# KYPT 2019 2 차 연구보고서

## 15. Newton's Cradle

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**1. Question**

The oscillations of a Newton's cradle will gradually decay until the spheres come to rest. Investigate how the rate of decay of a Newton's cradle depends on relevant parameters such as the number, material, and alignment of the spheres.

**2. Objective**

In this experiment, using balls of diverse materials, sizes, and numbers and strings of various lengths, the system of Newton's Cradle will be analyzed for diverse parameters in how the rate of decay of the system changes.

**3. Theory**

**3.1 F = ma**

$$F = m\ddot{x}_n \tag{1}$$

$F$ is the force applied on ball n,
$x_n$ is the position of ball n,
$m$ is the mass of the ball,

F = ma equation is usually utilized to find a force of or on an object in motion based on its mass and its acceleration. By definition, acceleration is the rate of change in velocity, which means that it can also be the rate of rate of change in position using two derivatives. therefore, if the velocity or the position of a ball in the Newton's Cradle can be analyzed, then the acceleration of the ball can be found.

In the experiment, the equation shows the force applied onto the nth ball using to the position of the ball. If the motion of the ball is analyzed, then the equation for the position of the ball can be derived to times to find the acceleration of the ball. Then, if the ball is weighed, then the force applied on the ball can be calculated.

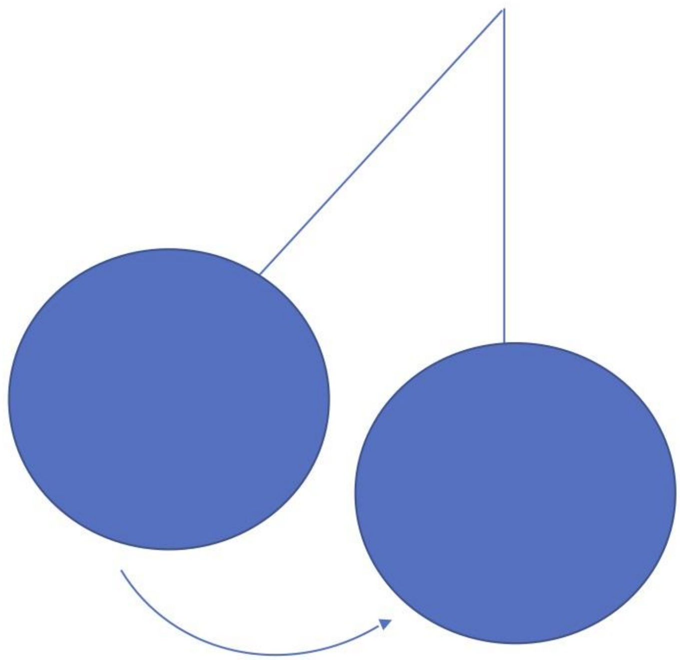

*Figure 1. Motion of the Newton's Cradle Ball*

**3.2 Hooke's Law:**

$$F = kx \tag{2}$$

$F$ is the force applied on ball n,
k is the spring constant,
$x$ is the amount of compression/extension.

The Hooke's Law is used to find the force applied on an object in a spring system. The x represents the amount of compression/extension of the spring, which is measured from the location where the spring is not compressed to the location of compression. Using this value and the spring constant, which every spring owns, the force applied on an object by a spring can be calculated.

In this experiment, the spring force is used in the interaction between two balls. When two balls collide within a Newton's Cradle, there is a brief second when the shape of the two balls distort. Then, the balls restore its shape after the collision. This part of the system can be seen as a spring-like interaction, thus the Hooke's Law can be used to find the force applied on the ball.

$$x_0 = 2r - x_{m,n} \tag{3}$$

$x_0$ is the overlapping distance between two balls during collision,
$r$ is the radius of the spheres
$x_{m,n}$ is the distance between the centers of two balls during collision.

The overlapping distance between two balls during collision can be found using this equation. 2r is the original distance between the centers of the two balls when the ball's shape is not distorted. The distance between the centers of the two balls after the distortion is measured by $x_{m,n}$. If this newly measured distance is subtracted from the original distance, then the overlapping distance, or the amount of distortion of the two balls can be calculated.

$$F = kx_0 \tag{4}$$

The spring force applied onto a ball can be found using the overlapping distance, because the overlapping distance is the displacement of a spring system from its uncompressed system. Before collision, the two balls are at their uncompressed state. While the two balls collide, the balls are compressed by a certain displacement, which is $x_0$. Using this, the spring force applied onto a ball in a Newton's Cradle can be found.

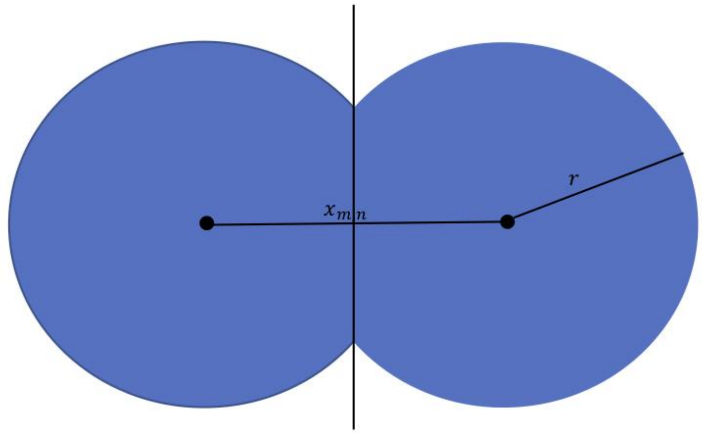


*Figure 2. Collision between the balls*

**3.3 Stokes Law**

$$F_{fr} = \eta v \tag{5}$$

$\eta$ is the damping constant, and for this experiment, this constant is set at 6.8,

$v$ is the velocity.

This force is known as drag force, which is the force of friction exerted on spherical objects moving through a fluid. As shown in the equation, this force is directly proportional to the velocity of the object.

**3.4 Viscoelastic Dissipation Force**

$$F_{diss} = -\gamma \frac{d}{dt}(\xi^{\beta}) \tag{6}$$

$\gamma$ is the viscoelastic dissipation parameter, and for steel, this parameter is set at 147,

$\xi$ is the overlap between two balls

The viscoelastic dissipation force is produced during the collision between two objects in fluid with viscosity. During collision, the viscosity of the fluid dissipates some of the forces in the motion of the newton's cradle, causing the motion to decay.

### 3.5 Motion of Newton's Cradle

$$m\ddot{x}_n = k\xi^{\alpha}_{n-1,n} - k\xi^{\alpha}_{n,n+1} + k_g\left(x_{o,n} - x_n\right) - \eta v - \gamma \frac{d}{dt}(\xi^{\beta}) \tag{7}$$

This equation represents the decaying motion of the Newtons Cradle. Of the variables of this equation, the last two terms are specifically responsible for the decay, thus the rate of decay in the Newton's Cradle was investigated specifically on the last two terms defined in equations 5 and 6.

### 3.6 Newton's Second Law for Rotation

$$\sum \tau = I\alpha \tag{8}$$

This equation represents the net torque of the system, which is the moment of inertia times the angular acceleration of the ball.

Using this equation and the forces acting on the ball of the Newton's Cradle, this equation can be derived using θ as a variable:

$$mL^2\ddot{\theta} = -mgL\sin\theta - \tau_s - F_{fr}L \tag{9}$$

This equation can be reduced to:

$$\ddot{\theta} + \frac{\eta}{m}\dot{\theta} + \frac{g}{L}\theta = 0 \tag{10}$$

The solution for this equation is proportional to:

$$\theta = \theta_0 e^{-\frac{\eta t}{2m}} \tag{11}$$

Therefore, the decay of the motion in Newton's Cradle is proportional to Equation (11).

## 4. Experiment design

### 4.1. Parameters

| **Experiment** | **#1** | | | **#2** |
|---|---|---|---|---|
| Ball Number | 3 | 4 | 5 | 5 |
| Ball Material | Steel | | | Steel |

| | | |
|---|---|---|
| Ball Mass | 10.3g | 5.9g |
| Ball Diameter | 1.346cm | 1.110 cm |
| String Length | 9.3 cm | 5.6 cm |

*Table 1. Experiment parameters*

**4.2. Hypothesis**

4.2.1 Ball Number Experiment

As the number of balls in the Newton's Cradle increases, the rate of decay in the motion of the cradle would get slower.

4.2.2 Ball Mass Experiment

If the mass of the balls in the Newton's Cradle are reduced, then the rate of decay in the motion of the cradle would be faster.

**5. Experiment**

**5.1 Required Materials**

- Large Newton's cradle with 5 balls
- Small Newton's cradle with 5 balls
- Tripod
- Video camera
- Tape

- Protractor

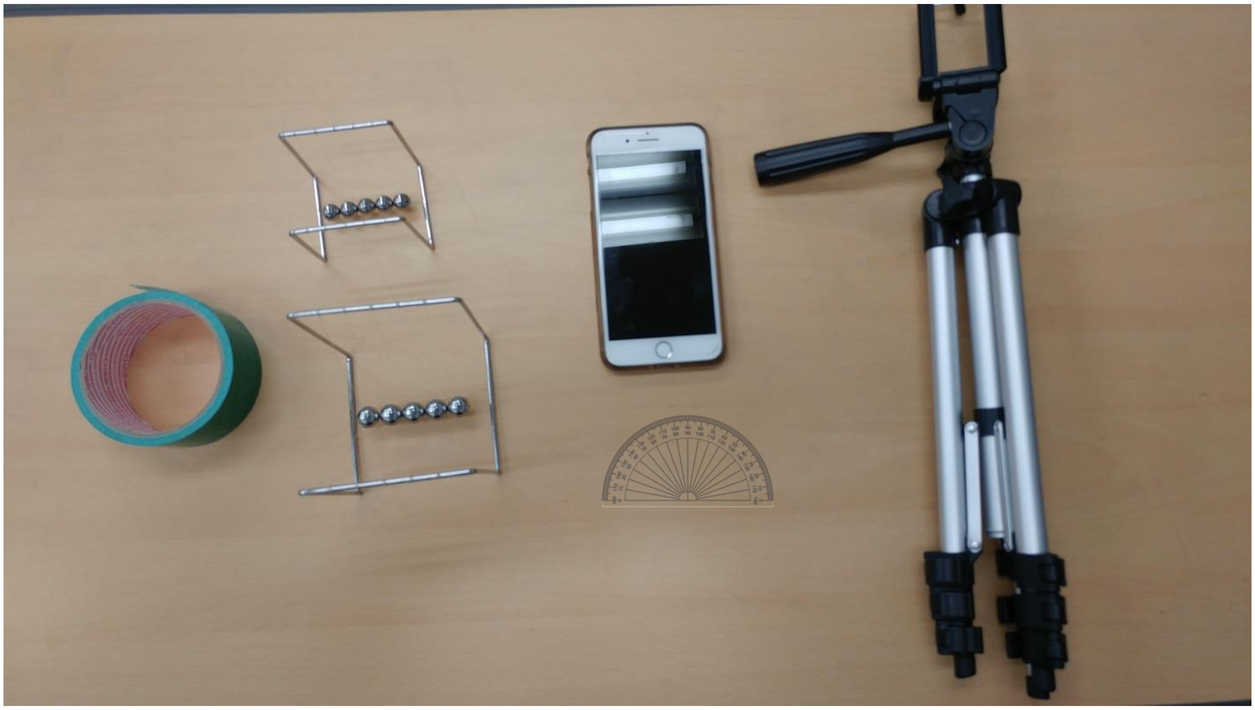

*Figure 3. Lab materials*

**5.2 Procedure**

**Set-up:**

1. Mount the video camera, which could also be a phone, on the tripod
2. Tape the base of the large Newton's cradle onto an edge a desk
3. Adjust the tripod so that the bottom of the camera is level with the desk
4. Move the tripod until the camera has a full side view of the large Newton's cradle

**Large Newton's Cradle Experiment:**

5 Ball Experiment:

1. Use the protractor to lift one of the balls at the end of the cradle to an initial angle of 30°
2. Start videotaping the cradle with the video camera set up on the tripod
3. Without applying any force, carefully release the ball
4. Stop videotaping once all the balls start swinging together

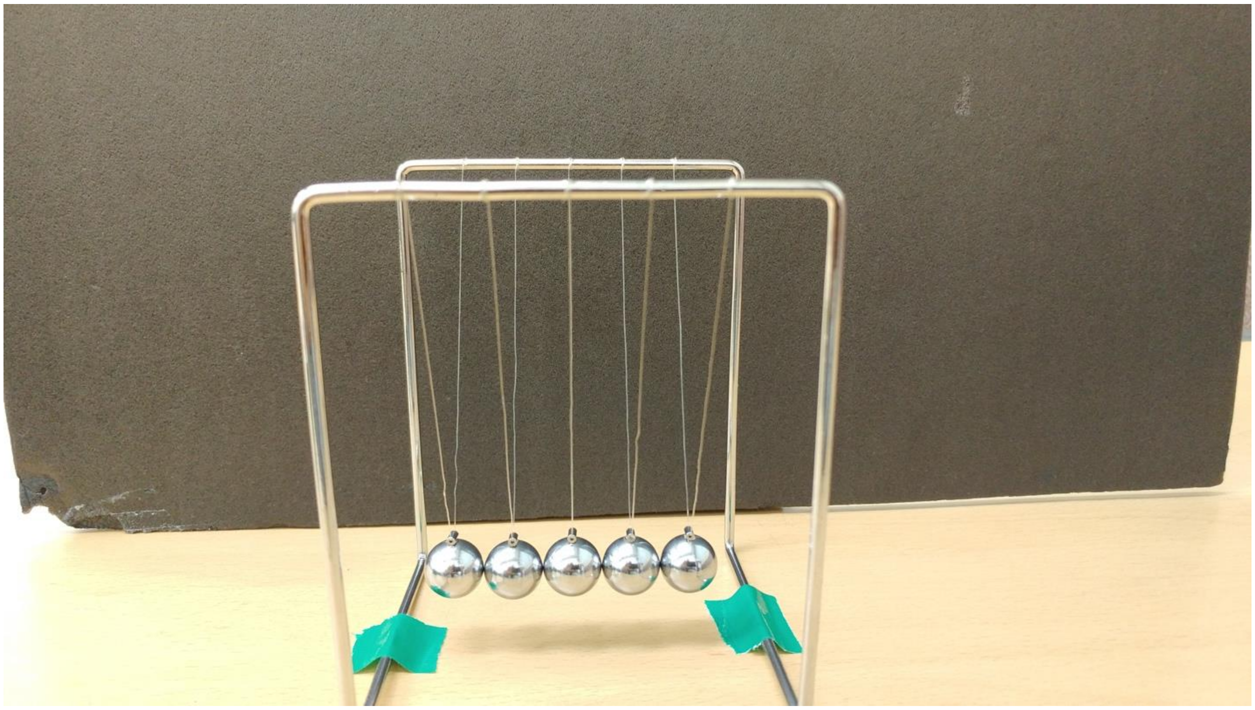

*Figure 4. 5 ball experiment lab setup*

4 Ball Experiment:

1. Lift up one of the balls at either ends of the cradle
2. Use the protractor to lift one of balls at the either end of the four balls still hanging from the cradle to an initial angle of 30°
3. Start videotaping the cradle with the video camera set up on the tripod
4. Without applying any force, carefully release the ball lifted in procedure (2)
5. Stop videotaping once all four balls start swinging together

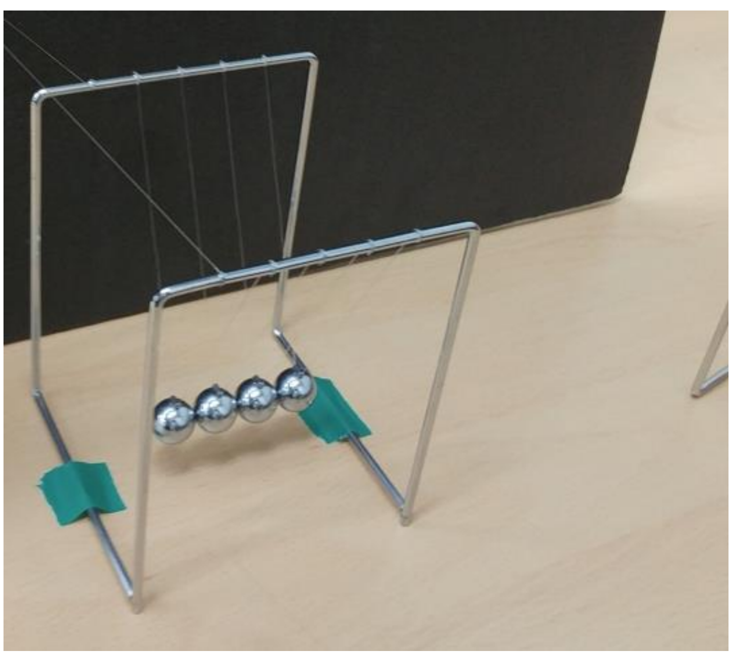

*Figure 5. 4 ball experiment lab setup*

3 Ball Experiment:
1. Lift up two of the balls from one end of the cradle
2. Use the protractor to lift one of balls at the either end of the three balls still hanging from the cradle to an initial angle of 30°
3. Start videotaping the cradle with the video camera set up on the tripod
4. Without applying any force, carefully release the ball lifted in procedure (2)
5. Stop videotaping once all three balls start swinging together

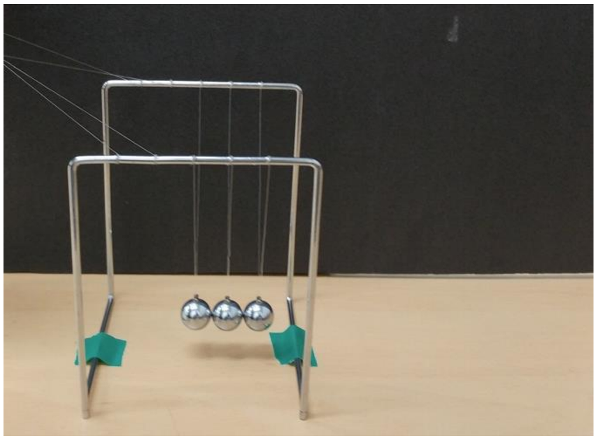

*Figure 6. 3 ball experiment lab setup*

**Small Newton's Cradle Experiment:**
1. Remove the large Newton's cradle from the table
2. Tape down the base of the small Newton's cradle to the same place
3. If necessary, adjust the tripod so that the video camera can fully capture the small Newton's cradle
4. Use the protractor to lift one of the balls at the end of the cradle to an initial angle of 30°
5. Start videotaping the cradle with the video camera set up on the tripod
6. Without applying any force, carefully release the ball
7. Stop videotaping once all the balls start swinging together

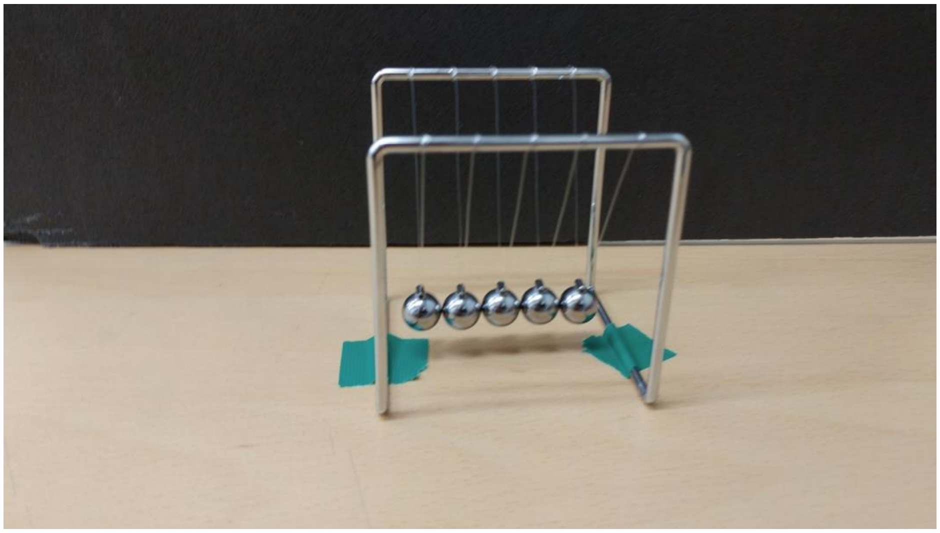

*Figure 7. Small cradle lab setup*

## 6. Experiment Results


*Figure 8. Ball numering*

### 6.1 Large Newton's Cradle Experiment

**5 Ball Experiment**

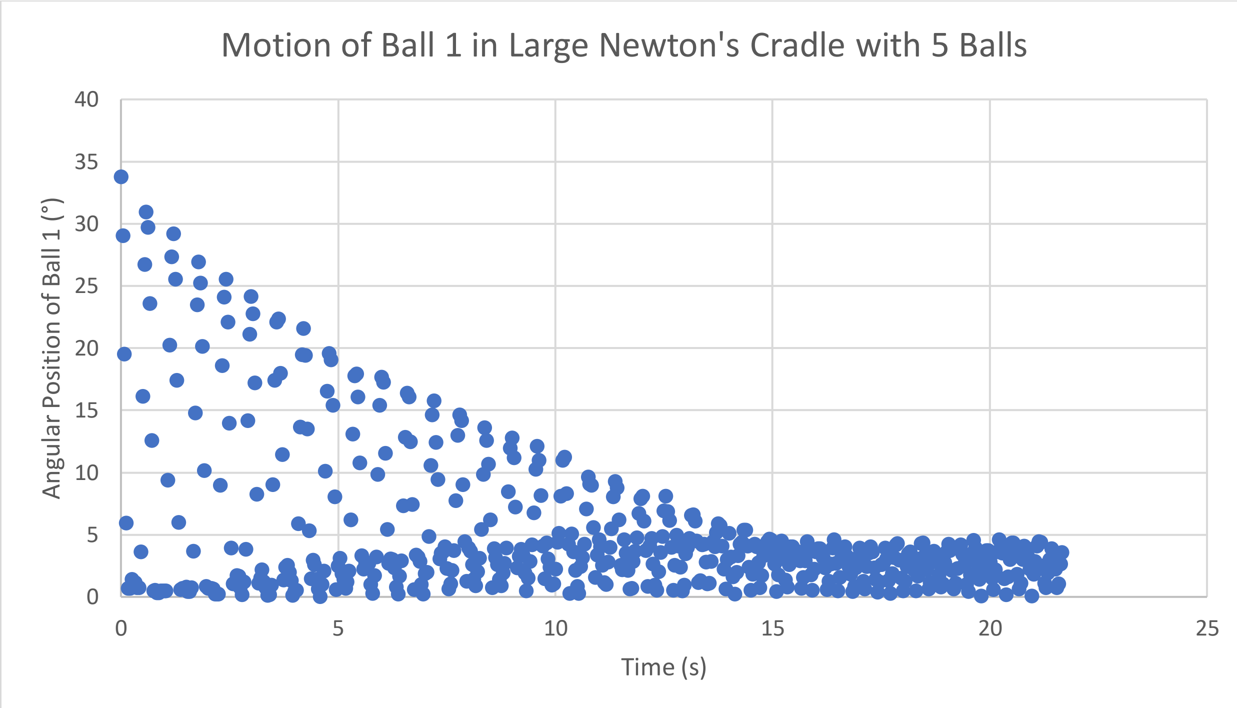


*Figure 9.*

The angle of ball one of the Newton's Cradle draws a decaying sinusoidal function. The amplitude of the angle continues decreasing until a certain point around 15 seconds, where the angle levels off.

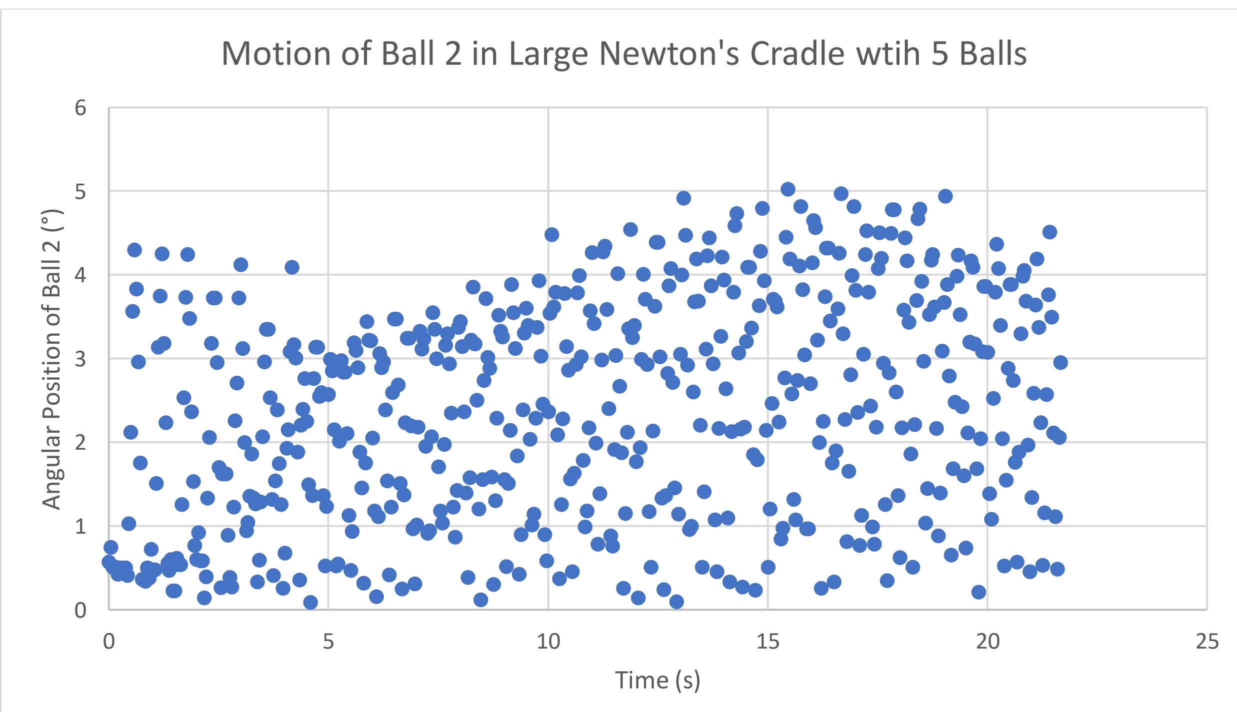


*Figure 10.*

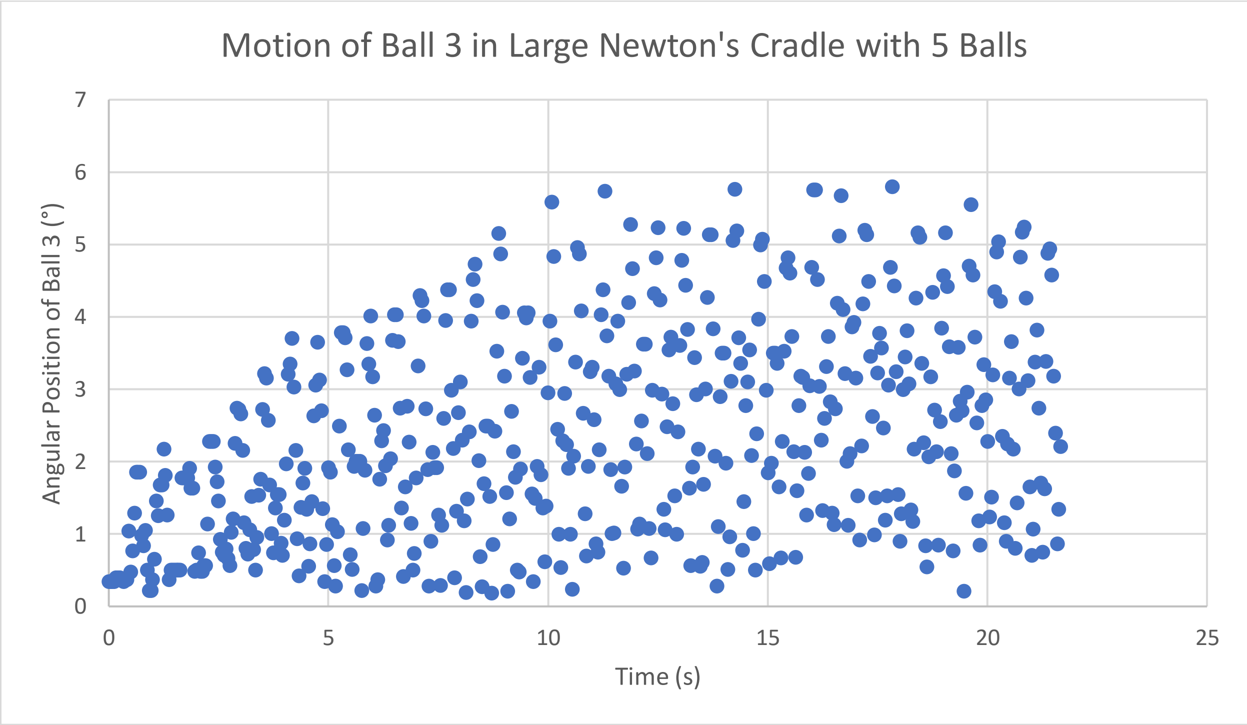


*Figure 11.*

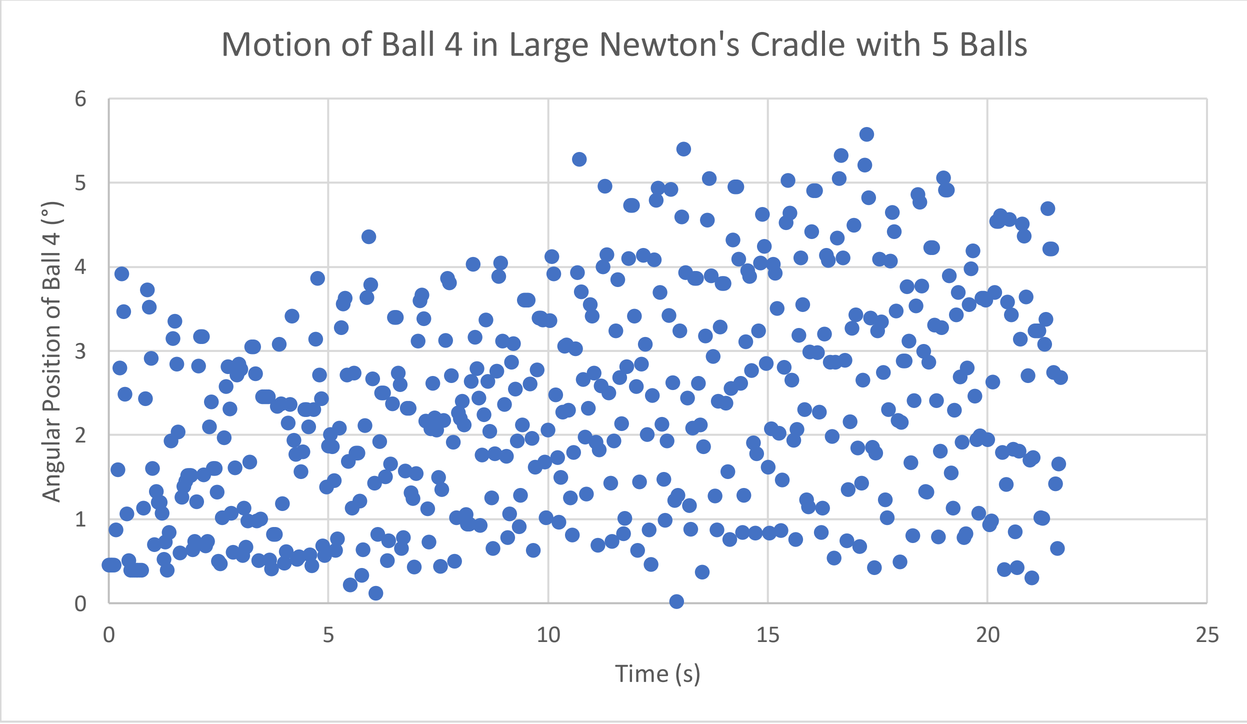


*Figure 12.*

The middle balls in the Newton's Cradle oscillates with an angle under 6 degrees. The pattern of the motions of the middle balls is observed to have weak correlation with the rate of decay in the Newton's Cradle.

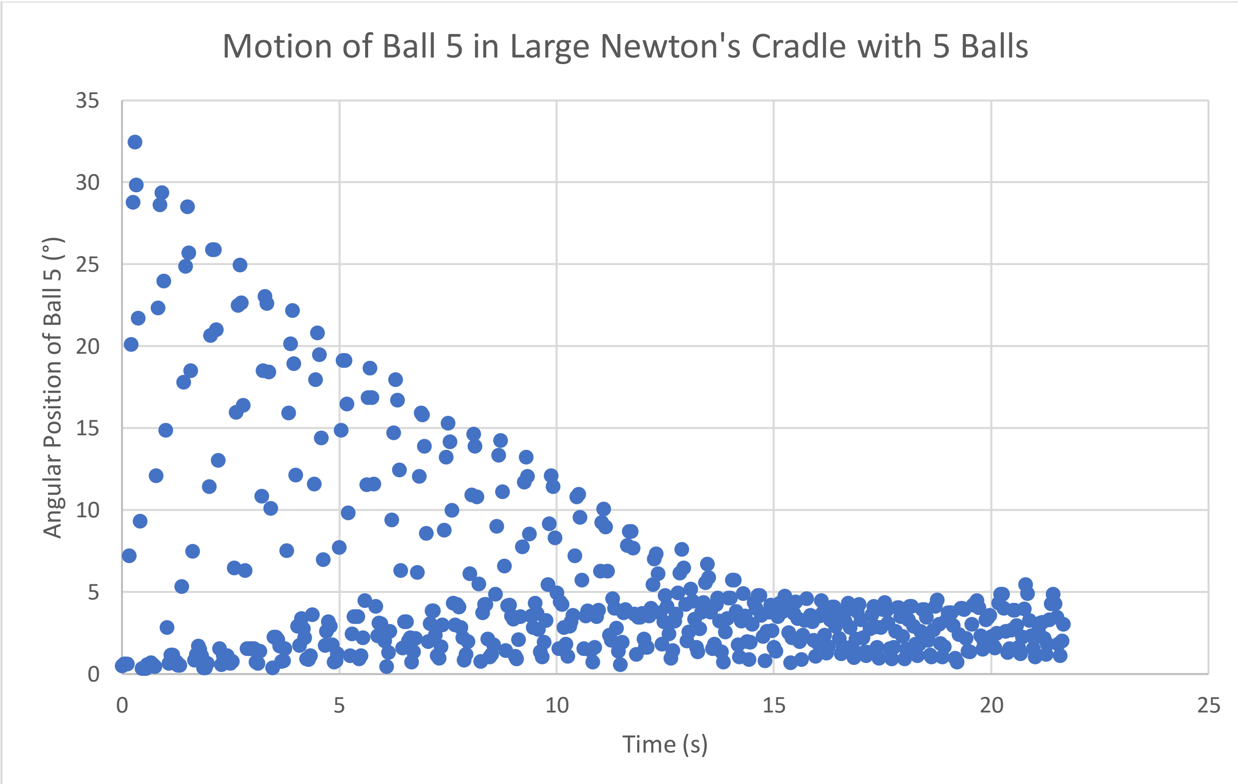


*Figure 13.*

**4 Ball Experiment**

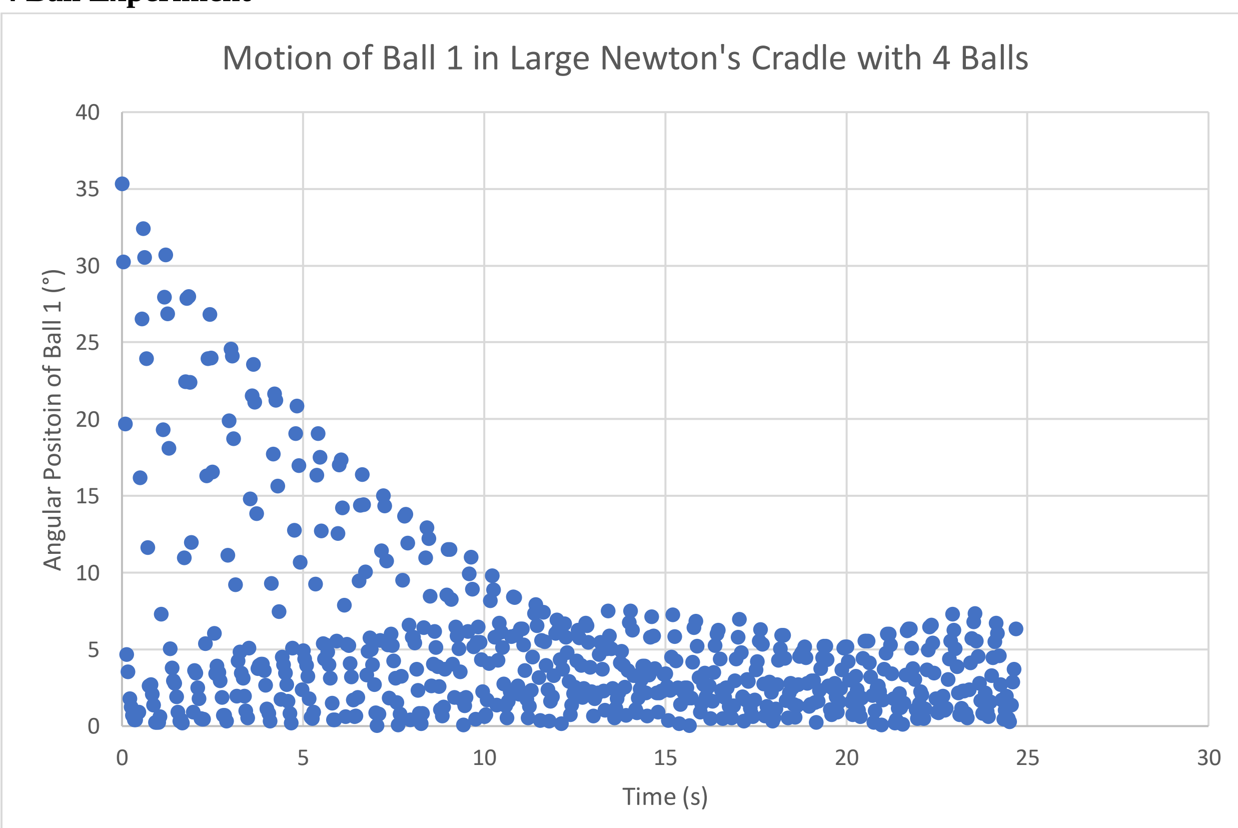

*Figure 14.*

The four ball newton's cradle had a little shorter decaying time than the five ball newton's cradle.

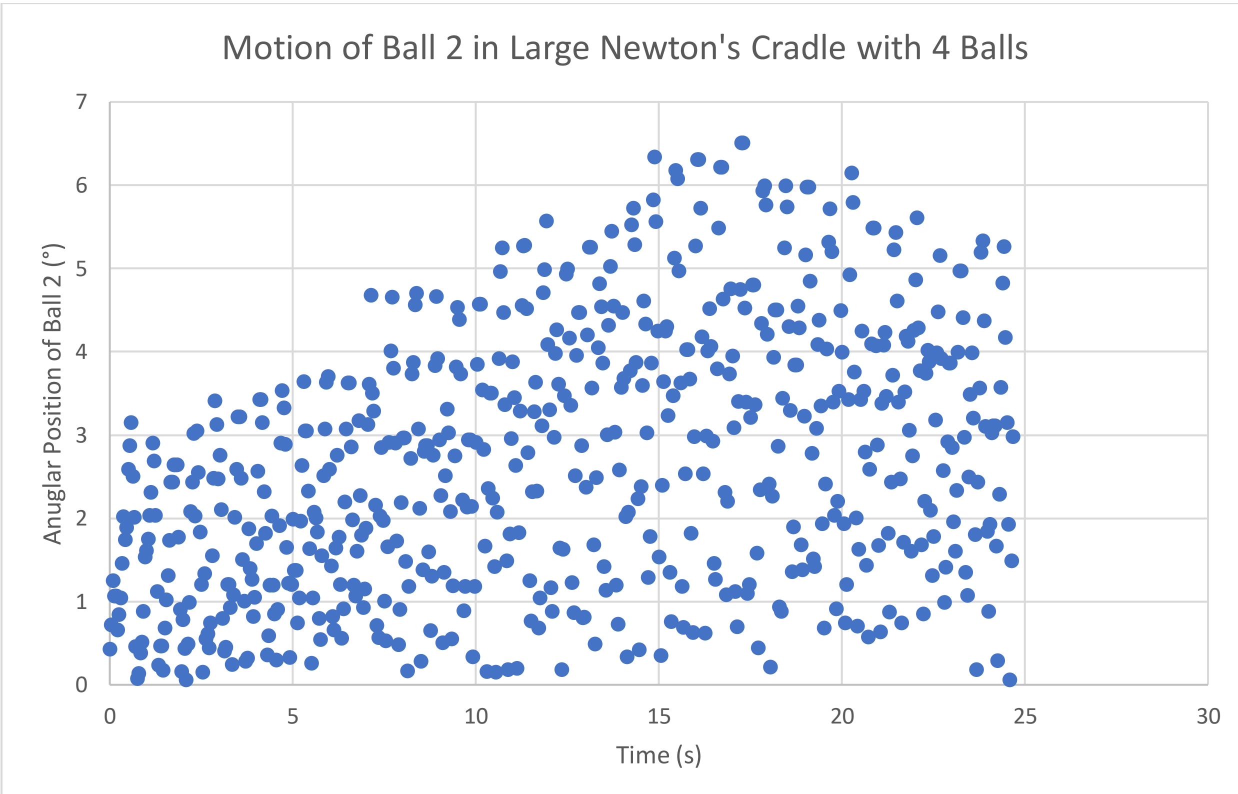


*Figure 15.*

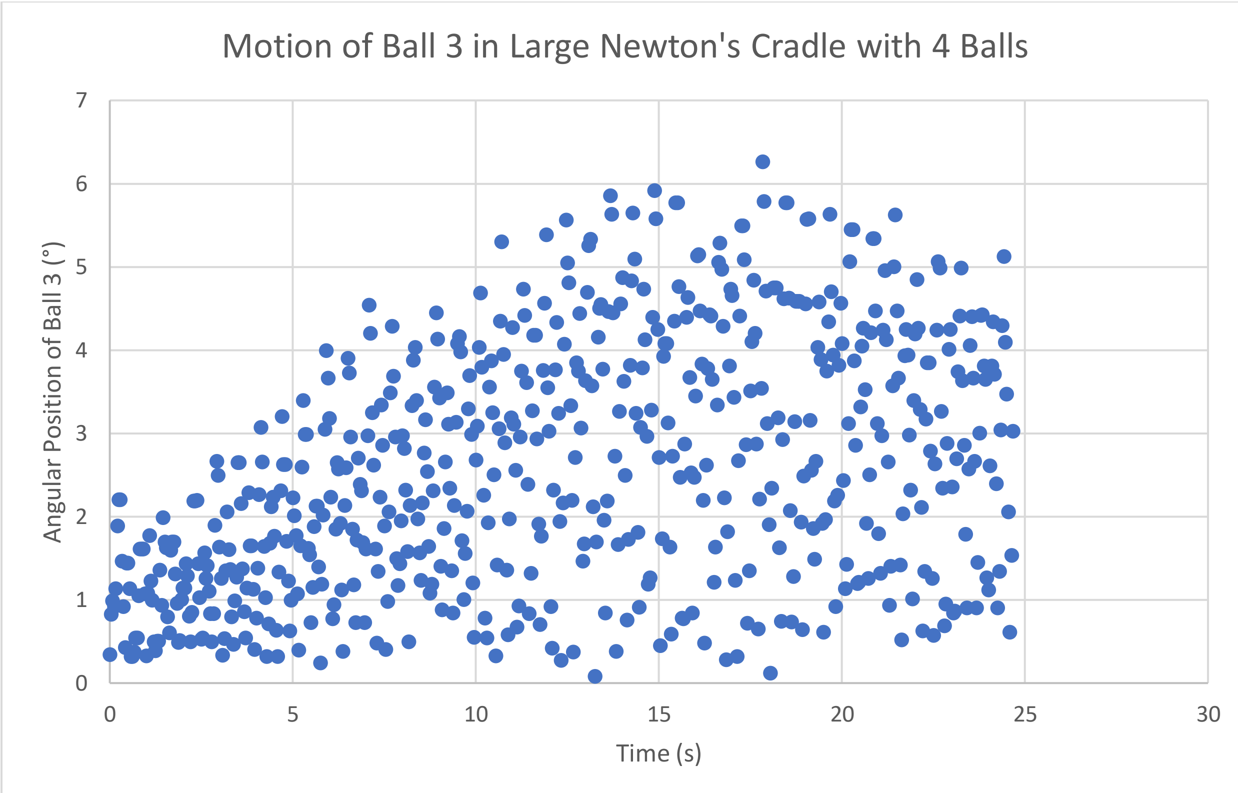


*Figure 16.*

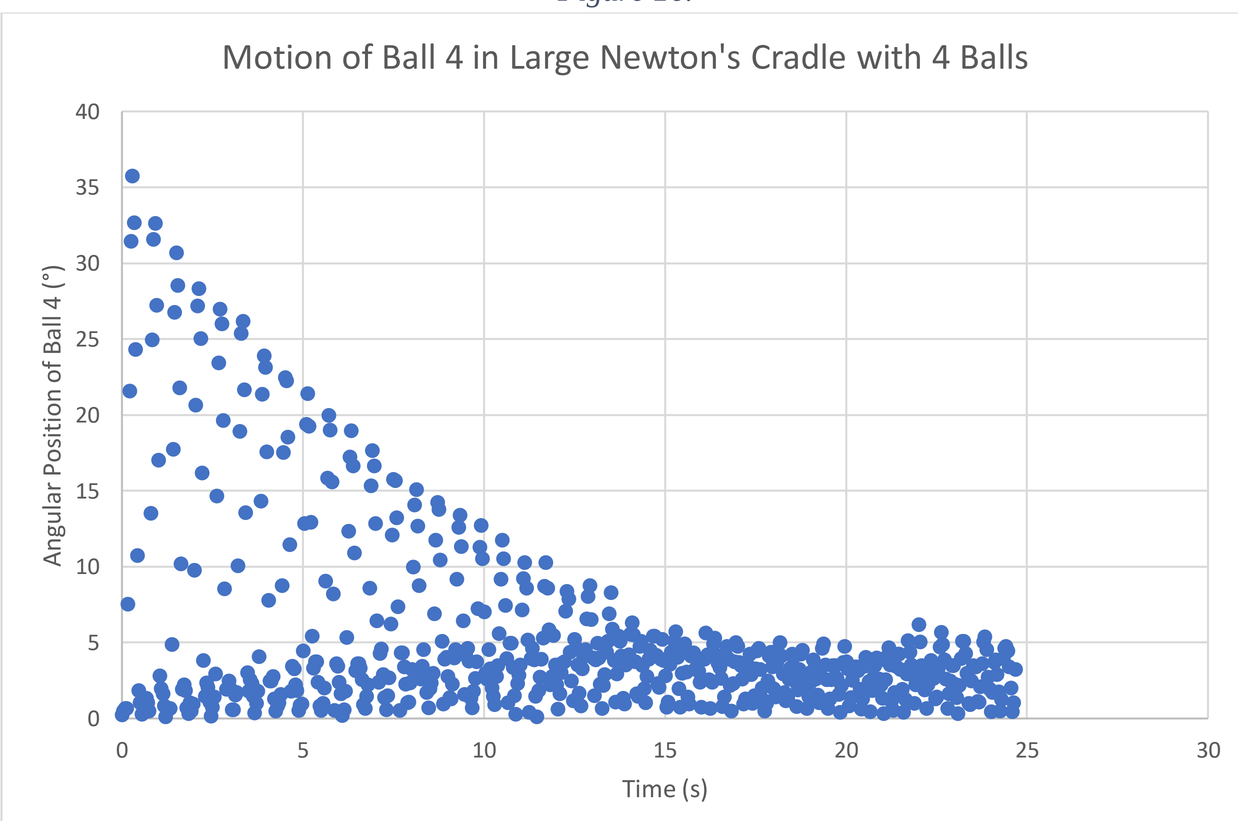


*Figure 17.*

**3 Ball Experiment**

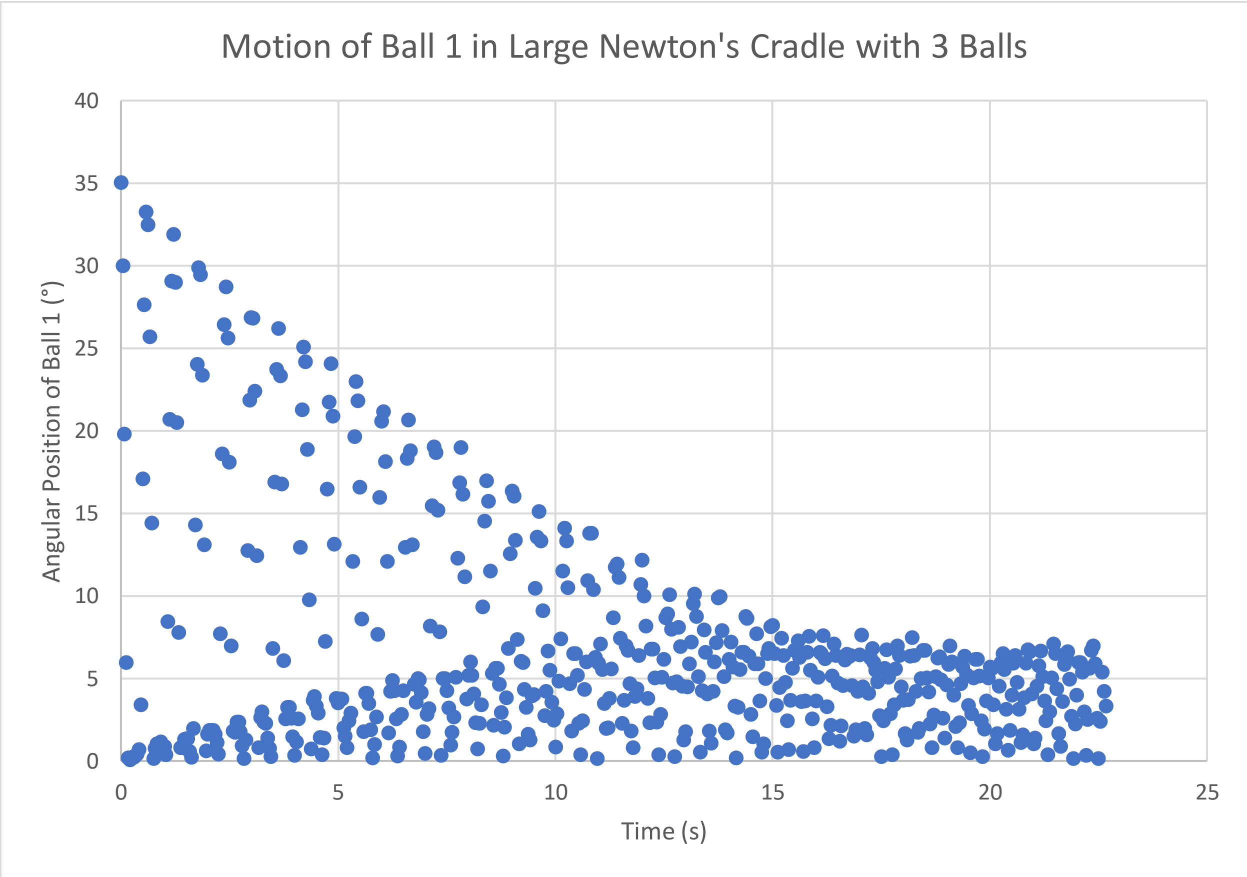


*Figure 18.*

The three ball Newton's Cradle has a decaying time a little longer than the five ball Newton's Cradle.

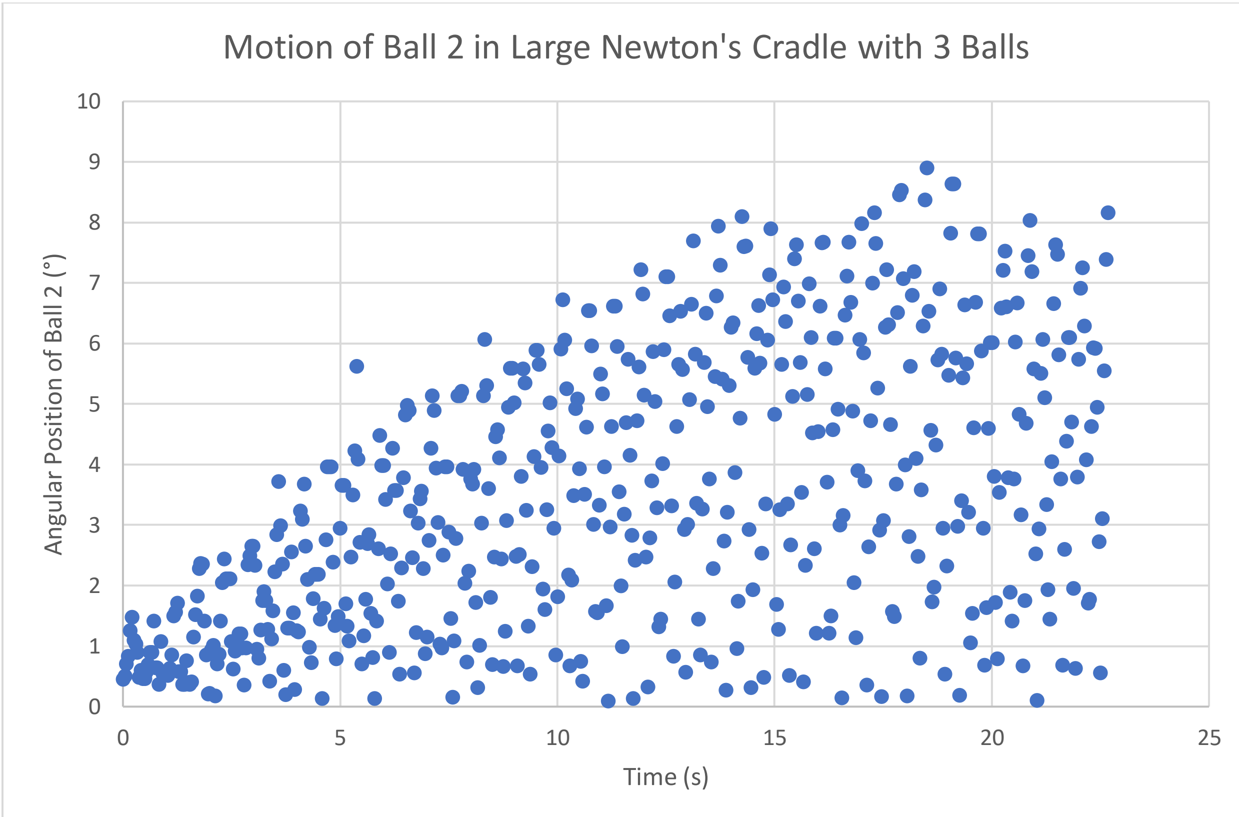


*Figure 19.*

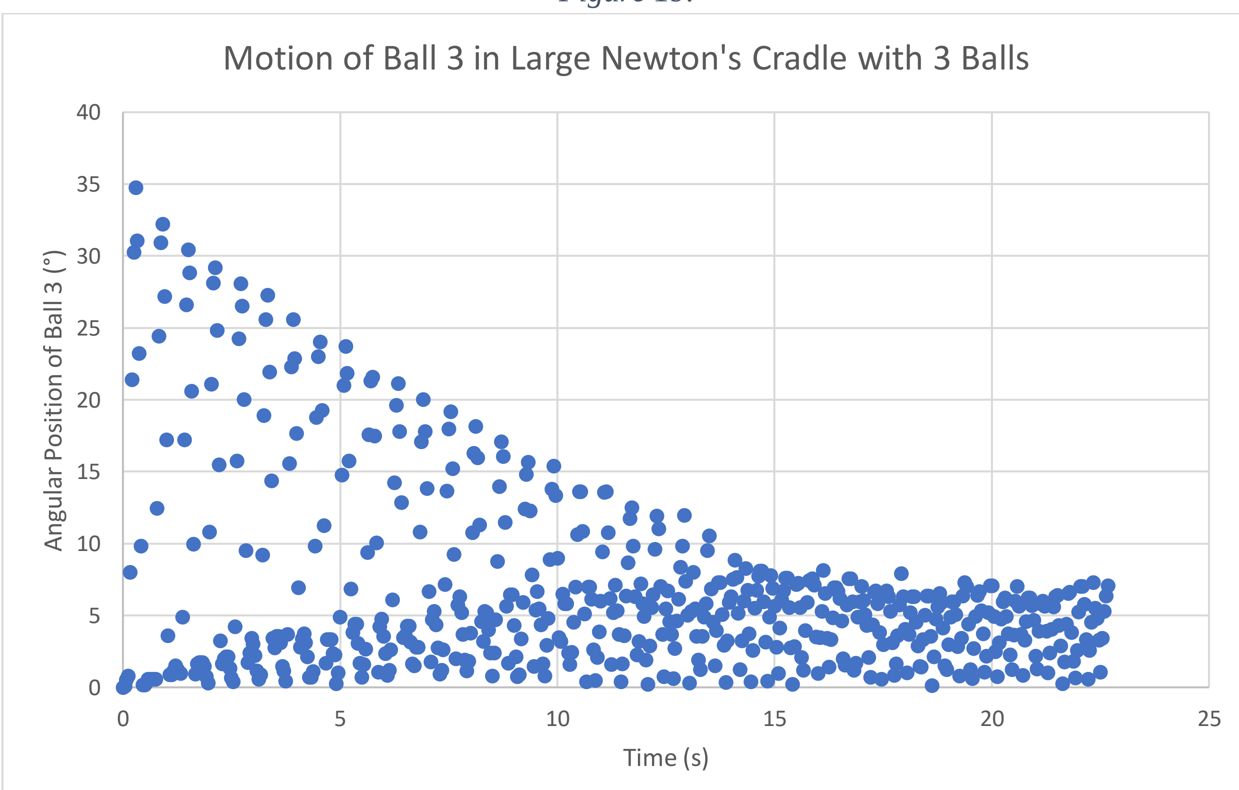


*Figure 20.*

### 6.2 Small Newton's Cradle Experiment

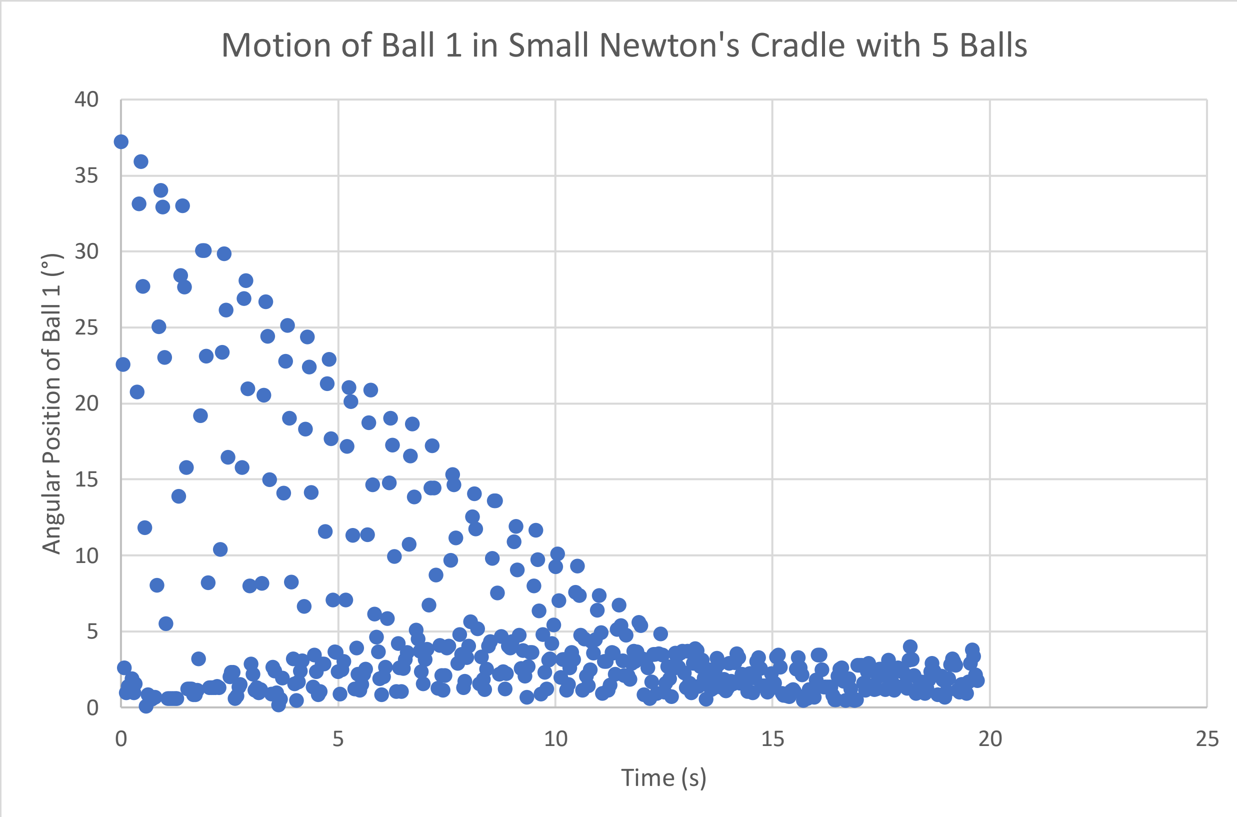


*Figure 21.*

The small Newton's Cradle has a significantly shorter decaying time than the bigger Newton's Cradle.

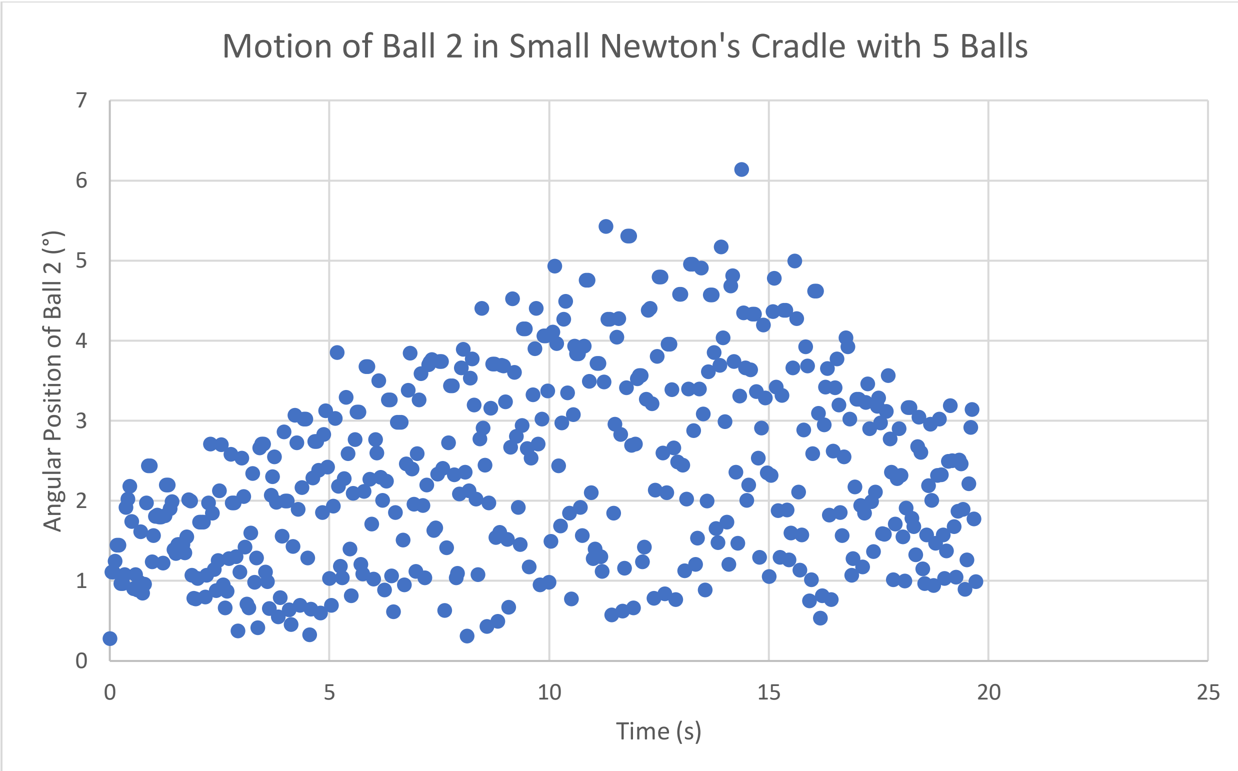


*Figure 22.*

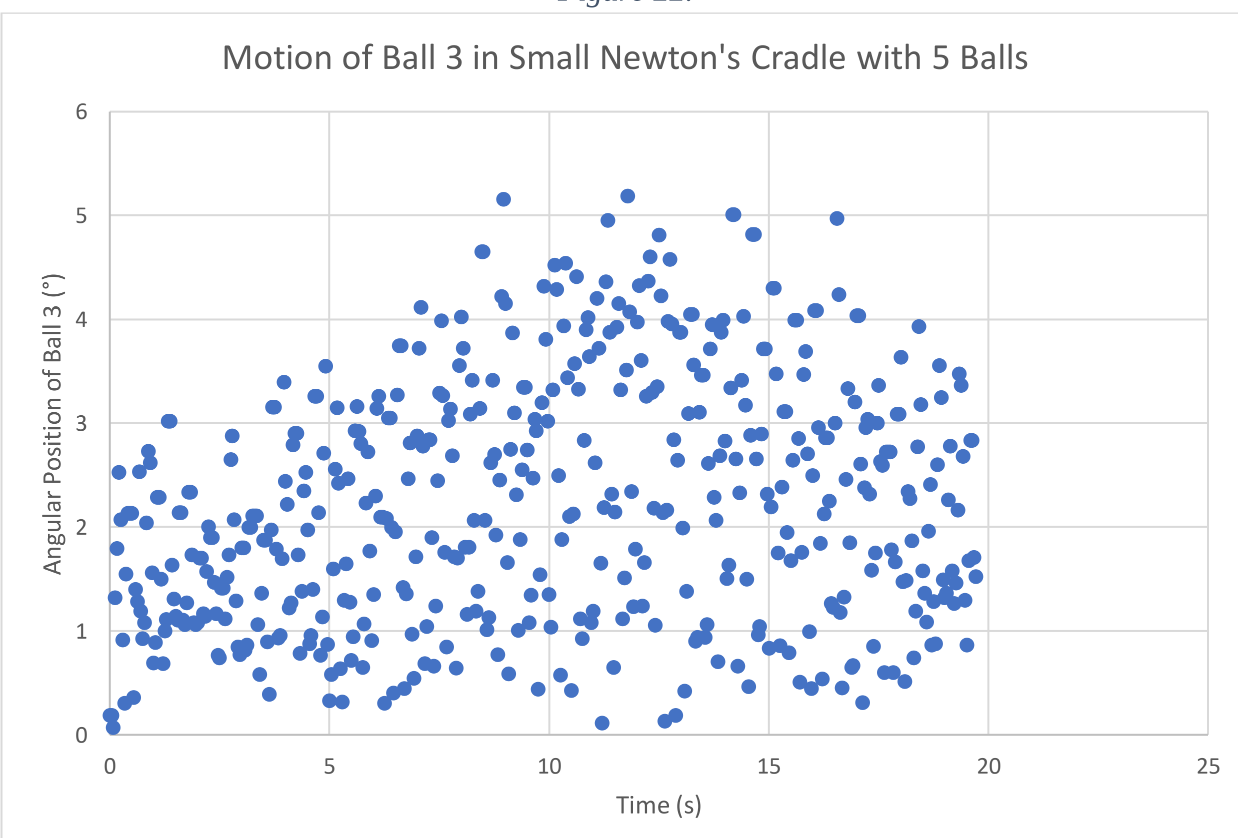


*Figure 23.*

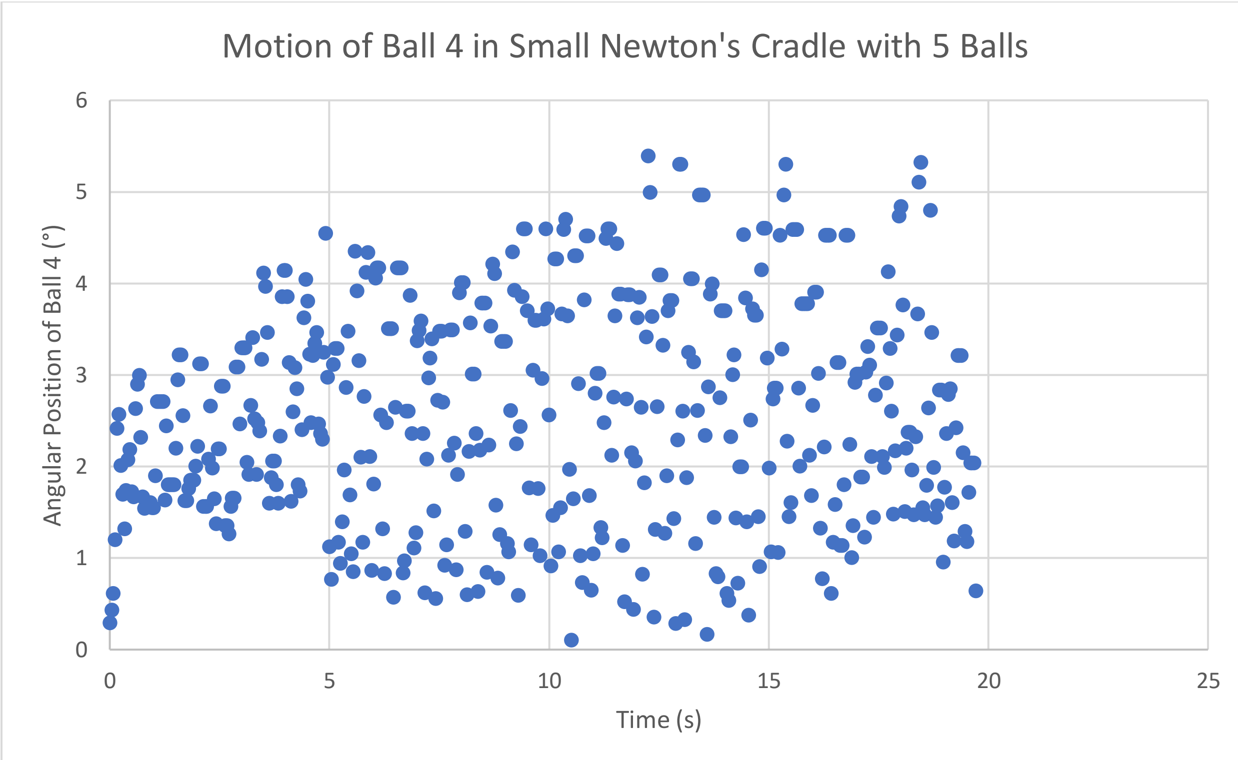


*Figure 24.*

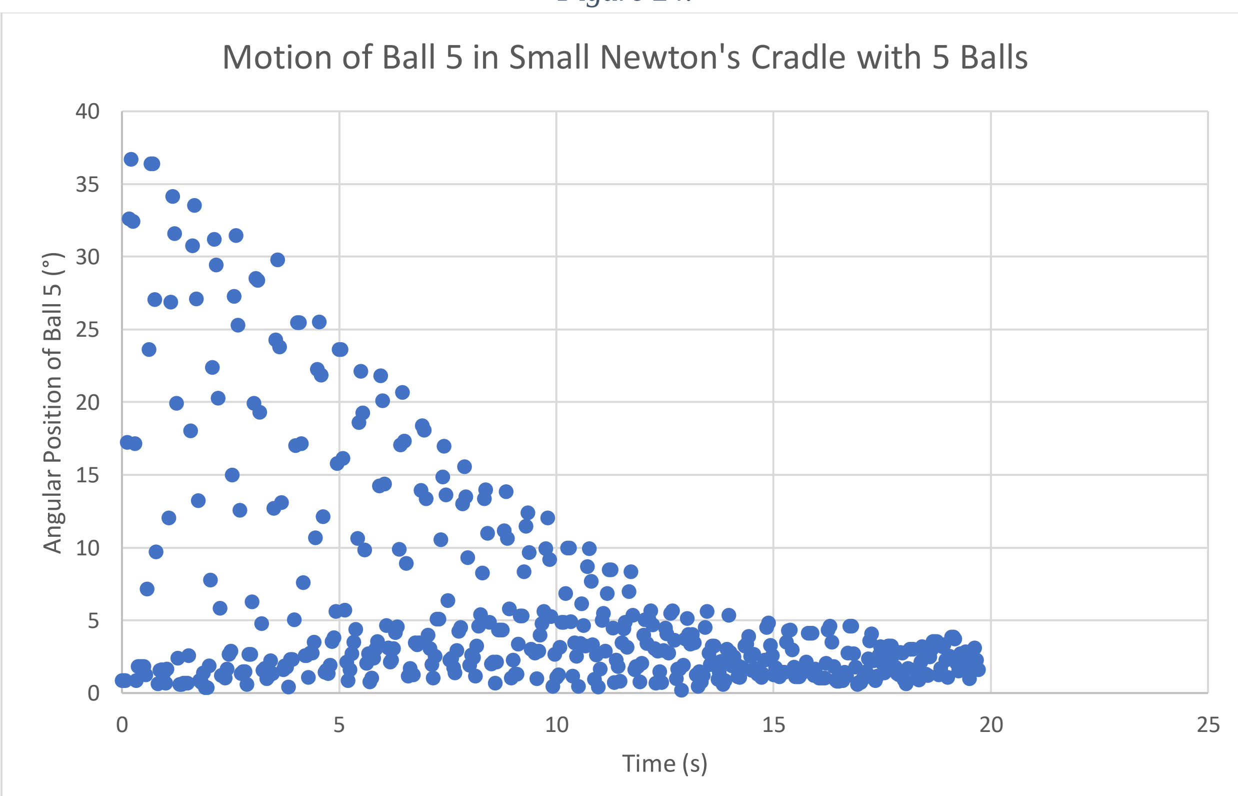


*Figure 25.*

Using Equation (11) I created the theoretical values with a simulation, which I compared with the experimental values of each experiment.

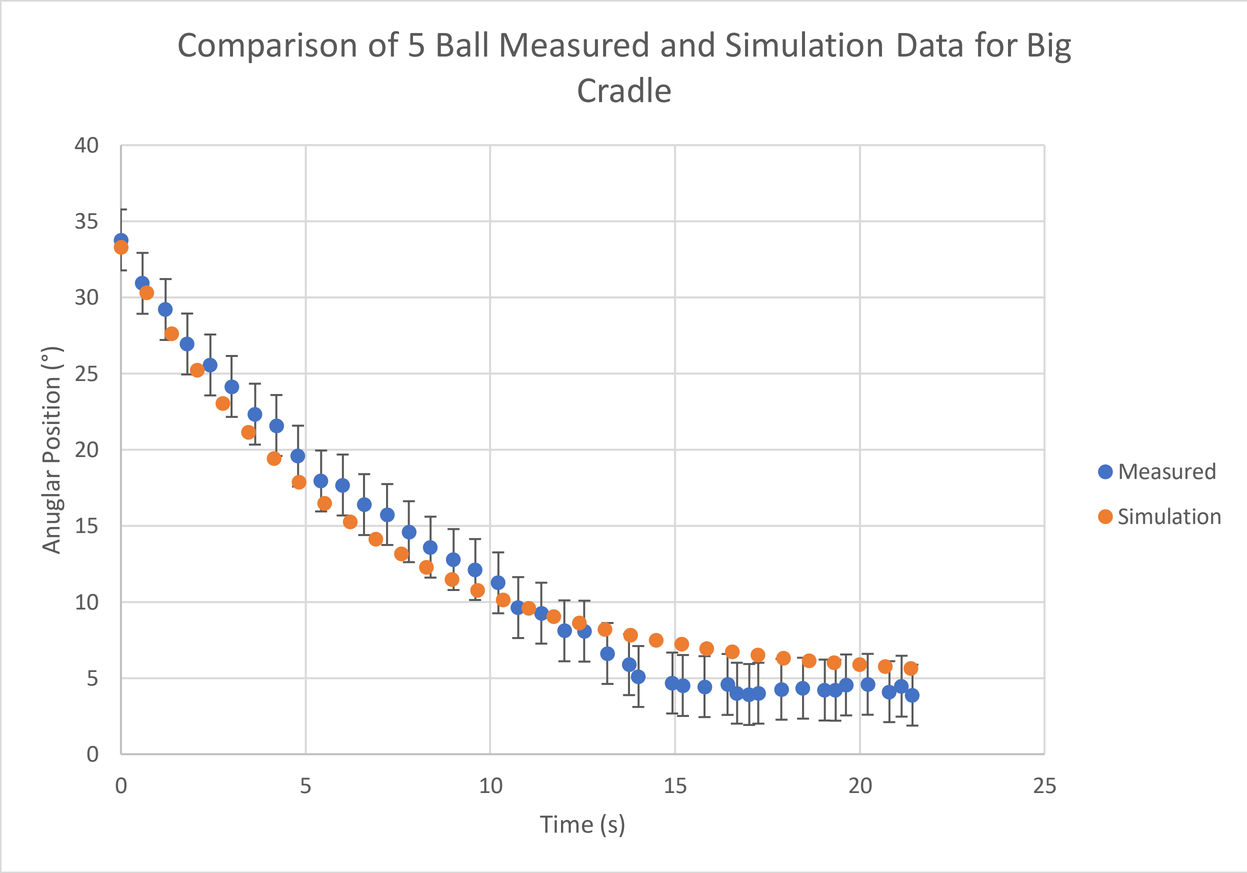


*Figure 26.*

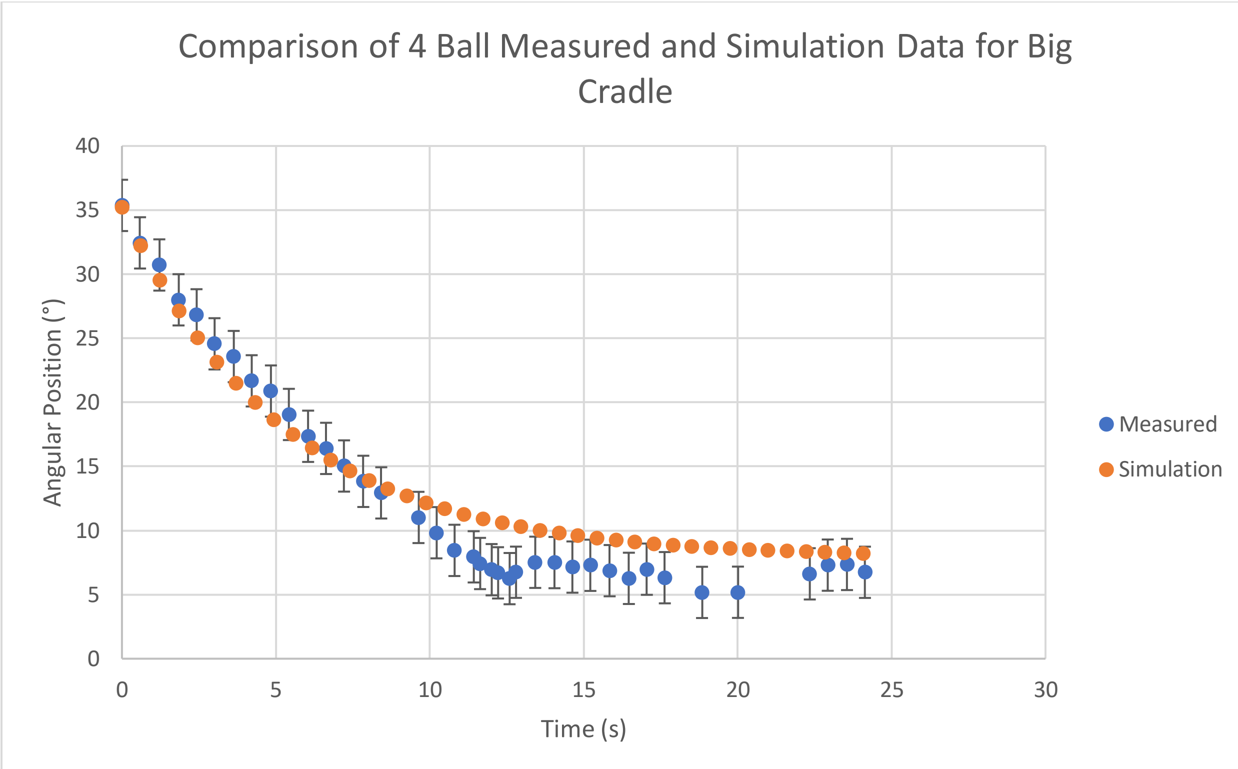


*Figure 27.*

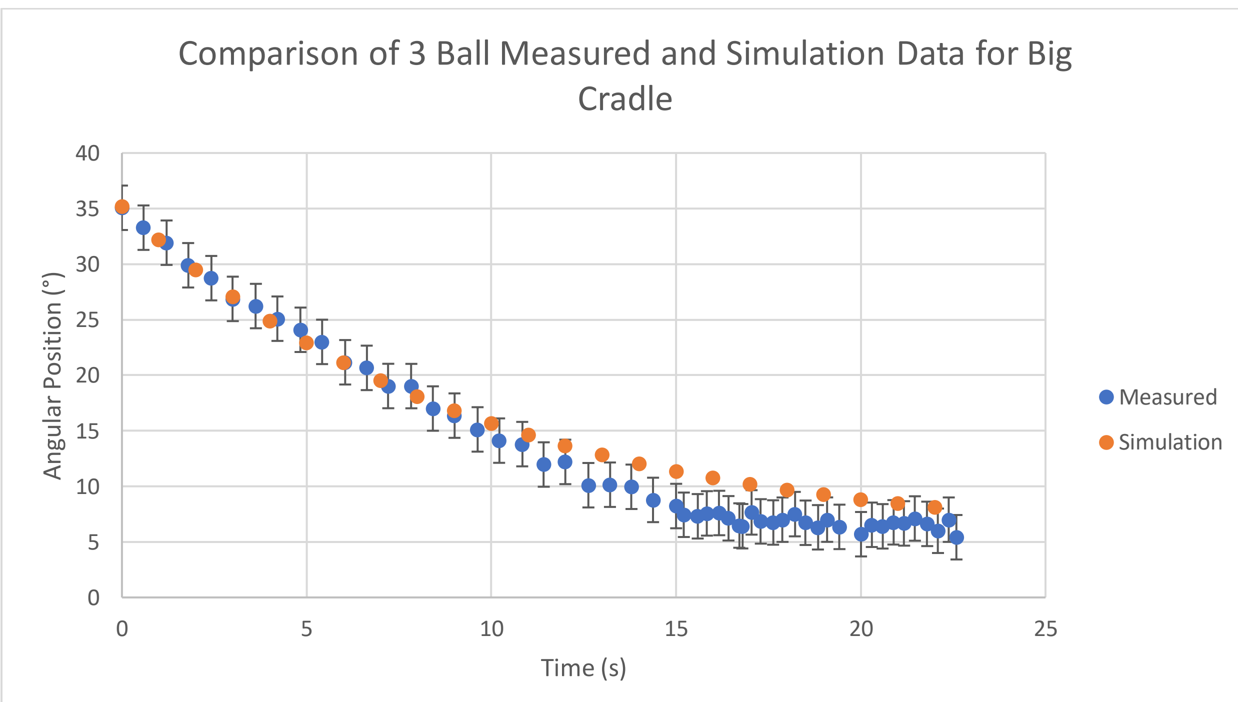


*Figure 28.*

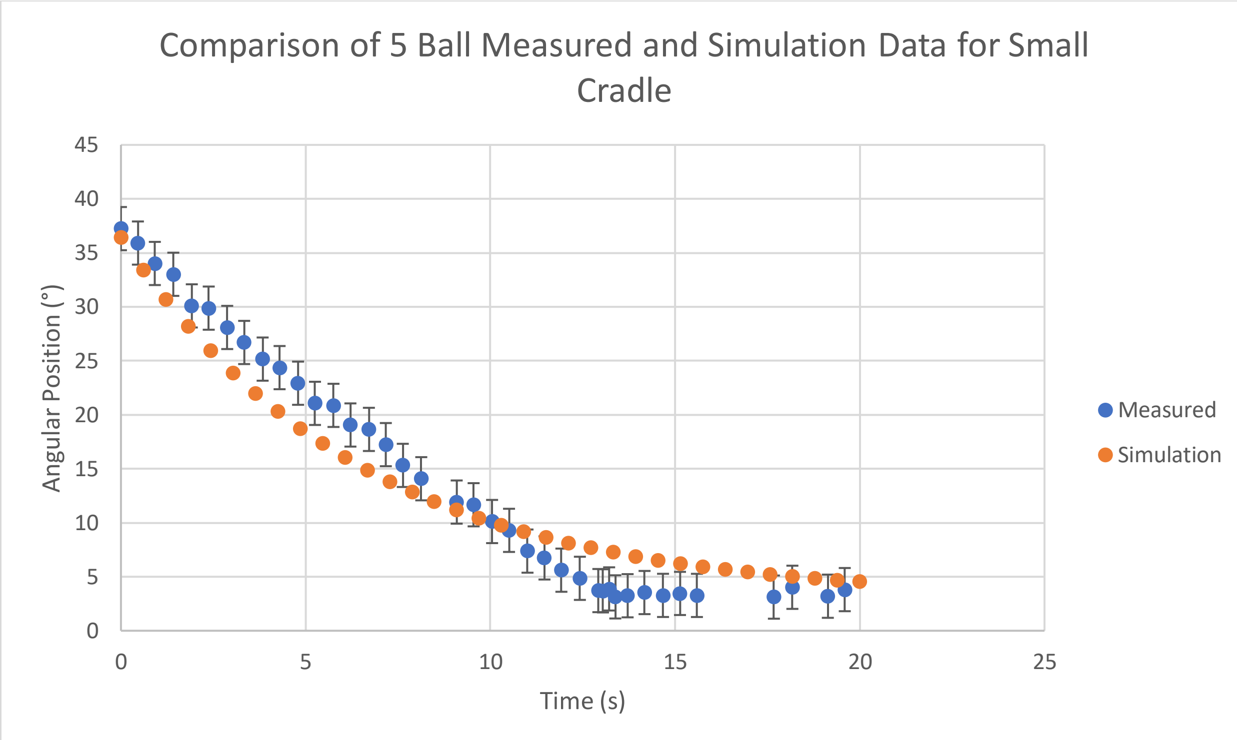


*Figure 29.*

**Conclusion:**

Through this investigation, I observed changes in the rate of decay in the motion of the Newton's Cradle depending on the number of balls of the cradle, and the mass of the balls in the cradle. For the number of balls experiment, I hypothesized the rate of decay of motion to decrease as the number of balls increased. However, the data did not show any certain patterns in the change of rate of decay of the motion of the cradle as the number of balls increased. The number of collisions in the motion of the cradle do not necessarily correlate with the time it takes for the cradle to come to a stop, which explains the absence of a pattern. For the mass of ball experiment, I hypothesized that the rate of decay in the motion of the Newton's Cradle would increase if the mass of the ball decreased, which the experimental data supported. This would be because the initial force in the motion of the system is smaller, thus the motion would decay faster due to Stokes law and viscoelastic dissipation forces.

**Discussion:**

In this experiment, the biggest limitation I faced was the difficulty in finding many variations of Newton's Cradles. Cradles made up of materials other than steel were not seen on the web, which meant that I had to build the Newton's Cradle out of scratch. However, if I made a

Newton’s Cradle, there would inevitably be holes in the precision of measurements, disrupting the theoretical physics of the Newton’s Cradle.

**References:**


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