Homework 6.1
Due: May 19, 2021

Competition with unlimited growth
Analyze the competition model

$$\frac{dN_1}{dT} = r_1 N_1 \left[1 - \frac{(N_1 + \alpha_{12} N_2)}{K_1}\right]$$
$$\frac{dN_2}{dT} = r_2 N_2 \left(1 - \alpha_{21} \frac{N_1}{K_2}\right)$$

In this model, only one of species, N_1, has a finite carrying capacity.

Solution.

Let us look at the zero-growth isoclines

$$\frac{dN_1}{dT} = 0 \longrightarrow N_1 = 0,\ N_1 = K_1 - \alpha_{12} N_2$$
$$\frac{dN_2}{dT} = 0 \longrightarrow N_2 = 0,\ N_1 = \frac{K_2}{\alpha_{21}}$$

If $\alpha_{21} < \frac{K_2}{K_1}$, i.e. species 1 has a small effect on species 2, then the two isoclines will not intersect. Species 1 will go extinct and species 2 will grow exponentially. If $\alpha_{21} \geq \frac{K_2}{K_1}$, i.e. species 1 has a large effect on species 2, then the two isoclines will intersect. If initially $N_1 < \frac{K_2}{\alpha_{21}}$, then species 1 will go extinct and species 2 will grow exponentially. If $N_1 > \frac{K_2}{\alpha_{21}}$, species 1 will grow to its carrying capacity and species 2 will go extinct. See Figure 1.

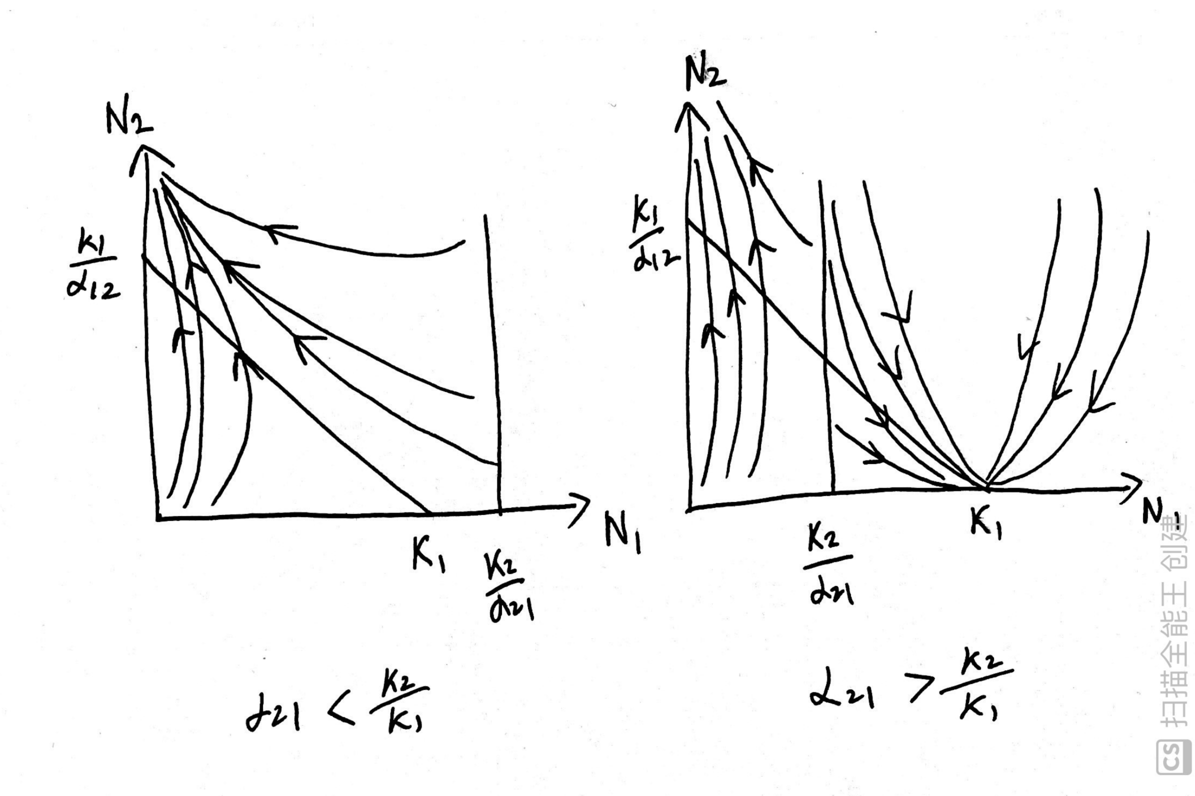

Figure 1: Phase portraits of the competition model