CS4780 Midterm

October 2015

NAME:	
Net ID:	
Email:	

GML	
Short	
kNN	
NB	
Perceptron + LR	
SVM	
Linear Regression	
TOTAL	

1 [??] General Machine Learning

Please identify if these statements are either True or False. Please justify your answer **if false**. Correct "True" questions yield 1 point. Correct "False" questions yield two points, one for the answer and one for the justification.

T/F Naïve Bayes makes the assumption that the individual features are independent, i.e. $p(\mathbf{x}) = \prod_\alpha p([\mathbf{x}]_\alpha)$.

T/F A linear regressor is parameterized by parameters $\mathbf{w}$ and b. The parameter b is the *bias*, which, together with *variance* and *noise* combine to the test error.

T/F If a classifier obtains 0% training error it cannot have 100% testing error.

T/F Any reasonable machine learning algorithm must make assumptions about the data.

T/F If data is drawn uniformly at random within a hypercube, increasing the dimensionality makes the data points more equidistant (i.e. they all have roughly the same distance from each other).

T/F Although MLE and MAP are different approaches to set model parameters θ, both of them do consider θ to be random variables.

T/F If a data set is linearly separable, the Perceptron is guaranteed to converge in a finite number of updates. Otherwise, it sometimes converges, but there are no guarantees.

T/F Assume there exists a vector $\mathbf{w}^*$ that defines a hyperplane that perfectly separates your data. Let the Perceptron vector be $\mathbf{w}$. As the algorithm proceeds, the two vectors must converge ($\mathbf{w} \to \mathbf{w}^*$) and in the limit (after possibly infinitely many updates) we have $\mathbf{w} = \mathbf{w}^*$.

T/F The Naïve Bayes classifier is a discriminative classification algorithm.

T/F The squared-hinge-loss SVM (with or without bias) can be solved very efficiently with various fast optimization methods. For the Elastic Net this is not the case because the l_1 regularizer (/constraint) is non-differentiable.

2 [22] Short Questions

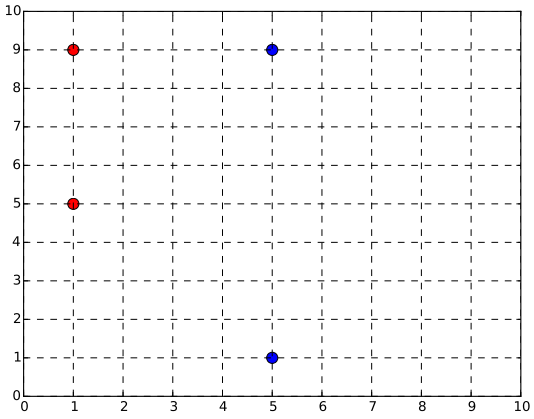
1. (4) Assume you want to model your data $y_1, \ldots, y_n$ with a Poisson distribution,

$$P(y; \theta) = \frac{\theta^y e^{-\theta}}{y!} \text{ for } y = 0, 1, 2, \ldots$$

derive the log-likelihood of your data as a function of θ.

2. (4) Briefly describe the Bayes Optimal classifier if $P(\mathbf{x}, y)$ is known.

3. (2) Draw the decision boundary for 1-NN for the following graph :



4. (2) What is the relationship between kNN and the Bayes Optimal classifier?

5. (2) What is a downside of kNN as the data set size gets very large $n \gg 0$?

6. (2) In what scenario would you want to use the Huber loss?

7. (2) Name an advantage and a disadvantage of the l_1 regularization over l_2 regularization?

8. (4) You have a box with two coins. One of them is red, the other blue. We have $P(head|red) = \frac{1}{4}$ and $P(head|blue) = \frac{1}{2}$. You pick a coin uniformly at random and toss it. It comes up heads. What's the probability that it was the red coin?

3 [12] Naive Bayes

Assume you are provided with the following data:

$$\begin{aligned}
\mathbf{x}_1 &= [1,1,2,1]^\top & y_1 &= +1 \\
\mathbf{x}_2 &= [1,2,2,1]^\top & y_2 &= +1 \\
\mathbf{x}_3 &= [2,2,1,1]^\top & y_3 &= -1 \\
\mathbf{x}_4 &= [2,2,2,1]^\top & y_4 &= -1 \\
\mathbf{x}_t &= [1,1,2,2]^\top & y_t &= ?
\end{aligned}$$

Throughout this question use *categorical* Naïve Bayes *with +1 smoothing*.

1. (2) What is the probability of $P([\mathbf{x}_t]_2 = 1 | y_t = -1)$, where $[\mathbf{x}_t]_2$ corresponds to the second feature value of $\mathbf{x}_t$.

2. (4) What is the probability of $P(\mathbf{x}_t | y_t = -1)$?

3. (6) What is the probability of $P(y_t = +1 | \mathbf{x}_t)$?

4 [17] Perceptron / LR

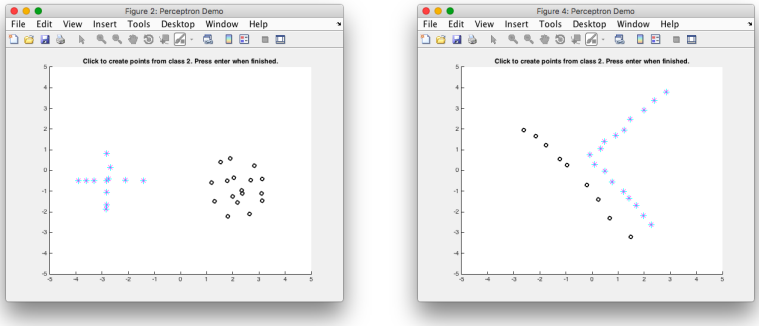
Assume you are provided with the following data:

$$\begin{aligned}
\mathbf{x}_1 &= [1,1,0,1]^\top & y_1 &= +1 \\
\mathbf{x}_2 &= [1,0,0,1]^\top & y_2 &= +1 \\
\mathbf{x}_3 &= [0,0,1,1]^\top & y_3 &= -1 \\
\mathbf{x}_4 &= [0,0,0,1]^\top & y_4 &= -1 \\
\mathbf{x}_t &= [1,1,0,0]^\top & y_t &= ?
\end{aligned}$$

1. (8) Assume you train the *Perceptron* classifier on this data set and you visit the training data in the order $\mathbf{x}_1, \mathbf{x}_2, \mathbf{x}_3, \mathbf{x}_4, \mathbf{x}_1, \mathbf{x}_2, \ldots$. How many updates do you need to make? What is the weight vector after each update?

2. (2) Name one advantage of Logistic Regression over Naïve Bayes, and one advantage vice versa.

3. (7) Consider the following two data sets (Left and Right).



For both data sets state if

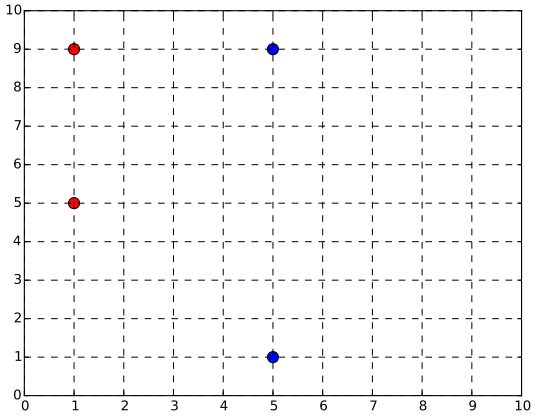
- . . . Gaussian Naïve Bayes will obtain zero training loss;
- . . . Logistic Regression will classify obtain zero training loss;
- . . . LR and NB will yield (almost) identical hyperplanes.

Justify your answers *briefly*.

5 [7] SVM

1. (2) Name one advantage of Logistic Regression over the SVM classifier, and one advantage vice versa.

2. (3) Assume you train an SVM (without slack variables) on the following data set. Draw in the decision boundary.



3. (2) Now you need to add two data points such that the problem still *yields a feasible solution* (without slack) (i.e. the SVM must still find a valid solution after the point is added). First add a "blue" data point (big X) that *would not affect* the decision boundary. Then add another "blue" data point (big circle) that *would* affect the decision boundary.

6 [11] Linear Regression

Remember the loss function of the ordinary least squares (OLS)

$$\ell(\mathbf{w}) = \frac{1}{n}\sum_{i=1}^{n}(\mathbf{x}_i^\top \mathbf{w} - y_i)^2 \tag{1}$$

for the data $\{\mathbf{x}_i, y_i\}$ where $\mathbf{x}_i \in \mathbb{R}^d$, $y_i \in \mathbb{R}$, $i \in \{1...n\}$ in terms of the matrix $\mathbf{X} := [x_1\ x_2\ x_3\ ...\ x_n]$, the vector $\mathbf{y} := [y_1\ y_2\ ...\ y_n]^T$ and the weight vector $\mathbf{w} \in \mathbb{R}^d$

1. (3) State a model assumption for $P(y|\mathbf{x}; \mathbf{w})$, under which the MLE estimate leads to the loss function $\ell(\mathbf{w})$ as stated above.

2. (2) Write down the gradient descent update for the OLS problem.

3. (4) If you were to optimize the OLS objective with Newton's Method, how many steps would you need until convergence (you can assume the Hessian is invertible)? (You can either derive the answer or state it clearly with justification.)

4. (2) Why can't you use Newton's Method to optimize the (standard) SVM hinge loss?

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