MECH4890
Final Year Design Project
Design a High-performance Airfoil with Advanced CFD and Machine-learning Methods

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Introduction
2D Mesh Building
2D Grid Validation
2D Classical Optimization
2D Data-driven Optimization
3D Validation and Optimization
Future Work

Overview

Introduction

2D Mesh Building

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3D Validation and Optimization

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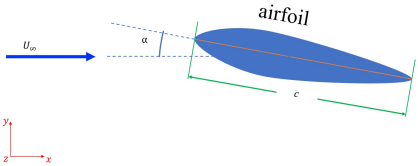
Introduction

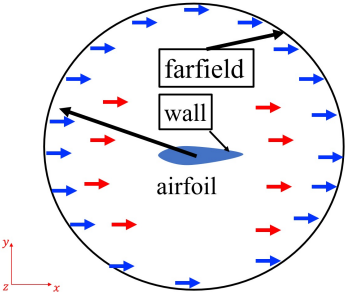
The project aims to increase the lift-to-drag ratio of an airfoil in the transonic region with SU2 and Post Process with ParaView by

1. Adjoint Method
2. Bayesian Optimization

Governing Equation

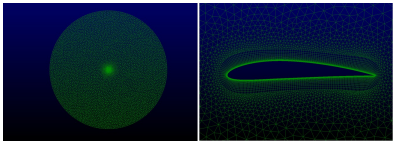
1. Euler Equation
2. Reynolds Averaged Navier- Stokes (RANS)



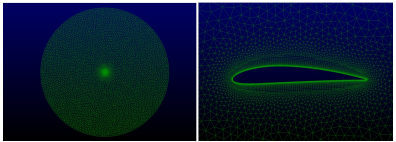


2D Grid Validation

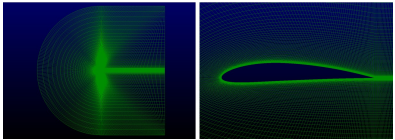
Mesh Groups



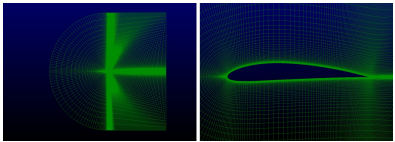
(a) O-Grid Hybrid



(b) O-Grid Hybrid TRex



(c) C-Grid Structured

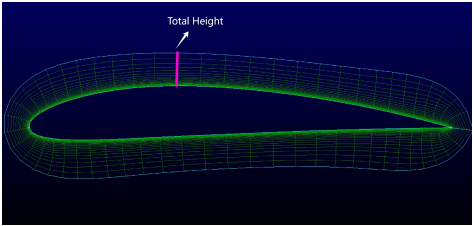


(d) C-Grid NASA

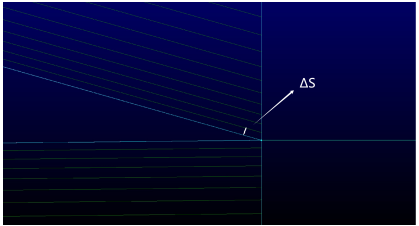
2D Mesh Building

Sketch of Boundary Layer Setting

Total height of layers:



ΔS: Initial distance at tailing edge



2D Mesh Building

Wall Spacing and Reynolds Number Determination

$$Re_x = \frac{\rho U_\infty L}{\mu} \quad (1)$$

$$C_f = \frac{0.026}{Re_x^{1/7}} \quad (2)$$

$$\tau_{wall} = \frac{C_f \rho U_\infty^2}{2} \quad (3)$$

$$U_{fric} = \sqrt{\frac{\tau_{wall}}{\rho}} \quad (4)$$

$$\Delta S = \frac{y^+ \mu}{U_{fric} \rho} \quad (5)$$

- $y+$ is a dimensionless parameter measures the distance from the first grid cell to the surface.
- Determines the accuracy of the boundary layer thickness prediction
- We set $y+$ to ONE

2D Mesh Building

Grid Setting Parameters

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Table: Unstructured O-Grid Properties Setting

# of Points on Airfoil	# of Points on Farfield	Spacing Value	ΔS	Height of Boundary Layer	Growth Rate
30	59	0.013	1.5678e-6	150	1.4
50	99	0.010	1.5678e-6	150	1.3
75	149	0.006	1.5678e-6	150	1.2
100	199	0.005	1.5678e-6	150	1.1

Table: Structured C-Grid Properties Setting

# of Points and Spacing on Airfoil	# of Points and Spacing on Arc	# of Points and Spacing on Vertical Bars	# of Points and Spacing on Horizontal Bars
50, 0.010	99, 1.0	100, 1.5678e-6	50, 1.0
75, 0.006	149, 1.0	170, 1.5678e-6	75, 1.0
100, 0.005	199, 1.0	180, 1.5678e-6	100, 1.0
125, 0.004	249, 1.0	190, 1.5678e-6	125, 1.0
150, 0.003	299, 1.0	200, 1.5678e-6	150, 1.0

2D Grid Validation

Settings for the Convergence Behaviour Test

Flow Property	Value
Mach	0.717
Sonic speed [m/s]	340
Velocity (U) $[m/s]$	244
Absolute Pressure (P_∞) $[Pa]$	101325
Air Density (ρ) $[kg/m^3]$	1.225
Dynamic Viscosity (μ) $[Pa \cdot s]$	$1.7893 \cdot 10^{-5}$
Chord Length (c) $[m]$	0.127
Reynolds Number ($Re = \frac{\rho U c}{\mu}$)	2162861.175
Temperature (T_∞) $[K]$	288.15

Simulation Settings	Value
Numerical Scheme	RANS
CFL	$1 \sim 1000$
Maximum iteration	99999
Convergence criteria	residual $< 10^{-8}$
Angle of Attack (α) [°]	-0.25

2D Grid Validation

Settings for the Convergence Behaviour Test

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Table: Number of Cells and Convergence Test Result

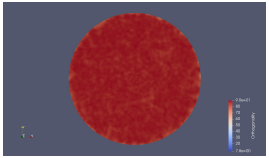
# of points	Unstructured O-grid	Unstructured O-grid T-Rex	Structured C-grid	Structured C-grid NASA
30	6998	7202	-	3584
50	11204	11291	19404	14336
75	17462	17325	50024	57344
100	25110	24606	70884	-
125	33868	32942	93744	-
150	43588	42045	118604	-

Number in blocks: Number of cells

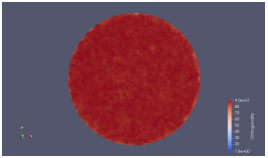
- GREEN block: Result is convergence
- YELLOW block: Reaches the maximum iteration(99999)
- RED block: Result is divergence

2D Grid Validation

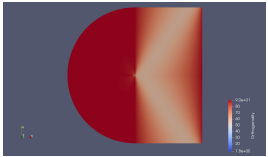
Results of Orthogonality Test



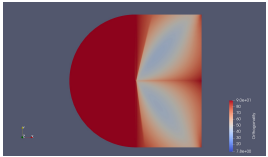
(a) O-Grid Hybrid



(b) O-Grid Hybrid T-Rex



(c) C-Grid Structured



(d) C-Grid NASA

Uniformly distributed orthogonality results in:

- Better gradient of properties captured
- Better stability of the numerical solution

2D Grid Validation

Grid Convergence Test

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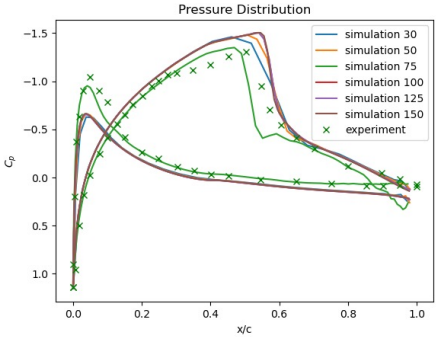
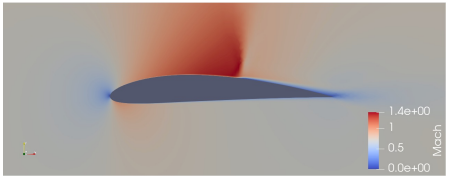
T-Rex method pressure distribution:





Generally fitting the experiment result.
(But not that well)
Tiny difference among different mesh

2D Classical Optimization

NACA4412 Optimization at Transonic Region Setting

Flow Property	Value
Mach	0.729
Sonic speed [m/s]	340
Velocity (U) $[m/s]$	247.86
Absolute Pressure (P_∞) $[Pa]$	101325
Air Density (ρ) $[kg/m^3]$	1.225
Dynamic Viscosity (μ) $[Pa \cdot s]$	$1.7893 \cdot 10^{-5}$
Chord Length (c) $[m]$	1
Reynolds Number ($Re = \frac{\rho U c}{\mu}$)	$6.5 \cdot 10^6$
Temperature (T_∞) $[K]$	288.15

Simulation Settings	Value
Numerical Scheme	RANS
CFL	10
Maximum iteration	9999
Convergence criteria	residual $< 10^{-12}$
Angle of Attack (α) [°]	0
Target C_L	0.622313
Relaxation Factor	10

2D Classical Optimization

NACA4412 Optimization at Transonic Region Result

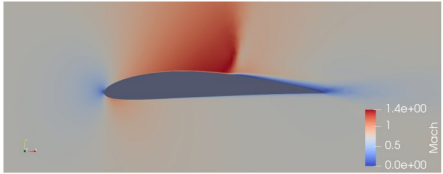


Figure 7.1: NACA4412 Visualization

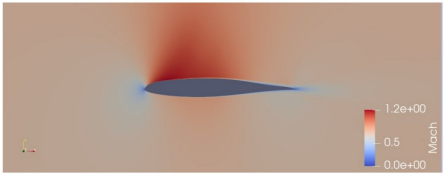
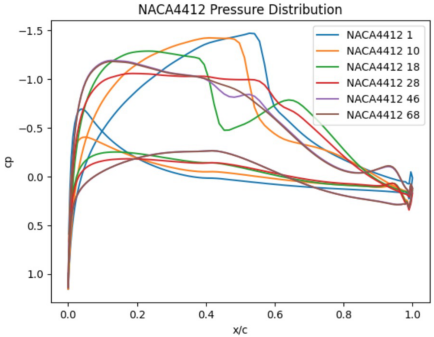


Figure 7.2: NACA4412 Optimised Visualization



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2D Classical Optimization

NACA4412 Optimization at Transonic Region Result

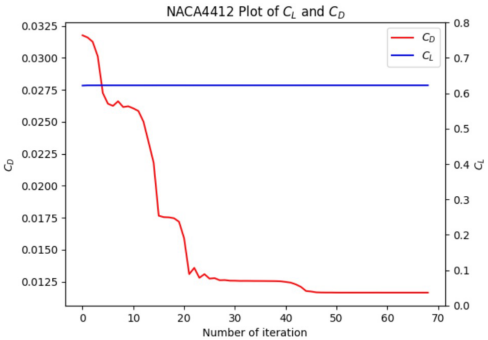


Table: NACA4412 Optimization Result

Case	C_L	C_D
Original	0.621615	0.031757
Optimized	0.622345	0.011637

C_D **Percentage Change** $= -63.356\%$

NACA4412 is not suitable for the Transonic region.

2D Classical Optimization

RAE2822 Optimization at Transonic Region Setting

Flow Property	Value
Mach	0.729
Sonic speed [m/s]	340
Velocity (U) $[m/s]$	247.86
Absolute Pressure (P_∞) $[Pa]$	101325
Air Density (ρ) $[kg/m^3]$	1.225
Dynamic Viscosity (μ) $[Pa \cdot s]$	$1.7893 \cdot 10^{-5}$
Chord Length (c) $[m]$	1
Reynolds Number ($Re = \frac{\rho U c}{\mu}$)	$6.5 \cdot 10^6$
Temperature (T_∞) $[K]$	288.15

Simulation Settings	Value
Numerical Scheme	RANS
CFL	10
Maximum iteration	9999
Convergence criteria	residual $< 10^{-12}$
Angle of Attack (α) [°]	0
Target C_L	0.622313
Relaxation Factor	10

2D Classical Optimization

RAE2822 Optimization at Transonic Region Result

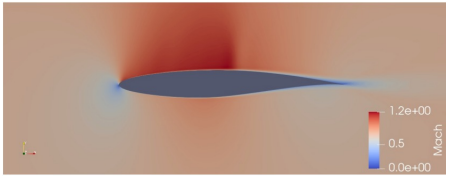


Figure 7.4: RAE2822 Visualization

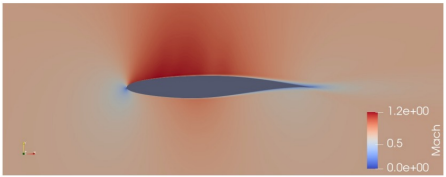
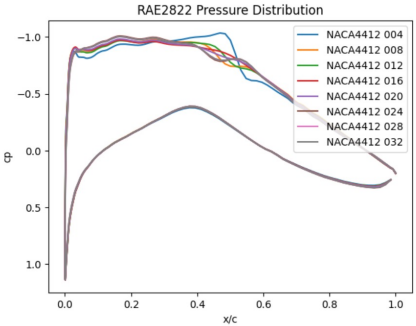


Figure 7.5: RAE2822 Optimised Visualization



2D Classical Optimization

RAE2822 Optimization at Transonic Region Result

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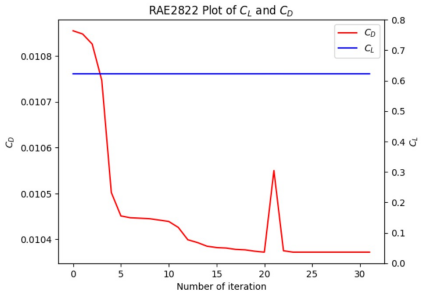


Table: RAE2822 Optimization Result

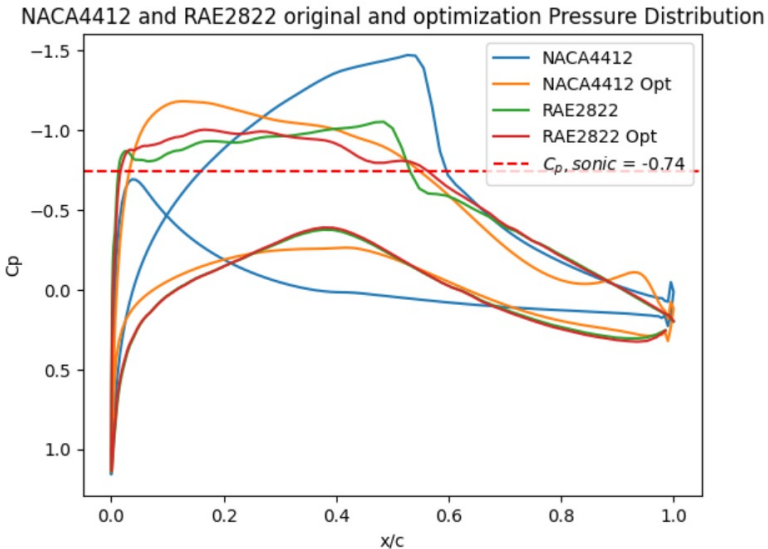
Case	C_L	C_D
Original	0.622313	0.010855
Optimized	0.622333	0.010372

C_D **Percentage Change** $= -4.44\%$

RAE2822 is suitable for the Transonic region.

2D Classical Optimization

NACA4412 and RAE2822 Comparision



2D Data-driven Optimization

Method: Bayesian Optimization

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Assumption: the objective function is a sample from a Gaussian Process (GP)

$$f(x)|f(x_{1:n}) \sim \mathcal{N}(\mu(x), \sigma^2(x))$$

$$\mu(x) = \Sigma_0(x, x_{1:n})\Sigma_0(x_{1:n}, x_{1:n})^{-1}(f(x_{1:n}) - \mu_0(x_{1:n})) + \mu_0(x)$$

$$\sigma^2(x) = \Sigma_0(x, x) - \Sigma_0(x, x_{1:n})\Sigma_0(x_{1:n}, x_{1:n})^{-1}\Sigma_0(x_{1:n}, x)$$

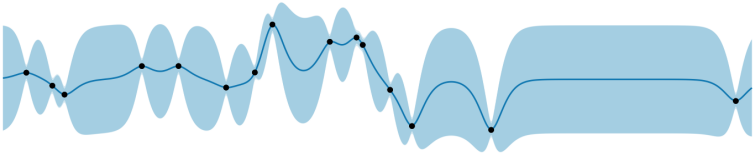

Figure: Example of the posterior distribution of a GP

- Blue line: posterior mean
- Shaded area: 95% confidence interval ($\mu(x) \pm 1.96\sigma(x)$)

2D Data-driven Optimization

Method: Bayesian Optimization

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Acquisition function: determines the next point to evaluate

- Expected Improvement (EI)

$$EI(x) = \mathbb{E}\left[[f(x) - f^*]^+\right]$$

  - $[a]^+$: defined as $[a]^+ = \max(a, 0)$, ensuring that the acquisition function is always positive

- Lower Confidence Bound (LCB)

$$LCB(x) = \mu(x) - \beta\sigma(x)$$

  - β: trade-off parameter, set to 1.96 in our implementation, corresponding to the 95% confidence interval

2D Data-driven Optimization

Implementation

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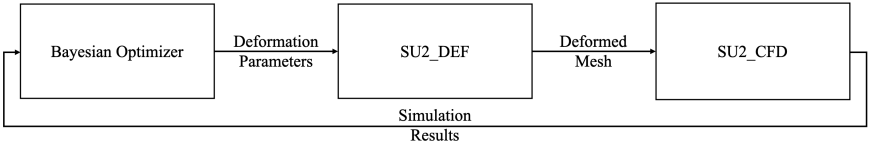
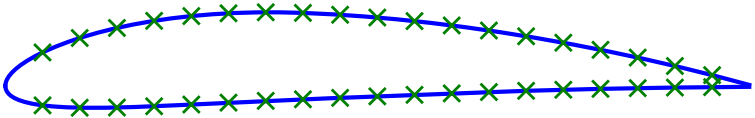
Communication between Components



Variables: 19 control points from $x/c = 0.05$ to $x/c = 0.95$, with a spacing of $x/c = 0.05$. In total, 38 variables.



2D Data-driven Optimization

Implementation

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Design goals and constraints

- Fixed C_l
- Minimize C_d

Objective function

$$f(x) = \begin{cases} 10 & \text{if } C_d \leq 0 \\ 5 & \text{if } C_d \geq 5 \\ 2.5 & \text{if } |C_l - C_{l,target}| > 0.2 \\ C_d & \text{otherwise} \end{cases}$$

2D Data-driven Optimization

Optimization Settings

Flow Property	Value
Mach	0.729
Sonic speed [m/s]	340
Velocity (U) $[m/s]$	247.86
Absolute Pressure (P_∞) $[Pa]$	101325
Air Density (ρ) $[kg/m^3]$	1.225
Dynamic Viscosity (μ) $[Pa \cdot s]$	$1.7893 \cdot 10^{-5}$
Chord Length (c) $[m]$	1
Reynolds Number ($Re = \frac{\rho U c}{\mu}$)	$6.5 \cdot 10^6$
Temperature (T_∞) $[K]$	288.15

Simulation Settings	Value
Numerical Scheme	RANS
CFL	$1 \sim 1000$
Maximum Iteration	9999
Convergence Criteria	residual $< 10^{-8}$
Angle of Attack (α) [°]	0.00
Optimization Settings	**Value**
Number of Iterations	600
Acquisition Function	EI, LCB
Deformation Bound	$-10^{-6} \sim 10^{-4}$

2D Data-driven Optimization

Results Based on NACA4412

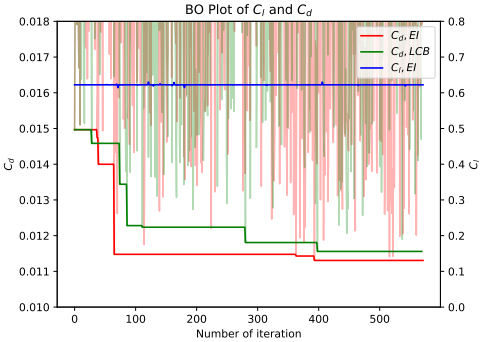


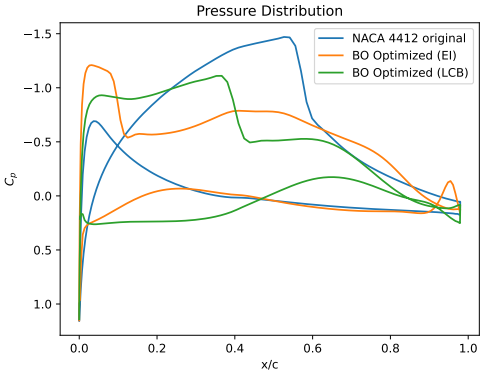
Figure: Optimization process

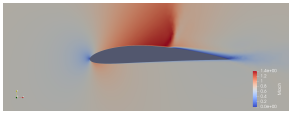
Table: BO Optimization Result

Cases	C_l	C_d
NACA4412	0.621615	0.031757
Adjoint	0.622345	0.011637
EI	0.622300	0.011306
LCB	0.622318	0.011558

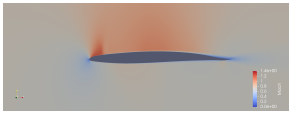
2D Data-driven Optimization

Results Based on NACA4412

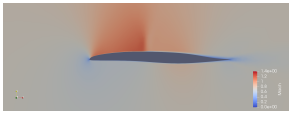




(a) NACA4412



(b) EI



(c) LCB

2D Data-driven Optimization

Results Based on RAE2822

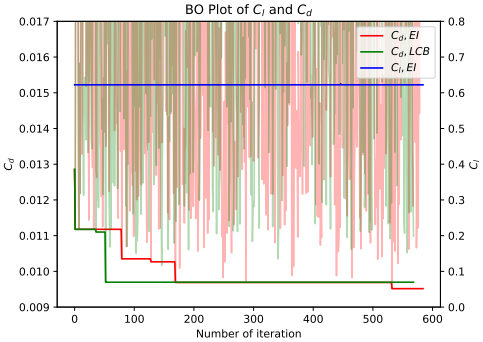


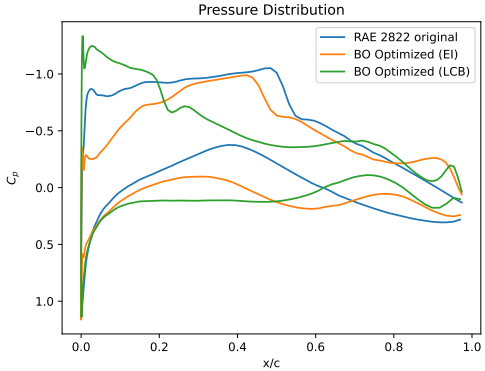
Figure: Optimization process

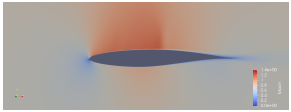
Table: BO Optimization Result

Cases	C_l	C_d
RAE2822	0.622318	0.010854
Adjoint	0.622333	0.010372
EI	0.622303	0.009516
LCB	0.622320	0.009696

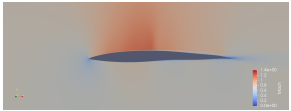
2D Data-driven Optimization

Results Based on RAE2822

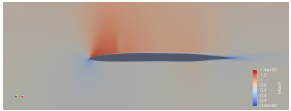




(a) RAE2822



(b) EI



(c) LCB

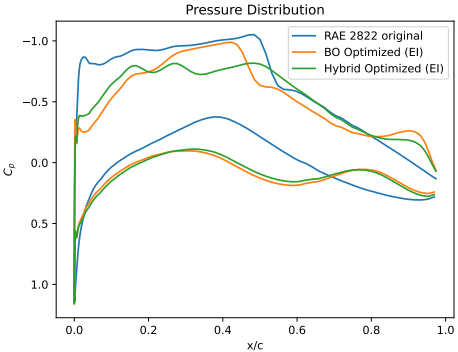
2D Data-driven Optimization

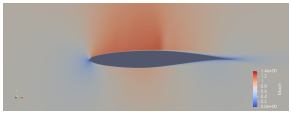
RAE2822 Hybrid Optimization

Hybrid Optimization:

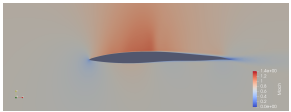
- BO - global optimum
- Adjoint - local refinement

Base Airfoil: BO-optimized (EI)

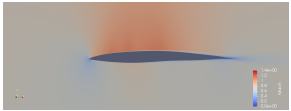




(a) RAE2822



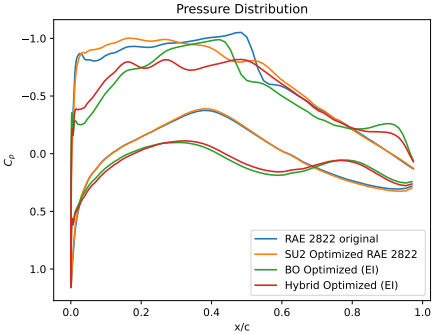
(b) EI

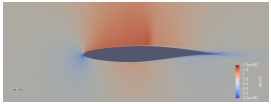


(c) Hybrid

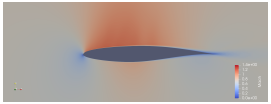
2D Data-driven Optimization

Comparison between Three Optimization Methods

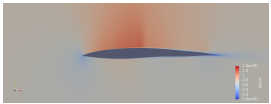




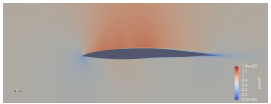
(a) RAE2822



(b) Adjoint



(c) EI



(d) Hybrid

Figure: Mach number distribution

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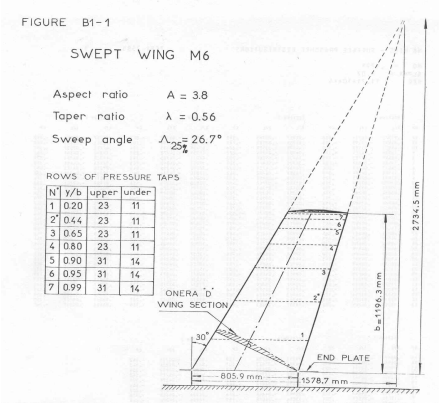
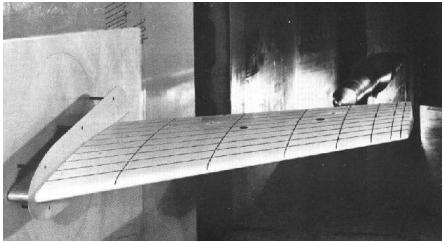
ONERA M6 Geometry

The ONERA M6 wing is a swept, half-span wing with no twist, using a symmetrical airfoil of the ONERA D section.



(a) Geometric layout



(b) The ONERA M6

ONERA M6 Simulation Validation

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Experiment comparison

- **Transonic**: Schmitt, V. and F. Charpin, "Pressure Distributions on the ONERA-M6-Wing at Transonic Mach Numbers," Experimental Data Base for Computer Program Assessment. Report of the Fluid Dynamics Panel Working Group 04, AGARD AR 138, May 1979.
- **Subsonic**: Hart, Pierce L., and Sven Schmitz. "Drag decomposition using partial-pressure fields: ONERA M6 wing." AIAA journal 60.5 (2022): 2941-2952.

ONERA M6 Simulation Validation

Transonic Settings

Flow Property	Value
Mach	0.8395
Sonic speed [m/s]	340
Velocity (U) $[m/s]$	285.68
Absolute Pressure (P_∞) $[Pa]$	101325
Air Density (ρ) $[kg/m^3]$	1.225
Dynamic Viscosity (μ) $[Pa \cdot s]$	$1.93 \cdot 10^{-5}$
Chord Length (c) $[m]$	0.6467
Reynolds Number $(Re = \frac{\rho U c}{\mu})$	11720000
Temperature (T_∞) $[K]$	288.15

Simulation Settings	Value
Numerical Scheme	RANS
CFL	100
Maximum iteration	999999
Convergence criteria	residual $< 10^{-12}$
Angle of Attack (α) $[^\circ]$	3.06
Relaxation Factor	10

ONERA M6 Simulation Validation

Subsonic Settings

Flow Property	Value
Mach	0.25
Sonic speed [m/s]	340
Velocity (U) $[m/s]$	85
Absolute Pressure (P_∞) $[Pa]$	101325
Air Density (ρ) $[kg/m^3]$	1.225
Dynamic Viscosity (μ) $[Pa \cdot s]$	$5.74 \cdot 10^{-6}$
Chord Length (c) $[m]$	0.6467
Reynolds Number ($Re = \frac{\rho U c}{\mu}$)	11720000
Temperature (T_∞) $[K]$	288.15

Simulation Settings	Value
Numerical Scheme	RANS
CFL	100
Maximum iteration	999999
Convergence criteria	residual $< 10^{-12}$
Angle of Attack (α) [°]	3.06
Relaxation Factor	10

Transonic Validation result

Simulation

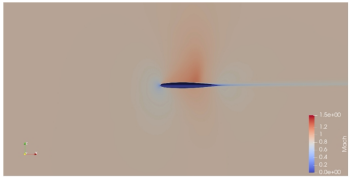


Figure: Mach

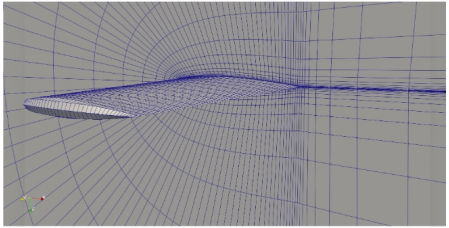
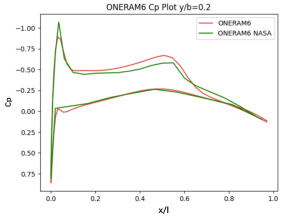


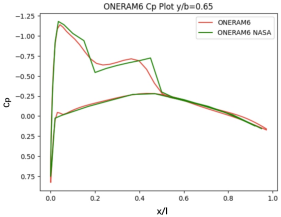
Figure: Mesh

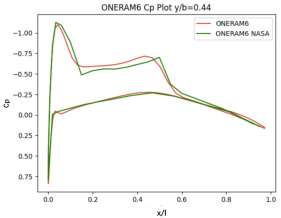
Figure: Pressure Coefficient

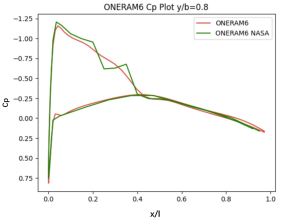
Transonic Validation result

Comparison



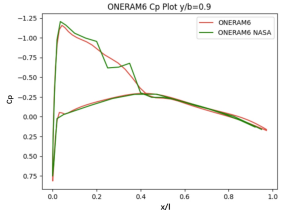


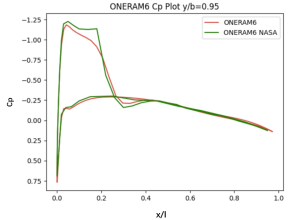


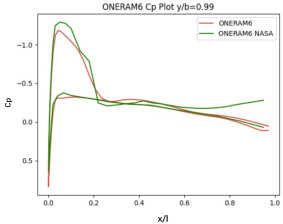


Transonic Validation result

Comparison







Subsonic Validation result

Simulation

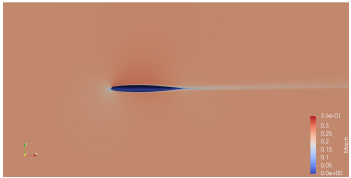


Figure: Mach

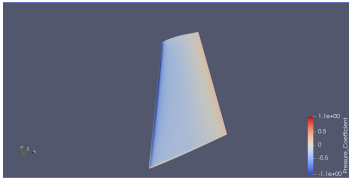


Figure: Pressure Coefficient

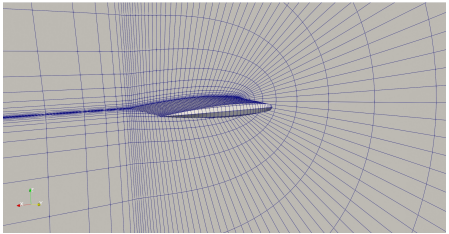
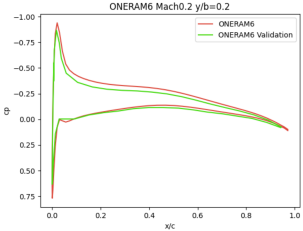
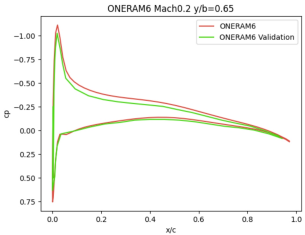


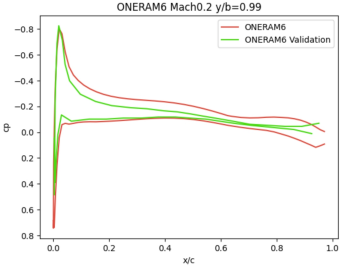
Figure: Mesh

Subsonic Validation result

Comparison







ONERA M6 Optimization at Transonic Region

Transonic Settings

Flow Property	Value
Mach	0.8395
Sonic speed [m/s]	340
Velocity (U) $[m/s]$	285.68
Absolute Pressure (P_∞) $[Pa]$	101325
Air Density (ρ) $[kg/m^3]$	1.225
Chord Length (c) $[m]$	0.6467
Temperature (T_∞) $[K]$	288.15

Simulation Settings	Value
Numerical Scheme	EULAR
CFL	7.5
Maximum iteration	999999
Convergence criteria	residual $< 10^{-12}$
Angle of Attack (α) [°]	3.06
Target C_L	0.286

ONERA M6 Optimization at Transonic Region

Optimization Result

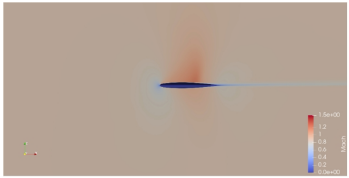


Figure: ONERA M6 MACH

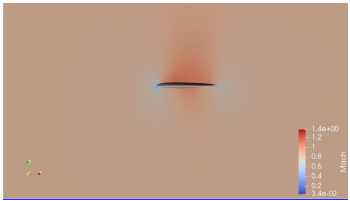
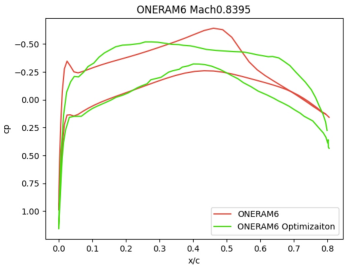
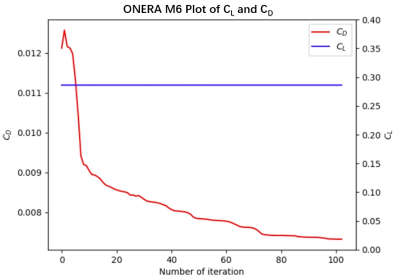


Figure: Optimized-ONERA M6 MACH

ONERA M6 Optimization at Transonic Region

Optimization Result





ONERA M6 Optimization at Subsonic Region

Subsonic Settings

Flow Property	Value
Mach	0.25
Sonic speed [m/s]	340
Velocity (U) $[m/s]$	85
Absolute Pressure (P_∞) $[Pa]$	101325
Air Density (ρ) $[kg/m^3]$	1.225
Chord Length (c) $[m]$	0.6467
Temperature (T_∞) $[K]$	288.15

Simulation Settings	Value
Numerical Scheme	EULAR
CFL	7.5
Maximum iteration	999999
Convergence criteria	residual $< 10^{-12}$
Angle of Attack (α) [°]	3.06
Target C_L	0.195

ONERA M6 Optimization at Subsonic Region

Optimization Result

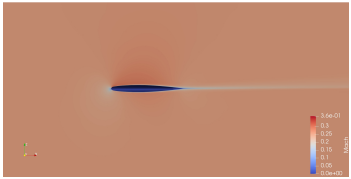


Figure: ONERA M6 MACH

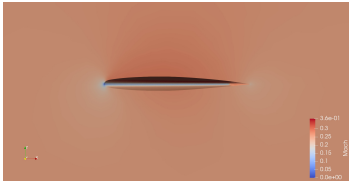
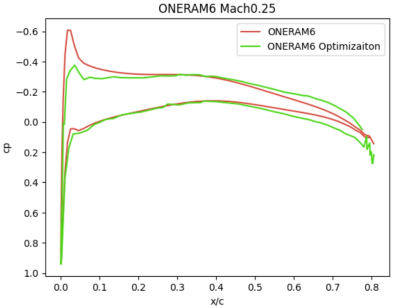


Figure: Optimized-ONERA M6 MACH



Future Work

Machine-learning track

- 2D high-lift device design at low-Mach region

Classical track

- 3D wing design at transonic region

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2D Grid Validation
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2D Data-driven Optimization
3D Validation and Optimization
Future Work

Future Work

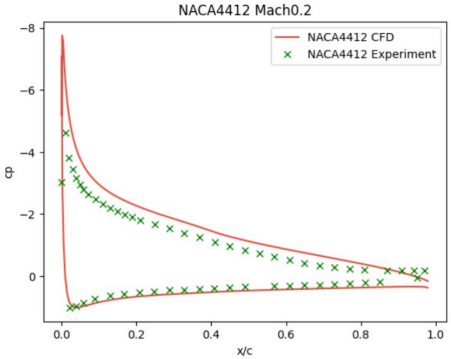
2D High Lift Devices with Data-Driven Methods

Flow Property	Value
Mach	0.2
Sonic speed [m/s]	340
Velocity (U) $[m/s]$	68
Absolute Pressure (P_∞) $[Pa]$	101325
Air Density (ρ) $[kg/m^3]$	1.14
Dynamic Viscosity (μ) $[Pa \cdot s]$	$1.61 \cdot 10^{-5}$
Chord Length (c) $[m]$	1
Reynolds Number $(Re = \frac{\rho U c}{\mu})$	1640000
Temperature (T_∞) $[K]$	297.15

Simulation Settings	Value
Numerical Scheme	RANS
CFL	1000
Maximum iteration	999999
Convergence criteria	residual $< 10^{-8}$
Angle of Attack (α) $[°]$	12

Future Work

2D High Lift Devices with Data-Driven Methods



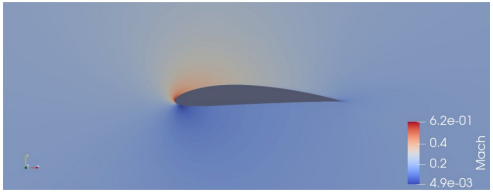
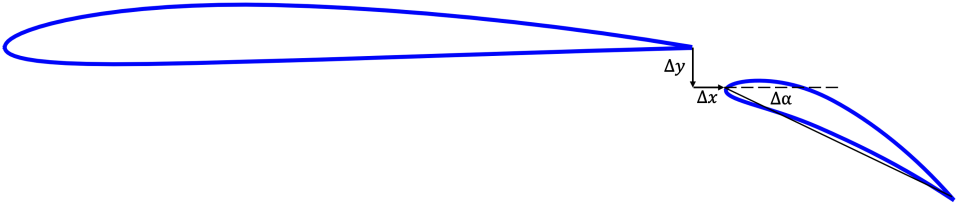


Figure 6.6: Visualization of NACA4412 at M=0.2

The validation can be considered satisfactory

Future Work

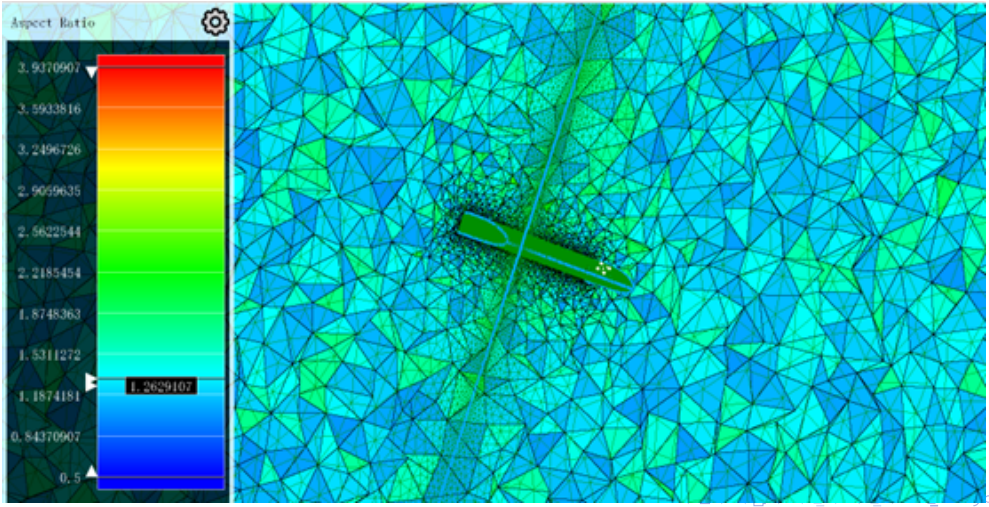
2D High Lift Devices with Data-Driven Methods



Future Work

3D wing design at transonic region

We currently have:



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Future Work

THANK YOU!

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