Worksheet 17: Bayes Rule

Name:

1. Three persons A, B and C have applied for a job in a private company. The chance of their selections is in the ratio $1 : 2 : 4$. The probabilities that A, B and C can introduce changes to improve the profits of the company are 0.8, 0.5 and 0.3, respectively. If the change does not take place, find the probability that it is due to the appointment of C.

   The given ratio 1:2:4 for A, B, and C tells us that the probabilities for A, B, and C, being appointed, respectively, are

   $$P(A) = \frac{1}{7}, P(B) = \frac{2}{7}, \text{ and } P(C) = \frac{4}{7}$$

   We are also given

   $$P(\text{change}|A) = .8, P(\text{change}|B) = .5, P(\text{change}|C) = .7$$

   Subtracting each of those from one, we get:

   $$P(\text{ no change }|A) = .2, P(\text{no change}|B) = .5, P(\text{no change}|C) = .3$$

   The question is asking us to find $P(C|\text{no change})$

   Applying Baye's Rule gives:

   $$P(C|\text{no change}) = \frac{P(\text{no change}|C) * P(C)}{P(\text{no change})}$$

   Applying the law of total probability to the denominator gives:

   $$P(C|\text{no change}) = \frac{P(\text{no change}|C) * P(C)}{P(\text{no change})} = \frac{P(\text{no change}|C) * P(C)}{P(\text{no change} \cap A) + P(\text{no change} \cap B) + P(\text{no change} \cap C)}$$

   Plugging in the values for for these expressions gives:

   $$P(C|\text{no change}) = \frac{.3 * \frac{4}{7}}{.2 * \frac{1}{7} + .5 * \frac{2}{7} + .7 * \frac{4}{7}} = .7$$

2. There are three identical cards except that both the sides of the first card is coloured red, both sides of the second card is coloured blue and for the third card one side is coloured red and the other side is blue. One card is randomly selected among these three cards and put down, visible side of the card is red. What is the probability that the other side is blue?

   The question is asking us to find the probability that the card we selected is the red and blue card given that the visible side of the card is red.

   Since there are 3 cards, each with 2 sides, there are 3*2=6 total possibilities for how this situation could play out (for example, drawing the red and blue card with the red side visible, drawing the red and blue card with the blue side visible, drawing the red card with its first side visible, drawing the red card with its second side visible,...)

   There are three possible ways for us to have a red side visible: we can draw the red card with its first side visible, the red card with its second side visible, or the red and blue card with its red side visible.

   In only one of those three cases is the randomly selected card the red and blue card. Therefore, the probability that the other, non-visible side is blue is 1/3.