Costly Attention and Retirement

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Outline

Introduction

Context + Data

Motivating Evidence

Model

Solution Method

Results + Detailed Evidence

Conclusion

Attention+Mistaken Beliefs+Policy Uncertainty

- **Inattention** ⇒ households cannot exploit features of benefits they are unaware of
- Well-documented unawareness of benefit system: **mistaken beliefs** about pensions
  - … e.g. Manski (2004;ECTA) uncertainty about current social security formula
- In my data I observe true and self-reported State Pension Age (SPA)
  - … 60% mistaken about SPA by over a year 2-4 years from SPA
- **Costly attention + policy uncertainty** ⇒ **mistaken beliefs** are less surprising

Can costly attention to **uncertain policy** explain **mistaken beliefs**? And how does this impact **retirement**?

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Can **costly attention** to **changeable policy** explain both observed **mistaken beliefs** and puzzling aspects of retirement behaviour (in the UK)?

Excess Employment Sensitivity: Why do people retire exactly at pension eligibility ages when so few incentives?

- ... UK very weak incentive to retire at SPA yet labour market exits concentrate there
- ... similar puzzle observed in multiple other countries (e.g. USA, Switzerland, Germany)
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... closes **43%-88%** of gap in employment drop at SPA between baseline model and data

Costly attention increases employment response at SPA but ...

... **selection into SPA knowledge** means model replicates negative relationship observed

Estimated costs of attention are small (mean WTP to learn SPA **£11-£83**) but

... large employment response at SPA because (1) those near retirement are close to participation margin

... and (2) response represent intertemporal shifting of employment, so welfare implications modest

To get to these conclusions I...

... develop solution technique for **dynamic RI** model with **history-dependent beliefs**

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Related Literature+Contributions

Related Literature: Builds on empirical and theoretical

- Excess sensitivity: Gruber and Wise, 2004 (multiple); Behaghel and Blau, 2012 (US); Cribb et al., 2016 (GB); Rabaté, 2019 (FR); Lalive et al., 2020 (CH); Seibold, 2021 (DE); Gruber et al., 2022 (FI)
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Contribution: Embed RI in dynamic retirement model to explain puzzle + beliefs. To do this:

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Many explanations for sensitivity of employment to UK SPA ruled out:

1. UK law prohibits compulsory retirement ages Caveats
2. The state pension is not conditional on employment status
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State pension is anticipatable increase in non-labour income

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Policy Variation

Use reform increasing female SPA from 60 to 66 in *monthly increments* from 2010 to 2020 to:

1. Causally estimates impact of SPA on employment
2. Estimate extent of policy uncertainty about the SPA

Legislated in 1995 $\Rightarrow$ anticipatable change with horizon $\geq$ 15 years

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Data

The English Longitudinal Study of Ageing (ELSA):

- Representative panel survey of population aged over 50
- Waves conducted every two years
- Similar to the HRS in USA or SHARE in Europe

ELSA is linked to administrative data containing:

- Work histories
- Benefit claiming histories

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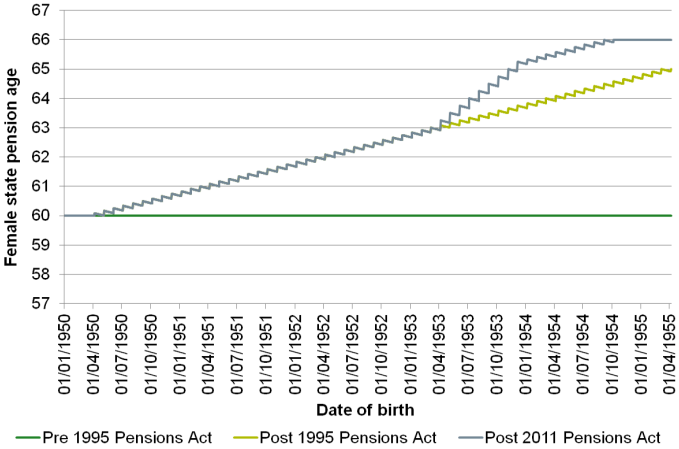
Reforms


Female state pension age
67
66
65
64
63
62
61
60
59
58
57
01/01/1950
01/04/1950
01/07/1950
01/10/1950
01/01/1951
01/04/1951
01/07/1951
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01/10/1954
01/01/1955
01/04/1955
Date of birth
Pre 1995 Pensions Act
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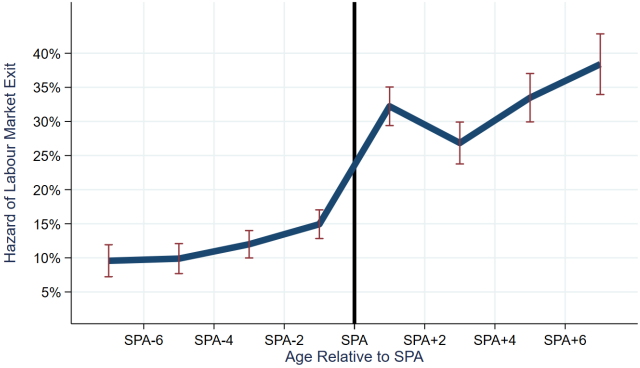

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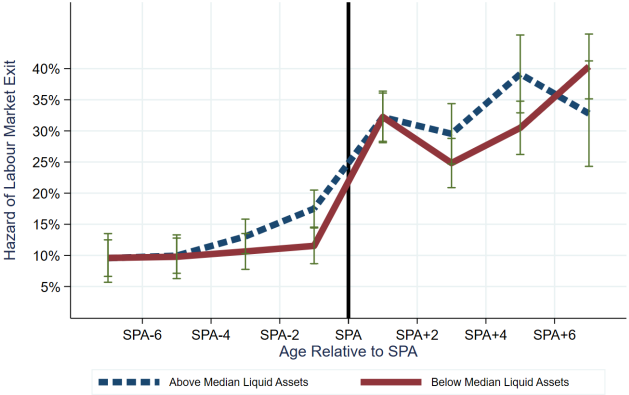
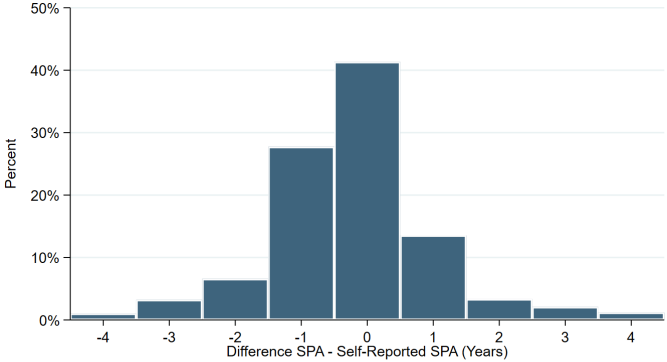


Mistaken Beliefs

Figure: Error in SPA self-reports at age 58



Beliefs by Cohort Monthly accuracy Beliefs+Emp. Sensitivity Beliefs by Educ.

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Get **utility** from:

- Consumption
- Leisure
- Bequests

Details

Policy uncertainty

The Government updates cohort's SPA following:

$$SPA_{t+1} = SPA_t + e_t$$

$$\text{where } e_t \in \{0, 1\} \text{ and } e_t \sim Bern(\rho)$$

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Group decisions into $d_t = (c_t, l_t)$ and states other than SPA into X_t:

- With policy uncertainty **states** are: X_t, and stochastic SPA SPA_t
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- $\underline{\pi_{t+1}}$ formed by Bayesian updating & SPA revealed upon reaching it Mutual info. Bayes

Rational Inattention

Within-period timing:

Start with states	Choose $Z_t \sim$	Receive realisation	Choose	States updated
$X_t, SPA_t, \underline{\pi_t}$	$\underline{f_t}[X_t, \underline{\pi_t}](z_t \mid SPA_t)$	z_t of Z_t	$d_t(X_t, \underline{\pi_t}, z_t)$	$X_{t+1}, SPA_{t+1}, \underline{\pi_{t+1}}$

t $\qquad$ $t+1$

How does all this help explain drop in employment at SPA? Arriving at the SPA becomes

... a positive information shock reducing precautionary labour supply

... a positive wealth shock reducing labour supply as leisure is normal good

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$X_t, SPA_t, \underline{\pi_t}$	$\underline{f_t}[X_t, \underline{\pi_t}](z_t \mid SPA_t)$	z_t of Z_t	$d_t(X_t, \underline{\pi_t}, z_t)$	$X_{t+1}, SPA_{t+1}, \underline{\pi_{t+1}}$

t $\qquad$ $t+1$

How does all this help explain drop in employment at SPA? Arriving at the SPA becomes

... a positive information shock reducing precautionary labour supply

... a positive wealth shock reducing labour supply as leisure is normal good

Rational Inattention

Within-period timing:

Start with states	Choose $Z_t \sim$	Receive realisation	Choose	States updated
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Problem: Complicated by high dimensional state (π_t) and choice (f_t)

Previous work: Restricts f_t (e.g. Maćkowiak et al. 2009;AER) or eliminates π_t (e.g. Miao et al 2023;QE)

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1. Analytic solution for f_t in dynamic discrete choice RI models (Steiner et al.,2017,Ecta)
2. RI $\Rightarrow$ consideration sets $\Rightarrow$ sparse solutions (Caplin et al.,2019,Restud)

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Choose $d_t \sim \underline{p_t}[X_t, \underline{\pi_t}](d_t|SPA_t)$

Replace ($\underline{f_t}$) by CCPs ($\underline{p_t}$) in cost of information:

$$I(\underline{p_t}; \underline{\pi_t}) = \sum_{spa} \pi_t(spa) \sum_d p_t(d|spa) \log(p_t(d|spa)) - \sum_d q_t(d) \log(q_t(d))$$

where:

$$q_t(d) = \sum_{spa} \pi_t(spa) p_t(d|spa)$$

Details

Analytic Result & Model Solution

The entropy-based cost of attention gives logit-like result:

$$p_t(d|spa) = \frac{q_t(d)\exp\left(\frac{(c^{\nu}I^{1-\nu})^{1-\gamma}}{\lambda(1-\gamma)} + \beta E\left[V_{t+1}(SPA_{t+1}, X_{t+1}, \underline{\pi_{t+1}})|d, spa\right]\right)}{\sum_{d'} q_t(d')\exp\left(\frac{(c'^{\nu}I'^{1-\nu})^{1-\gamma}}{\lambda(1-\gamma)} + \beta E\left[V_{t+1}(SPA_{t+1}, X_{t+1}, \underline{\pi_{t+1}})|d', spa\right]\right)} \quad (1)$$

Where the associated unconditional q_t solves:

$$\max_{\underline{q}} \sum_{spa} \pi_t(spa) \log\left(\sum_d q(d)\exp\left(\frac{(c^{\nu}I^{1-\nu})^{1-\gamma}}{\lambda(1-\gamma)} + \beta E\left[V_{t+1}(SPA_{t+1}, X_{t+1}, \underline{\pi_{t+1}})|d, spa\right]\right)\right) \quad (2)$$

Solution method:

1. Guess q_t to get $\underline{\pi_{t+1}}(d)$, solve (2) to get $q_t^{'}$ and iterate to convergence on $q_t^{'} - q_t$
2. Substitute q_t into (1) to solve for CCP

... avoid solving (2) at many points due to sparsity:

e.g. dropping dominated actions often leaves only one action Computational Details

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Estimate baseline model using two-stage simulated method of moments targeting pre-reform employment and asset profiles details Cohort Profiles

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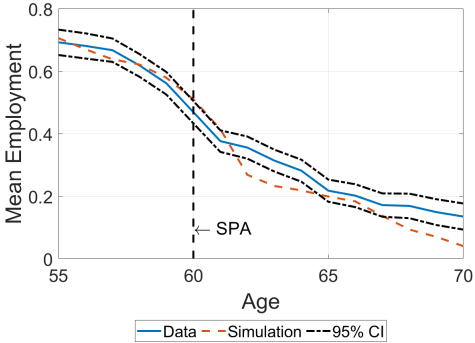

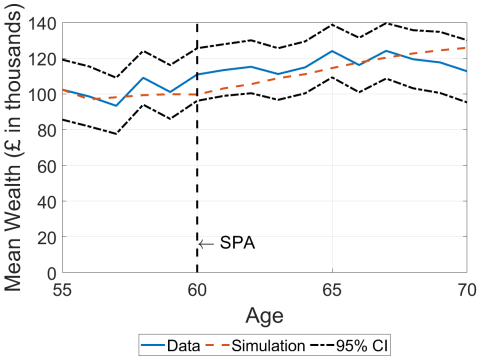

Excess Sensitivity Puzzle

Why do so many retire exactly at SPA? Regression details

	Whole Population	Wlth.>Med. (£29,000)
Above SPA(>SPA)	**0.128**	0.106
s.e.	(0.0239)	(0.0299)
P>|z|	0.000	0.000
Above SPA(>SPA)	0.156	–
s.e.	(0.0371)	–
(>SPA)×(Wlth.>Med.)	-0.050	–
s.e.	(0.0476)	–
P>|z|	0.299	–
Obs.	7,906	3,798

Significant response to anticipatable income shock of SPA on labour supply

As large for those with above median assets

⇒Unlikely to be liquidity constraints

Placebo tests indicate results are particular to SPA: Placebo

Other Specifications give similar results: Imputation-BJS Cont. interaction

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Table: Treatment effect from being above SPA on hazard rate

	Baseline	Policy Uncert.	RI($\lambda \approx$ £83)	RI($\lambda \approx$ £11)	Data
Entire Simulated Sample *[95% C.I]*	**0.019**	0.014	0.095	0.041	0.128 [0.077,0.174]
Assets >Median(£29,000) *[95% C.I]*	**0.003**	0.014	0.095	0.054	0.106 [0.046,0.168]

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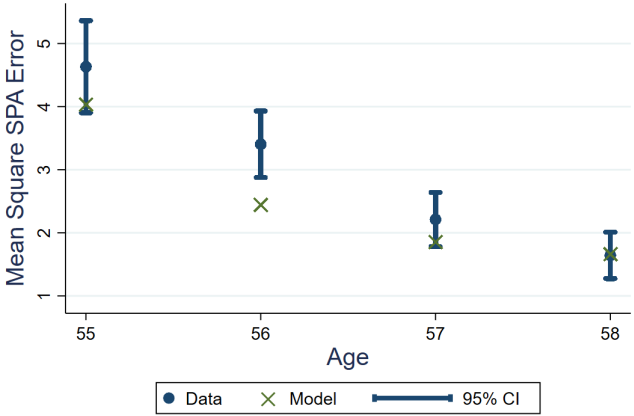
Mistaken Beliefs

Figure: Mean Squared Error of Reported SPA



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- If (like reference-dependence) target drop in employment $\Rightarrow$ attention Cost $\approx$ £83
  - ... maybe misbeliefs are a complimentary explanation to reference-dependence
  - ... or maybe I have underestimated policy uncertainty by only looking at the SPA

Excess Sensitivity + Mistaken Beliefs

	$\lambda \approx$ £11	$\lambda \approx$ £83	Data
Above SPA×abs(Self-report - SPA) *[95% C.I]*	-0.046	-0.002	**-0.052** [-0.102,-0.002]
Above SPA× 1[Self-report - SPA >0] *[95% C.I]*	-0.046	-0.002	**-0.078** [-0.262,0.106]

Regression

Is this consistent with model of costly attention?

- Ceteris Paribus worse informed ⇒ larger shock upon discovering SPA ✗
- SPA knowledge endogenous ⇒ if actions independent of SPA don't learn it ✓

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Policy Counterfactual

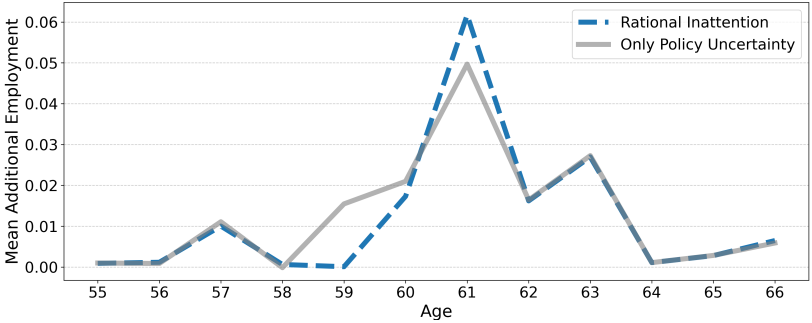
Table: Additional Mean Employment from Increasing SPA from 60

SPA increased from 60 to:	(1) - Informed Added Employment	(2) - Uninformed Added Employment	(3) MC	(4) WTP	(5) MR
61	0.07	0.06	£3.50	£4.22	£28.45
62	0.14	0.14	£4.00	£2.37	£11.78
63	0.18	0.16	£4.50	£18.34	£19.91
64	0.22	0.20	£5.00	£31.64	£4.31
65	0.31	0.27	£5.50	£44.41	£68.52

Notes: Employment increases over 56-65 from raising SPA from 60 to the age in Column (1) with costly attention and in Column (2) without it. Columns 3-5 show the financial impacts of an accompanying information letter campaign that moves people from uninformed to informed. Column (3) shows the marginal cost, Column (4) the willingness to pay, and Column (5) the marginal revenue.

Policy Counterfactual

Figure: Additional Employment resulting from Increasing the SPA from 60 to 62



Notes: For each model, the difference in employment increase between simulations of households with a female SPA of 60 and those with a female SPA of 62.

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Conclusion + Implications

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  - ... sensitivity of employment to SPA
  - ... mistaken beliefs about pension provisions
- Policy uncertainty + rational inattention helps with both issues
- Costly Attention increases employment sensitivity to SPA
  - ... whilst selection into SPA knowledge means worst informed respond less
- Policy experiment show information letters details :
  1. Increase welfare by more than their marginal cost
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Additional Slides

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Appendix

Intuition - Analytic Solution

Choice problem now selecting CCPs (p_t) to solve: Def. $\bar{V}$

$$\max_{p_t} \underbrace{E[u(d_t, p_t) + \beta \overbrace{\bar{V}_{t+1}(d_t, SPA_t)}^{=E[V_{t+1}(X_{t+1}, SPA_{t+1}, \pi_{t+1}(d_t)) | SPA_t, d_t]}]}_{=}$$

$$\sum_{spa} \pi_t^{spa} \sum_d p_t(d|spa)[\frac{(c^{\nu}/^{1-\nu})^{1-\gamma}}{\lambda(1-\gamma)} + \log(q_t(d)) - \log(p_t(d|spa)) + \beta \bar{V}_{t+1}(d, spa)]$$

Treating q_t as given the FOCs w.r.t p_t are:

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Excess Sensitivity Puzzle

	Whole Population	NHNBW >Median (£24,846)	VLA >Median (£23,800)
Below SPA	0.102	0.099	0.084
s.e.	(0.0312)	(0.0441)	(0.0443)
$P>\|z\|$	0.001	0.059	0.025
Below SPA	-	0.136	0.156
s.e.	-	(0.0461)	(0.0457)
Below SPA×(Assets >Median)	-	-0.036	-0.069
s.e.	-	(0.0640)	(0.0639)
$P>\|z\|$	-	0.572	0.284
Obs.(Indivs.)	5,710 (2,882)	2,573 (1,154)	2,563 (1,155)

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Excess Sensitivity Puzzle

- I use the follow specification to study the UK SPA reform:

$$y_{it} = \alpha \mathbb{1}[age_{it} > SPA_{it}] + \sum_{c \in C} \gamma_c \mathbb{1}[cohort_i = c] + \sum_{a \in A} \delta_a \mathbb{1}[age_{it} = a] + \sum_{d \in D} \kappa_d \mathbb{1}[date_{it} = d] + X_{it}\beta + \varepsilon_{it}.$$

- Interpret α as a DID estimator of the treatment effect of being below the SPA on the probability of being in work

Back

Mandatory Retirement + Age Discrimination

- Equality Act (2006) banned mandatory retirement below age 65
- Equality Act (2010) banned all ages with exceptions EJRA
- Seldon v Clarkson, Wright and Jakes (2012) an EJRA is justified if:
  1. Objective is of a public interest (e.g. intergenerational fairness)
  2. Consistent with the social policy aims of the state
  3. Proportionate
- Oxford and Cambridge have EJRA with objective of intergenerational sharing of job opportunities
  - Ewart v University of Oxford (2020) found this EJRA was not proportionate and so illegal

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National Insurance

- NI component of income tax (similar to US payroll tax) removed upon reaching SPA
- So tax effect ambiguous
- For some PTR $\uparrow$ and others $\downarrow$ and no systematic difference between groups in response to SPA (Cribb et al. 2013)

Back

Reduced Form Evidence

- Interaction term negative and significant
- However magnitudes are tiny (4.7 %-point reduction of treatment effect at the 95th percentile of the assets)

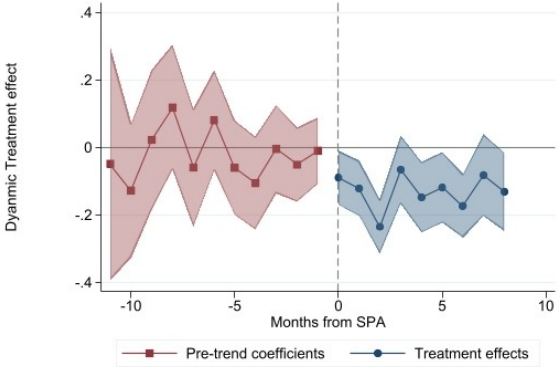
Table: Effect of SPA on Employment: Heterogeneity by Wealth

Below SPA	**0.145**
s.e	(0.0242)
p=	.000
Below SPA× NHNBW	**-1.77×10^{-7}**
s.e	(2.67e-08)
p=	.000
Obs.	7,906

back

Imputation

Figure: Dynamic Treatment Effects by Time from SPA

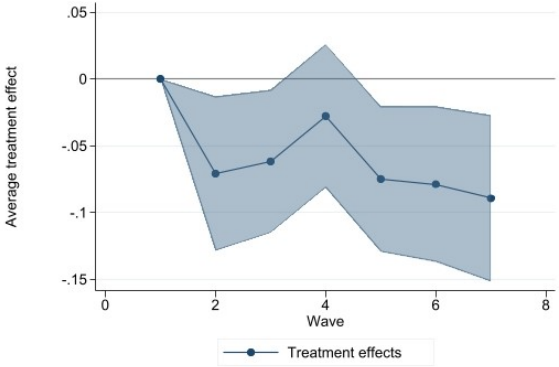


Notes: The average at a given time from SPA of the dynamic individual level treatment effects estimated using the imputation approach.

back

Imputation

Figure: Average Treatment Effect by Wave



Notes: The within wave average of the individual level treatment effects estimated using the imputation approach.

back

Parameter Estimates

ν: Consumption Weight	0.439
	(-)
β: Discount Factor	0.985
	(-)
γ: Relative Risk Aversion	3.291
	(-)
θ: Warm Glow bequest Weight	100
	(-)

Back

Distribution of Attention Costs

Table: Summary Statistics of Attention Cost Converted to Compensating Assets (£)

λ	Mean	SD	Median	5th Percentile	95th Percentile	Semi-elasticity (per 10k Assets)
1.3×10^{-7}	£15.37	£11.56	£12.43	£0.32	£37.74	-2..37%
3×10^{-6}	£59.93	£97.24	£23.75	£0.18	££262.34	-4.07%

Notes: Distribution of costs of attention as measured by compensating assets producing equivalent utility to learning your SPA today. Shown for two different costs of attention.

Back 1 Back 2

Baseline Model of Retirement

- Log earnings follows:

$$log(y_t) = \delta_{i0} + \delta_{i1} t + \delta_{i2} t^2 + \varepsilon_t$$

Where ε_t AR1 with normal error and an initial distribution $\varepsilon_1 \sim N(0, \sigma^2)$

- Unemployment shock u_t follows a conditional Markov process where probability conditional on type
- Spousal income is a fourth order polynomial in t,adjusted to account for male mortality
- The private and state pension have the functional form:

$$f_i(AIME_t) = \begin{cases} a_{i,1} AIME_t - a_{i,2} AIME_t^2 & AIME_t < A\bar{I}ME \equiv \frac{a_{i,1}}{2a_{i,2}} \\ a_{i,1} A\bar{I}ME - a_{i,2} A\bar{I}ME^2 & \text{otherwise} \end{cases}$$

back

Estimation of Baseline Model

1. **Outside the model**: estimate the parameters of the earning process, the private pension system, the unemployment transition matrix, and the state pension function

2. **Simulated method of moments**: Match 42 pre-reform moments of mean labour market participation and asset holdings between 55 and 75 (with $SPA = 60$ the pre-reform age) to estaimte the preference parameters $(\nu, \beta, \gamma, \theta)$. Most important source of identifying variation comes from:
   2.1 ν - Labour supply
   2.2 β - Wealth level
   2.3 γ - Wealth profile and reaction to changes in risk
   2.4 θ - Wealth decline in older age back

Mutual Information

Entropy is:

... a measure of uncertainty

.. lower bound on number of bytes needed to store or transmit

$$H(Z) = -E[log(Pr(Z))]$$

Mutual information ($I(SPA_t; Z_t)$) is:

... the expected reduction in uncertainty of SPA_t after receiving the signal Z_t measured using entropy

$$I(SPA_t; Z_t) = H(SPA_t) - E_z[H(SPA_t|Z_t)]$$

back

Mutual Information vs Correlation

back

Private Pension and SPA

1. DC pension wealth is accessible post 55 without penalty
2. Only DB pensions have accessibility ages that may correlate with SPA
3. Drop everyone with more that £2,000 in DB wealth

Below SPA		**0.117**
	s.e	(0.0369)
	p=	.002
Below SPA×(NHNBW.>Med.)		**-0.049**
	s.e	(0.0592)
	p=	.413
Obs.		3,735
Indv.		2,197

back

Private Pension and SPA

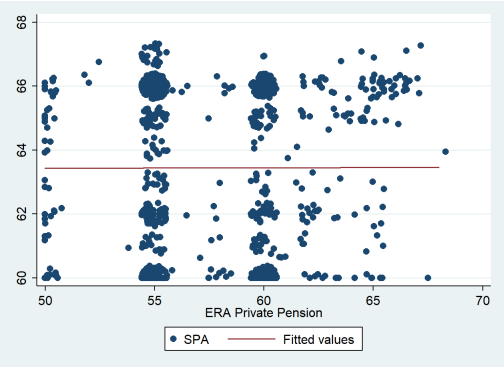


Figure: Scatter Plot of SPA and ERA of Private Pension

back

Cohort Profiles

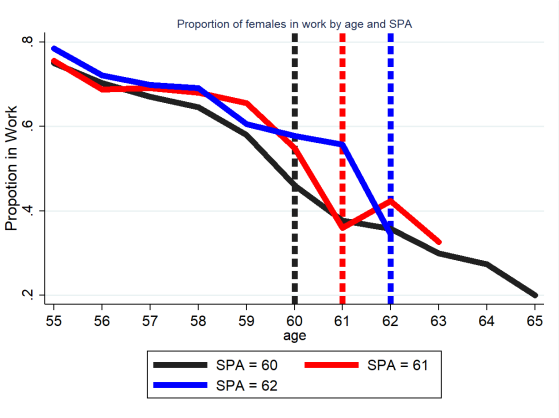
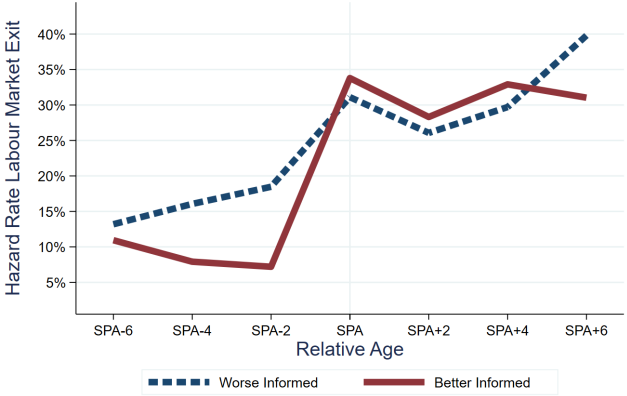


Figure: SPA-Cohort Employment Profiles

back

Mistaken Belief + Employment Sensitivity

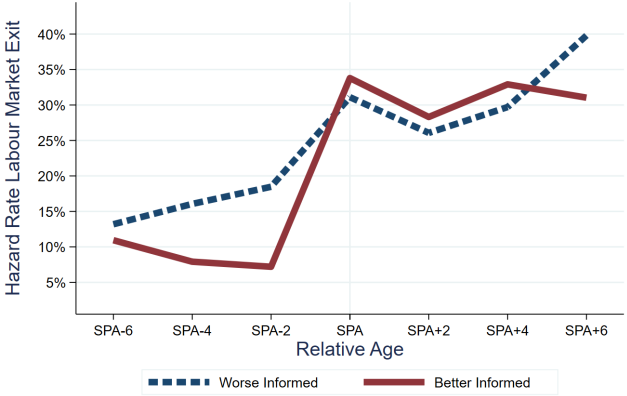


Is this consistent with model of costly attention?

- Ceteris Paribus worse informed ⇒ larger shock upon discovering SPA ✗
- SPA knowledge endogenous ⇒ if actions independent of SPA don't learn it ✓

back

Mistaken Belief + Employment Sensitivity



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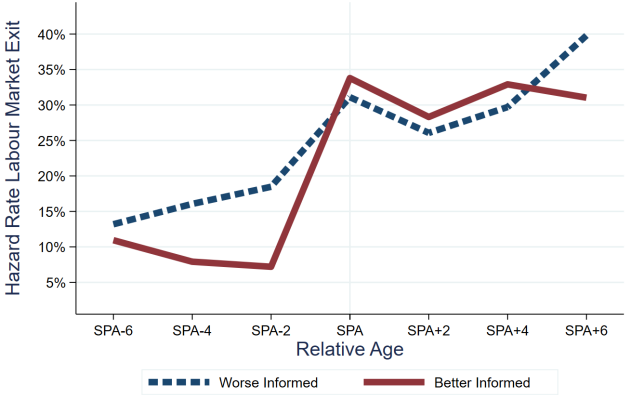

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Mistaken Beliefs

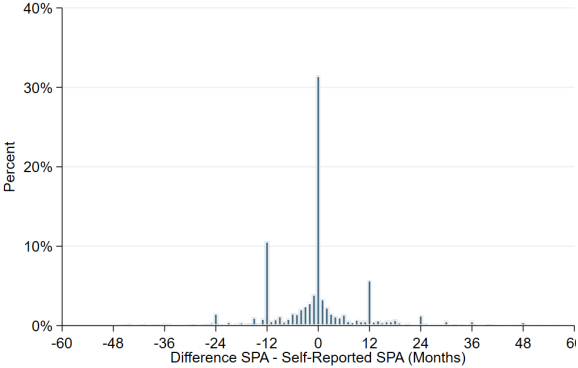
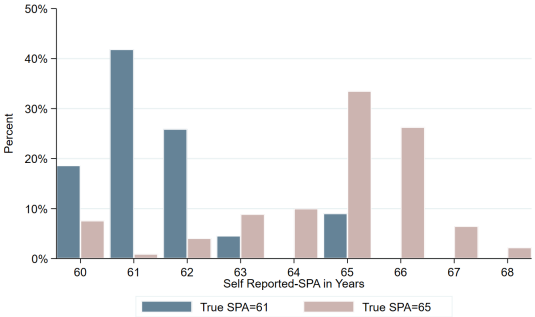


Figure: SPA Beliefs of Women Subject to the Reform Age 58

back

Mistaken Beliefs



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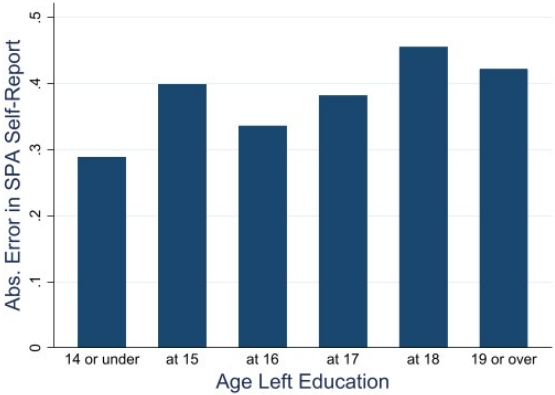

Figure: SPA Beliefs by Years of Schooling

back

Wealth Effect

An increased SPA decreases life-time earnings ⇒ wealth effect increasing labour supply

Could this wealth effect explain the labour supply response to SPA? Probably not

1. The wealth effect should occur over the whole planning horizon, not just immediately before SPA
2. Regression framework only identifies short-run response during quarter before SPA, not whole 15 year horizon (subsumed in cohort effect)
3. Treatment effect is a lower bound on the labour supply response to the SPA reform over the life-cylce
4. If this treatment effect was generated by the wealth effect it would imply a mean MPE of -0.3
5. An MPE of -0.3 is on the high end of modern estimates (e.g. Cesarini et al. 2017, AER) but should only captures a tiny fraction of the total wealth effect

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Health Profile

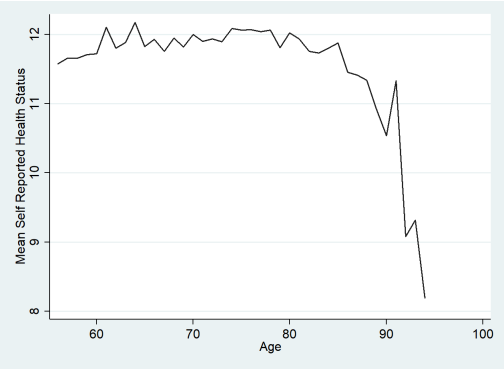


Figure: Self Reported Health Profile

Health Profile

Heterogeneity by health for Individuals with above median assets

	Coeff	s.e.	P >|z|
Below SPA	0.093	0.0429	0.030
Below SPA×(V.good Health)	-0.027	0.0371	0.461
Below SPA×(Good Health)	0.016	0.0390	0.689
Below SPA×(Fair Health)	-0.056	0.0422	0.186
Below SPA×(Poor Health)	-0.145	0.0495	0.003

- Appears only to be relevant for those in poor health
- These individuals have a lower treatment effect
- Poor health individuals make up <7 % of the sample

back

Placebo Test

Table: Placebo Tests

	One Year Below SPA	Two Years Below SPA
Placebo Test Coefficient	**0.031**	**0.005**
s.e	(0.0256)	(0.0230)
p=	.239	.831
Obs.	4,279	4,279

Notes: A placebo test for a violated parallel trends assumption. I drop observations over SPA and replace the treatment with an indicator for being past the date one or two years before SPA.

- Parallel trends is equivalent to cohort-and-date-constant age effects and age-and-date-constant cohort effects, and cohort-and-age-constant age effects
- A Wald test fails to reject the null that these interaction are zero ($p = 0.5377$)

back

Baseline Model of Retirement

- Solve following problem to find consumption $c_t(\boldsymbol{X_t})$, leisure $l_t(\boldsymbol{X_t})$, and savings $a_{t+1}(\boldsymbol{X_t})$ in terms of states $\boldsymbol{X_t} = (\boldsymbol{a_t}, \boldsymbol{w}_t, \boldsymbol{AIME_t}, \boldsymbol{u_t}, \boldsymbol{i})$ more details

$$V_t(\boldsymbol{X_t}) = \max_{c_t, l_t, a_{t+1}} \{u_i(c_t, l_t, \boldsymbol{a_t}) + \beta s_{i,t+1} E[V_{t+1}(\boldsymbol{X_{t+1}})|\boldsymbol{X_t}]\}$$

s.t

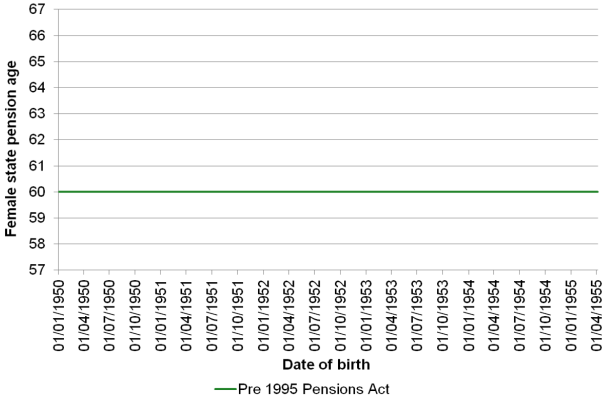
$$c_t + a_{t+1} = (1+r)\boldsymbol{a_t} + \boldsymbol{w_t}(1-l_t) + b\mathbb{1}[\boldsymbol{u_t} = 1] + \mathbb{1}[t \geq SPA]S_i(\boldsymbol{AIME_t})$$
$$+ y_i(t) + \mathbb{1}[t \geq PPA_i]P_i(\boldsymbol{AIME_t})$$

$$a_{t+1} \geq 0,\ (1-l_t)\boldsymbol{u_t} = 0$$

where $u_i(c_t, l_t, \boldsymbol{a_t}) = n_i \dfrac{((c_t/n_i)^{\nu} {l_t}^{1-\nu})^{1-\gamma}}{1-\gamma}|alive + \theta \dfrac{(\boldsymbol{a_t} + K)^{\nu(1-\gamma)}}{1-\gamma}|deceased$

back

Reforms


Female state pension age
67
66
65
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01/01/1951
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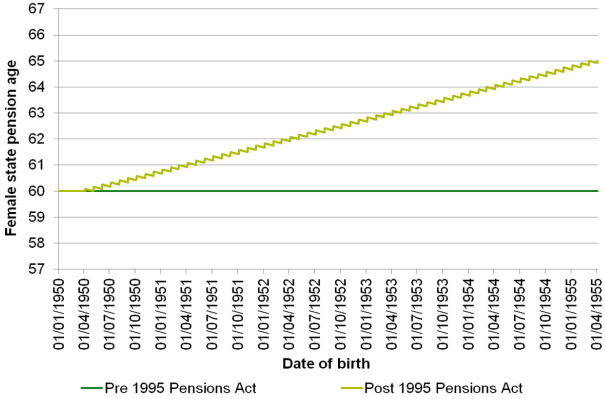
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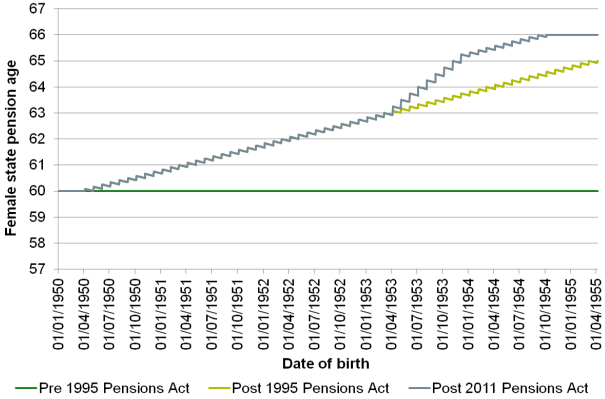

back

Rational Inattention: Modelling Assumptions

1. Model information acquisition as choice of signal function
2. Disutility from information acquisition $\propto$ expected reduction in uncertainty as measured by entropy
   2.1 **Tractability** and follows **convention**
   2.2 Endogenously generates **observed heuristics** e.g. Koszegi & Matějka (2020,QJE) show mental budgeting and naive diversification
   2.3 Entropy developed to explain how **computers** process **information**,

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back

Rational Inattention: Modelling Choices

Utility cost of information $\propto$ ...

$$I(\underline{f_t};\underline{\pi_t}) = \sum_z \sum_{spa} \pi_t(spa) f_t(z|spa) \log\left(\pi_t(spa) f_t(z|spa)\right) - \sum_{spa} \pi_t(spa) \log(\pi_t(spa))$$

(i.e. how much signal reduces uncertainty about SPA as measured by entropy entropy video)

... standard assumption in rational inattention literature details

... convex cost of choice $\underline{f_t}$ that depends on probabitlites not outcomes

back

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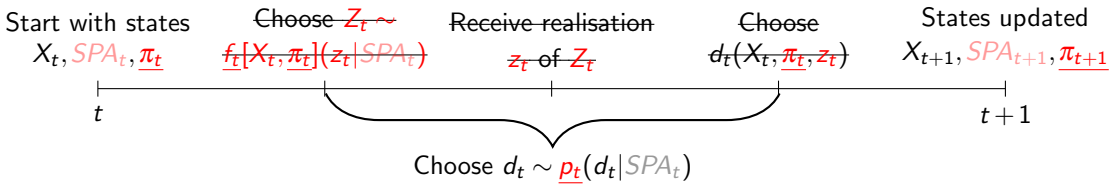
Rational Inattention



back

Employment Response by Learning

Table: Treatment Effect Heterogeneity by Learning

Treatment Effect (SPA Knowledge Gets Worse)	**0.054**
s.e	(0.0317)
p=	.091
Treatment Effect (SPA Knowledge Weakly Gets Better)	**0.116**
s.e	(0.0216)
p=	.000
Obs.	10,488
Cohorts	63

back

Employment Response by Error Direction

Table: Treatment Effect Heterogeneity by Learning

Treatment Effect (Weakly Overestimates SPA)	**0.115**
s.e	(0.0175)
p=	.000
Treatment Effect (Underestimates SPA)	**0.066**
s.e	(0.0396)
p=	.099
Obs.	10,488
Cohorts	63

back

Excess Sensitivity + Mistaken Beliefs

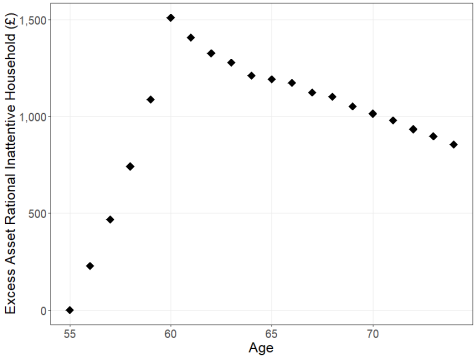
Table: Heterogeneity by SPA Knowledge

Below SPA		**0.202**
	s.e	(0.0385)
	p=	.000
Below SPA×(abs. Error in SPA report)		**-0.052**
	s.e	(0.0246)
	p=	.0039
Obs.		5,115

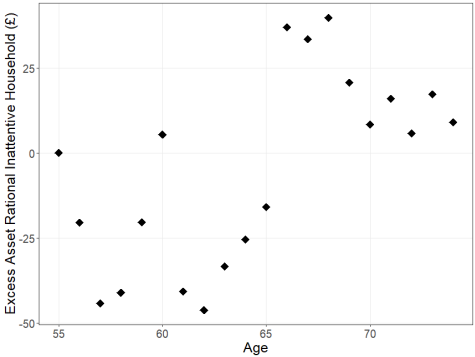
Learning Direction Back

Optimal Savings

Figure: Excess Saving



(a) $\lambda = 3 \times 10^{-6}$



(b) $\lambda = 1.3 \times 10^{-7}$

Back

Optimal Savings

Table: Regression Analysis of the Determinants of Learning

	Constant	Assets	AIME	Wage	Age = 56	Age = 57	Age = 58	Age = 59
Coefficient	0.18	-2.898e-07	2.826e-06	9.655e-08	-6.458e-02	-7.781e-02	-9.796e-02	-8.282e-02

Notes: Regression coefficient where the dependent variable is bits of information acquired

Back

Policy Experiment

- Introduce fraction of naive-passive agents who retire at SPA alongside costly attention
  - ... 8% passive allows $\lambda \approx$ £15 to match all key facts
- Compare to model that uses "only" naive-passive (12%) agents to explain puzzle
  - ... compare SPA's effectiveness at increasing old age labour force participation

Table: Additional Mean Employment (Years) from Increasing SPA from 60

Post-reform SPA	With Fraction of Passive Household			
	Policy Uncertainty		Costly Attention	
	Prime (55-65)	Work Life (55-79)	Prime (55-65)	Work Life (55-79)
61	0.11	0.12	0.11	0.12
62	0.23	0.26	0.23	0.26
63	0.32	0.35	0.30	0.34
64	0.63	0.71	0.50	0.59
65	0.63	0.70	0.50	0.58

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- Look at SPA's effectiveness at increasing old age labour force participation
- Increases the SPA from 60 when the cohort is 55
- Compare prediction of the model with and without costly attention

Table: Additional Mean Years of Employment between 55-65 due to Increasing SPA from 60

Post-reform SPA	Policy Uncertainty	Costly Attention
61	0.07	0.06
62	0.15	0.15
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Costly attention $\Rightarrow$ larger employment response at SPA but potentially smaller overall

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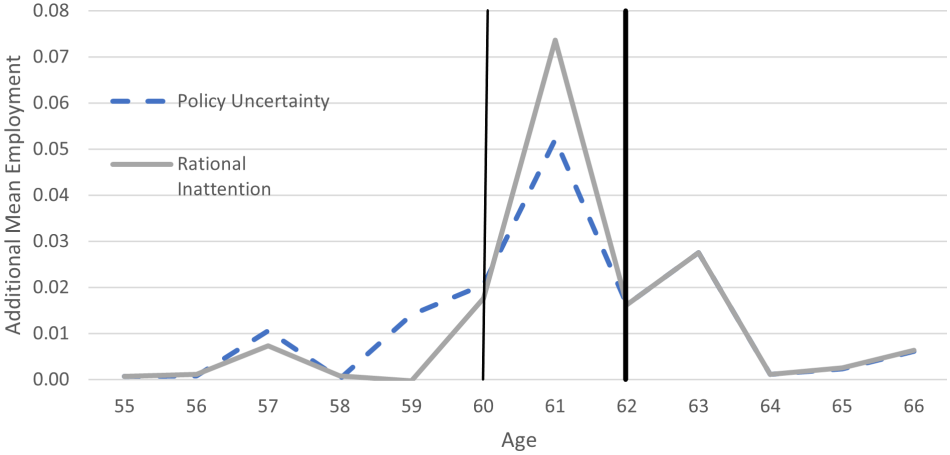
Employment Sensitivity

Figure: Increase in Employment from Increasing SPA 60 to 62



Back

Extension - Deferral

- Up to now have ignored benefit claiming and deferral adjustment
- **Deferral puzzle**: <13% defer despite >10% p.a. increase in benefit

Extension:

- Add claiming decision and cost of attention to stochastic adjustment
- Rational inattention $\Rightarrow$ incentive to claim early (removes attention cost)

Can this help with the deferral puzzle?

- Preliminary results indicate yes!
- Fraction voluntarily claiming pension rises from 6.2% to 22.2%

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Table: Model Predictions - Extension with benefit claiming and uncertain deferral

	Costly Attention	Data
Population	Treatment Effect for being below SPA on participation	
Entire Simulated Sample	0.0416	0.080
Assets >Median(£29,000)	0.0903	0.061
Age	Variance of SPA Answers	
55	2.985	2.852
58	1.795	1.180
Coefficient	Treatment Effect Hetrogeneity by SPA Error	
Treatment Effect	0.0532	0.132
Interaction	-0.0111	-0.066

Back

Analytic Solution - Outline

Specifically, the posterior is formed as:

$$Pr(spa|z) = \frac{f_t(z|spa)\pi_t(spa)}{\sum_{spa'} \pi_t(spa')f_t(z|spa')}$$

And next periods prior is found by applying the law of motion to the posterior

back

Belief Updating

Step one suppress signal function

1. Chooses stochastic signal (f_t) + deterministic (d_t) actions given realisation (z_t)

$\Leftrightarrow$ Choosing conditional choice probabilities (CCPs) $d_t \sim \underline{p_t}(d_t|SPA_t)$

2. Cost of info. $\Rightarrow$ chooses no superfluous info.

$\Rightarrow$ Actions reveal signal, CCPs ($\underline{p_t}$) are relabelling of signal function ($\underline{f_t}$) :

3. Can replace signal function ($\underline{f_t}$) by CCPs ($\underline{p_t}$) in cost of information

$$I(\underline{p_t};\underline{\pi_t}) = \sum_{spa} \pi_t(spa) \sum_d p_t(d|spa) \log(p_t(d|spa)) - \sum_d q_t(d) \log(q_t(d))$$

where q_t is unconditional corresponding to conditional $\underline{p_t}$: Timeline

$$q_t(d) = \sum_{spa} \pi_t(spa) p_t(d|spa)$$

Step two: express problem in terms of CCP ($\underline{p_t}$) and solve FOCs back

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$$I(\underline{p_t};\underline{\pi_t}) = \sum_{spa} \pi_t(spa) \sum_d p_t(d|spa) \log(p_t(d|spa)) - \sum_d q_t(d) \log(q_t(d))$$

where q_t is unconditional corresponding to conditional $\underline{p_t}$: Timeline

$$q_t(d) = \sum_{spa} \pi_t(spa) p_t(d|spa)$$

Step two: express problem in terms of CCP ($\underline{p_t}$) and solve FOCs back

Belief Updating

Step one suppress signal function

1. Chooses stochastic signal (f_t) + deterministic (d_t) actions given realisation (z_t)

$\Leftrightarrow$ Choosing conditional choice probabilities (CCPs) $d_t \sim \underline{p_t}(d_t|SPA_t)$

2. Cost of info. $\Rightarrow$ chooses no superfluous info.

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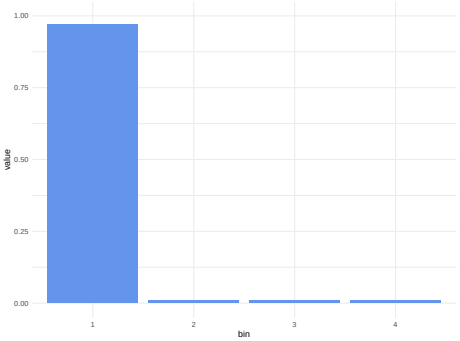

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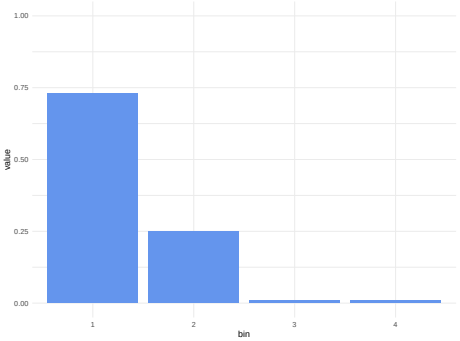

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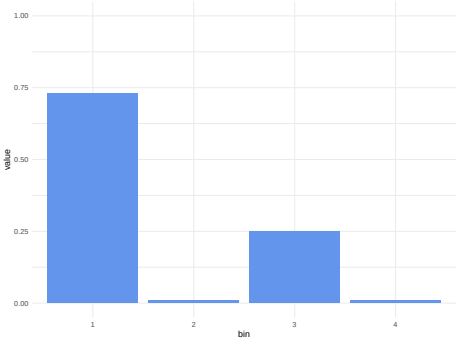

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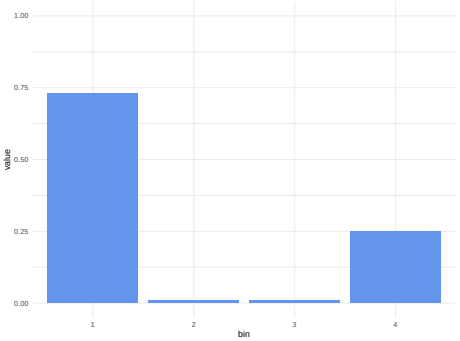

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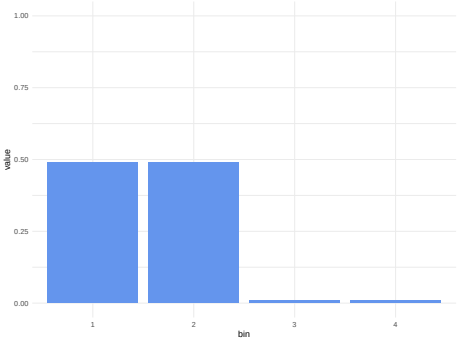

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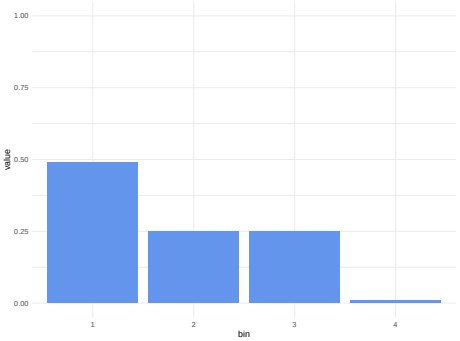

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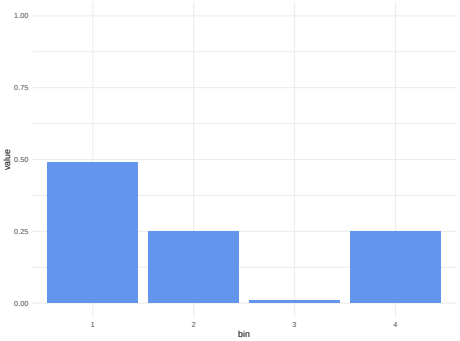

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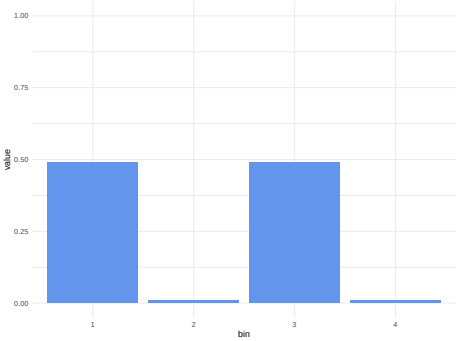

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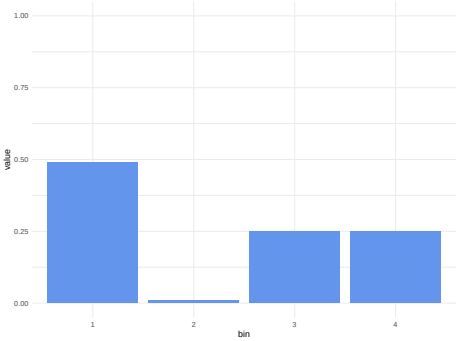

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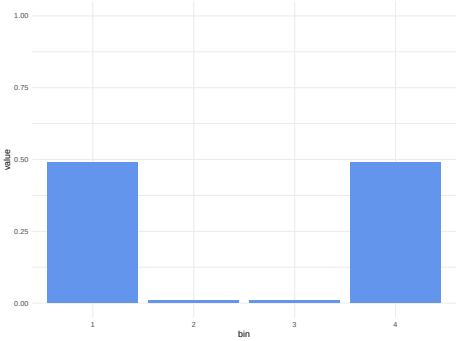

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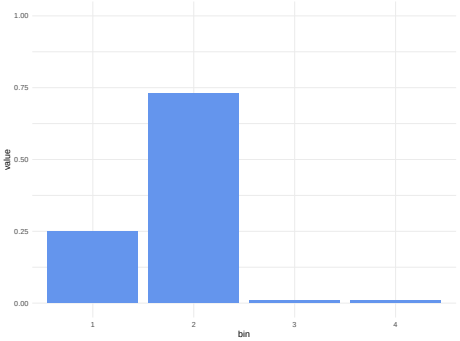

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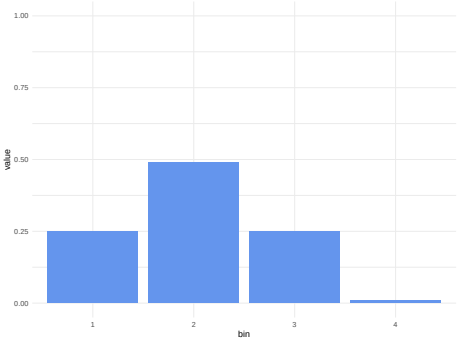

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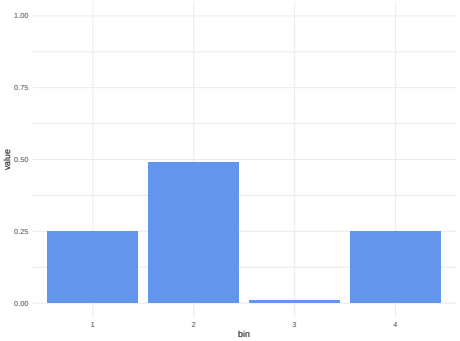

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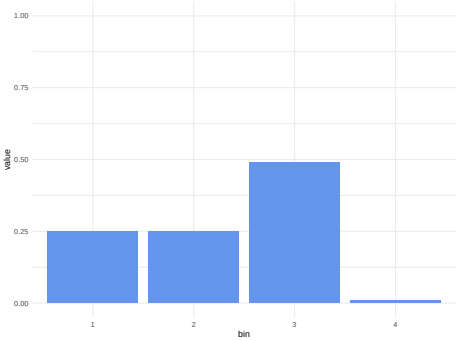

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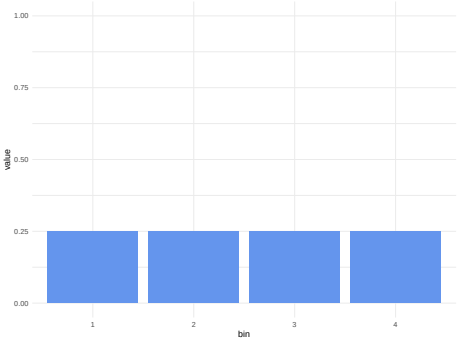

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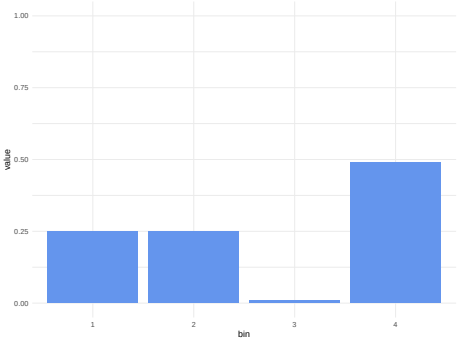

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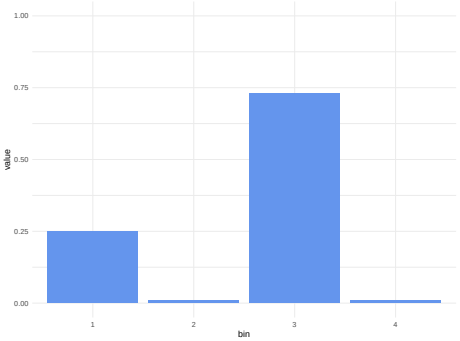

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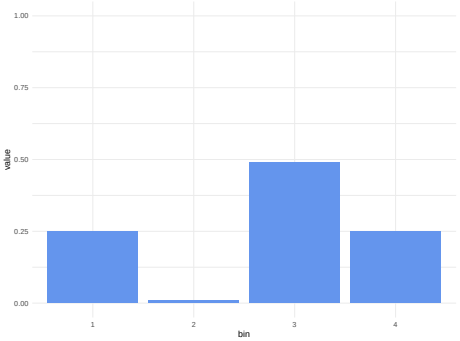

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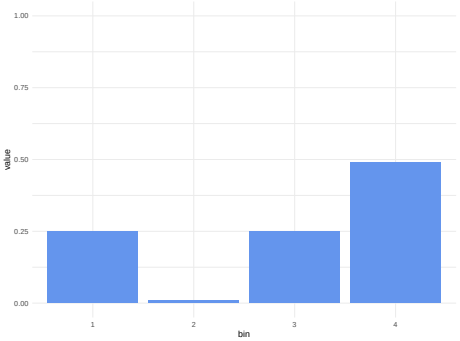

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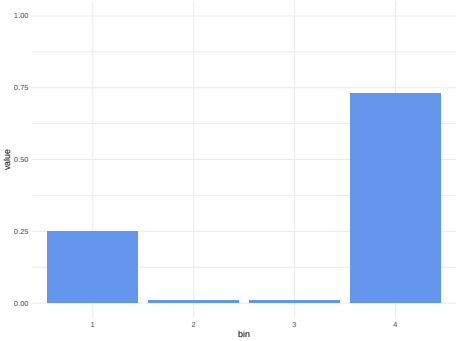

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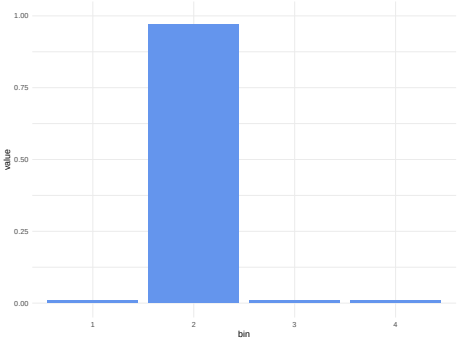

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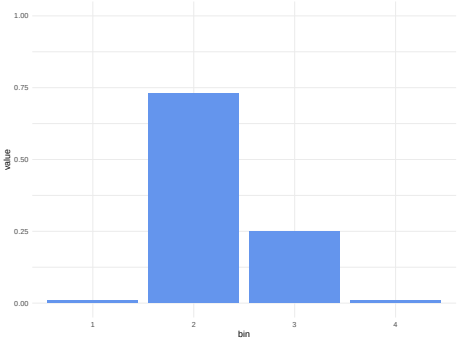

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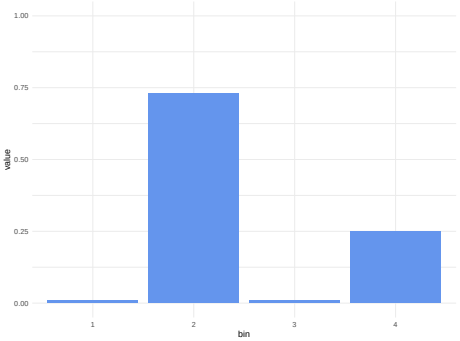

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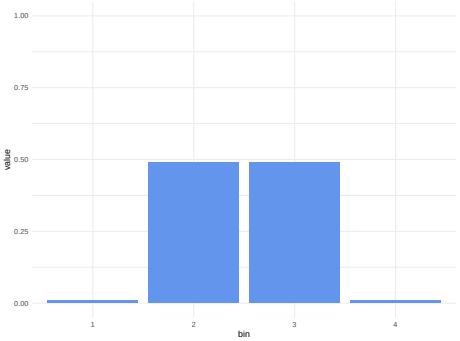

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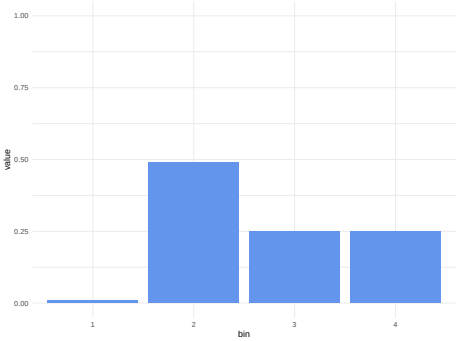

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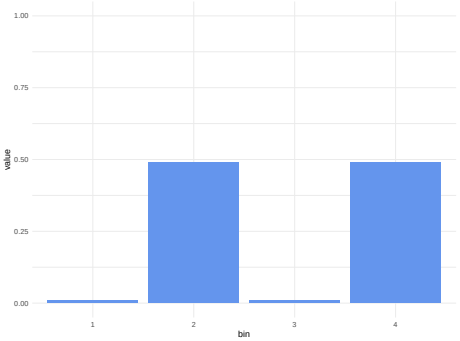

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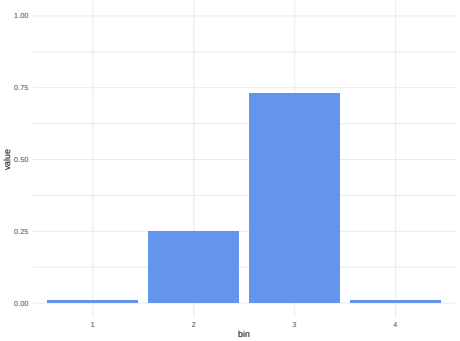

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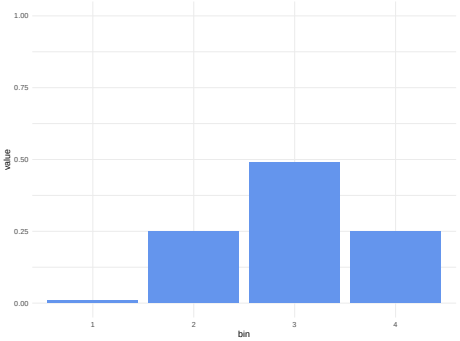

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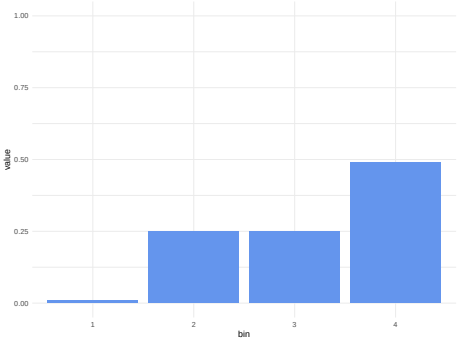

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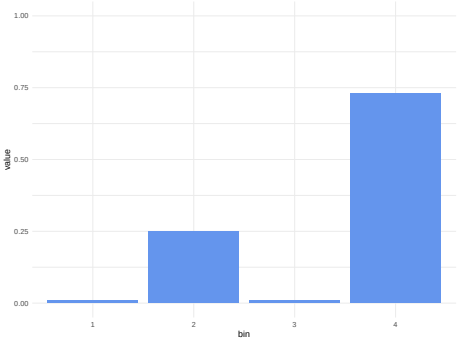

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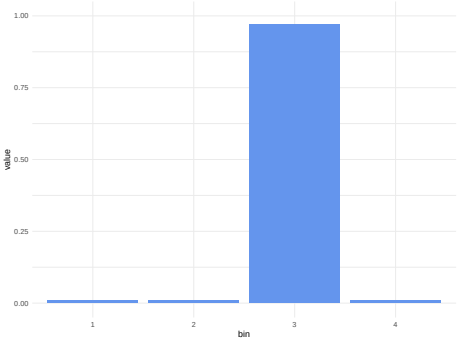

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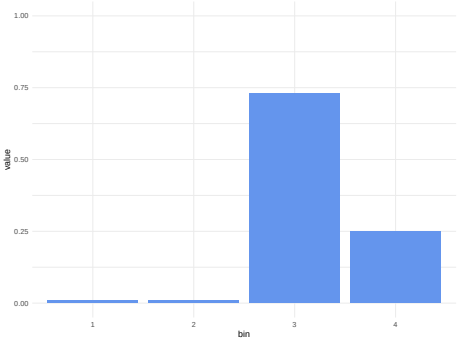

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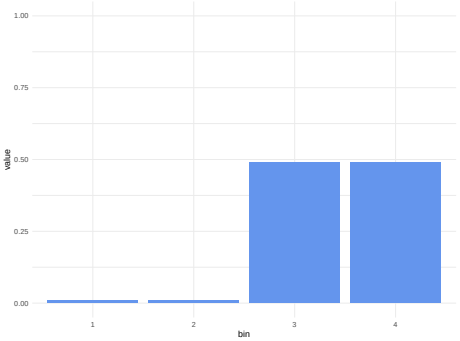

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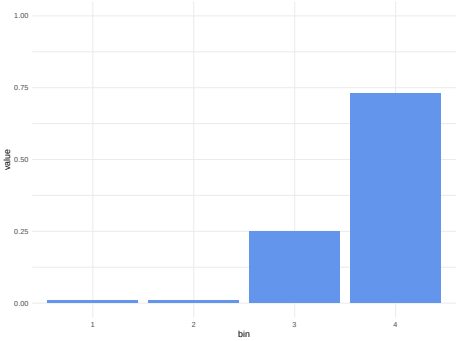

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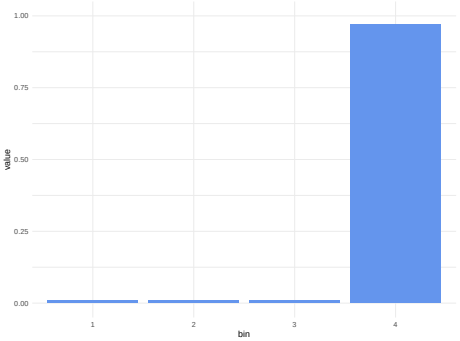

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