1 What Are Prime Numbers?

Lets start by looking at the most ordinary numbers we know, the **counting numbers**.

$$1, 2, 3, 4, 5, 6, 7, 8, \ldots$$

We became familiar with these numbers when we were just toddlers, counting apples in a bowl, for example.

Multiplication

We soon learned to add and multiply these numbers. Many of us learned our times tables by heart. Almost without thinking we could recite multiplications like $2 \times 4 = 8$, and $5 \times 5 = 25$.

When we multiply 3 by 4, the answer is 12. This 12 is called a **product**, and the 3 and 4 are called **factors**.

If we pick any two numbers a and b and multiply them, the result is another number, which we can call c.

$$a \times b = c$$

Because a and b are whole numbers, so is c

An Innocent Question

Those factors a and b can be any counting number we feel like choosing. Does this freedom apply to c as well?

Surely some combination of a and b can give us any number c that we desire. Let's try a couple of examples.

- If we want c to be 12, we could choose $a = 3$ and $b = 4$. We could have chosen $a = 2$ and $b = 6$, and that would work too.

- If we want c to be 100, we could choose $a = 2$ and $b = 50$. Another combination that works is $a = 10$ and $b = 10$.

What if we want c to be 7?

If we try for a short while, we'll find there doesn't seem to be a combination of factors a and b that gives 7 as a product. In fact we can try all the numbers in the range $2 \ldots 6$ we'll see for ourselves there really is no combination that gives $a \times b = 7$.

What if we want c to be 11? Again we'd find that no combination of whole number factors gives 11 as a product.

So the answer to our innocent-looking question is no, c can't be any whole number.

Numbers like 7 and 11 that don't have whole number factors, are called **prime numbers**. Here are the first few.

$$2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, \ldots$$

In short, if we multiply two counting numbers, the answer is never a prime number.

What About 1?

You might have spotted that when we were trying to find factors of 7 we didn't consider combinations like $a = 1$ and $b = 7$. That's because we exclude 1 as a legitimate factor. Why? Because every number has 1 as a factor, and that's not particularly interesting.

If we didn't exclude 1, there would be no prime numbers because every number c would have factors $a = 1$ and $b = c$.

Even worse, a number could have lots of factors of 1, which is also rather unhelpful. The number 12 could have an infinite number of factors.

$$12 = 4 \times 3 \times 1 \times 1 \times 1 \times 1 \times 1 \times 1 \times 1 \ldots$$

Negative Numbers?

Prime numbers were known about and discussed in ancient times, well before the idea of a negative number was accepted.

Over the hundreds of years since then, new ideas and insights were developed about prime numbers, and they were built on the original assumption that prime numbers could only be **positive** whole numbers.

Today almost all exploration of prime numbers continues under the same constraint that products, factors and primes are positive whole numbers greater than 1. This constraint really doesn't limit the mysteries and suprises that prime numbers hold.

Apparent Randomness

Looking back at the list of prime numbers, there doesn't seem to be a pattern to them. Apart from never being even numbers, they seem to be fairly randomly located.

For hundreds of years, mathematicians puzzled over the primes, attacking them with all sorts of exotic tools, trying to crack them open to reveal any elusive rules that govern their location. That endeavour continues to this day.