問 2　問 1 の同時確率分布から Xi の周辺確率分布を求めよ.

X_iの確率分布は j の出る回数によらずに全ての場合での和を求めれば良い。
したがって、問 1 の結果より、$i = k$ 回、すなわち $X_i = k$ の時の周辺確率分布は、

(a)i , j が共に奇数であるとき、

$$\begin{aligned}
&\sum_{l=1}^{n} \frac{n!}{k!\ l!\ (n-k-l)!} \left(\frac{1}{9}\right)^k \left(\frac{1}{9}\right)^l \left(\frac{7}{9}\right)^{n-k-l} \\
&= \sum_{l=1}^{n} \frac{n!(n-k)!}{k!\ l!\ (n-k-l)!(n-k)!} \left(\frac{1}{9}\right)^k \left(\frac{1}{9}\right)^l \left(\frac{7}{9}\right)^{n-k-l} \\
&= \frac{n!}{k!(n-k)!} \left(\frac{1}{9}\right)^k \sum_{l=1}^{n} \frac{(n-k)!}{l!(n-k-l)!} \left(\frac{1}{9}\right)^l \left(\frac{7}{9}\right)^{n-k-l} \\
&= {}_nC_k \left(\frac{1}{9}\right)^k \sum_{l=1}^{n} {}_{n-k}C_l \left(\frac{1}{9}\right)^l \left(\frac{7}{9}\right)^{n-k-l} \\
&= {}_nC_k \left(\frac{1}{9}\right)^k \left(\frac{1}{9} + \frac{7}{9}\right)^{n-k} \\
&= {}_nC_k \left(\frac{1}{9}\right)^k \left(\frac{8}{9}\right)^{n-k}
\end{aligned}$$

(b)i が奇数 , j が偶数であるとき、

$$
\begin{aligned}
&\sum_{l=1}^{n} \frac{n!}{k!\ l!\ (n-k-l)!} \left(\frac{1}{9}\right)^{k} \left(\frac{2}{9}\right)^{l} \left(\frac{2}{3}\right)^{n-k-l} \\
&= \sum_{l=1}^{n} \frac{n!(n-k)!}{k!\ l!\ (n-k-l)!(n-k)!} \left(\frac{1}{9}\right)^{k} \left(\frac{2}{9}\right)^{l} \left(\frac{2}{3}\right)^{n-k-l} \\
&= \frac{n!}{k!(n-k)!} \left(\frac{1}{9}\right)^{k} \sum_{l=1}^{n} \frac{(n-k)!}{l!(n-k-l)!} \left(\frac{1}{9}\right)^{l} \left(\frac{2}{3}\right)^{n-k-l} \\
&= {}_nC_k \left(\frac{1}{9}\right)^{k} \sum_{l=1}^{n} {}_{n-k}C_l \left(\frac{2}{9}\right)^{l} \left(\frac{2}{3}\right)^{n-k-l} \\
&= {}_nC_k \left(\frac{1}{9}\right)^{k} \left(\frac{2}{9} + \frac{2}{3}\right)^{n-k} \\
&= {}_nC_k \left(\frac{1}{9}\right)^{k} \left(\frac{8}{9}\right)^{n-k}
\end{aligned}
$$

(c)i が偶数 , j が奇数であるとき、

$$
\begin{aligned}
&\sum_{l=1}^{n} \frac{n!}{k!\ l!\ (n-k-l)!} \left(\frac{2}{9}\right)^{k} \left(\frac{1}{9}\right)^{l} \left(\frac{2}{3}\right)^{n-k-l} \\
&= \sum_{l=1}^{n} \frac{n!(n-k)!}{k!\ l!\ (n-k-l)!(n-k)!} \left(\frac{2}{9}\right)^{k} \left(\frac{1}{9}\right)^{l} \left(\frac{2}{3}\right)^{n-k-l} \\
&= \frac{n!}{k!(n-k)!} \left(\frac{2}{9}\right)^{k} \sum_{l=1}^{n} \frac{(n-k)!}{l!(n-k-l)!} \left(\frac{1}{9}\right)^{l} \left(\frac{2}{3}\right)^{n-k-l} \\
&= {}_nC_k \left(\frac{2}{9}\right)^{k} \sum_{l=1}^{n} {}_{n-k}C_l \left(\frac{1}{9}\right)^{l} \left(\frac{2}{3}\right)^{n-k-l} \\
&= {}_nC_k \left(\frac{2}{9}\right)^{k} \left(\frac{1}{9} + \frac{2}{3}\right)^{n-k} \\
&= {}_nC_k \left(\frac{2}{9}\right)^{k} \left(\frac{7}{9}\right)^{n-k}
\end{aligned}
$$

(d)i , j が共に偶数であるとき、

$$\sum_{l=1}^{n} \frac{n!}{k!\ l!\ (n-k-l)!} \left(\frac{2}{9}\right)^{k} \left(\frac{2}{9}\right)^{l} \left(\frac{5}{9}\right)^{n-k-l}$$

$$= \sum_{l=1}^{n} \frac{n!(n-k)!}{k!\ l!\ (n-k-l)!(n-k)!} \left(\frac{2}{9}\right)^{k} \left(\frac{2}{9}\right)^{l} \left(\frac{5}{9}\right)^{n-k-l}$$

$$= \frac{n!}{k!(n-k)!} \left(\frac{2}{9}\right)^{k} \sum_{l=1}^{n} \frac{(n-k)!}{l!(n-k-l)!} \left(\frac{2}{9}\right)^{l} \left(\frac{5}{9}\right)^{n-k-l}$$

$$= {}_{n}C_{k} \left(\frac{2}{9}\right)^{k} \sum_{l=1}^{n} {}_{n-k}C_{l} \left(\frac{2}{9}\right)^{l} \left(\frac{5}{9}\right)^{n-k-l}$$

$$= {}_{n}C_{k} \left(\frac{2}{9}\right)^{k} \left(\frac{2}{9} + \frac{5}{9}\right)^{n-k}$$

$$= {}_{n}C_{k} \left(\frac{2}{9}\right)^{k} \left(\frac{7}{9}\right)^{n-k}$$