2}{2\pi}\left\{\int_{-\pi/2}^{\pi/2}\cos nt\,dt + \int_{\pi/2}^{3\pi/2} -\cos nt\,dt\right\}$$

$$\Rightarrow a_n = \frac{1}{\pi}\left\{+\frac{\sin nt}{n}\Big|_{-\pi/2}^{\pi/2} - \frac{\sin nt}{n}\Big|_{\pi/2}^{3\pi/2}\right\}$$

$$\Rightarrow a_n = \frac{1}{\pi}\left\{\frac{1}{n}\left[\sin\frac{n\pi}{2} + \sin\frac{n\pi}{2}\right] - \frac{1}{n}\left[\sin\frac{3n\pi}{2} - \sin\frac{n\pi}{2}\right]\right\}$$

$$\Rightarrow a_n = \frac{1}{\pi}\left\{\frac{2}{n}\sin\frac{n\pi}{2} - \frac{1}{n}\sin\frac{3n\pi}{2} + \frac{1}{n}\sin\frac{n\pi}{2}\right\}$$

$$\Rightarrow a_n = \frac{1}{\pi}\left\{\frac{3}{n}\sin\frac{n\pi}{2} - \frac{1}{n}\sin\frac{3n\pi}{2}\right\}$$

$$\boxed{\Rightarrow a_n = \frac{1}{n\pi}\left\{3\sin\frac{n\pi}{2} - \sin\frac{3n\pi}{2}\right\}}$$

$$\Rightarrow b_n = \frac{2}{T}\int_0^T f(t)\sin n\omega t\,dt$$

$$\Rightarrow b_n = \frac{2}{2\pi}\left[\int_{-\pi/2}^{\pi/2} \sin nt\,dt \int_{\pi/2}^{3\pi/2} -\sin nt\,dt\right]$$

$$\Rightarrow b_n = \frac{1}{\pi}\left[\int_{-\pi/2}^{\pi/2} \sin nt\,dt - \int_{\pi/2}^{3\pi/2} \sin nt\,dt\right]$$

$$\Rightarrow b_n = \frac{1}{\pi}\left[-\frac{\cos nt}{n}\Big|_{-\pi/2}^{\pi/2} + \frac{\cos nt}{n}\Big|_{\pi/2}^{3\pi/2}\right]$$

$$\Rightarrow b_n = \frac{1}{\pi}\left[\frac{1}{n}\left(\cos\frac{3n\pi}{2} - \cos\frac{n\pi}{2}\right) - \frac{1}{n}\left[\cos\frac{n\pi}{2} - \cos\frac{n\pi}{2}\right]\right]$$

$$\Rightarrow \boxed{b_n = \frac{1}{n\pi}\left\{\cos\frac{3n\pi}{2} - \cos\frac{n\pi}{2}\right\}}$$

$$\Rightarrow \frac{1}{\pi}\int_{-\pi/2}^{3\pi/2} |f(t)|^2\,dt = \frac{a_0^2}{2} + \sum_{n=1}^{\infty} a_n^2 + b_n^2$$

$$\Rightarrow \frac{1}{\pi}\int_{-\pi/2}^{3\pi/2} |f(t)|^2\,dt = \sum_{n=1}^{\infty} \frac{1}{n^2\pi^2}\left(3\sin\frac{n\pi}{2} - \sin\frac{3n\pi}{2}\right)^2 + \frac{1}{n^2\pi^2}\left(\cos\frac{3n\pi}{2} - \cos\frac{n\pi}{2}\right)$$

$$\Rightarrow \frac{1}{\pi}\int_{-\pi/2}^{3\pi/2} |f(t)|^2 dt = \frac{1}{\pi^2}\sum_{n=1}^{\infty} \frac{\left(3\sin\frac{n\pi}{2} - \sin\frac{3n\pi}{2}\right)^2 + \left(\cos\frac{3n\pi}{2} - \cos\frac{n\pi}{2}\right)^2}{n^2}$$

The R.H.S can be re-written as

$$\Rightarrow \frac{1}{\pi^2}\sum_{n=1}^{\infty} \frac{9\sin^2\frac{n\pi}{2} + \sin^2\frac{3n\pi}{2} - 6\sin\frac{n\pi}{2}\sin\frac{3n\pi}{2} + \dots + \cos^2\frac{3n\pi}{2} + \cos^2\frac{n\pi}{2} - 2\cos\frac{n\pi}{2}\cos\frac{3n\pi}{2}}{n^2}$$

$$\Rightarrow \frac{1}{\pi^2}\sum_{n=1}^{\infty} \frac{\left(8\sin^2\frac{n\pi}{2} + 1 + 1 - 6\sin\frac{n\pi}{2}\sin\frac{3n\pi}{2} + \dots - 2\cos n\pi/2 \cos\frac{3n\pi}{2}\right)}{n^2}$$

$$\Rightarrow \frac{1}{\pi^2}\sum_{n=1}^{\infty} \frac{2 + 8\sin^2\frac{n\pi}{2} - 4\sin\frac{n\pi}{2}\sin\frac{3n\pi}{2} - 2\cos(n\pi)}{n^2}$$

$$\Rightarrow \frac{1}{\pi^2}\sum_{n=1}^{\infty} \frac{2 + 8\sin^2\frac{n\pi}{2} - 4\sin\frac{n\pi}{2}\sin\frac{3n\pi}{2} - 2(-1)^n}{n^2}$$

$$\Rightarrow \frac{1}{\pi^2}\sum_{n=1}^{\infty} \frac{2 + 2\sin^2\frac{n\pi}{2} - \sin\frac{n\pi}{2}\sin\frac{3n\pi}{2} - 2(-1)^n}{n^2}$$

$$\Rightarrow \frac{1}{\pi^2}\sum_{n=1}^{\infty}\frac{2+2(-1)^{n+1}+8\sin^2\frac{n\pi}{2}-4\sin\frac{n\pi}{2}\sin\frac{3n\pi}{2}}{n^2}$$

$$\Rightarrow \boxed{\frac{1}{\pi^2}\left[\frac{(4)^2}{1^2}+\frac{(4)^2}{3^2}+\frac{(4)^2}{5^2}+\cdots\right]}$$

The L.H.S can be simplified as.

$$\Rightarrow \frac{1}{\pi}\left[\int_{\pi/2}^{\pi/2} dt+\int_{\pi/2}^{3\pi/2} dt\right] \Rightarrow \frac{1}{\pi}\left[\pi+\pi\right]=2.$$

$$\Rightarrow 2=\frac{1}{\pi^2}\left[\frac{(4)^2}{1^2}+\frac{(4)^2}{3^2}+\frac{(4)^2}{5^2}+\cdots\right]$$

$$\boxed{\therefore \frac{1}{1^2}+\frac{1}{3^2}+\frac{1}{5^2}+\cdots=\frac{\pi^2}{8}}$$

Q-4) Find the first two harmonics for the following data.

(a)

t	0	$\pi/2$	π	$3\pi/2$
$f(t)$	1	2	3	4

A) as given.

Let's assume $f(t)$ is periodic function with period 2π, Hence.

$f(2\pi) = f(0) = 1$ $\qquad T = 2\pi,\ \omega = \frac{2\pi}{2\pi} = 1$

So, now. $\qquad \omega = 1$

t	$f(t)$.	$\cos t$	$\sin t$	$\cos 2t$	$\sin 2t$
0	1	1	0	1	0.
$\pi/2$	2	0	1	-1	0.
π	3	-1	0	1	0
$3\pi/2$	4.	0	-1	-1	0.

Also, N = number of divisions. $= 4$.

Hence,

$$\Rightarrow \boxed{a_0 = \frac{2}{N} \sum_{i=1}^{N} f(x_i)}$$

$$\Rightarrow a_0 = \frac{2}{4}\left[f(x_1) + f(x_2) + f(x_3) + f(x_4)\right]$$

$$\Rightarrow a_0 = \frac{1}{2}\left[1+2+3+4\right] = 5$$

$$\Rightarrow \boxed{a_n = \frac{2}{N} \sum_{i=1}^{N} f(x_i) \cos n x_i.}$$

$\Rightarrow a_1 = \frac{1}{2}\left[1-3\right] = -1.$

$\Rightarrow a_2 = \frac{1}{2}\left[1-2+3-4\right] = -1$

$$\Rightarrow \boxed{b_n = \frac{2}{N}\sum_{N=1}^{N} f(x_i)\sin(nx_i)}$$

$\Rightarrow b_1 = \frac{2}{4}\left[2-4\right] \Rightarrow b_1 = -1.$

$\Rightarrow b_2 = \frac{2}{4}\left[0\right] \Rightarrow b_2 = 0.$

Hence,

$$\boxed{f(t) = \frac{5}{2} - \cos t - \sin t - \cos 2t}$$

(b)

t	0	$\pi/3$	$2\pi/3$	π	$\frac{4\pi}{3}$	$5\pi/3$
$f(t)$	3	4	5	3	-4	-12

A) as given.

Let's assume $f(t)$ is periodic function with period 2π, Hence.

$f(2\pi) = f(0) = 3.$

$T = 2\pi, \ \omega = 1$

So, now.

t	f(t)	cost	sint	cos2t	sin2t
0	3	1	0.	1	0.
$\pi/3$	4.	$1/2$	$\sqrt{3}/2$	$-1/2$	$\sqrt{3}/2$
$2\pi/3$	5	$-1/2$	$\sqrt{3}/2$	$-1/2$	$-\sqrt{3}/2$
π	3	-1	0.	1	0
$4\pi/3$	-4	$-1/2$	$-\sqrt{3}/2$	$-1/2$	$\sqrt{3}/2$
$5\pi/3$	-12	$1/2$	$-\sqrt{3}/2$	$-1/2$	$-\sqrt{3}/2$

Also, N = number of divisions = 6.

Hence,

$\Rightarrow a_0 = \frac{2}{N}\sum_{i=1}^{N} f(x_i) = \frac{1}{3}\left[3+4+5+3-4-12\right] = -1/3$

$\Rightarrow a_1 = \frac{2}{N}\sum_{i=1}^{N} f(x_i)\cos(x_i) = \frac{1}{3}\left[3+\frac{4}{2}-\frac{5}{2}-3+\frac{4}{2}+\frac{12}{2}\right] = -3/2$

$\Rightarrow b_1 = \frac{2}{N}\sum_{i=1}^{N} f(x_i)\sin(x_i) = \frac{1}{3}\left[\frac{4\sqrt{3}}{2}+\frac{5\sqrt{3}}{2}+\frac{4\sqrt{3}}{2}+\frac{12\sqrt{3}}{2}\right] = \frac{25\sqrt{3}}{6}$

$\Rightarrow a_2 = \frac{2}{N}\sum_{i=1}^{N} f(x_i)\cos(2x_i) = 1/3\left[3-4/2-5/2+3+\frac{4}{2}+\frac{12}{2}\right] = \frac{7}{6}$

$\Rightarrow b_2 = \frac{2}{N}\sum_{i=1}^{N} f(x_i)\sin(2x_i) = 1/3\left[\frac{4\sqrt{3}}{2}-\frac{5\sqrt{3}}{2}-\frac{4\sqrt{3}}{2}+\frac{12\sqrt{3}}{2}\right] = \frac{7\sqrt{3}}{6}$

Hence,

$$\boxed{f(t) = -1/6 - \frac{3}{2}\cos t + \frac{25\sqrt{3}}{6}\sin t + \frac{7}{6}\cos 2t + \frac{7\sqrt{3}}{6}\sin 2t}$$