Lecture 27: Learning from relational data

STATS 202: Data mining and analysis

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Announcements

- ► Kaggle deadline is this Wednesday (Dec 4) at 4pm. If you haven't already, make a submission to earn credit.
- ▶ In order to earn credit for the Kaggle competition, submit "Homework 8" on the website by this Friday. It will ask for your user name, your team's name, and a brief description of what you did.
- My office hours this week have been rescheduled to Friday afternoon (see website).
- ▶ You will get Homeworks 6 and 7 back by the end of the week.

Announcements

Final exam:

- ▶ Monday, December 9, 8:30-11:30 am at Cubberley Auditorium.
- SCPD students: The exam will be sent out early Monday morning, and you must return it by Tuesday December 10th at noon PST. Local students may take the exam at Stanford with the rest of the class.
- Closed book, closed notes. No calculators or computers.
 Equation sheet provided.
- Practice problems on the website.
- ▶ At least one problem in the exam will be from the practice problems.

Announcements

Course evaluations are available on Axess. If you complete them by Dec 16, you will be able to see your grades as soon as we post them. If you don't complete the evaluations, you will have to wait until Dec 19.

Relational data

The observations have the form of a graph.

Examples.

- Links between websites.
- Relationships between accounts in social networks.
- Transmission networks for contagious diseases.

The links can be directed or undirected.

There can be different types of link (friend, follower, followed).

We can observe the graph in time (how do social networks grow?).

Each vertex can have additional features or **metadata**.

- Invented by Sergei Brin and Larry Page of Google.
- Uses a graph of links between websites to rank websites by "importance".

Motivation:

- Consider the problem of searching the web using the query "birth control".
- ▶ There are millions of pages containing the term.
- Analyzing the content of each website semantically to infer which one is more likely to satisfy the user is very expensive.
- ► We need a way to rank websites, to filter out all those that are rarely visited. This information is given by links.

Consider a hypothetical **surfer** who is jumping from website to website by clicking on random links. Intuitively, the websites that are visited more frequently can be considered more important in the network of links.

Will the surfer visit every website eventually? No. It is possible to get stuck in a website with no outgoing links, or to be stuck in a loop between two websites, for example.

To avoid this problem, we modify the random walk, such that at every step, with probability 1-q, we pick a website at random, and with probability q we go through one of the links in the current website at random.

- ► The surfer's random walk is a Markov chain on the set of websites.
- ▶ It is a fact that the frequency with which the surfer visits any website converges to some limit.
- ► The PageRank of a website is this limiting frequency.

Let P_{ij} be the probability of jumping from website i to website j, then

$$P_{ij} = (1 - q)\frac{1}{n} + q \left[\frac{\text{\# of links from } i \text{ to } j}{\text{\# of links out of } i} \right]$$

The limiting frequency of website j, π_j , must satisfy

$$\pi_j = \sum_{i=1}^n \pi_i P_{ij}$$

or in matrix notation $\pi=\pi P$. That is, π is an eigenvector of the transition probability matrix P with eigenvalue 1.

Finding the limiting frequencies π

In principle, finding the limiting frequencies could require solving the eigendecomposition of a matrix P which is $n \times n$, and this has a complexity which grows as n^3 .

However, it is possible to compute π by starting with the approximation $\pi^{(0)}=(1/n,\dots,1/n)$, and iterating:

$$\pi^{(t)} = \pi^{(t-1)} P.$$

The number of iterations necessary for convergence is typically small.

The matrix-vector multiplication in each iteration can be sped up using sparse matrix techniques.

How can PageRank be used in web search?

One idea:

- 1. Find all websites that contain all query terms.
- 2. Display them in order of their PageRank.

A more likely approach:

- 1. Use PageRank to select the 10,000 most important pages which contain the query terms.
- 2. Rank these 10,000 pages by analyzing their content, integrating information about the user, etc.

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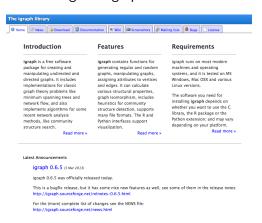
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PageRank in R

The package igraph in R has a function page.rank, among many other utilities for working with graphs.



Edge prediction in Facebook's social graph

For the rest of this lecture, we will review Edwin Chen's solution to a Kaggle contest organized by Facebook.



Link to blog post.