1. Unusual ,

a) how many unique subsets of 5 letters (of the 7) exist?

$$\frac{n!}{(n-r!)} \quad r\text{-perm} \atop w/o \text{ rep}$$

3 cases
1 u
2 u
3 u

Case 1: one "u"

$4C4 \gets$ 4 more to choose $= 1$

| 11 unique subsets ✓

Case 2: two "u"

$4C3 \gets$ 3 more to choose $= 4$

Case 3: 3 "u"

$4C2$

↗ 2 more to choose $= 6$

b) how many diff strings can be made 5 of those 7 letters

case 1:

$$\underline{5} \cdot \underline{4} \cdot \underline{3} \cdot \underline{2} \cdot \underline{1}$$

1 unique subsets

$5!(1) = 120C_1 \rangle = 120$

case 2: 4 unique subset

$$\frac{5!}{2!}(4) = 60(4) = 240$$

ↅ

case 3: 6 unique subsets

$$\frac{5!}{3!}(6) = 20(6) = 120$$

$= 120$

$= \boxed{480 \text{ ways } !}$

2: how many ways to form 5-card w/ 2 pairs & 1 stand alone

52 cards   2 pairs   one other
13 ranks $_{13}C_2 \cdot {}_4C_2 \cdot {}_4C_2 \cdot {}_{11}C_1 \cdot {}_4C_1$

$$= \frac{13!}{2! \cdot 11!} \times \frac{4!}{2! \cdot 2!} \cdot \frac{4!}{2 \cdot 2!} \cdot \frac{11!}{10!} \cdot \frac{4!}{3!}$$

$$= \boxed{123,552 \text{ ways}}$$

3. 16 songs/hr, 7 couples
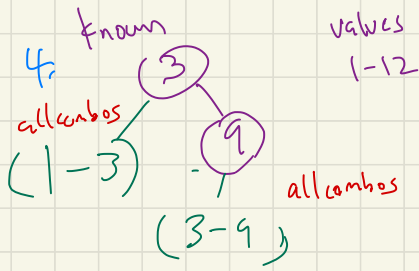
~ Combination w/ repetition

Case 1: play song 🎵
$$\frac{(6+14)!}{15!(5!)} = \frac{20!}{15!5!} \quad n=6 \quad r=15$$

Case 2: don't play song 🎵
$$n=6 \quad \frac{(6+15)!}{16!5!} \quad \begin{array}{l}\text{fighting}\\\text{couple}\end{array}$$
$$r=16$$

$$\frac{20!}{15!5!} + \frac{21!}{16!5!} = 15,504 + 20,349 = \boxed{35,853}$$

4.

known
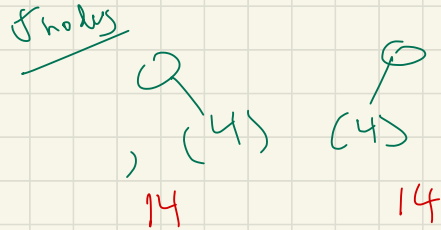values
1-12

all combos
$(1-3)$

(3)
(9)

all combos
$(3-9)$

unique combos, b/f choosing each one

( ) size of subtree
Sub-tree arrangements

4 nodes 4
/ \
(4) 5   6   7   8
/ \   (1) (3) (2) (2) (3) (1)
2   2   = 5   = 5
= 4

6 + 5 + 4 = 14 ways

Arrangements

2 nodes, 2 ways

(a)   (b)
 \   /
 (b)   (a)

3 nodes, 5 ways

(a)
 \
 (b)
  \
  (c)

(b)
/ \
(a) (c)

(b)
 \
 (c)
 /
(a)

(c)
/
(b)
/
(a)

(b)
/ \
(a) (c)

6 nodes

(4)   (4)
14   14

(2) (2) 14
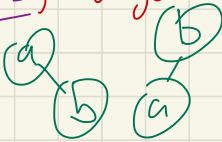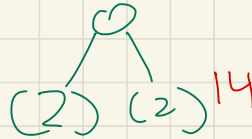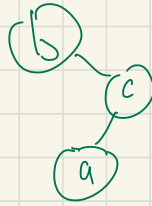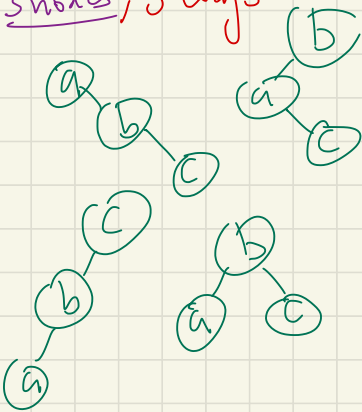
14 (5) ways · 2 ways · 5 ways

= 420 ways

5. serve at least one person   10 friends

case 1: no nurses scheduled for break:

   how many ways 10 → 4 groups
   divide

9 ways

```
2  3  2  3
3  3  3  1
1  1  1  7
1  2  1  6
1  3  1  5
4  4  1  1
2  2  1  5
3  4  2  1
2  2  2  4
```

Case 2: 1 nurse scheduled for break:

   how many ways 10 → 3 groups
   divide

```
1  1  8
1  2  7
1  3  6
1  4  5
2  2  6
2  3  5
2  4  4
2  5  3
```

8 ways

$$9 + 8 = \boxed{17 \text{ total ways}}$$