What Is A Prime?

From Primes To Riemann

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Counting Numbers

$$1, 2, 3, 4, 5, 6, 7, 8, \ldots$$

Multiplication

$$2 \times 4 = 8$$
$$5 \times 5 = 25$$

Factors & Products

$$3 \times 4 = 12$$

- 3 and 4 are **factors**
- 12 is a **product**

Multiplying Whole Numbers

$$a \times b = c$$

- if a and b are whole numbers, so is c

An Innocent Question

$$a \times b = c$$

- a and b can be any counting number we feel like choosing
- does c have this freedom too?

An Innocent Question

$$c = 12$$

- $a = 3, \; b = 4$
- $a = 2, \; b = 6$

$$c = 100$$

- $a = 2, \; b = 50$
- $a = 10, \; b = 10$

An Innocent Question

$$c = 7$$

- no solutions !

$$c = 11$$

- no solutions !

- These are **prime numbers**.

Prime Numbers

$$2, 3, 5, 7, 11, 13, 19, 23, 29, 31, 37, 41, 43, 47, 53, \ldots$$

What About 1?

$$c = 7 = 1 \times 7$$

- We exclude 1 as a legitimate factor.
- If we didn't, there would be no prime numbers because every number would have a factor of 1

What About 1?

- Even worse...

$$
\begin{aligned}
12 &= 3 \times 4 \\
12 &= 3 \times 4 \times 1 \\
12 &= 3 \times 4 \times 1 \times 1 \times 1 \times 1 \\
12 &= 3 \times 4 \times 1 \times 1 \times 1 \times 1 \times 1 \times 1 \times \ldots
\end{aligned}
$$

Negative Primes?

History:

- prime numbers were known about and discussed in ancient times ...
- .. before the idea of a negative number was accepted

Apparent Randomness

$$2, 3, 5, 7, 11, 13, 17, 19, 23, 29$$
$$31, 37, 41, 43, 47, 53, 59, 61, 67, 71$$
$$73, 79, 83, 89, 97, 101, 103, 107, 109, 113$$
$$127, 131, 137, 139, 149, 151, 157, 163, 167, 173$$
$$179, 181, 191, 193, 197, 199, 211, 223, 227, 229$$
$$233, 239, 241, 251, 257, 263, 269, 271, 277, 281$$

- No apparent pattern $\rightarrow$ hard to predict next prime