HW9

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December 16, 2019

1 Problem 1

1.1 Part i

$$\begin{array}{lrrcl} \min & \sum_j y_j & & & \\ \text{s.t.} & \forall i, & \sum_j x_{ij} & = & 1 \ ; \\ & \forall j & \sum_i x_{ij} \cdot a_i & \leq & V \ ; \\ & \forall j, \forall i, & y_j & \geq & x_{ij} \ ; \\ & \forall i, \forall j, & x_{ij}, y_j & \in & \{0,1\}. \end{array}$$

1.2 Part ii

For the IP problem, I had an optimal value of 3 bins where items 1 and 2 are in bin 1, item 3 is in bin 2, and item 4 is in bin 4. For the LP relaxation, I got an optimal value of 1 with $\mathbf{x} = \begin{pmatrix} .25 & .25 & .25 & .25 \\ .25 & .25 & .25 & .25 \\ .25 & .25 & .25 & .25 \\ .25 & .25 & .25 & .25 \end{pmatrix}$ and $\mathbf{y} = \begin{pmatrix} .25 \\ .25 \\ .25 \\ .25 \end{pmatrix}$. This is an integrality gap of 2.

2 Problem 2

2.1 Part i

Let $S_{ij} = 1$ if customer i has product j in his bundle and 0 otherwise.

$$\begin{array}{llrcl} \min & & \sum_i x_i \cdot v_i & & \\ \text{s.t.} & \forall j, & \sum_i x_i \cdot S_{ij} & \leq & B_j \ ; \\ & \forall i, & x_i & \in & \{0,1\} \ . \end{array}$$

2.2 Part ii

For the LP relaxation and for the IP, I got an optimal solution of 8 where the seller rejects customer 1's request but accepts the other 4. The integrality gap in this case is 0.

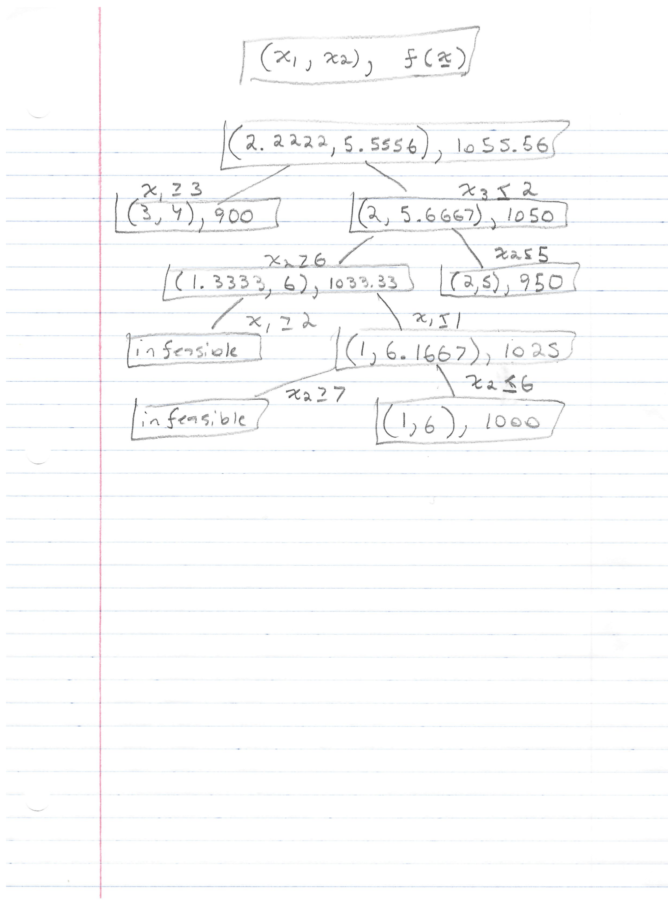

Figure 1: Branch and Bound Tree

3 Problem 3

I got an optimal value of 1000 with $\mathbf{x} = (1, 6)$).