

**Homework 1.1**

Due: April 7, 2021

**Monod's<sup>1</sup> nightmare**

Escherichia coli is a bacterium that has been used extensively in microbiological studies. *E. coli* cells are rod-shaped; they are  $0.75\ \mu\text{m}$  wide,  $0.75\ \mu\text{m}$  deep, and  $2\ \mu\text{m}$  long. Under ideal conditions, a population of *E. coli* doubles in just over 20 minutes.

(a) What is  $r$  for *E. coli*?

(b) If  $N_0 = 1$ , how long would it take for an exponentially growing population of *E. coli* experiencing ideal conditions to fill your room ?

**Solution.**

(a) Since the population doubles every 20 minutes, it becomes  $2^3 = 8$  times the initial population in one hour. We have  $\frac{dN}{dt} = 8N$  and  $r = \frac{1}{N} \frac{dN}{dt} = 8$ .

(b) A single *E. coli* is  $0.75 \times 0.75 \times 2 = 1.125\ \mu\text{m}^3 = 1.125 \times 10^{-18}\ \text{m}^3$  large. My room is  $5 \times 5 \times 2 = 50\ \text{m}^3$ . Thus  $4.4 \times 10^{19}$  *E. coli* are needed to fill my room. We have  $\exp(rt) = 4.4 \times 10^{19}$ . Solving yields  $t = 5.7$  hours.

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<sup>1</sup>Jacques Monod (1910-1976) was the recipient of a 1965 Nobel Prize for his work on gene regulation. He also invented the chemostat and conducted innovative experimental studies on microbial growth.