CS4780 Midterm

Spring 2017

NAME:	
Net ID:	
Email:	

1 [??] General Machine Learning

Please identify if these statements are either True or False. Please justify your answer **if false**. Correct "True" questions yield 1 point. Correct "False" questions yield two points, one for the answer and one for the justification.

1. (**T/F**) Naïve Bayes makes the assumption that the individual features are independent, i.e. $p(\mathbf{x}) = \prod_\alpha p([\mathbf{x}]_\alpha)$.

2. (**T/F**) The Perceptron classifier provably converges after a finite number of iterations in all cases.

3. (**T/F**) The best machine learning algorithm make no assumptions about the data.

4. (**T/F**) The higher dimensional the data, the less likely the Perceptron is to converge.

5. (**T/F**) If MAP is used with a prior that has non-zero support everywhere, it will converge to the same result as MLE, as $n \to \infty$ (where n is the number of training samples).

6. (**T/F**) As $n \to \infty$, the error of the Perceptron classifier is at most twice the error of the Optimal Bayes classifier (where n is the number of training samples).

7. (**T/F**) The SVM classifier is just a fancy way of saying you use the logistic loss (aka as log-loss) with an $l2$-regularizer.

8. (**T/F**) The Naïve Bayes classifier is a discriminative classification algorithm.

9. (**T/F**) One advantage of Adagrad is that each dimension has effectively its own separate learning rate, which is set automatically.

10. (**T/F**) If the Naive Bayes assumption holds, the Naive Bayes classifier becomes identical to the Bayes Optimal classifier.

11. (**T/F**) MAP estimation with a Beta prior to estimate the probability of a coin leading to heads, is identical to "halluzinating" coin toss results.

12. (**T/F**) In practice people often create *Train* and *Validation* sets out of their original data D. Each point $x_i \in D$ is placed either into $Train$, $Validation$ or into both.

13. (**T/F**) As the validation set becomes extremely large, the validation error approaches the test error.

14. (**T/F**) *Generalization error* is just another word for *validation error*.

15. (**T/F**) If a data set is linearly separable, the Perceptron and the SVM will often learn identical hyperplanes.

16. (**T/F**) If you were to use the "true" Bayesian way of machine learning you would put a prior over the possible models and draw one randomly during training.

2 [16] Short Questions

1. (3) Assume you observed n draws $x_1, \ldots, x_n$. State the log-likelihood function, and estimate the parameters for the distrubion

$$f(x; \lambda) = \lambda e^{-\lambda x}$$

using MLE.

2. (3) Why would someone chose l_1 over l_2 regularization, and vice versa?

3. (4) Recall the linear regression model

$$y_i = \mathbf{x}_i^\top \mathbf{w} + \epsilon_i \tag{1}$$

where $\mathbf{x}_1, \ldots, \mathbf{x}_n \in \mathbb{R}^d$ are input features, $\mathbf{w} \in \mathbb{R}^d$ is the model parameter and $\epsilon_i \sim N(0, \sigma^2)$ are iid Gaussian noise. Write down the loss function $\ell(\mathbf{w})$ whose minimizer results in the maximum a posteriori (MAP) estimate with Gaussian prior for the parameter vector $\mathbf{w}$, i.e. the entries of $\mathbf{w}$ are iid zero-mean Gaussians. Is the solution of this loss function unique if $d \gg n$?

4. The newly elected president to the international machine learning society just announced some *awesome* simplifications to ERM.

   - From now on losses are not computed over the entire training data sets, but sample by sample.
   - All labels must be chosen from $\{-1, +1\}$
   - The only valid loss function is: $\ell(\mathbf{x}, y; \mathbf{w}) = \max(0, -y\mathbf{w}^\top \mathbf{x})$.
   - The learning rate is always 1.

   To be precise, his new version of empirical risk minimization becomes:

   1: Shuffle the data set D randomly and set $\mathbf{w} = \vec{0}$
   2: for $(\mathbf{x}_i, y_i) \in D$:
      3: compute gradient $\mathbf{g} = \frac{\partial \ell(\mathbf{x}_i, y_i)}{\partial \mathbf{w}}$
      4: $\mathbf{w} \leftarrow \mathbf{w} - \mathbf{g}$
   5: Goto 1: until convergence

   The community is excited! Clearly this algorithmic framework is much simpler and more elegant. How is it possible that nobody has thought of this before?

   (a) (3) Compute the gradient $\mathbf{g}$ as a function of $\mathbf{x}, y, \mathbf{w}$.

   (b) (3) Imagine you have a binary data set D with $\mathbf{x}_i \in \mathcal{R}^d$. There exists a hyperplane $\mathbf{w}^*, b^*$, with $b^* \neq 0$ that perfectly separates the two classes. Would this algorithm be guaranteed to find a separating hyperplane? If not, what modification(s) would you have to make?

3 [14] kNN

1. (1) What is the modeling assumption of kNN?

2. (4) Suppose there are n training points $\mathbf{x}_1, \ldots, \mathbf{x}_n \in \mathbb{R}^d$. What is the time complexity of 1-NN (with $k = 1$) using the Euclidean distance during testing time (with a single test point) in terms of n and d? Explain.

3. (4) Suppose your features are age, number of years of education, and annual income in US dollars for an individual. Why is the Euclidean distance not a good measure of dissimilarity? Propose an alternative distance metric and explain why it solves the problem.

4. (5) Briefly describe the curse of dimensionality. How does it affect the kNN classifier? Why can k-NN still function well on some high dimensional data sets (such as images of faces or handwritten digits)?

4 [17] Naive Bayes

You're building a spam classifier and you've compiled counts for certain keywords in both spam and authentic messages. You've also assembled a set of categorical features for those same messages. These two data sets are given below. (The $\mathbf{x}^c$ vectors are word counts, and the $\mathbf{x}^d$ vectors hold discrete categorical features within $\{a, b\}$.)

Training data:

$$\begin{array}{lll} \mathbf{x}_1^c = [0, 4]^\top & \mathbf{x}_1^d = [b, a]^\top & y_1 = +1 \\ \mathbf{x}_2^c = [0, 2]^\top & \mathbf{x}_2^d = [a, a]^\top & y_2 = +1 \\ \mathbf{x}_3^c = [2, 0]^\top & \mathbf{x}_3^d = [a, b]^\top & y_3 = -1 \\ \mathbf{x}_4^c = [1, 1]^\top & \mathbf{x}_4^d = [a, b]^\top & y_4 = -1 \end{array} \tag{2}$$

1. (2) You decide to model the discrete categorial features using Laplace (aka +1) smoothing. What is the probability for a test point $P([\mathbf{x}_t^d]_2 = a | y_t = -1)$, where $[\mathbf{x}_t^d]_2$ corresponds to the second feature value of $\mathbf{x}_t^d$?

2. (5) For the following test point with discrete categorical features

$$\tag{3}$$

what is the posterior ratio

$$\frac{P(y_t = -1 | \mathbf{x}_t^d)}{P(y_t = +1 | \mathbf{x}_t^d)} \tag{4}$$

(Hint: Note that you just need to compute the ratio and not the individual probabilities. You also do not need to reduce the fraction to the simplest form.)

3. (5) You now also observe the word count features of the test point

$$\mathbf{x}_t^c = [3, 2]^\top, \tag{5}$$

which you decide to model with a multinomial distribution and Laplace (aka +1) smoothing. If you use both the word count and categorical features in your final classifier, what is the posterior ratio

$$\frac{P(y_t = -1|\mathbf{x}_t)}{P(y_t = +1|\mathbf{x}_t)}, \tag{6}$$

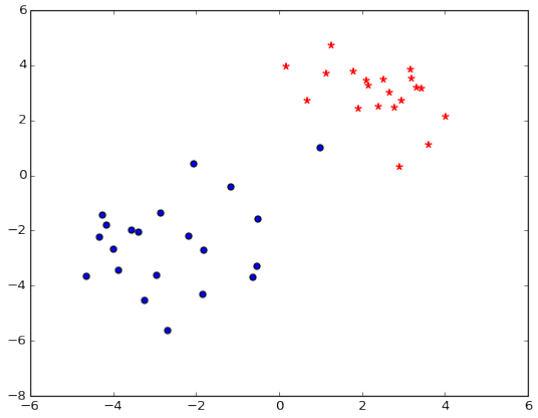
based on both feature sets. (You do not need to reduce the fraction to its simplest form.)

4. (5) Show that the Naive Bayes classifier with Multinomial feature distributions is a linear classifier with some weight vector $\mathbf{w}$ and offset b.

5 [16] Support Vector Machines

1. For this question, refer to the plot and dataset below, and recall that the soft-margin SVM optimization problem is equivalent to the following:

$$\min_{\mathbf{w},b} \mathbf{w}^\top \mathbf{w} + C \sum_{i=1}^{n} \max \left[1 - y_i(\mathbf{w}^\top \mathbf{x}_i + b), 0\right]$$



(a) (3) Suppose you train an SVM on the dataset above with a very *large* value for the regularization parameter C (e.g., $C \to \infty$). Explain what would happen to the learned decision boundary as C increases, and draw as a bold solid line a decision boundary most likely to correspond to a very large value of C.

(b) (3) Suppose that instead you were to use a *small* value of C. Explain what would happen to the learned decision boundary as C *decreases*, and draw as a dashed line on the plot above a decision boundary most likely to correspond to a very small value of C.

(c) (1) On the plot, draw an additional *circle* data point that would not affect the decision boundary of a hard margin SVM trained on the dataset.

(d) (1) On the plot, draw an additional *star* data point that would make the hard margin SVM optimization problem infeasible.

2. Suppose you hand your dataset to the exciting new startup Bad Machine Learning Solutions, Inc., and they give you back a hard margin SVM classifier with parameters $\mathbf{w}$ and b. However, upon inspection, you discover that for every training data point $(\mathbf{x}_i, y_i)$, $y_i(\mathbf{w}^\top \mathbf{x}_i + b) \geq 2$.

(a) (1) Write down an equation for the separating hyperplane constraint used in the hard margin SVM problem.

(b) (2) Write down an equation for the margin of a linear classifier, $\gamma(\mathbf{w}, b)$.

(c) (5) Prove that the parameters this company gave you cannot possibly correspond to an optimal maximum margin classifier.

This page is left blank for scratch space.

This page is left blank for jokes. (Nothing dirty.)

Please do not write on this page. For administrative purposes only.

GML	
Short	
kNN	
NB	
SVM	
TOTAL	