

Midterm

DUE: 10am Seattle time (PST), Saturday, November 7, 2020

I. Prove that the LU decomposition of a matrix \mathbf{A} is unique.

II. Show that the largest singular value of a matrix $\mathbf{A} \in \mathbb{C}^{m \times n}$ is given by

$$\sigma_{\max}(\mathbf{A}) = \max_{\mathbf{x} \in \mathbb{R}^n, \mathbf{y} \in \mathbb{R}^m} \frac{\mathbf{y}^T \mathbf{A} \mathbf{x}}{\|\mathbf{x}\|_2 \|\mathbf{y}\|_2}$$

III. What are the singular values of an orthogonal projection?

IV. Show that for a given norm $\kappa(\mathbf{A}\mathbf{B}) \leq \kappa(\mathbf{A})\kappa(\mathbf{B})$ and that $\kappa(\alpha\mathbf{A}) = \kappa(\mathbf{A})$ for a given (nonzero) constant α .

V. Write a python or matlab script that does an LU decomposition (including pivoting)