Full-rate STLC for Four Receive Antennas - Notes

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1 Problem Description

Formula (6) in [1] is given by

$$\xi = \rho \frac{\left(\sum_{p=0}^{M-1} \gamma_p \pm 2\mathcal{R}\{\epsilon_p\}\right)^2}{4\sum_{p'=0}^{M-1} \gamma_{p'}}, \tag{1}$$

where

$$\gamma_p = \sum_{q=0}^{3} |h_{p,q}|^2, \tag{2}$$

and

$$\epsilon_p = h_{p,0}h^*_{p,2} + h_{p,1}h^*_{p,3}. \tag{3}$$

Each complex channel gain $h_{p,q}$ is composed of two i.i.d. normal variables $a_{p,q}, b_{p,q} \sim \mathcal{N}(0, 1/2)$ which relate to channel gain by $h_{p,q} = a_{p,q} + jb_{p,q}$, where $j = \sqrt{-1}$. Formula (2) then evolves into

$$\gamma_p = \sum_{q=0}^{3} |a_{p,q}|^2 + |b_{p,q}|^2, \tag{4}$$

and $\mathcal{R}\{\epsilon_p\}$ is expressed as

$$\mathcal{R}\{\epsilon_p\} = a_{p,0}a_{p,2} + b_{p,0}b_{p,2} + a_{p,1}a_{p,3} + b_{p,1}b_{p,3}. \tag{5}$$

Formula (1) may be expanded into purely real-valued form

$$\xi = \frac{\rho}{4} \frac{\left(\sum_{p=0}^{M-1} \sum_{q=0}^{3} |a_{p,q}|^2 + |b_{p,q}|^2 \pm 2\mathcal{R}\{\epsilon_p\}\right)^2}{\sum_{p'=0}^{M-1} \sum_{q=0}^{3} |a_{p',q}|^2 + |b_{p',q}|^2}. \tag{6}$$

After Lemma 1 in [1] the receiver SNR is calculated via

$$\xi = \frac{\rho}{4} \sum_{p=0}^{M-1} \sum_{q=0}^{3} |a_{p,q}|^2 + |b_{p,q}|^2. \tag{7}$$

2 MATLAB Simulation

In MATLAB I created a script which generates the normal-distributed variables $a_{p,q}$ and $b_{p,q}$ in vectors of 10^7 samples each. This way I calculate the histogram of formula (6) and I attempt to approximate it with Gamma distribution with PDF given by

$$f_\Xi(\xi) = \frac{\xi^{nM-1} e^{-\frac{\xi m}{2}}}{(2/m)^{nM} \cdot \Gamma(nM)} \quad \forall \quad \xi \in \langle 0, \infty). \tag{8}$$

where n is related to the shape parameter and m to the Gamma scale parameter. Please note that for PDF approximations I set $\rho = 1$. As the error metric of the PDF approximation I used mean-squared-error defined as

$$J = \int \left(f_\Xi(\xi)' - f_\Xi(\xi)\right)^2 d\xi, \tag{9}$$

where $f_\Xi(\xi)'$ is the simulated data histogram and $f_\Xi(\xi)$ is the approximation.

Table 1: PDF fitting parameters of (6) at given M

M	m	n	J [dB]
1	1.530445	0.949676	−41.6985
2	1.574221	0.886598	−46.4452
4	1.596783	0.847111	−55.8000
8	1.600477	0.824861	−58.9625
16	1.606456	0.815408	−60.4560
32	1.610120	0.811129	−61.2368
64	1.608200	0.807196	−63.6193
128	1.613510	0.808271	−62.2914
256	1.612725	0.807231	−64.0791
512	1.610609	0.805685	−66.8909

Diversity gain of systems approximated by Gamma distribution is given by

$$\kappa = -\lim_{\rho \to \infty} \frac{d \log_{10}(\bar{\varepsilon}(\rho))}{d \log_{10}(\rho)} = nM, \tag{10}$$

where $\bar{\varepsilon}(\rho)$ is the average bit error rate at given ρ and n is the diversity gain coefficient. Table 1 then summarizes PDF approximation parameters n, m for several values of M.

Figures 1 to 5 I present the results for $M = \{1, 32, 64, 256, 512\}$ where formula (6) is fitted by (8) with m and n according to Table 1.

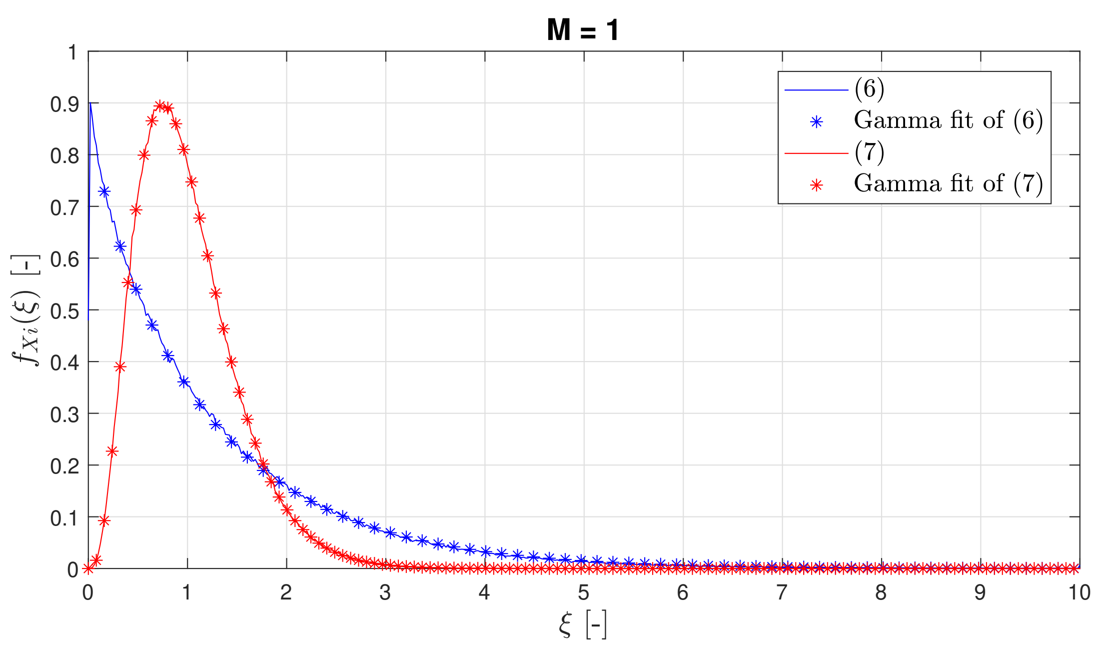

Figure 1: PDF of (6) approximated by (8) for $M = 1$.

3 Discussion

My MATLAB simulation attepts show that (6) is close to Gamma distribution with $n = 0.8$ and $m = 1.6$. The diversity gain is for Gamma distribution directly given by nM. Parameter n is decreasing with increasing value of M and it is approaching $n \sim 0.8$. I would be very grateful for your advice.

References

[1] S. -chan Lim and J. Joung, “Full-Rate Space–Time Line Code for Four Receive Antennas”, IEEE wireless communications letters, pp. 1-1, 2021.

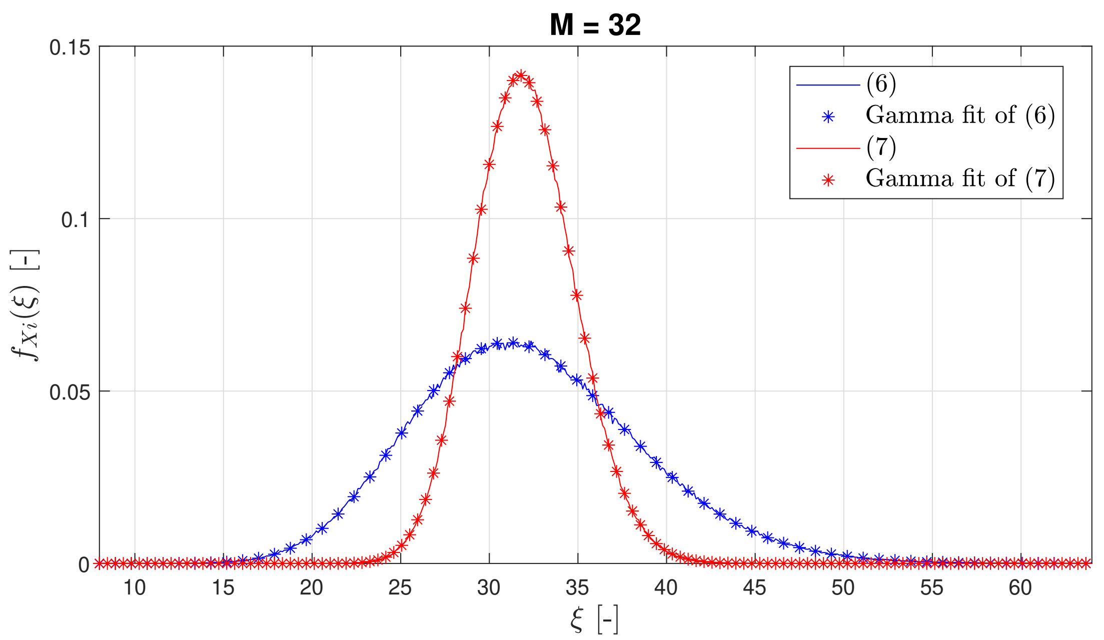

Figure 2: PDF of (6) approximated by (8) for $M = 32$.

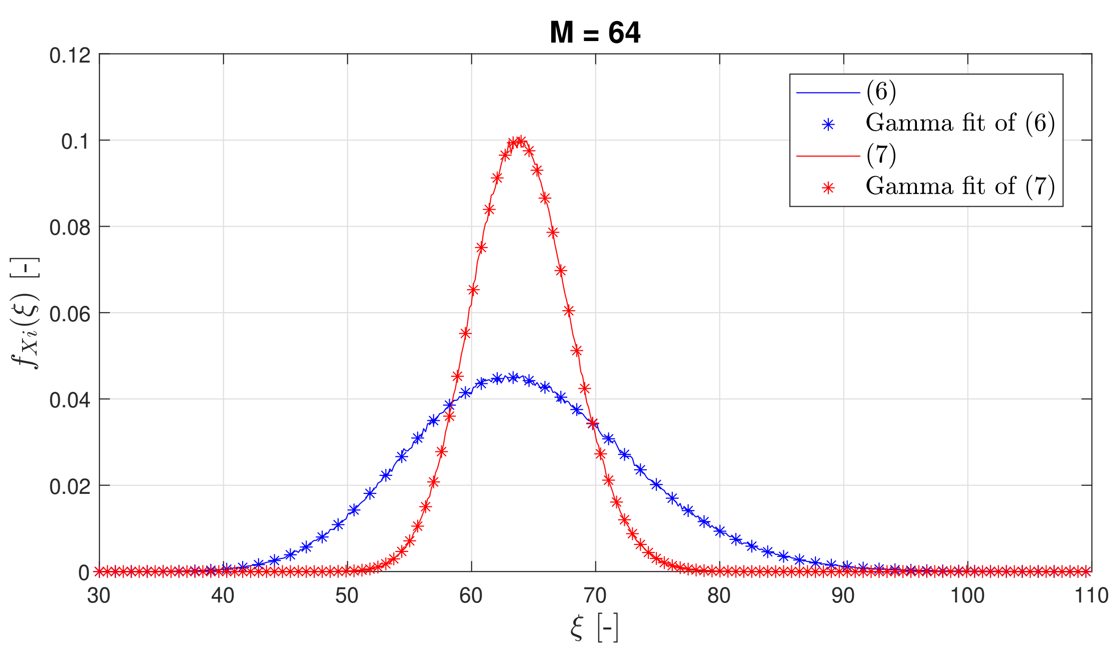

Figure 3: PDF of (6) approximated by (8) for $M = 64$.

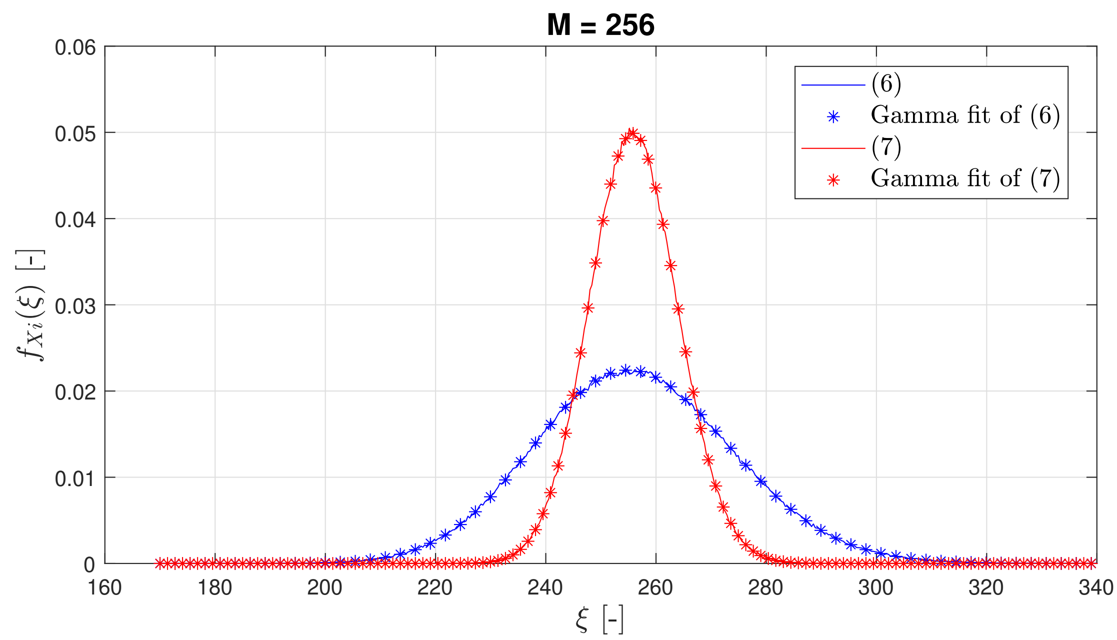

Figure 4: PDF of (6) approximated by (8) for $M = 256$.

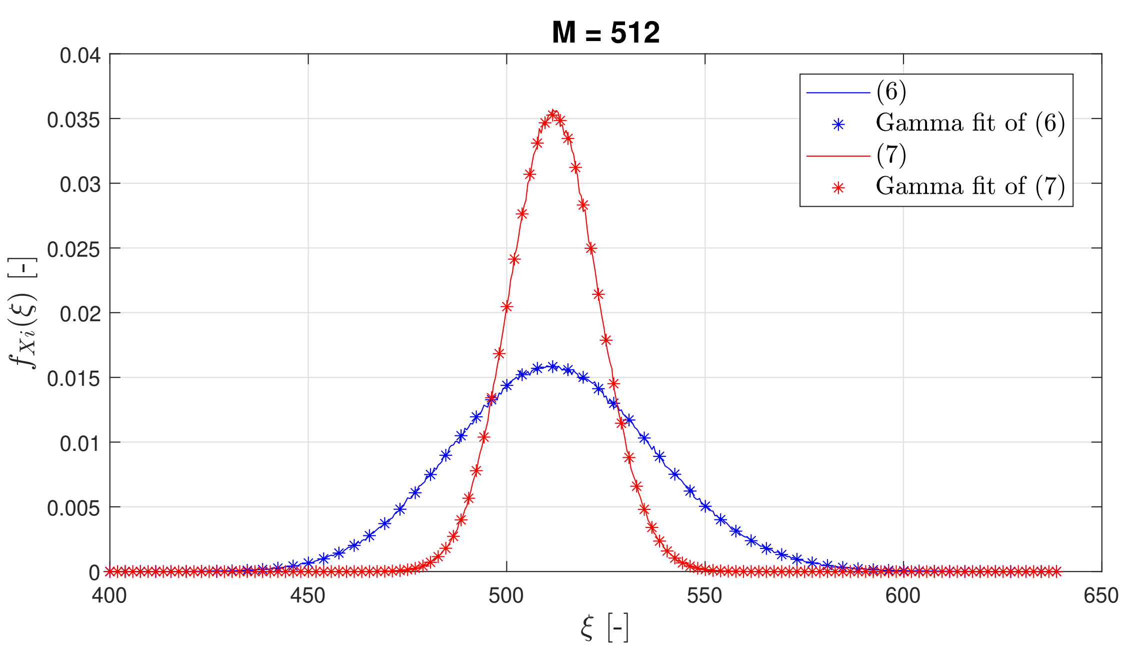

Figure 5: PDF of (6) approximated by (8) for $M = 512$.