14 - Disequality

Lean: First Steps

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Using Lemmas

- We've seen a lemma applied to a proof **goal**.
- We'll see a lemma applied to a **hypothesis**.

Task

- Given a natural number $n < 5$, show that

$$n \neq 5$$

- (the same task as before)

Maths

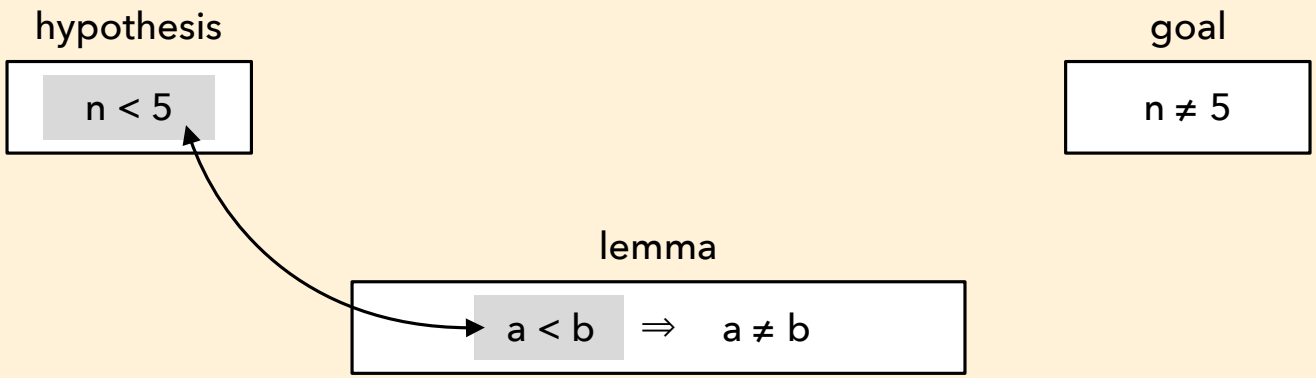
- We'll use the same lemma we used before. For natural numbers a and b,

$$a < b \implies a \neq b$$

- Previously we considered the proof goal $n \neq 5$, and used this lemma to say that proving $n < 5$ was a sufficient.

- This time we'll consider the hypothesis $n < 5$, and say that $n \neq 5$ follows directly as a result of that lemma.

Maths



Maths

$n < 5$	hypothesis	(1)
$n \neq 5$	proof objective	(2)
$a < b \implies a \neq b$	existing lemma	(3)
$n \neq 5$	lemma (3) applied to (1)	(4)
$n < 5 \implies n \neq 5$	by lemma (3)	$\square$

Maths

- We start with the hypothesis $n < 5$, and our proof objective $n \neq 5$.
- We know about a lemma (3) applicable to natural numbers, that if $a < b$ then $a \neq b$.
- The lemma's antecedent $a < b$ matches our hypothesis (1), which immediately gives us $n \neq 5$.

Code

```
-- 14 - Lemma: Not Equal from Less Than

import Mathlib.Tactic

example {n : ℕ} (h : n < 5): n ≠ 5 := by
  apply ne_of_lt at h
  exact h
```

Code

- The Lean proof is almost exacty the same as the previous tutorial.
- The lemma `ne_of_lt` is pointed at the hypothesis `h` by adding `at h`.
- It changes the hypothesis `h` from `h : n < 5` to `h : n ≠ 5`.
- This new hypothesis exactly matches the proof goal, so `exact h` works just as before to complete the proof.

Infoview

- Placing the cursor before `apply ne_of_lt at h` shows the original hypothesis.

```
n : ℕ
h : n < 5
⊢ n ≠ 5
```

- Moving the cursor to the beginning of the next line after `apply ne_of_lt at h` shows the hypothesis has been replaced.

```
n : ℕ
h : n ≠ 5
⊢ n ≠ 5
```

Easy Exercise

- Write a Lean program to prove $n \neq 5$, given $n > 5$, where n is a natural number.
- Use the same the lemma for "not equal from greater than" you found for the last chapter's exercise, and apply it to the hypothesis.