(14)

Según el libro, los polinomios de Legendre se definen de la siguiente manera: $P_n(x) = \frac{1}{2^n n!} \frac{d^n}{dx^n} (x^2-1)^n$

Los primeros polinomios son

$P_0 = 1$, $P_1 = x$; $P_2 = \frac{1}{2}(3x^2-1)$; $P_3 = \frac{1}{2}(5x^3-3x)$

Conjunto ortonormal.

La base de Legendre $\beta = \{P_0, P_1, P_2\}$

$3+5x+x^2 = \alpha_1 P_0 + \alpha_2 P_1 + \alpha_3 P_2$

$= \alpha_1 + \alpha_2 x + \frac{\alpha_3 (3x^2-1)}{2}$

$= \frac{2\alpha_1 + 2\alpha_2 x + 3\alpha_3 x^2 - \alpha_3}{2}$

$3+5x+x^2 = 1\left(\frac{2\alpha_1 - \alpha_3}{2}\right) + x(\alpha_2) + x^2\left(\frac{3\alpha_3}{2}\right)$

$2\alpha_1 - \alpha_3 = 6 \Rightarrow \alpha_1 = \frac{6+\alpha_3}{2} = \frac{6+\frac{2}{3}}{2} = \frac{18+2}{6} = \frac{20}{6} = \frac{10}{3}$

$\alpha_2 = 5 \Rightarrow \alpha_2 = 5$

$\alpha_3 = \frac{2}{3} \Rightarrow \alpha_3 = \frac{2}{3}$

$$3+5x+x^2 = \frac{10}{3} + 5x + \frac{2}{3}(3x^2-1)$$

1

$$\int_a^b P(x) = \frac{f(a)}{a-b}\int_a^b x-b + \frac{f(b)}{b-a}\int_a^b x-a$$

$$= \frac{f(a)}{a-b}\left(\frac{\cancel{(b-b)^2}-(a-b)^2}{2}\right) + \frac{f(b)}{b-a}\left(\frac{(b-a)^2-\cancel{(a-a)^2}}{2}\right)$$

$$= \frac{f(b)(b-a)^{\cancel{2}}}{2\cancel{(b-a)}} - \frac{f(a)(a-b)^{\cancel{2}}}{2\cancel{(a-b)}} = \frac{f(b)(b-a)}{2} - \frac{f(a)(a-b)}{2}$$

$$\frac{f(b)(b-a)+f(a)(b-a)}{2} = \frac{(b-a)}{2}\left(f(b)+f(a)\right)$$ ✓