Homework 2.1
Due: April 14, 2021

Fox Surplus-Yield Model
Consider the harvest model

$$\frac{dN}{dt} = \alpha N \ln\left(\frac{K}{N}\right) - qEN$$

for a fish population that is growing according to the Gompertz equation and that is being harvested so that the catch per unit effort is proportional to the stock size.
(a) Find the equilibria and plot the equilibria as a function of the fishing mortality qE.
(b) Determine and plot the yield curve.
(c) Determine the maximum sustainable yield.
(d) Compare and contrast your results with those for the Schaefer model,

$$\frac{dN}{dt} = rN\left(1 - \frac{N}{K}\right) - qEN.$$

Discuss the differences between the two models. In particular, highlight any significant qualitative differences between the two models and explain why they occur.

Solution.

(a) The equilibria is found by setting the RHS of the equation to 0

$$\alpha N^* \ln\left(\frac{K}{N^*}\right) - qEN^* = 0$$

Solving this yields two solutions. One is $N^* = 0$, but this would give $\ln(\infty)$. Thus there is only one equilibrium at $N^* = Ke^{-\frac{qE}{\alpha}}$. We plot this in Figure 1.

(b) The yield is defined as

$$Y = qEN^* = qEKe^{-\frac{qE}{\alpha}}$$

We plot this in Figure 2.

(c) For fixed catchability, the maximum sustainable yield occurs when

$$\frac{dY}{dE} = K(qe^{-\frac{qE}{\alpha}} - \frac{q^2E}{\alpha}e^{-\frac{qE}{\alpha}}) = 0$$
$$E = \frac{\alpha}{q}$$

This gives MSY $= \frac{\alpha K}{e}$.

(d) Some notable differences between these two models

- Schaefer model has two equilibria, whereas Fox model has only one. This is due to the logarithmic function in the Fox model. This also suggests that only an infinite fish mortality can drive the equlibrium to 0 in the Fox model.
- Schaefer model has a symmetric yield curve, whereas Fox model has an asymmetric one. This is due to the exponential function in N^*.

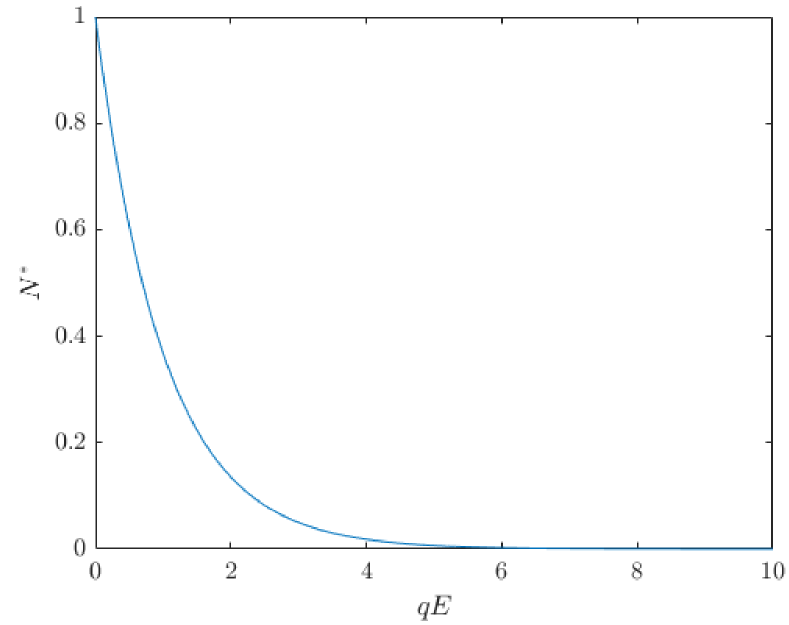

Figure 1: Equilibrium of the harvest model ($\alpha = 1,\ K = 1$)

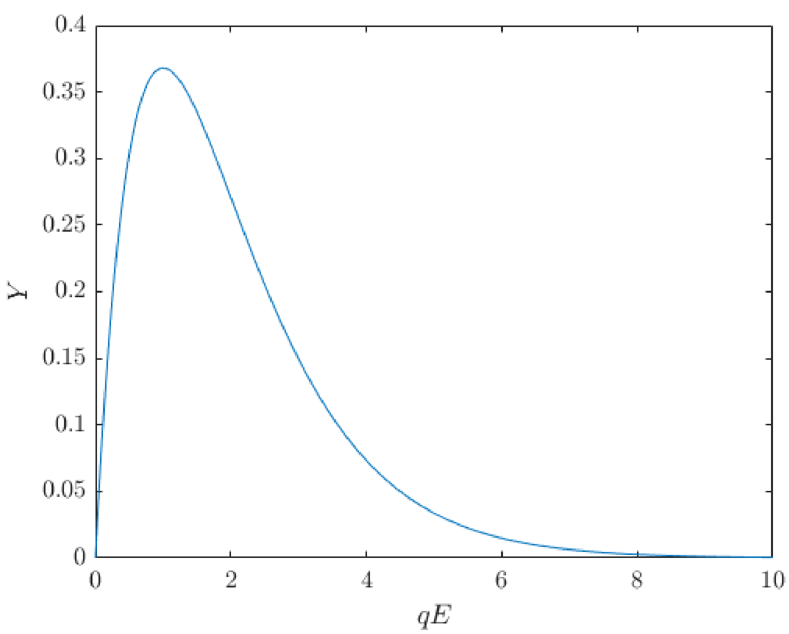

Figure 2: Yield curve of the harvest model ($\alpha = 1,\ K = 1$)