CS4780 Final

Spring 2018

NAME:	
Net ID:	
Email:	

1 [??] General Machine Learning

Please identify if these statements are either True or False. Please justify your answer **if false**. Correct "True" questions yield 1 point. Correct "False" questions yield two points, one for the answer and one for the justification.

1. (**T/F**)If we have a validation set, we do not need to do k-fold cross validation for hyperparameter tuning.
   True.

2. (**T/F**)As $n \to \infty$, the 1-NN error is less than thrice the error of Bayes Optimal Classifier.
   True.

3. (**T/F**) For perceptron algorithms, the order of datapoints do affect the number of misclassifications of each points.
   True.

4. (**T/F**) In MLE, the parameter we want to learn is a random variable.
   False. The model parameter do not have a distribution associated to it.

5. (**T/F**) A "True Bayesian" approach of learning will learn a point estimate of the model parameter.
   False. The model parameter has to be integrated out.

6. (**T/F**) Logistic Regression is a special case of Naive Bayes classifier.
False. Logistic regression do not assume features are independent given the label.

7. (**T/F**) For Logistic Regression, Newton's method will return the global optimum.
True. The loss function of Logistic Regression is convex.

8. (**T/F**) For AdaGrad, each feature has its own learning rate.
True.

9. (**T/F**) Linear regression will perform poorly if the relationship between the label and the features is not linear .
False. With an appropriate mapping, linear regression can model nonlinearity in data.

10. (**T/F**) In a linearly separable dataset, both linear SVM with hard and soft constraints will return the same solution.
False. Depending the C parameter set, linear SVM with soft margin constraints might return a solution that might misclassifiy a few points in the training set whereas linear SVM with hard constraints will never return a solution that will misclassify training points in this case.

11. (**T/F**) l1 regularizer encourage sparse solutions.
True.

12. (**T/F**) For SVM, setting the bias term to zero will increase the variance of the model
False. The bias term in SVM is different from the bias in bias variance decomposition

13. (**T/F**) Linear regression is kernelizable.
True.

14. (**T/F**) The marginal likelihood of Gaussian Process is not Gaussian.
False. By definition, any finite subset of GP has to be Gaussian.

15. (**T/F**) Gaussian Process is not a parametric model .
False. We have to store the full dataset in order to evaluate the posterior.

16. (**T/F**)To use CART models, we have to scale every dimension of the your data

to the range $[0, 1]$.
False, since we split based on features, the relative scales between features do not matter.

17. (**T/F**) In bagging, we use independent and identically distributed datasets to train our ensemble.

False, Each base classifier is trained using a dataset that is formed by sampling the training set with replacement.

18. (**T/F**) For boosting, we can learn multiple base classifiers in parallel.

False, you have to learn each base classifier sequentially

19. (**T/F**) For AdaBoost, the labels has to be $\{-1, +1\}$.

True.

20. (**T/F**) Deep learning allows us to implicitly learn a fixed high dimensional feature mapping

False, NN is essentially a feature mapping that has to be learned

2 [21] Bias Variance / Model Selection

1. Hyperparameter tuning is crucial in creating a powerful model. State two methods that we can use to do hyperparameter tuning for any models. 1. Telescopic search using k-fold cv or validation set 2. Bayes Opt

2. Suppose we are doing Gaussian Process regression. Unlike most of the models, there is a special method we can use to tune the model's hyperparameters. Explain how we could tune the hyperparameters of the Gaussian Process regression model. Optimize the marginal likelihood with respect to the hyperparameters.

3. Suppose we train a SVM classifier on n training data. After training, we obtain k support vectors and the training error (0-1 classification error) is zero.

   (a) Suppose we remove one non-support vector from the training set and train another SVM on the remaining $n-1$ training data. Then we test our new model on the removed point. What will the error be when we apply the SVM to the removed point? 1. We get the same model as we initally trained since non support vector are not needed to recover the initial model.

   (b) Suppose we remove one support vector from the training set, train another SVM model and test the new SVM model on the removed support vector. Will the error be similar to (a)? Why or why not? Nope. The error could be zero or one. Since we remove a support vector, we essentially have a model that is different from what we have initially and we cannot really conclude whether the model will correctly the removed support vector.

(c) Suppose we perform Leave-One-Out Cross Validation on the training set and produce an estimate of the real test loss. Give an upper bound of the test loss in terms of n and k. $testloss < \frac{k}{n}$

(d) Does SVM generalize better when we increase the number of training points? Yes. As $n \to \infty$, $testloss \to 0$

(e) Does SVM generalize better when we have a large number of support vectors?No. As $k \to n$, $testloss \to 1$

4. (13) For each of the following scenarios, determine if the model has low/high bias and variance. Explain your choice.

   (a) SVM with linear and RBF kernel.
   Linear: high bias, low variance
   RBF: low bias, high variance

   (b) Gaussian Naive Bayes, logistic regression
   Gaussian Naive Bayes: high bias, low variance
   logistic regression: low bias, high variance

3 [21] Kernel Methods

1. Being Cornellians, your friends heard that the perceptron algorithm was invented at Cornell. They want to try classifying the following dataset using the perceptron algorithm.

Order	$\mathcal{X}$	$\mathcal{Y}$
1	$[-1, 0]^T$	$+1$
2	$[0, 0]^T$	-1
3	$[1, 0]^T$	$+1$

   (a) Your friends tried running the perceptron algorithm multiple times and the algorithm did not converge. They asked you for help. Explain to them why the algorithm would not converge on this dataset. The datapoints are not linearly separable.

   (b) Your friends were convinced that the linear perceptron algorithm would not converge on this dataset and were very disappointed. However, equipped with the knowledge you learn in CS4780/5780, you know you can kernelize the perceptron algorithm to make it a more powerful algorithm. Suppose the kernel function is k. Fill in the missing pieces of the kernelized perceptron algorithm below:
   Solution in HW8.

Algorithm 1: Kernelized Perceptron

```
Initialize α⃗ = 0⃗ ;
while TRUE do
    m = 0 ;
    for (x_i, y_i) ∈ D do
        if _____ then
            _____;
            _____;
        end
    end
    if m = 0 then
        break
    end
end
```

(c) After looking at the dataset, you suggest your friends to use the kernelized perceptron using the quadratic kernel

$$k(\vec{x}_1, \vec{x}_2) = (\vec{x}_1^T \vec{x}_2 + 1)^2$$

Your friends run the kernelized perceptron and get $\vec{\alpha} = [1, 2, 1]^T$. However, when they applied the model to the training set, they could not get 100% accuracy. Again, knowing that you are the pro in ML, they ask for your help again. After talking to them, you realize that they might not have run the algorithm till convergence. So, you ran the kernelized perceptron algorithm with $\vec{\alpha}$ initialized to $[1, 2, 1]^T$. What is the α you get after you run the algorithm till convergence? $\alpha = [1, 3, 1]^T$. Students only need to go through the dataset twice - one for the update, another one for checking zero training error.

4 [21] CART

1. Suppose a classification tree with depth 1 can correctly classify our data. Is this classification tree a linear model? Explain your reasoning. Yes. The model has a linear decision boundary $x_i + c_i = 0$

2. CART models are highly flexible. Suggest three ways to reduce its variance. Fix the tree depth, bagging, ensembling

3. Under what condition will CART models be non-parametric. When CART models are trained to full depth, they are non-parametric since the depth of the CART models scales as a function of the training data.

4. Consider the following dataset: If we grow a classification tree using ID3 algorithm and Gini Impurity, what is the depth of the final tree? Depth = 1. Decision rule: Go to Class = Always $\Rightarrow$ Will Pass 4780

Feature			Label
Go to Class	Do Projects and HWs	Pass Final Exam	Will Pass CS4780
Always	Yes	No	Yes
Always	No	Yes	Yes
Always	No	No	Yes
Occasionally	No	Yes	No
Occasionally	No	No	No
Occasionally	No	Yes	No

5 [x points] AdaBoost

In this question, we are going to explore AdaBoost using classification tree of **depth 1** as our base classifier. For your reference, we have included the following pseudo-code for AdaBoost:

Algorithm 2: AdaBoost (Assume n training points.)

Initialize $H_0 = 0; \forall i : w_i = \frac{1}{n}$;
for *t = 0: T - 1* **do**
  h = the base classifier that minimizes the weighted classification error ϵ_t :

$$\epsilon = \sum_{i:h(x_i) \neq y_i} w_i$$

  if $\epsilon < \frac{1}{2}$ **then**
    $\alpha = \frac{1}{2} \ln(\frac{1-\epsilon}{\epsilon})$;
    $H_{t+1} = H_t + \alpha h$;
    $\forall i : w_i \propto w_i e^{-\alpha h(x_i) y_i}$;
  else
    return H_t
  end
  return H_T
end

Consider the dataset in figure TODO

1. (x points) In the figure above, draw the decision boundary of first classification tree that AdaBoost would choose. Please indicate the positive and negative side of the decision boundary. (Note: Throughout the whole question, please assume that your tree would split at integer values for each branch.) Any horizontal line between $x_2 = 1$ and $x_2 = 3$ will minimize the weighted error. However, due to the constraint we impose, $x_2 = 2$ is the only acceptable solution.

2. (x points) What is the weighted classification error **before** weights are renormalized. $\epsilon = \frac{1}{5}$

3. (x points) What is the weight α of the first classification tree? $\alpha = \ln 2$

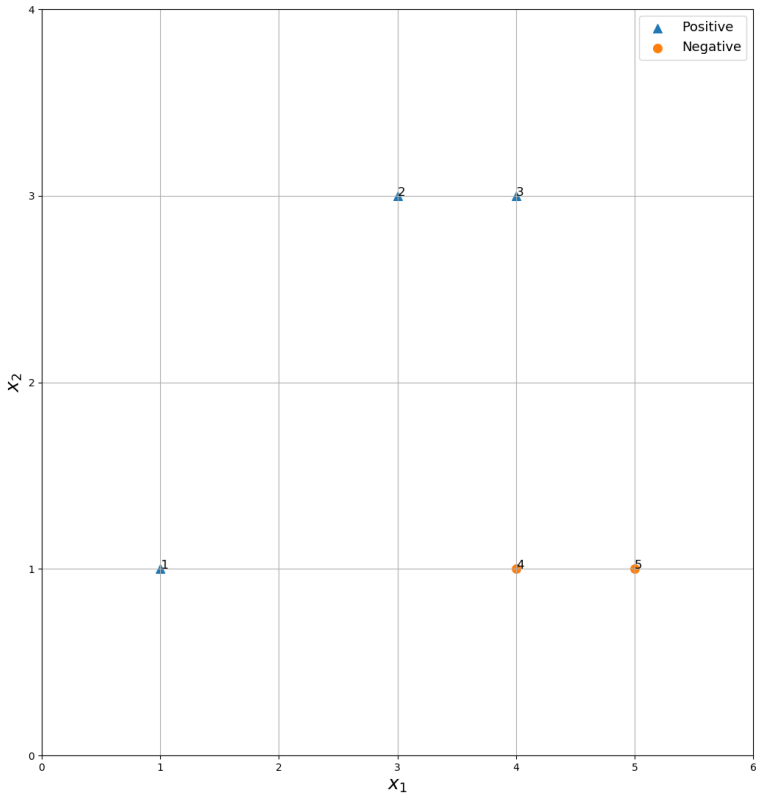


Figure 1: Dataset for AdaBoost

4. (x points) Circle the point that has the highest weight after renormalizing the weight. The first point since it is misclassified.

5. (x points) What is the weighted classification error of the first classification tree **after** weights are renormalized. 0.5

6. (x points) Draw the second classification tree that the algorithm would pick. Again, indicate the positive side and negative side of the decision boundary. Any horizontal line between $x_2 = 1$ and $x_2 = 3$ will minimize the weighted error. However, due to the constraint we impose, $x_2 = 2$ is the only acceptable solution.

6 [21] Deep Learning

1. Suppose you have a filter of size $k \times k$ and stride s. When you apply this filter to a $n \times n$ input, what is the dimension of the output feature map? $(\lfloor \frac{n-k}{s} \rfloor + 1) \times (\lfloor \frac{n-k}{s} \rfloor + 1)$

2. Suppose you have

$$input = \begin{bmatrix} 4 & 5 & 1 \\ 4 & 0 & 2 \\ 3 & 4 & 0 \end{bmatrix}$$

and

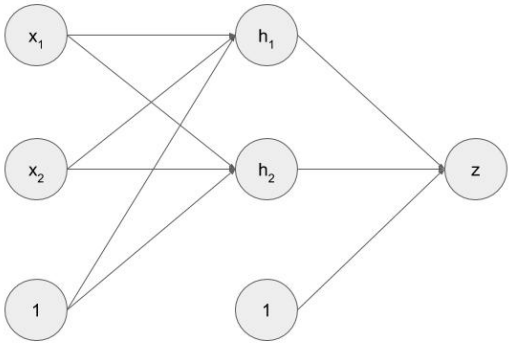
$$filter = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

What is the output of applying the filter to the input with stride 1?

$$\begin{bmatrix} 4 & 7 \\ 8 & 0 \end{bmatrix}$$

Yes, the answer is 4780!

3. Consider the following fully connected RELU network.



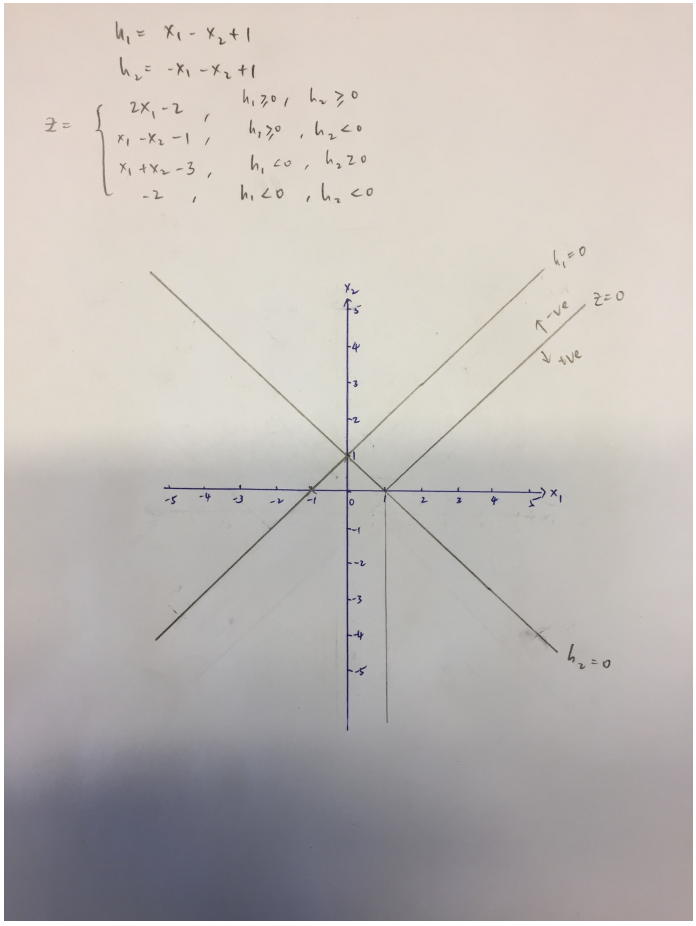
where

$$\begin{bmatrix} h_1 \\ h_2 \end{bmatrix} = \begin{bmatrix} w_{11} & w_{12} & w_{13} \\ w_{21} & w_{22} & w_{23} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ 1 \end{bmatrix}$$

$$z = \begin{bmatrix} v_1 & v_2 & v_3 \end{bmatrix} \begin{bmatrix} f(h_1) \\ f(h_2) \\ 1 \end{bmatrix}$$

$$t = z$$

where $f(h_i) = max(0, h_i)$ and t is the output of the network. Suppose $\begin{bmatrix} w_{11} & w_{12} & w_{13} \\ w_{21} & w_{22} & w_{23} \end{bmatrix} = \begin{bmatrix} 1 & -1 & 1 \\ -1 & -1 & 1 \end{bmatrix}$ and $\begin{bmatrix} v_1 & v_2 & v_3 \end{bmatrix} = \begin{bmatrix} 1 & -1 & -2 \end{bmatrix}$. Draw the decision boundary of the network, namely, $t = 0$ in the range $[-5, 5] \times [-5, 5]$. Please indicate the positive and negative side of the boundary.



4. We are going to use the network in (3) to do regression. Assume that our loss function is

$$l(y, t) = (t - y)^2$$

Show that for a single training example, $x = [x_1, x_2]$

$$\frac{\partial l}{\partial v_i} = 2(t - y) f(h_i) \text{ for } i \neq 3$$
$$\frac{\partial l}{\partial v_3} = 2(t - y)$$
$$\frac{\partial l}{\partial w_{ij}} = 2(t - y) v_i \mathbb{I}(h_i > 0) x_j \text{ for } j \neq 3$$
$$\frac{\partial l}{\partial w_{i3}} = 2(t - y) v_i \mathbb{I}(h_i > 0)$$

where $\mathbb{I}(\cdot)$ is the indicator function.

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Please do not write on this page. For administrative purposes only.

T/F	
BV	
Kernels	
CART	
ENSEMBLE	
DL	
TOTAL	