1. 15 students, 8 questions

prob students not getting asked Q yet?

situation where 8 students asked one Q, 7 students not asked Q yet

$$\frac{15}{15} \cdot \frac{14}{15} \cdot \frac{13}{15} \cdot \frac{12}{15} \cdot \frac{11}{15} \cdot \frac{10}{15} \cdot \frac{9}{15} \cdot \frac{8}{15}$$

$$= \frac{14 \cdot 13 \cdot 12 \cdot 11 \cdot 10 \cdot 9 \cdot 8}{15^7} = 0.10124 \rightarrow \sim 10.124\%$$

what students can be picked

2. 5 #'s / 8 criteria

0000 - 9999 → 10^5 numbers, all digits unique

Criteria: total # ints (every # unique), even & starting 2 odd digits

0.42

Ranges within 0000 - 9999

0-100: no #'s in criteria, b/c only 2 digits exist

100-1000: (3 digits), 0, 1, 2, 3, 4, 5, 6, 7, 8, 9

nC_r for each digit

$$_5C_1 \cdot {_4C_1} \cdot {_5C_1} = 5 \cdot 4 \cdot 5 = 100$$

1000 - 10,000 (4 digits)

$$_5C_1 \cdot {_4C_1} \cdot {_7C_1} \cdot {_5C_1} = 5 \cdot 4 \cdot 7 \cdot 5 = 700$$

10,000 - 100,000 (5 digits)

$$_5C_1 \cdot {_4C_1} \cdot {_7C_1} \cdot {_6C_1} \cdot {_5C_1} = 5 \cdot 4 \cdot 7 \cdot 6 \cdot 5 = 4200$$

$4200 + 700 + 100 = 5000$

$$\frac{5000}{100{,}000} = 0.05$$

2 cont random generate 8 Q's, exactly 5 #'s w/ 2 odd digits & even

$P(\#) = 0.05$ (from prev page)

X = # of integers that have criteria

$P(X=5) = {}_8C_5 \underbrace{(0.05)^5}_{\text{\# hit criteria}} \cdot \underbrace{(1-0.05)^3}_{\text{\# not hit criteria}} = 56 \cdot 3.125 \times 10^{-7} \cdot 0.857375$

$= \boxed{1.50040625 \times 10^{-5}}$

3. Event A:
X = dice that show 4 or above — 3/6 prob

$P(A) = P(X=2) + P(X=3)$

$P(X=2) = {}_3C_2 \cdot (3/6)^2 \cdot (3/6)^1$ — prob of 3rd dice not being ≥4 — $= 0.125 \cdot 3 = 0.375$

$P(X=3) = {}_3C_3 \cdot (3/6)^3 (3/6)^0 = 0.125$

$P(A) = 0.375 + 0.125 = 0.5$

Event B: $P(B) = \frac{6}{6^3}$ ← possible values that can all be the same; ← total dice rolls

$= 1/36$

Probability of A & B

$P(A \cap B)$, 3 same & val ≥4, X = all 3 dice same 4 > value

$P(X=4) = (1/6)^3$

$P(X=5) = (1/6)^3$ $\qquad (1/6)^3 + (1/6)^3 + (1/6)^3 = 3/216 = \frac{1}{72}$

$P(X=6) = (1/6)^3$

Compare

$P(A) \cdot P(B) = 1/2 \cdot 1/36 = 1/72 = P(A \cap B) = 1/72$

↳ Event A & B are $\boxed{\text{independent}}$, b/c the 2 probabilities are equal

4. new deck of cards for every hand

To get a flush: 4 suits, 13 cards in each suit

$$\frac{{}_{13}C_5 \cdot 4}{{}_{52}C_5} \approx 0.00198079$$

$$E(X) = \frac{1}{0.00198079} \approx \boxed{504.84}$$

5. $P(\text{no super} \mid \text{win}) = 1/2$

$P(\text{super} \mid \text{win}) = 0.7$

$P(\text{super} \mid \text{win } 4/5) = {}_5C_4 \cdot \overbrace{0.7^4}^{\text{wins}} \cdot \overbrace{0.3}^{\text{losses}}$

$P(\text{no super} \mid \text{win } 4/5) = {}_5C_4 \cdot 0.5^5$

Total Probability, 0.75 chance player does play, 0.25 doesn't

$$0.75\,({}_5C_4 \cdot 0.7^4 \cdot 0.3) + ({}_5C_4 \cdot 0.5^5)\,0.25$$

$$= 0.36015 \cdot 0.75 + 0.15625 \cdot 0.25$$

$$0.2701 + 0.0390 = 0.3091$$

Bayes Thm: $\dfrac{0.2701}{0.3091}$ ↙ superstar playing / total probability $\approx \boxed{0.873}$ (depending on rounding)