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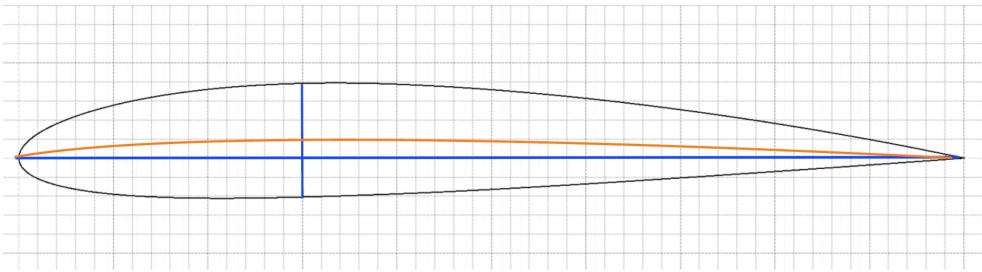
MECH3640 Course Project

1 Part I: Airfoil Aerodynamics

1.1 Determine NACA Number of the recovered wing



Take the width of each of the blocks as 1 unit length:

1. Chord length

   $c = 50$ unit lengths

2. Position of maximum thickness (by definition from 4-digit NACA airfoils)

   $x_{t_{max}} = 0.3c = 15$ unit lengths

3. Maximum thickness (by measuring at $x_{t_{max}}$)

   $t_{max} = 6$ unit lengths, $\frac{t_{max}}{c} = \frac{6}{50} = 0.12$

   The airfoil is NACA XX12.

4. Location of maximum camber

   By roughly draw out the camber line (in orange), you can find that the maximum camber occurs roughly at 0.4c.

   The airfoil is NACA X412.

5. Maximum camber (by measuring at 0.4c)

   $z_{max} = 1$ unit lengths, $\frac{z_{max}}{c} = \frac{1}{50} = 0.02$

   The airfoil is **NACA 2412**.

1.2 Using TAT to calculate

1. $\alpha_{L=0}$

   i) obtaining the airfoil equation

   For NACA 2412,

   location of maximum camber is 0.4c,
   so $p = 0.4$

   maximum camber is 0.02c,
   so $m = 0.02$

For $0 \leq x \leq 0.4c$:

$z = \frac{x}{10} - \frac{x^2}{8c}$

For $0.4c \leq x \leq c$:

$z = \frac{c}{90} + \frac{2x}{45} - \frac{x^2}{18c}$

ii) differentiate

For $0 \leq x \leq 0.4c$:

$\frac{dz}{dx} = \frac{1}{10} - \frac{x}{4c}$

For $0.4c \leq x \leq c$:

$\frac{dz}{dx} = \frac{2}{45} - \frac{x}{9c}$

iii) transforming variable

For $0 \leq x \leq 0.4c$, Substitute $x = \frac{c}{2}(1 - cos\theta)$:

From $0 \leq \frac{x}{c} \leq 0.4$, $0 \leq \theta \leq 1.37$ (rad)

$\frac{dz}{dx} = \frac{cos\theta}{8} - \frac{1}{40}$

For $0.4c \leq x \leq c$, Substitute $x = \frac{c}{2}(1 - cos\theta)$:

From $0 \leq \frac{x}{c} \leq 0.4$, $1.37 \leq \theta \leq \pi$ (rad)

$\frac{dz}{dx} = \frac{cos\theta}{18} - \frac{1}{90}$

iv) plug in the $\alpha_{L=0}$ formula

$\alpha_{L=0} = -\frac{1}{\pi}\int_0^{\pi} \frac{dz}{dx}(cos\theta - 1)d\theta$

$\alpha_{L=0} = -\frac{1}{\pi}[\int_0^{1.37}(\frac{cos\theta}{8} - \frac{1}{40})(cos\theta - 1)d\theta + \int_{1.37}^{\pi}(\frac{cos\theta}{18} - \frac{1}{90})(cos\theta - 1)d\theta]$

$\alpha_{L=0} \approx -0.0363(rad) = -2.0772^\circ$

2. $C_{m,c/4}$

$A_n = \frac{2}{\pi}\int_0^{\pi} \frac{dz}{dx} cosn\theta \, d\theta = \frac{2}{\pi}(\int_0^{1.37}(\frac{cos\theta}{8} - \frac{1}{40})cosn\theta \, d\theta + \int_{1.37}^{\pi}(\frac{cos\theta}{18} - \frac{1}{90})cosn\theta \, d\theta)$

$A_1 = \frac{2}{\pi}(\int_0^{1.37}(\frac{cos\theta}{8} - \frac{1}{40})cos\theta \, d\theta + \int_{1.37}^{\pi}(\frac{cos\theta}{18} - \frac{1}{90})cos\theta \, d\theta)$

$A_1 \approx 0.0815$

$A_2 = \frac{2}{\pi}(\int_0^{1.37}(\frac{cos\theta}{8} - \frac{1}{40})cos2\theta \, d\theta + \int_{1.37}^{\pi}(\frac{cos\theta}{18} - \frac{1}{90})cos2\theta \, d\theta)$

$A_2 \approx 0.0139$

From TAT: $C_{m,c/4} = \frac{\pi}{4}(A_2 - A_1)$,

$C_{m,c/4} \approx -0.0531$

3. x_{cp}

$C_l = 2\pi(\alpha - \alpha_{L=0}) = 2\pi(\alpha + 0.0363(rad))$

$x_{cp} = \frac{c}{4}[1 + \frac{\pi}{C_l}(A_1 - A_2)] = \frac{c}{4}(\frac{0.0338}{\alpha + 0.0363} + 1)$

4. x_{ac}

$x_{ac} = \frac{c}{4}$

Calculation data (with software MathStudio)

```
p=0.4
m=0.02
f1(x) = m/p^2 * (2*p*x - x^2/c)
f2(x) = m/(1-p)^2*((1-2*p)*c+2p*x - x^2/c)
df1(x) = D(f1, x)
df2(x) = D(f2, x)
dft1(t) = df1(c/2*(1-cos(t)))
dft2(t) = df2(c/2*(1-cos(t)))
dfta1(t) = dft1*(cos(t)-1)
dfta2(t) = dft2*(cos(t)-1)
rada0 = -1/@pi*(Int(dfta1, t, 0, 1.37)+Int(dfta2, t, 1.37, @pi))
dega0 = rada0*180/@pi
dftcos11 = dft1 * cos(t)
dftcos21 = dft2 * cos(t)
A1 = 2/@pi*(Int(dftcos11, t, 0, 1.37) + Int(dftcos21, t, 1.37, @pi)
dftcos12 = dft1 * cos(2t)
dftcos22 = dft2 * cos(2t)
A2 = 2/@pi*(Int(dftcos12, t, 0, 1.37) + Int(dftcos22, t, 1.37, @pi)
Cmc4 = @pi/4*(A2-A1)
Cl(a) = 2*@pi*(a - rada0)
xcp(a) = c/4*[1+@pi/Cl*(A1-A2)]
[(c*(0.0338/(a+0.0363)+1))/4]

df1
((-2x)/c+4/5)/8
df2
((-2x)/c+4/5)/18
dft1
(cos(t)-1/5)/8
dft2
(cos(t)-1/5)/18
rada0
-0.0363
dega0
-2.0772
A1
0.0815
A2
0.0139
Cmc4
-0.0531
Cl
6.2832(a+0.0363)
xcp
[(c*(0.0338/(a+0.0363)+1))/4]
```

1.3 Plots

1.3.1 Calculations

a) Details about TAT plots

1. Lift curve

   Range: due to the restrictions of TAT, we can only plot the lift curve with a relatively small angle of attack. Therefore, I am using $-4^\circ \leq \alpha \leq 10^\circ$ as the range of TAT lift curve.

   Equation: from above:

   $$C_l = 2\pi(\alpha + 0.0363(rad)) = 2\pi(\frac{\alpha_{degree} \cdot \pi}{180} + 0.0363)$$

2. Drag polar

   Not possible

   - This is because TAT is based on the assumption that the flow is inviscid. With no viscosity, we do not have viscous drag. In 2D condition, we do not have induced drag. Therefore, drag is not accessible in TAT.

b) Details about Wind tunnel experiments

Using the atmosphere data of my own group (group 14):

1. Calculating the local air density by:

   $\frac{P_1}{\rho_1 T_1} = \frac{P_0}{\rho_0 T_0}$

   From standard atmosphere table:

   $$P_o \approx 101kPa, \rho_0 \approx 1.225kg/m^3, T_0 \approx 288.16K$$

   By measurement:

   $$P_1 \approx 99.87kPa, T_1 \approx 297.4K$$

   Therefore:

   $$\rho_1 \approx 1.17kg/m^3$$

2. Calculate $\frac{2}{\rho_1 V_\infty^2 S} \approx 0.08155 \frac{1}{N}$

   Where $S = chord \times span \approx 0.0395m^2$ and $V_\infty \approx 23m/s$

3. Calculate coefficient of lift (denoted as C_L) and coefficient of drag (denoted as C_D) by:

   $C_L = \frac{2L}{\rho V_\infty^2 S} \approx 0.08155 \frac{1}{N} \cdot L$

   $C_D = \frac{2D}{\rho V_\infty^2 S} \approx 0.08155 \frac{1}{N} \cdot D$

4. By assumption:

   $C_l = C_L$, $C_d = C_D$

c) Details about XFLR5 analysis

1. Airfoil

   NACA 2412, 100 Panels

2. Raynolds number (following the experiment conditions)

   $Re = \frac{Vc}{\nu}$

   $\nu = \nu_{air} = \frac{\mu_{air}}{\rho_{air}} \approx 1.46 \times 10^{-5} m^2/s$

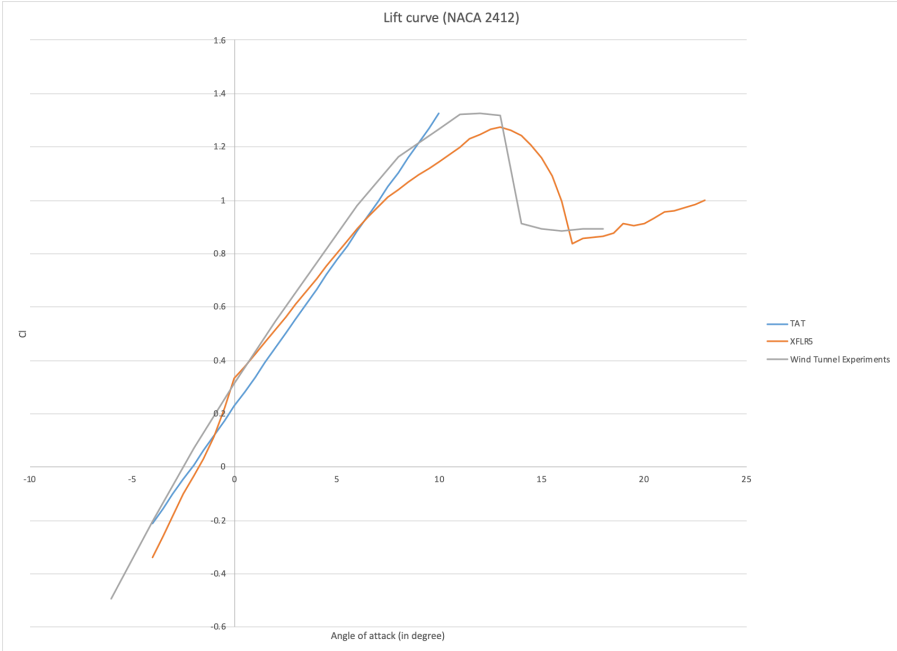
   $V = 23m/s$

   $c = 100mm = 0.1m$

   $Re \approx 157534$

1.3.2 Results

1.3.2.1 Lift curve



Comments

$-4^\circ \leq \alpha \leq 10^\circ$:

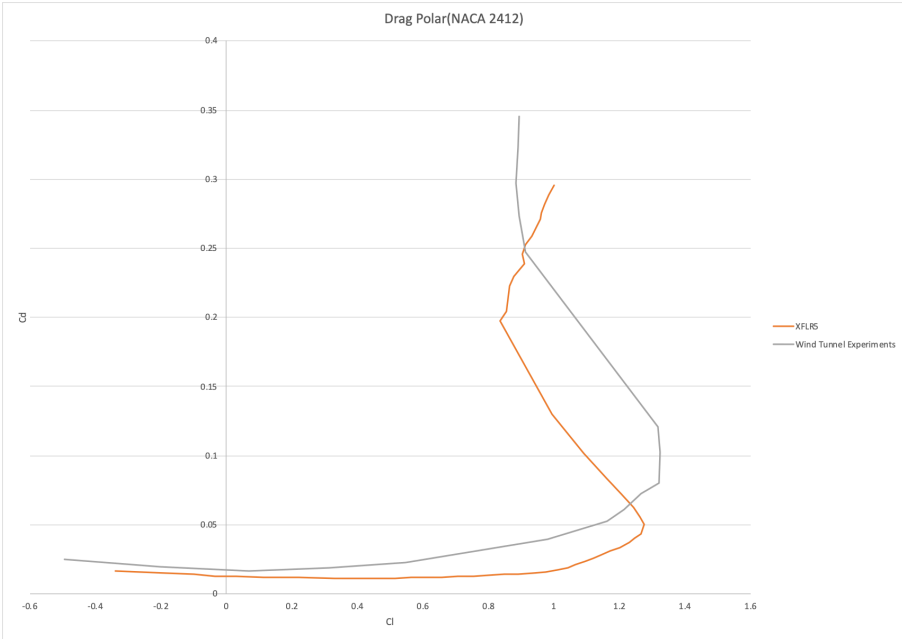
The results from the 3 methods are almost the same. The lift slope from the 3 results are around 2π(For α_{rad}).

$10^\circ < \alpha \leq 18^\circ$:

TAT is no longer applicable.

The lift curve from the numerical analysis and the lift curve from the wind tunnel experiments are almost the same, with some minor differences. Stall occurs at $\alpha \approx 10^\circ$ in wind tunnel experiments results, while in XFLR5 results, stall occurs at $\alpha \approx 13^\circ$, which is a little later. From either method, we found that the lift coefficient will drop relatively slowly after reaching $C_{l,max}$, and will reach a constant region after dropping to $C_l \approx 0.9$. After reaching $C_l \approx 0.9$, there is a growing tread in the XFLR5 results, while such trend is not revealed in the experiment results.

1.3.2.2 Drag polar



Comments

Before reaching $C_{l,max}$

In this region, the trend from the wind tunnel experiments results and the XFLR5 results are similar: when C_l is larger than 0, C_d would increase slightly with the increase of C_l, when C_l is smaller than 0, C_d would increase slightly with the decrease of C_l.

Overall, the C_d estimated by XFLR5 is slightly lower than that from the experiments, and this underestimation would grow with the increase of C_l.

After reaching $C_{l,max}$

In this region, from either method, C_d would rise dramatically while C_l is slowly dropping to $C_l \approx 0.9$. Still, the C_d estimated by XFLR5 is slightly lower than that from the experiments. Besides, after reaching $C_l \approx 0.9$, there is a growing tread in C_l in the XFLR5 results, while such trend is not revealed in the experiment results.

1.4 Strengths and weakness of three different approaches

a) TAT

1. Strengths:
    - Easy to conduct, available even if there is no computer.
    - The result is regular, simple and elegant, with more physics insight.
2. Weakness:
    - Only applicable to thin airfoil with a small camber and angle of attack.
    - Only applicable to incompressible and inviscid flow.
    - Has no access to drag coefficient.
    - Only applicable to 2D airfoil.

b) Wind tunnel experiments

1. Strengths:

   Can better reveal how models will react in real flow.

   Results more reliable.

2. Weakness:

   Costly and time consuming.

   Results sensible to experiment conditions (air pressure, humidity etc.).

   Some quantities hard to measure (pressure distribution, streamline etc.)

c) Numerical

1. Strengths:

   Providing more detailed results (pressure distribution, streamline etc., which are not easily accessible in wind tunnel experiments).

   Less costly and less time consuming.

2. Weakness:

   PDE hard to solve, sometimes does not converge.

   Requiring high computing power.

   Sometimes give unreliable results.

1.5 Effects of airfoil thickness

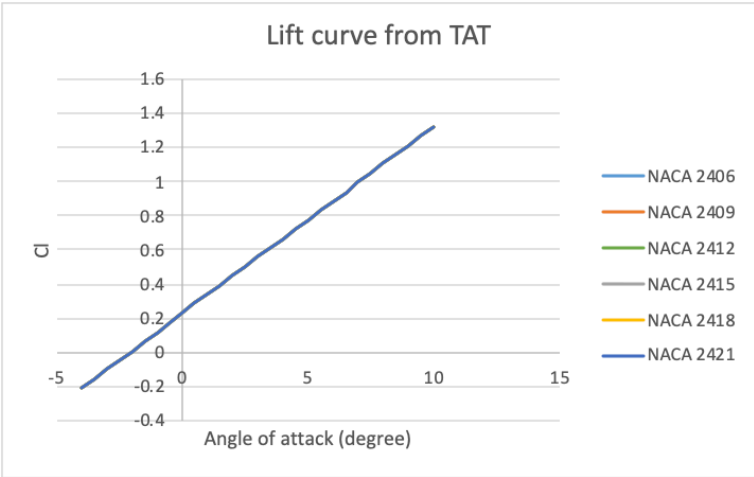
1.5.1 Table of NACA airfoils

	Airfoil	Description
ii)	NACA 2406	6% less thickness
i)	NACA 2409	3% less thickness
	NACA 2412	Baseline airfoil
iii)	NACA 2415	3% more thickness
iv)	NACA 2418	6% more thickness
v)	NACA 2421	9% more thickness

1.5.2 Plotting

1.5.2.1 TAT

Airfoil	Description	Lift curve
NACA 2406	6% less thickness	$C_l = 2\pi(\frac{\alpha_{degree} \cdot \pi}{180} + 0.0363)$
NACA 2409	3% less thickness	$C_l = 2\pi(\frac{\alpha_{degree} \cdot \pi}{180} + 0.0363)$
NACA 2412	Baseline airfoil	$C_l = 2\pi(\frac{\alpha_{degree} \cdot \pi}{180} + 0.0363)$
NACA 2415	3% more thickness	$C_l = 2\pi(\frac{\alpha_{degree} \cdot \pi}{180} + 0.0363)$
NACA 2418	6% more thickness	$C_l = 2\pi(\frac{\alpha_{degree} \cdot \pi}{180} + 0.0363)$
NACA 2421	9% more thickness	$C_l = 2\pi(\frac{\alpha_{degree} \cdot \pi}{180} + 0.0363)$



We can see that the lift curve of the 6 thicknesses are completely the same in TAT, which is not realistic. This is because TAT is not capable of examining the effect of thickness.

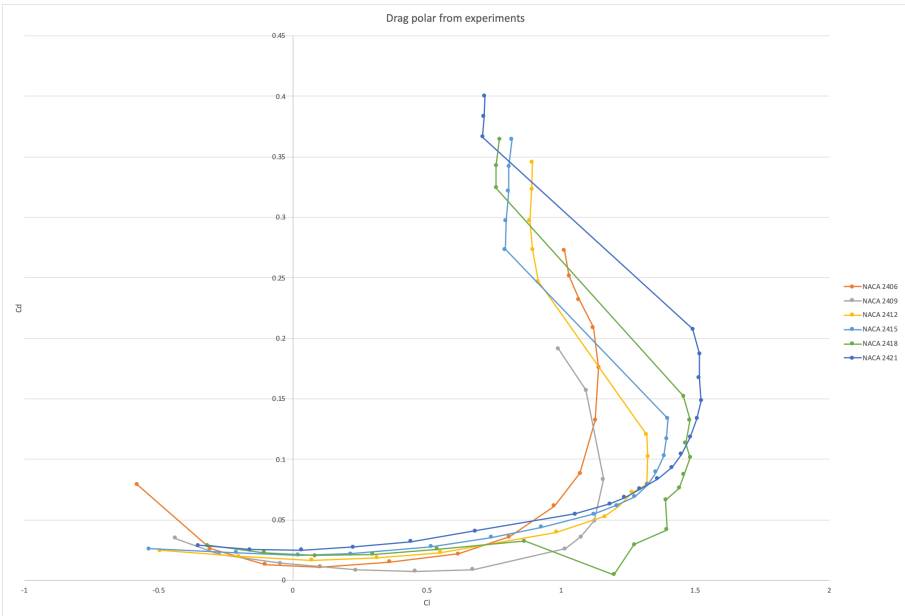
Drag polar not accessible by TAT.

1.5.2.2 Wind tunnel experiments

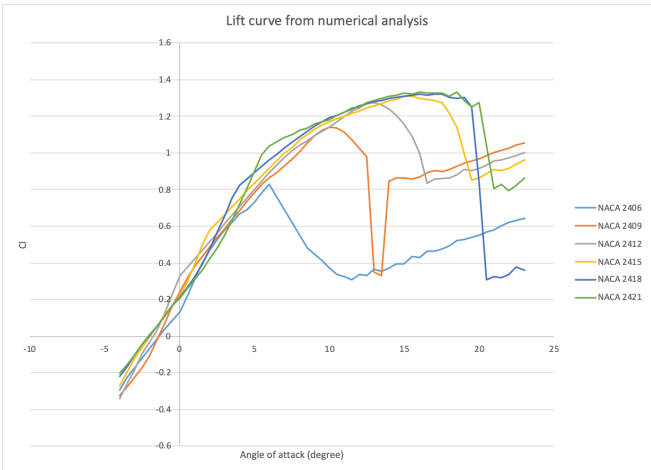
From above, we have:

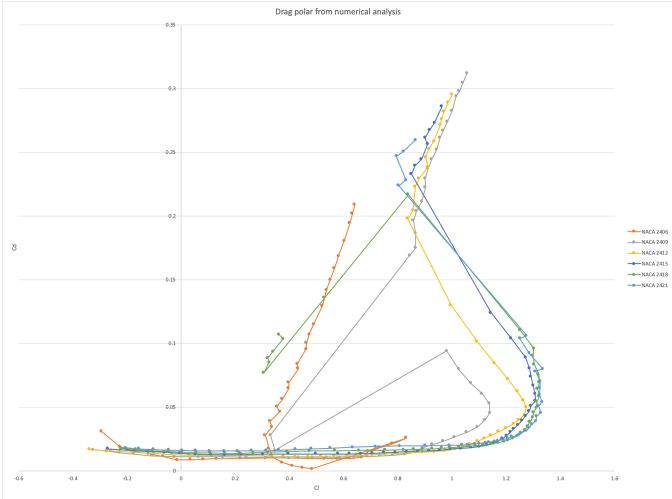
$$C_L = \frac{2L}{\rho V_\infty^2 S} \approx 0.08155 \frac{1}{N} \cdot L$$

$$C_D = \frac{2D}{\rho V_\infty^2 S} \approx 0.08155 \frac{1}{N} \cdot D$$



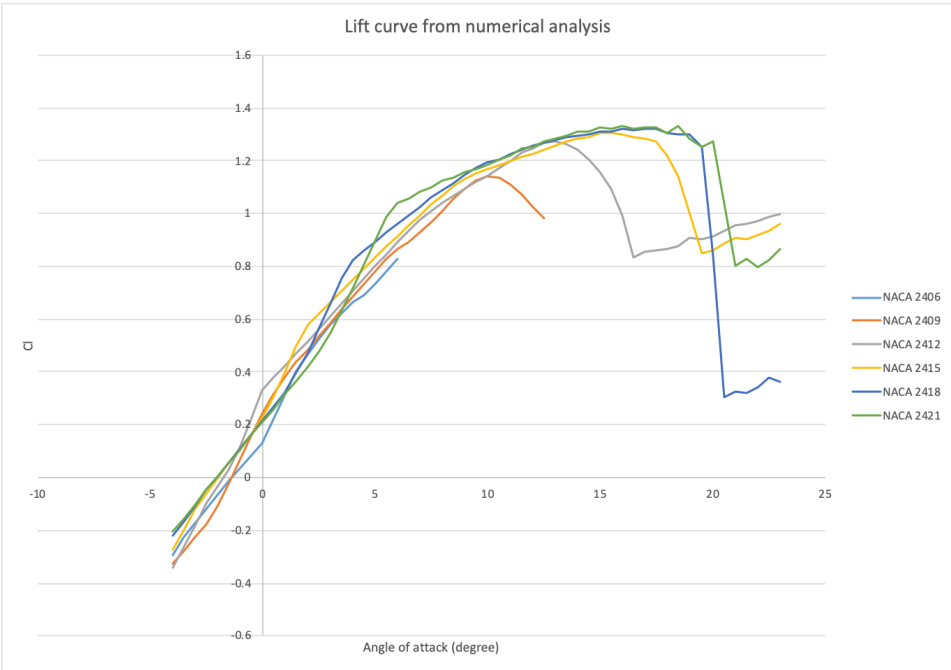
1.5.2.3 Numerical analysis (with XFLR5)



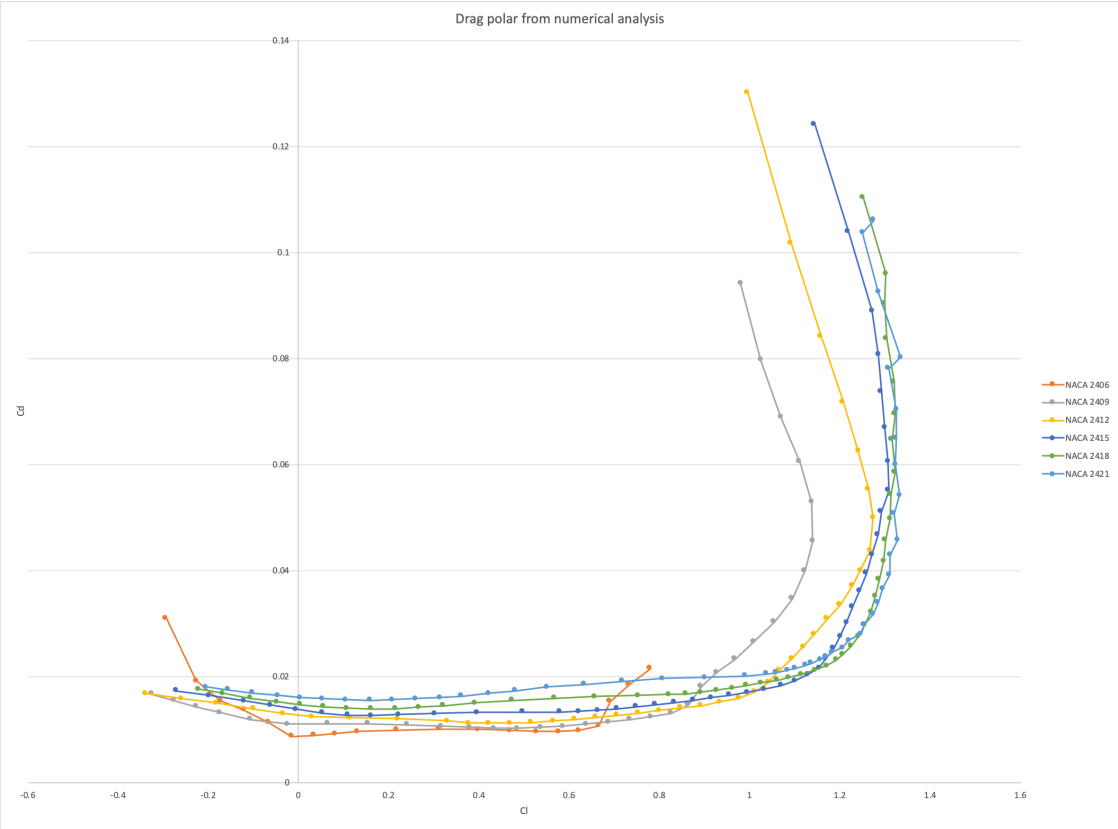


Some data of the lift curve from XFLR5 is suspicious. With some observation, we can easily find that some of the data for NACA 2406 and NACA 2418 is not reasonable. Therefore, it is proper to delete some of the data.

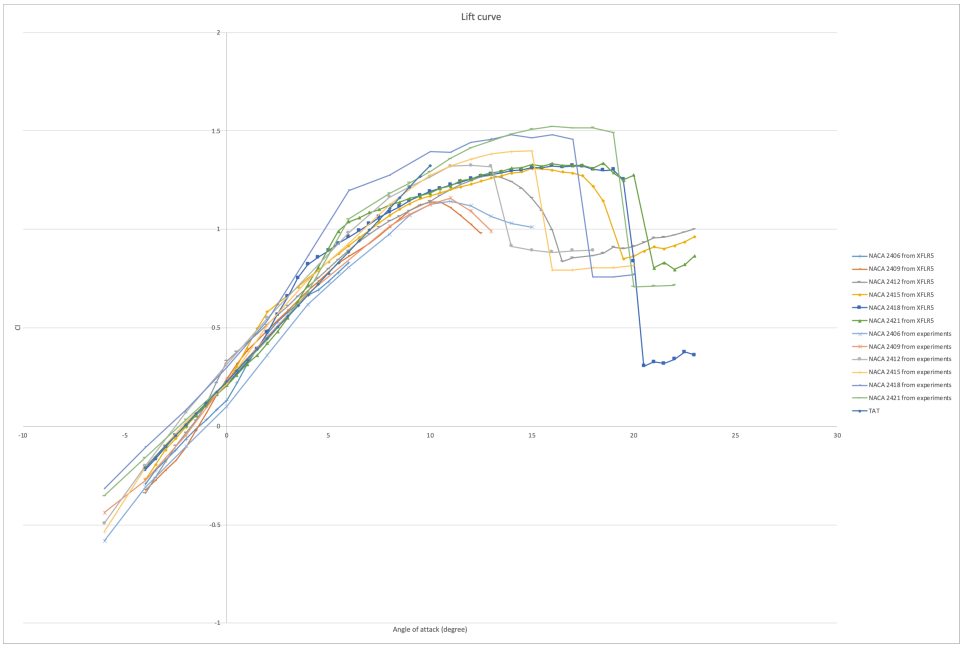
After deleting the data:

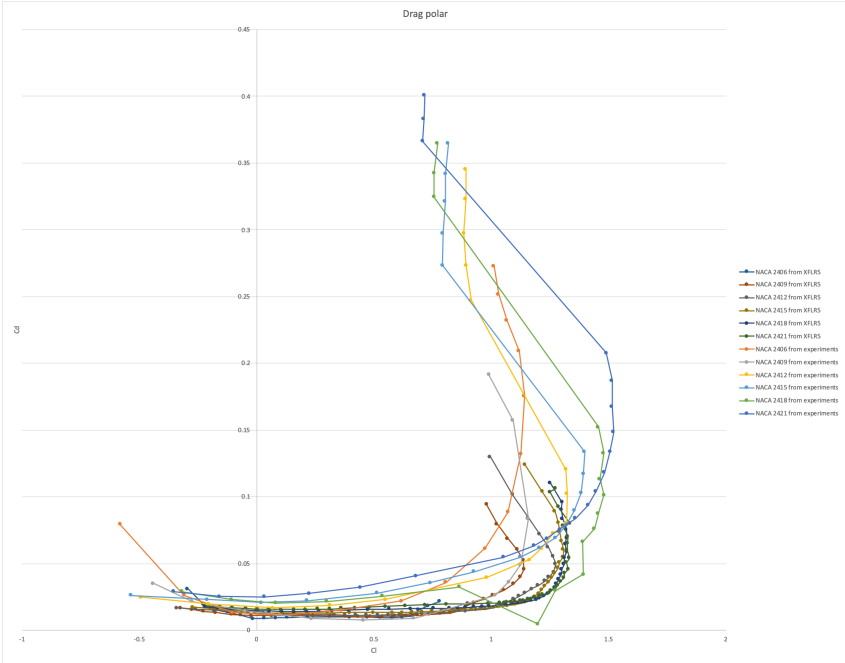


The drag polar from XFLR5 is also not so reliable. It might be the case that the way XFRL5 calculate drag is no longer reliable after stall. Therefore, it would be reasonable that some of the data should be deleted. After deleting some data:



1.5.2.4 Plotting all the results in one single graph:



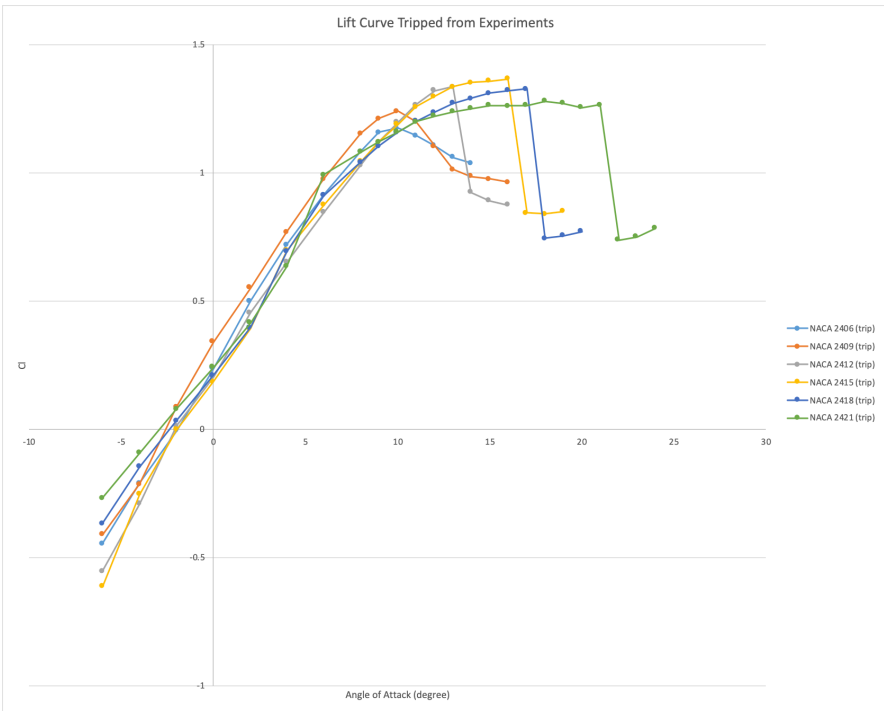


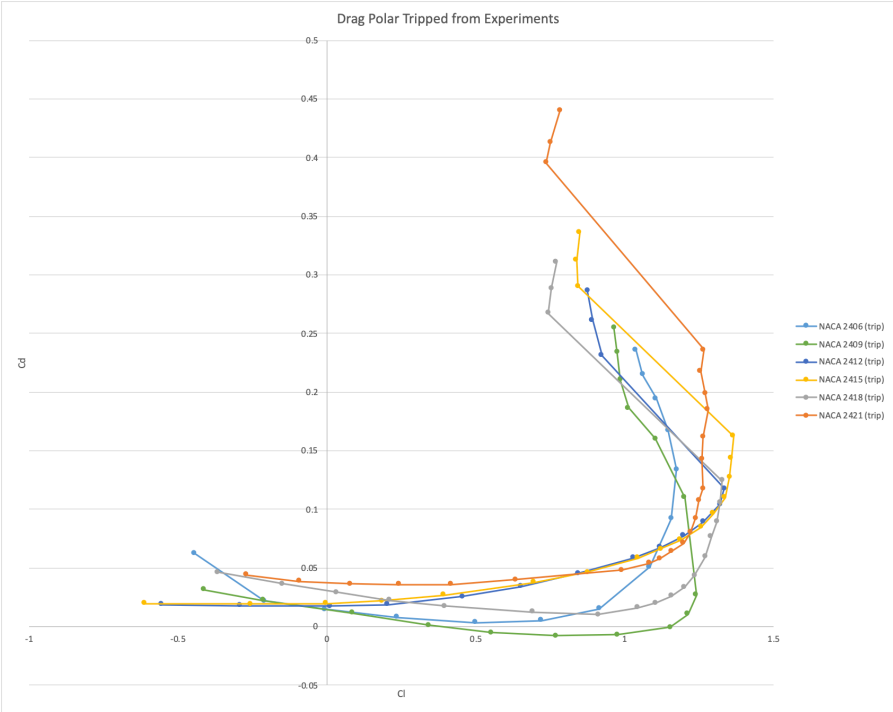
1.5.3 Tripped airfoil

1.5.3.1 TAT

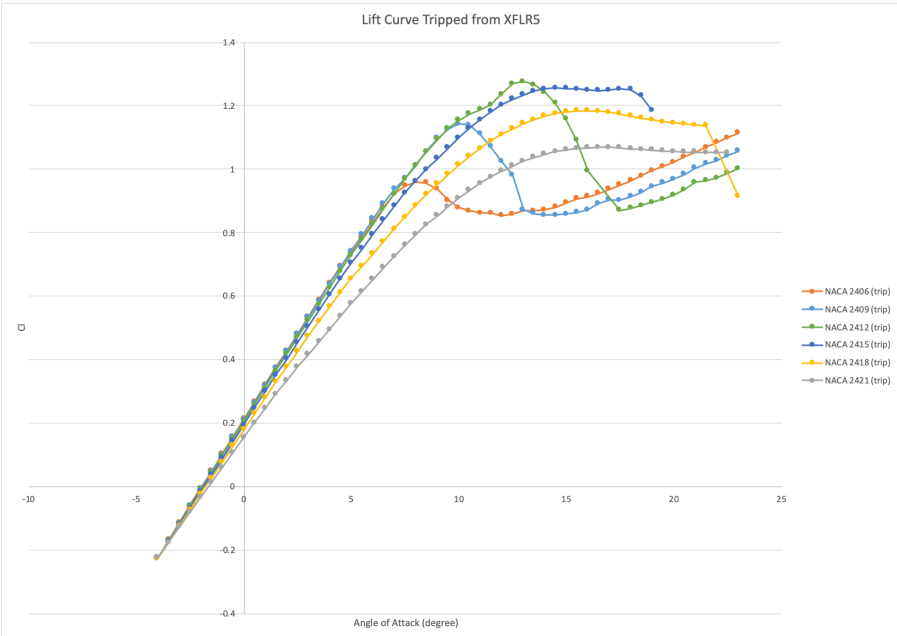
TAT assume that the flow is inviscid, therefore, it cannot account for the result of tripping the airfoil. Consequently, the result for tripped airfoil would be the same as the result for untripped airfoil, which is plotted in 1.5.2.1. As is stated above, drag is not accessible from TAT.

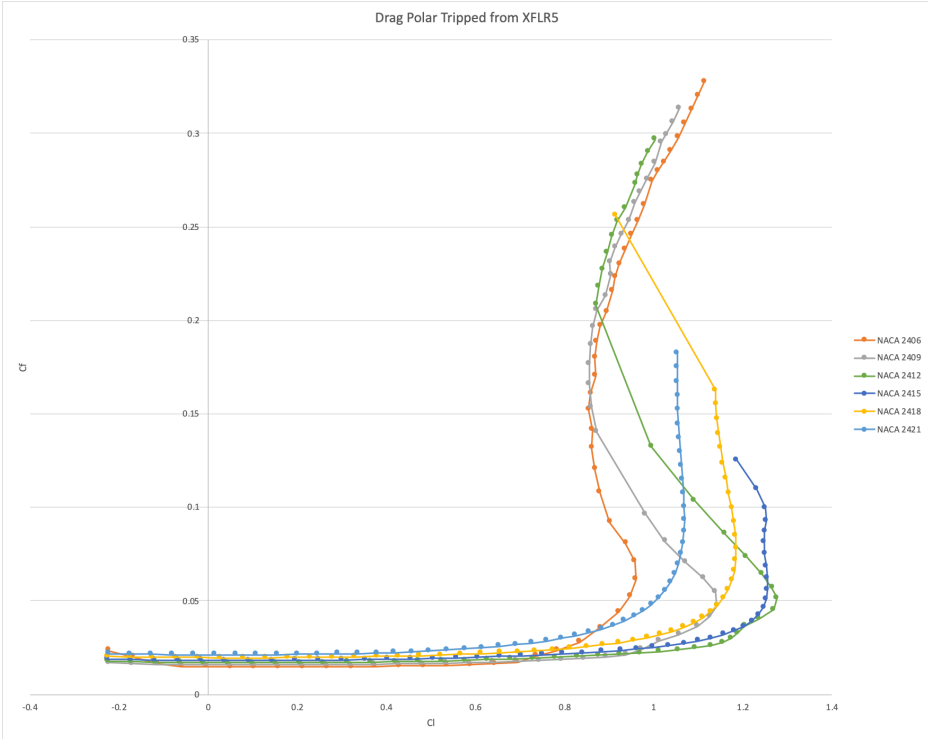
1.5.3.2 Wind tunnel experiments





1.5.3.3 Numerical analysis





1.5.4 Comments

1.5.4.1 Effect of thickness

- TAT can not taken into account the effect of thickness.
    - This is because, TAT is no longer eligible approaching stall, so can not evaluate the effect of thickness (which is increasing the stall angle, and will approach stall more gradually).
- All the three methods reveals that the lift slope would generally be the same for different thicknesses before reaching stall angle. The zero-lift angle of attack would also be generally the same.
    - This is because zero-lift angle of attack depends mostly on camber, and is generally independent of thickness.
- From lift curve of numerical analysis and experiments, we can find that the stall angle would increase with thickness. From the lift curve, we can also observe a more gradual stall.
    - This is because, the stall type of thicker airfoil is trailing edge stall, which begins at trailing edge and grows towards the leading edge gradually.

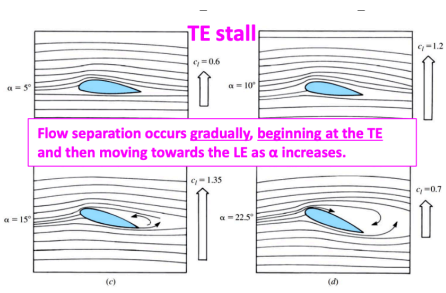
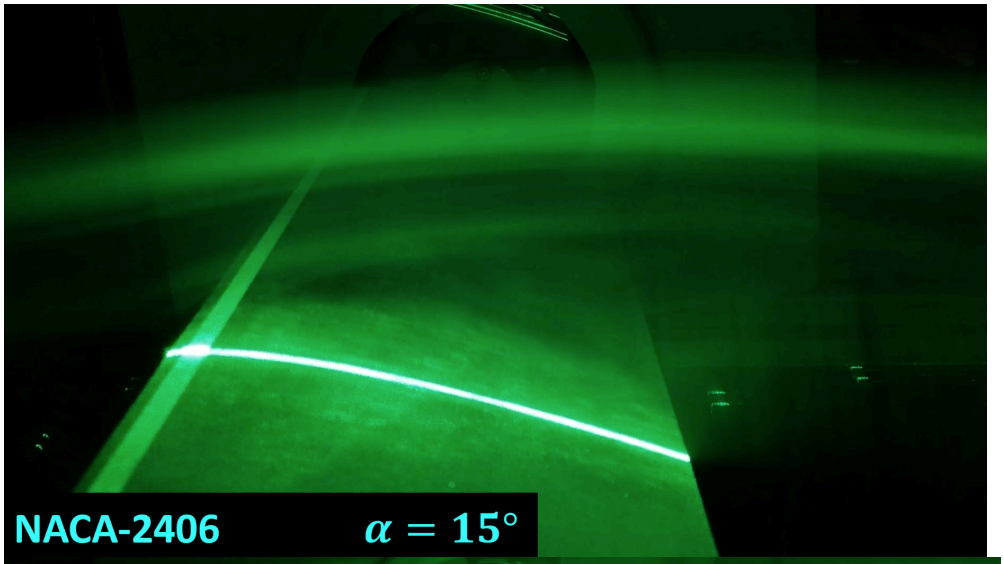
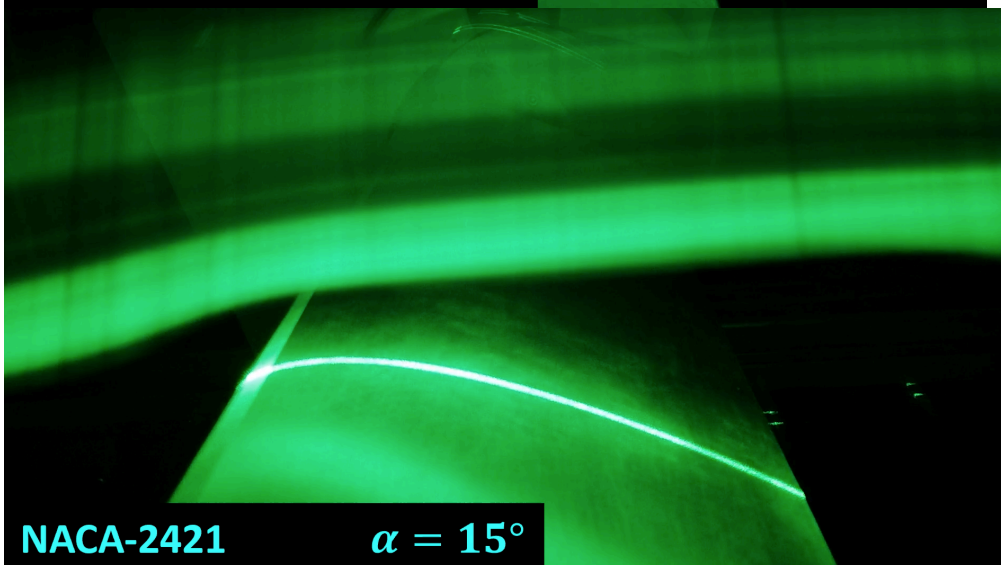


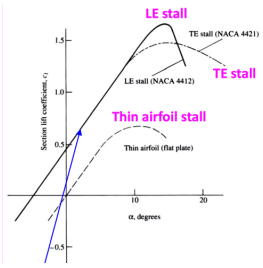
Figure 4.50 Example of trailing-edge stall. Streamline patterns for an NACA 4421 airfoil at different angles of attack. (The streamlines are drawn to scale from the experimental results of Nakayama, Y. (ed): *Visualized Flow*, compiled by the Japan Society of Mechanical Engineers, Pergamon Press, New York, 1988.) Re = 2.1×10^5 and $V_\infty = 8$ m/s in air.

- From the tuft-visualization videos, you can also see that at $\alpha = 15°$, the flow is already separated for NACA 2406, while the flow is still attached at the leading edge of NACA 2412.

- 

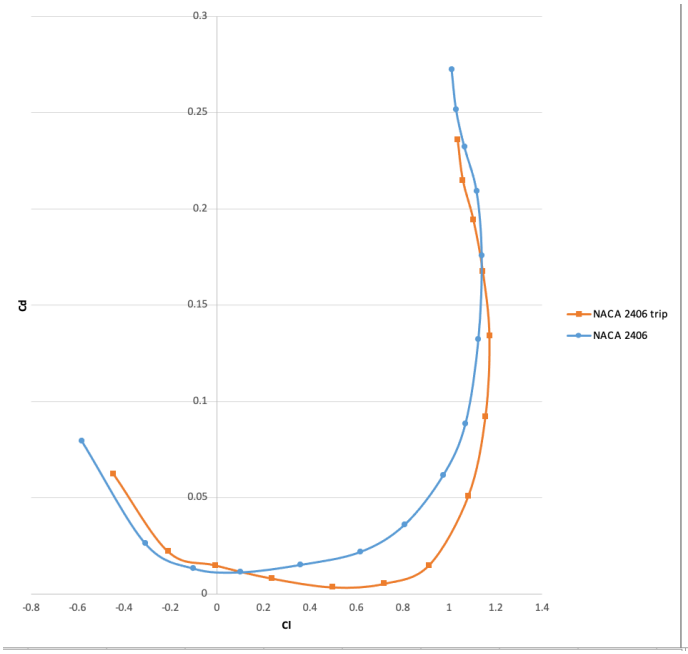
- 

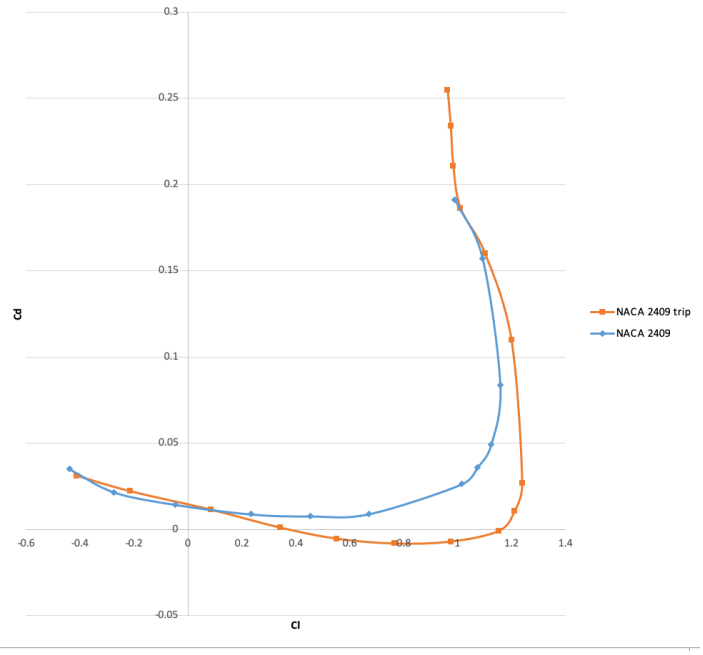
- From the experiment data and the XFLR5 results, the maximum lift coefficient gradually increases with the increase of thickness.
  - This is different from what we learnt during course:

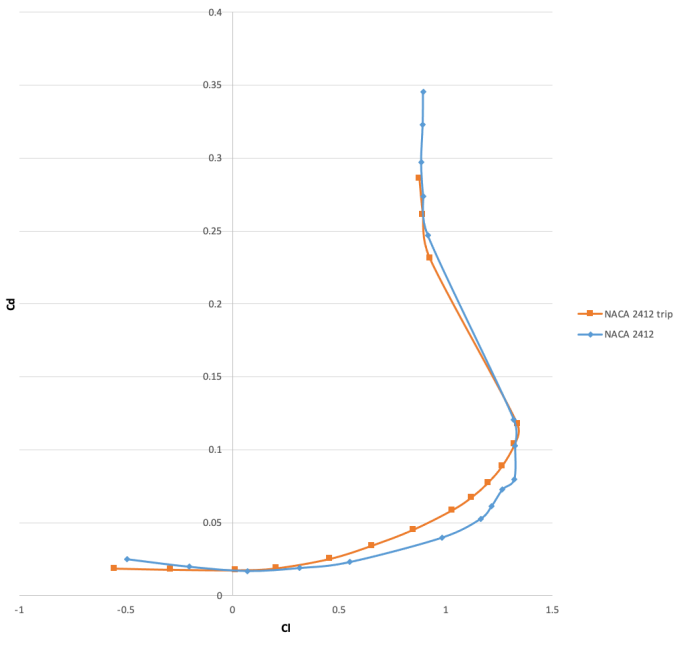


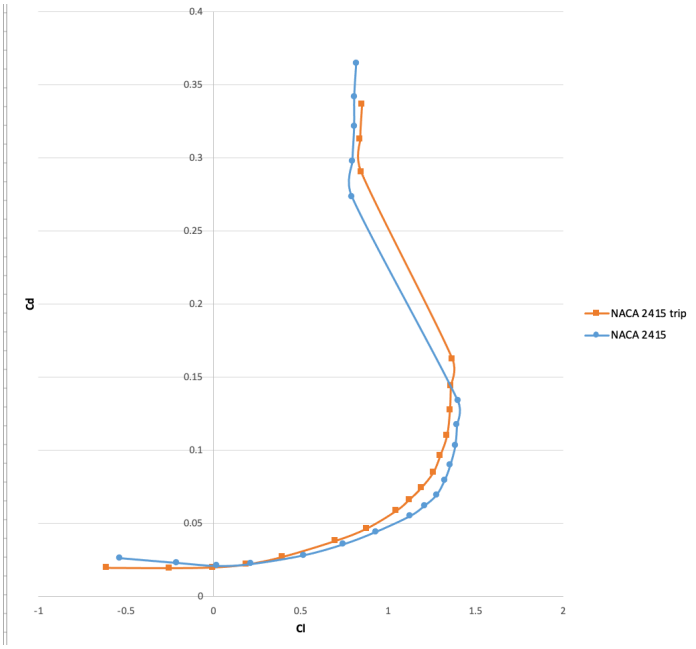
  - This is probably because the Reynolds number for our experiment is different from the Reynolds number for the graph on lecture slides.
- Under the same lift coefficient, the drag coefficient of thicker airfoils would generally be larger than that of thinner airfoils.
  - This is probably because the cross-section area is larger for thicker airfoil.

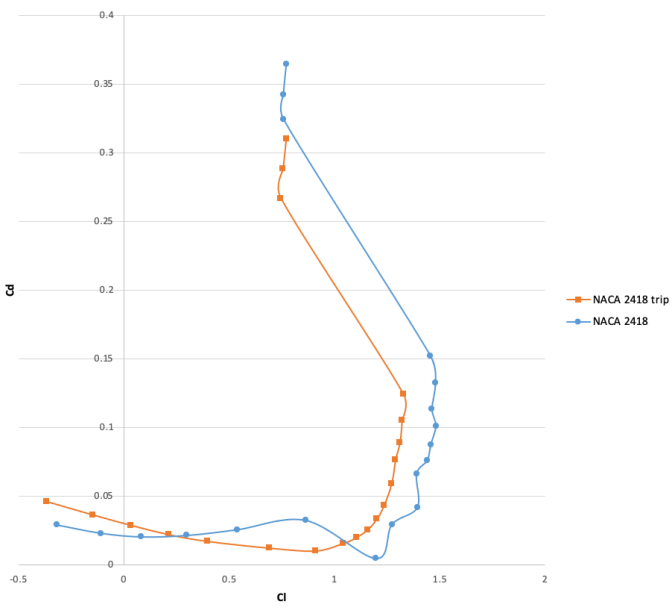
1.5.4.2 Effect of tripping the flow

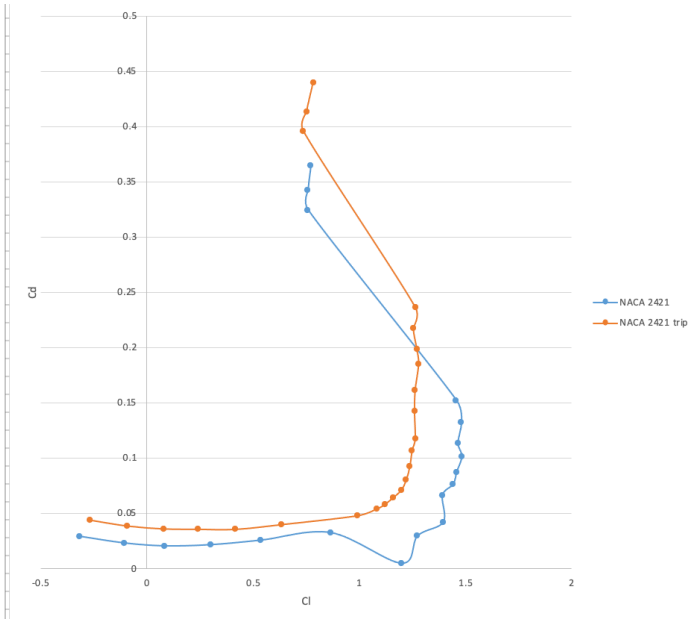


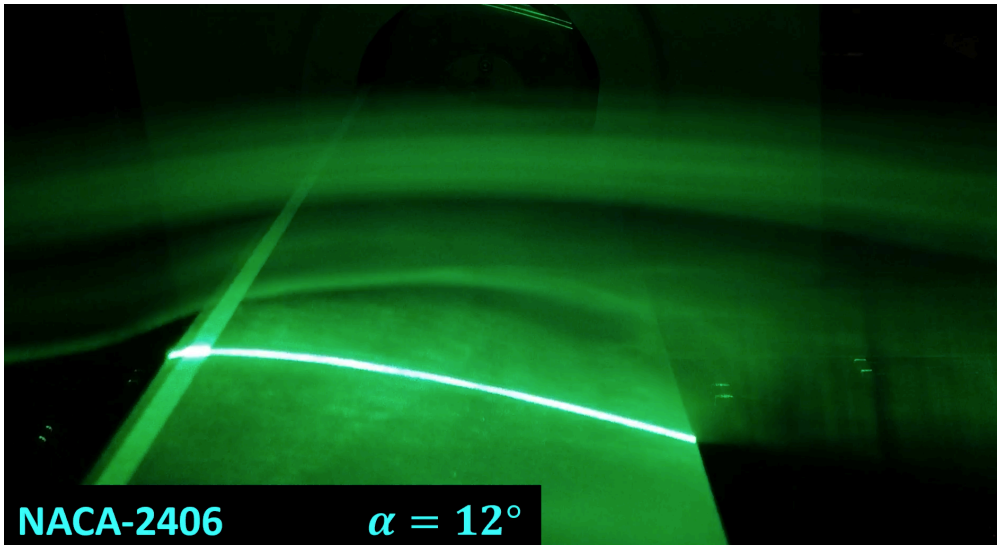










- For thinner airfoil (i.e. NACA 2406 and NACA 2409), tripping the flow results in lower drag.
  - This is probably because tripping the flow causes the flow to be more turbulent. As a result, the flow would stay attached to the wing longer, and therefore, the separation would happen later, lowering the drag. This is also why such effect of lowering the drag is most noticeable when we are approaching maximum C_l (i.e. approaching stall).
    - From the graph below, we can see that at a relatively high angle of attack, there is sign of flow separation. While very close to the airfoil, near the trip strip, you can see that the flow seems to attach more on the airfoil.
    - 

  - However, I must acknowledge that the data for NACA 2409 might be suspicious, since there is a negative region, which is impossible. However, the general trend (i.e. the envelope where there is lower drag) is correct.
- For thicker airfoil, the effect of tripping the flow is the opposite.
  - This might because, the effect of tripping the flow is not only making the flow more turbulent, but also rougher. As a result, the viscous drag might be larger.
    - This effect can be revealed in the graph below:

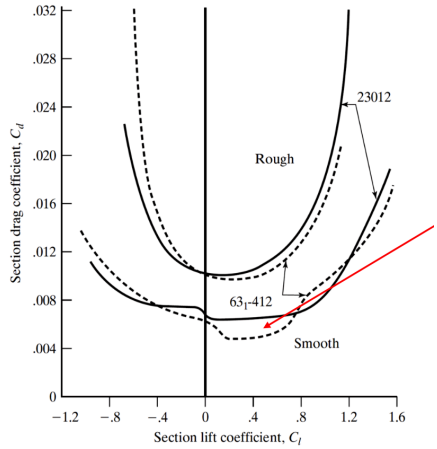


Figure 6.13 Drag characteristics of NACA laminar flow and conventional airfoils sections with both smooth and rough leading edges [from Loftin (1985)].

- The reason why the result of tripping the flow is different for thin airfoils and thick airfoils might be that for thicker airfoil, the flow is already more attached to the airfoil at leading edge (see 1.5.4.1 effect of thickness), and so it might not be necessary to use the trip strip to force attach the flow. As a result, the trip strip will only increase the roughness, leading to more drag.

1.6 Effects of airfoil camber

1.6.1 table of NACA airfoils

	Airfoil	Description
i)	NACA 1412	1% less camber
	NACA 2412	Baseline airfoil
ii)	NACA 3412	1% more camber

1.6.2 Plots

1.6.2.1 TAT

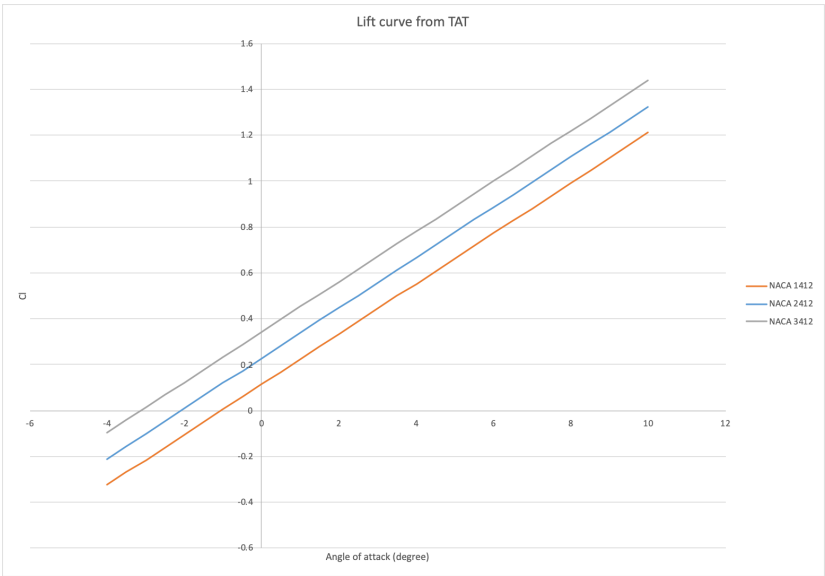
Steps already described in 1.2, slightly modify the codes in 1.2, we can obtain the lift curve for NACA 1412, NACA 2412 and NACA 3412:

Calculation data

```
f1(x) = m/p^2 * (2*p*x - x^2/c)
f2(x) = m/(1-p)^2*((1-2*p)*c+2p*x - x^2/c)
df1(x) = D(f1, x)
df2(x) = D(f2, x)
dft1(t) = df1(c/2*(1-cos(t)))
dft2(t) = df2(c/2*(1-cos(t)))
dfta1(t) = dft1*(cos(t)-1)
dfta2(t) = dft2*(cos(t)-1)
rada0(m,p) = -1/@pi*(Int(dfta1, t, 0, 1.37)+Int(dfta2, t, 1.37, @pi))
dega0(m,p) = rada0*180/@pi
```

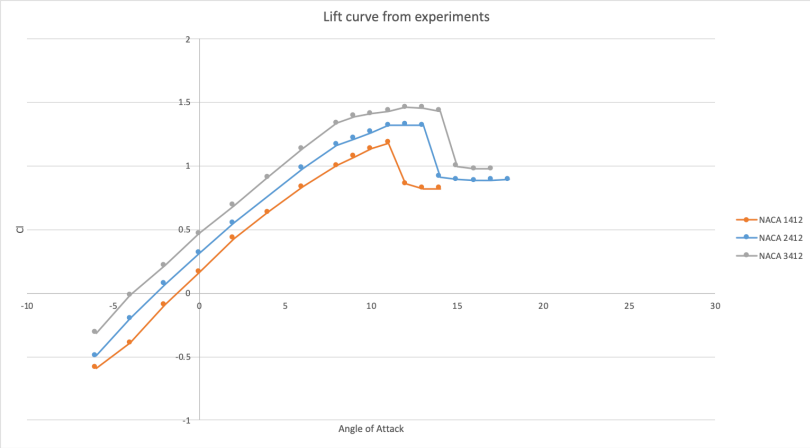
```
-18.2378((-0.1929m+0.7802m*p)/((-p+1)^2)+(0.1929m-0.7802m*p)/(p^2)+(4.7124m-6.2832m*p)/((-p+1)^2))

// results:

rada0(0.01,0.4)
-0.0181

rada0(0.02,0.4)
-0.0363

rada0(0.03,0.4)
-0.0544

Cld(a,m,p) = 2*@pi*(a*@pi/180 - rada0(m,p))
6.2832(0.0175a+0.3183((-0.1929m+0.7802m*p)/((-p+1)^2)+(0.1929m-0.7802m*p)/(p^2)+(4.7124m-6.2832m*p)/((-p+1)^2)))

Cld(a,0.01,0.4)
6.2832(0.0175a+0.0181)

Cld(a,0.02,0.4)
6.2832(0.0175a+0.0363)

Cld(a,0.03,0.4)
6.2832(0.0175a+0.0544)
```

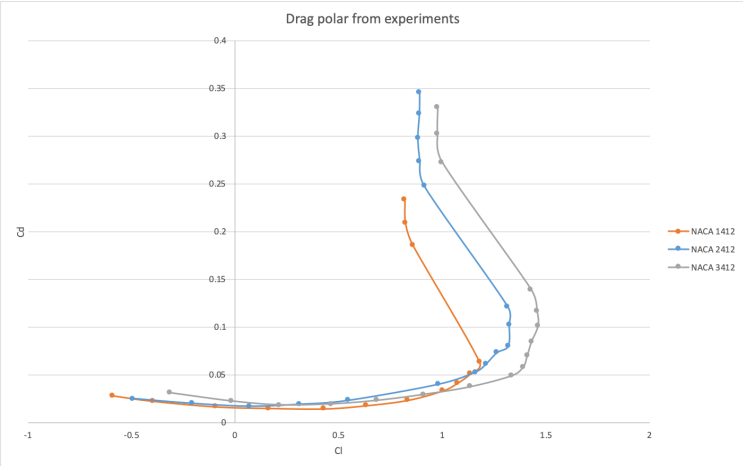
Airfoil	Description	$\alpha_{L=0}$	Lift curve
NACA 1412	1% less camber	-0.0181	$C_l = 2\pi(\frac{\alpha_{degree} \cdot \pi}{180} + 0.0181)$
NACA 2412	Baseline airfoil	-0.0363	$C_l = 2\pi(\frac{\alpha_{degree} \cdot \pi}{180} + 0.0363)$
NACA 3412	1% more camber	-0.0544	$C_l = 2\pi(\frac{\alpha_{degree} \cdot \pi}{180} + 0.0544)$



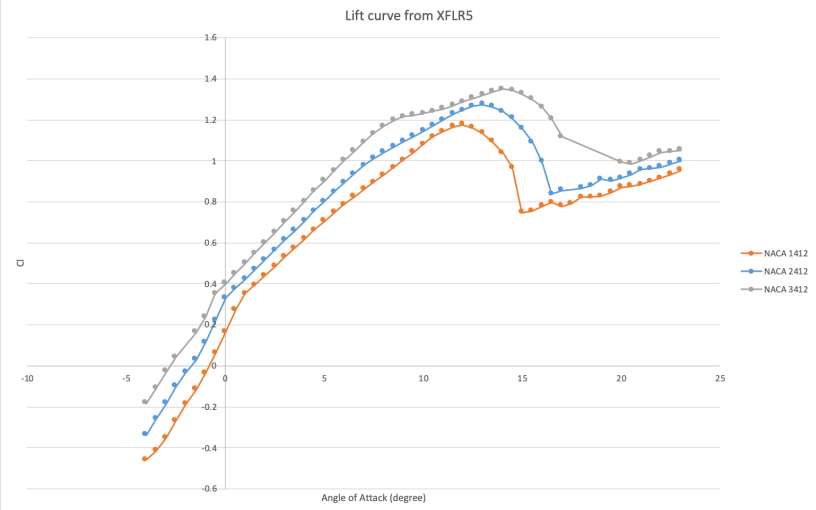
As stated above, drag is not accessible from TAT

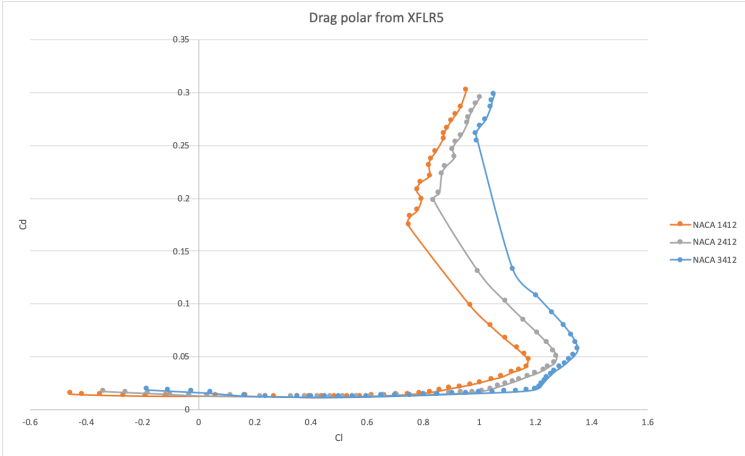
1.6.2.2 Wing tunnel experiments





1.6.2.3 Numerical analysis



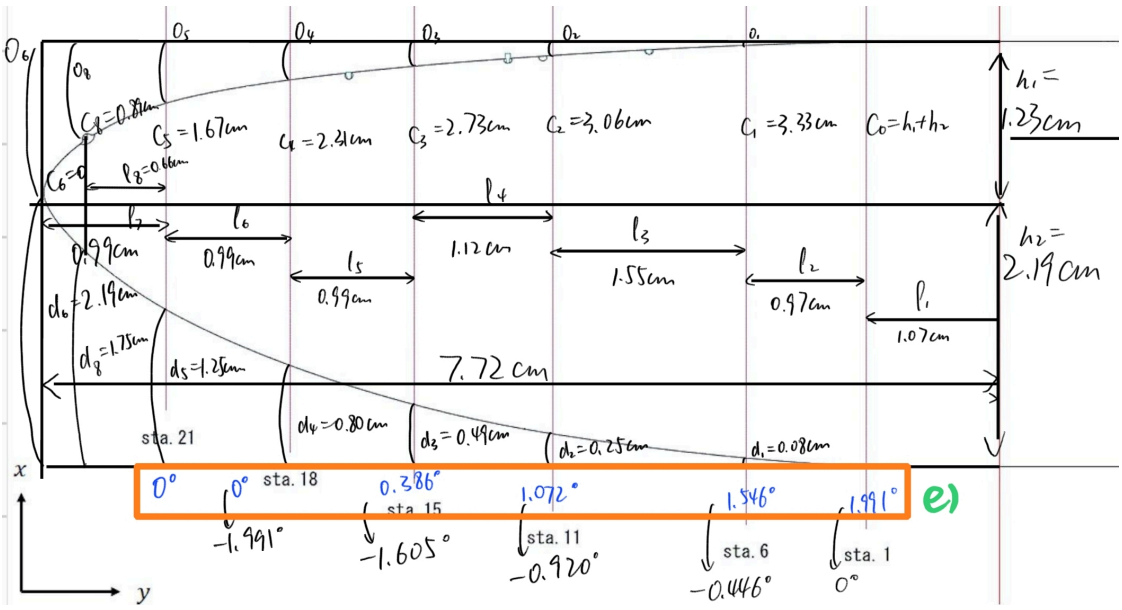


1.6.3 Comments

- From all the three methods:
  - The lift slope is almost the same for different cambers.
  - The zero lift angle of attack is smaller for a higher camber.
- From results from experiments and numerical analysis:
  - The maximum C_l and stall angle is also larger for a higher camber.
    - This is probably because, for a higher camber, air flow would blow on the leading edge more smoothly at higher angle of attack. As a result, stall might be delayed.
  - Before stall, the drag for different cambers are almost the same.
- Surprisingly, the results from numerical analysis fits perfectly well with experiment data.

2 Part II: Wing Aerodynamics

2.1 Measurement



The angles colored in blue are the angle directly measured from the graph, the angles colored in black are the angle inputted into XFLR5, which is 0 at wing root.

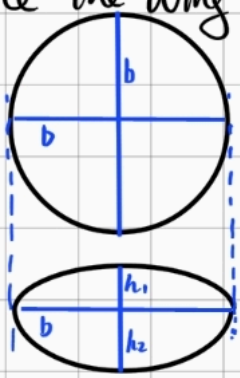
$$\lambda := \frac{\text{actual length}}{\text{length on graph}} = \frac{3m}{4.14cm} \approx 72.464$$

a) $\boxed{b = (7.72\,cm \cdot \lambda) \cdot 2 \approx 11.188\,m}$

$h_1 = (1.23\,cm \cdot \lambda) \approx 0.891\,m$

$h_2 = (2.19\,cm \cdot \lambda) \approx 1.587\,m$

Since the wing is elliptical:



$$S_{ellipse} = S_{circle} \cdot \frac{h_1 + h_2}{b} = \pi\left(\frac{b}{2}\right)^2 \cdot \frac{h_1 + h_2}{b} = \frac{\pi b (h_1 + h_2)}{4}$$

$\boxed{S_{ellipse} \approx 21.777\,m^2}$ b)

$\boxed{AR = \frac{b^2}{S} = 5.748}$ c)

$l_i = (\text{measured } l_i) \cdot \lambda$, $\begin{cases} |y_1| = l_1 \\ |y_i| = |y_{i-1}| + l_i \end{cases}$ $\quad O_i = h_1 + h_2 - C_i - d_i$

$l_1 = 0.775m$, $|y_1| = 0.775m$

$l_2 = 0.703m$, $|y_2| = 1.478m$

$l_3 = 1.123m$, $|y_3| = 2.601m$

$l_4 = 0.812m$, $|y_4| = 3.413m$

$l_5 = 0.717m$, $|y_5| = 4.130m$

$l_6 = 0.717m$, $|y_6| = 4.848m$

$l_7 = 0.717m$, $|y_7| = 5.565m$

$C_0 = h_1 + h_2 = 2.478m$ d)

$C_1 = 2.413m$, $O_1 = 0.007m$ ☆ starting from 0

$C_2 = 2.217m$, $O_2 = 0.080m$

$C_3 = 1.978m$, $O_3 = 0.145m$

$C_4 = 1.674m$, $O_4 = 0.225m$

$C_5 = 1.120m$, $O_5 = 0.362m$

$C_6 = 0.000m$, $O_6 = 0.891m$

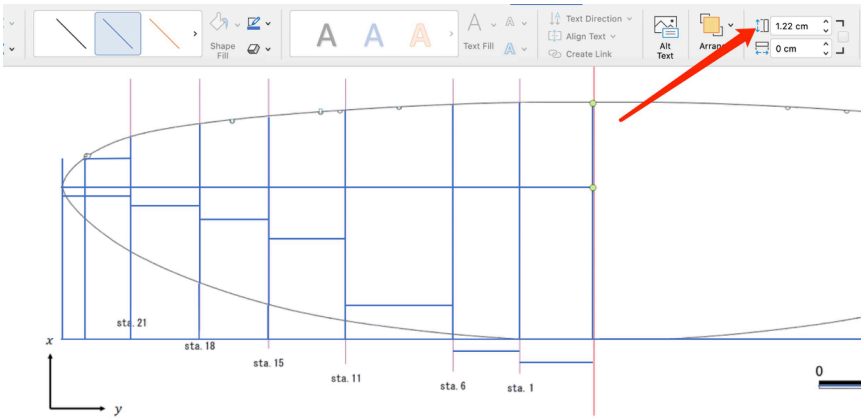
$l_8 = 0.478m$, $C_8 = 0.645m$, $d_8 = 1.268m$, $O_8 = 0.565m$

$|y_8| = |y_6| + l_8 = 5.326m$

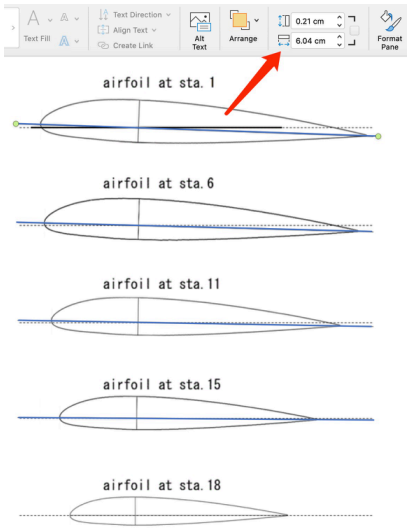
The indices of chord length starts from 0, where C_0 is the root chord length, C_6 is the wing tip chord length (0m, actually inputted as 0.1m in XFLR5), and C_8 is an extra station selected to improve accuracy.

How did I measure length and degree?
I did it with Microsoft Word:

1. Length measurement



2. Degree measurement




```
deg1 = atan(0.21/6.04)
= 1.991269695
```

2.2 Analytical LLT

a)

$L = \frac{1}{2}\rho V_\infty^2 S C_L$

$V_\infty = 140km/h = 38.89m/s$

$L = W = 5.7kN = 5700N$

$S = 21.777m^2$

At 2000m, from Appendix: $\rho = 1.0066kg/m^3$

Therefore,

$$C_L = 0.344$$

b)

$$C_{D_i} = \frac{C_L^2}{\pi AR} = 0.00655$$

c)

$$\alpha_i = \frac{C_L}{\pi AR} = 0.0190(rad)$$

d)

$$C_L = 2\pi(\alpha - \alpha_i - \alpha_{L=0})$$

From previous calculation in TAT: $\alpha_{L=0} = -0.0363(rad)$

From a): $C_L = 0.344$

From c): $\alpha_i = 0.0190(rad)$

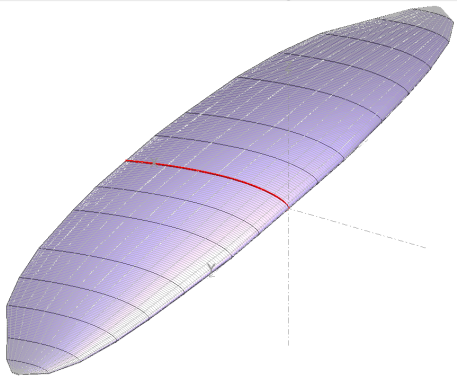
Therefore:

$$\alpha = 0.0375(rad)$$

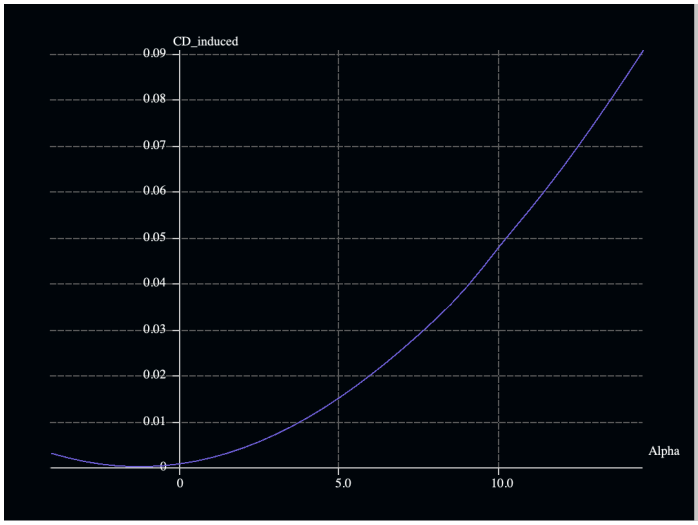
2.3 Numerical LLT

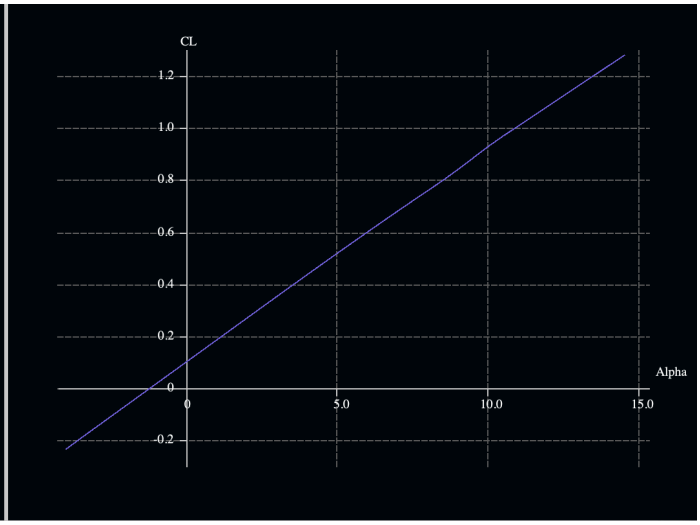
As is mentioned in 2.1, the number entered in the XFRL5 is already modified with an offset so that the angle of attack is measured from the root:

	y (m)	chord (m)	offset (m)	dihedral(°)	twist(°)	foil	X-panels	X-dist	Y-panels	Y-dist
1	0.000	2.478	0.000	0.0	0.00	NACA 2412	15	Cosine	16	Uniform
2	0.775	2.478	0.000	0.0	0.00	NACA 2412	15	Cosine	16	Uniform
3	1.478	2.413	0.007	0.0	-0.45	NACA 2412	15	Cosine	24	Uniform
4	2.601	2.217	0.080	0.0	-0.92	NACA 2412	15	Cosine	20	Uniform
5	3.413	1.978	0.145	0.0	-1.61	NACA 2412	15	Cosine	20	Uniform
6	4.130	1.674	0.225	0.0	-1.99	NACA 2412	15	Cosine	20	Uniform
7	4.848	1.120	0.362	0.0	-1.99	NACA 2412	15	Cosine	20	Uniform
8	5.326	0.645	0.565	0.0	-1.99	NACA 2412	15	Cosine	20	-Sine
9	5.565	0.100	0.891		-1.99	NACA 2412				

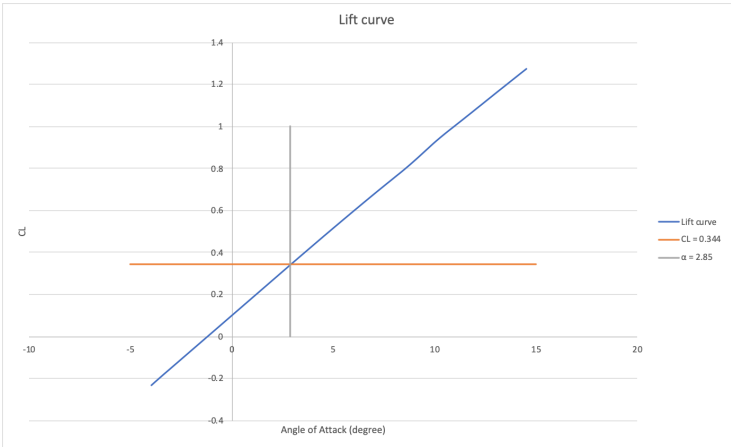


Results:





Angle of Attack:



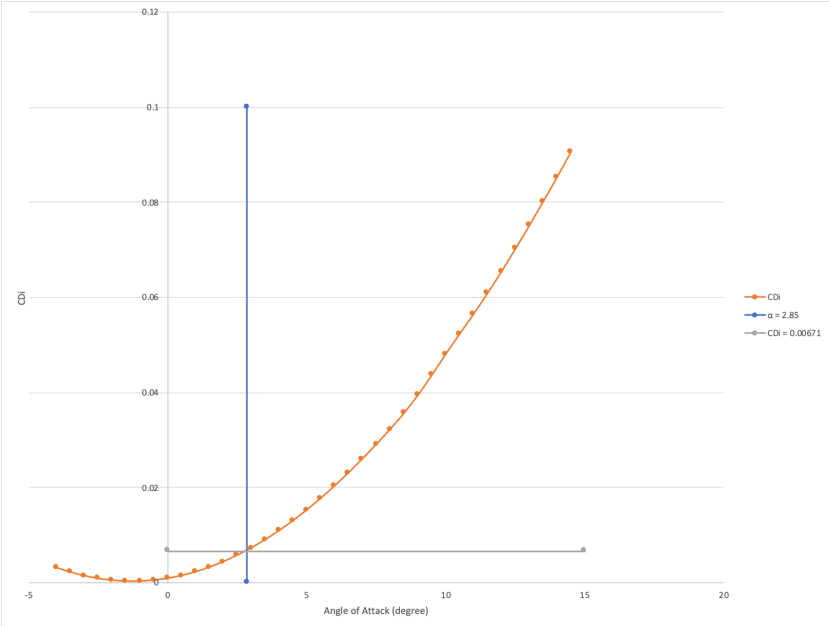
From the curve above, we can see that when $C_L = 0.344$,

$\alpha = 2.85^\circ = 0.0497(rad)$

2.4 Compare

a)

From the data in 2.3, at $\alpha = 2.85^\circ$:



$C_{D_i3} = 0.00671$

From 2.2:

$C_{D_i2} = 0.00655$

Percentage Difference $= \frac{C_{D_i3} - C_{D_i2}}{C_{D_i2}} = 2.44\%$

b)

In 2.2, we assume that the wing is perfectly elliptical:

$e_2 = 1$

Since $e = (1+\delta)^{-1}$: $\delta = \frac{1}{e} - 1$

$\delta_2 = 0$

In 2.3,

$C_{D_i3} = \frac{C_{L3}^2}{\pi e_3 AR}$

$C_{L3} = 0.344,\ AR = 5.748,\ C_{D_i3} = 0.00671$

Therefore,

$e_3 = 0.977$

$\delta_3 = 0.0235$

c)

- Less cost
  - Do not need to actually build a model in order to obtain the data needed.
- Less time consuming
- More data obtained

- E.g. pressure distribution, which is very difficult to measure in experiments.
- More accurate than analytical methods

2.5 Effects of taper ratio

a)

```
S=21.777;b=11.188;AR = b^2/S // since AR is the same, S is the same, b will be the same
```

= 5.748

```
Croot(lambda) = solve(((Cr + Cr* lambda) * b) / 2 = S)
C(lambda) = [lambda, Croot(lambda), Croot(lambda) * lambda]
```

= [lambda, **Solve**(5.594 (Cr + Crlambda) = 21.777), lambda**Solve**(5.594 (Cr + Cr lambda) = 21.777)]

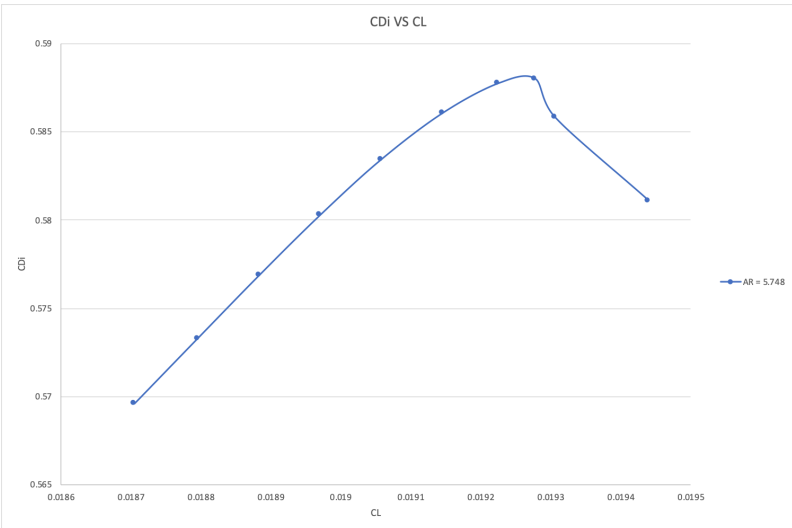
```
[["λ","Croot","Ctip"],C(0.1),
C(0.2),C(0.3),C(0.4),C(0.6),C(0.7),C(0.8),C(0.9),C(1.0)]
```

=

λ	Croot	Ctip
0.1	3.539	0.354
0.2	3.244	0.649
0.3	2.995	0.898
0.4	2.781	1.112
0.6	2.433	1.46
0.7	2.29	1.603
0.8	2.163	1.73
0.9	2.049	1.844
1	1.946	1.946

b)

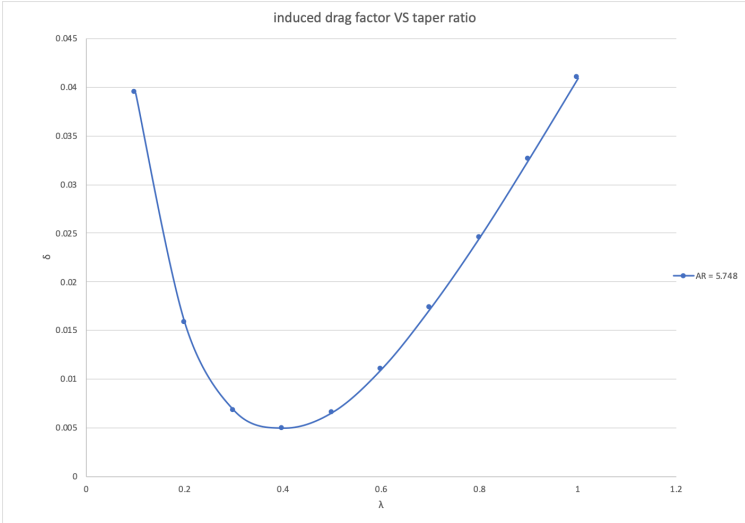
With XFLR5, running analysis on main wing of 10 different taper ratios, we can obtain this data:



Since $C_{D_i} = \frac{C_L^2}{\pi AR}(1+\delta)$,

$$\delta = \pi \cdot AR \cdot \frac{C_{D_i}}{C_L^2} - 1$$

Therefore, we can obtain the relationship between induced drag factor and taper ratio:



c)

From the above result, we can find that the induced drag factor reaches its minimum when $\lambda \approx 0.4$. Therefore, the optimal taper ratio is 0.4.

With a proper taper ratio, we can reach a very low induced drag factor ($\delta_{min} \approx 0.00492$), which is close to 0. Considering that it is much harder to build an elliptical wing, it is more preferred to build a properly tapered wing, instead of an elliptical wing. In this way, the wing's elliptical wing is not that meaningful.

2.6 Effects of aspect ratio

a)

From the calculation above, we should choose a taper ratio of 0.4.

```
lambda=0.4; S=21.777;
b(AR) = sqrt(AR*S)
Croot(AR) = solve(((_Croot + _Croot * lambda) * b(AR)) / 2 = S)
Ctip(AR) = Croot(AR)*lambda
```

= 0.4 **Solve**(0.7 $\sqrt{21.777\,AR}$ _Croot: 21.777)

```
C(AR) = [AR, b(AR), Croot(AR), Ctip(AR)]
```

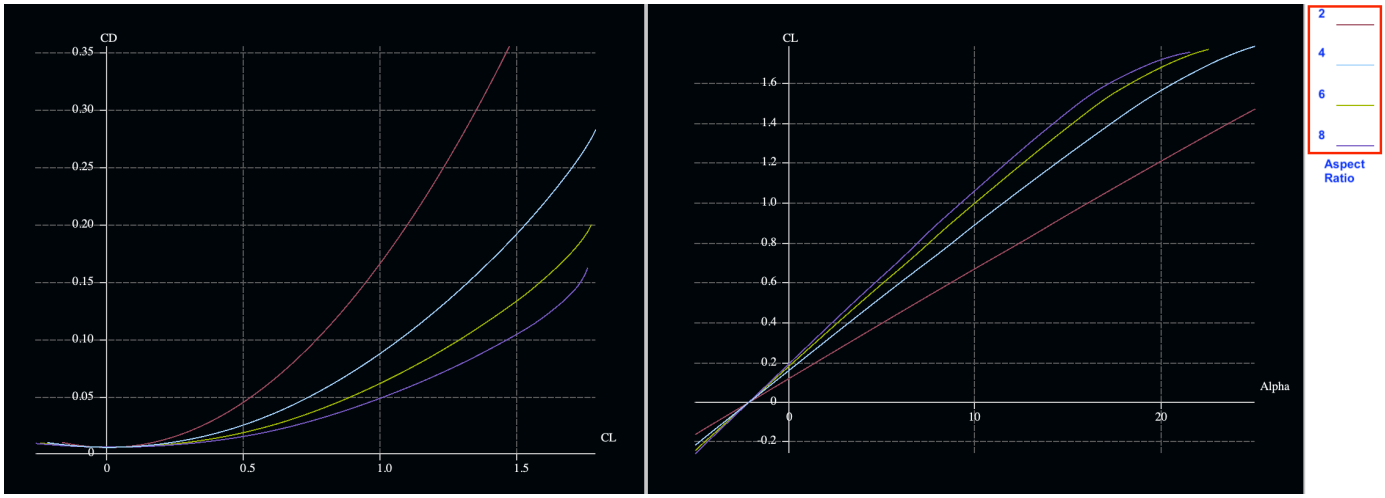
= [AR, $\sqrt{21.777\,AR}$, **Solve**(0.7 $\sqrt{21.777\,AR}$ _Croot: 21.777),
0.4 **Solve**(0.7 $\sqrt{21.777\,AR}$ _Croot: 21.777)]

```
[["Aspect Ratio(AR)","Wingspan(b)","Croot","Ctip"],C(2),C(4),C(6),C(8)]
```

=

Aspect Ratio(AR)	Wingspan(b)	Croot	Ctip
2	6.6	4.714	1.886
4	9.333	3.333	1.333
6	11.431	2.722	1.089
8	13.199	2.357	0.943

b)

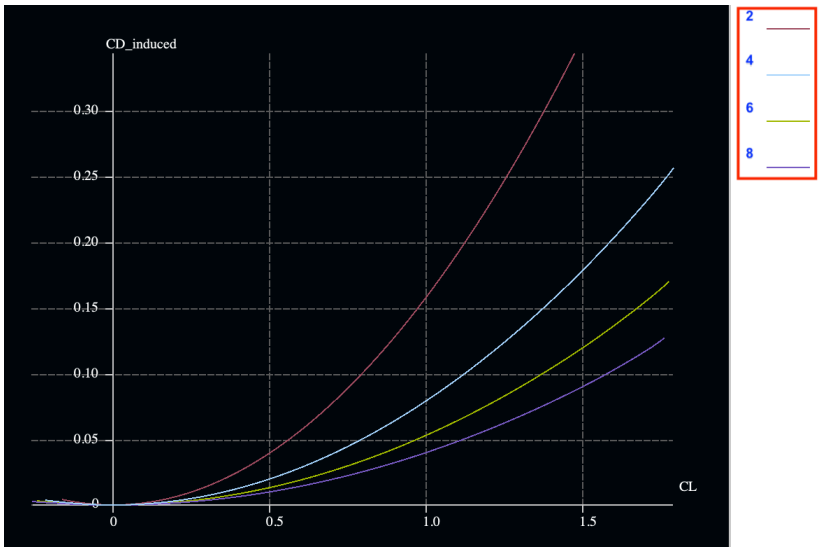


c)

From the results above, we can find that under the same wing area and an optimized taper ratio, the higher the aspect ratio is, the lower the drag coefficient is, and the lift coefficient will also increase with the increase of aspect ratio. However, such trend of lower drag coefficient and higher lift coefficient is not so noticeable when the aspect ratio is already high enough (i.e. in the graph above, from AR = 6 to AR = 8, the decrease in drag coefficient and the increase in lift coefficient is not so noticeable).

When we are designing an aircraft, it is desirable to have a larger aspect ratio (higher C_L, lower C_D). However, under the same wing area, a higher aspect ratio indicates that we need a larger wingspan. Considering that having a too large wingspan can be troublesome(structural problem, operating problem etc.), and that the trend of having a lower drag coefficient and higher lift coefficient is not so noticeable when the aspect ratio is already high enough, we have to choose a reasonably large aspect ratio.

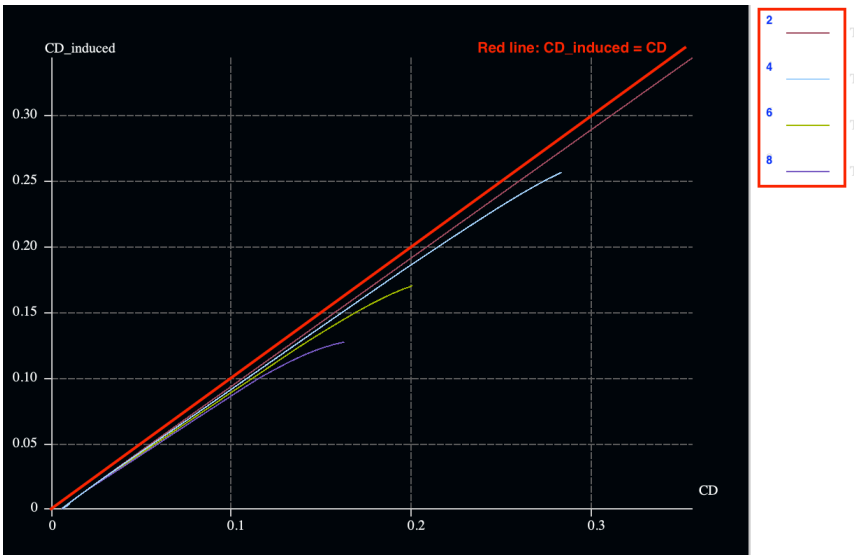
d)



- The plot below shows how C_{D_i} changes with C_D. We can identify that, compared to the red reference line ($C_{D_i} = C_D$), the real C_{D_i} vs C_D relationship has a offset, which correlates to the formula $C_D = C_{D_i} + C_{D_0}$, where the offset is C_{D_0}.

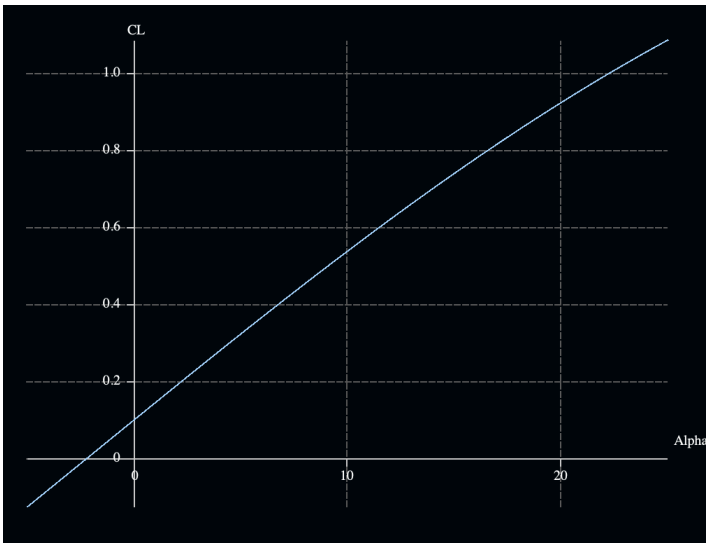
- However, the offset will slowly increase with C_D, this indicates that C_{D_0} is not strictly constant, but is slowly increasing. This might be the result of flow separation when the wing is approaching stall.
- The offset is very small compared to C_D, this indicates that C_{D_0} only account for a very small proportion of the total drag. Most of the drag comes from induced drag.
- Another effect is that, the offset (i.e. C_{D_0}) is larger for a larger aspect ratio. This might be attributed to a larger overall skin friction coefficient.
  - $C_F \approx 1.328\sqrt{\frac{\nu}{U_\infty L}}$

    For wings with a higher AR, the chord length is shorter, and therefore the L In the overall skin friction coefficient is smaller. Therefore, the overall skin friction coefficient is lower. As a result, viscous drag is larger.

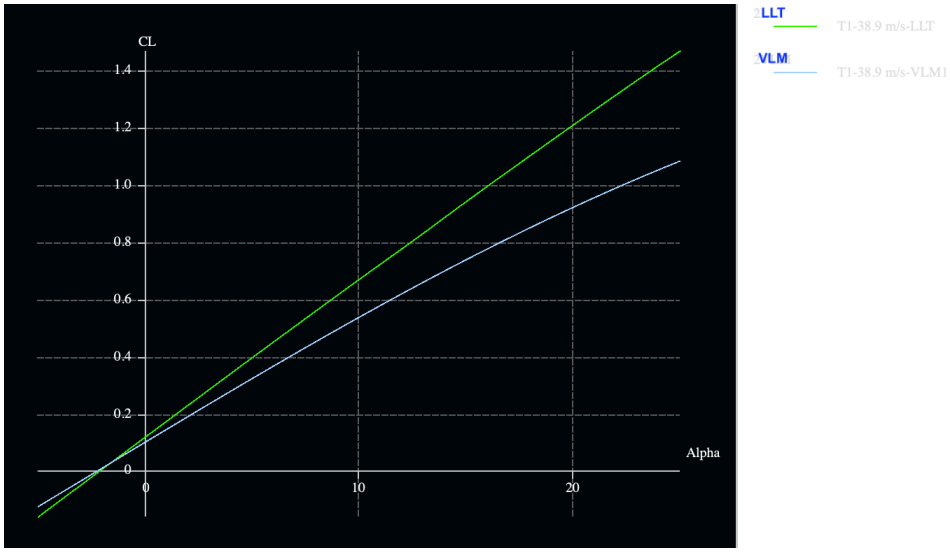


e)

VLM results:



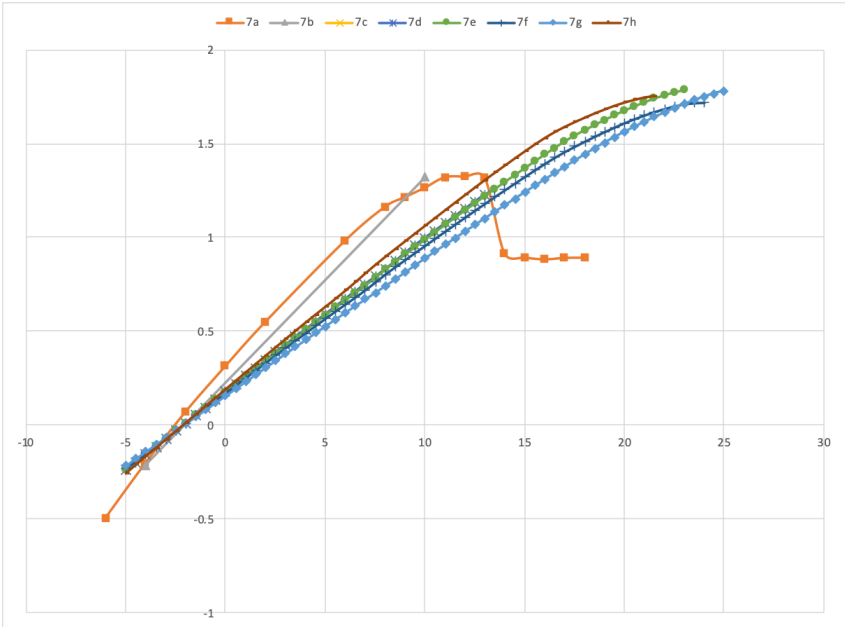
VLM vs LLT



I trust VLM method more.

- At lower AR, lift is more distributed, and so we can no longer assume lift is acting on the quarter chord, so LLT is no longer eligible.
- VLM consider wing as a surface, instead of a line, which is more realistic.
- The VLM can take into account the shape of the wings, not just the chord length of different y positions (e.g. it can take into account the sweep angle), so the result would be more accurate.
- From the result above, we can see that from LLT, the wing can reach a C_L of about 1.5, which is obviously unrealistic.

2.7 Comprehensive plotting



1. For all the graphs, the C_L or C_l would increase with angle of attack before stall.
2. 2D airfoil has a larger lift slope than 3D wings.
    - This is probably because there will be an induced angle of attack for 3D wings due to the downwash effect. As a result, the effective angle of attack is smaller than the geometric angle of attack, and the resultant lift coefficient is smaller.

3. For 2D foil, the result from TAT fits quite well with the experiment results when the AoA is small, but it becomes less reliable when the airfoil is approaching stall.
4. Among the 3D wings, the elliptical wing has the largest lift coefficient, while under the same wing area and aspect ratio, tapered wing with taper ratio 0.4 has a very similar lift coefficient (so similar that they can not even be distinguished in the graph).
5. The lift coefficient of the rectangular wing is a little lower than that of the properly tapered wing.
    - This is reasonable since it is not optimized.
6. The lift curve of AR = 4 is very similar to the lift curve of AR = 5.748 (Part II: Question 5).
7. The lift coefficient of AR = 8 is generally higher than that of AR = 4.
    - This is probably because for a higher AR, the induced angle of attack would be lower. As a result, there will be less lift decrease due to downwash effect.

3 Part I': Wind Tunnel Lab

Name: ZHANG Rushan Group: 14 Date: Oct 24th, 2021

Sample: Airfoil F, NACA2421 tripped

3.1 Calculation involved in sorting out the result

1. Summing up the x component and y component.
2. Calculating average by $average = \frac{sum}{number\ of\ lines\ of\ data}$
3. Calculating lift (denoted as L) and drag (denoted as D) by:

   $L = -(x_{avg} \cdot cos(AoA) - y_{avg} \cdot sin(AoA))$

   $D = -(y_{avg} \cdot cos(AoA) + x_{avg} \cdot sin(AoA))$
4. Calculating the local air density by:

   $\frac{P_1}{\rho_1 T_1} = \frac{P_0}{\rho_0 T_0}$

   From standard atmosphere table:

   $P_o \approx 101kPa, \rho_0 \approx 1.225kg/m^3, T_0 \approx 288.16K$

   By measurement:

   $P_1 \approx 99.87kPa, T_1 \approx 297.4K$

   Therefore:

   $\rho_1 \approx 1.17kg/m^3$
5. Calculate $\frac{2}{\rho_1 V_\infty^2 S} \approx 0.08155 \frac{1}{N}$

   Where $S = chord \times span \approx 0.0395m^2$ and $V_\infty \approx 23m/s$
6. Calculate coefficient of lift (denoted as C_L) and coefficient of drag (denoted as C_D) by:

   $C_L = \frac{2L}{\rho V_\infty^2 S} \approx 0.08155 \frac{1}{N} \cdot L$

   $C_D = \frac{2D}{\rho V_\infty^2 S} \approx 0.08155 \frac{1}{N} \cdot D$

7. By assumption:

   $C_l = C_L$, $C_d = C_D$

3.2 Codes for sorting out the result

```
#include <iostream>
#include <fstream>
#include <string>
#include <cmath>

const float PI = M_PI;

using namespace std;

double get_result(string num, double result[2], int &num_of_lines)
{
    string ip_file, op_file;

    ip_file = num + "aoa";

    ifstream ifs(ip_file);
    if (!ifs)
    {
        cerr << "Error: Can't open \"" << ip_file << "\"\n";
        return false;
    }
    string output = "N/A";

    char x[21];
    char y[21];
    char z[21];
    char tx[21];
    char ty[21];
    char tz[21];

    double sum_x = 0;
    double sum_y = 0;
    int count = 0;

    while (true)
    {
        ifs.getline(x, 20, ',');
        ifs.getline(y, 20, ',');
        ifs.getline(z, 20, ',');
        ifs.getline(tx, 20, ',');
        ifs.getline(ty, 20, ',');
        ifs.getline(tz, 20);
        if (ifs.eof())
        {
```

```
            break;
        }
        sum_x += stof(x);
        sum_y += stof(y);
        count++;
    }
    ifs.close();
    num_of_lines = count;
    result[0] = sum_x;
    result[1] = sum_y;
    return true;
}

bool get_curve()
{
    string op_file = "Result_output.txt";
    ofstream ofs(op_file);
    if (!ofs)
    {
        cerr << "Error: Can't open \"" << op_file << "\"\n";
        return false;
    }
    ofs << "degree\tsum_of_x\tsum_of_y\tnum_of_lines\tavg_of_x\tavg_of_y\treal_L\treal_D\tC_L\tC_D" << endl;

    double result[2];
    int num_of_lines;
    double result_avg[2];
    double result_real[2];
    double result_coefficient[2];
    double radius;
    for (int degree = -6; degree <= 24; degree++)
    {
        radius = degree * PI / 180;
        string num = to_string(degree);
        bool get_avg_success = get_result(num, result, num_of_lines);
        if (!get_avg_success)
            continue;
        result_avg[0] = result[0] / num_of_lines;
        result_avg[1] = result[1] / num_of_lines;
        result_real[0] = -(result_avg[0] * cos(radius) - result_avg[1] * sin(radius));
        result_real[1] = -(result_avg[1] * cos(radius) + result_avg[0] * sin(radius));
        result_coefficient[0] = result_real[0]*0.08155;
        result_coefficient[1] = result_real[1]*0.08155;
        ofs << degree << "\t"
            << result[0] << "\t"
```

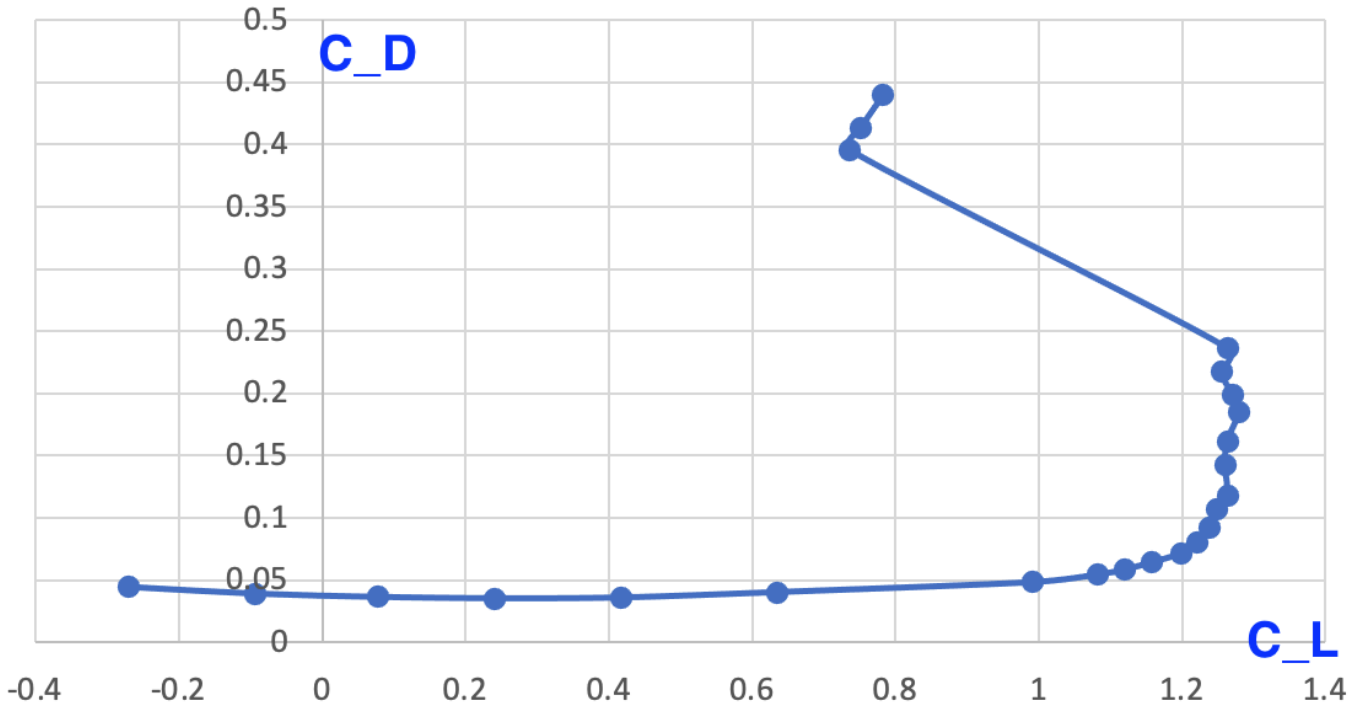
```
            << result[1] << "\t"
            << num_of_lines << "\t"
            << result_avg[0] << "\t"
            << result_avg[1] << "\t"
            << result_real[0] << "\t"
            << result_real[1] << "\t"
            << result_coefficient[0] << "\t"
            << result_coefficient[1] << "\t"
            << endl;
    }
    return true;
}

int main()
{
    get_curve();
}
```

3.3 Results

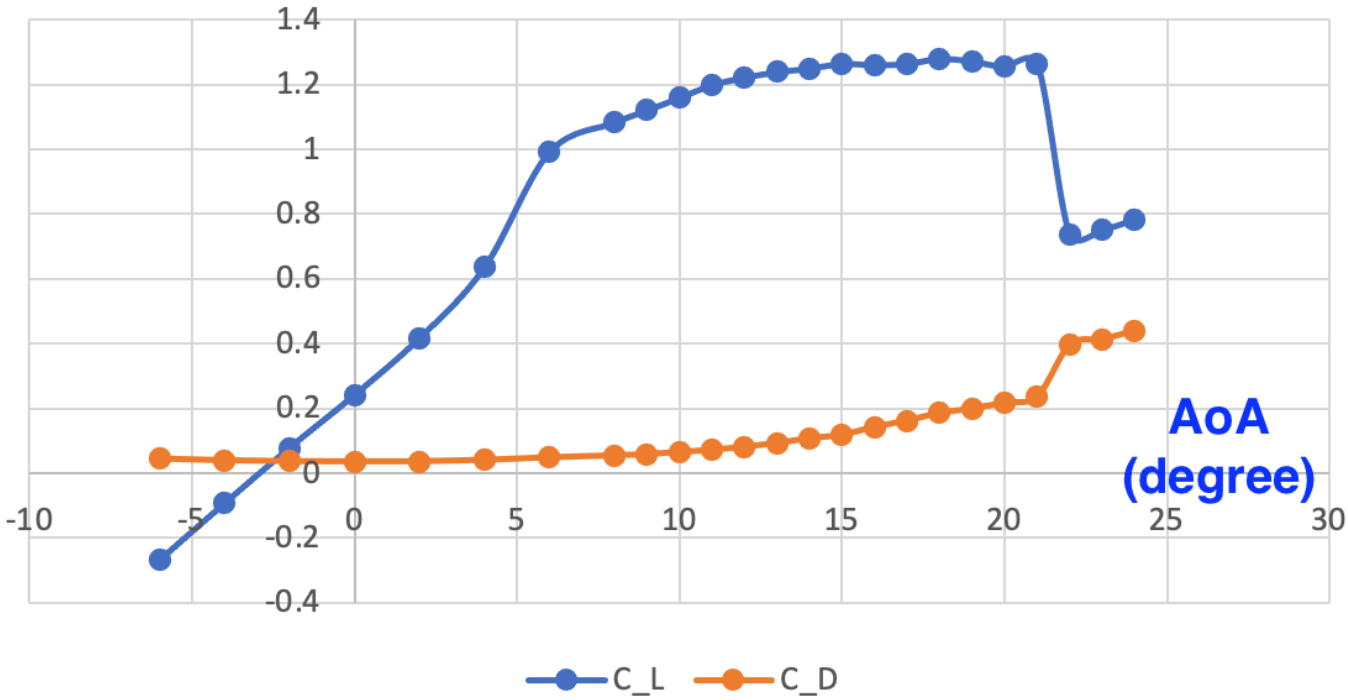
3.3.1 Table

degree	sum_of_x	sum_of_y	num_of_lines	avg_of_x	avg_of_y	real_L	real_D	C_L	C_D
-6	133608	-7746.76	40000	3.3402	-0.193669	-3.30166	0.541754	-0.26925	0.0441801
-4	47124.8	-15793.1	40000	1.17812	-0.394827	-1.14771	0.476046	-0.0935956	0.0388216
-2	-37602.3	-19082	40000	-0.940058	-0.477049	0.956134	0.443951	0.0779727	0.0362042
0	-236660	-34488.3	80000	-2.95824	-0.431104	2.95824	0.431104	0.241245	0.0351565
2	-204864	-10402	40000	-5.12159	-0.26005	5.10939	0.438632	0.416671	0.0357705
4	-312210	2135.52	40000	-7.80526	0.053388	7.78997	0.491209	0.635272	0.0400581
6	-486087	27374.4	40000	-12.1522	0.68436	12.1571	0.589637	0.991415	0.0480849
8	-529242	47555.9	40000	-13.231	1.1889	13.2677	0.664077	1.08198	0.0541554
9	-547040	57758.3	40000	-13.676	1.44396	13.7335	0.713216	1.11997	0.0581628
10	-565155	67669.5	40000	-14.1289	1.69174	14.208	0.787416	1.15866	0.0642138
11	-583592	77882	40000	-14.5898	1.94705	14.6933	0.872589	1.19824	0.0711596
12	-594152	86049.2	40000	-14.8538	2.15123	14.9765	0.984057	1.22133	0.0802498
13	-601983	92470.9	40000	-15.0496	2.31177	15.1839	1.13289	1.23825	0.0923876
14	-607337	97302.9	40000	-15.1834	2.43257	15.3209	1.31289	1.24942	0.107066
15	-613618	104658	40000	-15.3405	2.61646	15.4949	1.4431	1.26361	0.117684
16	-613497	103274	40000	-15.3374	2.58186	15.4549	1.74572	1.26035	0.142364
17	-615819	105405	40000	-15.3955	2.63512	15.4932	1.98122	1.26347	0.161568
18	-624976	107566	40000	-15.6244	2.68915	15.6907	2.27067	1.27958	0.185173
19	-621261	110751	40000	-15.5315	2.76877	15.5868	2.43865	1.2711	0.198872
20	-614855	110305	40000	-15.3714	2.75763	15.3875	2.66599	1.25485	0.217412
21	-620668	114192	40000	-15.5167	2.8548	15.5092	2.89551	1.26477	0.236128
22	-407873	-44534.9	40000	-10.1968	-1.11337	9.03726	4.8521	0.736989	0.395689
23	-418299	-42496.4	40000	-10.4575	-1.06241	9.21104	5.06402	0.751161	0.412971
24	-438475	-40830.3	40000	-10.9619	-1.02076	9.59899	5.3911	0.782798	0.439644

3.3.2 Drag polar



3.3.3 Lift curve and drag curve



We are assuming that $C_l = C_L$, $C_d = C_D$ here.