5.28

5.28 (a) $\pi_1 = 0.2$ $\quad \pi_2 = 0.3$ $\quad PWR = 0.8$ $\quad \beta = 1 - PWR = 0.2$ $\quad \alpha = 0.1$.

Equal sample size $= n_1 = n_2 = \dfrac{(z_{\alpha/2} + z_\beta)^2 \left(\pi_1(1-\pi_1) + \pi_2(1-\pi_2)\right)}{(\pi_1 - \pi_2)^2} \approx 228$.

(b) (i) Same Equation but $PWR = 0.9$ $\therefore \beta = 1 - PWR = 0.1$

$$n_1 = n_2 = \frac{(z_{\alpha/2} + z_\beta)^2 \left[\pi_1(1-\pi_1) + \pi_2(1-\pi_2)\right]}{(\pi_1 - \pi_2)^2} \approx 316.$$

(ii) α changed to 0.05. Same Equation $n_1 = n_2 \approx 291$

(iii) α, PWR changed. $\beta = 0.1$ $\quad \alpha = 0.05$. $\quad n_1 = n_2 \approx 389$.

6.1

6.1 a. $\log \dfrac{\hat{\pi}_R}{\hat{\pi}_D} = \log \dfrac{\hat{\pi}_R}{\pi_2} - \log \dfrac{\hat{\pi}_D}{\pi_2} = -2.3 + 0.5x$.

Slope here: log odds ratio of Republican to Democrat is 0.5 as one unit changed.

b. $\hat{\pi}_R > \hat{\pi}_D \Rightarrow \log \dfrac{\hat{\pi}_R}{\hat{\pi}_D} > 0$ $\quad \therefore x > 4.6$.

c. $\hat{\pi}_2 = \dfrac{1}{1 + e^{3.3-0.2x} + e^{1+0.3x}}$

6.3

a. x1: lake George; x2: lake Hancock; x3: lake Oklawaha; x4: size.

y=1: $log\frac{\pi_1}{\pi_5} = 0.0564 + 1.5164x_1 + 0.6902x_2 + 1.5107x_3 + 0.3316x_4$

y=2: $log\frac{\pi_2}{\pi_5} = 1.0875 + 0.3944x_1 - 2.0901x_2 + 1.3260x_3 - 1.1267x_4$

y=3: $log\frac{\pi_3}{\pi_5} = -0.6742 - 1.4183x_1 - 1.0023x_2 + 1.0343x_3 + 0.6828x_4$

y=4: $log\frac{\pi_4}{\pi_5} = -1.5796 + 0.4286x_1 + 0.2975x_2 - 0.2303x_3 + 0.9622x_4$

Analysis of Maximum Likelihood Estimates

Parameter		y	DF	Estimate	Standard Error	Wald Chi-Square	Pr > ChiSq
Intercept		1	1	0.0564	0.5022	0.0126	0.9107
Intercept		2	1	1.0875	0.4700	5.3530	0.0207
Intercept		3	1	-0.6742	0.6506	1.0739	0.3001
Intercept		4	1	-1.5796	0.7926	3.9717	0.0463
lake	George	1	1	1.5164	0.6214	5.9541	0.0147
lake	George	2	1	0.3944	0.6263	0.3965	0.5289
lake	George	3	1	-1.4183	1.1890	1.4229	0.2329
lake	George	4	1	0.4286	0.9383	0.2087	0.6478
lake	Hancock	1	1	0.6902	0.5597	1.5207	0.2175
lake	Hancock	2	1	-2.0901	0.7184	8.4653	0.0036
lake	Hancock	3	1	-1.0023	0.8297	1.4593	0.2270
lake	Hancock	4	1	0.2975	0.8342	0.1272	0.7214
lake	Oklawaha	1	1	1.5107	0.7532	4.0229	0.0449
lake	Oklawaha	2	1	1.3260	0.7468	3.1527	0.0758
lake	Oklawaha	3	1	1.0343	0.8402	1.5154	0.2183
lake	Oklawaha	4	1	-0.2303	1.3005	0.0313	0.8595
size		1	1	0.3316	0.4483	0.5471	0.4595
size		2	1	-1.1267	0.5049	4.9790	0.0257
size		3	1	0.6828	0.6514	1.0988	0.2945
size		4	1	0.9622	0.7127	1.8227	0.1770

b.

length ≤ 2.3, $x_3 = 0$, $\therefore \pi \approx 0.26$

length > 2.3, $x_3 = 1$ $\therefore \pi \approx 0.46$.

$$\pi_3(x) = \frac{e^{\alpha + \beta_3 x}}{1 + \Sigma e^{\alpha + \beta x}}$$

$$= \frac{e^{0.0564 + 0.3316x_4 + 1.5107x_3}}{1 + e^{0.0564 + 0.3316x_4 + 1.5107x_3} + e^{1.0875 - 1.1267x_4 + 1.3260x_3} + e^{-0.6742 + 0.6828x_4 + 1.0343x_3} + e^{-1.5796 + 0.9622x_4 + 0.2303x_3}}$$

Interpretation: As the length increased from below 2.3 to above 2.3, the probability of the choice on fish is increased from 0.26 to 0.46.

6.8

a.

Analysis of Maximum Likelihood Estimates						
Parameter		DF	Estimate	Standard Error	Wald Chi-Square	Pr > ChiSq
Intercept	4	1	-2.4221	0.3276	54.6609	<.0001
Intercept	3	1	-1.3713	0.3059	20.0903	<.0001
Intercept	2	1	0.1960	0.2947	0.4424	0.5060
trt		1	0.5807	0.2119	7.5131	0.0061
male		1	0.5414	0.2953	3.3619	0.0667

Deviance and Pearson Goodness-of-Fit Statistics				
Criterion	Value	DF	Value/DF	Pr > ChiSq
Deviance	5.5677	7	0.7954	0.5910
Pearson	5.3527	7	0.7647	0.6170

Model: $logit(\pi) = \alpha_i + 0.5807x_1 + 0.5414x_2 \;\; x_1: trt, x_2: gender\,;\; \alpha_2 = 0.1960;\; \alpha_3 = -1.3713;\; \alpha_4 = -2.4221$

Interpretation: The log odds of any in sequential trt is 0.5807 to the alternating trt.

As p-value here is 0.5910 > 0.05, we fail to reject H_0. So the null model fits well.

b.

Analysis of Maximum Likelihood Estimates						
Parameter		DF	Estimate	Standard Error	Wald Chi-Square	Pr > ChiSq
Intercept	4	1	-2.6978	0.4260	40.1002	<.0001
Intercept	3	1	-1.6484	0.4102	16.1462	<.0001
Intercept	2	1	-0.0770	0.3986	0.0373	0.8468
trt		1	1.0786	0.5498	3.8490	0.0498
male		1	0.8646	0.4309	4.0268	0.0448
trt*male		1	-0.5906	0.5935	0.9901	0.3197

Deviance and Pearson Goodness-of-Fit Statistics				
Criterion	Value	DF	Value/DF	Pr > ChiSq
Deviance	4.5209	6	0.7535	0.6066
Pearson	4.4151	6	0.7359	0.6207

Model: $logit(\pi) = \alpha_i + 1.0786x_1 + 0.8846x_2 - 0.5906x_1x_2 \;\; x_1: trt, x_2: gender\,; \alpha_2 = -0.0770;\; \alpha_3 = -1.6484;\; \alpha_4 = -2.6978$

If male: $x_2 = 1$: $logit(\pi) = \alpha_i + 0.4880x_1 + 0.8846x_2$

Interpretation: The log odds of any in sequential trt is 0.4880 to the alternating trt when gender is male.

If female: $x_2 = 0$: $logit(\pi) = \alpha_i + 1.0786x_1 + 0.8846x_2$

Interpretation: The log odds of any in sequential trt is 1.0786 to the alternating trt when gender is female.

As p-value here is 0.6066 > 0.05, we fail to reject H_0. So the null model fits well.

c. Diff = (Dev1-Dev2)/(df1-df2)=(5.5677-4.4151)/(7-6)=1.05 ~ chi-square distribution. Therefore, the p-value is >0.05, so we fail to reject H_0 here, there is no difference between these two models. So the null model (no interaction) fits well enough.

6.11

a.

Analysis of Maximum Likelihood Estimates						
Parameter		DF	Estimate	Standard Error	Wald Chi-Square	Pr > ChiSq
Intercept	1	1	-2.5795	0.5618	21.0840	<.0001
Intercept	2	1	-0.8940	0.3603	6.1569	0.0131
Intercept	3	1	2.0780	0.4206	24.4101	<.0001
incscore		1	-0.0444	0.0185	5.7372	0.0166
male		1	-0.0259	0.4274	0.0037	0.9516

Effect of income: As income score increased by one, the log odds of that is -0.0444 as gender controlled.

Goodness of fit: As row margins from the table 6.12 is small, so we cannot use the Pearson's test or deviance to test goodness of fit.

b.

Analysis of Maximum Likelihood Estimates						
Parameter		DF	Estimate	Standard Error	Wald Chi-Square	Pr > ChiSq
Intercept	4	1	-2.0582	0.4197	24.0511	<.0001
Intercept	3	1	0.9154	0.3616	6.4091	0.0114
incscore		1	0.0435	0.0186	5.4991	0.0190
male		1	0.0247	0.4286	0.0033	0.9541

Effect of income: As income score increased by one, the log odds of that is 0.0435 as gender controlled, which the correspond direction changed from negative to positive.

Goodness of fit: As row margins is still small, have the same conclusion as above, not applied.

c. Model w/t gender

Deviance and Pearson Goodness-of-Fit Statistics				
Criterion	Value	DF	Value/DF	Pr > ChiSq
Deviance	6.7494	8	0.8437	0.5639
Pearson	5.7584	8	0.7198	0.6743

w/ gender

Deviance and Pearson Goodness-of-Fit Statistics				
Criterion	Value	DF	Value/DF	Pr > ChiSq
Deviance	13.9519	19	0.7343	0.7865
Pearson	14.3128	19	0.7533	0.7652

Diff = (13.8519-6.7494)/(19-8) = 7.2 ~ chi-sqaured with degree of freedom of 11. Therefore, we cannot reject null hypothesis. So the model w/t gender is good, we can drop the gender.