Massachusetts Institute of Technology

Bayesian Modelling and Inference
6.435

Project proposal

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Applying Bayesian methods to estimate price elasticity at scale

Methodology and Motivation

We plan to apply general hierarchical Bayesian framework, specifically the general Heterogeneity Model introduced in [Weber and Steiner, 2021], to model and forecast price elasticity coefficients for various Walmart products sold across several Walmart stores.

The original paper applied a hierarchical Bayesian framework with latent class assignments to calculate the mean and variance of price elasticity coefficients. The analysis specifically focused on several brands of orange juice sold in the Dominick's Finer Food supermarket chain across variety of locations in the Chicago metropolitan area. The goal was to find the most accurate hierarchical representation, which is done by varying key parameters, eg. reducing variance to 0. In the paper, the model was solved using Gibbs Sampling.

The paper proposed that brands may have different elasticity values when sold at different locations, thus heterogeneity was introduced to model price elasticities at different store locations (discrete or continuous). However, the general findings of the paper suggest that for the specific dataset used, the introduction of heterogeneity did not add much value over a homogeneous model in terms of model fit measured by the model likelihood. Moreover, the discrete heterogeneity models clearly outperformed the continuous heterogeneity models in terms of the model fit. Finally, there was no clear winner in terms of forecasting accuracy, but it was concluded that adding continuous heterogeneity did not lead to better results than using discrete heterogeneity.

Proposed goals of the project

In parallel with 6.435, We hope to explore model building and sampling to accurately and efficiently retrieve information from the posterior at scale.

Specifically, we are interested in replicating the model approach across several different product categories of the Walmart Dataset (introduced in detail in the Data section) and investigating whether the conclusions regarding the benefits of heterogeneity hold even with this data. We expect that the results may differ as the Wallmart dataset contains stores located across 3 different states in the U.S. thus the location difference is geographically more significant than in term the original paper, where all stores are located in the metropolitan cluster of Chicago. Moreover, we believe that it will be interesting to investigate how the results differ across various product categories (e.g. food, furniture).

Data

We plan on extending this analysis to the much larger Walmart Dataset provided freely via the M5 Forecasting competition. This dataset contains over 30,000 products with prices and quantities sold across 10 different locations in 3 different U.S. states over a recent 5 year period. We want to see if the papers modelling approach is feasible at this scale and whether it remains the most accurate method of predicting price elasticities.

Project Plan

The general work division plan is thatt Jay will be responsible for the odd numbered steps and Filip will be responsible for the even numbered items.

1. Literature review [by March 20th]: in addition to summarizing findings from [Weber and Steiner, 2021], we plan to read more about: Heterogeniety of demand and its modelling (e.g.[Allenby et al., 1998], [Andrews et al., 2002]); Bayesian estimation of relevant models (e.g. [Hein et al., 2020], [Akinc and Vandebroek, 2018]); and solving models that involve discrete variables, per the example in the Stan user guide - we note that this is only tentative list
2. Data Processing [by March 27]: Processing data from kaggle and storing it on cloud local machine. Tidying up into a relevant dataframes and preparing for use in analysis
3. Benchmark Models [by April 3rd]: Constructing a benchmark model based on standard linear regression (non-bayesian) estimation and prediction of sales in advance. Perhaps also applying Prophet introduced in [Taylor and Letham, 2018]. Applying benchmark to one food sub-category in one region, aiming for 8 products for roughly 80 weeks for roughly 640 rows to replicate paper.
4. Write Project Progress Report [April 6th]: To update progress on project for instructors
5. Initial bayesian model [by April 10th]: Replicate the model from [Weber and Steiner, 2021] using the same dataset as for the benchmark. We will start by using STAN since there are a wider range of tutorials available.
6. Expand model [by April 17th]: To more categories and subcategories, while recording the speed and accuracy under various assumptions. Up to a million rows. Here we will consider Edward and Variational Bayes as alternatives.
7. Visualise results and compare model results [by April 24th]: Create final visualizations comparing Bayesian estimates of elasticity and accuracy of time series predictions of various models
8. Finish project report [by April 27st]: Collect project results, graphs, tables and describe findings with enough details for another team to replicate our experiments

Risks

The main concern is the computational limitations of STAN or other MCMC samplers, which we can mitigate by incrementally increasing our scale. It is also worth exploring newer probabilistic languages which use variational bayes. Finally we can consider accessing cloud based computational resources, such as Kaggle or AWS.

References

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