AMATH 584 Autumn Quarter 2020

**Homework 1**
DUE: Friday, October 9, 2020

1. Show that if matrix $\mathbf{A}$ is triangular and unitary, then it is diagonal
2. Consider that the matrices $\mathbf{A} \in \mathbb{C}^{n\times m}$ and $\mathbf{B} \in \mathbb{C}^{n\times m}$ are Hermitian (self-adjoint)
   - Prove that all eigenvalues $\lambda_k$ of $\mathbf{A}$ are real
   - Prove that if $\mathbf{x}_k$ is the $k$th eigenvector, then eigenvectors with distinct eigenvalues are orthogonal
   - Prove the sum of two Hermitian matrices is Hermitian
   - Prove the inverse of an invertible Hermitian matrix is Hermitian as well
   - Prove the product of two Hermitian matrices is Hermitian if and only if $\mathbf{AB} = \mathbf{BA}$.
3. Consider the matrix $\mathbf{U} \in \mathbb{C}^{n\times m}$ which is unitary
   - Prove that the matrix is diagonalizable
   - Prove that the inverse if $\mathbf{U}^{-1} = \mathbf{U}^*$
   - Prove it is isometric with respect to the $\ell_2$ norm, i.e. $\|\mathbf{Ux}\| = \|\mathbf{x}\|$.
   - Prove that all eigenvalues have modulus unity