

# Primes Are The Building Blocks Of Numbers

## From Primes To Riemann

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# Breaking A Number Into Its Factors

- Let's think about the number 12 and its factors.

$$12 = 2 \times 6$$

$$12 = 3 \times 4$$

# Breaking A Number Into Its Factors

- That 6 can be broken down further. So can that 4.

$$12 = 2 \times (3 \times 2)$$

$$12 = 3 \times (2 \times 2)$$

- Can't break these down any further  $\rightarrow$  prime.

# Breaking A Number Into Its Factors

- If we order those factors by size, we can see the lists are the **same**.

$$12 = 2 \times 2 \times 3$$

$$12 = 2 \times 2 \times 3$$

- Perhaps every number has a **unique** breakdown of factors ... like DNA is unique to people?

# Fundamental Theorem of Arithmetic

- Proof part 1:
  - Show any number can be broken down into a list of **factors that are all prime**.
- Proof part 2:
  - Show this list of primes is **unique**.

- Imagine a counting number  $N$ , and write it as a product of factors.

$$N = f_1 \cdot f_2 \cdot f_3 \cdot \dots \cdot f_n$$

- Looking at each factor in turn, if it isn't prime we can break it down into smaller factors.

- $f_1$  might be broken down as  $f_1 = g_1 \cdot g_2$
- $f_2$  might be prime  $p_1$ .

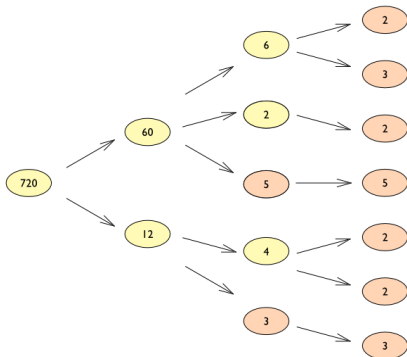
$$\begin{aligned} N &= f_1 \cdot f_2 \cdot f_3 \cdot \dots \cdot f_n \\ &= (g_1 \cdot g_2) \cdot p_1 \cdot (g_3 \cdot g_4 \cdot g_5) \cdot \dots \cdot (g_x \cdot g_y) \end{aligned}$$

- We can keep repeating this process of breaking down numbers into smaller factors.
- We only stop when we can't break numbers down further → **when they're all prime.**



# FTA Part 1

- Let's try this process with an example number, 720.



- Imagine  $N$  can be written as a product of primes in two different ways.

$$N = p_1 \cdot p_2 \cdot p_3 \cdot \dots \cdot p_a$$

$$N = q_1 \cdot q_2 \cdot q_3 \cdot \dots \cdot q_a \cdot q_b \cdot q_c \cdot q_d$$

- For generality, that second list is longer than the first.

- If  $p_1$  is a factor of  $N$  from the first list, then it must be in the second list too.

$$\cancel{p_1} \cdot p_2 \cdot p_3 \cdot \dots \cdot p_a = \cancel{q_1} \cdot q_2 \cdot q_3 \cdot \dots \cdot q_a \cdot q_b \cdot q_c \cdot q_d$$

- We can repeat the same logic. If  $p_2$  is a factor of  $N$  then it must be in the second list.

$$\cancel{p_1} \cdot \cancel{p_2} \cdot p_3 \cdot \dots \cdot p_a = \cancel{q_1} \cdot \cancel{q_2} \cdot q_3 \cdot \dots \cdot q_a \cdot q_b \cdot q_c \cdot q_d$$

- We can repeat the process until we've cancelled all the prime factors in the shorter first list.

$$\cancel{p_1} \cdot \cancel{p_2} \cdot \cancel{p_3} \cdot \dots \cdot \cancel{p_a} = \cancel{q_1} \cdot \cancel{q_2} \cdot \cancel{q_3} \cdot \dots \cdot \cancel{q_a} \cdot q_b \cdot q_c \cdot q_d$$

- We can simplify this.

$$1 = q_b \cdot q_c \cdot q_d$$

- All the remaining factors  $q$  are 1. So the **two lists are equal**.

1. Any whole number  $N > 1$  can be broken down into a list of prime factors.
2. ... and that list of primes is unique to that number.

person  $\leftrightarrow$  DNA

whole number  $\leftrightarrow$  prime factors