KYPT 2019 2차 연구보고서

3. Undertone Sound

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1. Question

Allow a tuning fork or another simple oscillator to vibrate against a sheet of paper with a weak contact between them. The frequency of the resulting sound can have a lower frequency than the tuning fork's fundamental frequency. Investigate this phenomenon.

2. Objective

In this experiment, using various tuning forks and papers of different sizes, the phenomenon of the resulting frequency decreasing when the tuning fork is in contact with paper in how much the frequency is lowered from the fundamental frequency.

3. Theory

3.1 Harmonic:

$$f_n = nf_1 \tag{1}$$

f_1 is the fundamental frequency of an wave,
n is the harmonic number of a wave,
f_n is the frequency of a nth harmonic of a wave.

Harmonics is the component frequency of an oscillation or a wave. In the natural form, a resonating object will vibrate at a fundamental frequency. In the example of sinusoidal waves, which is the form single-frequency waves take, the period of the fundamental frequency spans out from one end to the other end. Another name for the fundamental frequency is 1st harmonic, because only a single period exists between two endpoints of a wave. Other types of harmonics also exist, in which the frequency is the nth multiple of the fundamental frequency. For example, if the fundamental frequency of a wave is 440 Hz, then the 2nd harmonic would have a frequency of 880 Hz, the 3rd harmonic would have a frequency of 1320 Hz and so on. As the n value rises, the number of periods between the two endpoints of a wave becomes n periods, and a node is created at the point where the periods are divided.

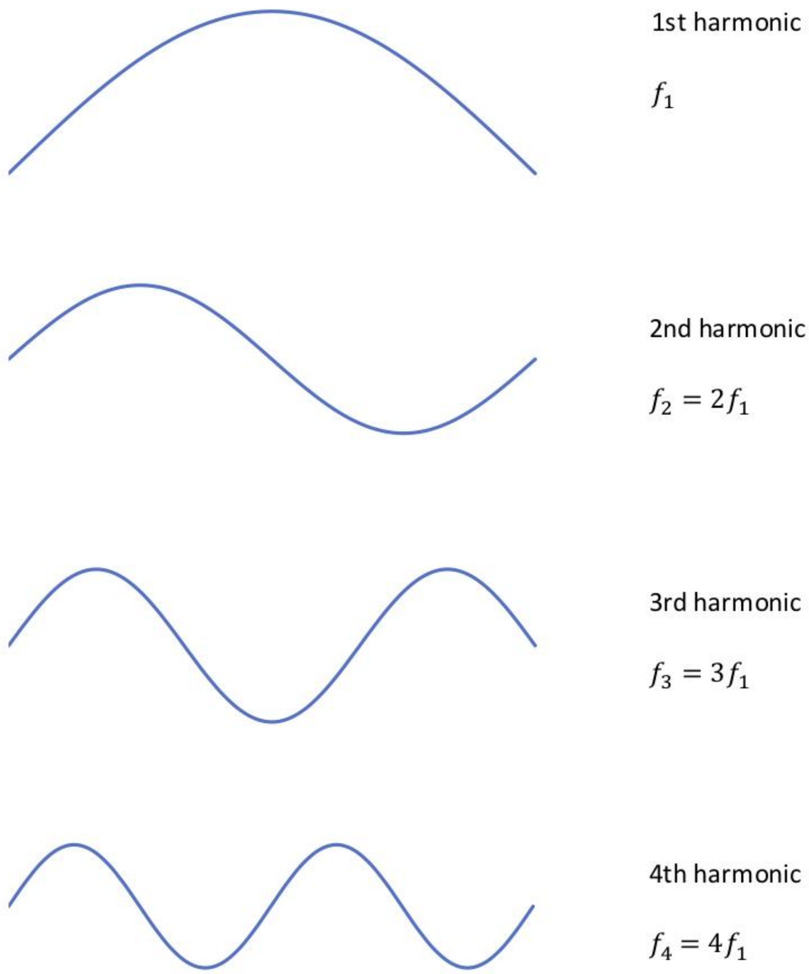

Figure 1. Harmonics

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3.2 Tuning Fork Vibration:

Sound radiation from tuning forks:

When the tuning fork is hit, the air pressure between the two tines decrease while the pressure on the outer surfaces of the tines increases. This difference in the pressure creates soundwaves, making tuning forks a linear quadrupole source of sound, which means that sound is radiated in a linear manner.

According to Rossing, Russell, and Brown (1991), there are two linear modes of the motion of tuning forks, the asymmetrical mode, in which the stem vibrates horizontally with the tines, and the symmetrical mode, in which only the tines vibrate horizontally.

Within the symmetrical modes of the tuning fork, there are two patterns of motion. One is the principal mode, in which only the tip of the tines vibrate horizontally, and the clang mode, in which the whole body of the tines vibrate left and right. For the purpose of this experiment, the tuning forks will be hit at the tips of the tines, thus the symmetrical principal mode of the tuning fork will be investigated.

Fundamental frequency of tuning fork in symmetrical principal mode :

$$f_n = \frac{\pi K}{8L^2}\sqrt{E/\rho}\,[1.194^2, 2.988^2, 5^2, 7^2, \ldots, (2n-1)^2] \tag{2}$$

K is the radius of gyration of the beam cross section,
L is the length of the tines,
E is the Young's elastic modulus.
ρ is the density of the tuning fork.

Tuning forks carry overtones, which are created by the longitudinal motion along the stem of the tuning fork. When the tuning fork makes one full trip back and forth, the stem moves up and down twice, creating an overtone. Although this generated overtone is very small, if the amplitude of the fundamental frequency is high, a significant level of overtone is produced.

This is the fundamental frequency and the overtones produced when a 370 Hz tuning fork was hit with a mallet.

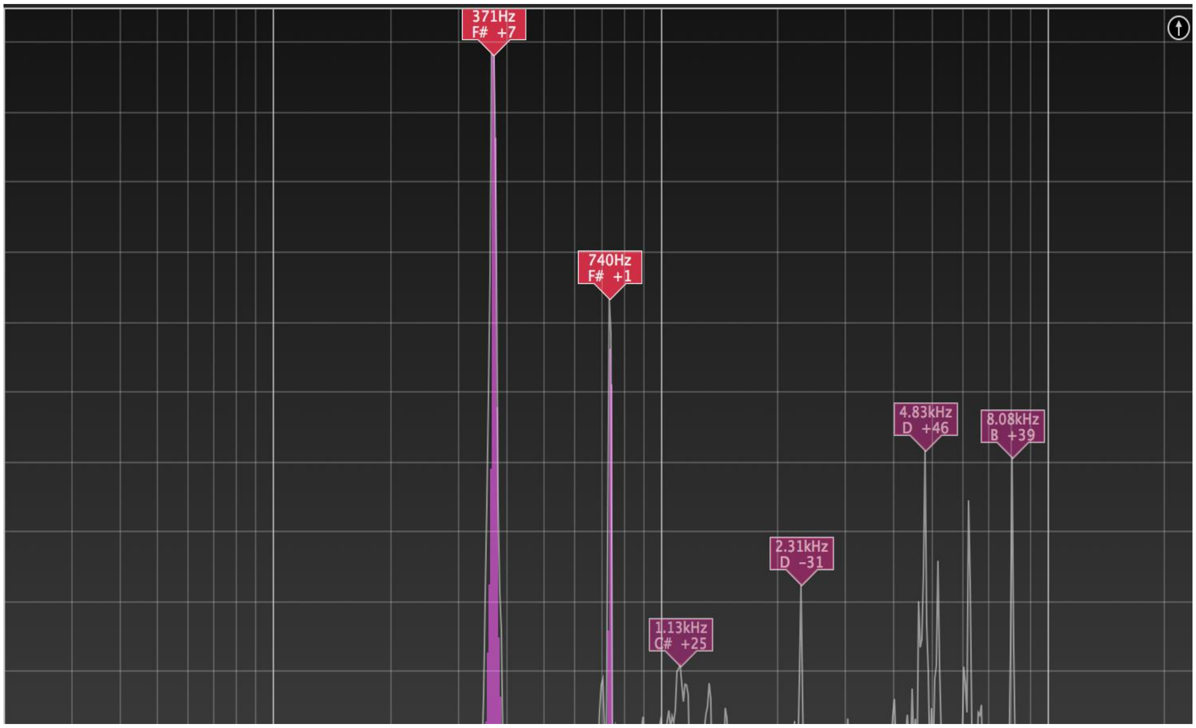

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Figure 3. Spectrum of a tuning fork (Created using Spectrum Analyzer)

When the vibrating tuning fork was put in contact with an A4 paper, the intensity of the overtones of the tuning fork were amplified.

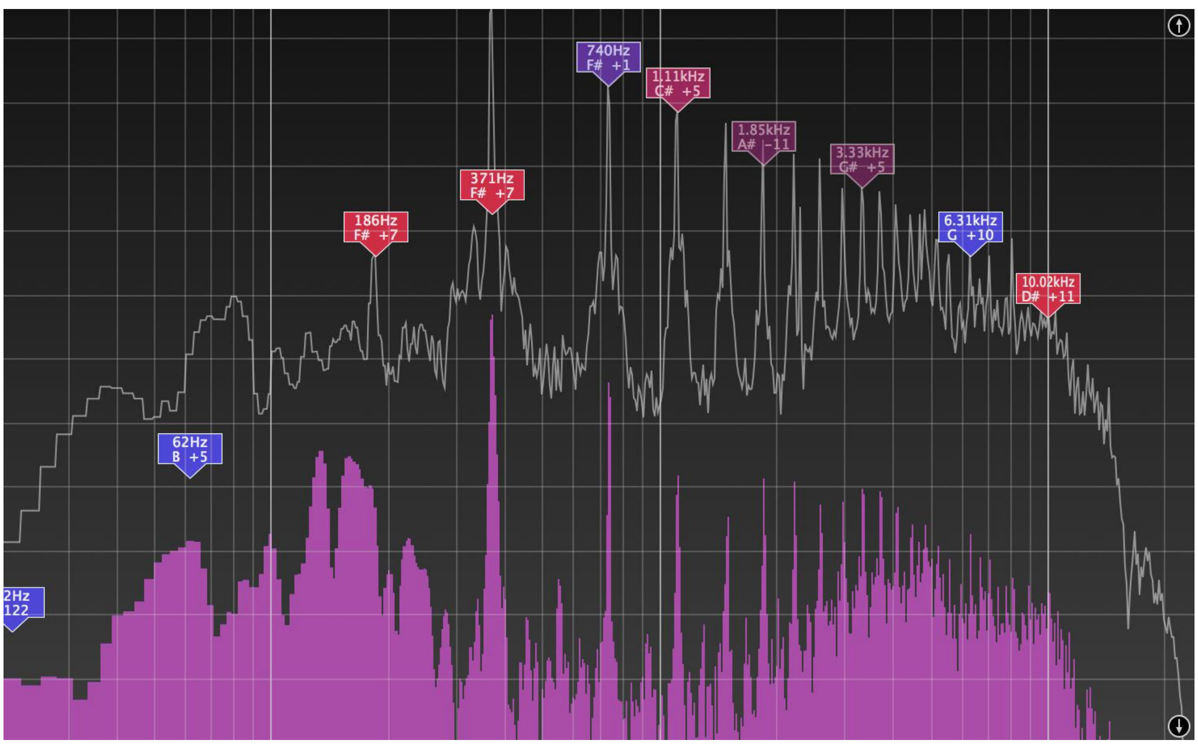

Figure 4. Spectrum of the sound created by tuning fork hitting A4 paper (Created using Spectrum Analyzer)

As shown, when the tuning fork made contact with the paper, the sound spectrum it created had amplified versions of the tuning fork's overtone frequencies. This signifies that there was energy transfer into the nth harmonic series of the tuning fork when the tuning fork interacted with the paper.

3.3 Undertone:

$$f_{\frac{1}{n}} = \frac{1}{n} f_1 \tag{3}$$

f_1 is the fundamental frequency of an wave,
n is the subharmonic number of a wave,
f_n is the frequency of a $\frac{1}{n}$ th harmonic of a wave.

Undertones, or subharmonics, are waves that have a frequency that is lower than the fundamental frequency. Under normal circumstances, the lowest frequency of a vibrating object is the fundamental frequency. However, if an external entity, such as the paper in the experiment, intercepts with the object, an undertone can be created.

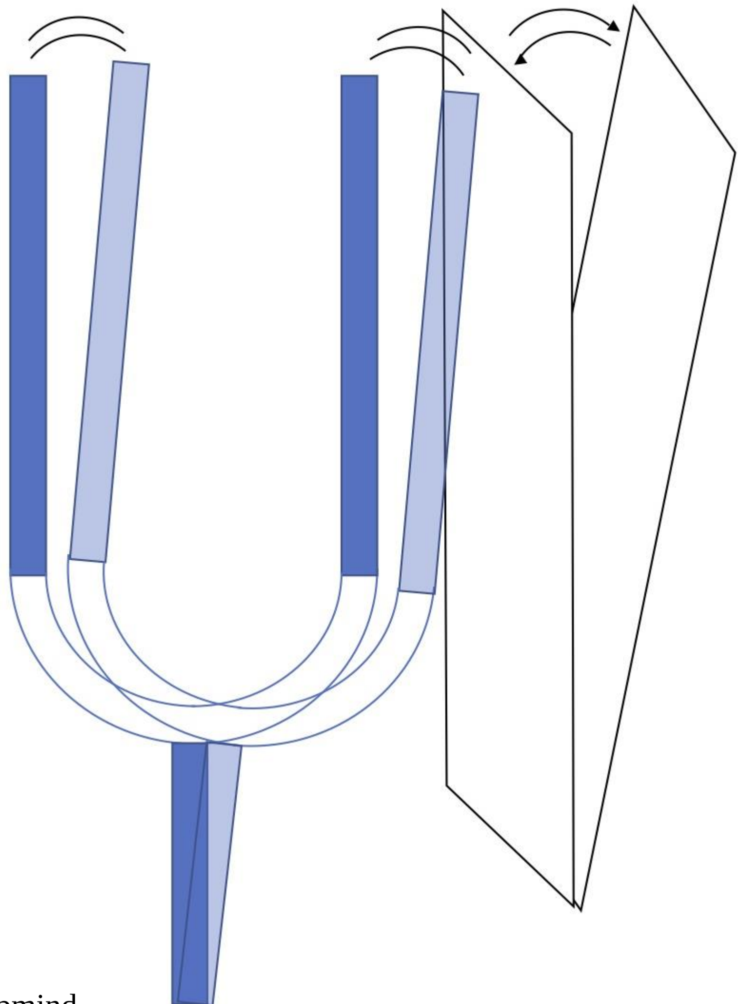

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Figure 5. Interactions between tuning fork and paper

Undertone is created in this system due to the differences in the motion of the tuning fork and paper. In this system, the tuning fork oscillates in its fundamental frequency. When the tuning fork comes in contact with the paper, the paper is pushed back by the impact. By the time of the next contact, the tuning fork would have gone through multiple oscillations. This shows that the resulting wave has a lower frequency than the fundamental frequency, because the contact between the paper and the tuning fork has a lower period than the oscillation of the tuning fork, an undertone is created when the tuning fork touches the paper. The n value of the subharmonic would need to be determined through the experiment.

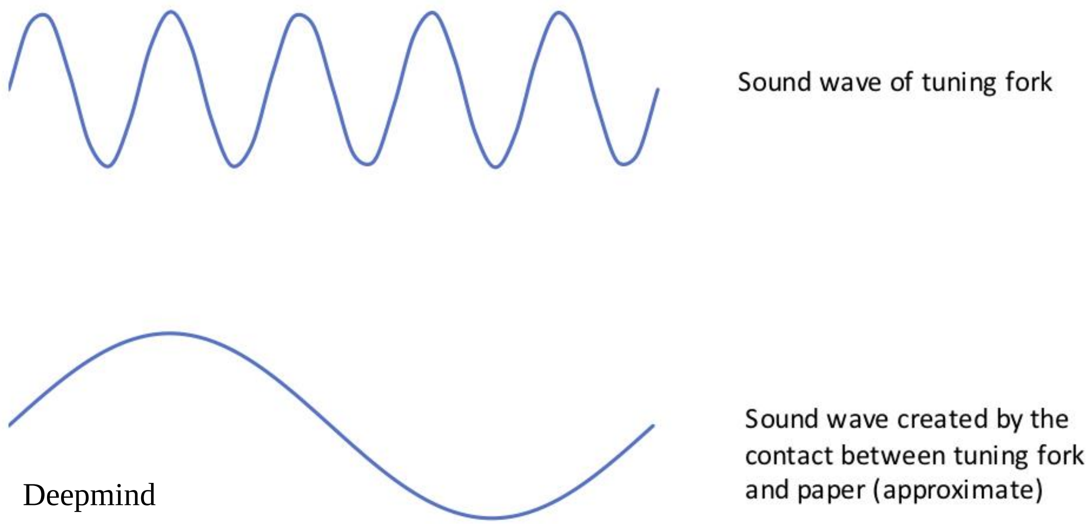

Figure 6. Predicted resulting soundwaves

3.4 Intensity:

$$I = \frac{P}{A} \tag{4}$$

P is the power transmitted by a sound wave,
I is the intensity of the sound wave,
A is the area the power of the sound wave is applied onto.

The intensity of a sound wave is directly proportional to the power transmitted by the sound wave, which is proportional to the amplitude of the sound wave. Therefore, if the amplitude of a sound wave is increased, the intensity of the sound wave will increase along with it.

In the experiment, although the sound wave created by the contact between the tuning fork and the paper has a lower frequency than the fundamental frequency of the tuning fork, the sound wave created by the contact between the tuning fork has a bigger intensity, because the impact created from tuning fork hitting the paper increases the amplitude of the sound wave.

4. Experiment design

4.1. Parameters

Experiment	**#1**		**#2**				**#3**				
Tuning Fork Size	11.9cm	8.8cm	11.7 cm				11.7 cm				
Tuning Fork Frequency	440 Hz		370 Hz				370 Hz				
Paper Size	A4		A4	¼ A4	½ A4	¾ A4	½ A4				
Material Type	A4 Paper		Normal Paper				Plastic	A4 Paper	Hardboard	Thick Paper	Paper File

Table 1. Experiment Parameters

4.2 Hypothesis

4.2.1 Tuning Fork Size Experiment

The bigger tuning fork would produce an undertone with a lower frequency, because the bigger the tuning fork is, the force that hits the paper would be stronger, thus the paper would be pushed back further, reducing the frequency of the contact between the tuning fork and the paper.

4.2.2 Paper Size Experiment

The smaller paper would produce an undertone with a lower frequency, because smaller paper would have a lower mass, which would cause it to be more affected by the force of the tuning fork hitting the paper.

4.2.3 Material Type Experiment

The more rigid materials would produce an undertone with a higher frequency, because its rigid qualities would lessen the movement caused by the contact with the tuning fork.

5. Experiment

Figure 7. Lab Materials

5.1 Required Materials

- 4 A4 Papers
- Paper File
- Plastic File
- Thick Paper
- Hardboard
- USB Mic
- Tape
- Scissors
- Mallet
- Computer
    - Used Logic Pro for recording frequencies
- 3 Tuning Forks
    - One 11.7cm, 370 Hz
    - Two 440 Hz
        - 8.8 cm, 11.9 cm

5.2 Procedure

Set-up:

1. Gather the materials

Experiment 1: Tuning Fork Size Experiment

1. Tape one A4 size paper vertically to the edge of a table
2. Hold the 440 Hz 8.8cm tuning fork
3. Hit the tuning fork with the mallet
4. Hold the mic close to the middle of the A4 paper
5. Gently press the tuning fork on the middle of the A4 paper
6. Repeat steps 2-5 with the 440 Hz 11.9cm tuning fork

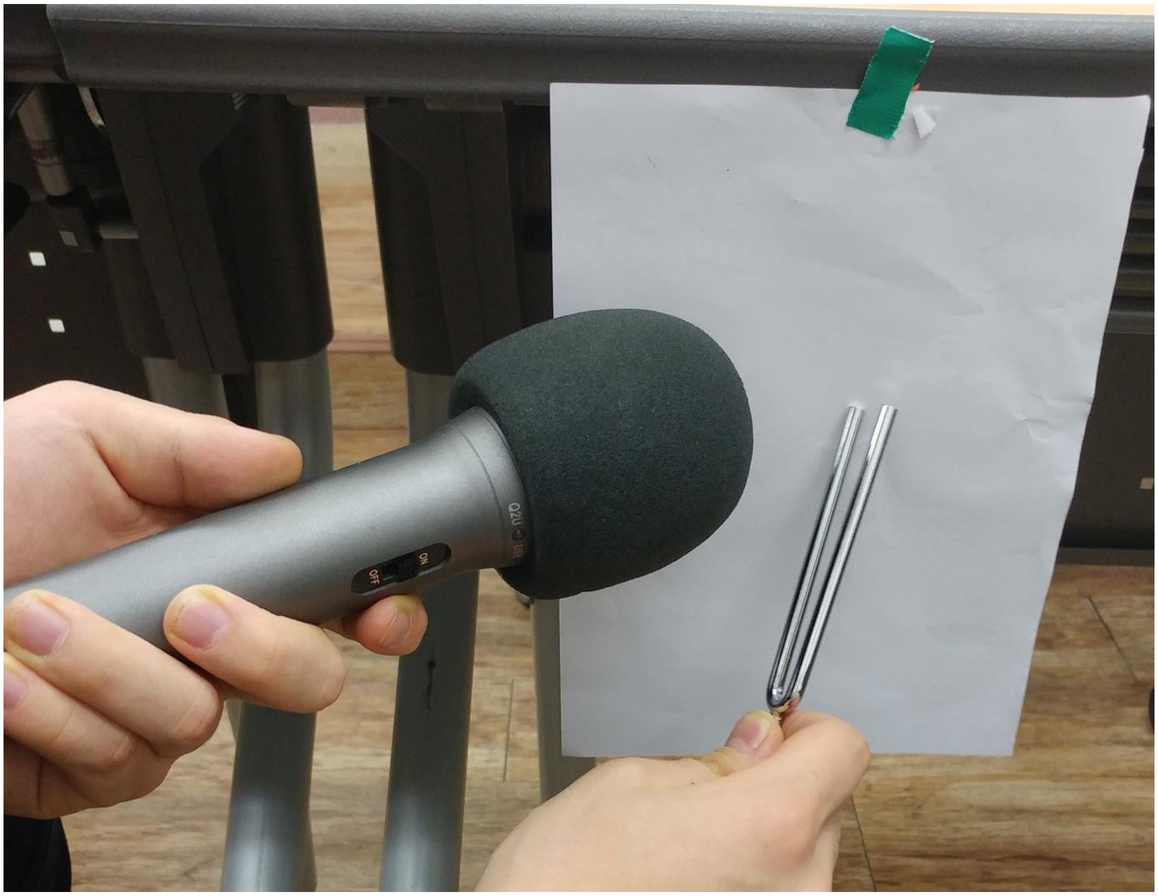

Figure 8. Tuning fork size experiment setup

Experiment 2: Paper Size Experiment

1. Cut the 3 A4 papers into 3 sizes
    a. 1/4, 1/2, and 3/4 size
2. Tape the 1/4 paper onto the edge of the table
3. Hold the 370 Hz 11.7cm tuning fork
4. Hit the tuning fork with a mallet
5. Hold the mic close to the middle of the paper
6. Gently press the tuning fork on the middle of the paper
7. Repeat steps 2-6 with 1/2, 3/4, and A4 size papers

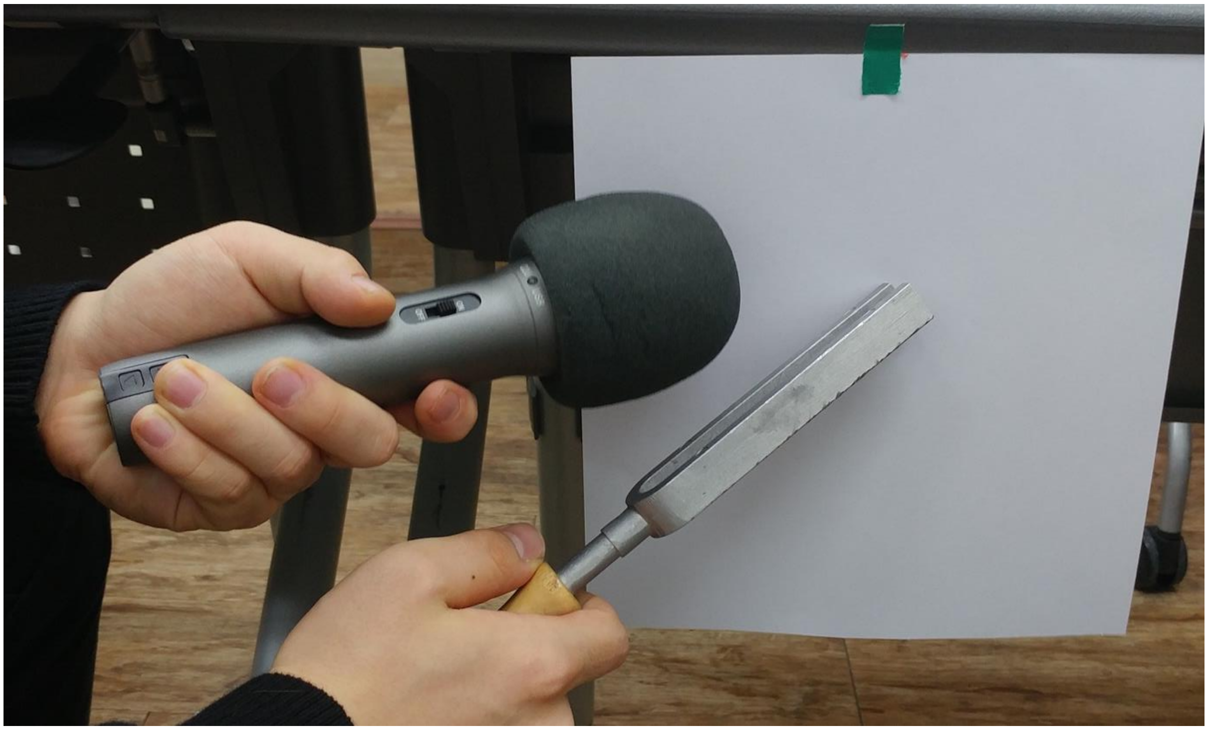

Figure 9. Paper size experiment setup

Experiment 3: Material Type Experiment

1. Cut the plastic file, hardboard, paper file, thick paper and A4 paper into 1/2 A4 size.
2. Tape the plastic file onto the edge of the table
3. Hold the 370 Hz 11.7cm tuning fork
4. Hit the tuning fork with a mallet
5. Hold the mic close to the middle of the paper

6. Gently press the tuning fork on the middle of the paper
7. Repeat steps 2-6 with hardboard, paper file, thick paper and A4 paper

Figure 10. Material type experiment setup

Only the primary undertone frequency or the loudest undertone frequency was recorded for all three experiments.

6. Experiment Results

Tuning Fork Size Experiment

Length (cm)	Trial 1 (Hz)	Trial 2 (Hz)	Trial 3 (Hz)	Trial 4 (Hz)	Trial 5 (Hz)	Trial 6 (Hz)	Trial 7 (Hz)	Average Frequency (Hz)	Standard Error
11.9	135	135	140	139	147	132	140	138.2857	± 1.848

8.8	153	156	154	151	149	154	154	153	± 0.808

Table 2. Undertone frequencies caused by two different-sized tuning forks

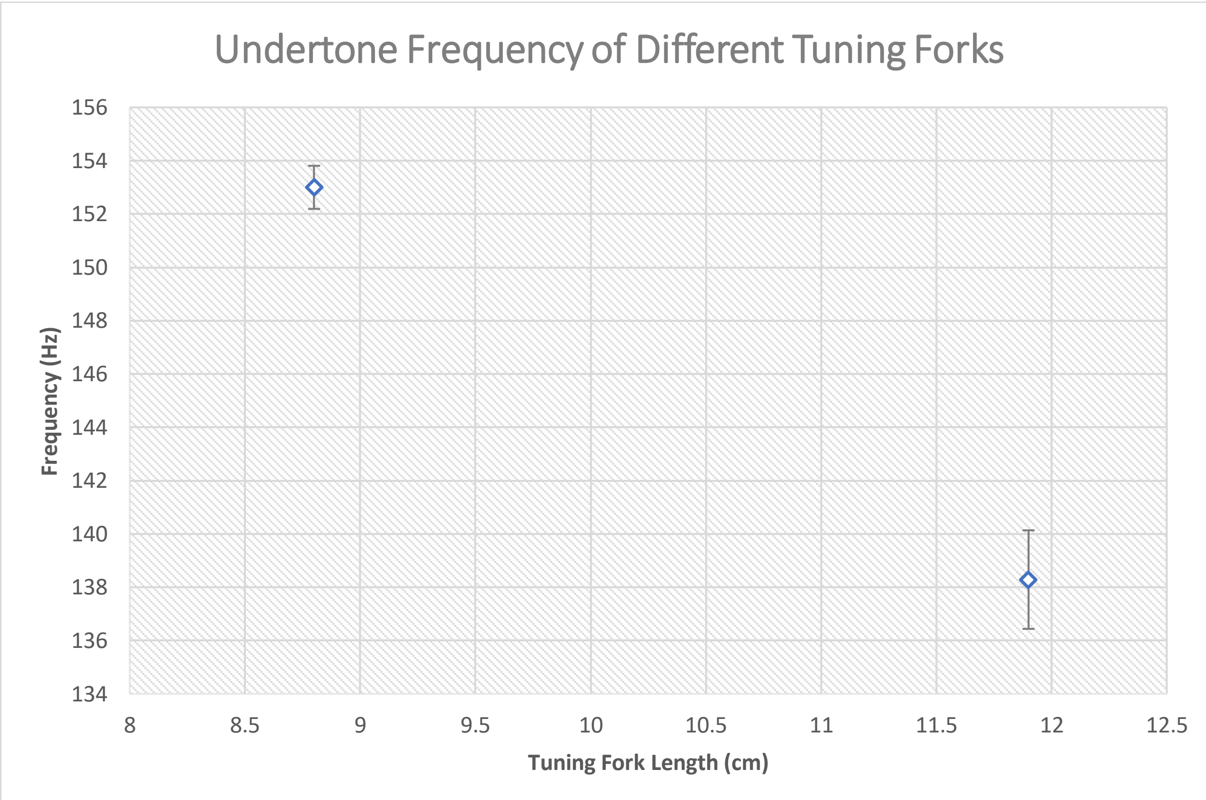

Figure 11. Average undertone frequencies caused by two different-sized tuning forks

The primary undertone frequency of the longer tuning fork was smaller than that of the shorter tuning fork as supported by the theory. Although the resulting frequencies deviates from the average value, the difference between the average frequencies is big enough to make a separation.

Paper Size Experiment

Paper Area (cm^2)	Trial 1 (Hz)	Trial 2 (Hz)	Trial 3 (Hz)	Trial 4 (Hz)	Trial 5 (Hz)	Trial 6 (Hz)	Trial 7 (Hz)	Average Frequency (Hz)	Standard Error
623.7	183	181	180	183	180	182	180	181.286	± 0.522

467.775	196	194	186	187	193	194	188	191.143	± 1.519
311.85	194	194	186	188	194	188	192	190.857	± 1.299
155.925	186	186	186	188	184	188	190	186.857	± 0.738

Table 3. Undertone frequencies caused by four different-sized papers

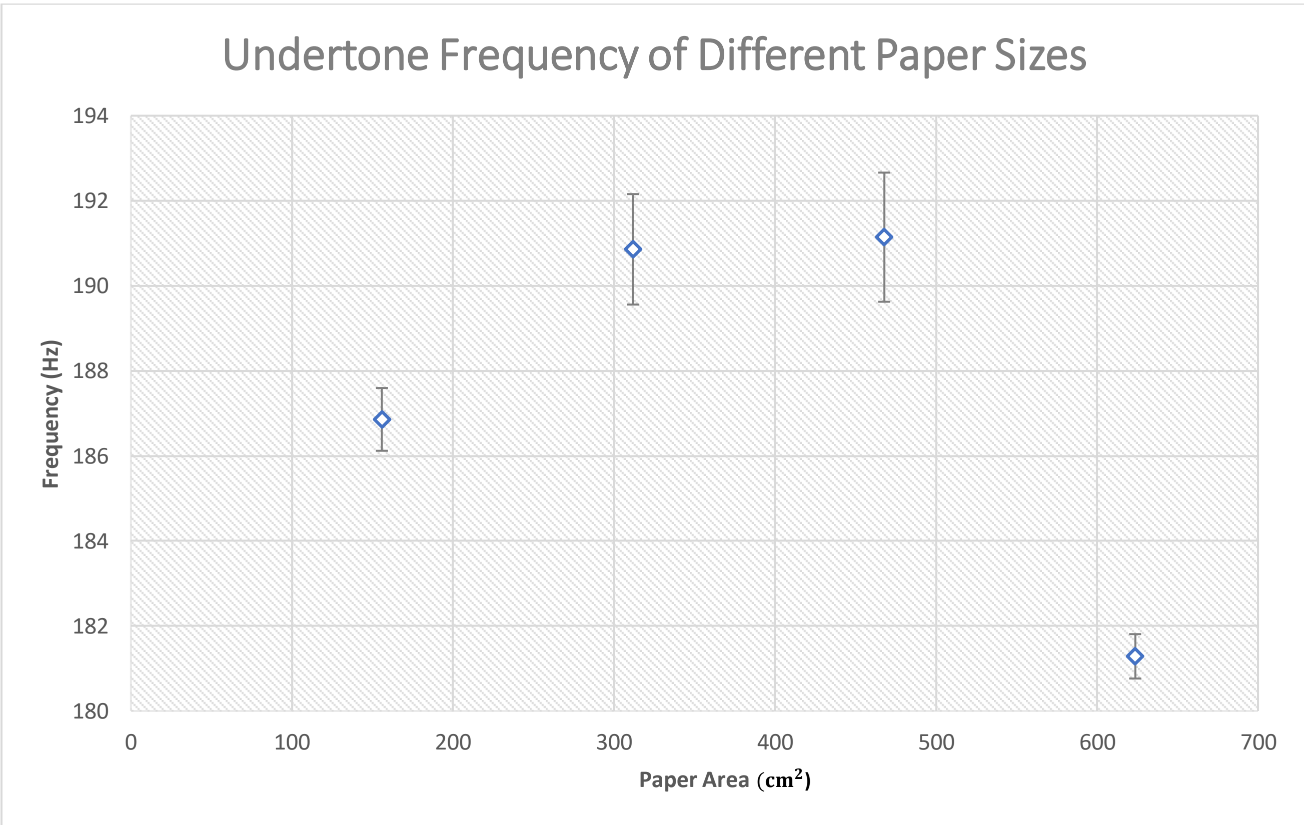

Figure 12. Average undertone frequencies caused by four different-sized paper

The size of the paper creates some changes to the average frequency of the resulting sound, but all the frequencies stay in the same ½ subharmonic level. Looking at the standard error, there is not a significant enough difference between the undertone frequencies of the ¾ A4 paper and the ½ A4 paper. There is no observable pattern in the changes in undertone frequencies as the area of paper increases.

Material Type Experiment

Material Type	Trial 1 (Hz)	Trial 2 (Hz)	Trial 3 (Hz)	Trial 4 (Hz)	Trial 5 (Hz)	Trial 6 (Hz)	Trial 7 (Hz)	Average Frequency (Hz)	Standard Error
Paper	194	194	186	188	194	188	192	190.857	± 1.299
Plastic File	135	135	137	137	124	129	129	132.286	± 1.886
Paper File	92	92	84	83	81	75	78	83.571	± 2.458
Hardboard	73	71	69	75	70	69	71	71.143	± 0.829
Thick Paper	113	116	105	110	110	110	115	111.286	± 1.409

Table 4. Undertone frequencies caused by five different materials

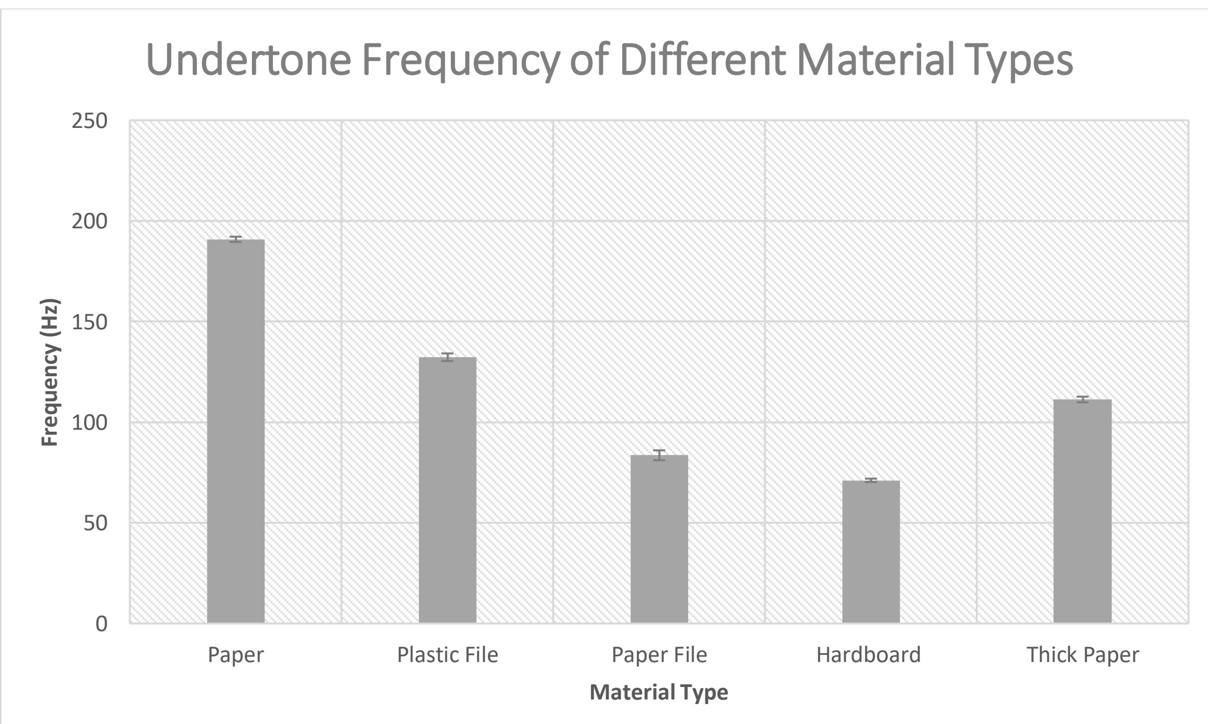

Figure 13. Average undertone frequencies caused by five different materials

A correlation between the rigidity of the materials and the primary undertone frequencies is observed, although not in the direction of the hypothesis. There is a general trend of rigid materials having a lower average undertone frequency than those of flexible materials such as normal paper.

7. Conclusion:

Through this investigation, I observed differences in the primary undertone frequency of sound created by the contact between tuning forks of different sizes and papers of different materials and sizes. For the paper size experiment, I had speculated in my hypothesis that smaller sized papers would produce a sound with lower frequencies, because smaller papers have a smaller surface area and mass, which would cause it to go further back when hit by the tuning fork, reducing the frequency of the sound of contact. However, in the data, no significant patterns were found in the data from the paper size experiment, suggesting that the differences in surface area and mass may not be significant enough to produce a difference in the frequency of the resulting sound. In the tuning fork size experiment, I hypothesized that the larger tuning fork would produce a sound with a lower frequency. My experimental data supported the hypothesis, showing a significant drop in the produced undertone frequency when the larger tuning fork was used. This difference in the undertone frequency is attributed to the larger tuning fork applying a larger momentum onto the paper, causing the paper to move back further than when experimenting with the smaller tuning fork. For the material type experiment, I hypothesized that the more rigid materials would produce a higher undertone frequency. The data supported the opposite; the more rigid materials produced a lower undertone frequency. The more rigid materials carried more mass, thus the decrease in the primary undertone frequency is caused by the rigid materials carrying more inertia, resisting the change in the state of motion.

8. Discussion:

I faced a few limitations while executing this investigation. One limitation I faced was the lack of variation in the tuning forks. Since I was not able to create tuning forks, all the tuning forks had to be bought off the internet. However, I was not able to find many variations of tuning forks in the internet that fit my budget, especially regarding the size of the tuning fork. Additionally, I could not produce perfectly controlled results in these experiments, because although I did try my best, the tuning fork could not be hit with the same force every time. I tried other methods other than my hand to produce a steady intensity, but using the mallet with my hand created the most punctual results.

9. References:

Harmonic. (2018, August 27). Retrieved November 16, 2018, from https://en.wikipedia.org/wiki/Harmonic

Knapman, H. (1904). An Experiment Illustrating Harmonic Undertones. *Proceedings of the Royal Society of London, 74*(497-506), 118-120. doi:10.1098/rspl.1904.0092

Rossing, T. D., Russell, D. A., & Brown, D. E. (1992). On the acoustics of tuning forks. *American Journal of Physics, 60*(7), 620-626. doi:10.1119/1.17116

Undertone series. (2018, June 12). Retrieved November 10, 2018, from https://en.wikipedia.org/wiki/Undertone_series