CSCI 2824
Spring 2020

Write **clearly**:

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Section number:	002
Assignment:	Homework #2

Read the following:

- This cover sheet must be included as the first page for all written homework submissions to CSCI 2824.
- Fill out all of the fields above.
- Submit your written homework assignment to the electronic dropbox. You will receive graded feedback through the same mechanism.
- If you type up your homework assignment using MS Word or LaTeX, then you can earn two extra credit points per homework assignment. You **must** use properly formatted equations and nice-looking text in order to be eligible for this extra credit point. If you type it up and do not format equations properly or do not use the cover sheet (for example), you might still lose the style/neatness points.
- By submitting this assignment, you are agreeing that you have abided by the **CU Honor Code**, and that this submission constitutes your own original work.

CSCI 2824 – Spring 2020 **Homework 2**

This assignment is due on Friday, January 31 to Gradescope by 11:59pm. You are expected to write up your solutions neatly and **use the coverpage**. Remember that you are encouraged to discuss problems with your classmates, but you must work and write your solutions on your own.

Important: On the **official CSCI 2824 cover page** of your assignment clearly write your full name, the lecture section you belong to (001 or 002), and your student ID number. You may **neatly** type your solutions for +2 extra credit on the assignment. You will lose *all* 5 style/neatness points if you fail to use the official cover page.

(1) In lecture we covered a couple of ways to convert numbers from base-10 in base-2. Adapt one or both of these techniques to convert $(472)_{10}$ into base-5. Make sure to show all steps.

Solution:

- $472 \div 5 == 94$ w/ remainder $2 \Rightarrow$ base-5 $==$ _ _ _ 2
- $94 \div 5 == 18$ w/ remainder $4 \Rightarrow$ base-5 $==$ _ _ 4 2
- $18 \div 5 == 3$ w/ remainder $3 \Rightarrow$ base-5 $==$ _ 3 4 2
- $3 \div 5 == 0$ w/ remainder $3 \Rightarrow$ base-5 $==$ **3342** $\Leftarrow$ **ANSWER**

(2) Suppose that the domain of the propositional function $P(x)$ consists of the integers 1, 4, 9, and 16. Express the following statements without using quantifiers, instead using negations, disjunctions, and conjunctions. [e.g. $\exists x P(x)$ would be $P(1) \vee P(4) \vee P(9) \vee P(16)$]

(a) $\forall x P(x)$

(b) $\neg\exists x P(x)$

(c) $\neg\forall x P(x)$

Solution:

(a) $P(1) \wedge P(4) \wedge P(6) \wedge P(16)$

(b) $\neg(P(1) \vee P(4) \vee P(6) \vee P(16))$

(c) $\neg(P(1) \wedge P(4) \wedge P(6) \wedge P(16))$

(3) You're going to go on spring break vacation! Your destination: the island of Knights & Knaves. On this island, there are only two types of native inhabitants; Knights, who always tell the truth, and Knaves, who always lie. For each of the following problems, determine which of the islanders you encounter are Knights and which are Knaves, if possible. If multiple solutions may exist, fully describe each of them. Include a full truth table, and justify and explain your answer.

(a) As you are finding a nice spot on the beach to set up a picnic, you are approached by 3 of the native inhabitants. We'll call them Alfred, Batman, and Catwoman. Batman tells you: "we are all knaves." Catwoman tells you: "exactly one of us is a knave."

(b) You decide you need a tasty beverage, but on your way you now encounter Poseidon, Quetzalcoatl, and Rah. Poseidon tells you "we are all knaves." Rah corrects him: "exactly one of us is a knight."

Solution:

(a)
- Batman's statement (SB) $== (\neg A \wedge \neg B \wedge \neg C)$
- Catwoman's statement (SC) $== (\neg A \oplus \neg B \oplus \neg C)$

- Let A, B, and C mean Alfred, Batman, and Catwoman are knights respectively.

A	B	C	SB	SC	$B \Leftrightarrow SB$	$C \Leftrightarrow SC$	$\wedge$
F	F	F	T	F	F	T	F
F	F	T	F	F	T	F	F
F	T	F	F	F	F	T	F
F	T	T	F	T	F	T	F
T	F	F	F	F	T	T	**T**
T	F	T	F	T	T	F	F
T	T	F	F	T	F	T	F
T	T	T	F	F	F	F	F

A: **Alfred is a knight, Batman and Catwoman are both knaves**

(b)
- Poseidon's statement (SP) == $(\neg P \wedge \neg Q \wedge \neg R)$
- Rah's statement (SR) == $(P \oplus Q \oplus R)$
- Let P, Q, and R mean Poseidon, Quetzalcoatl, and Rah are knights respectively.

P	Q	R	SP	SR	$P \Leftrightarrow SP$	$R \Leftrightarrow SR$	$\wedge$
F	F	F	T	F	F	T	F
F	F	T	F	T	T	F	F
F	T	F	F	T	T	T	**T**
F	T	T	F	F	T	F	F
T	F	F	F	T	F	F	F
T	F	T	F	F	F	T	F
T	T	F	F	F	F	F	F
T	T	T	F	F	F	F	F

A: **Quetzalcoatl is knight, Poseidon and Rah are both knaves**

(4) (a) Consider the logical proposition $[(l \vee m) \wedge (l \rightarrow n) \wedge (m \rightarrow n)] \rightarrow n$.

(i) Show that the proposition is a tautology using a truth table.

(ii) Show that the proposition is a tautology using a chain of logical equivalences. Note that you may only use logical equivalences from Table 6 (p. 27 of the Rosen textbook) and the other four starred equivalences given in lecture. At each step you should cite the name of the equivalence rule you are using, and please only use one rule per step.

(iii) Is this compound proposition satisfiable? Why or why not?

(iv) Create a plain English sentence explaining this tautology.

(b) Show that $(p \rightarrow q) \rightarrow r$ and $p \rightarrow (q \rightarrow r)$ are not logically equivalent. Show all work, but your choice of method is your own.

Solution:

(a) (i)

l	m	n	$l \vee m$	$l \to n$	$m \to n$	$[(l \vee m) \wedge (l \to n) \wedge (m \to n)] \to n$
F	F	F	F	T	T	T
F	F	T	F	T	T	T
F	T	F	T	T	T	T
F	T	T	T	T	T	T
T	F	F	T	F	F	T
T	F	T	T	T	T	T
T	T	F	T	F	F	T
T	T	T	T	T	T	T

(ii) $[(l \vee m) \wedge (l \to n) \wedge (m \to n)] \to n$
$[(l \vee m) \wedge (\neg l \vee n) \wedge (\neg m \vee n)] \to n$ :: Relation by Implication
$[(l \vee m) \wedge ((\neg l \wedge \neg m) \vee n)] \to n$ :: Distributive Law
$[(l \vee m) \wedge (\neg(l \vee m) \vee n)] \to n$ :: De Morgan's Law
$[((l \vee m) \wedge \neg(l \vee m)) \vee ((l \vee m) \wedge n] \to n$ :: Distributive Law
$[False \vee ((l \vee m) \wedge n)] \to n$ :: Negation Law
$[(l \vee m) \wedge n] \to n$ :: Identity Law
$\neg\, [(l \vee m) \wedge n] \vee n$ :: Relation by Implication
$[\neg(l \vee m) \vee \neg n] \vee n$ :: De Morgan's Law
$\neg(l \vee m) \vee (\neg n \vee n)$ :: Distributive Law
$\neg(l \vee m) \vee True$ :: Negation Law
$True$ :: Domination Law

(iii) **Yes**, a tautology is always satisfiable because there will always be ≥ 1 true in its truth table.

(iv) The series of OR statements within the brackets is never true at the same time that n is false, meaning the final conditional statement outside the brackets will always yield true.

(b)

p	q	r	$(p \to q) \to r$
F	F	F	F
F	F	T	T
F	T	F	F
F	T	T	T
T	F	F	T
T	F	T	T
T	T	F	F
T	T	T	T

$\neq$

p	q	r	$p \to (q \to r)$
F	F	F	T
F	F	T	T
F	T	F	T
F	T	T	T
T	F	F	T
T	F	T	T
T	T	F	F
T	T	T	T

(5) It's time to unwind by playing Pokémon. You and your #squad (your friends: Brock, Ash, Giovanni, and Misty) sit down and debate which Pokémon you should bring on your shared team, which must contain anywhere from 1 to 3 monsters. Ash just loves to catch 'em all, so he's good with anything. The rest are quite particular.

(i) Misty insists that we bring Mudkip and definitely not bring Blastoise.

(ii) Giovanni really doesn't want us to bring Sandshrew.

(iii) You think that if the team has Blastoise, we shouldn't bring Mudkip.

(iv) Brock thinks that we should bring Mudkip if and only if we also bring Pikachu or Sandshrew.

Can we meet all of their standards? Let $T(x)$ represent the propositional function "monster x is on our team," where the domain of x is the 4 possible Pokémon we have available: Mudkip, Blastoise, Sandshrew, and Pikachu (or M, B, S, and P).

(a) Translate each of the 4 statements (i)-(iv) into a proposition, using the logical notation covered in class.

(b) Is it possible to create a team under these conditions? If this problem is satisfiable, what is that team or those teams? If not, give a clear written argument explaining why or why not. Do not use a truth table.

Solution:

(a)
- $\mathrm{T}_1 = M \wedge \neg B$
- $\mathrm{T}_2 = \neg S$
- $\mathrm{T}_3 = (\neg B) \vee (B \wedge \neg M)$
  Reducing using distribution
  $\mathrm{T}_3 = (B \vee \neg B) \wedge (\neg M \vee \neg B)$
  $B \vee \neg B$ is always true
  $\mathrm{T}_3 = \neg M \vee \neg B$
- $\mathrm{T}_4 = (\neg M) \vee (M \wedge (P \vee S))$
  Reducing using distribution
  $\mathrm{T}_4 = (\neg M \vee M) \wedge (\neg M \vee (P \vee S))$
  $M \vee \neg M$ is always true
  $\mathrm{T}_4 = (\neg M) \vee (P \vee S)$

(b) Team $= T_1 \wedge T_2 \wedge T_3 \wedge T_4$

$T_1 \wedge T_3 = (M \wedge \neg B) \wedge (\neg M \vee \neg B)$
Distribute
$((M \wedge \neg B) \wedge (\neg M)) \vee ((M \wedge \neg B) \wedge (\neg B))$
$((M \wedge \neg B) \wedge (\neg M)) = ((M \wedge \neg M) \wedge (\neg B))$
$T_1 \wedge T_3 = (M \wedge \neg B) \wedge (\neg B)$
$T_1 \wedge T_3 = (M \wedge \neg B)$

$T_2 \wedge T_4 = (\neg S) \wedge (\neg M \vee (P \vee S))$
$T_2 \wedge T_4 = (\neg S) \wedge (S \vee (\neg M \vee P))$
$T_2 \wedge T_4 = ((\neg S) \wedge S) \vee ((\neg S) \wedge (\neg M \vee P))$
$\neg S \wedge S$ is always false
$T_2 \wedge T_4 = (\neg S) \wedge (\neg M \vee P)$

$T_1 \wedge T_2 \wedge T_3 \wedge T_4 = (T_1 \wedge T_3) \wedge (T_2 \wedge T_4)$
$= (M \wedge \neg B) \wedge ((\neg S) \wedge (\neg M \vee P))$
Distribute
$= (\neg S) \wedge (((M \wedge \neg B) \wedge (\neg M)) \vee ((M \wedge \neg B) \wedge P))$
$(M \wedge \neg B) \wedge \neg M$ is always false
$= (\neg S) \wedge ((M \wedge \neg B) \wedge P)$
$= (\neg S) \wedge (M) \wedge (\neg B) \wedge (P)$
Team will have Mudkip and Pikachu, and leave Blastoise and Sandshrew at home.