Full-rate STLC for Four Receive Antennas - Notes

radim.zedka@vut.cz

March 2022

1 Problem Description

Formula (6) in [1] is given by

$$\xi = \rho \frac{\left(\sum_{p=0}^{M-1} \gamma_p \pm 2\mathcal{R}\{\epsilon_p\}\right)^2}{4\sum_{p'=0}^{M-1} \gamma_{p'}}, \tag{1}$$

where

$$\gamma_p = \sum_{q=0}^{3} |h_{p,q}|^2, \tag{2}$$

and

$$\epsilon_p = h_{p,0}h_{p,2}^* + h_{p,1}h_{p,3}^*. \tag{3}$$

Each complex channel gain $h_{p,q}$ is composed of two i.i.d. normal variables $a_{p,q}, b_{p,q} \sim \mathcal{N}(0, 1/2)$ which relate to channel gain by $h_{p,q} = a_{p,q} + jb_{p,q}$, where $j = \sqrt{-1}$. Formula (2) then evolves into

$$\gamma_p = \sum_{q=0}^{3} |a_{p,q}|^2 + |b_{p,q}|^2, \tag{4}$$

and $\mathcal{R}\{\epsilon_p\}$ is expressed as

$$\mathcal{R}\{\epsilon_p\} = a_{p,0}a_{p,2} + b_{p,0}b_{p,2} + a_{p,1}a_{p,3} + b_{p,1}b_{p,3}. \tag{5}$$

Formula (1) may be expanded into purely real-valued form

$$\xi = \frac{\rho}{4} \frac{\left(\sum_{p=0}^{M-1}\sum_{q=0}^{3} |a_{p,q}|^2 + |b_{p,q}|^2 \pm 2\mathcal{R}\{\epsilon_p\}\right)^2}{\sum_{p'=0}^{M-1}\sum_{q=0}^{3} |a_{p',q}|^2 + |b_{p',q}|^2}. \tag{6}$$

After Lemma 1 in [1] the receiver SNR is calculated via

$$\xi = \frac{\rho}{4} \sum_{p=0}^{M-1}\sum_{q=0}^{3} |a_{p,q}|^2 + |b_{p,q}|^2. \tag{7}$$

2 MATLAB Simulation

In MATLAB I created a script which generates the normal-distributed variables $a_{p,q}$ and $b_{p,q}$ in vectors of 10^7 samples each. This way I calculate the histogram of formula (6) and I attempt to approximate it with Gamma distribution with PDF given by

$$f_\Xi(\xi) = \frac{\xi^{k/2-1} e^{-\frac{\xi}{2\cdot c}}}{(2c)^{k/2} \cdot \Gamma(k/2)} \quad \forall \quad \xi \in \langle 0, \infty). \tag{8}$$

The parameters k and c are related to another pair of parameters - n, m - (for greater convenience) according to $k = 2nM$ and $c = \rho/m$. Please note that for PDF approximations I set $\rho = 1$. As the error metric of the PDF approximation I used mean-squared-error defined as

$$J = \int \left(f_\Xi(\xi)' - f_\Xi(\xi)\right)^2 d\xi, \tag{9}$$

where $f_\Xi(\xi)'$ is the simulated data histogram and $f_\Xi(\xi)$ is the approximation.

Table 1: Diversity gain and PDF approximation parameters of (6) at given M

M	n	m	$\kappa = nM$	J [dB]
1	0.947	1.528	0.947	-39.47
2	0.879	1.562	1.658	-44.92
4	0.846	1.595	3.384	-51.27
8	0.828	1.609	6.624	-54.65
16	0.818	1.613	13.088	-57.12
32	0.812	1.613	25.984	-59.82
64	0.810	1.612	51.840	-60.86
128	0.809	1.615	103.55	-61.15

Diversity gain of systems approximated by Gamma distribution is given by

$$\kappa = -\lim_{\rho\to\infty} \frac{d\log_{10}(\bar{\varepsilon}(\rho))}{d\log_{10}(\rho)} = k/2 = nM, \tag{10}$$

where $\bar{\varepsilon}(\rho)$ is the average bit error rate at given ρ and n is the diversity gain coefficient. Table 1 then summarizes PDF approximation parameters n, m for many values of M.

In Figure. 1, Figure. 2 and Figure. 3 I present the results of my MATLAB simulations for $M = 1$, $M = 2$ and $M = 128$. Formula (6) is approximated by (8) with $k = 2nM$ and $c = 1/m$ according to Table 1.

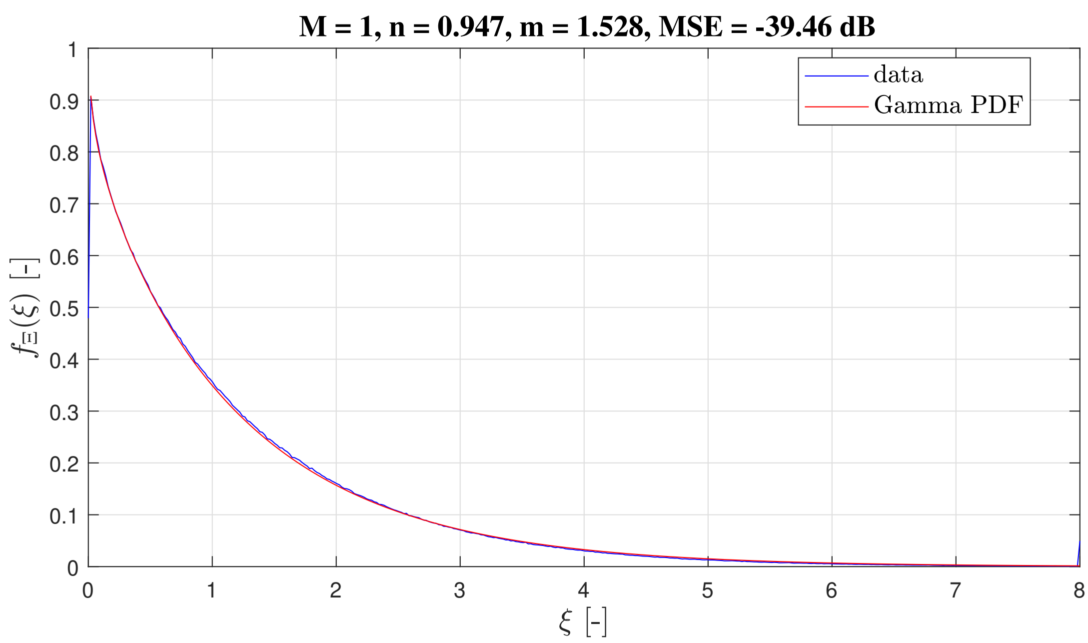

Figure 1: PDF of (6) approximated by (8) for $M = 1$.

3 Discussion

My MATLAB simulation attepts show that (6) is close to Gamma distribution with $k = 2nM$ and $c = \rho/m$. However, the diversity gain is not approaching $4M$, i.e. parameter n is not approaching 4, to the contrary, with rising number of antennas M parameter n approaches value ~ 0.81. I would be very grateful for your advice.

References

[1] S. -chan Lim and J. Joung, "Full-Rate Space–Time Line Code for Four Receive Antennas", IEEE wireless communications letters, pp. 1-1, 2021.

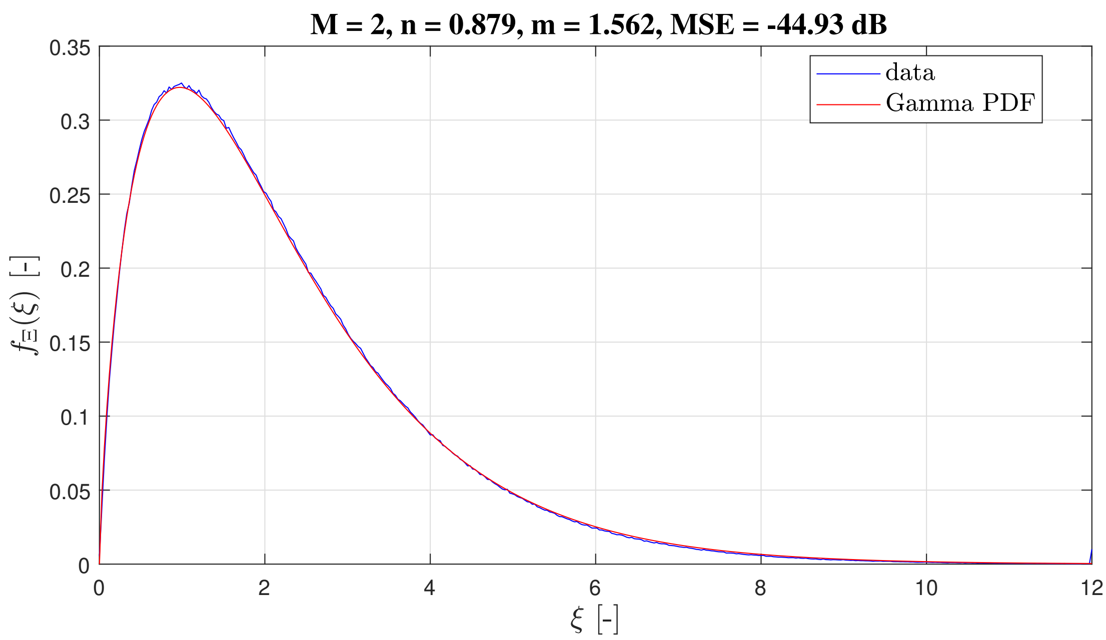

Figure 2: PDF of (6) approximated by (8) for $M = 2$.

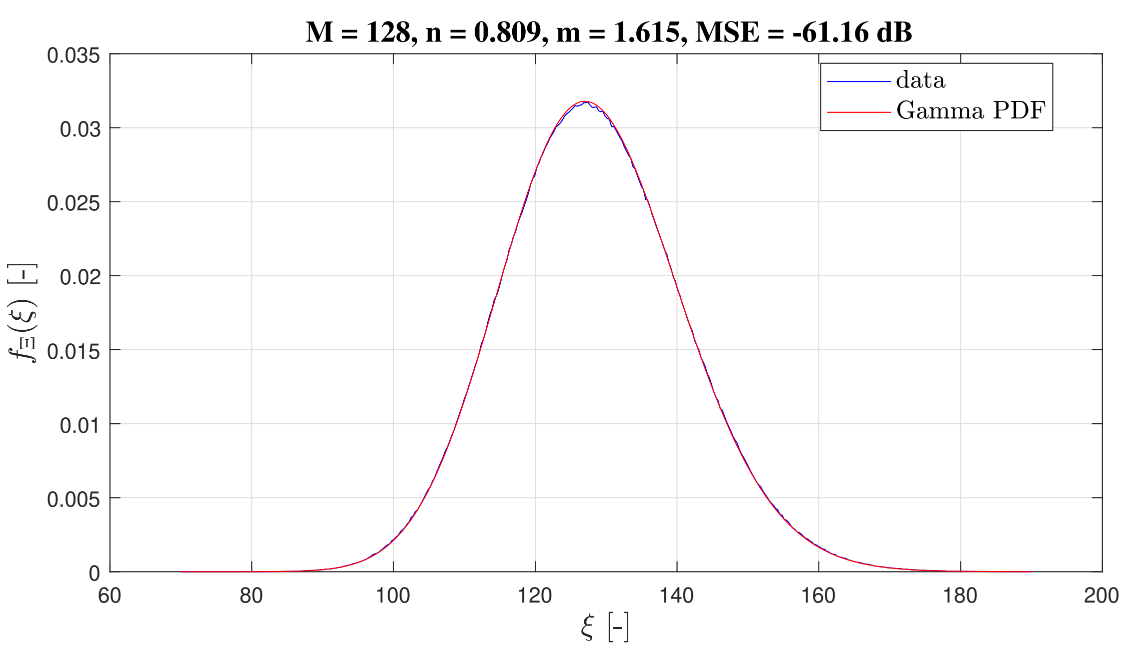

Figure 3: PDF of (6) approximated by (8) for $M = 128$.