

# What Is A Prime?

## From Primes To Riemann

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# Counting Numbers

1, 2, 3, 4, 5, 6, 7, 8, ...

# Multiplication

$$2 \times 4 = 8$$

$$5 \times 5 = 25$$

# Factors & Products

$$3 \times 4 = 12$$

- 3 and 4 are **factors**
- 12 is a **product**

# Multiplying Whole Numbers

$$a \times b = c$$

- if  $a$  and  $b$  are whole numbers, so is  $c$

# An Innocent Question

$$a \times b = c$$

- $a$  and  $b$  can be any counting number we feel like choosing
- does  $c$  have this freedom too?

# An Innocent Question

$$a \times b = 12$$

- $a = 3, b = 4$
- $a = 2, b = 6$

# An Innocent Question

$$a \times b = 100$$

- $a = 2, b = 50$
- $a = 10, b = 10$



# An Innocent Question

$$a \times b = 7$$

- $a = ?$ ,  $b = ?$
- no solutions !

# An Innocent Question

$$a \times b = 11$$

- $a = ?$ ,  $b = ?$
- no solutions !
- These numbers 7 and 11 are **prime numbers**.

# Prime Numbers

2, 3, 5, 7, 11, 13, 19, 23, 29, 31, 37, 41, 43, 47, 53, ...

# What About 1?

$$1 \times 7 = 7$$

- We exclude 1 as a legitimate factor.
- If we didn't, there would be no prime numbers because every number would have a factor of 1

# What About 1?

- Even worse...

$$12 = 3 \times 4$$

$$12 = 3 \times 4 \times 1$$

$$12 = 3 \times 4 \times 1 \times 1 \times 1 \times 1$$

$$12 = 3 \times 4 \times 1 \times 1 \times 1 \times 1 \times 1 \times 1 \times \dots$$

# Negative Primes?

History:

- prime numbers were known about and discussed in ancient times ...
- .. before the idea of a **negative number** was accepted

# Apparent Randomness

2, 3, 5, 7, 11, 13, 17, 19, 23, 29

31, 37, 41, 43, 47, 53, 59, 61, 67, 71

73, 79, 83, 89, 97, 101, 103, 107, 109, 113

127, 131, 137, 139, 149, 151, 157, 163, 167, 173

179, 181, 191, 193, 197, 199, 211, 223, 227, 229

233, 239, 241, 251, 257, 263, 269, 271, 277, 281

- No apparent pattern → hard to predict next prime