Are People Rational?

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July 6, 2025

I will discuss 3 of my recent papers

- Disequilibrium Play in Tennis (with Axel Anderson, Jeremy Rosen and Kin-ping Wong)
- Explaining Early Bidding in Informationally Restricted Ascending Bid Auctions (with Sungjin Cho and Harry Paarsch)
- Who is More Bayesian? Humans or ChatGPT? (with Tianshi Mu, Pranjal Rawat, Qixuan Zhong and Chengjun Zhang)

Can we do "structural behavioral economics"?

- That is, can we relax the rationality assumption? **Answer:** yes, but strong assumptions are required due to the identification problem
- The *structure* is the triple $\{\beta, u, p\}$ where β is discount factor, u is utility function, and the transition probability $p(s'|s, a)$ are beliefs
- Under *rational expectations* we can estimate p from the data on (s', s, a), so only $\{\beta, u\}$ are the remaining "unknowns"
- If we impose optimality and relax rationality, then the CCP is $P(a|s, \{\beta^*, u^*, p^*\})$. Then subjective beliefs p^* might not equal p, the "objective" or "rational" belief
- Identification problem in discrete state case: is there a unique solution to the system of $(|A| - 1)|S|$ equations in the $1 + |A||S| + |S|^2|A|$ unknowns $\{\beta, u, p\}$?

$$P(a|s, \{\beta^*, u^*, p^*\}) = P(a|s, \{\beta, u, p\})$$

- Answer generally no: more unknowns than equations, so we have impose prior assumptions to identify subjective beliefs, or else use data that *elicits* beliefs

Paradox of the "Information Age"

Justin Smith, *Irrationality A History of the Dark Side of Reason* 2019

It is a great paradox of the present age that, even though the totality of all human learning is more accessible than ever before in history, indeed billions of us on earth can now easily access it with a special device we carry in our pockets, nonetheless false beliefs are as epidemic as ever.

- Of course irrationality/superstitition occurred well before the "information age" but modern information technology may paradoxically exacerbate false/irrational beliefs, overwhelming us with "too much information" i.e. *information pollution*
- There is increasing concern that AI and "virtual reality" will only dramatically magnify our current problems of misinformation and the risk it poses for contagious mass irrationality. Examples: automatic creation of fake news and "deepfake" images, even AI-generated newscasters.

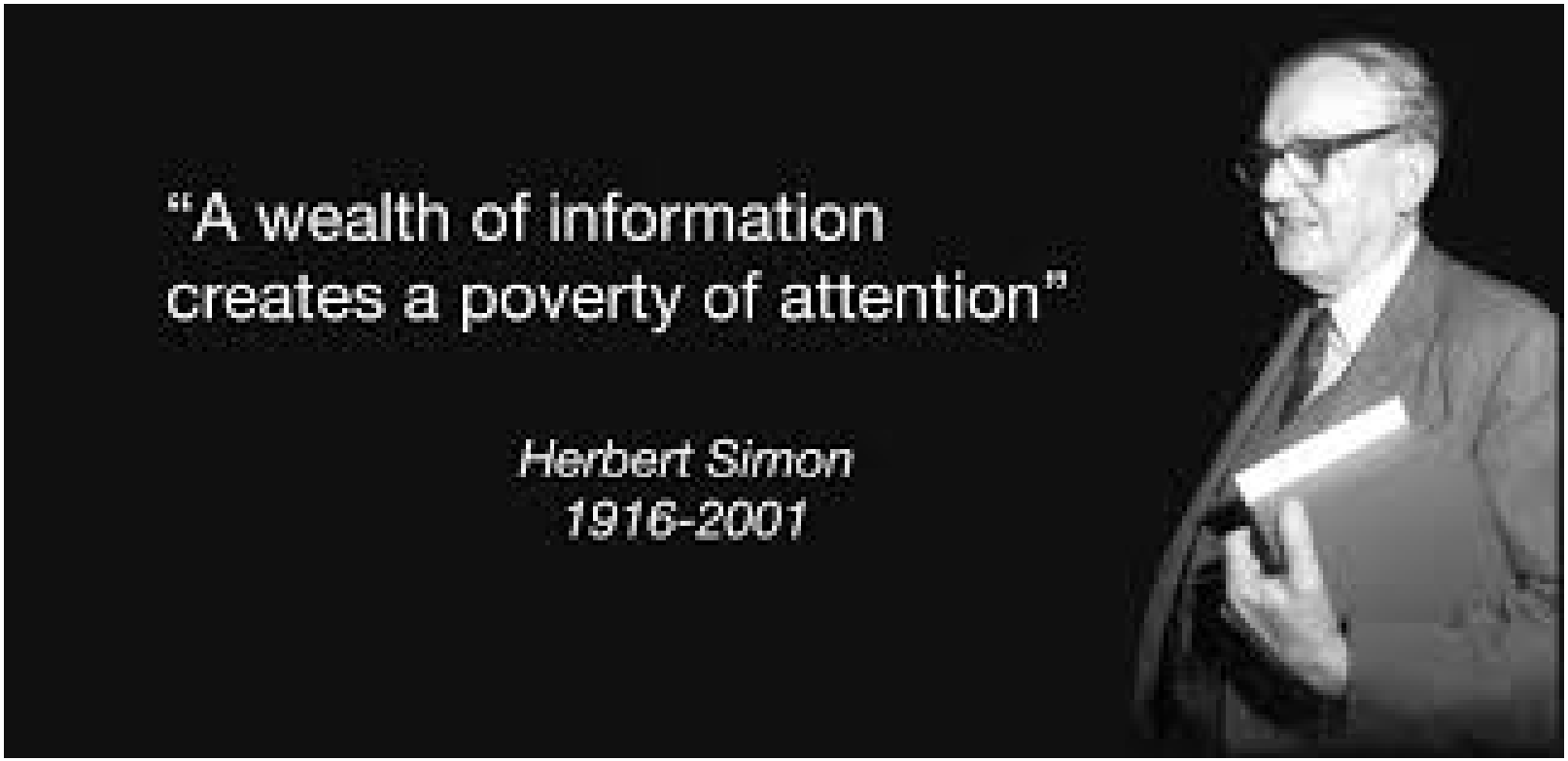
Herbert Simon: people are "boundedly rational"

Herbert Simon's definition of "bounded rationality" (*Wikipedia*)

Bounded rationality is the idea that rationality is limited by computational constraints and limited information and knowledge. In such situations, rational individuals will select a decision that is satisfactory rather than optimal. Decision-makers, in this view, act as *satisficers*, seeking a satisfactory solution using heuristics or intelligent "rules of thumb" rather than attempting to compute an optimal solution.

- Simon's notion is intended to be a more realistic alternative to the idea that people are *perfectly rational* (also known as *homo economicus*) that is so widely adopted among economists.
- "by the middle 1950s, a theory of bounded rationality had been proposed as an alternative to classical omniscient rationality [and] a significant number of empirical studies had been carried out that showed actual business decision making to conform reasonably well with the assumptions of bounded rationality but not with the assumptions of perfect rationality." (from Simon's Nobel Prize speech)

Herbert Simon



It takes energy to be rational

- The human brain operates on approximately 20 watts of energy, and even though it is 2% of our body weight, it takes 20% of our overall energy consumption
- LLMs models, on the other hand, require immense amounts of energy. According to Google AI "It's estimated that training a large language model like GPT-3 can consume up to 10 gigawatt-hours, and daily queries can cost around 1 GWh"
- Maybe evolution did not "design" us to be perfectly individually rational

Hutchinson and Meyer 1994

From a broader perspective, however, one can argue that optimal solutions are known for a relatively small number of similar, well-specified problems whereas humans evolved to survive in a world filled with a large and diverse set of ill-specified problems. Our 'suboptimality' may be a small price to pay for the flexibility and adaptiveness of our intuitive decision processes.

Misspecified models and biased learning

- The standard model for "rational learning" is Bayesian updating.
- The assumption is that you have the right likelihood function and a possibly subjective prior, and update this via Bayes Rule.
- Bayesian DP adds an exploration/exploitation tradeoff: if it is costly to learn more, sometimes it is optimal to remain less than fully informed (example Easley and Kiefer (1988) "Controlling a stochastic process with unknown parameters")
- Frick, Iijima and Ishi (2024) "Welfare Comparisons for Biased Learning" if your "mental model" of the problem is misspecified, your choices will be suboptimal relative to a person with a correctly specified model.
- Insights: 1) misspecified beliefs do not always lead to suboptimal decisions, 2) agents with misspecified likelihoods may outperform agents with correctly specified likelihood but high prior uncertainty

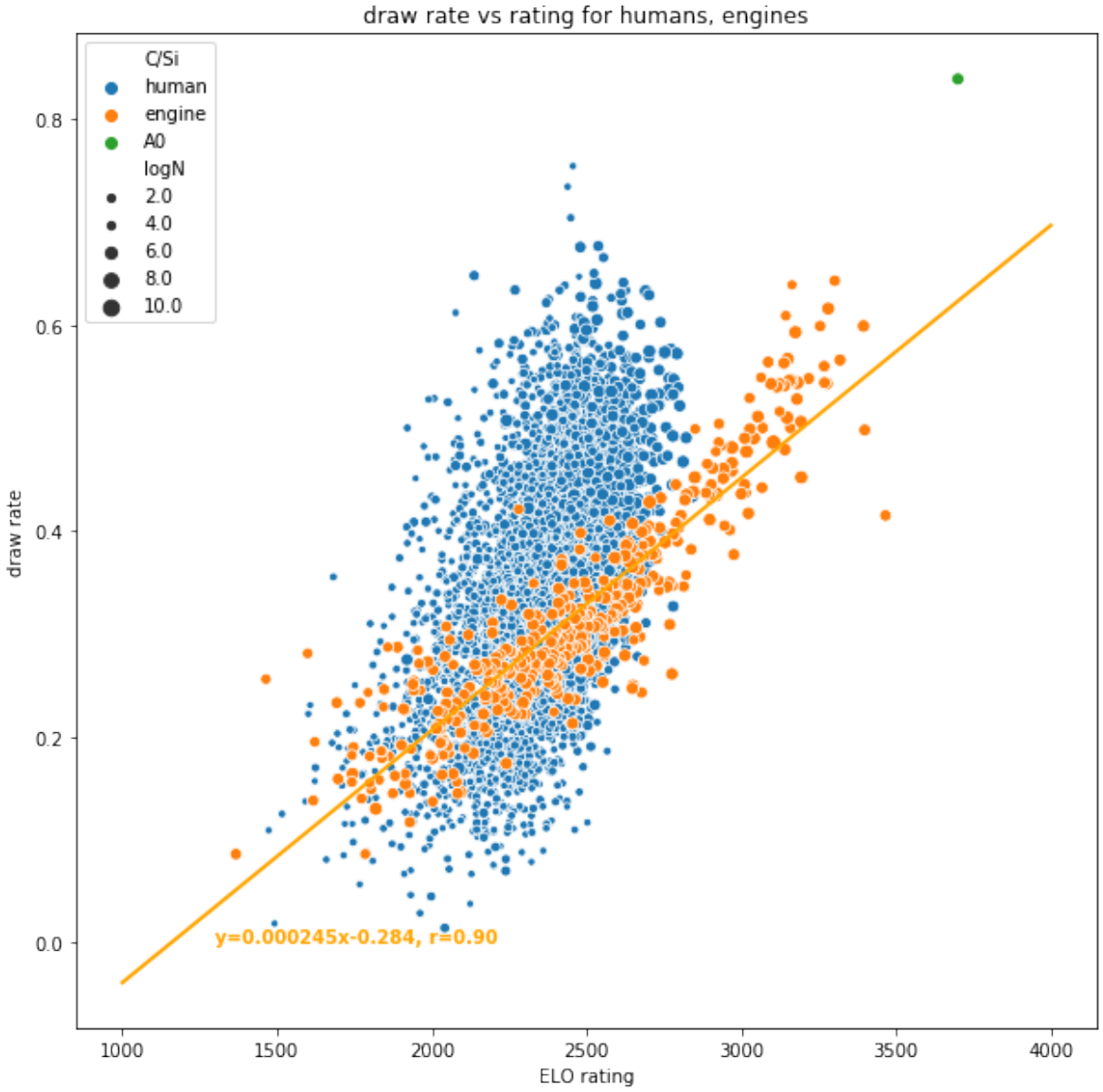
Do humans play perfect chess?

- Do professional chess players (e.g. Magnus Carlson) play perfectly rationally (i.e. play dynamically optimal Nash equilibrium chess strategies)?
- Answer: obviously not! Ever since *Deep Blue* beat Garry Kasparov in 1997, the world's best chess players have been computers!
- But do computers play "perfect" chess? **Answer:** not yet, but according to an analysis by Jeremy Rutman they will get there by 2059.
- **Zermelo's theorem (1913)** *In an alternating move, two player game of perfect information with no chance moves, if both players play "perfectly" (i.e. both use dynamic Nash equilibrium strategies) then there are only 3 possible outcomes:*
  - 1. the game always ends in a draw
  - 2. the first mover always wins
  - 3. the second mover always wins
- Example: tic tac toe played perfectly always ends in a draw

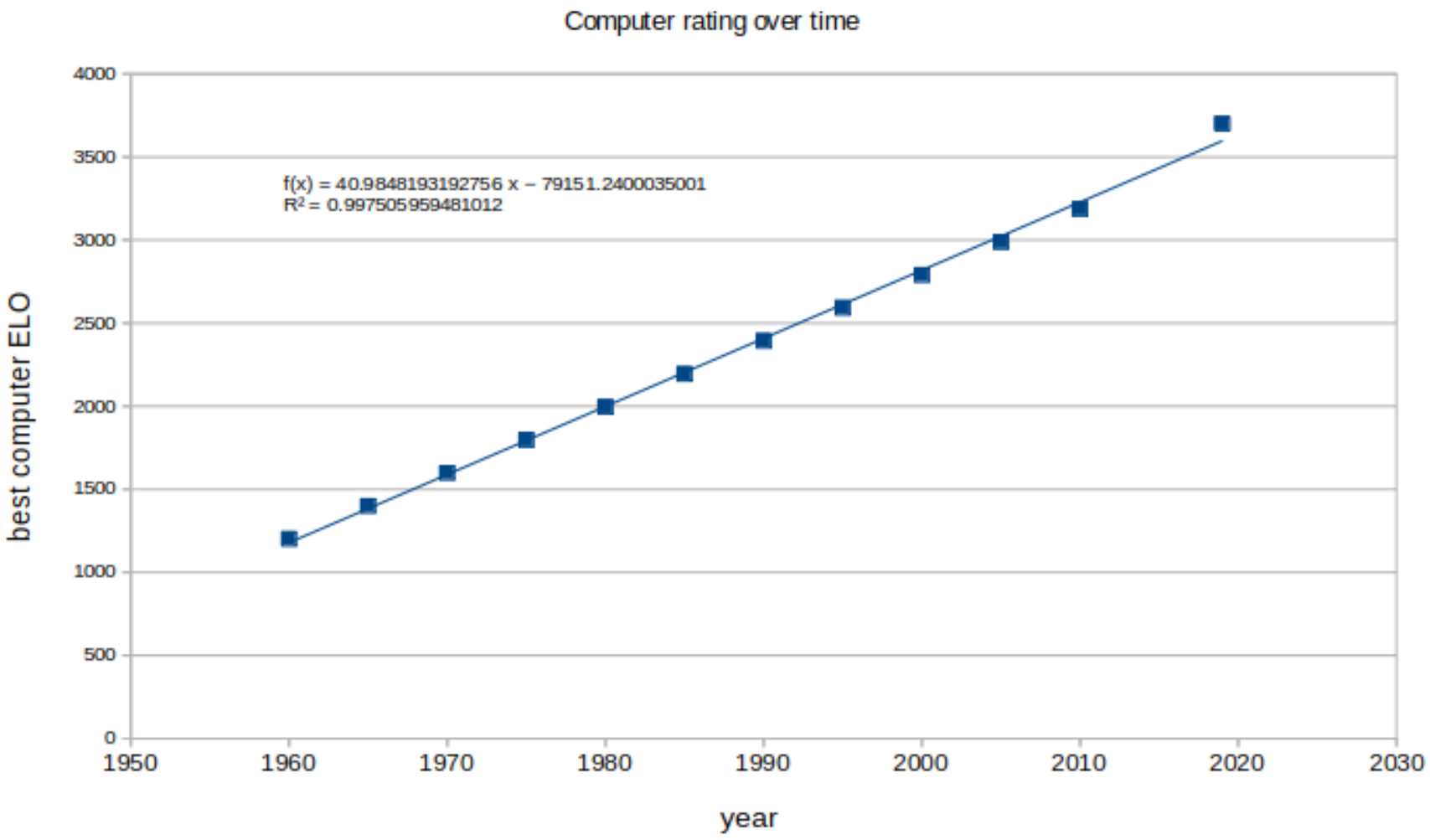
Which outcome holds for perfect chess?

- In 2007 Schaeffer *et. al. Science* solved checkers and showed *that it always ends in a draw.*
- The combinatorics of chess is still too large for us to "solve" it (though end game solutions with incresing numbers of remaining pieces have been solved).
- The empirical evidence suggests that perfectly played chess will also always end in a draw.
- Jeremy Rutman has compiled data on chess outcomes against the ELO score (a standard measure of ability in chess) and shown as the players' ELO scores improve, the chance of a draw increased.
- Given the current technological rate of improvement in ELO score, he predicts that we will achieve "perfect chess" around 2059 with an ELO score of around 5200 (for comparison Magnus Carlson's ELO score is 2840).

Draw rating vs ELO score in chess



Computer chess ability is increasing faster than humans



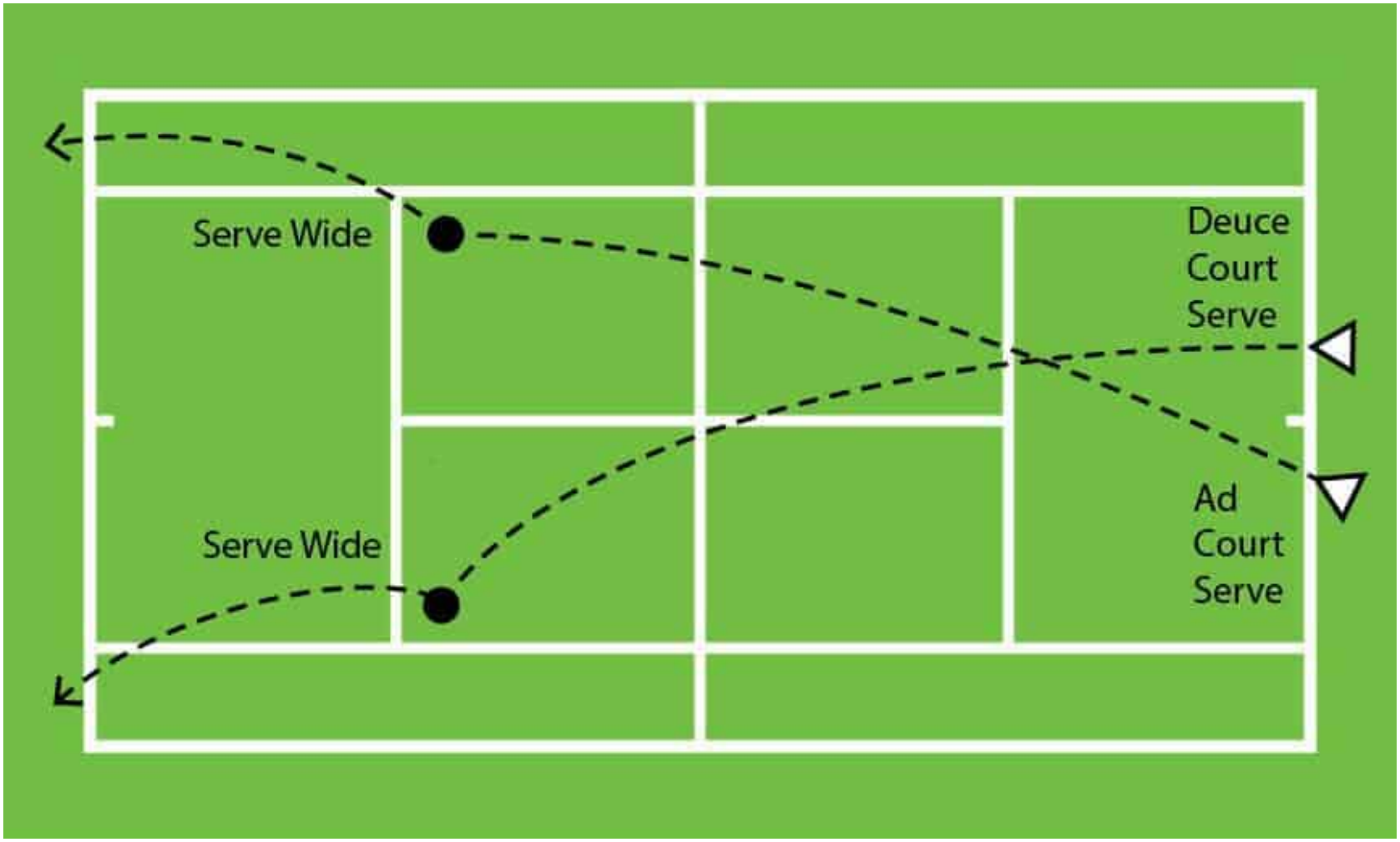
Chess versus tennis

- Both are alternating move, two player constant sum games.
- Does Zermelo's Theorem apply to tennis? No: there is a significant role of "chance" in tennis, both in terms of mixed serve strategies as well as in "imperfect muscle control" (e.g. faulting a first serve)
- Also there is no possibility of a tie in tennis: the rules of tennis lead to only 2 possible outcome of a game: 1) server wins, or 2) server loses.
- Brains vs brawn: unlike chess, which is purely a test of mental ability in reasoning and ability to "look ahead", physical ability plays a huge role in tennis.
- Can game theory acommodate differences in physical ability? Is it possible to quantify the relative roles of mental ability versus physical ability in success in tennis?

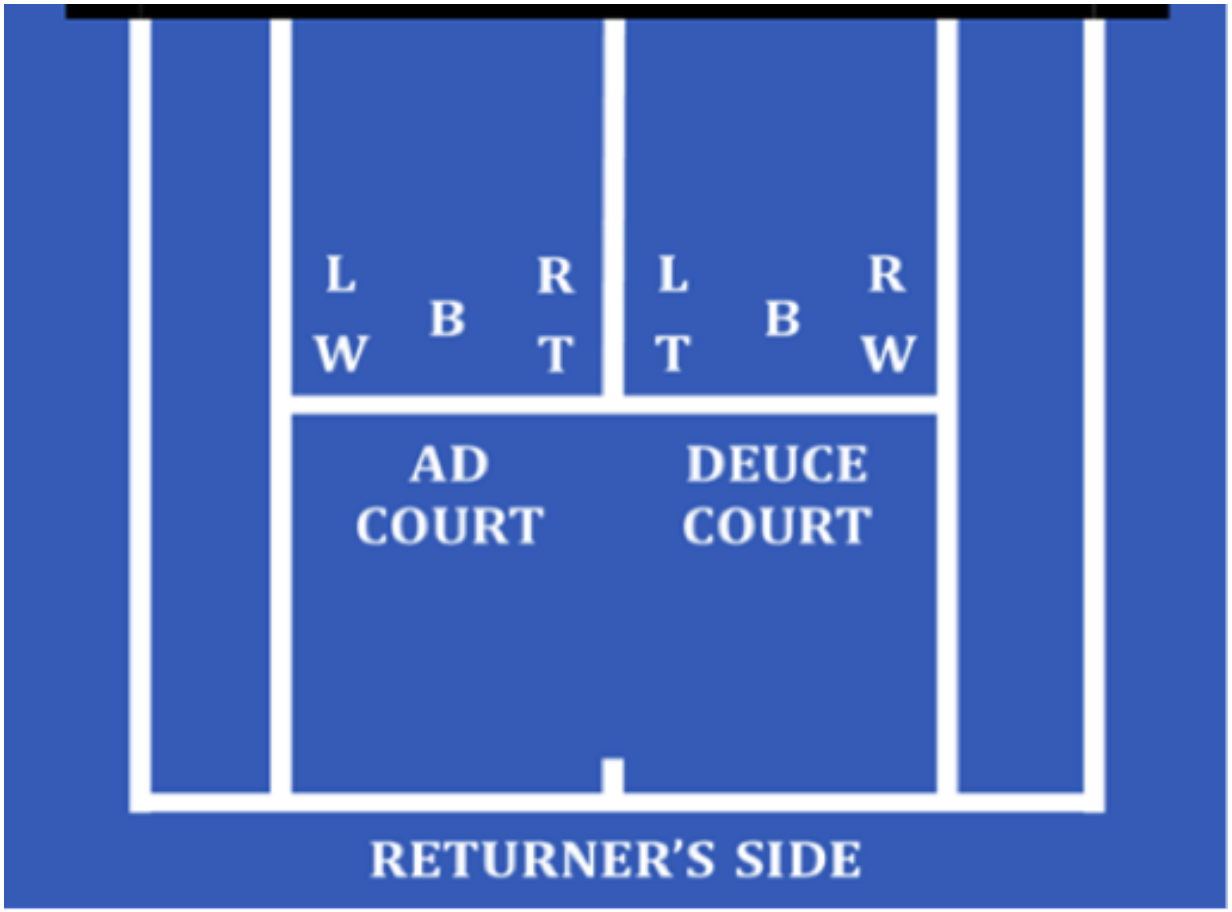
Disequilibrium play in tennis

- Testing whether firm behavior is consistent with MPE is difficult since we don't know their "production function" or their "objective function" (i.e. maximizing stock market value or present discounted value of dividends?)
- I have focused on simpler problems where the technology is much simpler and where the objective is known.
- Example: tennis or chess: objective is to adopt a best response strategy to maximize the probability of winning.
- Rewards to winning are very high and the game is played by professionals so we expect to see Nash/MPE behavior in these circumstances
- Walkers and Wooders (2001) "Minimax Play at Wimbledon" or Palacios-Huerta (2003) "Professionals Play Minimax"

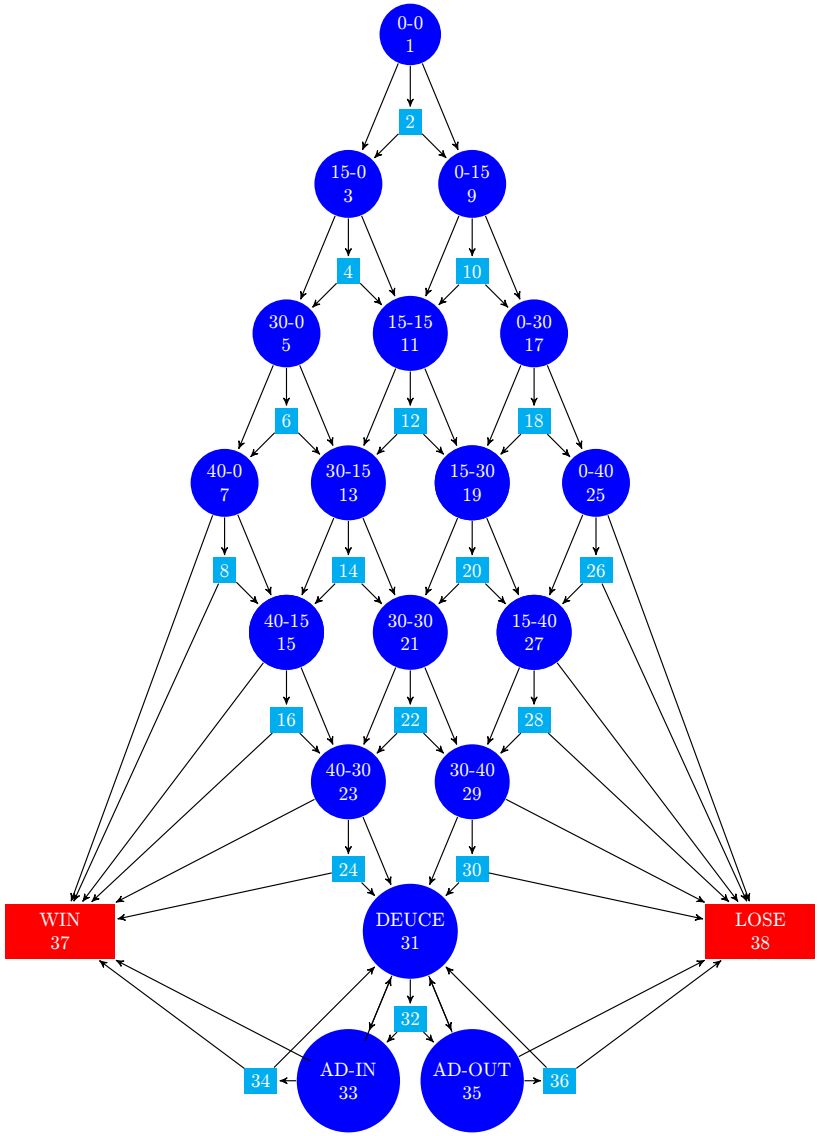
Serve directions: Wide, to the body, and down the T



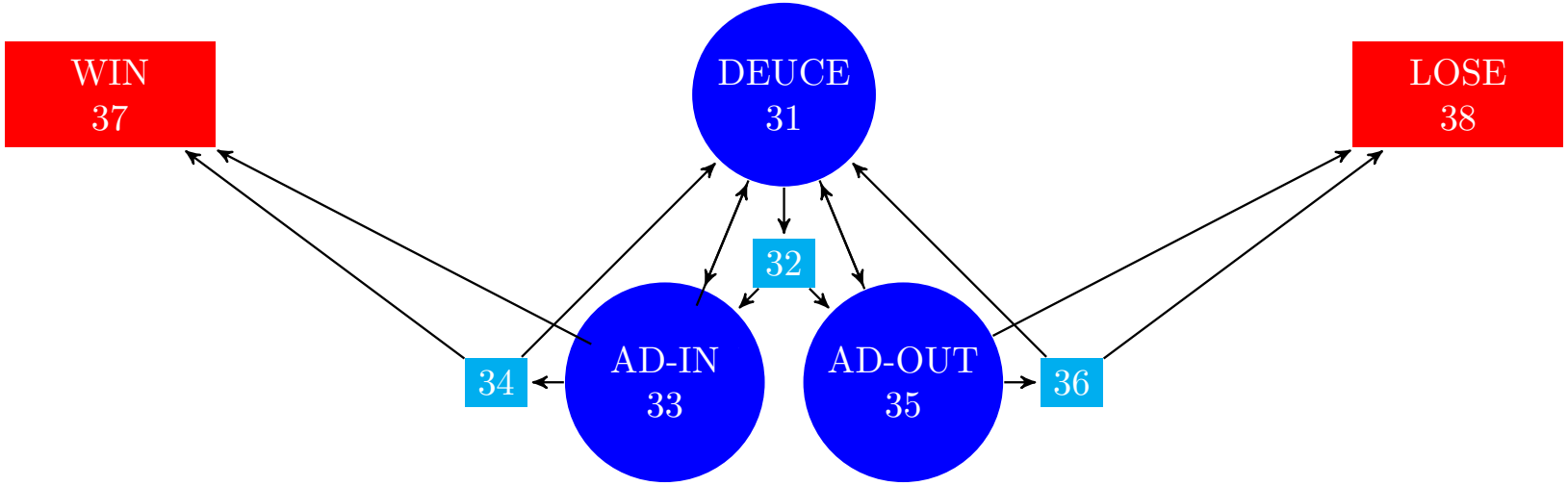
Serve directions from the returner's perspective



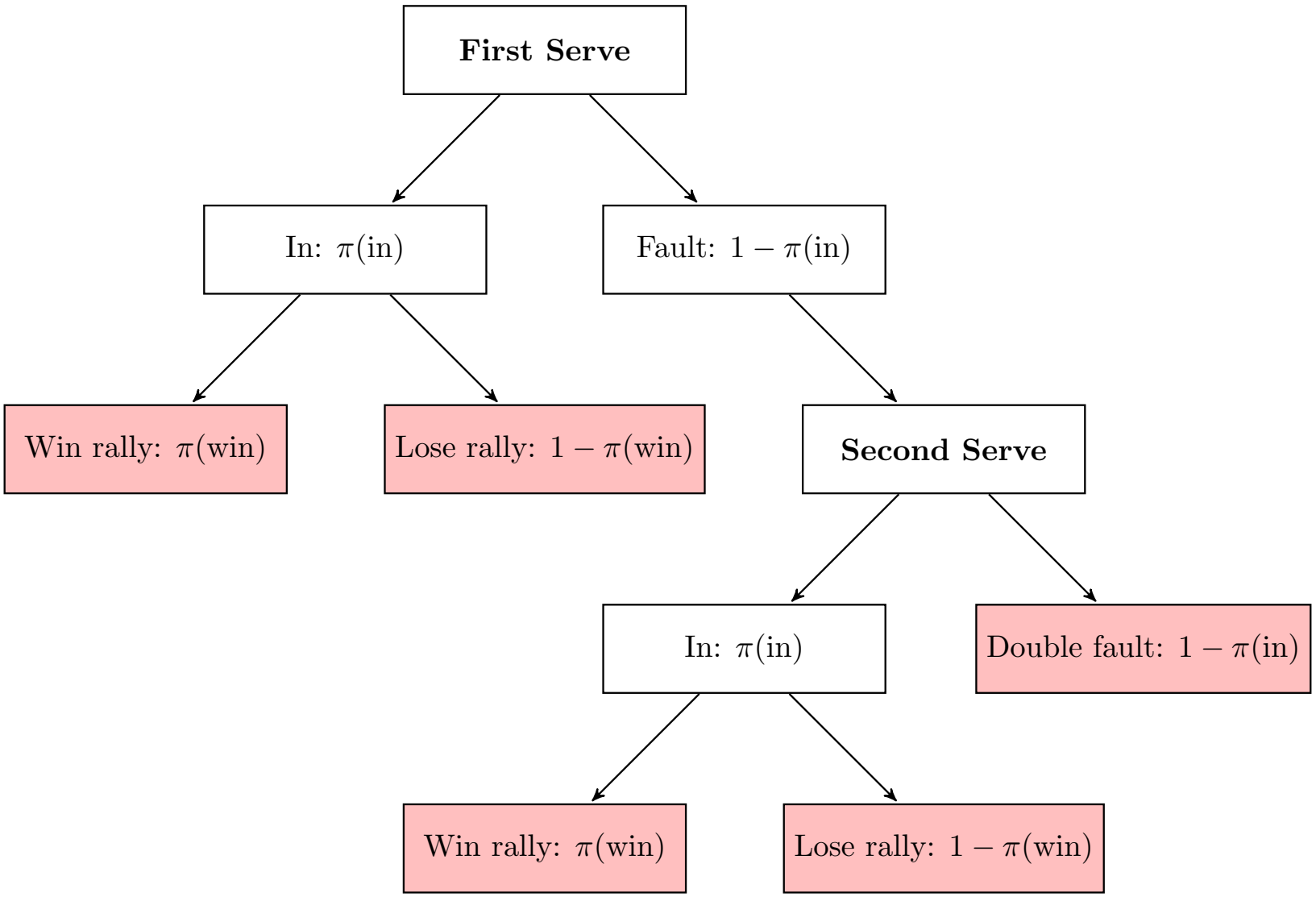
Graph of state transitions in the service game



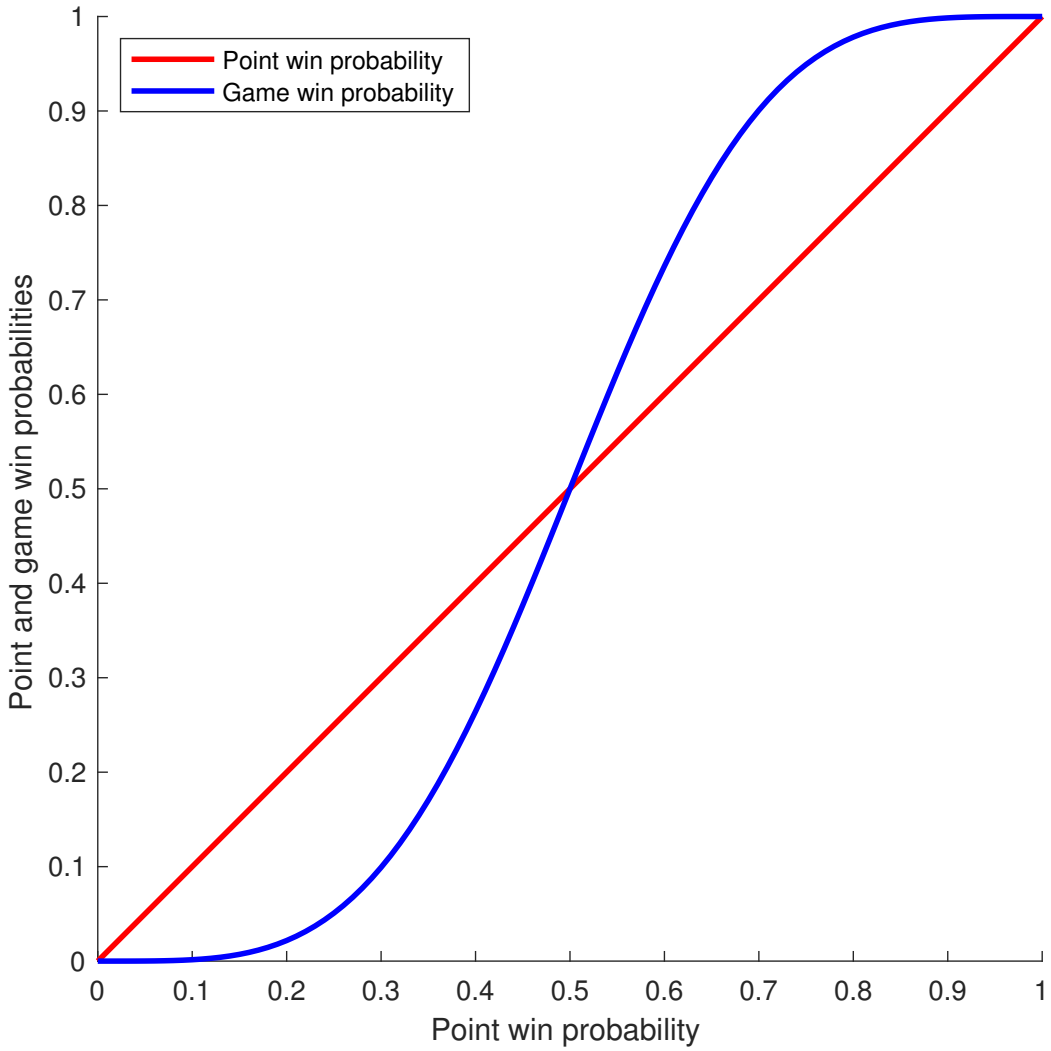
Detail on the "deuce endgame"



Detail on the "point stage game"



Point win vs game win probabilities



Two key implications of equilibrium play in tennis

- **Equal win probabilities** In a mixed strategy equilibrium the payoff to server should be the same regardless of which direction he/she serves
- **One shot deviation principle** At any point in a tennis game, there should be no deviation in the strategy of the server that should increase his/her win probabilities

Modeling tennis as a dynamic game

- Unlike Walker and Wooders, who focused on the point stage game as their unit of analysis, we focus on the entire service game.
- Let $x \in \{1, \ldots, 38\}$ denote the *score state*. Via our numbering of states in the previous graph, $x = 1$ is the initial node of the game, corresponding to a score of $0 - 0$.
- If the server wins the first serve, the state transits to $x = 3$ corresponding to a score of 15-0. If the server faults the first serve, the state transits to $x = 2$, corresponding to a second serve at score 0-0, etc.
- There are two *absorbing states:* $x = 37$ (server wins the game) and $x = 38$ (server loses the game).

Server and returner choices in tennis

- At each state x except the two absorbing states, the server chooses a *serve type* $t = (s, d)$ where $d \in \{l, r, b\}$ is the serve direction (left, right or to the body), and $s \in \mathcal{S} \subset R^2$ denotes *serve speed and spin.*
- The returner anticipates the direction choice of the server, and this includes observable choices (i.e. where to stand) and unobservable choices (i.e. where to focus his/her attention).
- We model anticipation with an attention vector $(a_l, a_r, a_b) \geq 0$ where a_d is the fraction of the returner's "attention budget" to anticipating a serve to direction d. We normalize the attention budget to 1, so $1 = a_l + a_r + a_b$.
- We assume that each serve the returner chooses a and where to stand, and the server moves next, choosing a serve type t.

Accounting for *muscle memory*

- We account for an additional dynamic in tennis via a *muscle memory state* m.
- We operationalize muscle memory by setting $m = (d_{-2}, d_{-1})$, i.e. the vector of the previous two *first serve directions.* We have two lags because serves alternate between the deuce and ad courts and we hypothesize that the relevant muscle memory is for the previous serve to the same court.
- At the start of each game muscle memory is null, $m = (\emptyset, \emptyset)$. After a first serve to direction d muscle memory updates from

$$m = (d_{-2}, d_{-1}) \longrightarrow m' = (d_{-1}, d) = m'(m, d)$$

- We do not update muscle memory after a second serve.

Point Outcome Probabilities (POPs)

Definition

If the stationarity assumption holds and (ρ, α) are implied by the MPE to the service game, then the *point outcome probabilities* (POPs) implied by these MPE strategies are given by

$$\begin{aligned} \pi(in|x, m, d) &\equiv \int l(x, m, d, s)\rho(ds|x, m) \\ \pi(win|x, m, d) &\equiv \int \int \omega(x, m, d, s, a)\rho(ds|x, m)\alpha(da|x, m) \end{aligned}$$

The induced dynamic program

- Now, assume that the play we observe in tennis is consistent with an MPE, i.e. we observe *equilibrium play.*
- We now show that the server's conditional win probability $W_s(x, m)$ is the solution to a dynamic program, which provides the optimal serve directions as a best response to the returner's strategy.
- If play is in equilibrium, we can treat the returner's mixed strategy as embodied, in reduced-form, via the POPs, and the server's strategy and win probability will be a "best reply" to these POPs.
- Bellman's equation for the server is

$$W_s(x, m) = \max_{d \in \{l, b, r\}} W_s(x, m, d)$$

where $W_s(x, m, d)$ is the server's conditional win probability from serving to direction d in state (x, m).

The server's Bellman equation

- Let $x^+(x)$ and $x^-(x)$ denote the successor point states conditional on the server winning and losing, respectively, the serve at point state x. Using these we can express $W_s(x, m, d)$ recursively as

$$
\begin{aligned}
&W_s(x, m, d) = \\
&\pi(in|x, m, d)\pi(win|x, m, d)W_s(x^+(x), m'(d, m)) \\
&+\pi(in|x, m, d)[1 - \pi(win|x, m, d)W_s(x^-(x), m'(d, m)) \\
&+[1 - \pi(in|x, m, d)]W_s(x + 1, m'(d, m))
\end{aligned}
$$

for a first serve state x, where $x + 1$ denotes the second serve state in the event of a faulted first serve and $m'(d, m)$ is the new muscle memory state after serving to direction m, $m'(d, m) = (d_{-1}, d)$ where $m = (d_{-2}, d_{-1})$.

The server's Bellman equation

- If x is a second serve state (i.e. x is an odd number in our score state numbering), then

$$
\begin{aligned}
&W_s(x, m, d) = \\
&\pi(in|x, m, d)\pi(win|x, m, d)W_s(x^+(x), m) \\
&+\pi(in|x, m, d)[1 - \pi(win|x, m, d)W_s(x^-(x), m) \\
&+[1 - \pi(in|x, m, d)]W_s(x^-(x), , m)
\end{aligned}
$$

  where we note that following a second serve, the muscle memory state is not updated and remains at m.

- The optimal serve strategy, $\sigma_s(x, m)$ is the set of maximizers of the Bellman equation

$$\sigma_s(x, m) = \underset{d \in \{l, b, r\}}{argmax}\ W_s(x, m, d).$$

Solving the server's DP problem

- We can write the Bellman equation for the server's conditional win probability more compactly as

$$W_s = \Gamma(W_s)$$

  where Γ is the *Bellman operator.* Even though there is no discounting in this DP problem, the POPs $(\pi(in|x, m, d), \pi(win|x, m, d))$ (which are generally strictly less than 1) play the equivalent role of a discount factor, ensuring the existence of a unique solution to the Bellman equation via the Contraction Mapping Theorem.
- In practice, we calculate the server's DP problem by *backward induction* but the induction is not *over time* but rather *over states* using an algorithm Iskhakov, Schjerning and Rust (IRS,2016) refer to as *state recursion.*

Testable Implications of Nash Equilibrium

- Suppose we could calculate the server's conditional win probabilities $W_s(x, m, d)$. If the equilibrium is in mixed strategies, then a key implication of a mixed strategy Nash equilibrium is

Testable Implication of Equilibrium (1)

Equal win probabilities In all states (x, m) we have:

$$W_s(x, m, l) = W_s(x, m, b) = W_s(x, m, r)$$

Valuing disequilibrium serve strategies

- Let $W_P(x, m)$ be the conditional win probability of a server whose potentially disequilibrium Markovian serve direction strategy is $P(d|x, m)$. We have

$$W_P(x, m) = \sum_{d \in \{l, b, r\}} W_P(x, m, d) P(d|x, m)$$

  where $W_P(x, m, d)$ is the conditional probability of winning under the strategy P given the server serves to direction d in state (x, m).
- The values $W_P(x, m, d)$ are given by the same recursive equations as we derived for the optimal (equilibrium) serve strategy above in Bellman's equation.

Linear equation system for conditional win probabilities

- We can show that $W_P(x, m)$ is the solution to the following system of linear equations

$$W_P = w_P(P, \Pi) + M_P(P, \Pi) W_P \tag{1}$$

  where $P = \{P(d|x, m)\}$, $\Pi = \{\pi(in|x, m, d), \pi(win|x, m, d)\}$ and $w_P(P, \Pi)$ is a 298×1 vector of probabilities of winning the service game directly (equal to zero for most states except the deuce endgame states and states where the server is within one shot of winning), and $M_P(P, \Pi)$ is a 298×298 Markov sub-transition probability matrix.
- Because $\|M_P(P, \Pi)\| < 1$, this system will have a unique solution W_P implied by any (P, Π).
- Note there are a total of $894 = 3 \times 298$ conditional win probabilities $W_P(x, m, d)$. Thus if we can estimate (P, Π) well using fewer parameters, we can derive more efficient estimates of $W_P(x, m, d)$.

Non-parametric vs Recursively calculated win probabilities

TABLE 1

ESTIMATED WIN AND CONDITIONAL WIN PROBABILITIES AT FIRST SERVE OF SERVICE GAME

SERVER → RECEIVER	WIN PROBABILITY, FIRST SERVE	CONDITIONAL WIN PROBABILITY, FIRST SERVE			DHW TEST *p*-VALUE
		Left	Body	Right	
Federer → Nadal					.004
NP estimate	.796 (.026)	.816 (.025)	.650 (.030)	.803 (.025)	
RF estimate	.829 (.023)	.828 (.024)	.819 (.027)	.833 (.022)	
Nadal → Federer					.107
NP estimate	.786 (.026)	.748 (.028)	.896 (.020)	.762 (.028)	
RF estimate	.807 (.023)	.808 (.023)	.807 (.025)	.803 (.025)	
Federer → Djokovic					.504
NP estimate	.810 (.024)	.844 (.022)	.867 (.021)	.767 (.025)	
RF estimate	.818 (.020)	.826 (.020)	.812 (.023)	.813 (.021)	
Djokovic → Federer					.910
NP estimate	.782 (.025)	.769 (.026)	.710 (.028)	.815 (.024)	
RF estimate	.781 (.022)	.792 (.022)	.769 (.026)	.774 (.024)	
Nadal → Djokovic					.992
NP estimate	.712 (.035)	.685 (.036)	.726 (.035)	.750 (.034)	
RF estimate	.712 (.034)	.712 (.034)	.701 (.035)	.718 (.034)	

Wald Tests for Equal Win Probabilities

TABLE 6
WALD TESTS OF EQUAL SERVICE-GAME WIN PROBABILITIES, MUSCLE-MEMORY SPECIFICATION

Server → Receiver	Four Fixed Serve Strategies at Three States (9 df)	Reduced-Form Serve Strategy at Four States (12 df)
Federer → Nadal	1.4×10^{-11}	.605
Nadal → Federer	.873	.018
Federer → Djokovic	6.5×10^{-30}	1.6×10^{-68}
Djokovic → Federer	.0009	.220
Nadal → Djokovic	2.2×10^{-254}	.526
Djokovic → Nadal	4.0×10^{-91}	4.5×10^{-44}
Djokovic → Murray	1.4×10^{-69}	.018
Murray → Djokovic	9.1×10^{-91}	.00001
Sampras → Agassi	.787	.003
Agassi → Sampras	.764	.667

Second Testable Implication of Nash Equilibrium

- Suppose we could estimate the server's conditional win probabilities $W_P(x, m, d)$ *without imposing optimality/equilibrium.* Then a key implication of a mixed strategy Nash equilibrium is

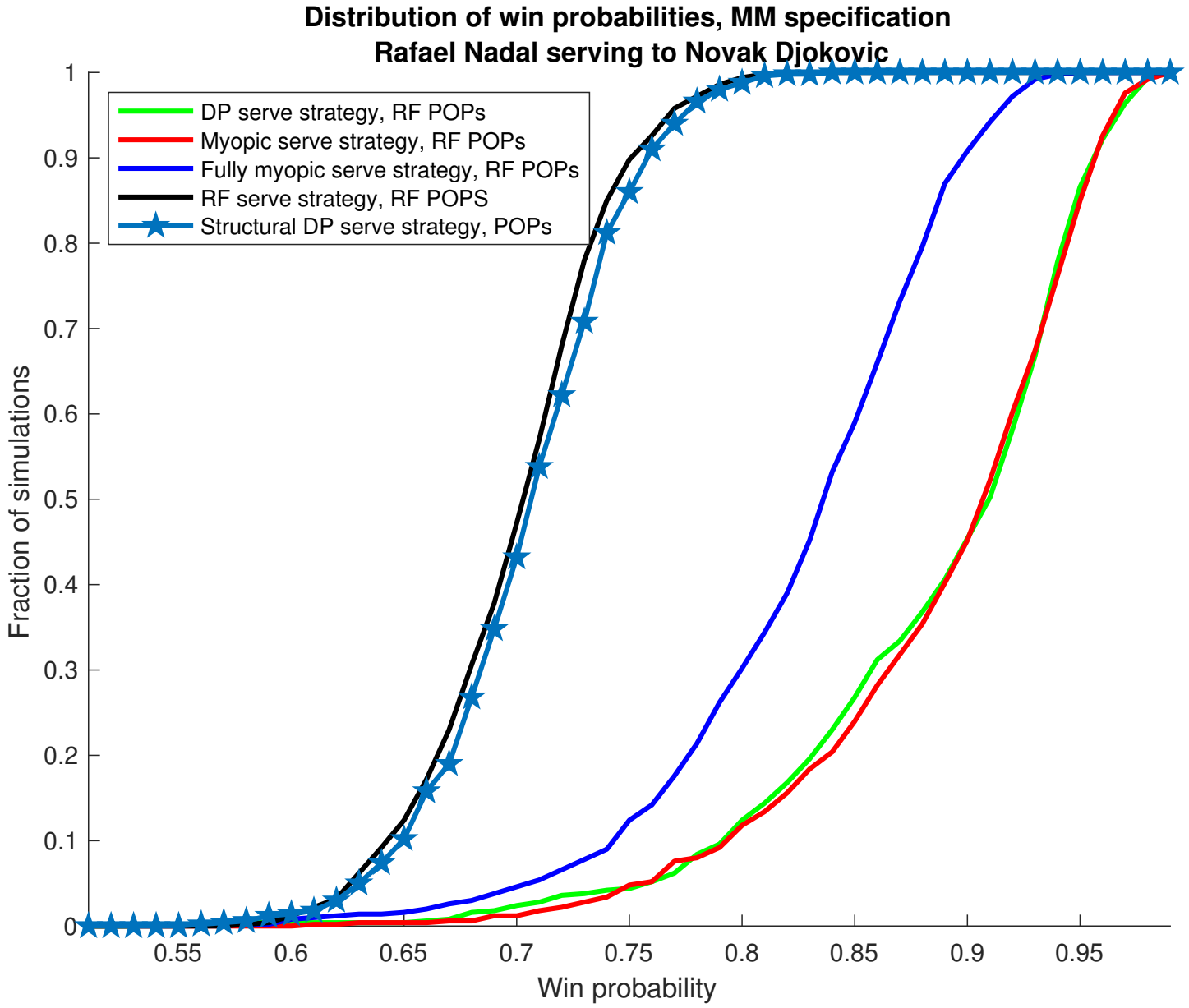
Testable Implication of Equilibrium (2)

One shot deviation principle In all states (x, m) and for all serve directions $d \in \{l, b, r\}$ we have:

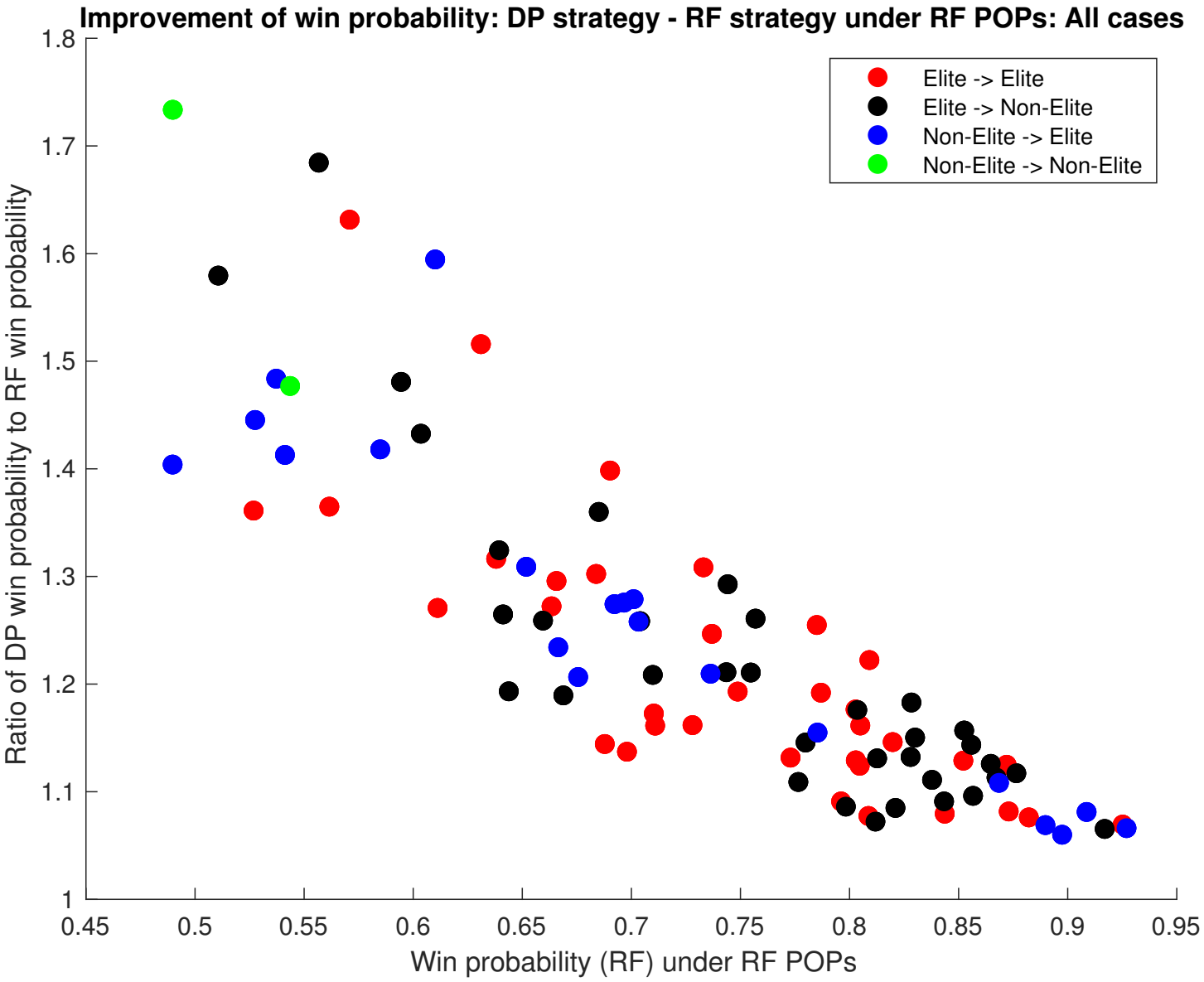
$$W_P(x, m, d) = W_s(x, m, d)$$

- According to *Wikipedia* that One shot deviation principle "is the principle of optimality of dynamic programming applied to game theory. It says that a strategy profile of a finite extensive-form game is a subgame perfect equilibrium (SPE) if and only if there exist no profitable one-shot deviations for each subgame and every player."

Distributions of win probs: Nadal serving to Djokovic



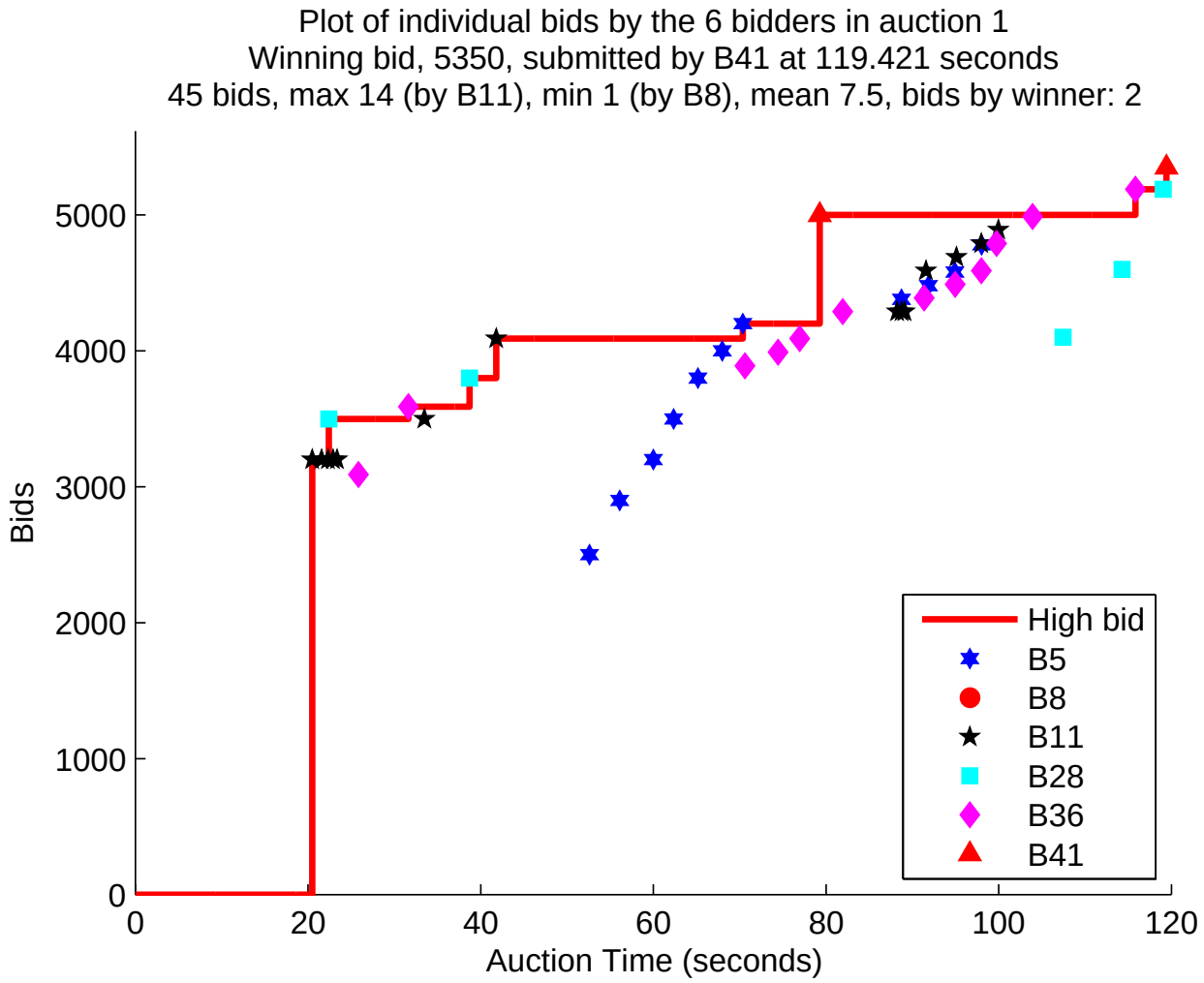
Disequilibrium play for both elite/non-elite pros



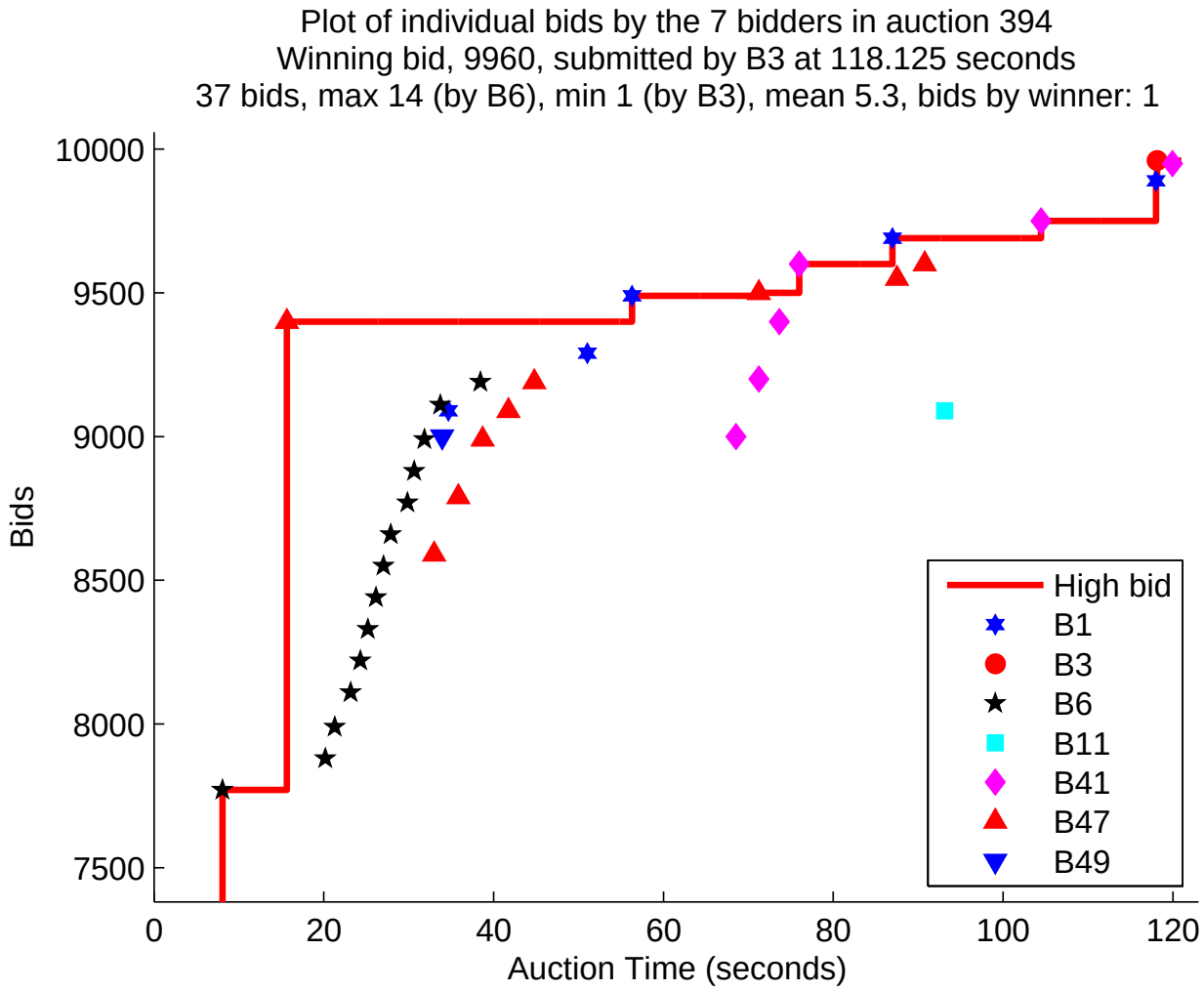
Early Overbidding in Informationally Restricted Auctions

- We analyze a large number of continuous time ascending bid auctions of rental cars conducted online over 2 minute intervals.
- Due to concerns about bidder collusion, the rental company designed a unique dynamic auction: the *Korean auction*.
- Bidders only know the amount of their own bids and an indicator whether their bid is the highest so far.
- *Bidders cannot observe the identities or bids placed by competing bidders, and thus do not even know the number of other bidders bidding in any given auction.*
- We are aware of only one other paper that analyzed auctions with informational restrictions similar to the Korean auctions: auctions of certificates of deposit (CDs) by the state of Texas.
- Barkley, Groeger and Miller *Journal of Econometrics* (2021) provide an empirical of analysis bidding in these auctions.
- "This market features frequent jump bidding and winning bids well above the highest losing bid, suggesting standard empirical approaches for ascending auctions may not be suitable."

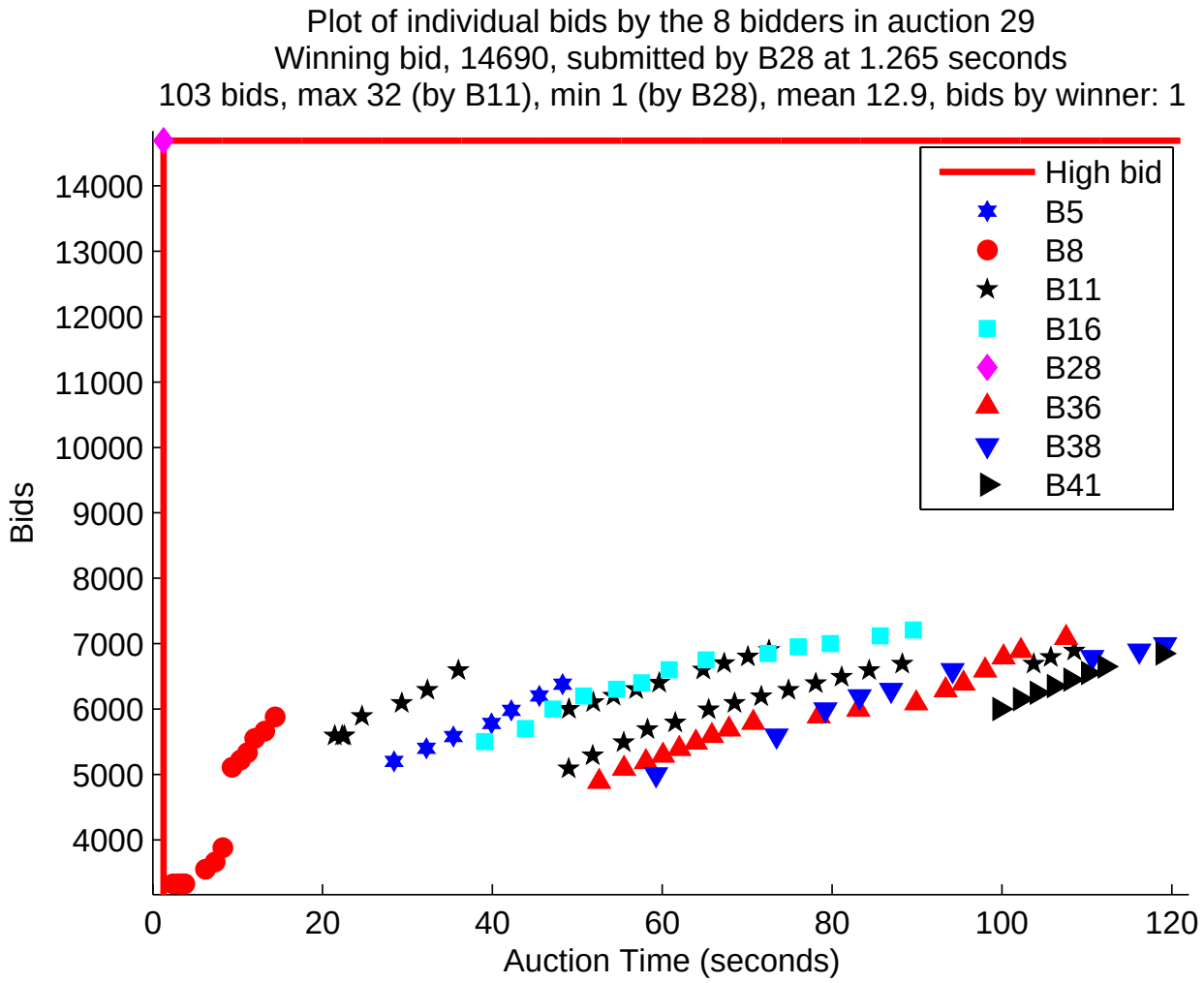
Auction 1, January 26, 2005



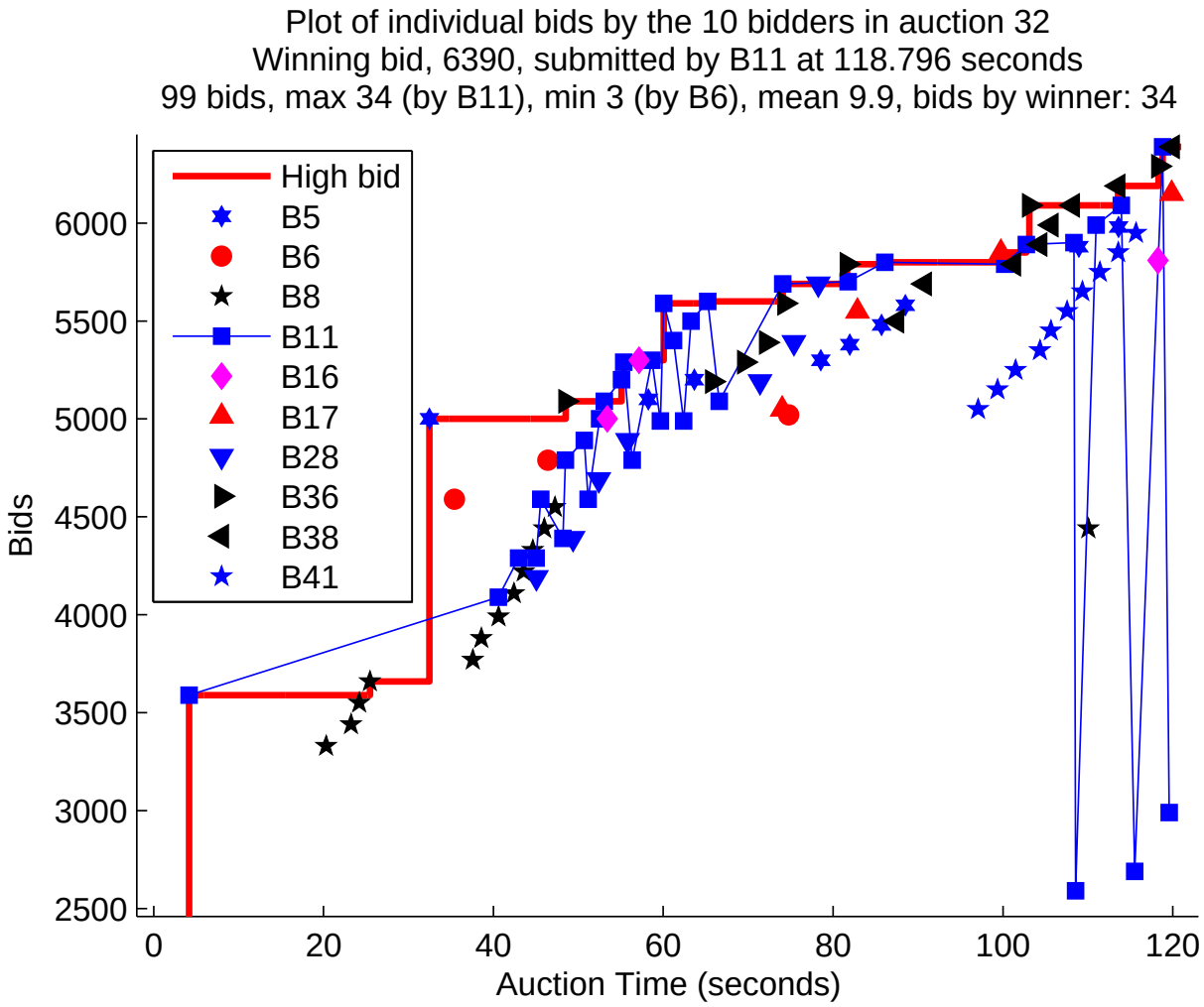
Auction 394 — bid sniping



Auction 29 — early high bidder



Auction 32 — a "crazy" bidder B11



Summary of bidding behaviors

- **Bid creeping** – most common behavior: due to desire to learn?
- **Bid jumping** like bid creeping but fewer bids/bigger jumps
- **Bid sniping** observed in 5% of all auctions
- **Pre-emptive early bidding** observed in 3% of all auctions
- **Non-monotonic bidding** boredom or stupidity?

Key finding and contribution of this paper

We show that pre-emptive early bidding is an extreme version of something we find much more commonly in these questions, which we refer to as *early overbidding*. We argue that it is hard to find a rational explanation for early overbidding, but we do find one, by appealing to *rational inattention* and *bidding frictions* and a weaker notion of equilibrium we call *Anonymous Equilibrium*.

Preview of our conclusions

- We conjecture that *early bidding will not occur in a PBE.* We provide a simple 2×2 example where there is no bidding in the first period. The only PBE is an *uninformative PBE* where all players snipe, so it is strategically equivalent to an (anonymized version) of a static first price sealed bid auction.
- We introduce a DP model of *rationally inattentive bidding* with *bidding frictions* in *Anonymous Equilibrium* (AE) that can explain early bidding.
- The DP model captures bidders' incentive to bid early to learn the current high bid and try to win the auction without overpaying.
- Our theory implies that as a result of a combination of bidding mistakes/inefficiencies and learning that occurs during the auction, human bidders pay less on average, but their profits are also significantly lower compared to what frictionless "bidding robots" could earn in AE.

Other results

- Our model provides a *qualitative explanation* for the "informative early bidding" in these auctions, but ...
- Human bidders tend to *bid too high, too fast* compared to what our model predicts, a phenomenon we call *early overbidding*.
- We show that our frictionless DP bidding strategies *outperform* human bidders in terms of expected profits.
- We blame the rejection of our model on the bounded rationality of the human bidders in the face of the difficult dynamic learning/DP problem in these continuous time auctions.
- This creates *substantial inefficiencies* that result in lower bidder profits and winning auction prices compared to a static first price sealed bid format.

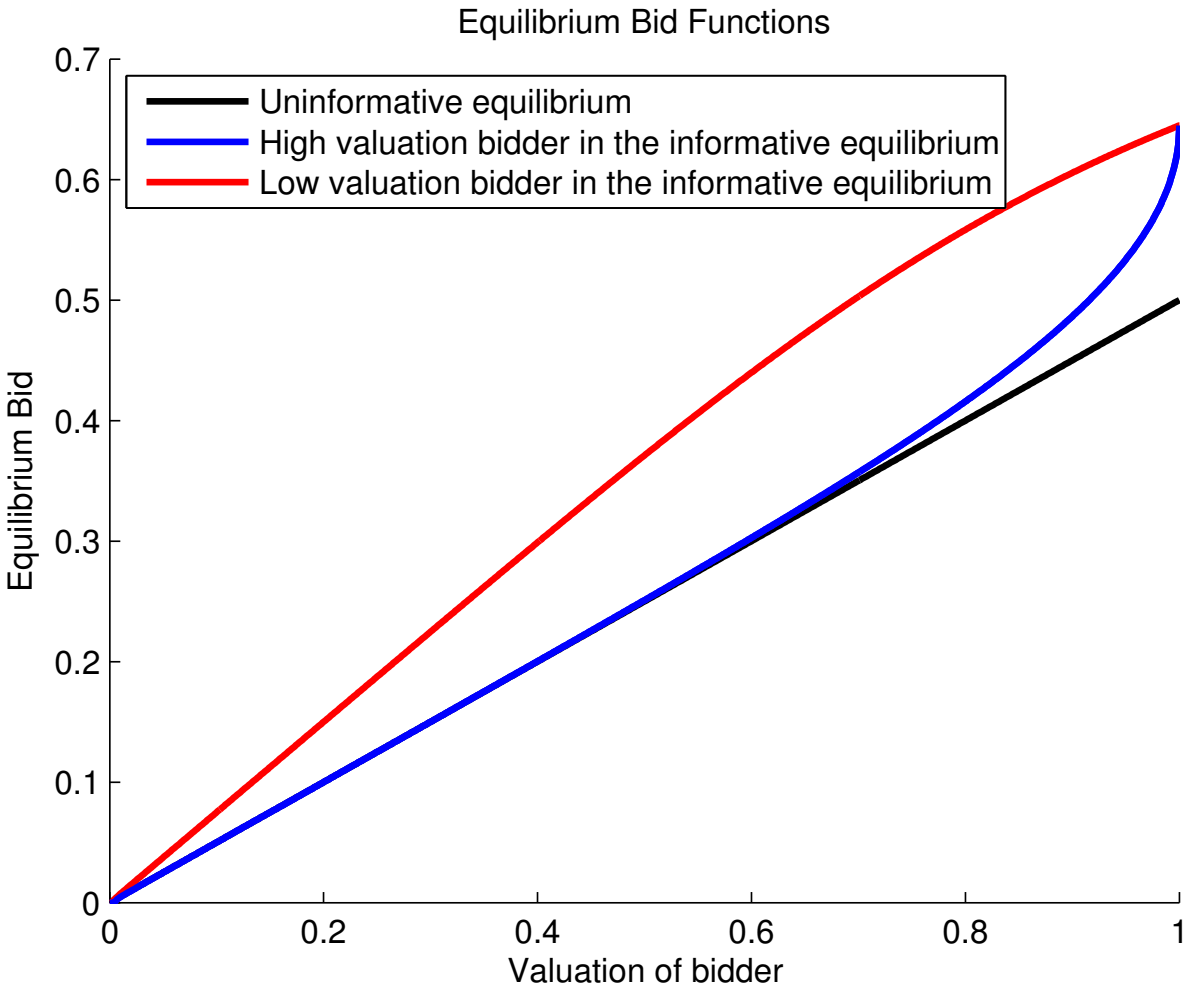
Gains to computerized trading

Key prediction

We show that auction prices and bidder profits would both increase if the human bidders used rational computerized bidding algorithms to serve as their 'bidding agents'. In the Anonymous Equilibrium of the Korean market, with rational computerized bidding agents, efficiency is 100%, bidders' profits nearly double, and auction prices are 13% higher than in auctions with human bidders.

- We argue that human traders can achieve approximately the same efficiency and outcomes as above in a simpler static sealed-bid auction mechanism.
- No good rationale for adopting a more complicated dynamic auction format.

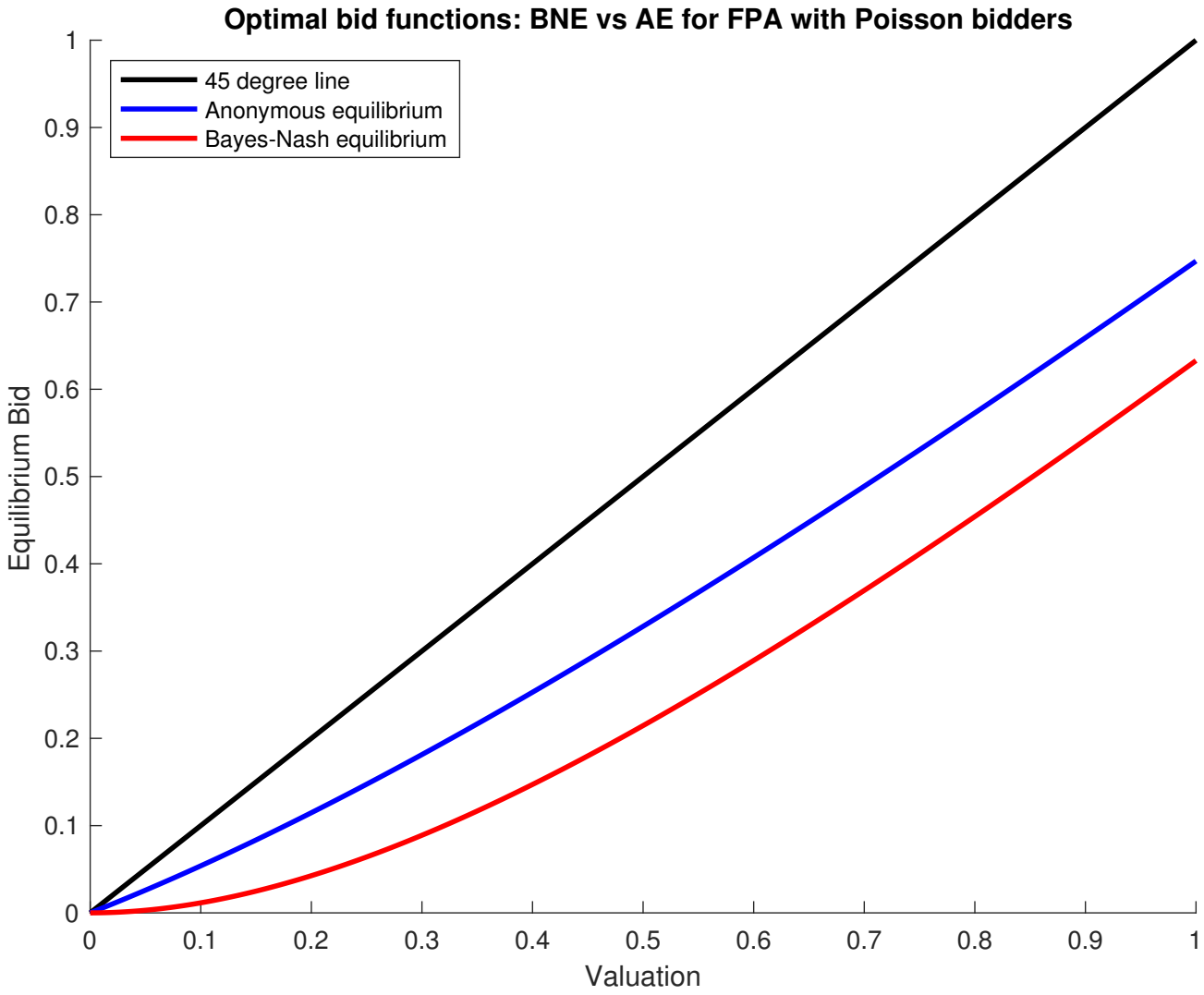
Equilibrium bid functions in period 2



Rationally inattentive bidding in Anonymous Equilibrium

- Due to the difficulty of computing PBE and because it is not clear that there is an informative PBE that would be consistent with the bidding behavior we observe in these auctions, we adopt an alternative modeling approach.
- We develop a *behavioral DP bidding model* that assumes *bidders have rational beliefs* about the stochastic process for the high bid price in the auction.
- Using these beliefs, we solve a dynamic program to determine the optimal bidding strategy implied by these beliefs.
- We define a concept of ϵ-*Anonymous Equilibrium* to define approximately self-confirming beliefs of bidders in these auctions.
- It is similar to a *rational expectations equilibrium* which is also a self-confirming system of beliefs. In an ϵ-AE the actual stochastic process for the highest bid during the auction is approximately equal to bidders' beliefs about this stochastic process.

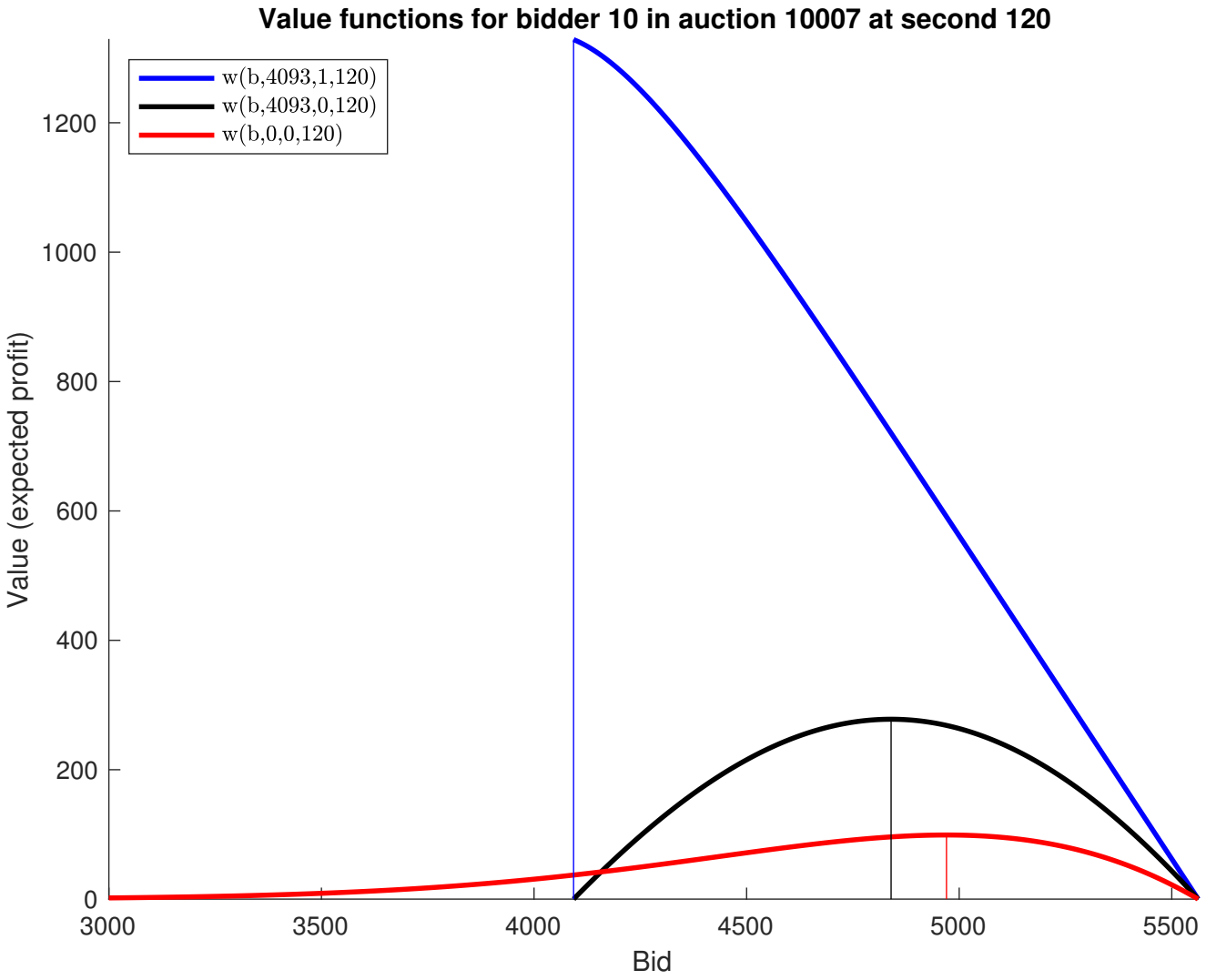
AE vs BNE bids in a static auction with Poisson arrivals



Accounting for learning

- Our approach involves *learning* but employs a simpler model of *experiential learning* rather than full Bayesian updating.
- Our model is appropriate for *experienced bidders* who have participated in many auctions, and thus have well-defined and fixed beliefs about the stochastic process for the high bid in the auction.
- If a bidder has the highest bid at t, then they know it. But if the bidder does not have the highest bid at t, they must predict it based on a rational belief of the probability distribution of the high bid.
- We show that there is a significant reduction in uncertainty by learning that one has the high bid prior to the end of the auction.
- Thus the motivation for early bidding is to gather information about the current high bid, helping bidders to win the auction without paying more than necessary.

The value of learning via early bidding, frictionless case



The value of learning from early bidding

- Bidder obtains valuable information from placing a bid of $b_{120} = 4093$ compared to not bidding and being uninformed (i.e. sniping) where $b_{120} = 0$.
- This true even if you you are not the high bidder, so $h_{120} = 0$. But clearly, the best case is when you know you are the high bidder at $t = 120$.

State ($v = 5562$)	$E\{\overline{b}_{121} \mid b_{120}, h_{120}\}$	$\beta_{120}(b_{120}, h_{120})$	$W_{120}(b_{120}, h_{120})$
$b_{120} = 0$, $h_{120} = 0$	5825	4970	99
$b_{120} = 4093$, $h_{120} = 0$	5173	4839	278
$b_{120} = 4093$, $h_{120} = 1$	4113	4093	1329

- More information: 1) lowers your expectation of the final high bid, 2) lowers your optimal bid, and 3) significantly increases your expected profits in the auction.

What happens if we remove the informational restriction?

- Suppose drop the informational restriction so all bidders can see the high bid in the auction at any moment *regardless of whether they hold the high bid or not.*
- However bidder identities are still suppressed, so it continues to be an *anonymous game.*
- The resulting auction is an anonymized, electronic version of an *open outcry auction.*

Theorem 2

There is no early bidding by rational, frictionless bidders in the AE of an anonymous open outcry auction. All bidders wait to the last instant T to place their bids, so the AE is strategically equivalent to the AE of an anonymous static first price sealed bid auction.

- Intuition: because of *informational free-riding* no rational bidder has an incentive to bid early in AE.

We also need bidding frictions to explain early bidding

- Let's return to the Korean auctions, with its informational restrictions. Do these restrictions always succeed in generating the early bidding we observe?
- **NO** We can find uninformative AE where there is no early bidding.

Theorem 3

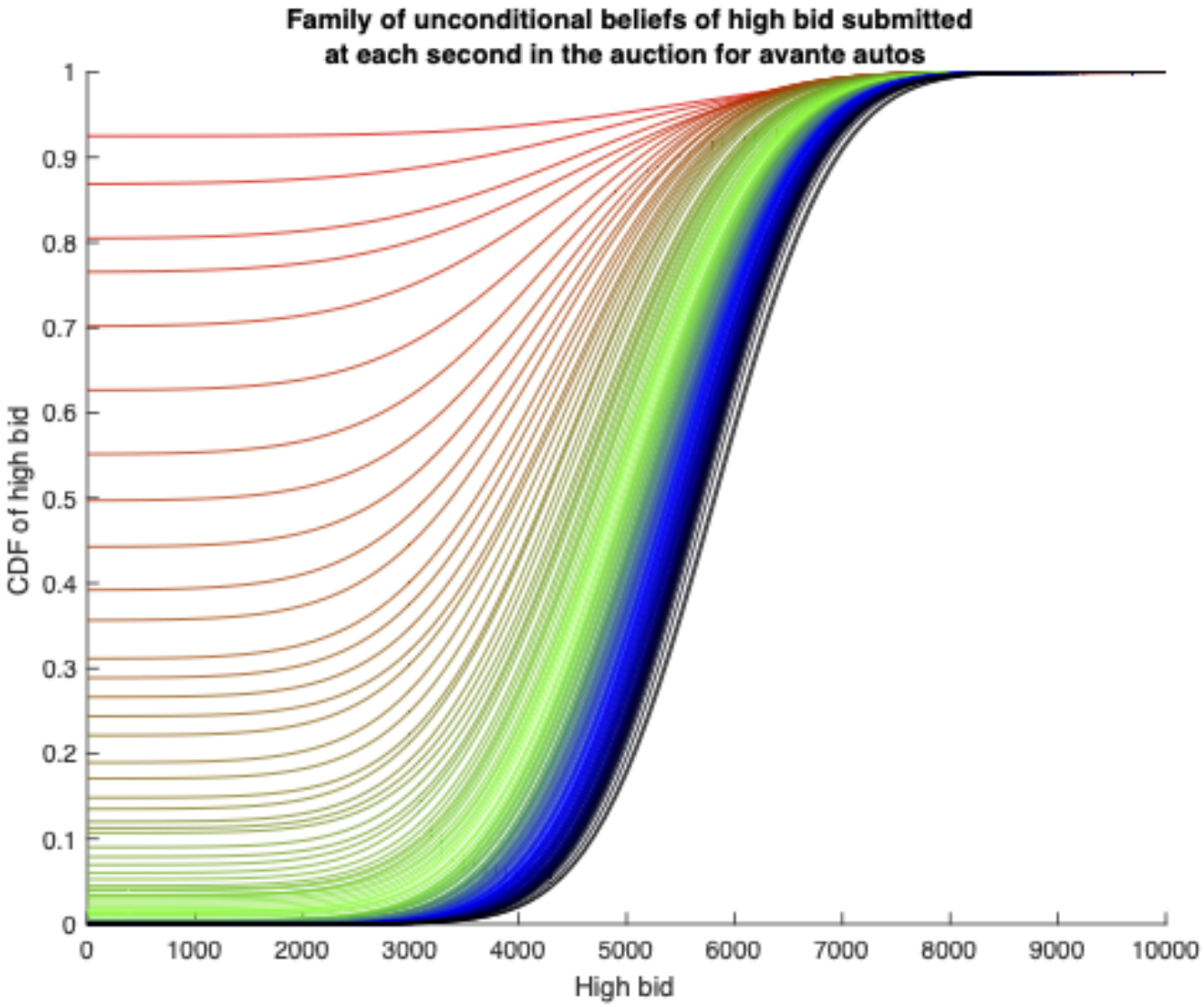
Suppose all bidders in the Korean auction are rational and frictionless and the auction has a "hard close." Then there exists an uninformative AE where all bidders only submit their bids in the last instant T of the auction.

A 4 parameter DP model of rationally inattentive bidding

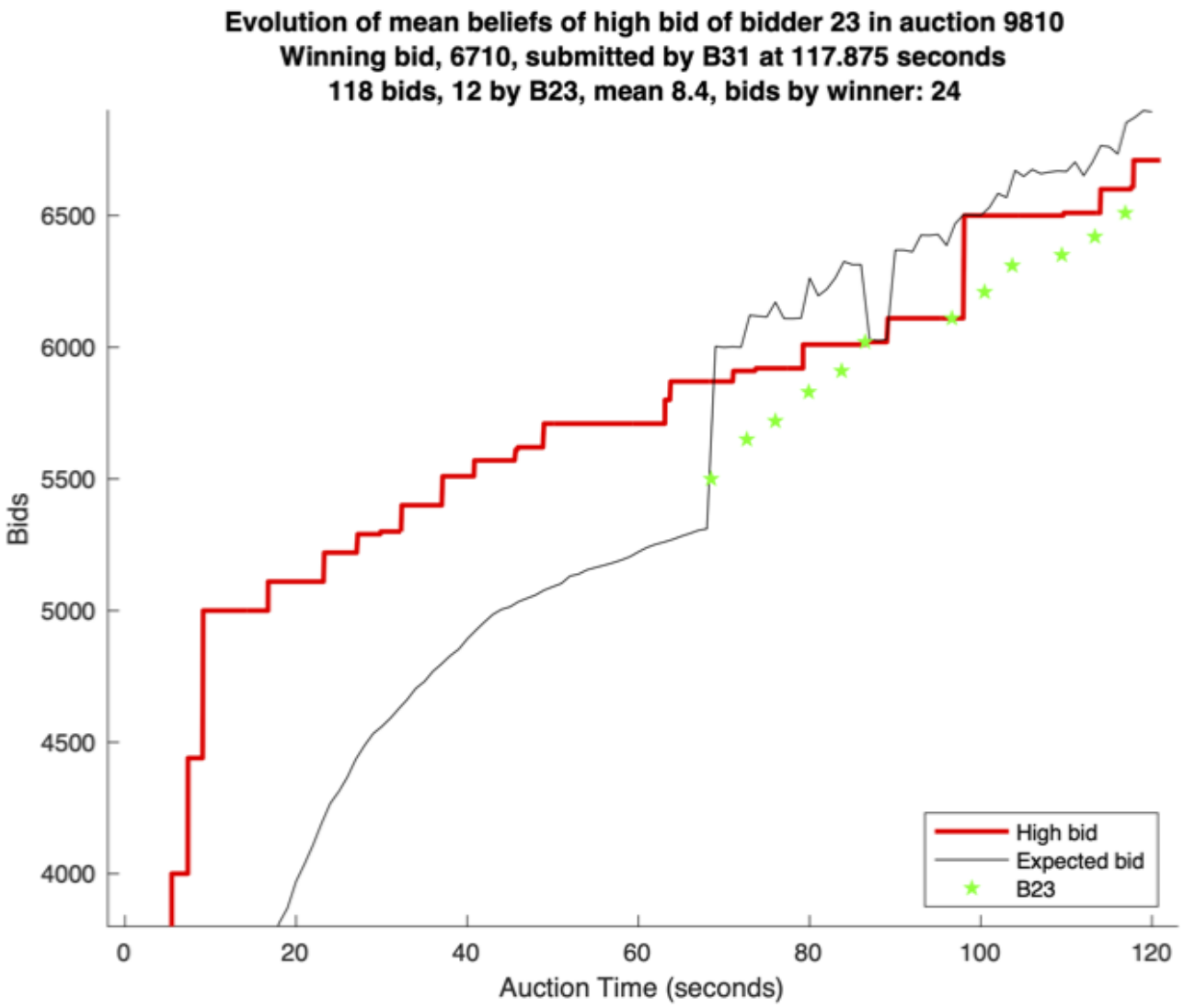
- Discretize the two minute auction into $T = 120$ one second *bidding instants.*
- Let $\tau = (v, c, p, \sigma)$ denote the *type* of the bidder, where v is the bidder's valuation of the car being auctioned, and c is the bidder's psychic *cost of submitting a bid,* p is a probability the bidder is *distracted* and cannot submit a bid, and σ is an extreme value scale parameter.
- Bidder beliefs $\mathcal{B}$ are captured by a family of conditional probability distributions $\{\lambda_t(b|b_t, h_t)\}$ where $\lambda_t(b|b_t, h_t)$ is a CDF for the high bid at during the interval $(t, t+1]$ of the auction, conditioned on b_t, the bidder's highest bid up to time t (or 0 if the bidder has not bid yet), and h_t is an indicator of whether the bidder holds the high at t.

2005 Hyundai Avante Elantra XD 1.6L

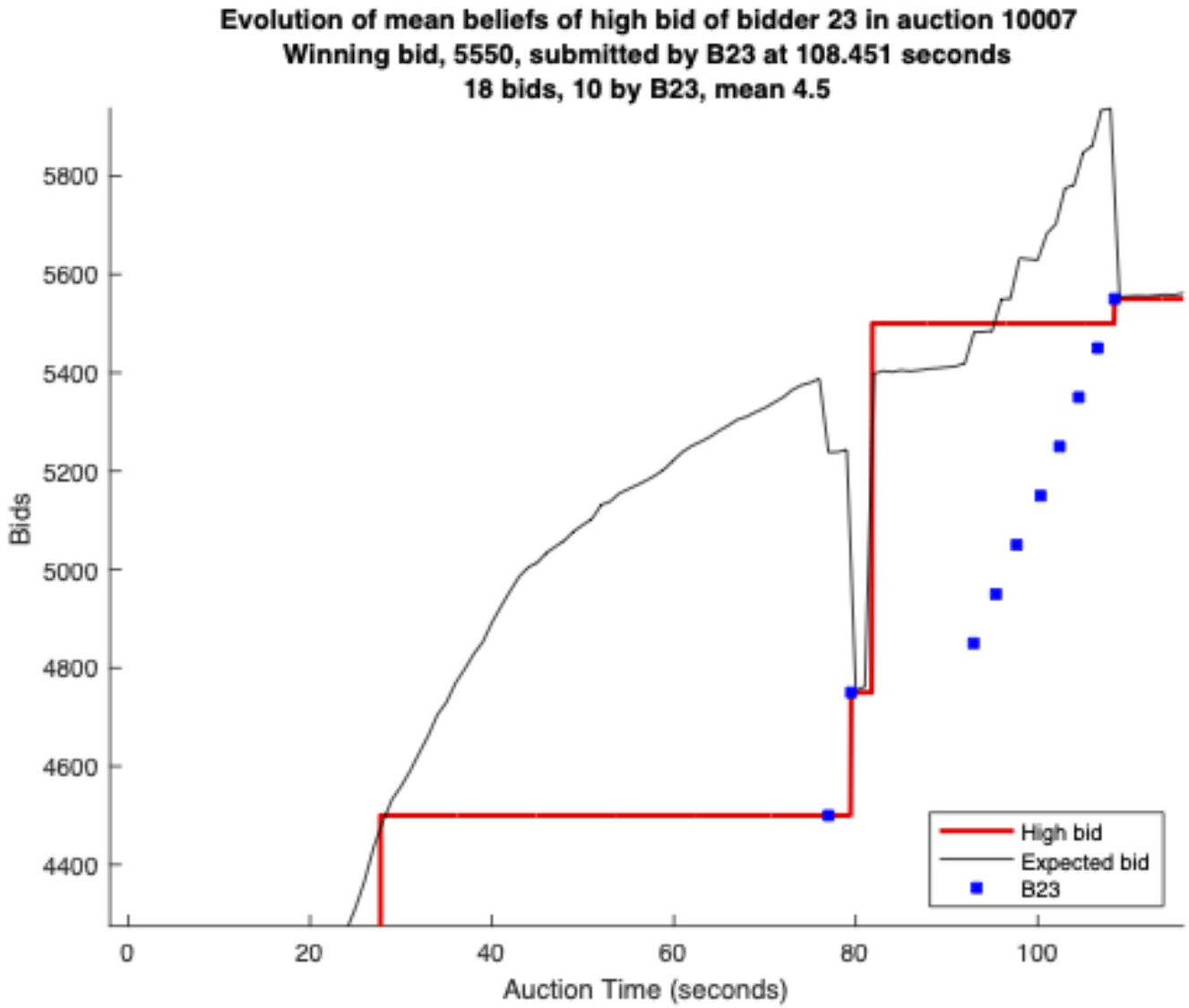
Unconditional beliefs about the high bid in the auction



Evolution of beliefs of B23 in auction 9810



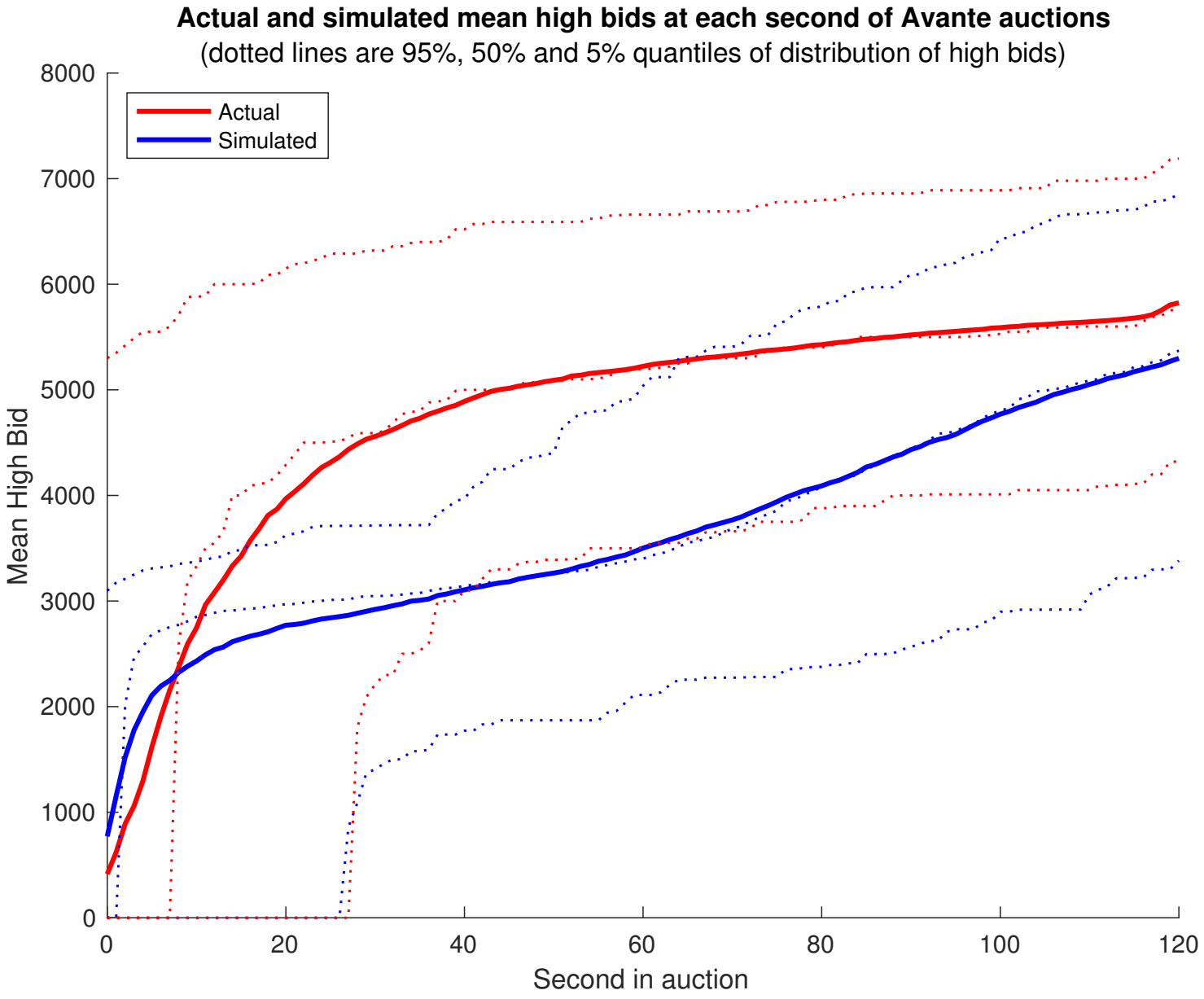
Evolution of beliefs of B23 in auction 10007



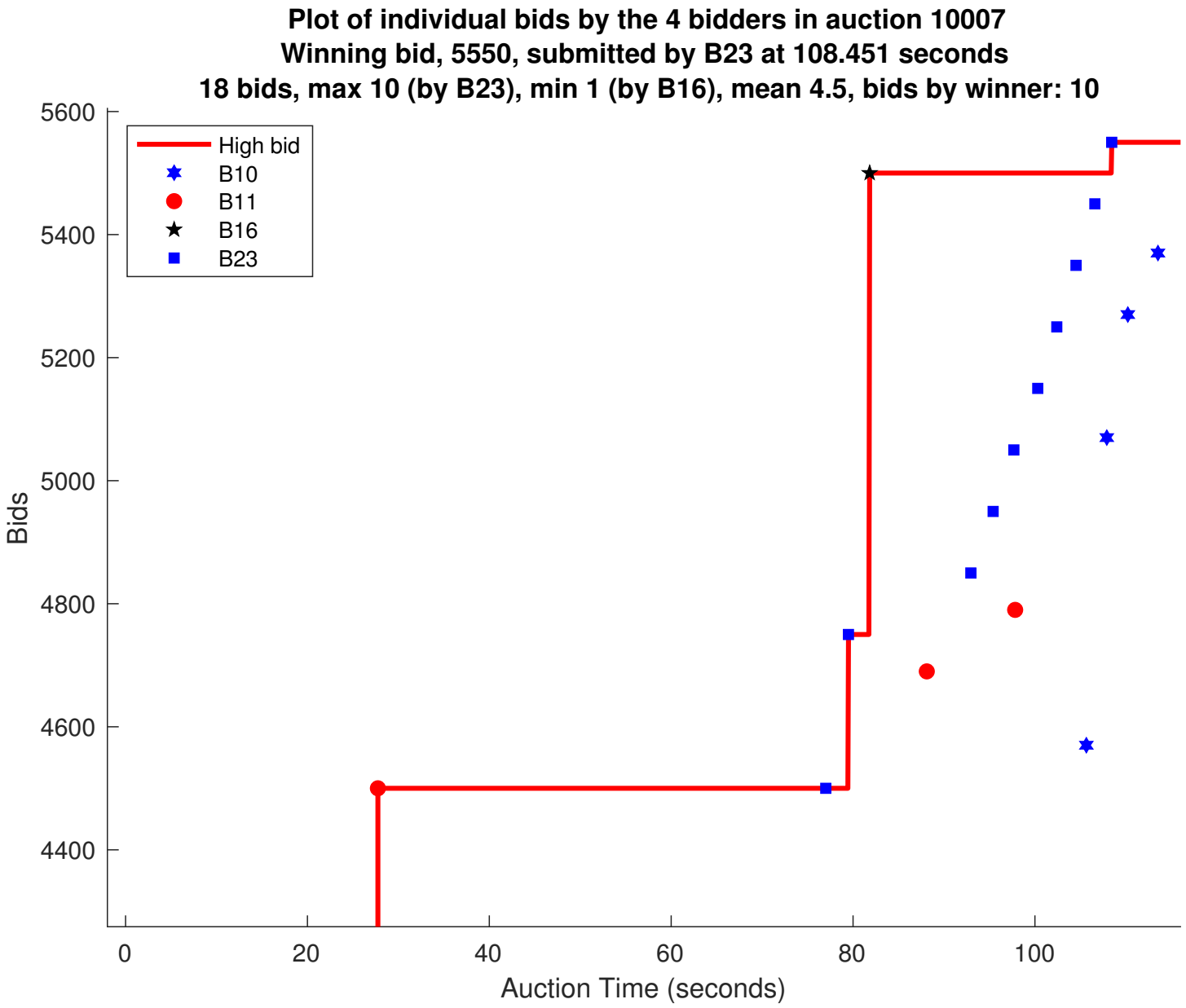
Two Step Estimation Strategy

- **Step 1** Using data on 533 auctions of Avante cars, we estimate the family of beliefs $\{\lambda_t\}$ about the stochastic process for the high bids in the auction.
- **Step 2** Using $\{\hat{\lambda}_t\}$ we solve a discrete dynamic programming problem determining the optimal bidding of the bidder at each second of the auction, resulting in a family of bid functions $\{\beta_t\}$ where $\beta_t(b_t, h_t)$ is the optimal bid at the start of second t in the auction when the bidder's high bid so far is b_t and h_t is an indicator of whether the bidder has the high bid or not.
- Note the only unknown parameters in step 2 are $\tau = (v, c, p, \sigma)$. Thus, we form a likelihood $L(\tau)$ for *each bidder in each auction* resulting in auction-by-auction bidder-specific estimates of valuations, v and other parameters τ characterizing the bidder's type.
- Our goal is to see if such a model is capable of explaining the early bidding we observe in these auctions.

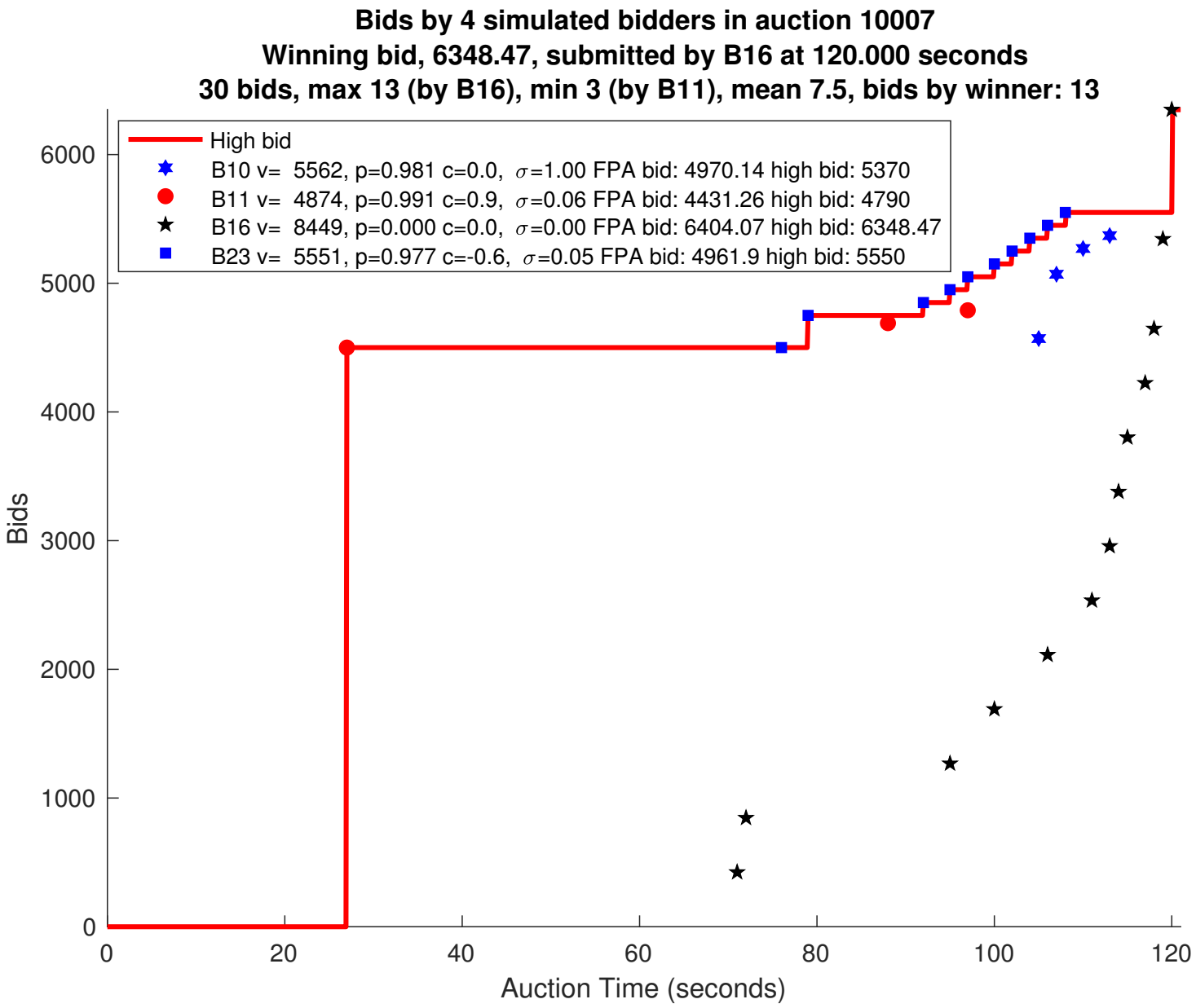
Actual vs simulated high bid trajectories



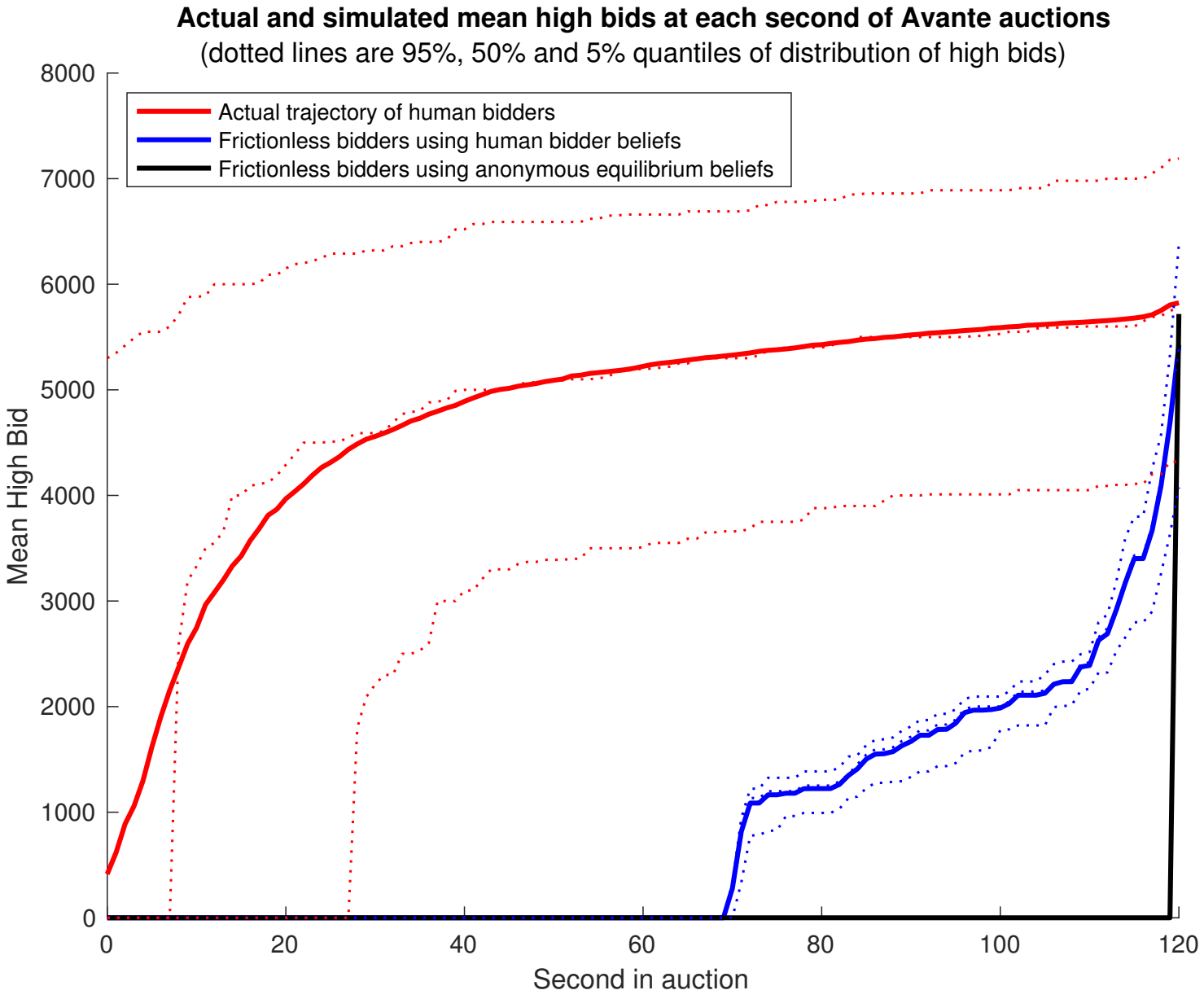
Actual bids by B16 in auction 10007



Counterfactual bids by algorithmic B16 in auction 10007



Convergence of beliefs to anonymous equilibrium



Empirical Auction Design Under Bounded Rationality

- Could the rental car company do even better by switching to a second price auction?
- By the Corollary to Theorem 1 the answer is no: expected revenues from a second price auction are lower than expected revenues from the AE of a first price sealed bid auction. Bidders are shading their bids less in AE than they do in a BNE.
- In AE bidders do not know the distribution of their opponents, so they have no way of adjusting the probability of winning to only consider the effect of their opponents and not their own impact of the distribution of winning bids.
- If the average number of bidders in a typical auction is sufficiently large, the impact of failing to make this adjustment is second order.

Table: Expected winning bids under different scenario and auction mechanisms

	Mean valuations	Mean profits	Mean winning bid	Efficiency
Actual	6843 (64)	659 (45)	5824 (38)	85.0%
First price	7888 (93)	1283 (41)	6604 (55)	100.0%
Second price	7888 (93)	1355 (53)	6532 (72)	100.0%

Who is more Bayesian Humans or ChatGPT?

Tianshi Mu, Pranjal Rawat
John Rust, Chengjun Zhang and Qixuan Zhong
Georgetown University

July 6, 2025

Are People Bayesian?

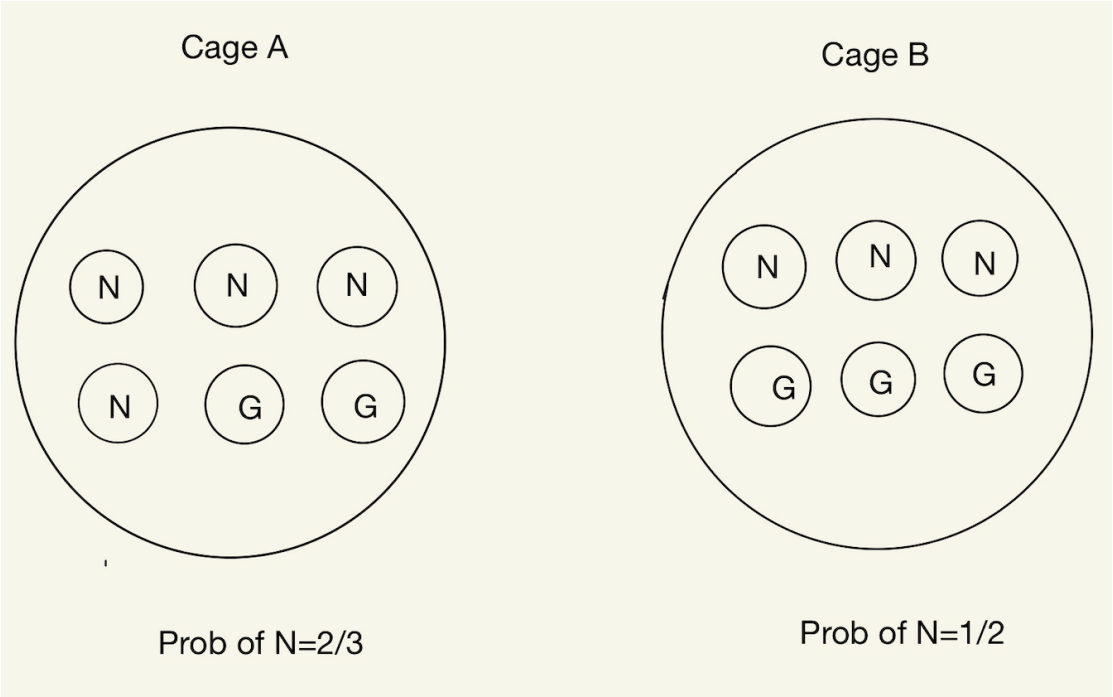
- Do untrained humans act like "intuitive statisticians" and make decisions under uncertainty "as if" they knew statistical decision theory and Bayes Rule?
- This question has been subject to vigorous debate for at least the last 6 decades and has motivated dozens of laboratory experiments.
- Bottom line answer: it depends

Daniel Kahneman, 1934-2024

David Grether, 1938-2021

Are People Bayesian?

- Title of the classic 1995 article by El-Gamal and Grether in *Journal of American Statistical Association*
- Experiment with 257 subjects from 4 UCLA, Pasadena Community College, Occidental College and Cal State Univ Los Angeles
- Subjects were shown samples of 6 balls drawn at random (with replacement) from one of two cages, A or B, and asked to predict which cage the sample came from.



Why should people use Bayes Rule?

- Because we're *greedy*
- A rational person should use an *optimal decision rule* for choosing A or B, one that maximizes their expected payoff from the choice.
- Mathematically, this can be represented by a function, $\delta(d, \pi) \rightarrow \{A, B\}$ that maximizes the probability of selecting the correct cage from which the sample was drawn.
- An optimal decision rule can be expressed in terms of Bayes Rule where $\Pi(A|d, \pi)$ is the posterior probability of A given (d, π).

Theorem *The optimal decision rule is given by*

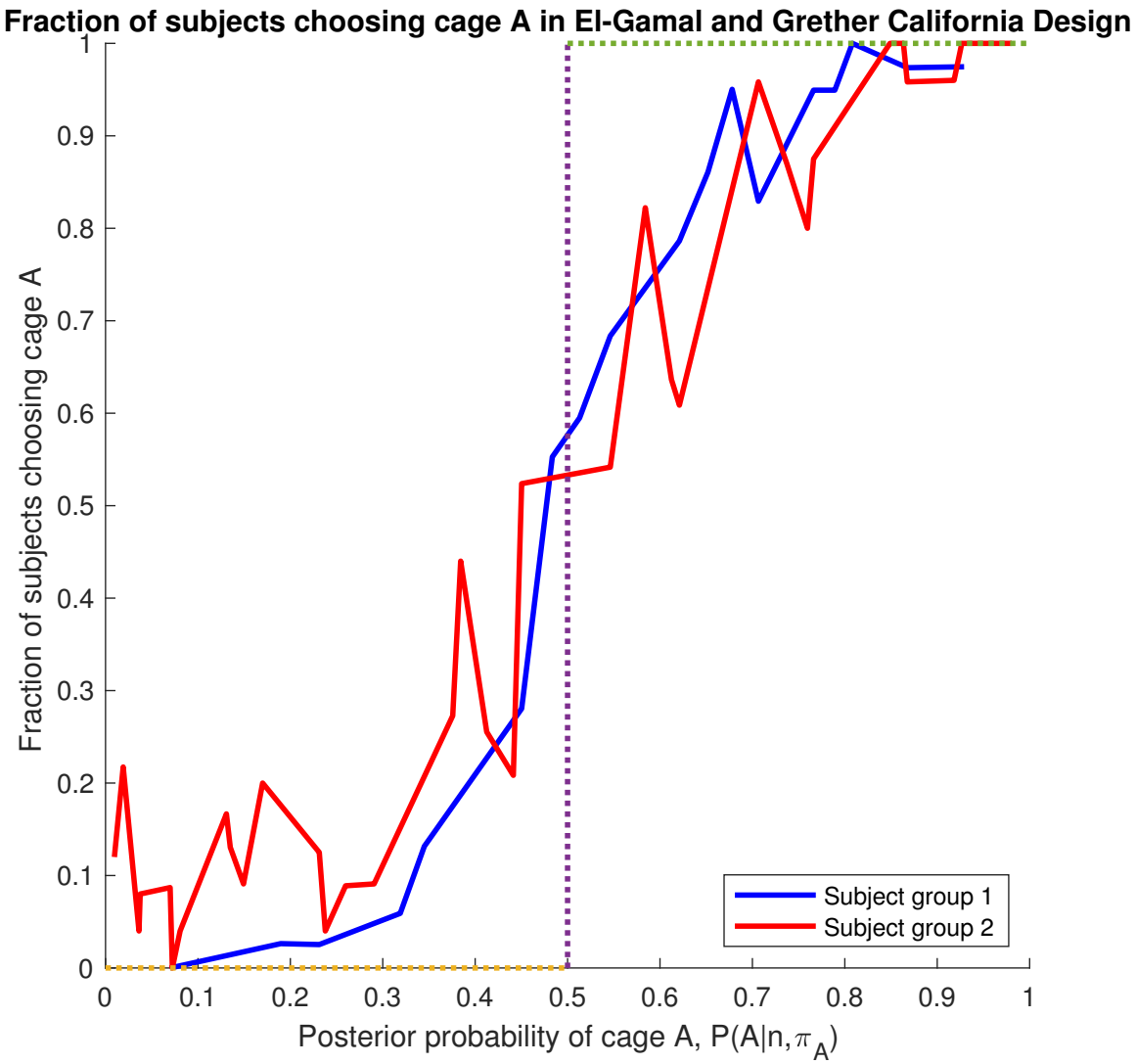
$$\delta(d, \pi) = \begin{cases} A & \text{if } \Pi(A|d, \pi) \geq 1/2 \\ B & \text{otherwise} \end{cases}$$

- However we will show that *you can behave like a Bayesian even if your don't believe like a Bayesian*

Is ChatGPT Bayesian?

- We replicated the experiments of El-Gamal and Grether (using subjects from California and a subsequent experiment at University of Wisconsin in 1999) that involved only binary choices of which bingo cage is more likely as well as experiments by Holt and Smith (2009) that used the *Becker-Degroot-Marshak* (BDM) mechanism to elicit subjective posterior probabilities of each bingo cage, but instead of using human subjects we used various versions of ChatGPT.
- The versions we studied were
  1. ChatGPT 3.5 (introduced in November 2022),
  2. ChatGPT 4 (introduced in March 2023),
  3. ChatGPT 4o (introduced in May 2024).
- We exactly replicated the number of subjects and trials used in El-Gamal and Grether and Holt and Smith to serve as an *exact matching control group*

Which are humans and which is ChatGPT?



Who is more Bayesian: Humans or ChatGPT?

Table: Performance of GPTs and humans in the 6-Ball Experiments

Model number	GPT 3.5	GPT-4	GPT-4o	Homo sapiens
Release date	11/2022	3/2023	5/2024	-300,000 BCE
Weights	175 billion	1.8 trillion	3 trillion?	100 trillion?
Efficiency	84.9 (0.6%)	96.0 (0.3%)	97.5 (0.2%)	96.5 (0.5%)
Accuracy	59.4 (8.4%)	77.7 (7.4%)	85.5 (6.3%)	81.9 (9.1%)
No. of Types	2	1	1	3

Why do we care if ChatGPT is more Bayesian than Humans?

- Because the rapid improvement in the capability and reasoning power of GPTs is starting to threaten human livelihoods. "The Machines They Will Replace Us!"
- The bingo cage example is a simple metaphor or prototype for more complicated, important "high stakes" decision problems where GPT may outperform humans as well.
- For example, consider medical *differential diagnoses* (DDx) is a name for a diagnostic procedure when multiple different diseases or conditions could be the cause of a set of observed symptoms in a patient.
- A good DDx is one where the doctor determine the *most likely disease or medical problem that is consistent with observed symptoms*
- Recent studies (e.g. Goh et al. (2024) and McDuff et al. (2023)) have demonstrated that LLMs and GPTs can outperform human physicians in the quality and accuracy of their differential diagnoses.

The logit model: the simplest possible neural net

- A natural alternative but *reduced-form model* of subject choices is the *binary logit model* which is a 3 parameter model of the probability that a subject chooses cage A given by

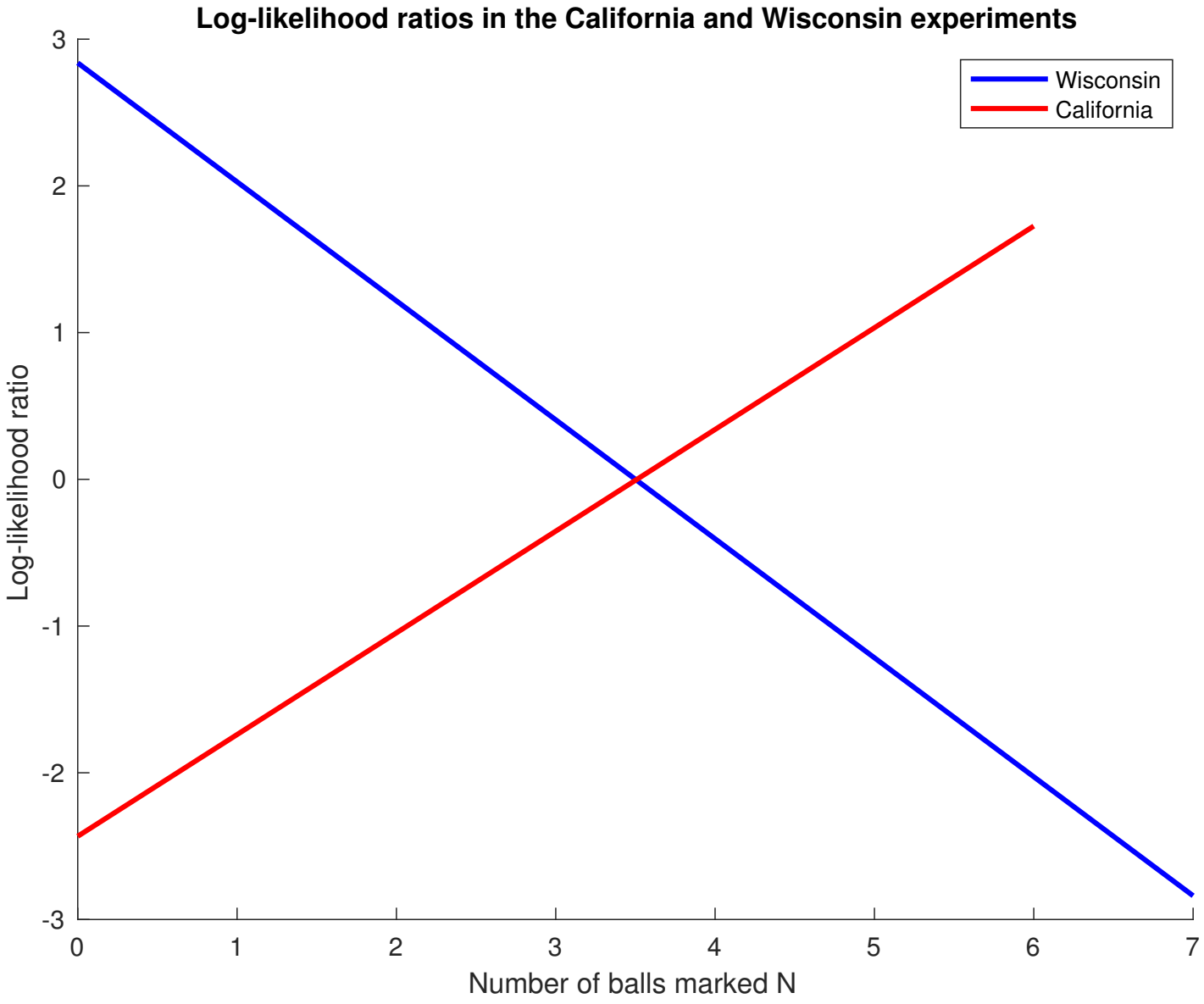
$$P(A|d,\pi) = \frac{1}{1+\exp\{\beta_0 + \beta_1 d + \beta_2 \pi\}}. \tag{1}$$

- The logit model can be regarded as *flexible functional form for approximating the probability of choosing cage A*. But it doesn't have a direct interpretation as a structural model or theory of how people learn. But it is the simplest example of a *single layer feedforward neural network with 2 inputs, 1 output, and the "softmax" or logistic "squashing function."*
- The logit/NN model fits the data significantly better than the EG (single type) model: the log-likelihood for the EG model (4 parameters) is -1942.63 whereas the logit/NN model results in a log-likelihood of -1811.31. A 2 type EG model (7 parameters) fit by EC fits even better: log-likelihood -1699.39. A 2 type logit/NN model (6 parameters) also fit by EC fits even better, as is evident from comparing predicted probabilities to non-parametric CCP.

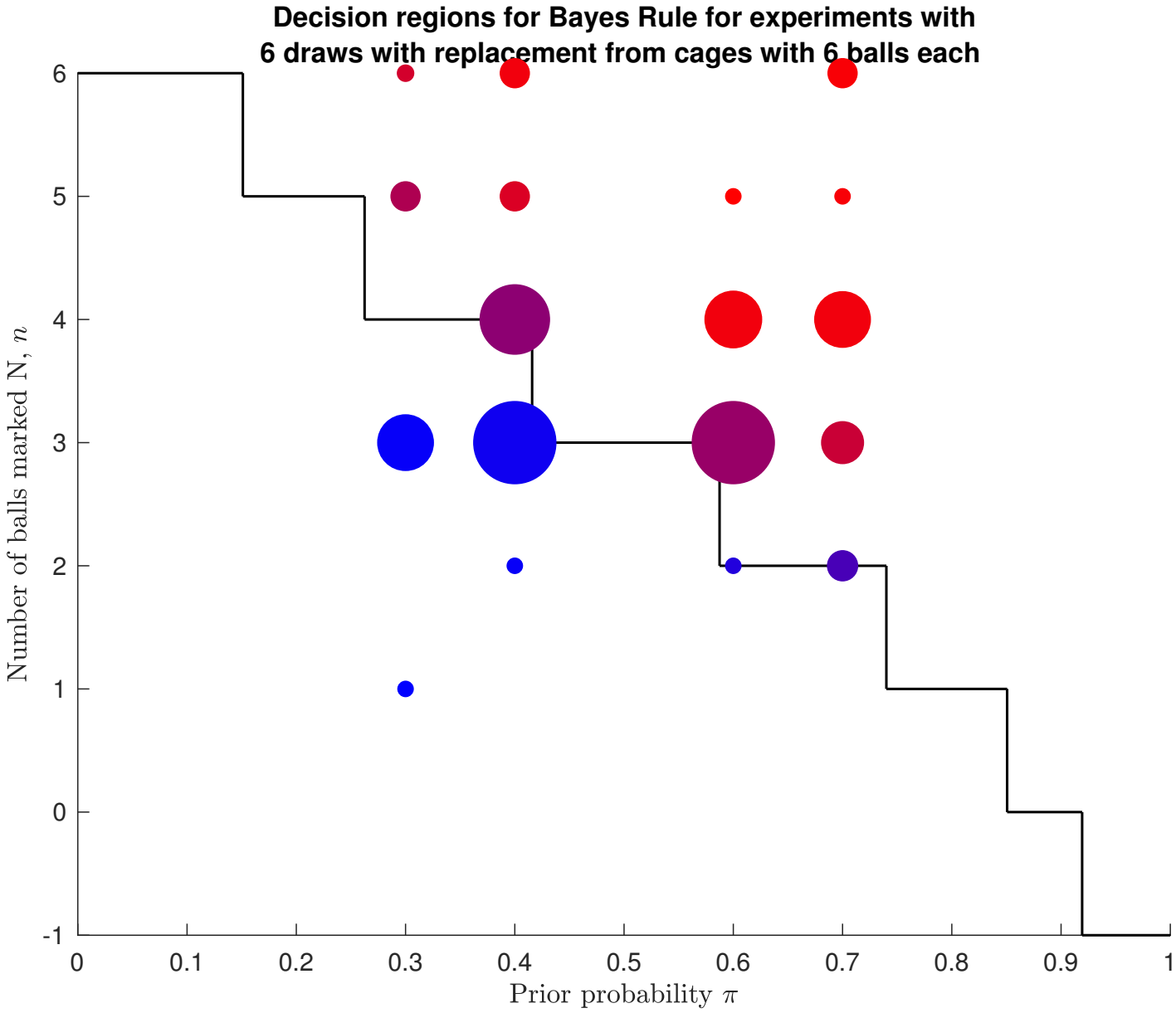
Changing decision rules: the Wisconsin experiments

- in 1999 El-Gamal and Grether conducted further experiments on learning and decision making at the University of Wisconsin involving 81 subjects.
- Subjects participated in two different experimental designs over two successive days:
- **California design:** 6 balls drawn with replacement from either Cage A (4 N balls and 2 G balls) or Cage B (3 N balls and 3 G balls)
- **Wisconsin design:** 7 balls drawn with replacement from either Cage A (4 N balls and 6 G balls) or Cage B (6 N balls and 4 G balls)
- Cage A was selected based on the draw of a 10 sided die, and π was chosen from the 4 values $\pi \in \{.3, .4, .6, .7\}$.
- Under the California design $p_A = .6667$ and $p_B = .5$, while under the Wisconsin design $p_A = .4$ and $p_B = .6$.
- This flips the likelihood ratios, so observing large values of d makes cage A more likely under the California design, but less likely under the Wisconsin design.

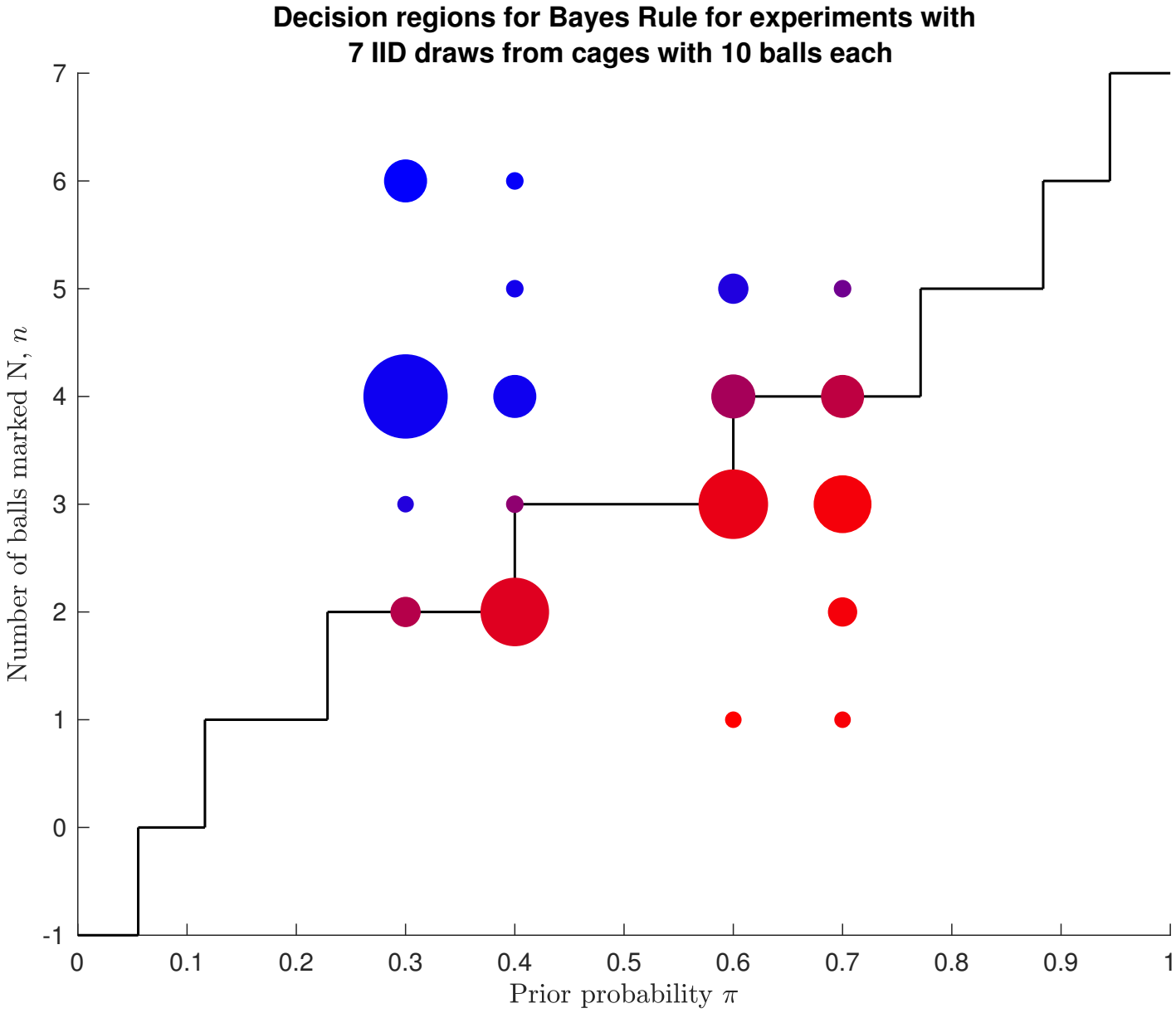
Likelihood ratios in Wisconsin experiments



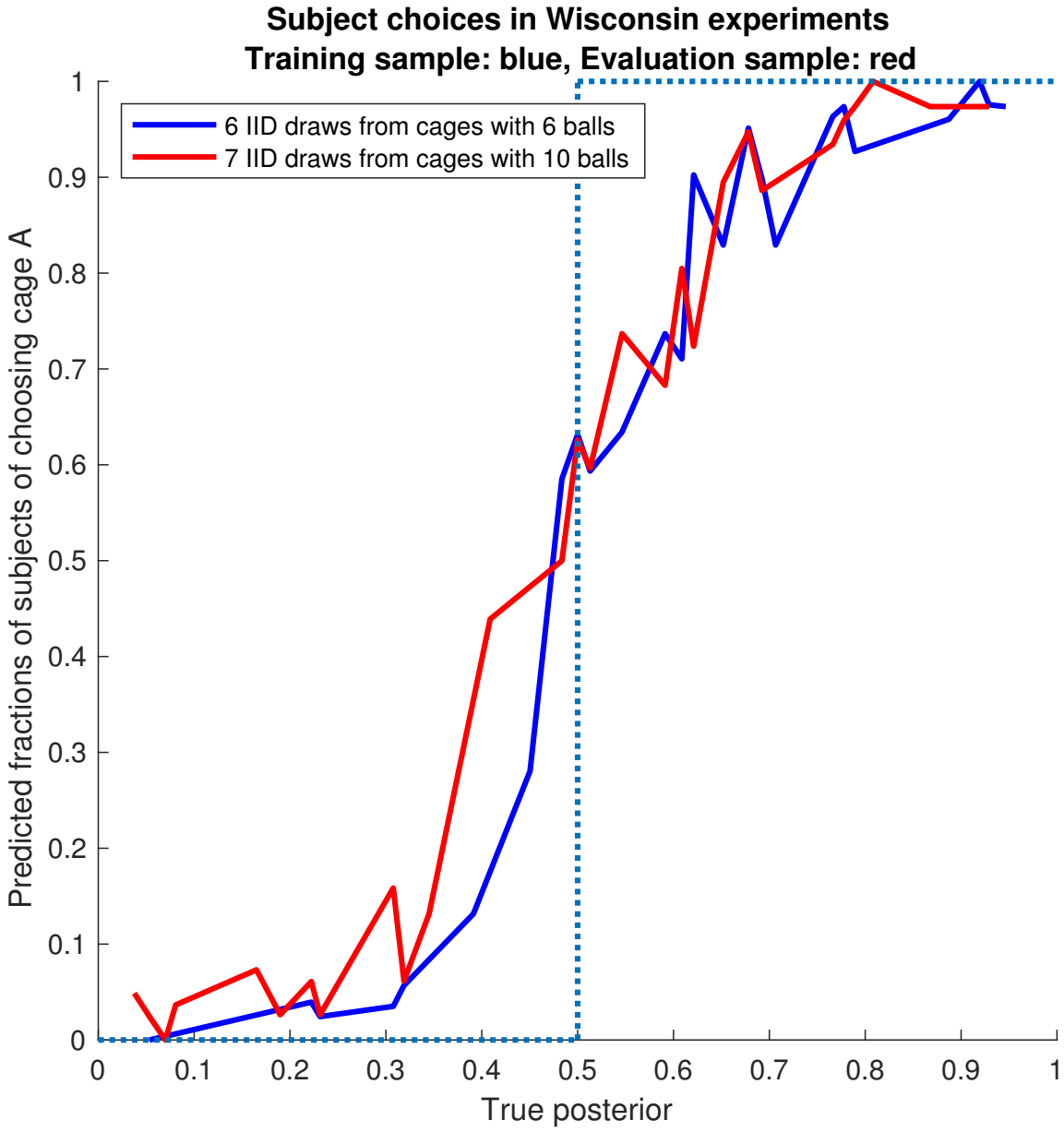
Subject choices under the California design ($D = 6$)



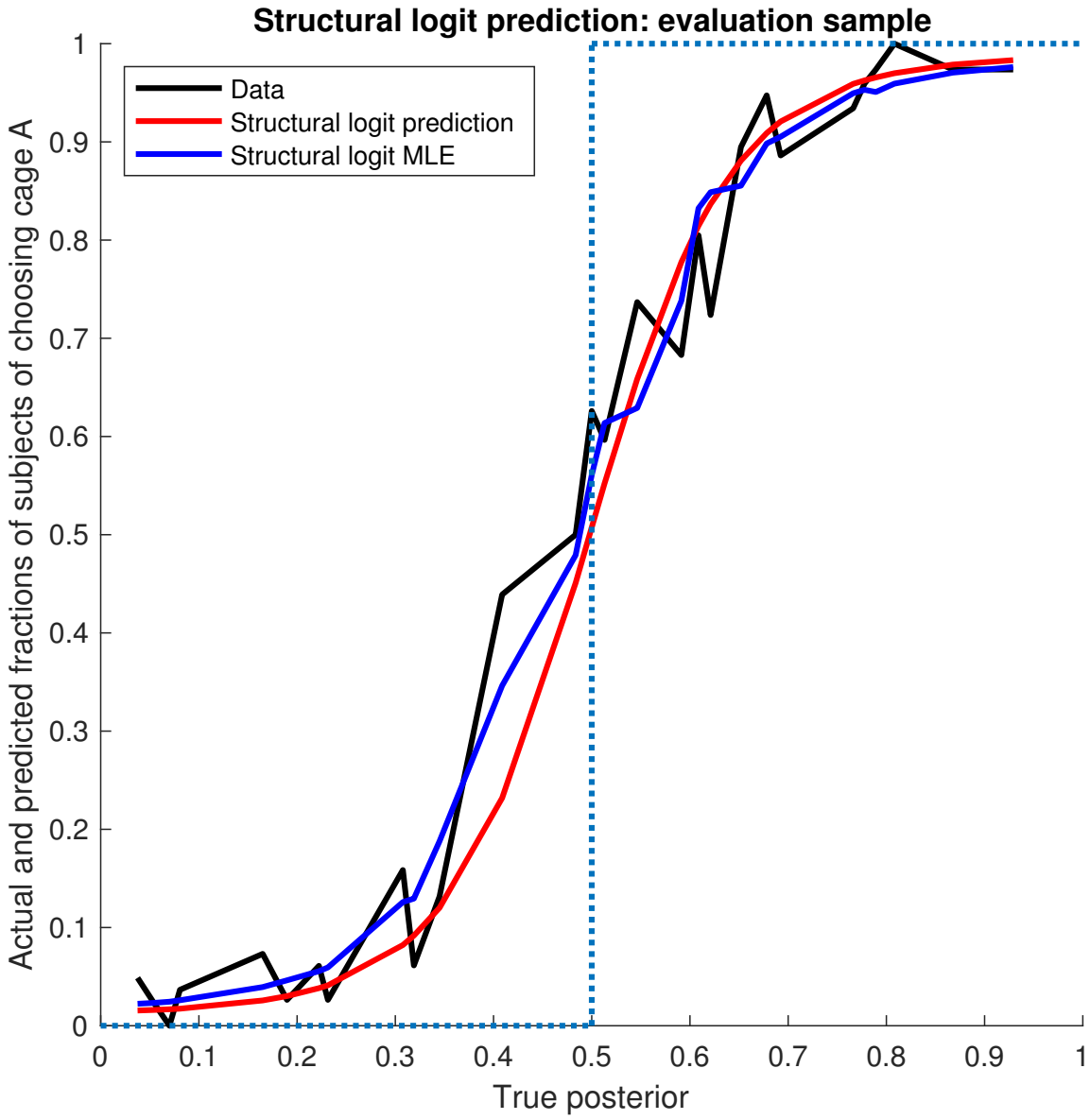
Subject choices under the Wisconsin design ($D = 7$)



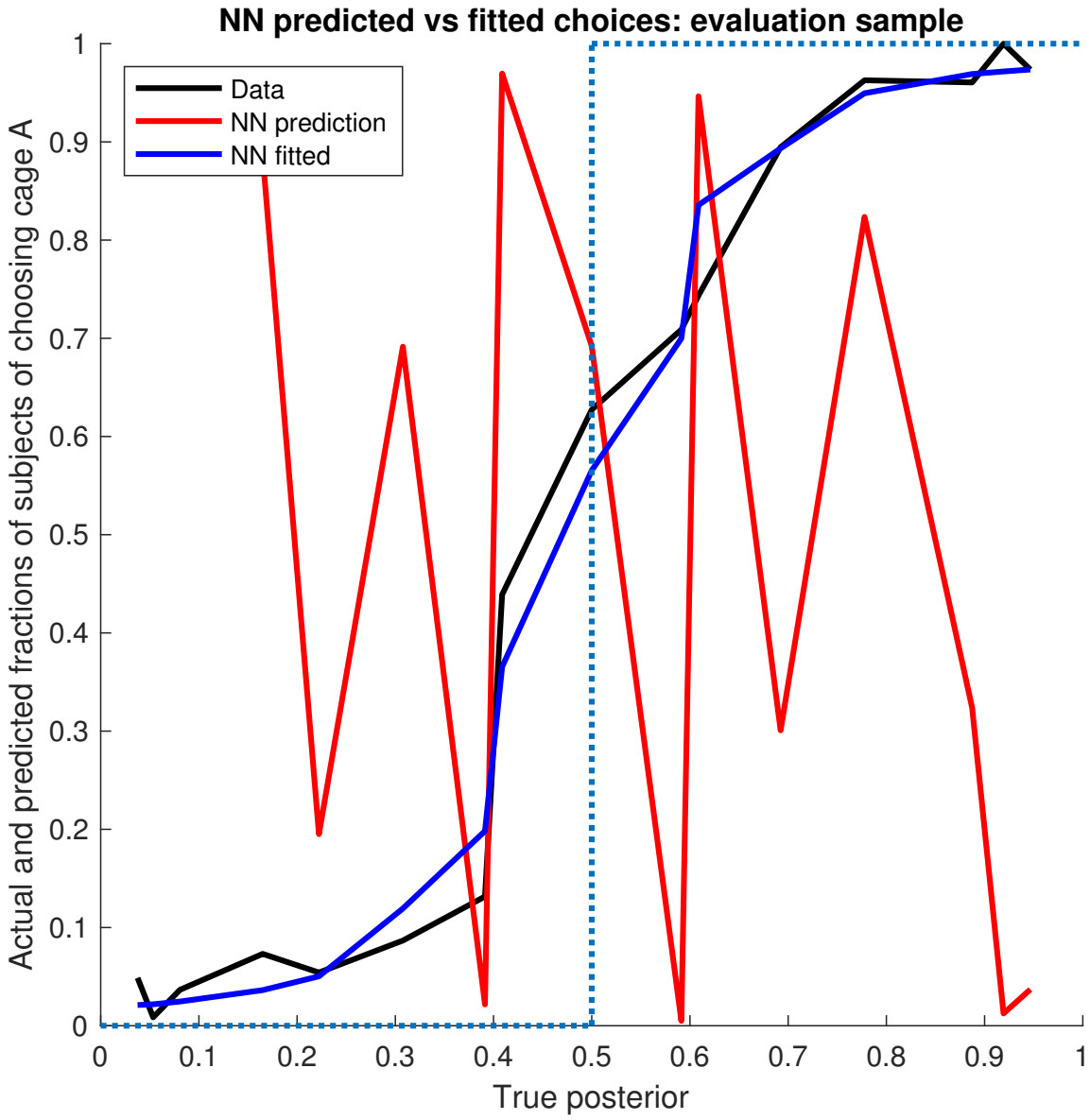
Subject choices in the Wisconsin experiments



The structural logit predicts the change in decision rules



But a reduced-form neural net specification does not



Why does the logit model do so poorly?

- Because subjects pay attention to more than (d, π) but the reduced form logit/neural net model only has (d, π) as inputs.
- The Wisconsin experiments convincingly demonstrate that human subjects are paying attention to *5 pieces of information* (d, π, p_A, p_B, D)
- In the California design, $D = 6$, $p_A = 2/3$ and $p_B = 1/2$.
- In the Wisconsin design, $D = 7$, $p_A = .4$ and $p_B = .6$.
- A neural net that does not use all 5 pieces of information cannot approximate human behavior in new experimental designs (policy shifts) where D, p_A, p_B have changed!

Structural logit model of subjects' choice

- We posit that subjects' beliefs and therefore their choices are function of *transformed inputs* (LLR, LPR) where LLR is the log likelihood ratio and LPR

$$\begin{aligned} \text{LLR} &= f(d|p_A, D)/f(d|p_B, D) \\ \text{LPR} &= \pi/(1-\pi) \end{aligned}$$

- Notice that (LLR, LPR) are sufficient statistics for all 5 pieces of information needed to make an informed prediction of the cage that was selected to draw the sample.
- We allow for *calculational errors* in computing (LLR, LPR) from the 5 inputs (d, π, p_A, p_B, D).
- We assume that subjects have a *subjective posterior probability* that depends on a random *calculational error* ν given by

$$\Pi_s(A|d, \pi, p_A, p_B, D, \nu) = \frac{\exp\{\beta_0 + \beta_1 \text{LLR} + \beta_2 \text{LPR} + \nu\}}{1 + \exp\{\beta_0 + \beta_1 \text{LLR} + \beta_2 \text{LPR} + \nu\}}.$$

- Notice the correct Bayesian posterior is nested in this specification when $(\beta_0, \beta_1, \beta_2) = (0, 1, 1)$ and $\nu = 0$ with probability 1.

Structural logit model of subjects' choice

- Given the subjective posterior beliefs, the choice of A or B depends on which gives the subject a greater expected payoff. Assume the payoff for a correct choice is R but also assume that there are additional idiosyncratic "choice distractions" $(\epsilon(A), \epsilon(B))$ that affect the final choice. Then the decision rule is given by

$$\delta(I, \nu, \epsilon) = \begin{cases} A & \text{if } R\Pi_s(A|I, \nu) + \epsilon(A) \geq R\Pi_s(A|I, \nu) + \epsilon(B) \\ B & \text{otherwise} \end{cases}$$

  where $I = (d, \pi, p_A, p_B, D)$ is the observed public information in each trial and (ν, ϵ) is private information only observed by the subject.

- Assuming ϵ and ν are independently distributed and ϵ is bivariate extreme value with scale parameter σ, we get the following logit CCP or decision rule

$$P(A|d, \pi, p_A, p_B, D, \nu) = \frac{1}{1 + \exp\{[1 - 2\Pi_s(A|d, \pi, p_A, p_B, D, \nu)]\, R/\sigma\}}.$$

Structural logit model of subjects' choice

- Since ν is also unobservable we need to integrate it out of the decision rule to get the final CCP or "mixed" decision rule. Assume that $\nu \sim N(0, \eta^2)$. Then we have

$$P(A|d, \pi, p_A, p_B, D) = \int_\nu P(A|d, \pi, p_A, p_B, D, \eta\nu)\phi(\nu)d\nu.$$

- This is a 5 parameter model where $\theta = (\beta_0, \beta_1, \beta_2, \sigma, \eta)$ are "structural parameters"
- Let $y_{ts} = 1$ if subject s chose cage A in trial t in response to information (d_{ts}, π_{ts}). Then the likelihood function is given by

$$\begin{aligned} L(\theta) = \quad & \prod_{s=1}^{S}\prod_{t=1}^{T_s} P(A|d_{ts}, \pi_{ts}, p_A, p_B, D, \theta)^{y_{ts}} \times \\ & [1 - P(A|d_{ts}, \pi_{ts}, p_A, p_B, D, \theta)]^{1-y_{ts}}. \end{aligned}$$

where (p_A, p_B, D) are the experimental design parameters.

The structural logit model is a two layer neural network

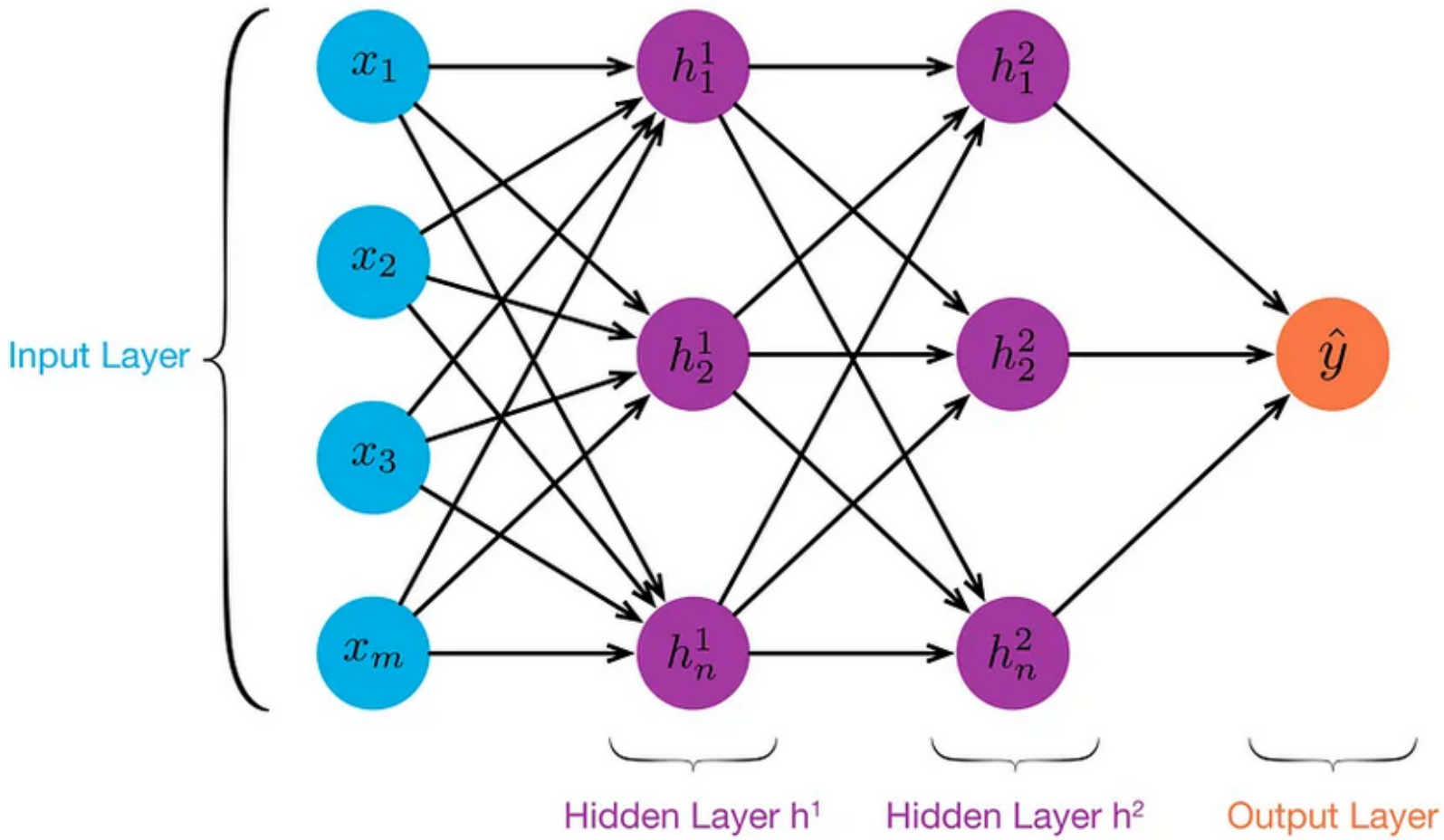
When $\eta = 0$, the structural model can be viewed as simple two layer feedforward neural network: a "logit inside a logit".

1. **Layer 1** uses the "transformed inputs" (LLR, LPR) to produce the subjective posterior, $\Pi_s(A|\text{LLR}, \text{LPR}, \beta)$ as output, where β_0 is the bias term and (β_1, β_2) are the "input weights".
2. **Layer 2** uses the output from layer 1 as input and generates the logit CCP as the single output.

$$P(A|\text{LLR}, \text{LPR}, \sigma, \beta) = P(A|\Pi_s(A|\text{LLR}, \text{LPR}, \beta), \sigma) \tag{2}$$

   where the 2nd layer bias term is R/σ and weight on the layer 1 input is $2R/\sigma$.
3. If we add additional layers, we can use them to approximate the transformed inputs (LLR, LPR) from the "raw inputs" (d, π, p_A, p_B, D).

Example classification net for learning Bayes Rule



Identification of subjective beliefs

- Maximum likelihood estimation of the structural logit model can be viewed as an exercise in "revealed beliefs" using subjects' binary choices only. Are these beliefs identified?
- No, not in general. For example the optimal Bayesian decision rule δ^* is nested within the structural logit model when $\theta^* = (\beta_0, \beta_1, \beta_2, \sigma, \eta) = (0, 1, 1, 0, 0)$. But now consider for any $\lambda > 0$ a non-Bayesian decision maker given $\theta_\lambda = (0, \lambda, \lambda, 0, 0)$. Then it is not hard to show that for any (d, π, p_A, p_B, D) we have

$$\Pi(A|d, \pi, p_A, p_B, D) \leq 1/2 \iff \Pi_s(A|d, \pi, p_A, p_B, D, \theta_\lambda) \leq 1/2.$$

- Thus *you don't have to believe like a Bayesian to act like a Bayesian!*

Identification of subjective beliefs when $\sigma > 0$

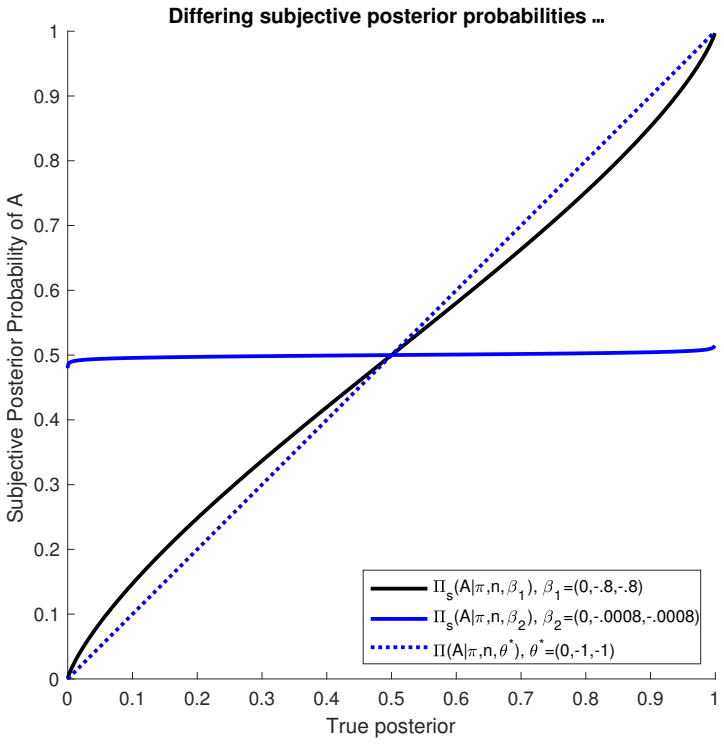
- When $\sigma > 0$ we can use the random decision mistakes to show that the model is identified if we have enough observations at extreme values of π where the true posterior is close to 0 or 1 to identify σ.

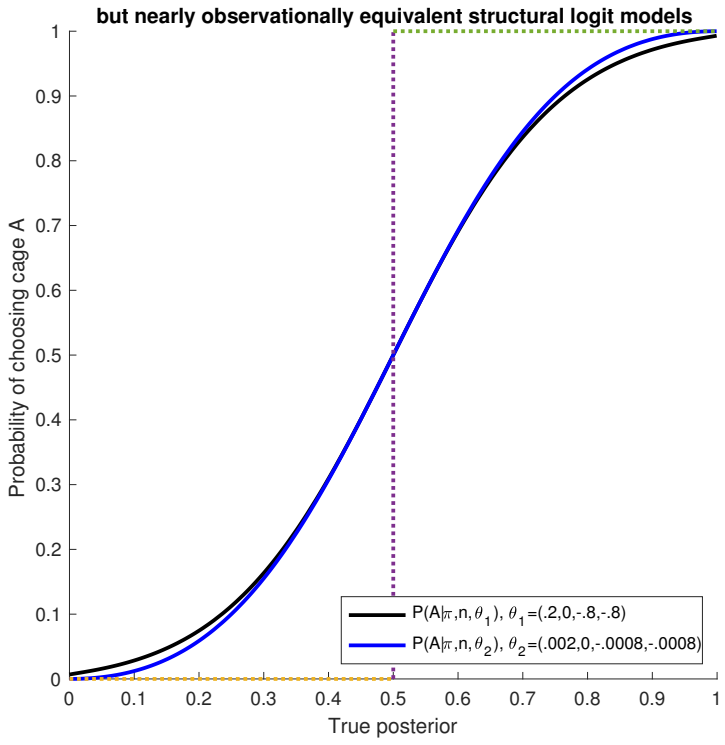
Lemma

Identification of subject beliefs when $\sigma > 0$ *Assume that $\eta = 0$. When $\sigma > 0$, all four parameters of the structural logit model are identified, so the subject's subjective beliefs can be identified from knowledge of their decision rule $P(A|d, \pi, p_A, p_B, D)$, where the latter is identifiable given sufficient experimental data on a subject's choices for different values of (d, π, p_A, p_B, D).*

- But this result is unrealistic in practice since cannot do that much experimentation with a given subject.

Example of weak identification of beliefs

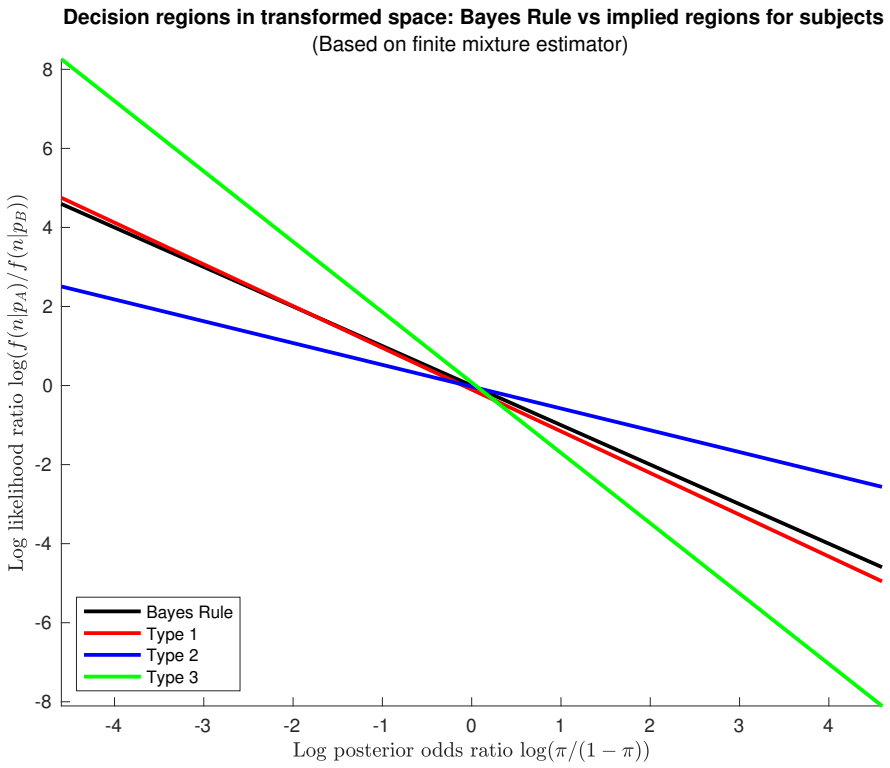


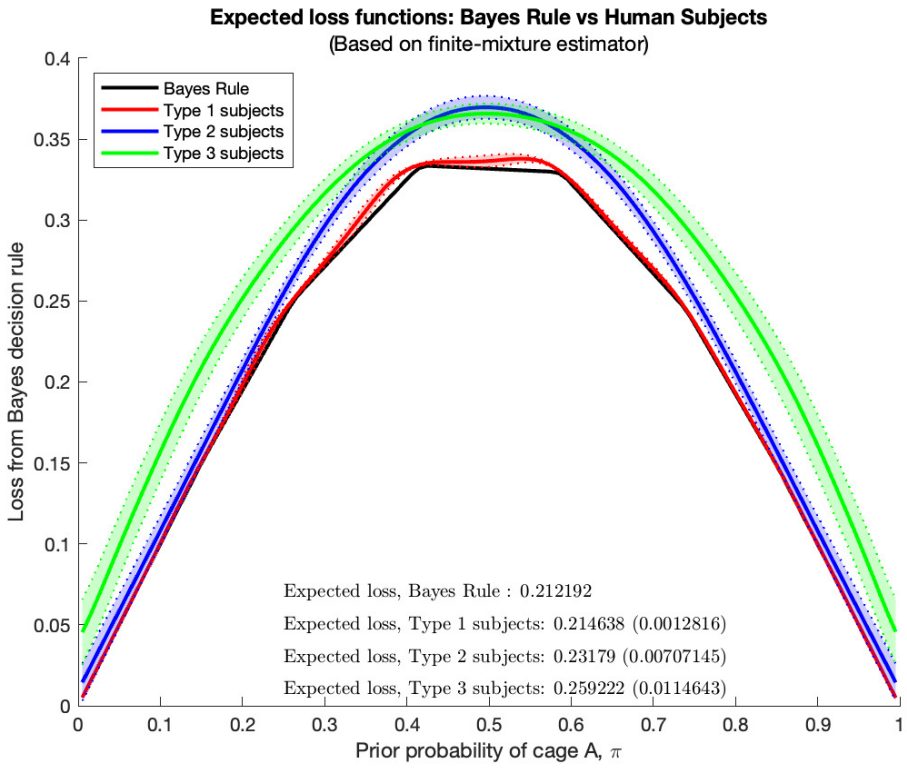


Results from reanalysis of California data

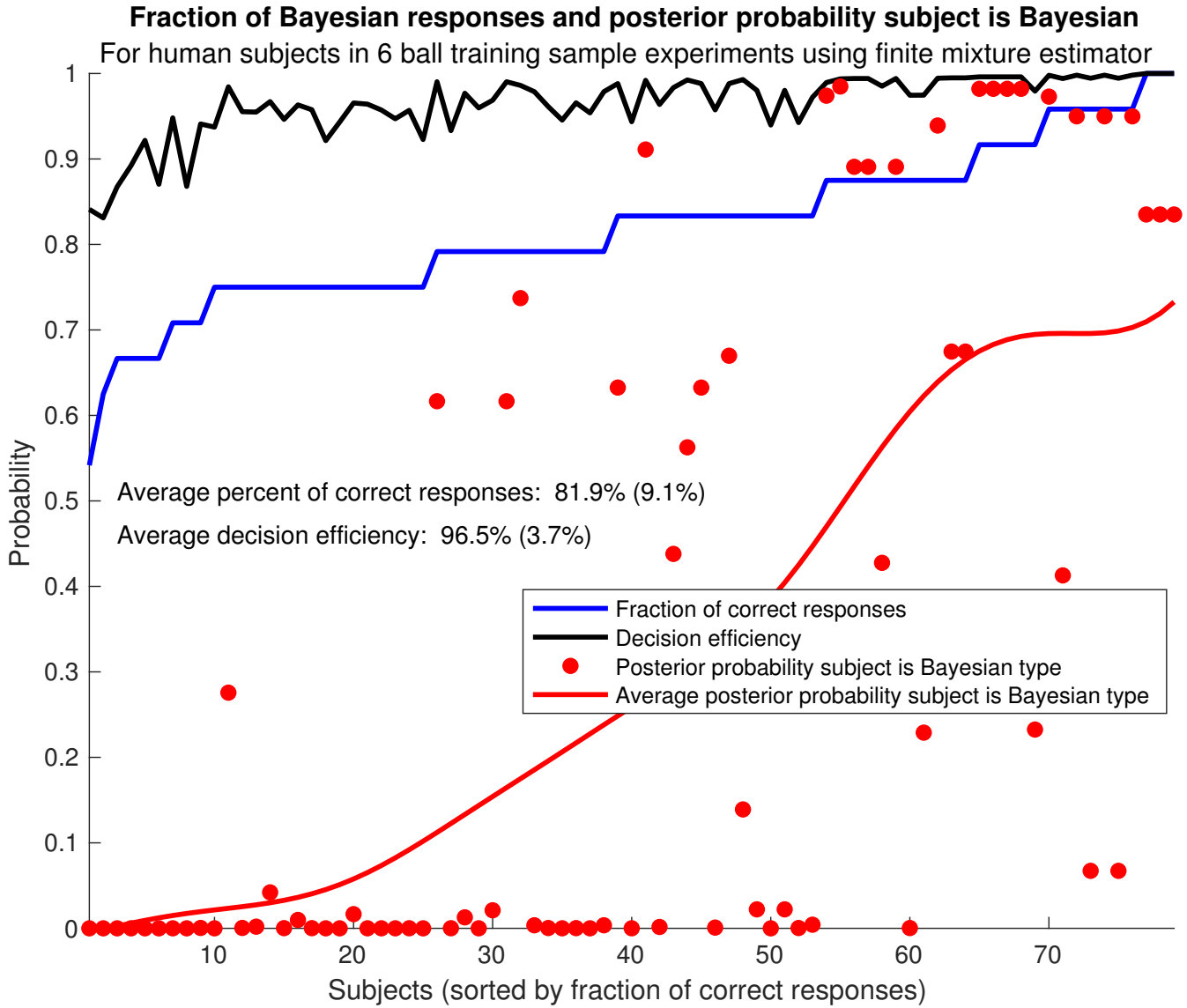
Model	Parameters	Log-likelihood	AIC
El-Gamal/Grether	4	-1952	3912
Structural probit	3	-1847	3700
Reduced-form logit	3	-1821	3648
Noisy Bayesian	1	-1801	3604
Structural logit	4	-1773	3554
Neural network	5	-1772	3554

Classification hyperplanes and Loss functions

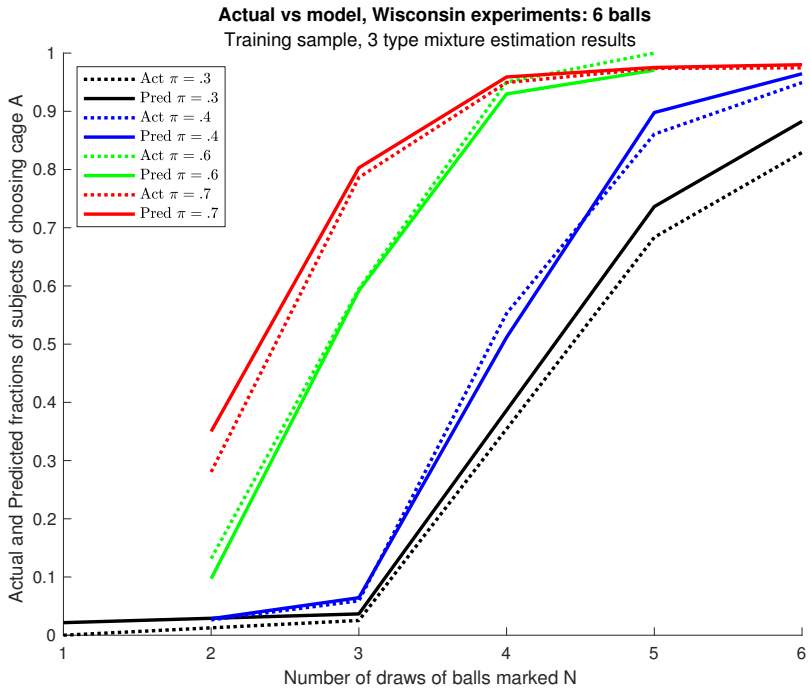


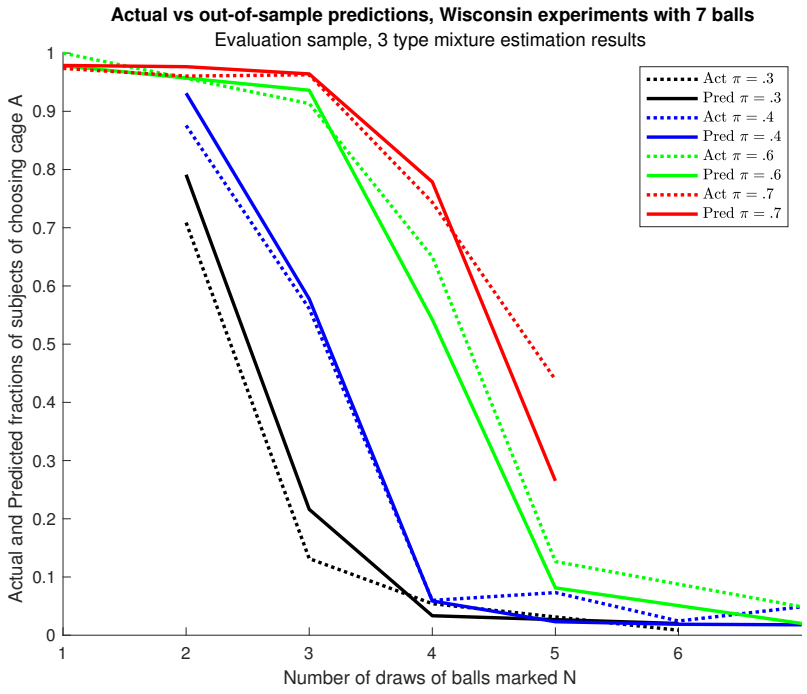


Subject accuracy/efficiency: Wisconsin 6 ball experiments



Out-of-sample predictive accuracy of structural logit model





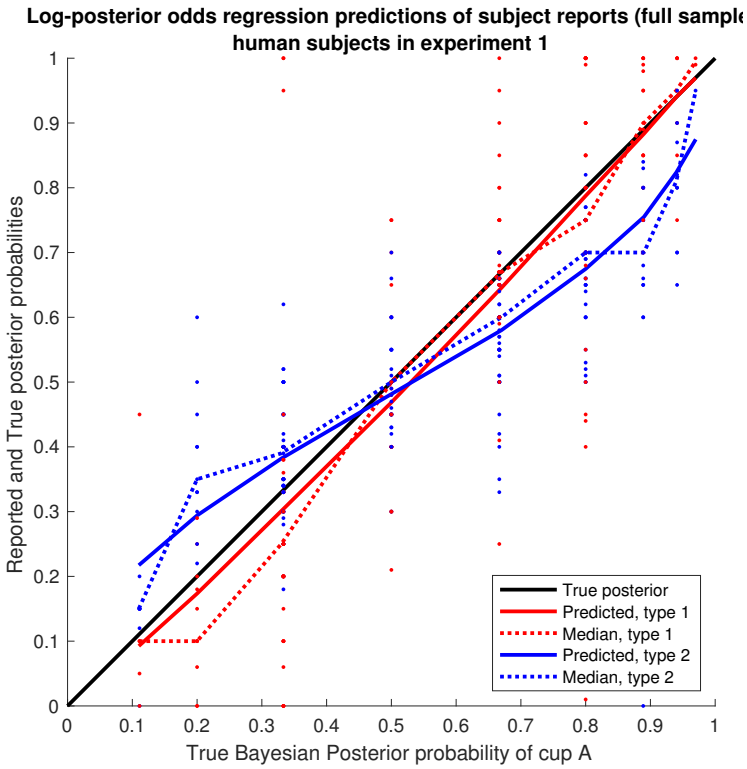
Reanalysis of Holt and Smith experiments

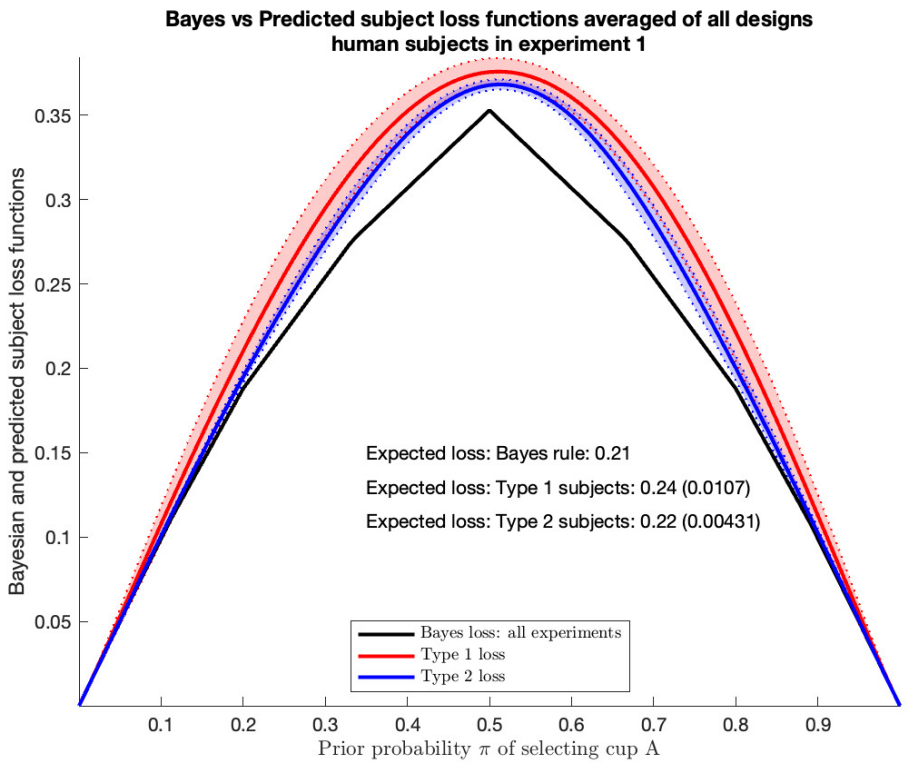
- These experiments directly elicted subjective posterior probabilities using the *Becker-Degroot-Marshak* (BDM) mechanism.
- It involves a second stage gamble to incentivize truthful reporting of the subjective posterior p.
- If subject reports a probability p_r that cage A is the correct cage, this leads to a gamble $\tilde{G}_R(p_r)$ that pays off amount R based on a random draw $\tilde{p} \sim U(0,1)$.
  1. if $\tilde{p} \leq p_r$, then the subject receives payment R if the true cage is cage A
  2. if $\tilde{p} > p_r$, then the subject receives R with probability $\tilde{p}$
- The subject's expected payoff from reporting p_r is

$$E\{\tilde{G}_R | p_r, d, \pi, p_A, p_B, D\} = R\left[\frac{1-p_r^2}{2} + p_r \Pi_s(A|d, \pi, p_A, p_B, D, \nu)\right],$$

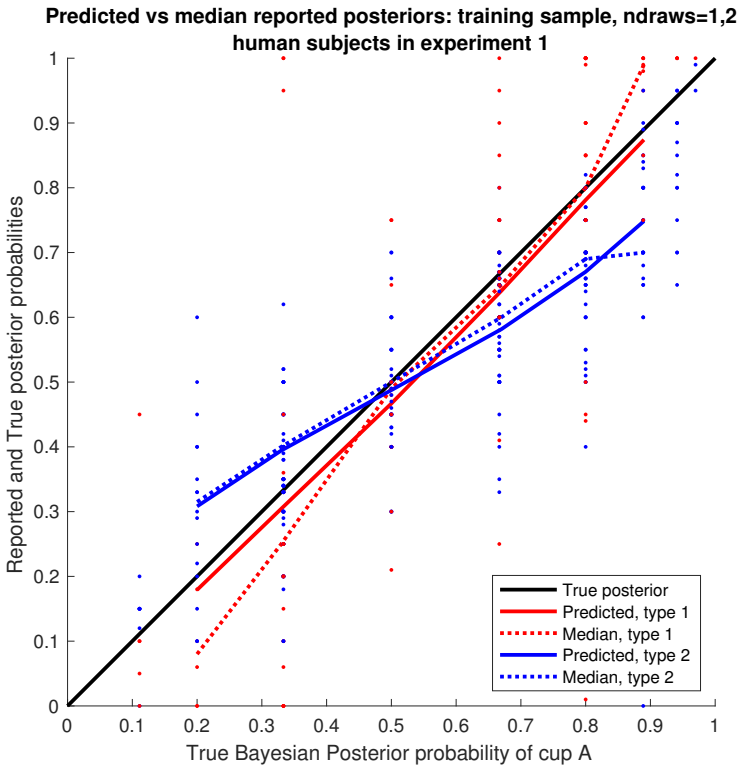
- The optimal report is the subject's true posterior probability $p_r^* = \Pi_s(A|d, \pi, p_A, p_B, D, \nu)$.

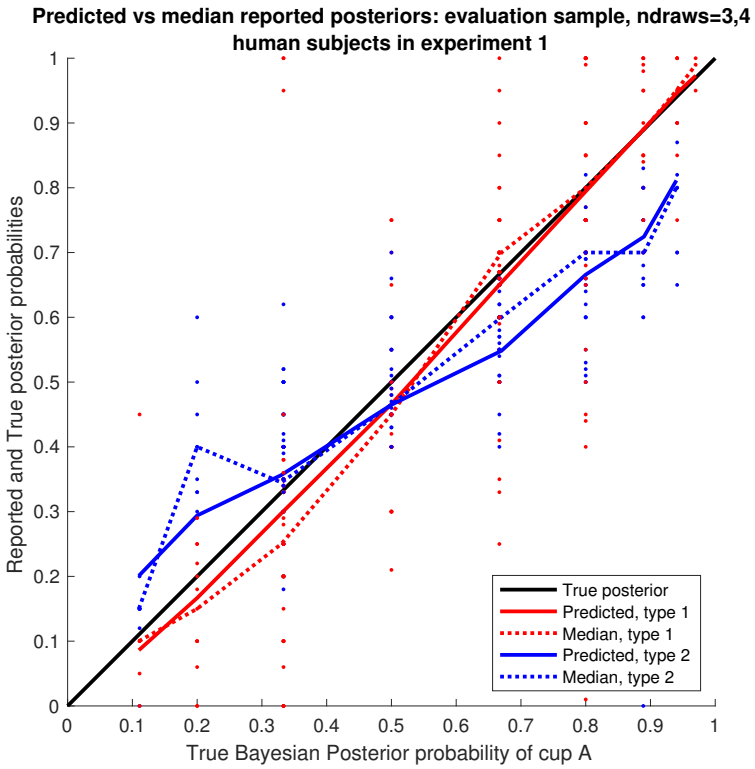
Beliefs and loss functions: Holt and Smith experiment 1





Stability of Beliefs: Holt and Smith experiment 1





Conclusions from reanalysis of human subject data

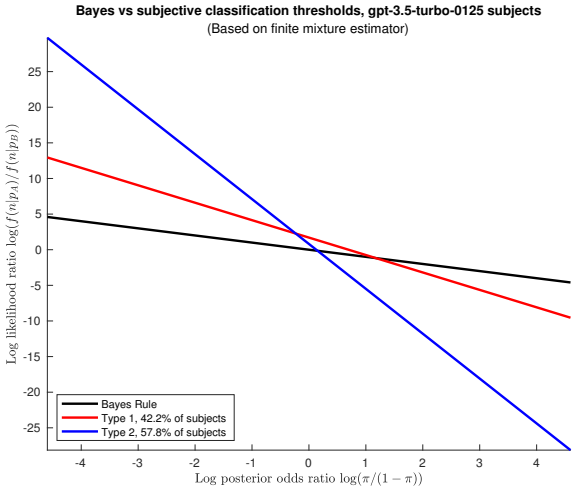
- We reject the hypothesis that all subjects are Bayesians or even noisy Bayesians
- However we do find significant subsets of subjects who are noisy Bayesians
  1. 49 to 79% in El-Gamal and Grether's California experiments
  2. 32% in their Wisconsin 6 ball experiments
  3. 77% in their Wisconsin 7 ball experiments
  4. 45% in Holt and Smith's experiment 1
  5. but none in their web-based experiment 2
- Decision efficiency is high overall
  1. 93% in El-Gamal and Grether's California experiments
  2. 97% in their Wisconsin 6 ball experiments
  3. 96% in their Wisconsin 7 ball experiments
  4. 96% in Holt and Smith's experiment 1
  5. 91% in their web-based experiment 2

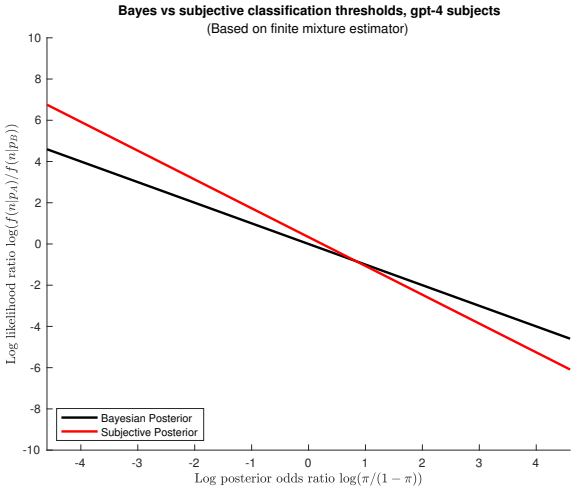
Results for ChatGPT in 6 ball design

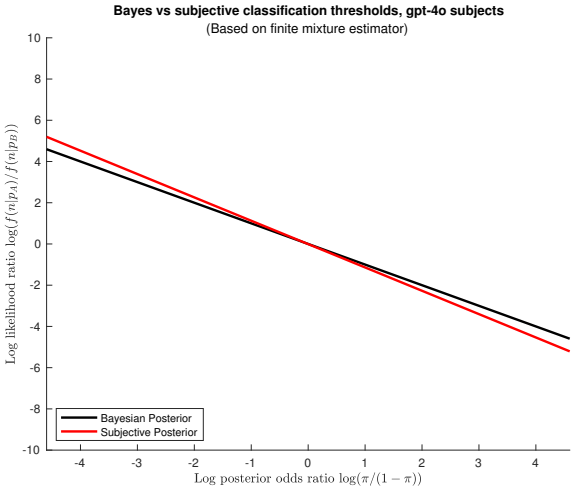
- We created a matching set of trials for GPT subjects using the actual trial outcomes for the 6 ball design of El-Gamal and Grether (1995 and 1999)
- We intentionally introduced subject-specific heterogeneity in the GPT subjects by randomly choosing different *temperature* settings.
- Higher temperature implies more randomness in the GPT response so it is akin to adding heterogeneity in the σ or η noise parameters of the structural logit model.
- Unlike human subjects, this is *observed heterogeneity.*
- However we analyzed the results using the same methods we used to handle unobserved heterogeneity in human subjects, i.e. the EC algorithm and finite mixture methods for unobserved heterogeneity.

Estimated classification hyperplanes

Classification hyperplanes for GPT 3.5, 4 and 4o

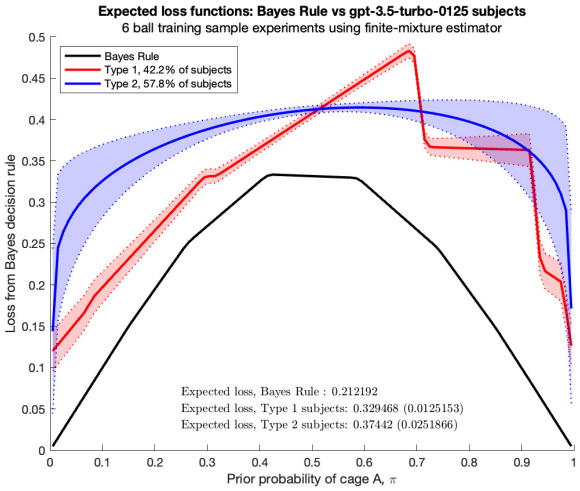


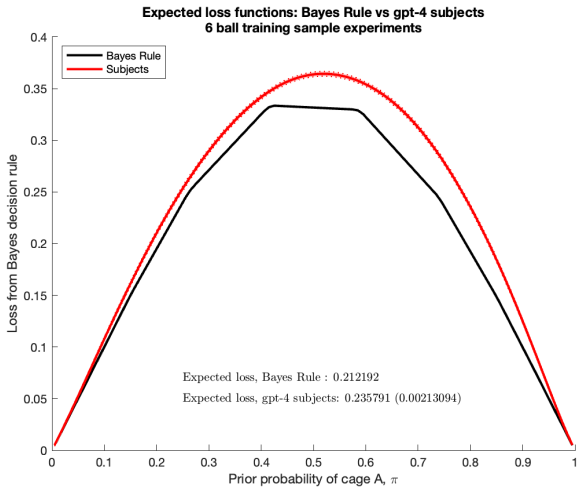


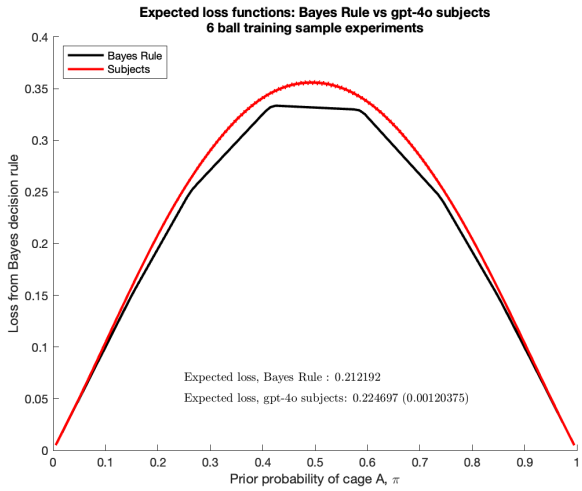


Loss Funtions

Estimated loss functions for GPT 3.5, 4 and 4o

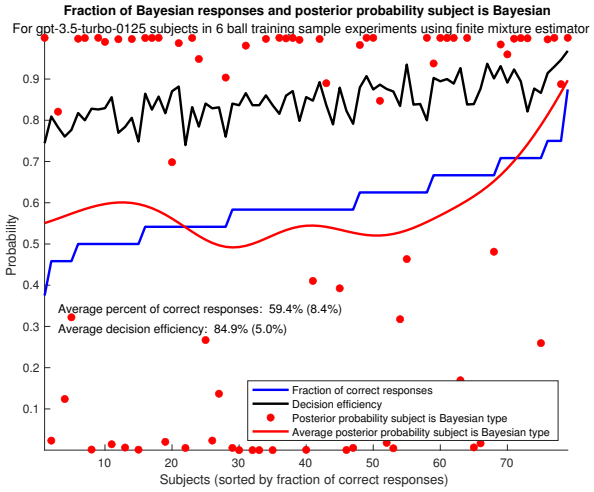


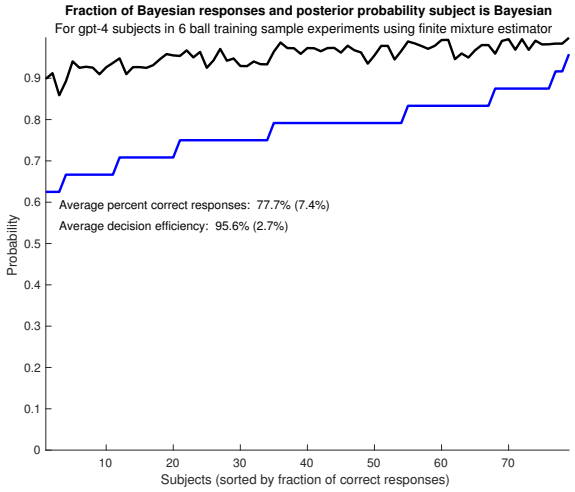


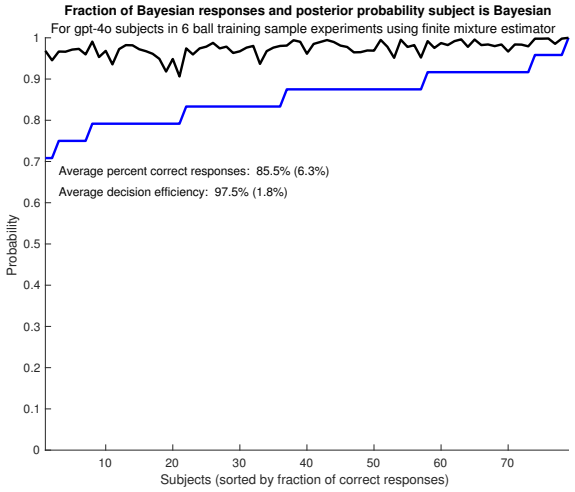


Accuracy and Efficiency

Subject-specific accuracy and efficiency for GPT 3.5, 4 and 4o







Comparison of Human and GPT performance

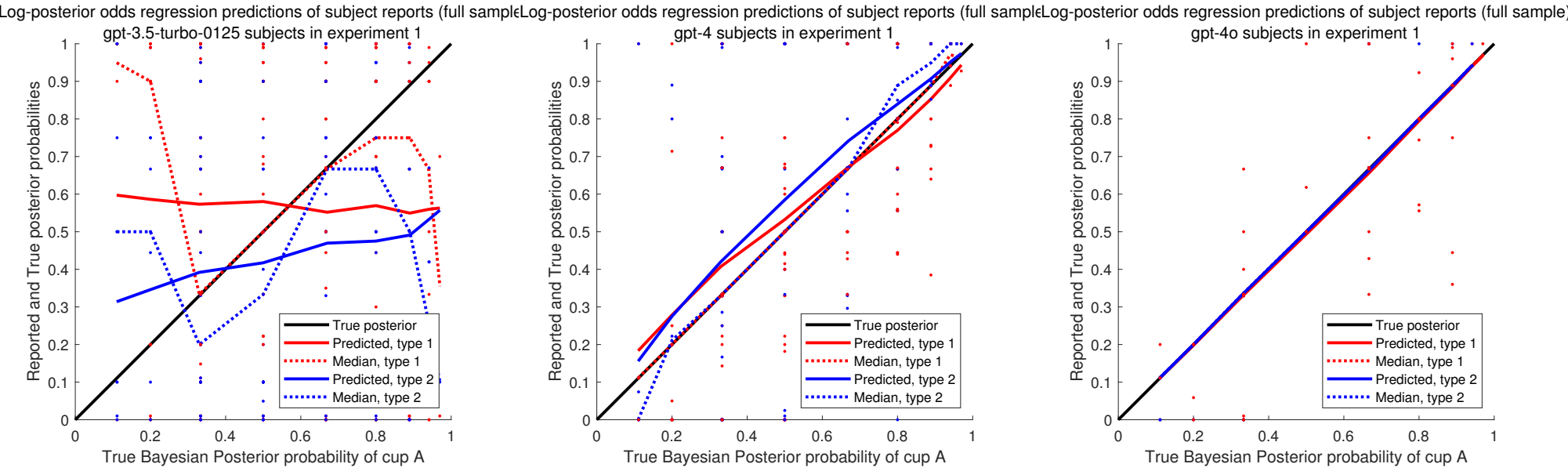
	GPT 3.5	GPT-4	GPT-4o	Humans
Efficiency	84.9 (0.6%)	96.0 (0.3%)	97.5 (0.2%)	96.5 (0.5%)
Accuracy	59.4 (8.4%)	77.7 (7.4%)	85.5 (6.3%)	81.9 (9.1%)
No. of Types	2	1	1	3

Results for ChatGPT in Holt and Smith design

- Recall that Holt and Smith directly elicited subjective posterior values using BDM mechanism
- Probably they gave human subjects some coaching: "trust us, it is in your best interest (to maximize your payout) to report your best estimate of the probability the sample was from cup A"
- But we did not give ChatGPT this sort of coaching. It had to be able to do the calculus to figure out the optimal report p_r in the second stage gamble $\tilde{G}_R(p_r)$.

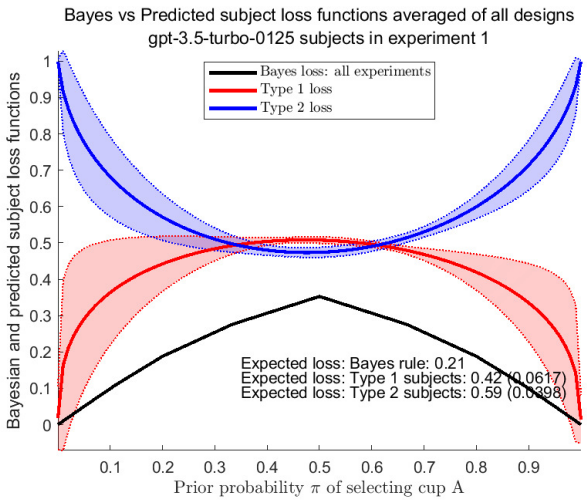
Elicited Subjective Posteriors

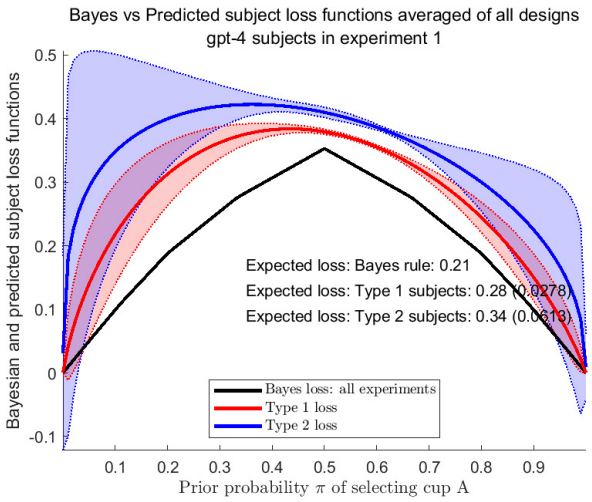
Estimated vs actual elicited posteriors for GPT 3.5, 4 and 4o

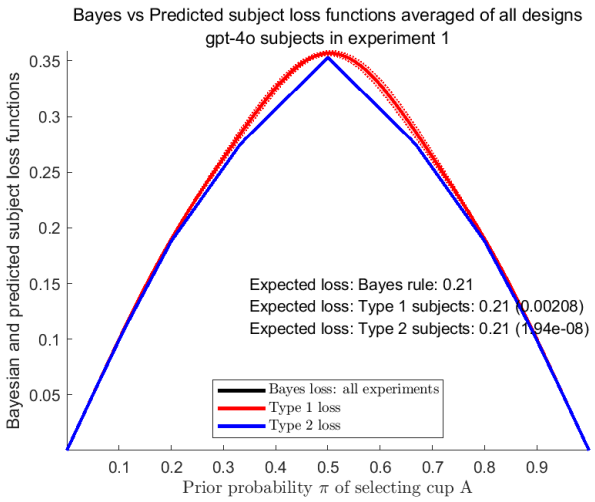


Implied Loss Functions

Implied loss functions for GPT 3.5, 4 and 4o







Comparison of Human and GPT performance

	GPT 3.5	GPT-4	GPT-4o	Humans
Efficiency	75.0 (1.0%)	93.0 (0.4%)	99.4 (0.3%)	96.0 (0.7%)
Accuracy	58.1 (0.1)	84.0 (0.1)	98.2 (0.02)	87.4 (0.1)
R^2	0.7	41.8	88.0	63.5
No. of Types	2	2	2	2

Testing for context effects in GPT responses

Model	Accuracy (Bingo)	Accuracy (Pond)	t-stat	p-value
GPT-4o	0.94	0.92	0.63	0.53
GPT-4	0.85	0.83	0.45	0.66
GPT-3.5	0.63	0.69	-0.84	0.40

Conclusions from comparison of human and GPT

- We find a rapid improvement in the accuracy and decision efficiency in successive versions of GPT, from sub-human performance for GPT 3.5, to approximately human for GPT-4, and super-human performance for GPT-4o.
- Three key aspects are behind this improvement: (i) decreasing bias, (ii) increasingly equal weight on the data and the prior, and (iii) lower levels of idiosyncratic calculational and decision noise.
- We find less unobserved heterogeneity for GPTs than humans in the Wisconsin experiments, especially for GPT-4 and 4o, where we find only one type of subject, noisy Bayesians.
- In the Holt and Smith experiment, 40% of subjects are classified as perfectly Bayesian, while the remaining 60% are best described as noisy Bayesians.
- The structural logit model provides an increasingly good approximation to the behavior of successive generations of GPT.

Error analysis of GPT responses

- Most econometric models treat humans as black-boxes and apart from some research from neuroscience, we do not know exactly how humans process information.
- However, with GPTs we have the unique advantage of knowing both how their neural "brains" compute information and the textual response which contains their reasoning behind their answers.
- Therefore, it makes logical sense to try to parse this text to understand how and why GPTs made mistakes, create a taxonomy of errors and obtain error distributions across models.
- There are some complications. There is more than one way to answer the question correctly and even more ways to make mistakes. The approach we adopted was twofold: (1) create a process for automated grading using a superior GPT model, o1, and (2) manual (human) grading of a smaller sample of GPT4o's responses.

Automated grading of GPTs by GPTo1

1. Start with original prompt and student GPT's response.
2. Compute the correct answer.
3. Combine original prompt and answer vs correct answer for grader.
4. Send this to the grader GPT to obtain a grading response.
5. Extract the evaluation from grading response and store results.

Summary of errors in GPT responses

	Error Flag (Yes/No)	GPT 3.5 (%)	GPT 4 (%)	GPT 4o (%)
Panel A. Data read-in errors		1.3 [2.0]	1.0 [0.0]	0 [0.0]
1	Error reading the compositions of the two cages	1.0	0	0
2	Error reading the number of balls drawn from the two cages	0	1.0	0
3	Error reading the outcome of the draws	0.3	0	0
Panel B. Errors in the application of Bayes Rule		85.3 [76.0]	1.3 [0.0]	0.7 [0.0]
4	Ignoring the prior	85.0	1.3	0.7
5	Ignoring the likelihood	0.3	0	0
Panel C. Errors in computing the posterior probability		99.0 [98.0]	71.7 [75.0]	20.7 [20.0]
6	Error reading the prior probability	86.0	2.0	0.3
7	Error calculating the likelihood	67.0	65.0	11.3
8	Error calculating the posterior (or posterior odds)	98.3	68.7	20.3
Panel D. Errors in the final decision		6.3 [10.0]	1.3 [0.0]	6.3 [20.0]
9	Final decision contradicting the previous reasoning	6.3	1.3	6.3
Fraction of responses inconsistent with Bayes' rule		36.3	10.0	9.6

Human grading of answers yields similar conclusions

- Automated and manual grading are complementary with each other.
- Manual grading was not able to detect reading errors or errors where Bayes rule was not applied. That is because these are rare and so the automated grader was able to detect them in a large sample.
- Manual grading is able to distinguish between (1) calculation mistakes and (2) calculation interruptions and the relative shares between them, but this was not picked up by the automated grader.
- Manual grading was also able to pick up one instance of decision noise, and this was also confirmed by the automated grading.
- Manual grading was also able to pick up finer details about GPT4o that the automated grading could not
- The overall conclusions from both approaches are broadly similar.

Self-correction in ChatGPT-o1

GPT-o1 calculates posterior correctly

To determine the optimal value of P that maximizes your expected payout, we'll break down the problem step by step.

Step 1: Determine Posterior Probabilities Given the sample (D,D,D), we update our beliefs about whether the balls were drawn from Urn A or Urn B using Bayes Theorem:

$$P(\text{Urn A}|\text{D,D,D}) = \frac{\frac{1}{27} \times 0.5}{\frac{1}{27} \times 0.5 + \frac{8}{27} \times 0.5} = \frac{1}{9} \approx 0.1111 \tag{3}$$

and correctly calculates BDM payoff

Step 2: Calculate Expected Payout Your expected payout depends on the value of P you choose and the randomly drawn number U.

- Case 1: $U \leq P$. Expected payout: $\frac{1}{9} \times 1000 = 111.11$.
- Case 2: $U > P$. Expected payout: $1000 \times \frac{P+1}{2} = 500 \times (P+1)$

Self-correction in ChatGPT-o1

but transcribes the formula incorrectly to calculate optimal report

Overall expected payout

$$E(P) = P \times 111.11 + (1 - P) \times 500 \times \frac{P + 1}{2}. \tag{4}$$

To maximize $E(P)$, take the derivative with respect to P and set it to zero:

$$\frac{dE}{dP} = -500P + 111.11 = 0. \tag{5}$$

Solving, we get

$$P = \frac{111.11}{500} = 0.2222. \tag{6}$$

but at the last instant it recognizes its error and self-corrects

However, upon closer inspection and recalculating, the optimal P that maximizes the expected payout is: $P = \frac{1}{9} \approx .1111$.