How Many Primes Are There?

From Primes To Riemann

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Do Primes Fizzle Out?

- Every multiple of 2 is not a prime.
- Every multiple of 3 is not a prime.
- Every multiple of 4 is not a prime.
- Every multiple of 5 is not a prime. ...

- All these multiples are reducing the probability that a large number is prime.

Intuition Is Not Proof

- Intuition is not proof.
- Lots of examples are not proof.

- A proof is a watertight logical argument that leads to a conclusion we can't argue with.

Euclid's Proof (300BC)

- Let's start by assuming there are a **finite** number of primes.

$$p_1, p_2, p_3, p_4 \ldots p_n$$

Euclid's Proof (300BC)

- Let's create a new number x by multiplying all those primes together.

$$x = p_1 \cdot p_2 \cdot p_3 \cdot p_4 \cdot \ldots \cdot p_n$$

- This x is clearly not a prime number. It's full of factors like p_1 , p_3 and p_n.

Euclid's Proof (300BC)

- Let's create another y in the same way, but this time add 1.

$$y = p_1 \cdot p_2 \cdot p_3 \cdot p_4 \cdot \ldots \cdot p_n + 1$$

- Is y prime? There are only two options - yes or no.

Euclid's Proof (300BC)

- Option 1 - yes, y is prime.
  - We just found a new prime that isn't in the original list !
- Option 2 - no, it is not prime.
  - That means it must have factors which are primes amongst

Euclid's Proof (300BC)

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Proof By Contradiction pictre