Statistics Refresher Notes

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Probability

Definition. Rules of probability for discrete variables The probability $p(x = x)$ of variable x being in state x is represented by a value between 0 and 1. $p(x = x) = 1$ means that we are certain x is in state x. Conversely, $p(x = x) = 0$ means that we are certain x is not in state x. Values between 0 and 1 represent the degree of certainty of state occupancy.
The summation of the probability over all the states is 1:

$$\sum_{x \in dom(x)} p(x = x) = 1$$

This is called that normalisation condition.

Definition. Marginals Given a joint distribution $p(x, y)$ the distribution of a single variable is given by

$$p(x) = \sum_y p(x, y)$$

Here $p(x)$ is termed as marginal of the joint probability distribution $p(x, y)$

Definition. Conditional probability If $P(B) > 0$ then the **conditional probability** of A given B is

$$\mathbb{P}(A|B) = \frac{\mathbb{P}(AB)}{\mathbb{P}(B)}$$

Theorem. The Law of Total Probability *Let $A_1, ..., A_k$ be a partition of Ω. Then, for any event B,*

$$\mathbb{P}(B) = \sum_{i=1}^{k} \mathbb{P}(B|A_i)\mathbb{P}(A_i)$$

Theorem. Bayes' Theorem *Let $A_1, ..., A_k$ be a partition of Ω such that $\mathbb{P}(A_i) > 0$ for each i. If $\mathbb{P}(B) > 0$ then, for each $i = 1, ..., k$,*

$$\mathbb{P}(A_i|B) = \frac{\mathbb{P}(B|A_i)\mathbb{P}(A_i)}{\sum_j \mathbb{P}(B|A_j)\mathbb{P}(A_j)}$$

Important. We call $\mathbb{P}(A_i)$ the **prior probability of** A and $\mathbb{P}(A_i|B)$ the **posterior probability of A**

Definition. Independence Two events A and B are **independent** if

$$\mathbb{P}(AB) = \mathbb{P}(A)\mathbb{P}(B)$$

and we write $A \amalg B$. A set of events $\{A_i : i \in I\}$ is independent if

$$P(\bigcup_{i\in J} A_i) = \prod_{i\in J} \mathbb{P}(A_i)$$

Definition. Conditional Independence

$$X \amalg Y | Z$$

denotes that the two sets of variable X and Y are independent of each other provided we know the state of the set of variables Z. For conditional independence, X and Y must be independent given all states of Z. Formally, this means that

$$P(X, Y|Z) = p(X|Z)p(Y|Z)$$

for all states of X, Y, Z. In case the conditioning set is empty we may also write $X \amalg Y$ for $X \amalg Y|$, in which case X is (unconditionally) independent of Y.

Definition. CDF *The **cumulative distribution function**, or CDF, is the function $F_X : \mathbb{R} \to [0, 1]$ defined by*

$$F_X(x) = \mathbb{P}(X \leq x)$$

Definition. Probability function *X is discrete if it takes countably many values $\{x_1, x_2, ...\}$. We define the **probability function** or **probability mass function** for X by $f_x(x) = \mathbb{P}(X = x)$*

Definition. Probability density function *A random variable X is continuous if there exists a function f_X such that $f_X(x) \geq 0$ for all x, $\int_{-\infty}^{\infty} f_X(x)dx = 1$ and for every $a \leq b$,*

$$\mathbb{P}(a < X < b) = \int_a^b f_X(x)dx$$

*The function f_X is called the **probability density function** (PDF). We have that,*

$$F_X(x) = \int_{-\infty}^{x} f_X(t)dt$$

and $f_X(x) = F_X'(x)$ and all points x at which F_X is differentiable

Definition. Quantile function *Let X be a random variable with CDF F. The **inverse CDF** or **quantile function** is defined by*[4]

$$F^{-1}(q) = \inf\{x : F(x) > q\}$$

for $q \in [0, 1]$. If F is strictly increasing and continuous than $F^{-1}(q)$ is the unique real number x such that $F(x) = q$