ST 437/537: Applied Multivariate and Longitudinal Data Analysis

Summary of inference in the one sample case

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Univariate case

Let $X_1, X_2, \ldots, X_n$ be a sample from a normal distribution with mean μ and unknown variance σ^2. An estimator of μ is the sample mean $\bar{X}$.

Confidence Interval *Let $t_{n-1}(\alpha/2)$ be the upper-tail probability corresponding to the t_{n-1} distribution. Then the $100(1-\alpha)\%$ confidence interval (CI) for μ is*

$$\left(\bar{X} - t_{n-1}(\alpha/2)\frac{s}{\sqrt{n}},\ \bar{X} + t_{n-1}(\alpha/2)\frac{s}{\sqrt{n}}\right).$$

*In absence of normality, we can construct **a large sample interval ($n \geq 40$)** using the same formula above with replacing $t_{n-1}(\alpha/2)$ by $z(\alpha/2)$.*

One sample t-test: *We reject $H_0 : \mu = \mu_0$ in favor of $H_a : \mu \neq \mu_0$, if*

$$\left|\frac{\bar{X} - \mu_0}{s/\sqrt{n}}\right| > t_{n-1}(\alpha/2);$$

we fail to reject H_0 otherwise.

R function: `t.test()` ; it does both estimation and testing.

Multivariate Inference

Suppose we have random sample $\boldsymbol{X}_1, \ldots, \boldsymbol{X}_n$, each of them is a $p \times 1$ vector (in our example, $p = 4$), generated from a p-variate normal distribution with mean $\boldsymbol{\mu} = (\mu_1, \ldots, \mu_p)^T$ and unknown covariance matrix $\boldsymbol{\Sigma}$. We want to form confidence intervals for the mean parameters $\mu_1, \ldots, \mu_p$.

Simultaneous confidence intervals: *The simultaneous $100(1-\alpha)\%$ confidence intervals for $\mu_1, \ldots, \mu_p$ are*

$$\text{For } \mu_k\colon \left(\bar{X}_k - \sqrt{\frac{(n-1)p}{(n-p)} F_{p,n-p}(\alpha) \frac{S_{kk}}{n}},\quad \bar{X}_k + \sqrt{\frac{(n-1)p}{(n-p)} F_{p,n-p}(\alpha) \frac{S_{kk}}{n}}\right),$$

where S_{kk} is the kth element of the sample covariance $\boldsymbol{S}$.

Large sample simultaneous intervals: *When n is large, the approximate simultaneous $100(1-\alpha)\%$ confidence intervals for $\mu_1, \ldots, \mu_p$ are*

$$\text{For } \mu_k\colon \left(\bar{X}_k - \sqrt{\chi_p^2(\alpha) \frac{S_{kk}}{n}},\quad \bar{X}_k + \sqrt{\chi_p^2(\alpha) \frac{S_{kk}}{n}}\right),$$

where S_{kk} is the kth element of the sample covariance $\boldsymbol{S}$.

The Bonferroni method for multiple correction: *The Bonferroni $100(1-\alpha)\%$ confidence intervals for μ_k, $k = 1, \ldots, p$ are*

$$\bar{X}_k \pm t_{n-1}\left(\frac{\alpha}{2p}\right) \sqrt{S_{kk}/n}, \qquad k = 1, \ldots, p$$

where S_{kk} is the kth element of the diagonal of the sample covariance $\boldsymbol{S}$.

Hotelling's T^2 test: *We reject $H_0 : \boldsymbol{\mu} = \boldsymbol{\mu}_0$ at level α if*

$$\frac{n(n-p)}{(n-1)p}(\bar{\boldsymbol{x}} - \boldsymbol{\mu}_0)^T \boldsymbol{s}^{-1}(\bar{\boldsymbol{x}} - \boldsymbol{\mu}_0) > F_{p,n-p}(\alpha).$$

When we have a large sample size, n, we can again relax the normality assumption and conduct an approximate test: reject H_0 at level α if

$$n(\bar{\boldsymbol{x}} - \boldsymbol{\mu}_0)^T \boldsymbol{s}^{-1}(\bar{\boldsymbol{x}} - \boldsymbol{\mu}_0) > \chi^2_p(\alpha).$$

R function: `HotellingsT2()` in the library `ICSNP` for Hotelling's T^2 testing.

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