Full-rate STLC for Four Receive Antennas - Notes

radim.zedka@vut.cz

March 2022

1 Problem Description

Formula (6) in [1] is given by

$$\xi = \rho \frac{\left(\sum_{p=0}^{M-1} \gamma_p \pm 2\mathcal{R}\{\epsilon_p\}\right)^2}{4\sum_{p'=0}^{M-1} \gamma_{p'}}, \tag{1}$$

where

$$\gamma_p = \sum_{q=0}^{3} |h_{p,q}|^2. \tag{2}$$

and

$$\epsilon_p = h_{p,0}h_{p,2}^* + h_{p,1}h_{p,3}^*. \tag{3}$$

Each complex channel gain $h_{p,q}$ is composed of two i.i.d. normal variables $a_{p,q}, b_{p,q} \sim \mathcal{N}(0, 1/2)$ which relate to channel gain by $h_{p,q} = a_{p,q} + jb_{p,q}$, where $j = \sqrt{-1}$. Formula (2) then evolves into

$$\gamma_p = \sum_{q=0}^{3} |a_{p,q}|^2 + |b_{p,q}|^2, \tag{4}$$

and $\mathcal{R}\{\epsilon_p\}$ is expressed as

$$\mathcal{R}\{\epsilon_p\} = a_{p,0}a_{p,2} + b_{p,0}b_{p,2} + a_{p,1}a_{p,3} + b_{p,1}b_{p,3}. \tag{5}$$

Formula (1) may be expanded into purely real-valued form

$$\xi = \frac{\rho}{4} \frac{\left(\sum_{p=0}^{M-1}\sum_{q=0}^{3} |a_{p,q}|^2 + |b_{p,q}|^2 \pm 2\mathcal{R}\{\epsilon_p\}\right)^2}{\sum_{p'=0}^{M-1}\sum_{q=0}^{3} |a_{p',q}|^2 + |b_{p',q}|^2}. \tag{6}$$

After Lemma 1 in [1] the receiver SNR is calculated via

$$\xi = \frac{\rho}{4} \sum_{p=0}^{M-1}\sum_{q=0}^{3} |a_{p,q}|^2 + |b_{p,q}|^2. \tag{7}$$

2 MATLAB Simulation

In MATLAB I created a script which generates the normal-distributed variables $a_{p,q}$ and $b_{p,q}$ in vectors of 10^6 samples each. This way I calculate the histogram of formulas (6) and (7) and I attempt to approximate it with analytical formulas of probability density functions of the Chi-squared distribution with k degrees of freedom

$$f_X(\xi) = \frac{1}{2^{k/2} \cdot \Gamma(k/2)} \xi^{k/2-1} e^{-\frac{\xi}{2}} \quad \forall \quad \xi \in \langle 0, \infty), \tag{8}$$

and since variance of the normal variables $a_{p,q}$ and $b_{p,q}$ equals 1/2 the chi-squared variable must be multiplied by some positive real scaling factor c. The PDF of such a scaled chi-squared random variable is given by

$$f_{cX}(\xi) = \frac{\xi^{k/2-1} e^{-\frac{\xi}{2\cdot c}}}{(2c)^{k/2} \cdot \Gamma(k/2)} \quad \forall \quad \xi \in \langle 0, \infty). \tag{9}$$

In Figure. 1 I present the results of my MATLAB simulations for $M = 256$ and $M = 512$. Formula (6) is approximated by (9) with $k = 2M$ and $c = \rho/2$. Formula (7) is approximated by (9) with $k = 8M$ and $c = \rho/8$. All simulations are done with $\rho = 1$.

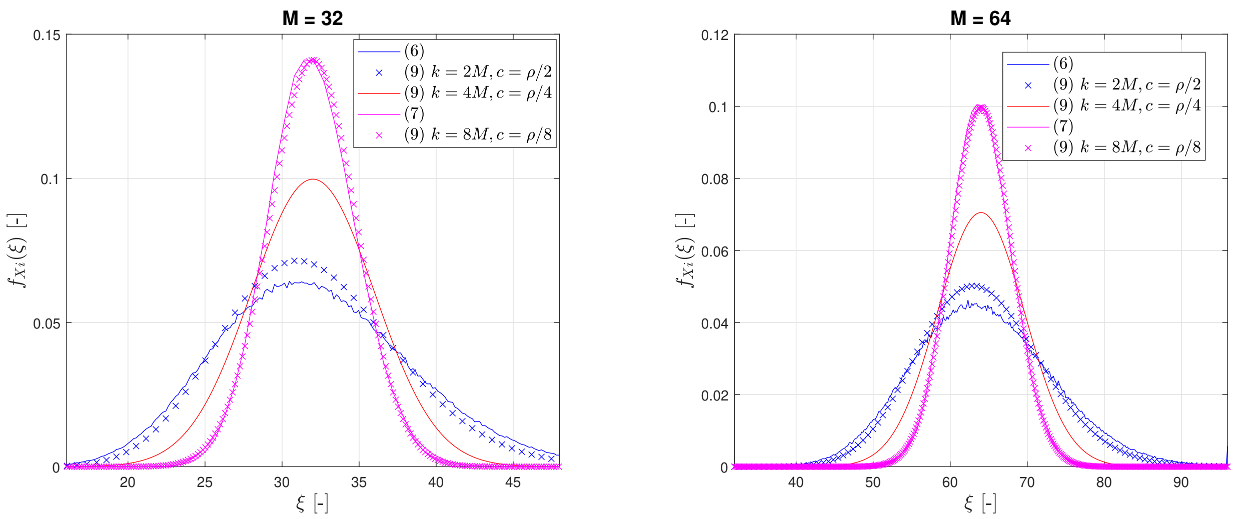

Figure 1: PDF of (6) and (7) with scaled chi-squared PDFs (9) for $M = 32$ and $M = 64$.

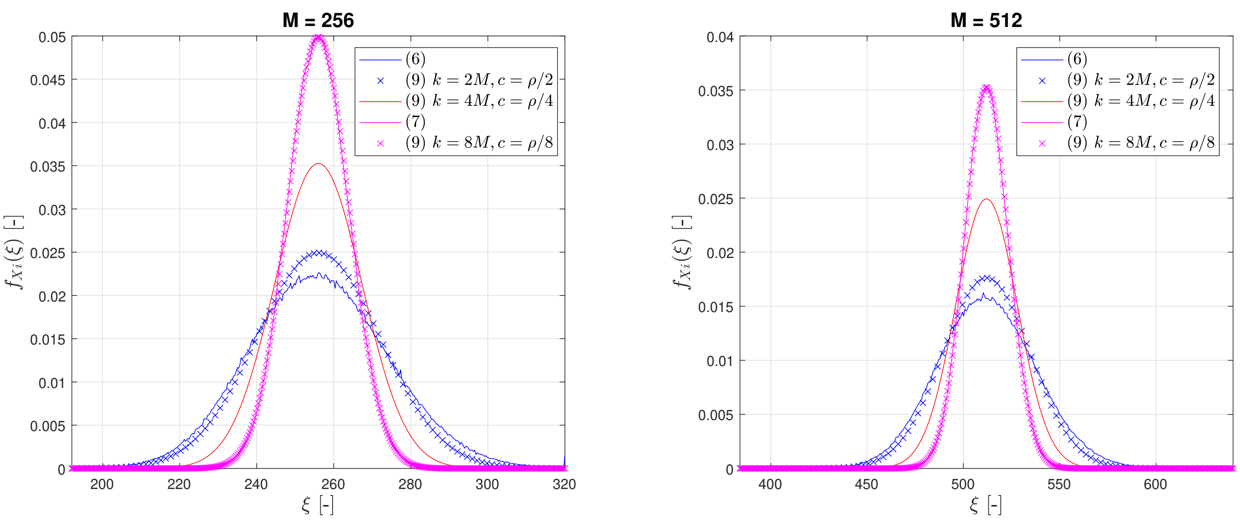

Figure 2: PDF of (6) and (7) with scaled chi-squared PDFs (9) for $M = 256$ and $M = 512$.

3 Discussion

My MATLAB simulation attepts show that (6) is close to scaled chi-squared distribution with $k = 2M$ and $c = \rho/2$. We can also see that even for M rising from 32 to 512 the PDF of (6) is not approaching that of (7). I would be very grateful for your advice.

References

[1] S. -chan Lim and J. Joung, "Full-Rate Space–Time Line Code for Four Receive Antennas", IEEE wireless communications letters, pp. 1-1, 2021.