Trade-Offs Between Ranking Objectives: Reduced-Form Evidence and Structural Estimation

DSE2025: Expectations and Learning in Dynamic Structural Models

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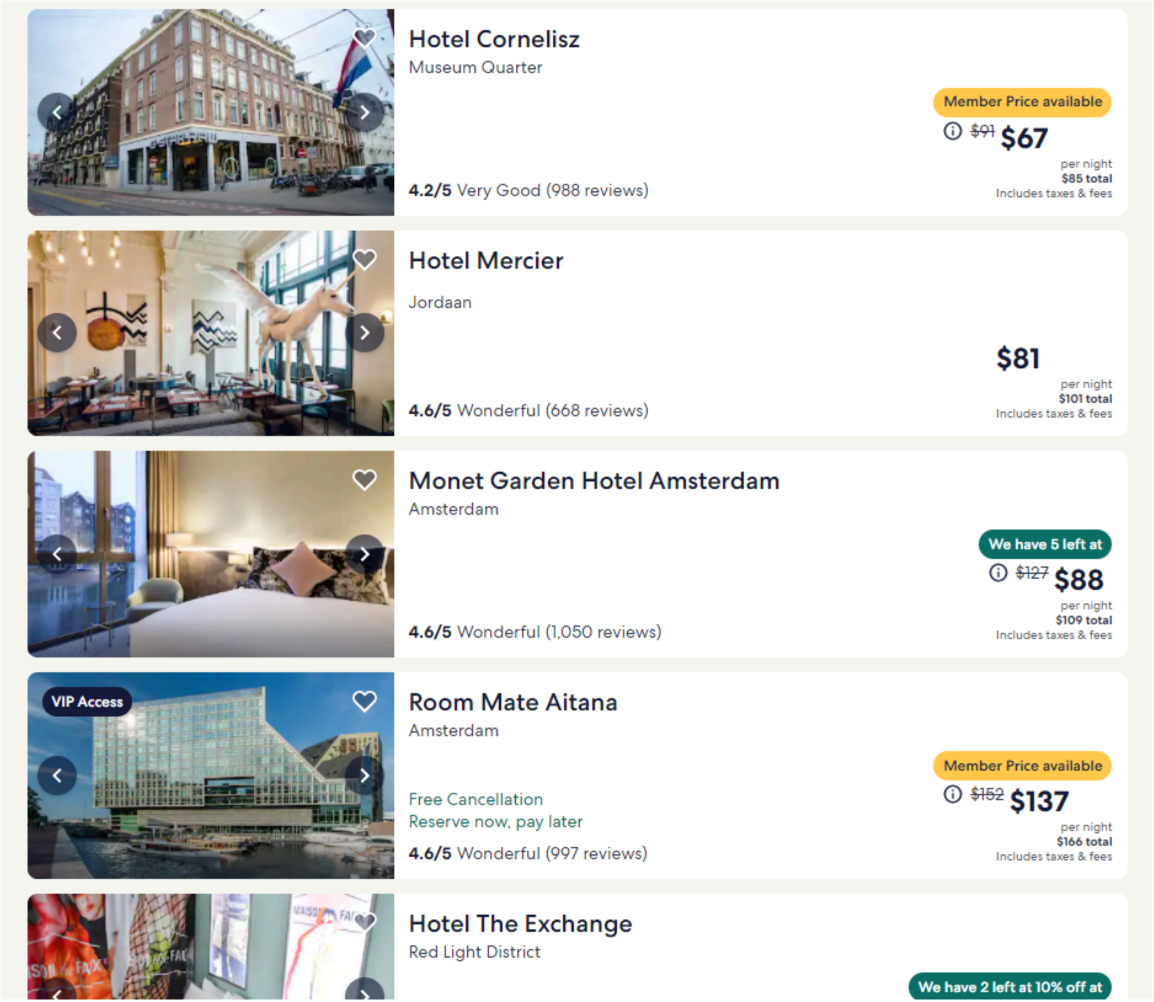
Ranked Product Lists



Research Questions

- Ranking algorithms can serve different objectives.
  - Maximize platform revenues/profits (=share of total revenues across products).
  - Maximize consumer welfare.

1. What determines the trade-offs between objectives?
2. How can we quantify these trade-offs?

- Focus: revenue and consumer welfare effects through influencing consumers' choices.
  - Relatively involved model of consumer search.
  - Taking supply side as given (other than ranking).

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This Paper

- Large literature empirically studying various aspects of rankings:
  - Ursu (2018); Choi and Mela (2019); Donnelly, Kanodia, and Morozov (2023); Compiani et al. (2024)
  - And more…
- So what exactly is new here?
  1. Reduced-form evidence for **heterogeneous position effects** (= key factor shaping trade-offs).
     - Position effect: increase in searches/purchases for an alternative moving up.
  2. Novel structural model with **product discovery** as mechanism for position effects.
     - Matters for model fit and quantifying trade-offs.
- *Today: focus on structural model.*

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Expedia Data

- Background: competition on Kaggle.com, first used by Ursu (2018).
- Click-stream data of hotel search sessions on Expedia.com.
  - 166,039 search sessions, 788 (overlapping) destinations, 55 countries.
- Main feature: subsample of $\approx 30\%$ sessions with a **randomized ranking**.

$\implies$ Allows identifying position effects separately from consumers' preferences.

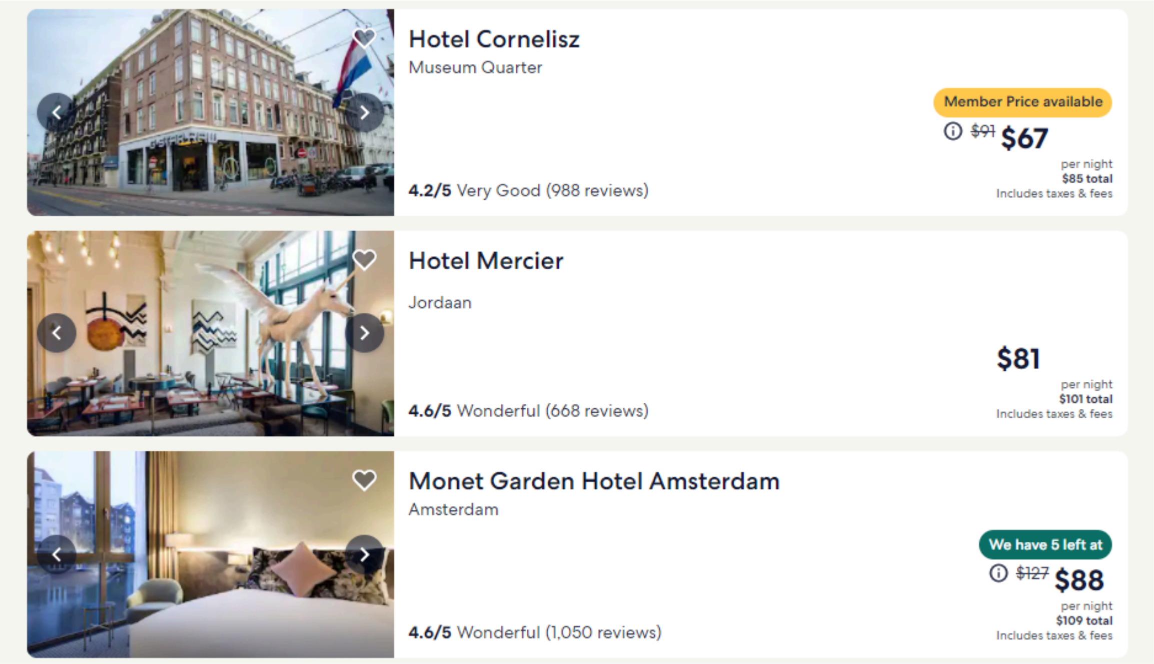
- Data contains information on:
  - Clicks and bookings (but not search order).
  - Positions.
  - Hotel & query characteristics.
- Only sessions that did not sort results.

Search and Discovery Model: Overview

- Quantifying the effects of different (counterfactual) rankings requires a model that captures how consumers interact with a ranked product list.

- I develop an empirical implementation of the **search and discovery** model of Greminger (2022).

Model: Decision Process



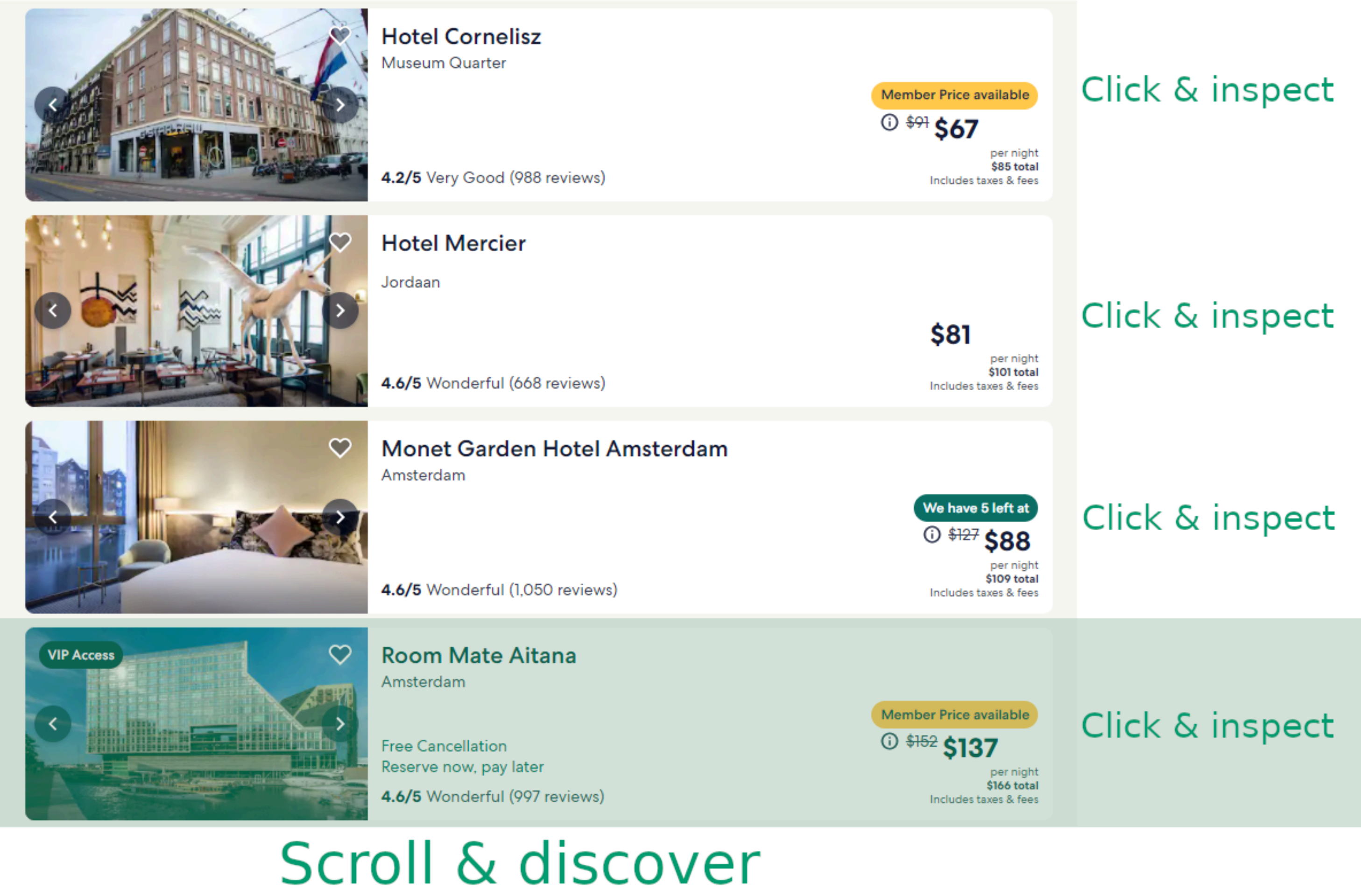
Click & inspect

Click & inspect

Click & inspect

Scroll & discover

Model: Decision Process



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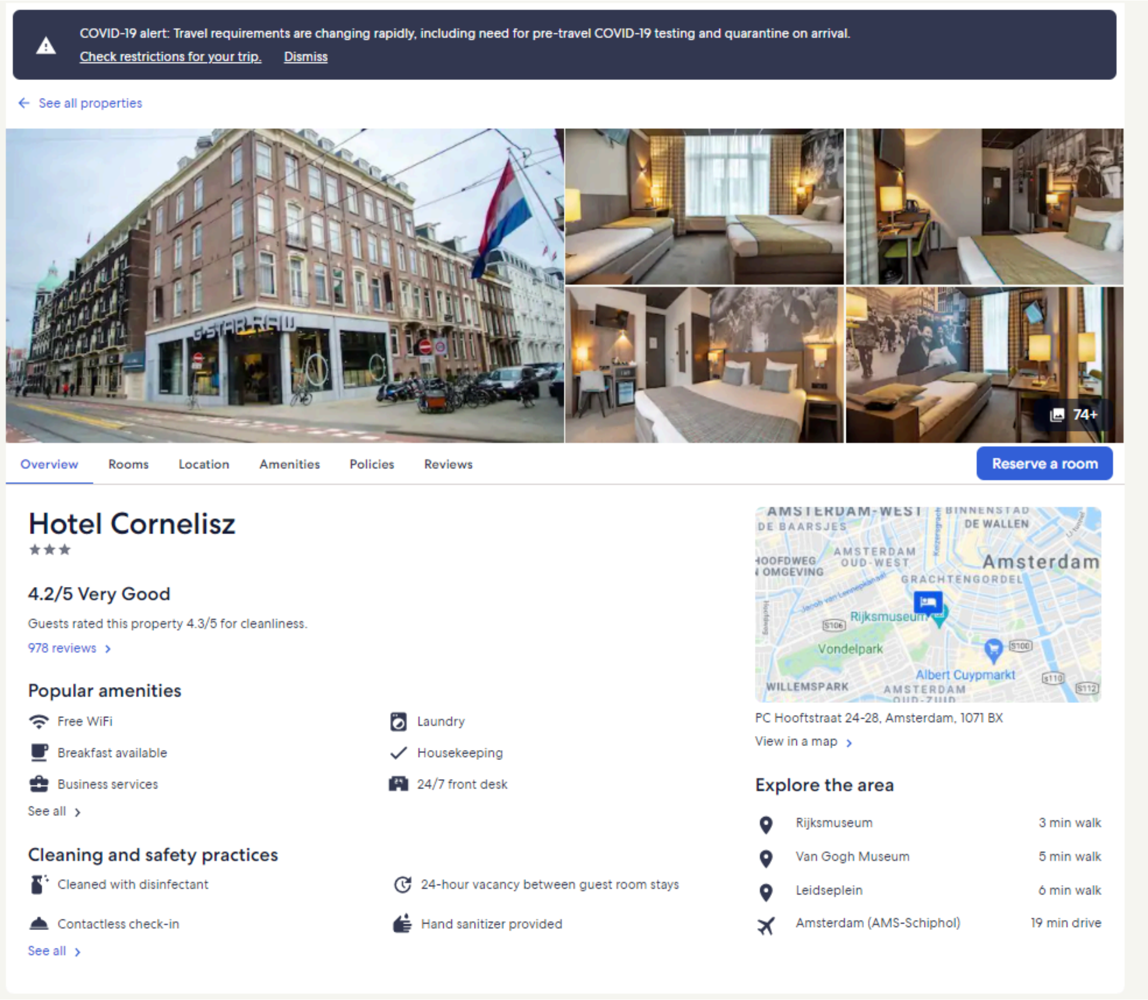

Model: Specification

- Utility of consumer i when booking hotel j:

$$u_{ij} = \underbrace{x_j^l \beta + \nu_{ij}}_{u_{ij}^l\text{: list page}} + \underbrace{x_j^d \kappa + \delta_j + \varepsilon_{ij}}_{u_{ij}^d\text{: detail page}}$$

  - x_j^l, x_j^d: observed hotel attributes.
  - δ_j: fixed effect for hotel j (estimated only for frequent hotels).
  - $\varepsilon_{ij} \sim \text{Normal}(0, 1)$
  - $\nu_{ij} \sim \text{Normal}(0, 1)$

- Utility of outside option : $u_{i0} = \beta_0 + \eta_i$, $\eta_i \sim \text{Uniform}(0, 1)$.

Model: Specification

- Actions are costly:
  - Discovery costs: c^d
  - Search costs (product-specific): c_j^s
- Free recall: no cost to go back.
- Precedence constraints:
  - Searching a hotel requires it to have been revealed.
  - Choosing a hotel requires it to have been searched.

Model: Beliefs About Detail Page Utility

- Initially, consumers only observe list page utility u^l_{ij} of first alternative j on list.
- Based on x^l_j, consumers have belief about the utility of the detail page U^d_{ij}.
  - $E[U^d_{ij}|x^l_j] = x^l_j\gamma$.
  - (unconditional) U^d_{ij} follows empirical distribution.
  - γ is estimated (captures perceived correlation between x^l_j and U^d_{ij}).

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Model: Beliefs About Alternatives to Be Revealed

- Consumers believe they are randomly sampling from the joint distribution of $(x_j^l, x_j^d, c_j^s, \nu_{ij}, \varepsilon_{ij})$.
- Equivalent to consumers believing that they sample from a distribution of *effective values*.
  - Expected benefit of revealing alternative depends only on distribution of effective values (Greminger (2022)).
  - Effective value combines $(x_j^l, x_j^d, c_j^s, \nu_{ij}, \varepsilon_{ij})$ in a way that accounts for search behavior.

Model: Beliefs About Alternatives to Be Revealed

- Belief about ranking: changes distribution of effective values across positions h.
  - To simplify, assume only mean $\mu(h)$ of the distribution changes and specify functional form.

$$\mu(h) = \overline{\mu} + \rho \log(h+1)$$

  - ρ is estimated and captures beliefs about ranking algorithm.
  - $\rho < 0$ means consumers expect alternatives further down to be worse *on average*.
  - $\overline{\mu}$ is determined by assumption that consumers know the overall distribution across positions.
  - Agnostic as to why distribution changes across positions.

Model: Consumer Decision Problem

- Rational consumers sequentially choose one of the available actions.
- Greminger (2022): the optimal policy is based on reservation values.

  $\Longrightarrow$ Always optimal to choose the action with the highest reservation value.
- Three reservation values:

  1. Purchasing: $z_{ij}^{p} = u_{ij}$
  2. Searching / clicking: $z_{ij}^{s} = x_j^l \beta + \nu_{ij} + x_j^l \gamma + \xi(c_j^s)$
  3. Discovering / scrolling: $z^d(h) = \mu(h) + \Xi(c^d)$
- $\xi(c_j^s)$ and $\Xi(c^d)$ capture respective net benefits.

Estimation Approach: Main Innovation

- Simulated maximum likelihood estimation approach:
  - Parameters to estimate: $\theta = (\beta_0, \beta, \kappa, \gamma, c^d, c_j^s, \delta_j, \rho)$.
  - Optimal policy implies inequalities for reservation values given observed actions.
  - Maximize likelihood of all inequalities holding given observed actions.
  - Example: i searching but not choosing hotel j requires $z_{ij}^s \geq u_{i0}$ and $u_{ij} \leq u_{i0}$ (and more).
- Resulting likelihood function can be difficult to compute, even in simpler search models.
  - High-dimensional integral over shocks of many alternatives.
  - See Ursu, Seiler, and Honka (2024).

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Estimation Approach: Main Innovation

- New result simplifying simulating the likelihood function.
- Main idea: inequalities can be expressed relative to the effective value $\tilde{w}_{ij}$ of the chosen option j.

$$\tilde{w}_{ij} = x_j^l\beta + \nu_{ij} + \min\{x_j^l\gamma + \xi(c_j^s), x_j^d\kappa + \varepsilon_{ij}\}$$

$$\tilde{w}_{i0} = u_{i0}$$

  - Probability of all inequalities holding given $\tilde{w}_{ij}$ has closed form (after another result).
  - Only requires numerically integrating over $\tilde{w}_{ij}$.

Estimation Approach: Two More Innovations

1. Partition probability space to obtain likelihood function that is smooth in all parameters.
   - Depending on $\tilde{w}_{ij}$, consumers discover different number of alternatives and inequalities change.
   - $\tilde{w}_{ij}$ has kink at $x_j^l \gamma + \xi(c_j^s) = x_j^d \kappa + \varepsilon_{ij}$.

2. Estimate $\xi(c_j^s)$ and $\Xi(c_d)$ and back out costs (c_j^s, c^d) post-estimation.
   - Works because only these values enter consumer decisions through reservation values.
   - Avoids costly computation of $\xi(c_j^s)$ and $\Xi(c^d)$ during estimation.
   - Allows demand predictions with few assumptions on beliefs.

- Julia and Python packages coming in the near future!

Identification

- Standard arguments for identification of parameters governing search (conditional on discovery).
- Discovery process is latent, but position effects only explained through discovery.
  - Ranking is randomized.
  - Only change in discovery value $z^d(h) = \mu(h) + \Xi(c^d)$ across positions h explains position effects.
  - $z^d(h)$ and consequently stopping probabilities are identified from the data.

Position Effect Mechanism

- Search and discovery model: **product discovery** explains position effects.
  - Consumers leave list page before reaching the end.
- Classic Weitzman model: **position-specific search costs** explain position effects.
  - Consumers observe the entire list page (all x_j^l and ν_{ij}).
  - It is more costly to search products in lower positions ($c_j^s(h)$ depends on position h).
- Can we differentiate the two mechanisms for position effects?

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Position Effect Mechanism

- A Weitzman model predicts the following:
  - Searches in lower positions are more likely to convert to a purchase.
  - (except without position effects).

- Rationale:
  - Searching in a lower position is more costly.

  $\Rightarrow$ Only happens when list utility $x_j^l\beta + \nu_{ij}$ is already large.

- No similar prediction from search and discovery model (costs do not depend on position).

- Prediction is not supported by the Expedia data $\Rightarrow$ suggests product discovery mechanism.
  - Ursu (2018): conditional on search, the purchase probability is constant across positions.

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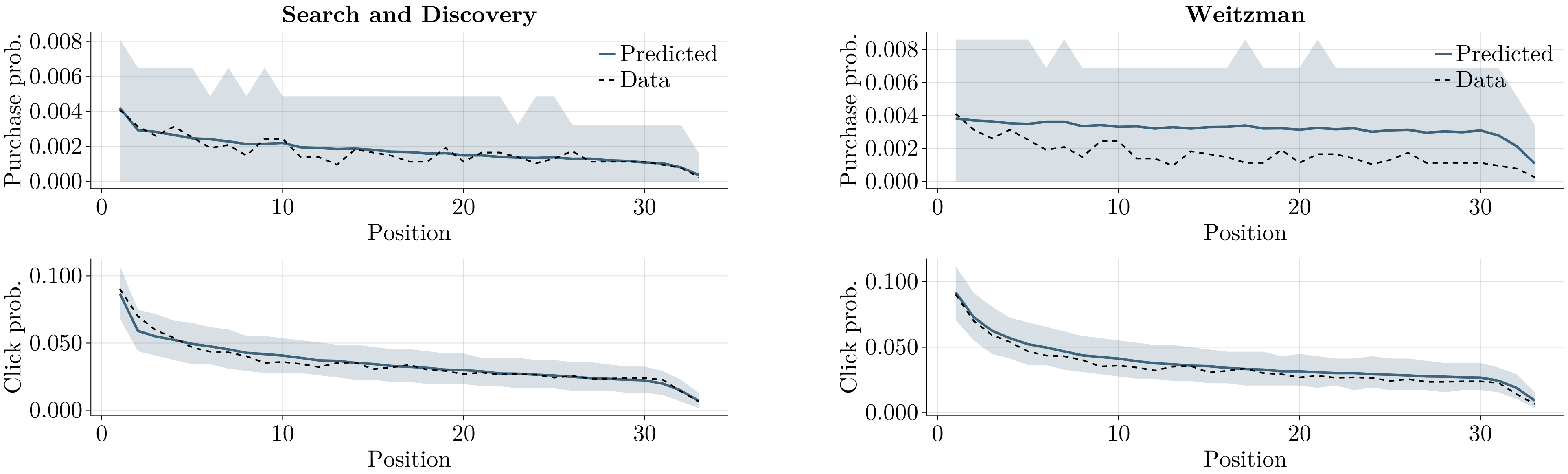

Counterfactual Rankings

- Quantifying trade-offs between ranking objectives requires optimal rankings for each objective.
  - Difficult combinatorial optimization problem.

1. New heuristic for maximizing revenues: **Bottom Up Ranking**.
   - Rationale: ordering of alternatives on top does not matter for demand at bottom.
2. New heuristic for maximizing consumer welfare: **Effective Value Ranking**.
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- Focus on case where all products are displayed and consumers update beliefs about the ranking.
- Estimate model and run counterfactuals on pooled sample.

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Counterfactual Results

Changes relative to Expedia Ranking

	Eff. Value	Bottom Up Ranking
Expedia		
Total revenues (%)	10.99	12.93
Number of transactions (%)	17.08	11.06
Avg. price of booking (%)	−5.20	1.67
Consumers		
Consumer welfare ($, per consumer)	0.06	0.04
Consumer welfare ($, per booking)	13.58	7.73
Discovery costs ($, per booking)	−0.14	−0.10

- Trade-offs are smaller than prior work suggests:
  - Ursu (2018) and Compiani et al. (2024) also use Expedia data and find larger trade-offs.
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Reduced-Form Evidence: Summary

- Heterogeneity in position effects shapes trade-offs between ranking objectives.
  - Ranking products with large position effect higher increases transactions.

$$\Delta_{\text{Switch}}\ \text{Demand} = \Delta_{\text{Up}}\text{Demand}_B + \Delta_{\text{Down}}\text{Demand}_A$$

$\Rightarrow$ Heterogeneity in position effects determines transactions and revenues under different rankings.

- Using simple reduced-form model (LPM), I find that:

1. Cheaper hotels tend to have larger positions effects (conditional on other attributes).
2. "Desirable" hotels tend to have larger position effects (desirable = more clicks/bookings).

$\Longrightarrow$ Moving cheaper/desirable hotels up can increase revenues through more transactions.

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Conclusion

- Novel structural model with product discovery mechanism for position effects.
  - Mechanism matters for model fit and counterfactual results.
- Trade-offs between ranking objectives are limited.

Thank You!

Working paper: https://arxiv.org/abs/2210.16408

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Position Effect Mechanism

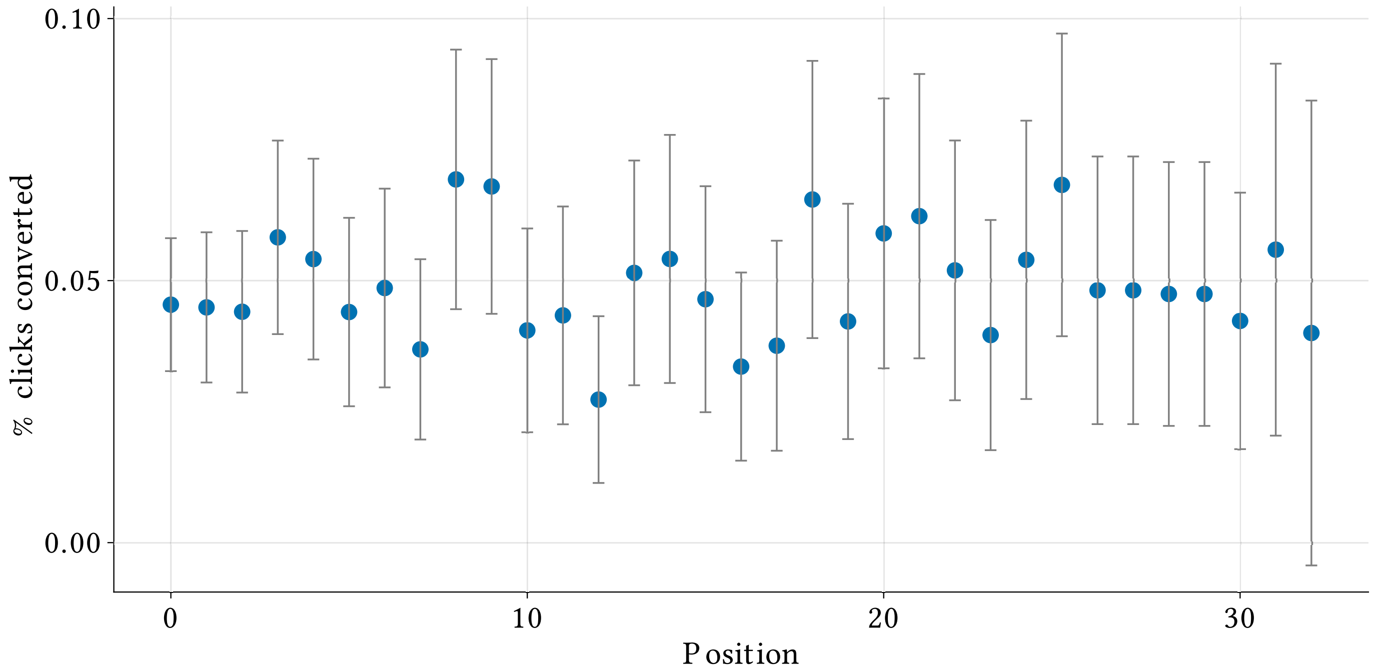
- Probability to book a hotel conditional on having clicked on it:

$$P(\text{choose } j \mid \text{search } j) = P(U_{ij}^l + U_{ij}^d \geq \overline{w}_{-j} \mid U_{ij}^l + \xi(h) \geq \overline{w}_{-j})$$

- U_{ij}^l: part of utility from list page.
- U_{ij}^d: part of utility from detail page.
- $\xi(h)$: part of search value that is not part of utility.
- $\overline{w}_{-j}$: maximum effective value of alternatives other than j.

Position Effect Mechanism

– Pattern first highlighted by Ursu (2018): conditional on search, the purchase probability is constant across positions.



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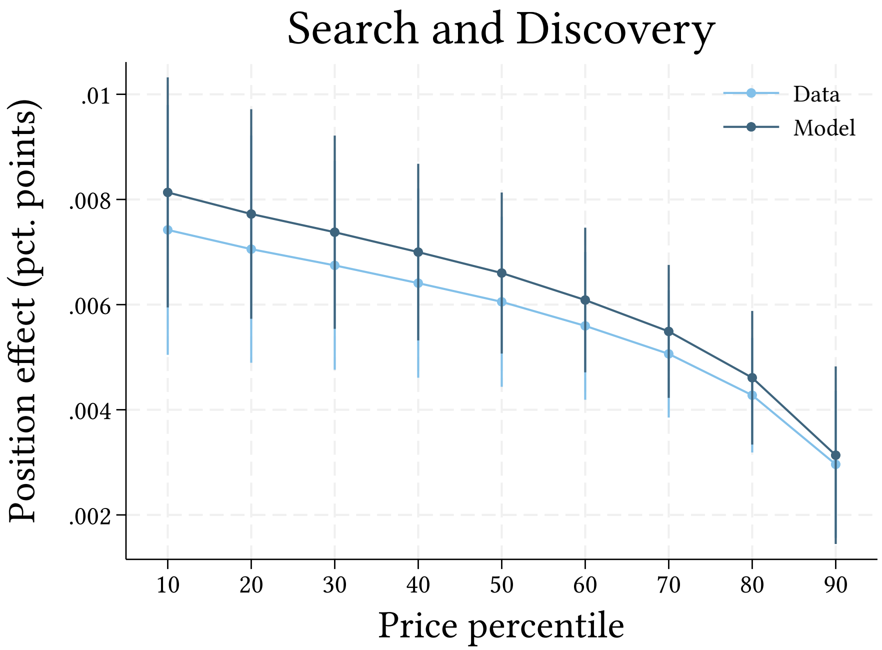

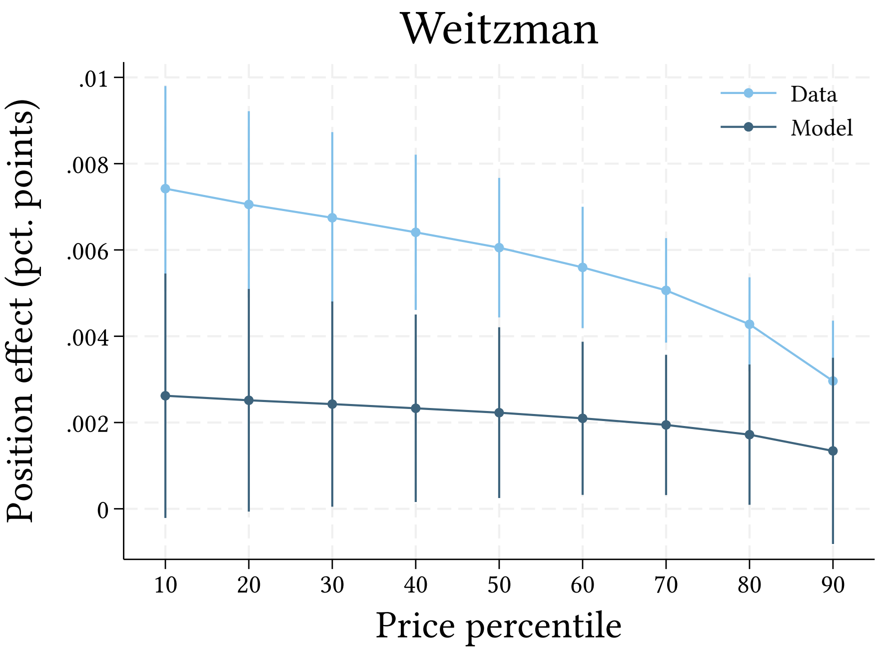

Position effect in booking probability at different prices.

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