

# HW6: Probability

CMSC250, Spring 24

Due: May 1st, 11:59PM

## Instructions:

- Submit your solutions on Gradescope before 11:59PM on May 1st.
- Tag all problems and ensure all work is legible. **Mistagged problems will receive no credit.** Please ask us to check over your submission if you're worried.
- **Show your work.** Correct answers with no work shown will not receive credit.
- You may work with other people, but your final answers should be your own.
- Draw Max as a camel for extra credit. We'll make a collage of those drawings to give to him.

## Suggestions:

- Put each problem on a separate page. Importantly, do NOT put all problems on one page.
- Make very clear where each problem is on its page.
- Write large enough so your work is easy to read.
- Check your work with the resources you have available. We also highly recommend discussing your answers with others who've already tried the problems.

# 1 Problems

$$\sum_{i=0}^3 2 = 2 + 2 + 2 + 2 = 2 * 4 = 8$$

$$\sum_{i=1}^4 2 = 2 + 2 + 2 + 2 = 2 * 4 = 8$$

$$\sum_{i=0}^3 i = 0 + 1 + 2 + 3 = 6$$

$$\sum_{i=1}^4 (i - 1) = (1 - 1) + (2 - 1) + (3 - 1) + (4 - 1) = 0 + 1 + 2 + 3 = 6$$

## 1. Counting problems:

- Given the word DISCRETEMATH, how many permutations of the letters are there?
- Given the letters A,E,M,C,L, how many total words can you make? A word is any ordering of 1 or more letters (so, M is a word, as is CM, as is ELMC).
- You have a bag with 11 red, 7 white, numbered marbles (meaning all elements are distinct). You pick 5 of them. How many samples contain at least one red marble? *Hint: think about the complement.*
- Your closet has pants, shirts, and hats; counting up the clothes, the number of pants is the smallest, while the number of hats is the biggest. I tell you that you can make 357 outfits with the clothes you have, where an outfit is any combination of 1 pant, 1 shirt, and 1 hat. How many pants, shirts, and hats do you have? Explain.
- Your sock drawer contains 10 pairs of red socks, 13 white, and 3 black, but they're all separated from each other. Your roommate is sleeping, so you have to grab them in the dark. How many socks should you grab to guarantee you have a matching pair of socks? How about so you guarantee you have a pair of black socks?

## 2. Probability and expected value problems:

- You pay \$10 to play the following game: a bag contains 5 red balls, 3 yellow, 4 green. You draw one ball, and win \$14 if its red, \$12 if its yellow, and nothing otherwise. How much do you expect to win/lose if you play this game 100 times?
- You are a small business owner, and you want to try selling a new product. Based on past experience, you approximate that you have between a 10%-30% chance of losing \$10,000, 50% of making \$6,000, and 20%-40% of making \$15,000. Assuming the worst chances, what is your expected value? Assuming the best?

- (c) Say every day of the year (365, non leap-year) has an equal chance of being someone's birthday. Given  $k$  people in a room, let  $P_k$  be the probability that at least one pair shares a birthday. How many people do you need for  $P_k > 0.5$ ? *Hint: think about the complement.*

3. Bayes rule problems

- (a) You are insurance company, and your customers come in two variants: careful and reckless. Over the course of the year, the careful customer has an accident with a probability of 0.01, while the reckless customer has an accident with probability 0.05. 75% of your customers are careful, while the rest are reckless. Suppose a randomly chosen customer has an accident this year. What is the probability that this customer is careful?
- (b) A certain store has three suppliers of chocolate: company A, company B, and company C. 80% of their chocolate comes from company A, 15% from company B, and 5% from company C. The chocolates made by company A have a 4% rate of defective wrapping, company B's have a 6% rate, and company C's have a 9% rate. If a chocolate is randomly selected from the store and has a defective wrapping, what is the probability that it was made by company A?
- (c) Suppose a disease affects 0.1% of the population, and a test has been developed for it. Anyone who has the disease will test positive, and a person who does not have the disease has a 5% chance of testing positive. Suppose a randomly selected person tests positive. What is the probability this person actually has the disease?
- (d) Suppose the probability of passing Math class is 0.5 at a certain high school. The probability that a student passed Science given they passed Math is 0.8, while the probability a student passed Math given they passed Science is 0.7. What is the probability that a random student passed Science?