KYPT 2019 2 차 연구보고서

4. Funnel and Ball

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1. Question

A light ball (e.g. ping-pong ball) can be picked up with a funnel by blowing air through it. Explain the phenomenon and investigate the relevant parameters.

2. Objective

In this experiment, using funnels of various dimensions and materials, the system of funnel and ball levitation will be investigated for diverse parameters in how the pressure on the ball changes.

3. Theory

3.1 Bernoulli's Equation:

$$P_1 + \frac{1}{2}\rho {v_1}^2 + \rho g h_1 = P_2 + \frac{1}{2}\rho {v_2}^2 + \rho g h_2 \tag{1}$$

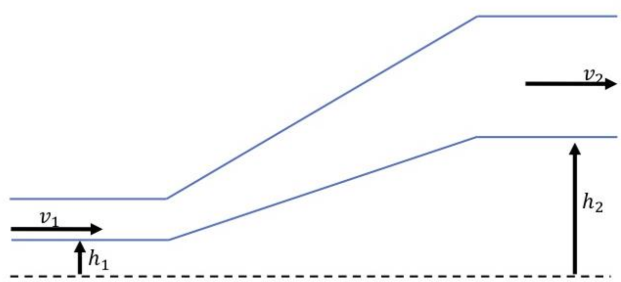

Figure 1. Bernoulli's Equation diagram

P is the pressure of the fluid at a given point in kPa,
ρ is the density of the fluid in kg/m^3,
v is the velocity of the fluid at a given point in m/s,
g is the gravitational acceleration in m/s^2,
h is the vertical displacement of the fluid at a given point in m.

Bernoulli's principle states that the speed of a fluid is indirectly proportional to pressure or the fluid's potential energy. Assuming that the vertical displacement of the fluid stays constant, if the velocity of a fluid is higher at one point than at another point, then the pressure is lower at that point. The mechanism of Bernoulli's Equation is seen in many places, including the lifting mechanism of airplanes. Air flows faster above the plane than below the plane, causing the pressure to be higher below the plane, thus lifting the airplane.

In the experiment, the velocity of air is faster between the ball and the funnel than below the ball, thus the pressure is higher below the ball, causing the ball to levitate off the ground when air is blown through the funnel.

3.2 Continuity Equation:

$$A_1 v_1 = A_2 v_2 \tag{2}$$

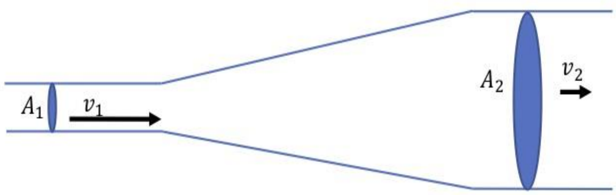

Figure 2. Continuity Equation diagram

A is the surface area of the fluid at a given point,
v is the velocity of the fluid at a given point.

The Continuity Equation states that the volume rate flow (m^3/s) is constant along the flow tube. If at one point in the tube, the surface area is larger than at another point, then the velocity of the fluid at that point would be slower, because velocity (m/s) times surface area (m^2) equals volume rate flow (m^3/s), which must be kept constant.

In the experiment, the surface area of the path of air is smallest between the ball and the funnel, and the largest below the ball, thus the velocity of air would be the fastest between the ball and the funnel and the slowest below the ball.

Theoretical Modeling:

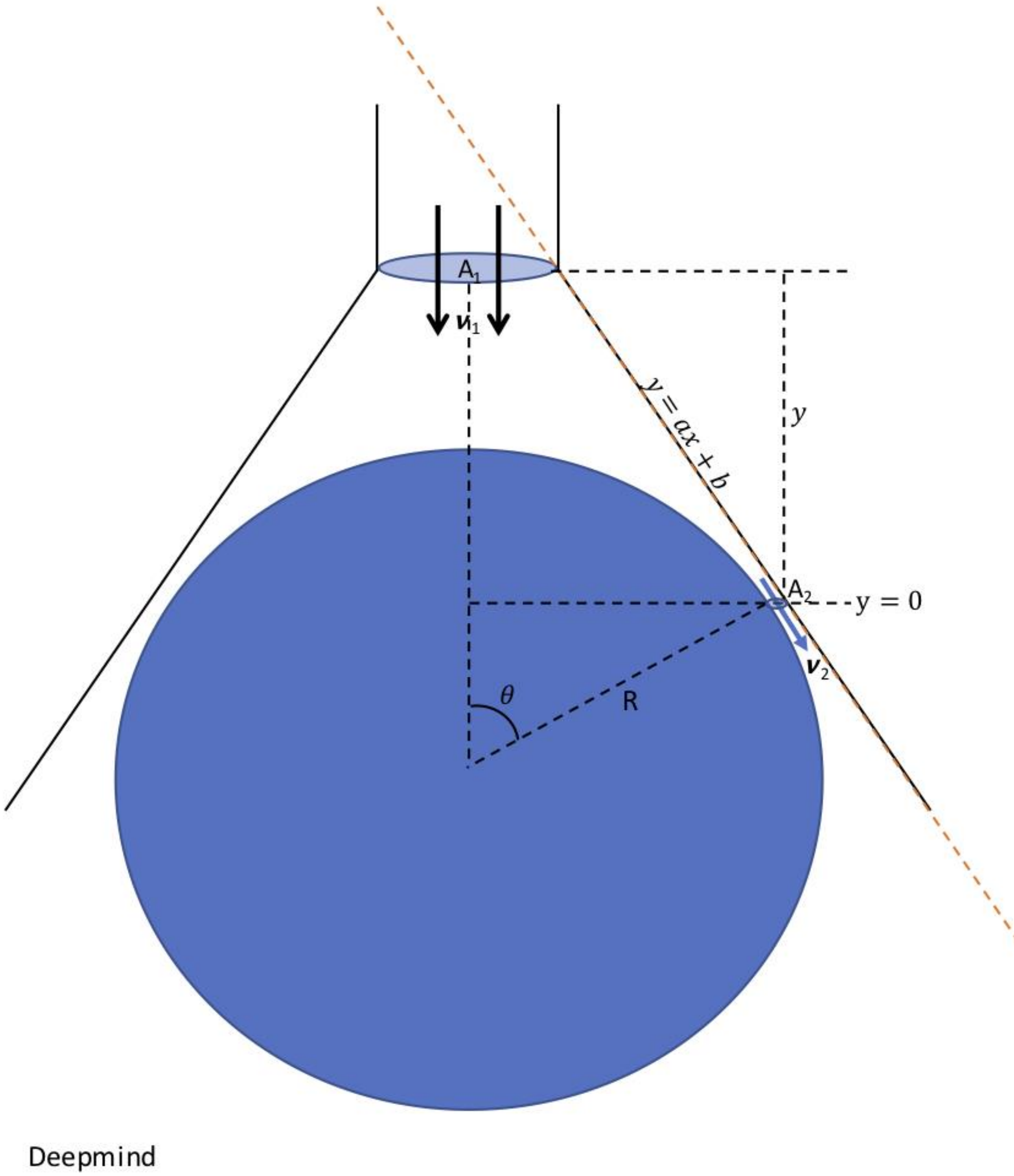

Figure 3. Theoretical Model of the funnel and ball system

To begin with, one of the funnel's legs is defined with a linear function with undetermined coefficients a and b. Using this is linear equation and the continuity equation:

$$\pi x^2 - \pi (R \sin \theta)^2 = A_2 \tag{3.1}$$

$$\upsilon_2 = \frac{A_1 \upsilon_1}{\pi x^2 - \pi (R \sin \theta)^2} \tag{3.2}$$

Substitute the υ_2 value into the Bernoulli's equation between υ_1 and υ_2.

$$P_2 = \rho_0 + \frac{1}{2}\rho_0 {\upsilon_1}^2 + \rho_0 g y - \frac{1}{2}\rho_0 (\frac{A_1 \upsilon_2}{\pi x^2 - \pi (R \sin \theta)^2})^2 \tag{4}$$

The pressure on the sphere has to be integrated with dA to calculate the force applied on to the sphere's top half.

$$f = \int P_2 cos\theta \;\; dA \tag{5.1}$$

$$f = \rho_0 r^2 \int_0^{\frac{\pi}{2}} (\left(\frac{A_1 \upsilon_2}{2(\pi x^2 - \pi (R \sin \theta)^2)}\right)^2 + gy + \frac{1}{2{\upsilon_1}^2} + 1)(\sin\theta \cos\theta) \;\; d\theta \int_0^{2\pi} d\,\varphi \tag{5.2}$$

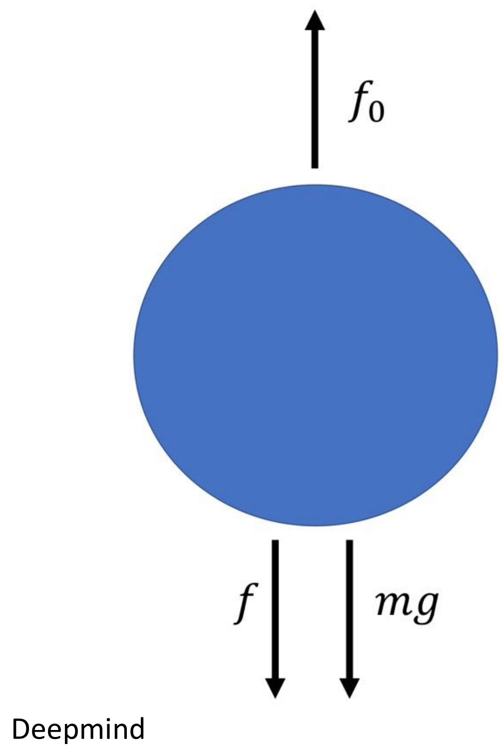

Figure 4. Forces acting on the ball

The f_0 value minus the $f + mg$ value would equal the net force on the ball upwards.

$$\Sigma F = f_0 - (f + mg) \tag{6}$$

4. Experiment design

4.1. Parameters

Experiment	**#1**				**#2**	**#3**
Ball Mass (g)	2.7				2.7	2.7
Ball Diameter (cm)	4				4	4
Funnel Height (cm)	14	18	22	26	18	18

<table>
<tr><td>Funnel Angle (degrees)</td><td>10</td><td>10</td><td>15</td><td>20</td><td colspan="4">10</td></tr>
<tr><td>Wind Velocity (m/s)</td><td>25</td><td colspan="3">25</td><td>15</td><td>20</td><td>25</td><td>30</td></tr>
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Table 1. Experimental Parameters

4.2. Hypothesis

4.2.1 Wind Velocity Experiment

As the wind velocity increases, the pressure at the top will decrease, causing the ping pong to be lifted easily.

4.2.2 Funnel Height Experiment

As the funnel height increases, the pressure at the top will decrease, causing the ping pong to be lifted easily.

4.2.3 Funnel Angle Experiment

As the funnel angle increases, the pressure at the top will decrease, causing the ping pong to be lifted easily.

5. Experiment

5.1 Required Materials

- Vacuum Cleaner
- Long pipe with three openings
- Tape
- Plastic wrapping
- Pressure gauge
- Protractor
- Paper
- Scissor

- Ping pong ball
- Wind speed gauge

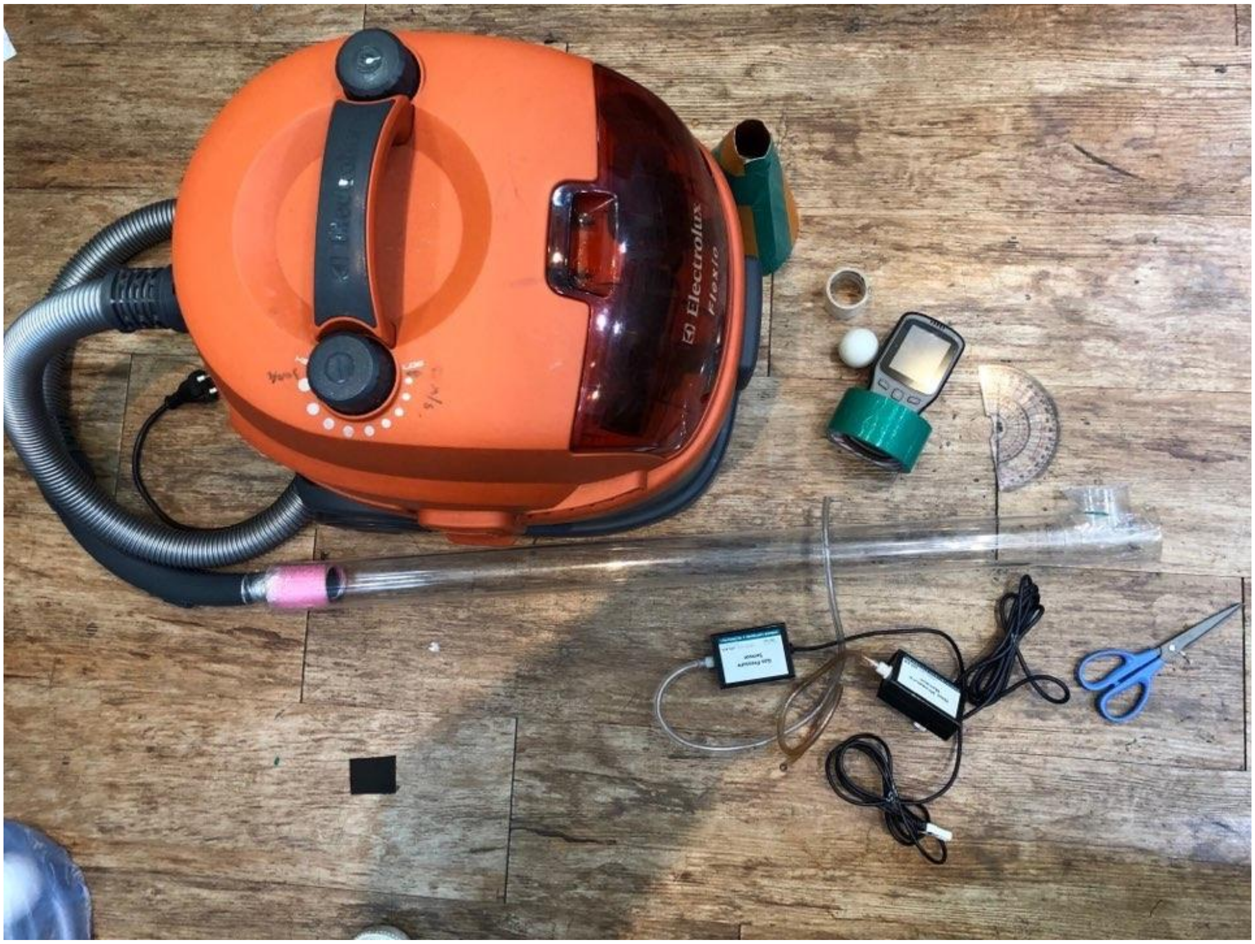

Figure 5. Lab materials

5.2 Procedure

Set-up:

1. Make 3 funnels of height 18 cm and angles 10, 15, and 20

2. Make 1 funnel with height 26 cm and angle 10

3. Attach the opening of the vacuum cleaner to the furthest hole in the pipe

Wind Velocity Experiment:

1. Attach the 26 cm funnel to the bottom hole of the pipe with tape and plastic wrap

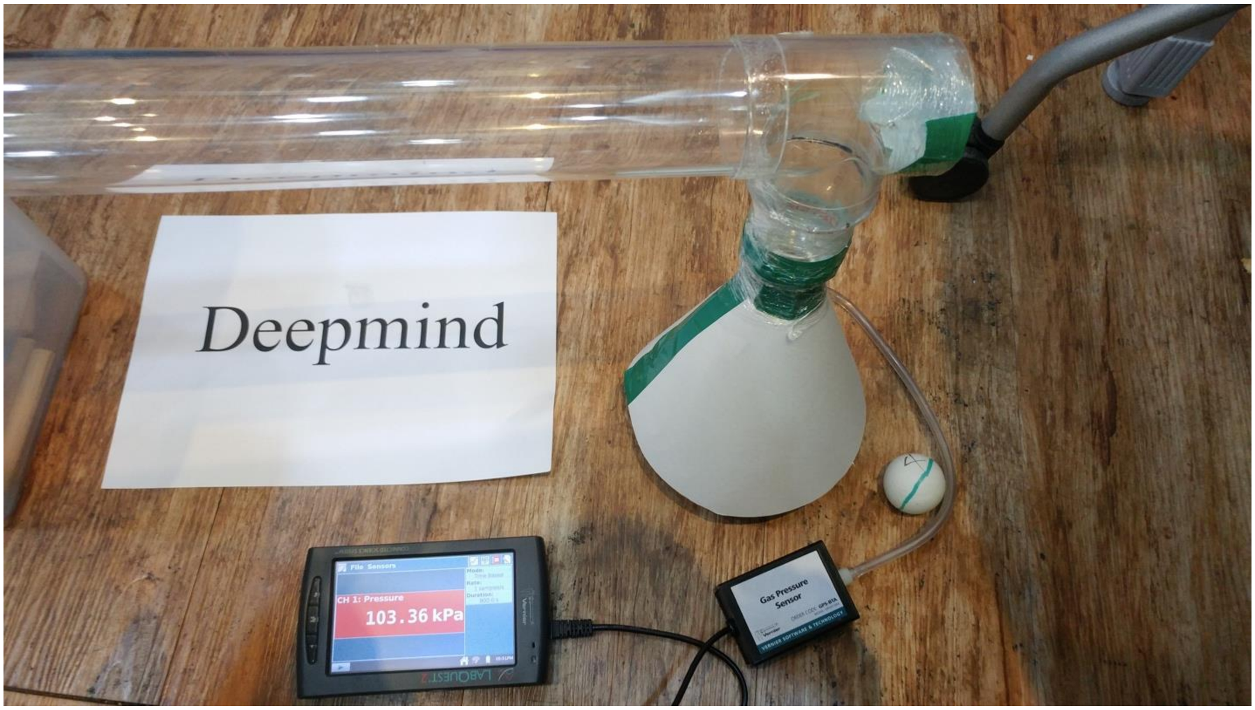

Figure 6. Lab setup

2. Block the other hole of the pipe with tape
3. With the wind gauge, measure 15 m/s wind speed
4. Tape the sensor for the pressure gauge at the top of the inside of the funnel

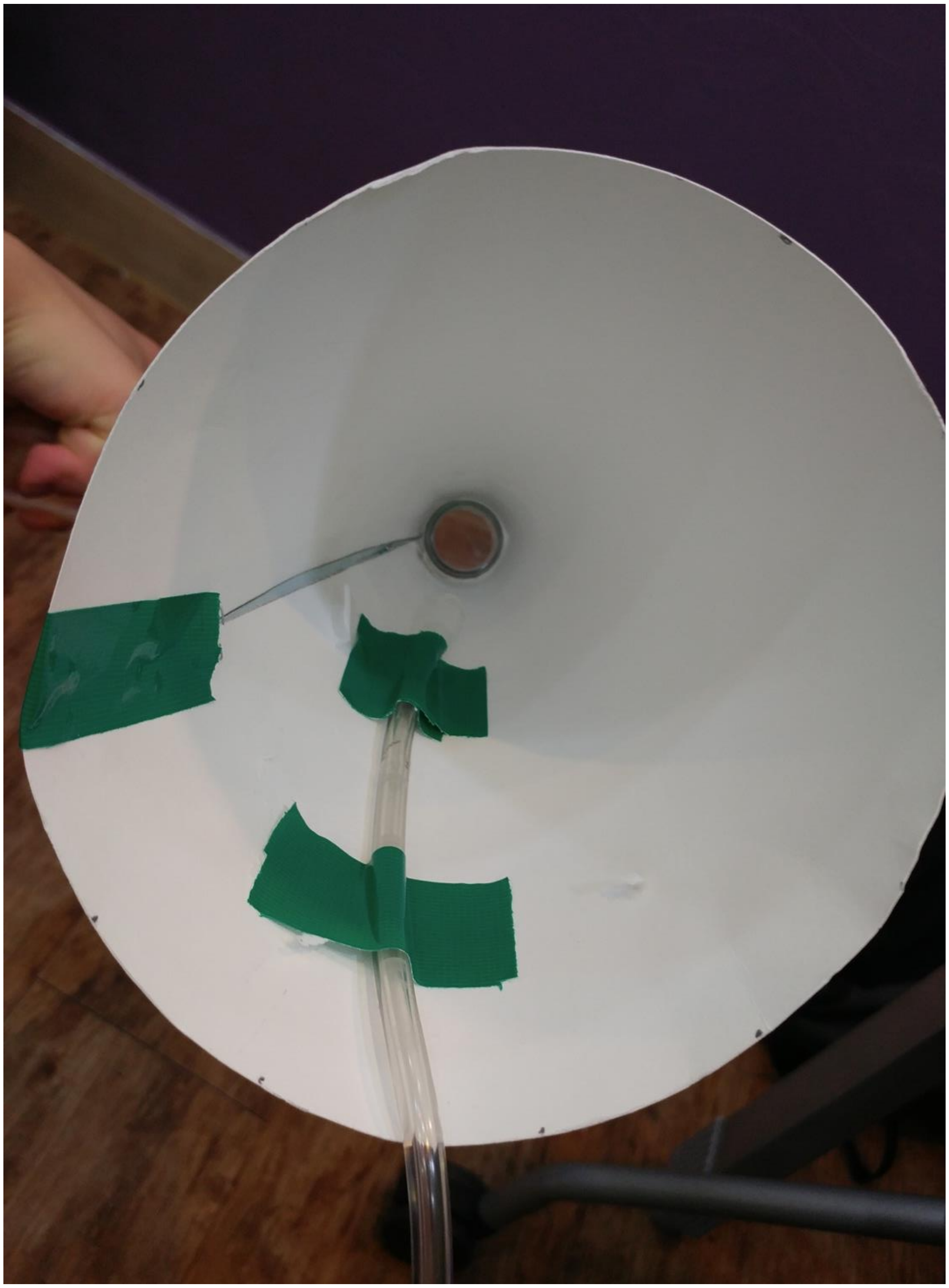

Figure 7. Pressure gauge placement

5. Set the funnel on the ground

6. Turn the vacuum cleaner on

7. Record the observed pressure

8. Repeat steps 5-7 with 20, 25, and 30 m/s wind speed

Funnel Height Experiment:

1. Attach the 26 cm funnel to the bottom hole of the pipe with tape and plastic wrap
2. Measure 25 m/s wind speed
3. Repeat steps 4-7 of the previous experiment
4. Repeat step 3 after cutting out 4cm of the funnel with a scissor

Funnel Angle Experiment:

1. Attach the 18cm 10 degrees funnel to the bottom hole of the pipe with tape and plastic wrap
2. Measure 25 m/s wind speed
3. Repeat steps 4-7 of the previous experiment
4. Repeat step 3 with the 18cm 15 degrees and 20 degrees funnel

6. Experiment Results

Pressure below the ball had a steady pressure of 104 kPa.

6.1 Wind Velocity Experiment

Wind Speed (m/s)	Trial 1 (kPa)	Trial 2 (kPa)	Trial 3 (kPa)	Average Pressure (kPa)	Standard Error	ΣF Upwards (N) (mg = 0.02646N)
15	**103.88**	**103.91**	**103.87**	**103.887**	**± 0.012**	**0.08654**
20	**103.73**	**103.78**	**103.77**	**103.76**	**± 0.0153**	**0.2135**
25	**103.66**	**103.57**	**103.63**	**103.62**	**± 0.0265**	**0.3535**
30	**103.21**	**103.19**	**103.17**	**103.19**	**± 0.01155**	**0.7835**

Table 2. Wind velocity lab experimental data

Using the Equation (5.2), the theoretical values for the pressure of air in the funnel was calculated:

Wind Speed (m/s)	Theoretical Pressure (kPa)
15	**103.81**
20	**103.72**
25	**103.55**

30	**103.24**

Table 3. Wind velocity lab theoretical data

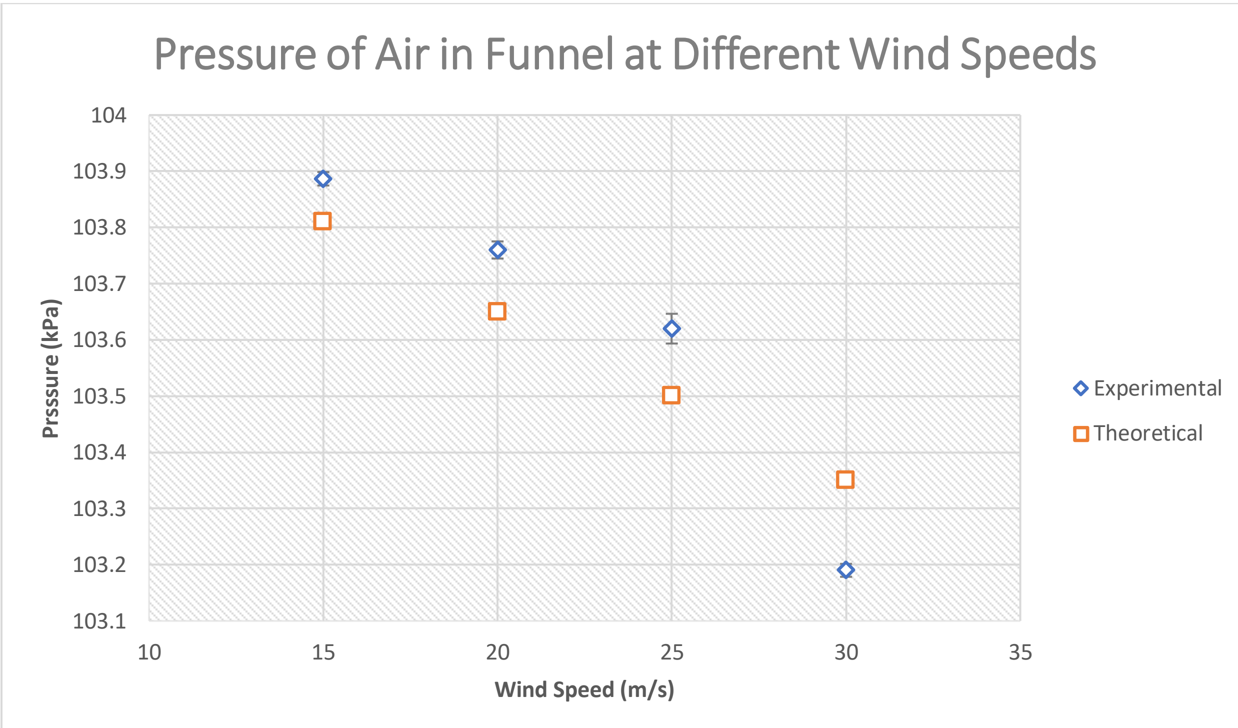

Figure 8.

6.2 Funnel Height Experiment

Funnel Height (cm)	Trial 1 (kPa)	Trial 2 (kPa)	Trial 3 (kPa)	Trial 4 (kPa)	Average Pressure (kPa)	Standard Error	ΣF Upwards (N) (mg = 0.02646N)
14	**103.05**	**103.11**	**103.09**	**103.16**	**103.1025**	**± 0.0229**	**0.8710**
18	**103.02**	**103.06**	**103.07**	**103.05**	**103.05**	**± 0.0108**	**0.9235**
22	**102.96**	**103.01**	**102.97**	**102.95**	**102.9725**	**± 0.0131**	**1.0010**
26	**102.90**	**102.92**	**102.92**	**102.93**	**102.9175**	**± 0.0063**	**1.0560**

Table 4. Funnel height lab experimental data

Using the Equation (5.2), the theoretical values for the pressure of air in the funnel was calculated:

Funnel Height (cm)	Theoretical Pressure (kPa)
14	**103.81**
18	**103.72**
22	**103.55**
26	**103.24**

Table 5. Funnel height lab theoretical data

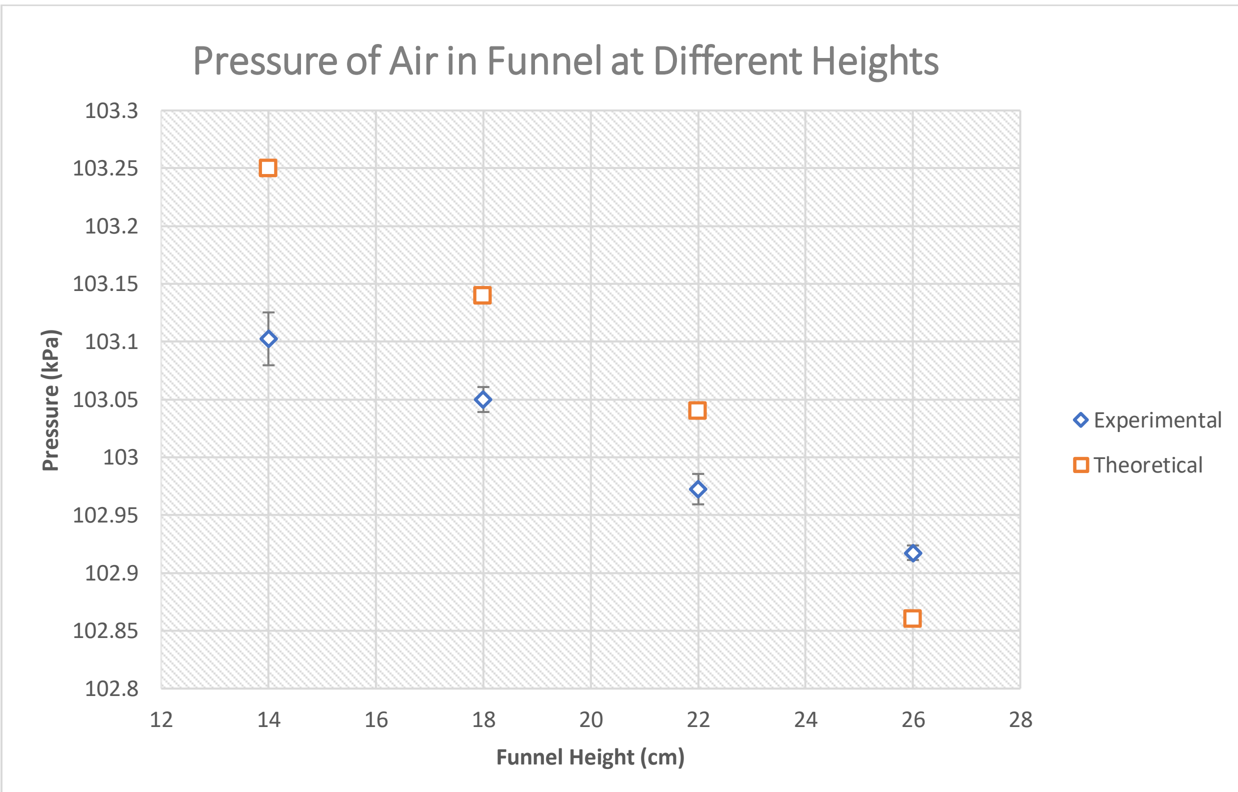

Figure 9.

6.3 Funnel Angle Experiment

Funnel Angle (degrees)	Trial 1 (kPa)	Trial 2 (kPa)	Trial 3 (kPa)	Trial 4 (kPa)	Average Pressure (kPa)	Standard Error	ΣF Upwards (N) (mg = 0.02646N)
10	**103.02**	**103.06**	**103.06**	**103.1**	**103.06**	**± 0.0163**	**0.3735**
15	**103.02**	**102.96**	**102.95**	**103.08**	**103.0025**	**± 0.0301**	**0.9710**

20	**102.96**	**102.92**	**102.95**	**102.94**	**102.9425**	**± 0.0085**	**1.0310**

Table 6. Funnel angle lab experimental data

Using the Equation (5.2), the theoretical values for the pressure of air in the funnel was calculated:

Funnel Angle (degrees)	Theoretical Pressure (kPa)
10	**103.81**
15	**103.72**
20	**103.55**

Table 7. Funnel angle lab theoretical data

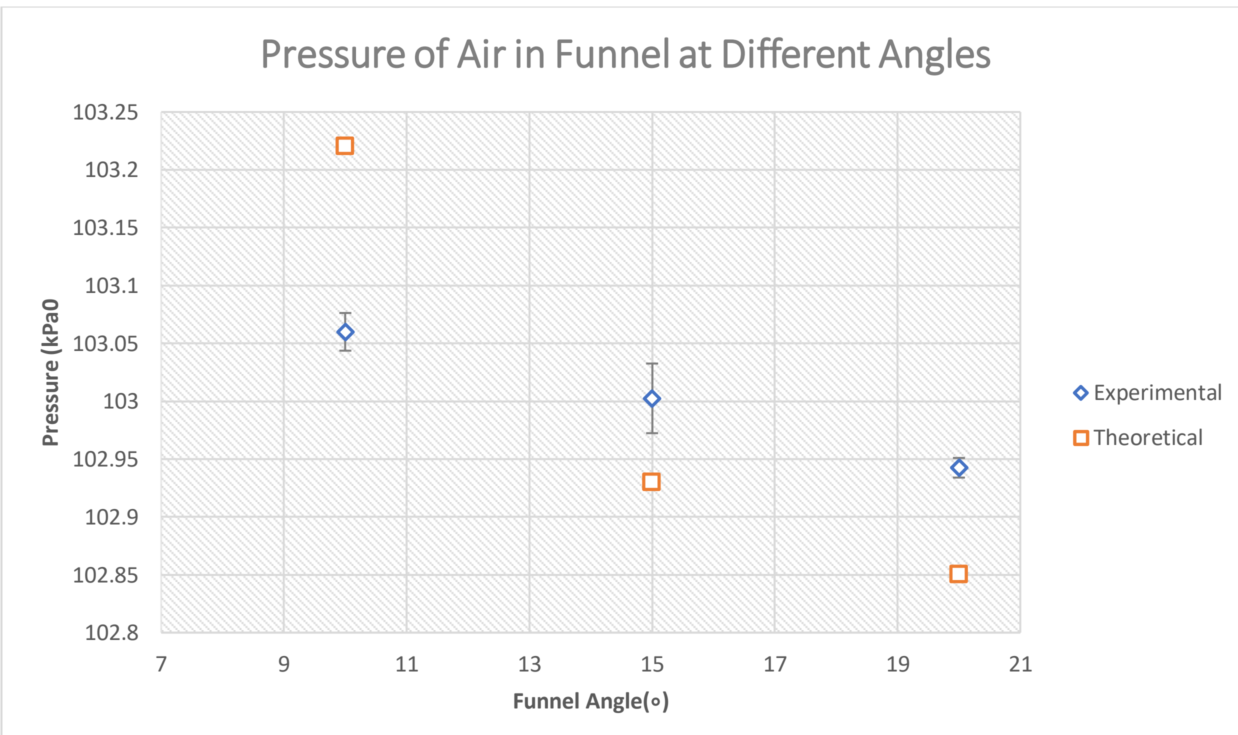

Figure 10.

Conclusion:

Through this investigation, changes in the difference between the pressure above the ball and the pressure below the ball were found in the funnel and ball system when the wind speed, funnel angle, and funnel height were altered. The pressure below the ball stayed steady at the pressure of 104 kPa. For the wind speed experiment, I hypothesized that the pressure above the ball would decrease as the wind speed increased. The data supported my hypothesis; as the wind speed increased, the pressure above the ball decreased, causing the ball to levitate with more force acting upwards. In the funnel height experiment, I hypothesized that the pressure above the ball would decrease as the funnel height increased. The data also showed that pressure above the ball inversely correlates with the funnel height. For the funnel angle experiment, I hypothesized also that the pressure above the ball would decrease as the funnel angle increased. This data also displayed an inverse correlation of pressure above the ball with the funnel angle. For all three experiments, when the data was compared to the theoretical value calculated using the derived Equation (5.2), the data points showed similar correlation to the theoretical data.

Discussion:

The limitations I found in this experiment was difficulties in measuring the pressure above the ball. The pressure gauge was always placed in the same location, but depending on the funnel angle and the wind speed, the movement of the ball varied, making it extremely hard to produce steady data. The thickness of the pressure gauge disrupted the levitation of the ball at times, causing the ball to fall back after hitting the gauge tube. The experiment would run better if the experiment is done with a pressure gauge with a thinner tube that does not disrupt the motion of the ball.

References:

[1] Bernoulli's principle. (2018, November 19). Retrieved from https://en.wikipedia.org/wiki/Bernoulli's_principle

[2] Giancoli, D. C. (2016). *Physics: Principles with applications*. Boston: Pearson.

[3] Munson, B. R., Okiishi, T. H., Huebsch, W. W., Rothmayer, A. P., & Munson, B. R. (2013). *Fundamentals of fluid mechanics, seventh edition: Student solutions manual and study guide*. Hoboken, NJ: John Wiley & Sons.

[4] Waltham, C., Bendall, S., & Kotlicki, A. (2003). Bernoulli levitation. *American Journal of Physics,71*(2). doi:10.1119/1.1524162.