Primes Are Rather Elusive From Primes To Riemann

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Simple Pattern, Simple Formula

- A simple pattern in the primes would mean a simple generating formula.
- Eg the triangle numbers 1,3,6,10,15,... are $\frac{1}{2}n(n+1)$

Polynomials¹

Polynomials are both simple and rather flexible.

$$P(n) = a + bn + cn^2 + dn^3 + \ldots + \alpha n^{\beta}$$

- Can a polynomial generate the nth prime?
 - Are primes simple enough to be modelled by a polynomial?

$$P(n) = a + bn + cn^2 + dn^3 + \ldots + \alpha n^{\beta}$$

- Simple polynomial:
 - coefficients $a, b, c \dots \alpha$ are whole numbers.
 - also $b, c, d, \dots \alpha$ are not all zero \rightarrow to avoid eg P(n) = 7

- Proof by contradition .. again!
- Assume P(n) does generates only primes.
- So when n=1, it generates a prime, which we can call p_1

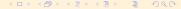
$$p_1 = P(1) = a + b + c + d + \ldots + \alpha$$

• Now let's try $n = (1 + p_1)$

$$P(1+p_1) = a + b(1+p_1) + c(1+p_1)^2 + d(1+p_1)^3 + \dots$$

- Looks scary but .. if we expand, we'll have terms with p_1 and those without.
- Let's collect all those p_1 terms and call them $p_1 \cdot X$

$$P(1+p_1) = (a+b+c+d+e+...\alpha) + p_1 \cdot X$$



• That $(a+b+c+d+e+\ldots\alpha)$ is actually p_1 .

$$P(1 + p_1) = p_1 + p_1 \cdot X$$

= $p_1(1 + X)$

- This is divisible by p_1 ... and it shouldn't be because P(n) is supposed to generate only primes!
- Contradiction means assumption P(n) only generates primes is wrong.

Stronger Proof Than Intended

- We wanted to prove
 - "P(n) can't generate n^{th} prime".
- We proved
 - "P(n) can't generate only primes"

What about Rational Coefficients?

• This time we'll consider $n = (1 + k \cdot p_1)$

$$P(1 + k \cdot p_1) = a + b(1 + k \cdot p_1) + c(1 + k \cdot p_1)^2 + \dots$$

= $p_1 + k \cdot p_1 \cdot X$
= $p_1(1 + k \cdot X)$

- X contains terms that are combinations of rational coefficients a, b, c . . . α multiplied together.
- We can choose a k which cancels all the denominators of the rational coefficients leaving $k \cdot X$ as an integer.
- .. and the proof continues as before.

