Big Data and Security

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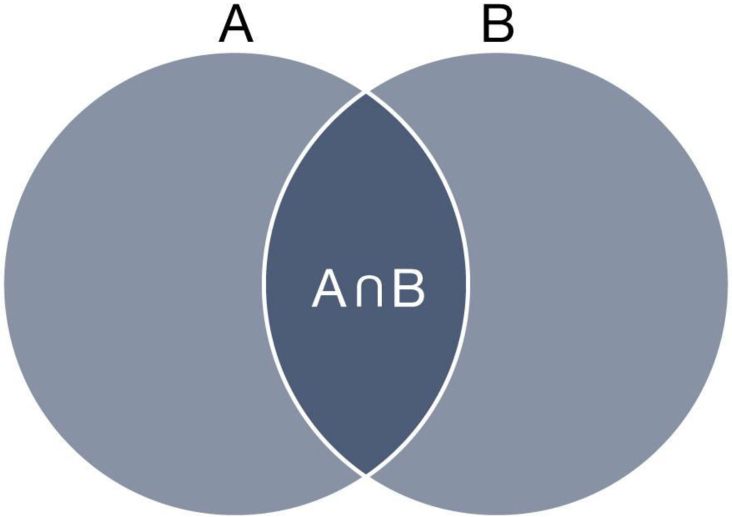
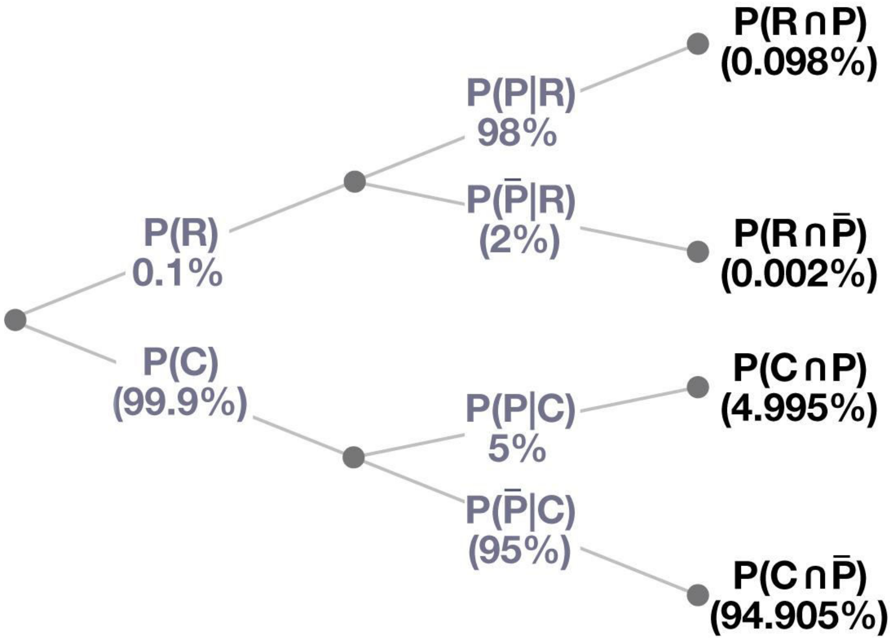
Probability and Bayes' Rule

Total Probability

- There are two ways to make A:
    - A and B
    - A and B^C



Conditional Probability



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Bayes' Rule

- Bayes' rule provides a way to incorporate new information into existing prior beliefs to form updated beliefs
- What are the chances that B will happen, given that A happened.
- Rule:

$$P(B|A) = \frac{P(A|B)\ \cdot P(B)}{P(A)} = \frac{P(A|B)\ \cdot P(B)}{P(A|B)P(B) + P(A|B^C)P(B^C)}$$

Bayes' Rule Example: Context Clues

- There's an incoming Georgia Tech undergrad, and we're trying to predict his or her major
- Somebody mentions that the person is really interested in liberal arts
  - What do you think he or she will major in?

Bayes' Rule Example: Context Clues

- There's an incoming Georgia Tech undergrad, and we're trying to predict his or her major
- Somebody mentions that the person is really interested in liberal arts
  - What do you think he or she will major in?
- We want to calculate the probability of majoring in liberal arts at Georgia Tech given an interest in liberal arts
  - A = person is interested in liberal arts
  - B = person majors in liberal arts
- Assume event probabilities:
  - $P(B) = 622/14,558 = .042$
  - $P(A|B) = .95$
  - $P(A|B^C) = .1$

Bayes' Rule Example

$$P(B|A) = \frac{P(A|B)\ \cdot P(B)}{P(A|B)P(B) + P(A|B^C)P(B^C)}$$

$$= \frac{(.95) \cdot (.042)}{(.95) \cdot (.042) + (.1) \cdot (.958)}$$

$$P(B|A) = .29$$

- Now it is more likely (.29 instead of .042) that this student will major in liberal arts
- But it's still more likely they won't major in liberal arts

How Does This Help Us Modeling?

- First, we have a model for the data
- We form **prior distribution** assumptions about our parameters of interest: p(parameters)
  - This takes into account our prior beliefs about how likely particular things are to be true
- We observe data, so we can use Bayes' rule to calculate:

$$p(parameters|data) = \frac{p(data|parameters) \cdot p(parameters)}{p(data)}$$

  - p(parameters) is just our prior probability distribution
  - p(data|parameters) is just plugging in each data point to our probability distribution
  - p(data) is all the different ways the data could have been generated

Frequentists vs. Bayesians

- Think about estimating β as the mean of a sample
    - Frequentists
        - β is a fixed, unknown parameter
        - When we estimate the mean, this depends on the data and is drawn from a distribution
        - So our estimate is a random variable
        - We think about our estimate as if it is drawn from a distribution
    - Bayesians
        - Before starting, we have some beliefs about the distribution of β
        - Once we see our data, we have a new belief, incorporating the data
        - E.g. We used to think β was normally distributed with mean 1, standard deviation 3, now we think it's normally distributed with mean 2, standard deviation 1.5

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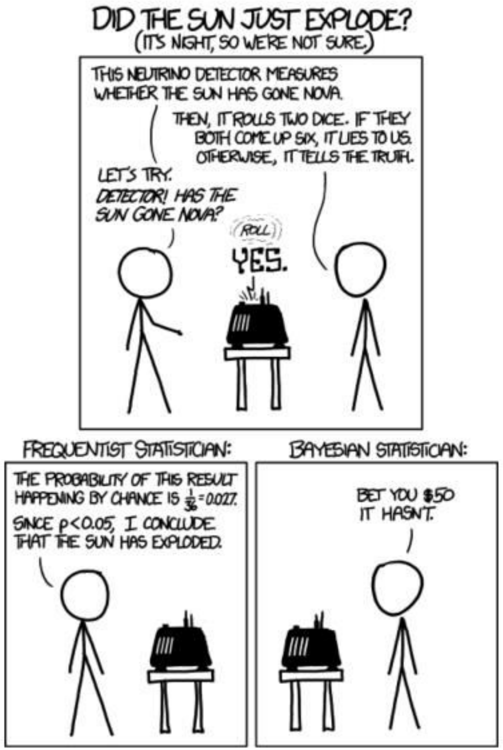
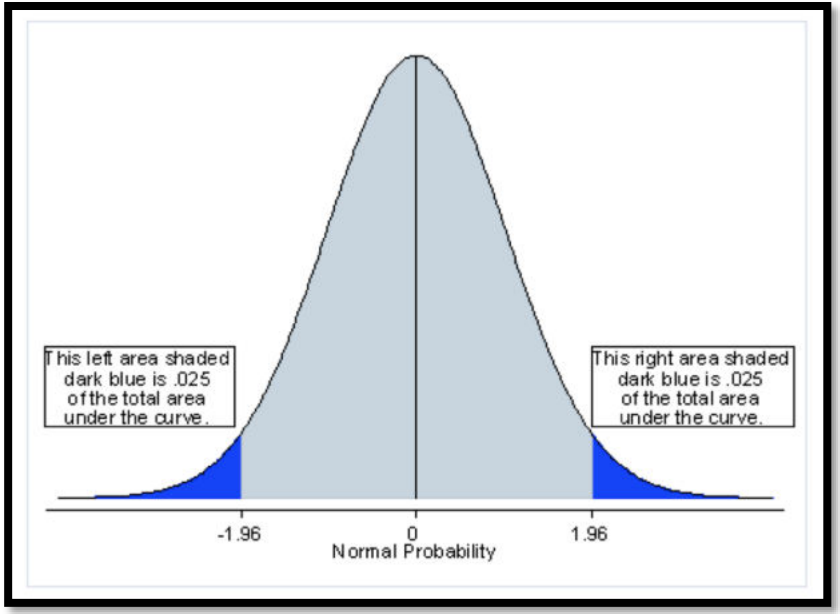
Frequentists vs. Bayesians



Frequentists vs. Bayesians. (n.d.). Retrieved from https://xkcd.com/1132/

The Frequentist Approach: p-Values

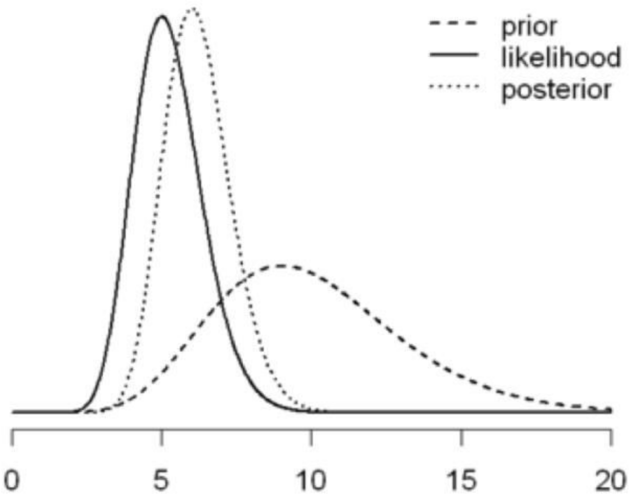
- Frequentists think about $\hat{\beta}$ compared to a specific **null hypothesis**, and reject if it's unlikely that this null hypothesis is true



(n.d.). Retrieved from https://saylordotorg.github.io/text_introductory-statistics/s09-04-areas-of-tails-of-distribution.html

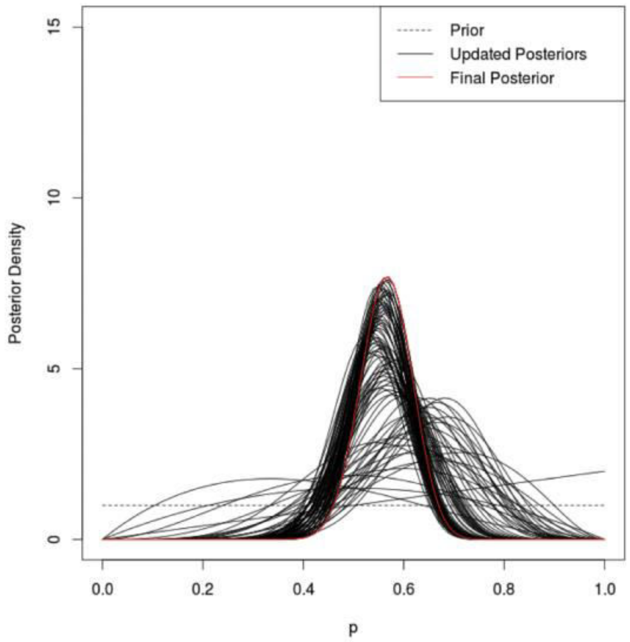
Bayesian Methods: More Intuition

- Bayes rule provides a “compromise” between what you thought beforehand (the “priors”) and what you think now



Matthew s, R. A. J. (n.d.). Methods for Assessing the Credibility of Clinical Trial Outcomes - Robert A. J. Matthew s, 2001. Retrieved from https://journals.sagepub.com/doi/abs/10.1177/009286150103500442

Bayesian Methods: More Intuition



Chivers, C. (2011, September 12). Visualizing Bayesian Updating. Retrieved from https://bayesianbiologist.com/2011/09/10/visualizing-bayesian-updating/

Lesson Summary

- Bayes' rule provides a way to incorporate new information into existing prior beliefs to form updated beliefs
- The rule is based on pure probability theory
- Bayesians have some beliefs about the distribution of β
    - Once seeing the data, we have a new belief, incorporating the data

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