HW1

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1 Problem 1

1.1 Part a

$$\begin{array}{rrcl} \max & 7.8x_1 + 7.1x_2 & & \\ \text{s.t.} & \frac{1}{4}x_1 + \frac{1}{3}x_2 & \leq & 90\ ; \\ & \frac{1}{8}x_1 + \frac{1}{3}x_2 & \leq & 80\ ; \\ & \mathbf{x} & \geq & \mathbf{0}\ . \end{array}$$

1.2 Part b

$$\mathbf{b} = \begin{pmatrix} 90 \\ 80 \end{pmatrix},\ \mathbf{c} = \begin{pmatrix} -7.8 \\ -7.1 \\ 0 \\ 0 \end{pmatrix},\ A = \begin{pmatrix} \frac{1}{4} & \frac{1}{3} & 1 & 0 \\ \frac{1}{8} & \frac{1}{3} & 0 & 1 \end{pmatrix}$$

$$\begin{array}{rrcl} -\min & \mathbf{c}^T\mathbf{x} & & \\ \text{s.t.} & A\mathbf{x} & = & \mathbf{b}\ ; \\ & \mathbf{x} & \geq & \mathbf{0}\ . \end{array}$$

1.3 Part c

$$\begin{array}{rrcl} \max & 7.8x_1 + 7.1x_2 - 7x_3 & & \\ \text{s.t.} & \frac{1}{4}x_1 + \frac{1}{3}x_2 & \leq & 140\ ; \\ & \frac{1}{4}x_1 + \frac{1}{3}x_2 - x_3 & \leq & 90\ ; \\ & \frac{1}{8}x_1 + \frac{1}{3}x_2 & \leq & 80\ ; \\ & \mathbf{x} & \geq & \mathbf{0}\ . \end{array}$$

Here, x_3 is the number of overtime hours. The first constraint still represents the number of assembly hours in total, but this time that number is allowed to be 140 instead of 90 as a result of the 50 overtime hours. The second constraint represents the total number of non-overtime assembly hours, which still must be less than 90. The cost of $7 per overtime hour has been reflected in the objective function, and the constraint that the number of overtime hours must be non-negative has been added

1.4 Part d

The optimal solution is 360 of product 1 and 0 of product 2, for an optimal value of 2808.

2 Problem 2

$$
\begin{array}{lrcccl}
\min & \mathbf{e}^T\mathbf{y} & & & & \\
\text{s.t.} & -\mathbf{y} & \leq & \mathbf{x} & \leq & \mathbf{y}\ ; \\
 & -\mathbf{e} & \leq & \mathbf{x} & \leq & \mathbf{e}\ ; \\
 & A\mathbf{x} & = & \mathbf{b}\ . & &
\end{array}
$$

I got an optimal value of 10 from an optimal solution of

$$x = \begin{pmatrix} 1.0000 \\ 1.0000 \\ 1.0000 \\ 1.0000 \\ 1.0000 \\ 1.0000 \\ 1.0000 \\ 1.0000 \\ 1.0000 \\ 1.0000 \\ 0.0000 \\ -0.0000 \\ 0.0000 \\ -0.0000 \\ 0.0000 \\ 0.0000 \\ 0.0000 \\ 0.0000 \\ 0.0000 \\ 0.0000 \\ -0.0000 \\ 0.0000 \\ 0.0000 \\ -0.0000 \\ 0.0000 \\ -0.0000 \\ -0.0000 \\ -0.0000 \\ -0.0000 \\ -0.0000 \\ 0.0000 \\ 0.0000 \\ 0.0000 \\ -0.0000 \\ 0.0000 \\ 0.0000 \\ 0.0000 \\ 0.0000 \\ 0.0000 \\ -0.0000 \end{pmatrix}$$

3 Problem 3

$$\begin{array}{llcll} \min & \sum_{i=1}^{I}\sum_{j=1}^{J}\sum_{g=1}^{G} x_{ijg}\cdot d_{ij} & & & \\ \text{s.t.} & \sum_{i=1}^{I} x_{ijg} & \leq & C_{jg} & \forall j\in\{1,..,J\},\forall g\in\{1,..,G\}\ ; \\ & \sum_{j=1}^{J} x_{ijg} & = & S_{ig} & \forall i\in\{1,..,I\},\forall g\in\{1,..,G\}\ ; \\ & x & \geq & 0_{I,J,G}\ . & \end{array}$$

4 Problem 4

Let C_{ij} denote the cost of moving a car from i to j defined in Table 1. Let x_{ij} denote the number of cars moved from i to j. Let P_i denote the number of cars presently in i and let N_i denote the number of cars needed in i. The problem can be represented by

$$\begin{array}{lrcll} \min & \sum_i \sum_j C_{ij}x_{ij} & & & \\ \text{s.t.} & \sum_j x_{ji} - \sum_j x_{ij} & \leq & N_i - P_i & \forall i\in\{1,..,5\}\ ; \\ & x & \geq & 0_{5,5}\ . & \end{array}$$

which gives an optimal value of 11370 for an optimal solution of

$$x = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 115 & 165 & 0 & 0 \\ 85 & 0 & 225 & 0 & 0 \end{pmatrix}$$

5 Problem 5

5.1 Part 1

Define $x \in \mathbb{R}^{T\times K}$ with x_{tk} denoting the amount of money invested in vehicle k at the beginning of time t. Define $h \in \mathbb{R}^{T}$ with h_t denoting the amount of cash held at the beginning of time t.

Then, for any t, the amount of cash available at the start of t is equal to

$$p_t + \sum_{i=1}^{t-1}\sum_{k=1}^{K} v_{i,t-1}^{k}\cdot x_{ik} + (1+.01q)h_{t-1}$$

and the total amount of cash allocated or held at the start of t is

$$\sum_{k=1}^{K} x_{tk} + h_t.$$

We can think of the total money made at the end of T as being the same as the amount of cash available at a theoretical $T+1$, with $p_{T+1}=0$. Thus the total money made, which we are trying to maximize, is

$$\sum_{i=1}^{T}\sum_{k=1}^{K} v_{i,T}^{k} \cdot x_{ik} + (1+.01q)h_T.$$

We also require as a constraint that, for each t, the amount of cash available at the start of t is the same as the amount of cash allocated or held at the start of t.

Thus, the problem can be formulated as

$$\begin{array}{lrcll} \max & \multicolumn{4}{l}{\sum_{i=1}^{T}\sum_{k=1}^{K} v_{i,T}^{k} \cdot x_{ik} + (1+.01q)h_T} \\ \text{s.t.} & \sum_{k=1}^{K} x_{tk} + h_t - (\sum_{i=1}^{t-1}\sum_{k=1}^{K} v_{i,t-1}^{k} \cdot x_{ik} + (1+.01q)h_{t-1}) & = & p_t & \forall t \in \{1,..,T\} \ ; \\ & x & \geq & 0_{T,K} \ ; & \\ & h & \geq & \mathbf{0} \ . & \end{array}$$

5.2 Part 2

The optimal value is 25.9892 for an optimal solution of

$$x = \begin{pmatrix} 10 & 0 & 0 \\ 0 & 4.8 & 0 \\ 0 & 0 & 8.292 \end{pmatrix}, \ h = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$