

Worksheet 15: Permutations and Combinations

Name: Solution

1. There are 7 distinct books on a shelf, 3 of which are by Brandon Sanderson.

- (a) How many ways can the books be arranged?

There are seven distinct books, and we wish to permute all seven of them, so:

$$P(7, 7) = \frac{7!}{(7-7)!} = 7! = 5040$$

- (b) How many ways can the books be arranged if all the books by Brandon Sanderson have to be together (in any order)?

We can consider the Sanderson books as one unit and do the permutation, then multiply by the number of permutations that we can make among the Sanderson books to get:

$$P(5, 5) \cdot P(3, 3) = 5! \cdot 3! = 720$$

- (c) How many ways can the books be arranged if one of the Brandon Sanderson books has to be the leftmost book?

We choose one of the Sanderson books to be leftmost, then permute the rest:

$$\binom{3}{1} \cdot P(6, 6) = 3 \cdot 6! = 2160$$

2. Now suppose that we're putting these same books in a box to donate to children in need.

- (a) How many ways are there to put all 7 books in the box?

$$\binom{7}{7} = 1$$

This should make sense intuitively.

- (b) How many ways are there to put one of the Brandon Sanderson books and three of the other books into the box?

We choose one from the three Sandersons, and choose three from the four other books.

$$\binom{3}{1} \cdot \binom{4}{3} = 3 \cdot 4 = 12$$

3. There are 10 six-sided dice, and each one is a different color (i.e. each die is distinct). How many ways are there to roll one 5 and the rest even?

We choose one of the dice to be 5, each additional die has three possibilities to be even (2, 4, 6), and there are nine of them.

$$\binom{10}{1} \cdot 3^9 = 196830$$