Learning and Dynamic Schooling Decisions

DSE Summer School

Arnaud Maurel

Duke University, NBER and IZA

July 2, 2025

Introduction

- Structural estimation of dynamic discrete choice models of schooling decisions started in the 90's.
- Builds on estimation methods for dynamic discrete programming models (Wolpin, 1984; Pakes, 1986; Rust, 1987...).
- Initial applications in labor focused on post-schooling decisions (e.g. labor supply, Eckstein and Wolpin, 1989).
- Keane and Wolpin (1997): dynamic life-cycle model of schooling, labor force participation and occupational decisions.

Introduction (Cont'd)

- Dynamic structural approach: consistent with Ben Porath's dynamic human capital accumulation model.
- Explicitly model human capital investment as resulting from an intertemporal optimization problem.
- At any given period t, forward-looking agents make the decision that maximizes their expected lifetime utility.
- **Typically assume that agents perfectly know their own characteristics.**
- **Today: relax this assumption, allowing agents to have imperfect information and learn about their own characteristics (e.g. ability/productivity).**

Relevant literatures

- **Schooling investments under imperfect information about ability**: Altonji (1993); Arcidiacono (2004); Stange (2012); Stinebrickner and Stinebrickner (2012, 2014), Thomas (2019); Proctor (2022); Larroucau and Rios (2022); Arcidiacono et al. (2025)...
- **Occupational choice with learning**: Miller (1984); James (2012); Antonovics and Golan (2012); Papageorgiou (2014); Sanders (2014); Hincapié (2020); Pastorino (2024); de Paula et al. (2025)...
- **Dynamic learning models**: Erdem and Keane (1996); Crawford and Shum (2005); Ching et al. (2013); Aguirregabiria and Jeon (2020); Bunting et al. (2025)...

We will now cover in detail Arcidiacono, Aucejo, Maurel and Ransom (*JPE*, 2025).

Motivation

- Substantial returns to receiving a four-year college degree (Heckman et al., 2006, Goldin and Katz, 2008, Bound and Turner, 2011, Ashworth et al., 2021).
- Low bachelor's completion rates: in the US, 40% of the students entering four-year college do not earn a bachelor's degree within six years (NCES, 2021).
- Imperfect information, and learning, likely to play a role in the decision to leave college.

Motivation (Cont'd)

- Learning can take place both in college and in the labor market.
- Students often take breaks before completing their degree.
  - In the NLSY97, 43% of the individuals who left college before graduation returned to college at some point (*stopouts*).
- Descriptive evidence suggests that learning about schooling ability and labor market productivity plays a role.
  - Those with negative GPA shocks more likely to leave college in the following period.
  - Those with negative wage shocks more likely to come back to college in the following period.

Overview

What are the extent of imperfect information on ability, and its impact on schooling/occupational choices & school-to-work transitions?

Using the NLSY97, we estimate a dynamic model that allows:

- Individuals to enroll or re-enroll in two or four-year colleges (Science/non-Science majors) in every period.
- Work part-time, full-time, or not at all - potentially while in college.
- Two sectors: blue- and white-collar. White-collar jobs only available with probability λ.
- Learn about their ability through GPA and wages.

Choice sets and expectations

- Choice set before college graduation:
  - School: 2-year; 4-year science, 4-year non-science, none.
  - Work: i) Part-time, Full-time, none; and ii) blue-collar or white-collar job (prob. $\lambda < 1$ if haven't worked in white-collar in $t-1$).
  - Home: reference alternative.
- College graduates choose among the different work alternatives.
- Time is discrete, individuals choose the sequence of actions maximizing the present value of *expected* lifetime utility.
- Expectation wrt. the distribution of:
  - Future idiosyncratic shocks;
  - **Ability signals associated with the alternative choice paths**;
  - Aggregate labor market shocks;
  - Receiving an offer in white-collar sector;
  - College graduation.

Information structure

Two main dimensions of unobserved heterogeneity:

1. Individuals have imperfect information about their schooling ability and labor market productivity.
   - College: learn about their ability by observing their performance (GPA) at the end of the year.
   - Gap between observed and expected GPA is an ability signal, which is used to update their belief in a Bayesian fashion.
   - Also provides some information about their ability in other types of colleges and majors, along with their productivity $\rightarrow$ *correlated learning*.
   - Same for wages.
2. Permanent unobserved (*to the econometrician only*) heterogeneity.

Throughout the analysis, maintain the assumption that agents have rational expectations.

Learning - notations

- Focus on the unknown portion of ability.
- Ability is multidimensional, given by $A = (A_{i2}, A_{i4s}, A_{i4ns}, A_{iw}, A_{ib})'$.
- Denote S_{it} as the vector of noisy signals the individual has received in period t.
    - Signals from grades, wages, or both.
- The prior ability mean is $E_{it}(A_i)$ where at $t = 1$ expected ability is zero.
- Denote the prior variance $\Sigma_{it}(A_i)$ where at $t = 1$ the prior variances is the population variance.
- Denote Ω_{it} as a 5×5 matrix with 1 divided by the transitory variances on the diagonal for signals received and zeros everywhere else.

Learning - updating

Assuming ability and signals are normally distributed, $E_{it}(A_i)$ and $\Sigma_{it}(A_i)$ are updated according to (DeGroot, 1970):

$$
\begin{aligned}
E_{it}(A_i) &= \left(\Sigma_{it-1}^{-1}(A_i) + \Omega_{it}\right)^{-1} \left(\Sigma_{it-1}^{-1}(A_i) E_{t-1}(A_i) + \Omega_{it} S_{it}\right) \\
\Sigma_{it}(A_i) &= \left(\Sigma_{it-1}^{-1}(A_i) + \Omega_{it}\right)^{-1}
\end{aligned}
$$

With additional signals, prior variance moves towards zero, giving more weight to the prior and less to the signal.

Unidimensional case: reduces down to the usual formula

$$E_{it}(A_i) = \frac{\sigma_A^2}{\sigma_A^2 + \sigma_\varepsilon^2} S_t + \frac{\sigma_\varepsilon^2}{\sigma_A^2 + \sigma_\varepsilon^2} E_{t-1}(A).$$

Grades

- Grades in two-year colleges and in the first two years of four-year colleges are given by, denoting by τ the period of college enrollment and j the type of college/major :

$$G_{ij\tau} \quad = \quad \gamma_{0j} + X_{ij\tau}\gamma_{1j} + A_{ij} + \varepsilon_{ij\tau}$$

  where $X_{ij\tau}$ is known to the individual i (demographics, ability, work while in college), A_{ij} is an unknown ability factor, and $\varepsilon_{ij\tau}$ is a noise distributed $\mathcal{N}(0, \sigma^2_{j\tau})$.

- Noisy signal on ability A_{ij} given by:

$$G_{ij\tau} - \gamma_{0j} - X_{ij\tau}\gamma_{1j} \quad = \quad A_{ij} + \varepsilon_{ij\tau}$$

Grades (Cont'd)

- Let $AI_{ij\tau} = \gamma_{0j} + X_{ij\tau}\gamma_{1j} + A_{ij}$ (academic index). For four-year colleges and periods $\tau > 2$, grades are given by:

$$G_{ij\tau} = \lambda_{0j} + \lambda_{1j} AI_{ij\tau} + \varepsilon_{ij\tau}$$

- Parsimonious location-scale specification that allows returns to ability to vary across periods of college enrollment and majors.

Wages

- Wages in the white-collar sector:

$$\ln(W_{iwt}) = \delta_t + X_{iwt}\gamma_w + A_{iw} + \varepsilon_{iwt}$$

- Wages in the blue-collar sector:

$$\ln(W_{ibt}) = \delta_t + X_{ibt}\gamma_b + A_{ib} + \varepsilon_{ibt}$$

- Productivity shocks $(\varepsilon_{iwt}, \varepsilon_{ibt})$: normally distributed (with sector-specific variances), independent over time, across sectors, and of the other state variables.
- Aggregate shocks: individuals form expectations over the $(\delta_{t'})_{t' \geq t+1}$ assuming an AR(1) process.

Wages (Cont'd)

- For in-school work, log-wages in sector l given by:

$$\ln(W_{ilt}) = \delta_{lt} + \lambda_{0l} + \lambda_{1l} PI_{ilt} + \varepsilon^{s}_{ilt}$$

  where $PI_{ilt} = \gamma_{0l} + X_{ilt}\gamma_{1l} + A_{il}$ (sector-l productivity index).

- Allow for noisier signals from in-school vs. out-of-school work.
- Controls: demographics, completed years of college (and major for white-collar wages), HS GPA, experiences in blue- and white-collar sectors, and work part-time.

Flow utilities and consumption

Flow utility for a particular schooling (j) /work (intensity k, sector l) combination is:

$$U_{jkl}(Z_{it}, \varepsilon_{ijklt}) \quad = \quad \alpha_{jkl} + \alpha_C E\left(U(C_{ijklt})\right) + Z_{1it}\alpha_j + Z_{2it}(\alpha_k + \alpha_l) + \varepsilon_{ijklt}$$

where the (ε_{ijklt})'s are distributed i.i.d. Type 1 extreme value.

- CRRA utility of consumption (θ set using grid search):

$$U(C_{ijklt}) = \frac{C_{ijklt}^{1-\theta}}{1-\theta}.$$

- Consumption depends on labor income, parental transfers, tuition fees and educational grants/loans.
- Decisions based on *expected* utility of consumption, as individuals don't see their actual wages until after the choice is made.

Consumption

$$C_t = \max(C_t^*, \underline{C})$$

with

$$C_t^* = \begin{cases} W_t & \text{if working \& not in school \& t} < T^* \\ W_t + PT_t + G_t + L_t - TF_t & \text{if working while in school \& t} < T^* \\ PT_t + G_t + L_t - TF_t & \text{if in school \& not working \& t} < T^* \\ W_t - SLR_t & \text{if working \& not in school \&} \ t \geq T^* \\ 0 & \text{if not in school \& not working} \end{cases}$$

where W_t denotes labor income, PT_t parental transfers, G_t educational grants, L_t educational loans, TF tuition and fees and SLR_t loan repayment.

Consumption floor $\underline{C} = \$2,800$ (Hai and Heckman, 2017); assume loan repayment begins after T^*.

Flow utilities and consumption (Cont'd)

- State variables that transition over time are:
  1. **Prior ability means and variances**.
  2. Age, experiences, accumulated schooling and four-year college degree.
  3. Lagged decisions (switching costs).
  4. Accumulated debt.
  5. Aggregate shocks to the labor market.

- Also allow for unobserved (*to the econometrician only*) heterogeneity in preferences.

Value functions

- The conditional value function is given by:

$$v_{jkl}(Z_{it}) = u_{jkl}(Z_{it}) + \beta E_t\left[V_{t+1}(Z_{it+1})\middle|Z_{it}, d_{it} = (j,k,l)\right],$$

- Given that the ε's are i.i.d. Type 1 extreme value and white collar offers are uncertain

$$\begin{aligned} v_{jkl}(Z_{it}) = u_{jkl}(Z_{it}) &+ \beta\lambda^{(jkl)}_{i,t+1} E_t\left[\ln\left(\sum_j\sum_k\sum_l \exp(v_{jkl}(Z_{it+1}))\right)\middle|Z_{it}, d_{it} = (j,k,l), \text{Offer}_{ilt+1} = 1\right] \\ &+ \beta\left(1 - \lambda^{(jkl)}_{i,t+1}\right) E_t\left[\ln\left(\sum_j\sum_k \exp(v_{jkB}(Z_{it+1}))\right)\middle|Z_{it}, d_{it} = (j,k,l), \text{Offer}_{ilt+1} = 0\right] + \beta\gamma \end{aligned}$$

- Information set of the individual at the beginning of period t includes the ability signals received from periods 1 to $t-1$.

Finite dependence

- We re-express the future payoffs to avoid solving the full backward recursion problem.
- **This is key to the computational feasibility of estimation for this type of correlated learning model.**
- Conditional on the one-period-ahead state variables besides ε, the following equalities hold:

$$\ln\left(\sum_{j,k,l}\exp(v_{jkl}(\mathbf{Z}_{it+1}))\right) = v_{j'k'l'}(\mathbf{Z}_{it+1}) - \ln(p_{j'k'l'}(\mathbf{Z}_{it+1}))$$

  for any choice $\{j', k', l'\}$.
- Allows to reformulate the estimation problem such that the differenced future utility term depends only on CCPs a few periods ahead and flow utilities.
- Need to choose choice sequences, given two different initial choices, such that the same distribution of states results from both paths (*finite dependence paths*).

Finite dependence (Cont'd)

- For alternatives (say, a) that do not involve white-collar work, this is straightforward.
- Consider the paths $\{a, home, home\}$ and $\{home, a, home\}$.
- One can check that both choice paths result in the same distribution of states, and hence the same expected future utility at $t + 3$.
- $v_a - v_{\text{home}}$ obtained as a function of the flow utilities and CCPs two periods ahead.
- For choices involving white-collar work, this requires more work since prob. of receiving a white-collar offer $\lambda < 1$.
- 2-period finite dependence still holds using a *weighted choice sequence* (weight above 1 on taking a white-collar job).

Data: NLSY97

- The NLSY97 is a nationally representative sample of 8,984 Americans born between Jan 1, 1980 and Dec 31, 1984.
- Respondents were first interviewed in 1997 and have continued to be interviewed annually (for a total of 17 Rounds as of 2016, which is the last point in our sample).
- Restrict to males and drop individuals who did not complete high school or who were missing key variables such as AFQT, SAT, HS grades or parental education.
- 22,398 person-year observations containing 2,300 people.

Identification

- Key difficulty: tell apart heterogeneity (known by the agents) from uncertainty.
- This paper: identification exploits a measurement system for the (known) heterogeneity components.
- Three (binary) dimensions, 8 types in total: schooling ability, preferences and work motivation.
  - Schooling ability: ASVAB, SAT scores.
  - Schooling preferences: School tardiness, discipline, took extra classes.
  - Work motivation: self-assessment of work standard and work ethics, beliefs about future labor supply.

▸ Measurements

- Use results from Allman et al. (2009) on finite mixture models with multiple measurements to identify the distribution of het. types.

Identification (Cont'd)

- Conditional on known heterogeneity, selection on observables only.
- Selection on ability based on ability beliefs.
- Outcome equation parameters (grades and wages) identified via a control function argument.
- Cond. value functions and flow utilities: identified from the type-specific CCPs, using standard inversion results (Hotz and Miller, 1993).
- Choice probabilities conditional on obs. state variables and measurements: finite mixture of the type-specific CCPs.
- Key identifying assumption: measurements $\perp\!\!\!\perp$ choices $|$ het. types.

Identification (Cont'd)

- Beyond this application, general challenge for learning models in general.
- Bunting, Diegert and Maurel (2025) study identification of a general class of learning models.
- Relative to Arcidiacono et al. (2025): (much) more flexible on the decision process, and does not rely on measurements for the latent factors.
- Normality maintained for the distribution of unknown heterogeneity and the signals, but distribution-free on the know heterogeneity component.
  - Things are simpler in a "pure learning" (no known het.) environment, allowing for distribution-free identification.

Estimation steps (pure learning case)

Consider first the case without (known) unobserved heterogeneity.

Step 1: Use the EM algorithm to estimate parameters of the wage and grade equations, along with the ability covariance matrix.

Step 2 (Flow utility parameters):

- Estimate reduced-form conditional choice probabilities (CCP) using flexible logits.
- Use the CCPs and distributions of the signals to form expected future value terms.
- Estimate a multinomial logit with a pre-calculated adjustment term on the school/work choices (the future value term).

$\rightarrow$ Estimation is very quick (few min. only).

Step 1: Estimation of outcome and learning parameters

Use the EM algorithm (Dempster et al., 1977) applied to a case where unobserved heterogeneity is *continuous*.

- Iterate, until convergence, between E(xpectation)- and M(aximization)-steps.
- **M-step**: taking as given the posterior ability distribution, maximize the expected complete log-likelihood of the outcomes (separable across sectors).
- For grades (lower-classmen), boils down to OLS on obs. characteristics and expected ability.
- **E-step**: given the outcome parameters, update posterior ability distribution (mean and variance) using the previous Bayesian updating formulas.

Adding heterogeneity types

- Breaks down the separability between choice and outcome components of the likelihood.
- Use an adaptation of the EM algorithm to reinstate the separability between the two stages at the maximization step.
- Two-stage estimation (Arcidiacono and Miller, 2011):
  1. EM estimation of the measurement system, unobserved heterogeneity distribution and CCPs.
  2. Estimation of the learning and structural parameters, using as weights the posterior type probabilities.
- Step 2: similar to the case w/o types, but now maximize instead the *expectation* of the *type-specific* log-likelihood.

Estimation results: parameters

Takeaways:

- Returns to college higher in the white-collar sector than in the blue-collar sector.
- Same is true for majoring in STEM.
- Expected academic abilities significantly affect the utilities of the college options $\rightarrow$ significant cost of effort.
- Working in college lowers expected grades, and working full time makes college unattractive.
- High switching costs associated with changing work or schooling options.
- College graduates have higher white-collar offer rates.

Abilities and learning

	White-collar	Blue-collar	4 yr Sci.	4 yr Non-Sci.	2 yr
Correlation matrix					
White-collar	1.00	0.62	0.28	0.19	0.15
	(—)	(0.04)	(0.09)	(0.06)	(0.08)
Blue-collar	0.62	1.00	0.24	0.05	0.16
	(0.04)	(—)	(0.10)	(0.06)	(0.08)
4 yr Sci.	0.28	0.24	1.00	0.83	0.47
	(0.09)	(0.10)	(—)	(0.05)	(0.11)
4 yr Non-Sci.	0.19	0.05	0.83	1.00	0.67
	(0.06)	(0.06)	(0.05)	(—)	(0.08)
2 yr	0.15	0.16	0.47	0.67	1.00
	(0.08)	(0.08)	(0.11)	(0.08)	(—)
(Population) Variances	0.13	0.07	0.21	0.27	0.35
Variances of Raw Outcomes	0.36	0.27	0.84	0.83	1.03

Abilities and learning (Cont'd)

- Students have a substantial amount of uncertainty about their own abilities by the end of high school.
- 1 std. increase in ability accounts for about half of a grade point.
- In the white-collar (blue-collar) sector: 36% (27%) increase in wages.
- Speed of learning heterogeneous across choice paths.
- Wages are less informative when working while in college: signal-to-noise ratio 0.20 vs. 0.49.

Counterfactual simulations

- We consider two counterfactual scenarios:
  1. Individuals have full information about their abilities in all sectors.
  2. (1) + always possible to work in the white-collar sector.
- Requires solving the model, done backwards from retirement age ($T = 65$).
- In both scenarios, agents are fully informed about their abilities $\rightarrow$ no need to integrate over the prior ability distribution to solve the DP problem.
- Fairly high dimensional state space, kept tractable using simplifying assumptions.
- In particular: no schooling decisions after 28, (data-driven) caps in school and work experience variables, discretized AR(1) aggregate labor market shock process...(see Appendix J for more details).

Information and educational choices

Status	Baseline model	Counterfactuals: Full info. alone	Counterfactuals: Full info. & no search frictions
Continuous completion (CC), Science	6.39	10.30	11.09
Continuous completion (CC), Non-Science	14.86	13.74	14.31
Stop out (SO) but graduated Science	0.97	1.74	1.94
Stop out (SO) but graduated Non-Science	3.45	4.33	4.72
Stop out (SO) then drop out	9.02	6.71	6.50
Truncated	5.76	4.73	5.07
Drop out (DO)	32.44	26.10	25.17
Never went to college	27.11	32.35	31.20
Graduate from 4-year college	25.67	30.11	32.07
Ever Switch Major	25.37	25.13	26.31
Time to degree	5.16	4.99	5.02

Information and labor market outcomes

Sector and Education Level	Average full-time log wage, relative to blue-collar non-graduates in baseline			Choice shares (%)		
	Baseline	Counterfactual	No Frictions Cfl	Baseline	Counterfactual	No Frictions Cfl
White collar, Science graduate	0.46	0.90	0.87	3.44	5.54	6.87
White collar, Non-Science graduate	0.32	0.63	0.56	8.15	5.28	7.25
White collar, Non-graduate	0.11	0.35	0.15	5.46	0.96	4.72
Blue collar, Science graduate	0.20	0.30	0.30	2.45	5.38	4.87
Blue collar, Non-Science graduate	0.11	0.08	0.08	6.62	9.62	8.46
Blue collar, Non-graduate	0.000	0.01	0.07	41.4	49.6	43.8
Remainder	—	—	—	32.5	23.7	24.0

Concluding remarks

- Number of exciting research avenues ahead.
- Recent advances on the econometric front and on the data/measurement side allow to make progress on important questions.
- Consumption value of schooling and preferences are key determinants of educational and occupational choices (Carneiro et al, 2003; Wiswall and Zafar, 2015; Arcidiacono et al, 2020).
- What is the role played by learning about preferences in schooling decisions?
- Much of the literature assumes Rational Expectations and Bayesian updating.
- More research needed to understand, in particular:
  - How agents form and update their beliefs.
  - Incorporate departures from RE/Bayesian updating in dynamic models of schooling decisions.

Grade shocks and college attrition

Difference between actual and expected period-t grades (by $t+1$ period college decision):

	Residual	Std Dev	Mean diff T (p-val)
Dropout from 4-year college & science	-0.203	0.629	1.87
Stay in school (any type)	0.014	0.593	(0.06)
Dropout from 4-year college & non-science	-0.104	0.686	2.00
Stay in school (any type)	0.014	0.530	(0.05)
Dropout from 2-year college	-0.163	0.901	3.78
Stay in school (any type)	0.076	0.654	(0.00)

Wage shocks and college attrition

Log-wage residuals at time t conditional on $t+1$ decision and stopping-out at some point during the sample period:

	Residual	Std Dev	Mean diff T (p-val)
Stay in the labor force	0.100	0.533	2.24
Return to school	-0.054	0.346	(0.03)

Missing majors and GPAs

- GPA and college majors are missing at a non-trivial rate.
- We deal with this issue by treating the (first) instance of unobserved GPA or major as another latent variable. Simple extension of the previous procedure.
- First stage: now also estimate the distribution of unobserved majors and GPAs (discretized into quartiles), conditional on each heterogeneity type.
- Second stage: weighted multinomial logit, where the weights are given by $Pr(Type, Major, GPA|data)$ (instead of $Pr(Type|data)$).

Back

Transitory variances

Period	Skilled	Unskilled	4 yr Sci.	4 yr Non-Sci.	2 yr
1	0.148	0.161	0.646	0.670	0.897
	(0.004)	(0.002)	(0.057)	(0.039)	(0.050)
2			0.160	0.212	0.352
			(0.026)	(0.017)	(0.029)
3			0.140	0.135	0.372
			(0.044)	(0.024)	(0.028)
4			0.120	0.111	
			(0.041)	(0.022)	
5+			0.321	0.199	
			(0.085)	(0.033)	

Back

Measurements

Schooling ability:

- ASVAB Arithmetic Reasoning
- ASVAB Coding Speed
- ASVAB Mathematical Knowledge
- ASVAB Numerical Operations
- ASVAB Paragraph Comprehension
- ASVAB Word Knowledge
- SAT I Math
- SAT I Verbal

Measurements (Cont'd)

Schooling preferences:

- Number of times the individual reported being late for school without excuse
- How strongly the individual agrees with the following statement: "When I was in school, I broke the rules regularly"
- How many hours per week the individual spent taking extra classes (such as music lessons, etc.)
- If the individual ever took classes during a school break (this could either be for remedial or accelerative reasons)
- If the individual took classes during break, the reason for doing so (e.g. "To accelerate, for fun, for enrichment" or "To make up classes")

Measurements (Cont'd)

Work motivation:

- How strongly the individual agrees with the following statement: "I have high standards at work"
- How strongly the individual agrees with the following statement: "I make every effort to do what is expected of me"
- The individual's subjective likelihood of working part- or full-time at age 30 (reported as a percent chance on a scale from 0-100)
- The parent's subjective likelihood of the individual working part- or full-time at age 30 (reported on the same scale as above)

▸ Back