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Decision theory: probability and bets Feb 24

Suppose G is an action of betting on A or $-A$. The payoff for A is 1, -A is 0, we then have expected value:

$$Exp(G) = P(A)\text{Payoff}(A) + P(-A)\text{Payoff}(-A) = P(A)$$

If we substitute payoff of A being $1 - P(A)$, -A being $-P(A)$:

$$\begin{aligned} Exp(G) &= P(A)\text{Payoff}(A) + P(-A)\text{Payoff}(-A) \\ &= P(A)[1 - P(A)] + [1 - P(A)][-P(A)] \\ \therefore Exp(G) &= 0 \end{aligned}$$

which gives a fair bet between A and -A.

We write the fair betting result into a decision matrix

	payoff
A	$1 - P(A)$
$-A$	$-P(A)$

If we do the computation differently by supposing the fair betting rate to be x, we have the following decision matrix:

	payoff
A	$1 - x$
$-A$	$-x$

Setting the expected value to be 0, we arrive at the same result that $x = P(A)$

If we suppose a more general decision matrix: Setting the expected value being 0 gives:

	payoff
A	W
$-A$	$-L$

$$Exp = 0 = P(A)W + [1 - P(A)](-L)$$

We solve for $P(A)$:

$$P(A) = \frac{L}{W + L}$$

We define L as potential loss, $W + L$ as stake. We can also define $P(A)$ as the agent's **betting rate**

Dutch books

Sure-loss contract could be formed once the agent does not obey the probability calculus axioms, assuming the agent is a utility maximizer.

For example, one probability axiom is $P(\text{logical truth}) = 1$, say $P(A \vee -A) = 1$, we assume someone who does not follow the probability calculus to have

$$P(A \vee -A) < 1$$

but otherwise obeys the axioms. We therefore can write, based off the agent's probability, that

$$P(A \vee -A) + P[-(A \vee -A)] = 1$$

Define $x \equiv P[-(A \vee -A)]$, $1 - x \equiv P(A \vee -A)$

So we have decision matrix

	payoff
$A \vee -A$	$-x$
$-(A \vee -A)$	$1 - x$

$$\begin{aligned} EV &= P(A \vee -A)\text{Payoff}(A \vee -A) + P - (A \vee -A))\text{Payoff}(-(A \vee -A)) \\ &= (1 - x)(-x) + x(1 - x) \\ &= 0 \end{aligned}$$

It is a fair bet for the agent. However, the only way for the agent to win money is for a contradiction to come true. The agent is guaranteed to lose money despite of a fair contract. This results in a dutch book, or a sure-loss contract.

<u>**Dutch Book**</u>: a collection of bets that the agent regards as fair, but inflicts a sure-loss.

Example: Suppose A and B are mutually exclusive. Say John violates additivity and has $P(A \vee B) < P(A) + P(B)$ such that the decision matrix for fair bets based off these probabilities are

	payoff
A	$1 - P(A)$
$-A$	$-P(A)$

	payoff
B	$1 - P(B)$
$-B$	$-P(B)$

	payoff
$A \vee B$	$P(A \vee B) - 1$
$-(A \vee B)$	$-P(A \vee B)$

Notice that we revert the last table by multiplying all payoff by -1, the expected value remains 0. Such operation reverts the position of agent and bookie, but keeps the bets same and fair.

	Total payoff	Simplified total payoff
$A\&-B$	$1-P(A)-P(B)+P(A\vee B)-1$	$P(A\vee B)-P(A)-P(B)<0$
$-A\&B$	$-P(A)+1-P(B)+P(A\vee B)-1<0$	$P(A\vee B)-P(A)-P(B)<0$
$-A\&-B$	$-P(A)-P(B)+P(A\vee B)<0$	$P(A\vee B)-P(A)-P(B)<0$

We then calculate the net payoff for all three bets for the Final decision matrix by considering 3 scenarios. Notice that since they are mutually exclusive we don't consider the case $A\&B$.

Now all outcomes have utility smaller than 0. We have a fair but sure-loss contract for John.

More on Dutch Books Mar 1

Probabilism: agents' credence in Bayesian framework should conform to the probability axioms.

Rejections:

- Diminishing marginal utility...(Pettigrew - Dutch Book arguments)
- The sure-loss contract is visible, but conflict with expected utility theory
- Dutch book arguments give a practical but not epistemic defect. i.e nothing wrong with rational beliefs, it is merely a bad consequence.

Dutch Book Theorem: if credences violate the probability calculus axioms, then the agent is dutch-bookable.

Converse Dutch Book Theorem : if credence satisfies the probability calculus axioms, then the agent is not dutch-bookable.

Conditionalization Mar 3

Conditionalization is a dynamic norm, and gives a dynamic/time evolution result of belief. Ex. P at t_1 and P_E at t_2, where P_E is derived from conditionalizing P on E. Whereas Bayes theorem is purely syn-chronic, only gives belief in instances of time.

Construct Dutch-book for conditionalization

Suppose an agent has probability P at t_1, with possible evidences $E_1, E_2, ..., E_n$. Upon learning E, the agent update the belief to C_E

If $P_E = C_E$, the agent is a conditonalizer.
If $P_E \neq C_E$, the agent is NOT a conditonalizer.

Now we suppose that agent is not a conditionalizer, that $P_E \neq C_E$. With no loss of generality, we assume $P_E > C_E$

Suppose decision matrix

	payoff
$H\&E$	$1 - P(H\|E)$
$-H\&E$	$-P(H\|E)$
$-E$	0

We confirm it's a fair best by confirming $Exp = 0$.

Notice that the agent and the bookie are uncertain at time t_1, they learn which partition comes at t_2, the bookkeeper does not know what bet to give prior to t_2.

At t_2, the bookie may or may not offer this bet if E becomes true.

	payoff
H	$C_E(H) - 1$
$-H$	$C_E(H)$

Summarize bookie's strategy: offer the above bet only iff E is revealed at t_2. If not revealed, bookie does nothing.

The agent would take this bet of E is revealed, since the given bet is fair to him.

Net payoff for both bets: (if E revealed, bet 2 is offered and accepted) for $-E$, the second bet if not offered.

	Net payoff
$H\&E$	$1 - P(H\|E) + C_E(H) - 1 = C_E(H) - P(H\|E)$
$-H\&E$	$-P(H\|E) + C_E(H)$
$-E$	0

Notice $C_E(H) < P(H|E) = P_E(H)$ but NOT a sure-loss! There could be no money loss at $-E$.

To make a sure-loss, we need the third "side bet" ALSO at t_1 on E. However, we can't have a fair bet, need a "backwards" sweetener in cyan

	payoff
E	$[P(H\|E) - C_E(H)][1 - P(E)]$
$-E$	$[C_E(H) - P(H\|E)][-P(E)]$

Let's check if the outcome is negative, we make algebra easy by define $x = -P(H|E) + C_E(H)$,

$$\begin{aligned} C_E(H) - P(H|E) + [P(H|E) - C_E(H)][1 - P(E)] &= x - x[1 - P(E)] \\ &= xP(E) \\ &= P(E)[-P(H|E) + C_E(H)] < 0 \end{aligned}$$

from our assumption that $P_E > C_E$

The Final decision matrix

	Net payoff
$H\&E$	$C_E(H) - P(H\|E)$
$-H\&E$	$-P(H\|E) + C_E(H)$
$-E$	$[-P(H\|E) + C_E(H)]P(E)$

Now it's a sure-loss, a dutch-book theorem. If the agent violates conditionalization, there is a strategy for invoking a dutch book strategy.

Dutch book on violation of conditionalization Mar 8

Dutch book theorem for conditionalization by David Lewis: if agent does not follow the conditionalization rules, the agent is dutch-bookable.

Converse Dutch book theorem for conditionalization by Brian Skyrms UCI: if agent follows the conditionalization rules, the agent is NOT dutch-bookable.

Therefore from the two theorems, conditionalization is the unique way to prevent Dutch-book - a privilege about conditionalization.

Elements of Bayesian Decision Theory:

- Probablism: probability calculus axioms are norms on credences
- Conditionalization as a diachronic (across time, as opposed to synchronic at one point in time) norm
- Expected utility maximization: supported the long-run arguments. If the situation repeats, maximization of utility will be beneficial.

Remaining problems of BDT

1) Problems of the priors: for $P(H|E)$, where do we get $P(H)$, the prior?

We could get the priors from:
Laplace's Principle of indifference (of insufficient reason): if there are n possible outcomes, each outcome should have probability $\frac{1}{n}$.
Rejection: infinite number of outcomes? We then separate them into intervals

Laplace's Principle of indifference (Continuous version): if there is an interval of possible outcomes from a to b, the probability of any subinterval c and d, the probability is $\frac{d-c}{b-a}$

Rejection: Bertrand's paradox - Example by van Fraassen's box factory.

You have a factory that makes square pieces of paper with side length 1 to 3 ft. We want the probability $P(\text{length between 1 and 2 ft})$. The principle of indifference says:

$$P(\text{length between 1 and 2 ft}) = \frac{d-c}{b-a} = \frac{2-1}{3-1} = \frac{1}{2}$$

Now another question: what is $P(\text{area between 1 and 4 sq. ft})$ Principle of Indifference says:

$$P(\text{area between 1 and 4 sq. ft}) = \frac{d-c}{b-a} = \frac{4-1}{9-1} = \frac{3}{8}$$

Same situation described differently should have the same probability!

What can we do about it? Solution by E.T. Jaynes: through some fancy math we can prove some options are privileged.

More problems with infinite domain: probability is often zero. For a roulette wheel with infinite number of angles, probability of landing on any specific angle $P(\theta) = 0$ due to the normality axiom.

Beyond Bayesian Mar 10

Regularity

Propose: Adding additional norm of regularity
Regularity: If A is possible, then $P(A) > 0$

Construct a dutch-book for A is possible but agent take $P(A) = 0$
We have the agents probability $P(-A) = 1$, decision matrix:

	Payoff
A	-1
$-A$	0

Expected value:

$$EV = P(A)\text{Payoff}(A) + P(-A)\text{Payoff}(-A) = 0 + 0 = 0$$

It is not sure-loss, but either loss or gain nothing, or "semi-dutch book."

However, regularity is in conflict with elements of BDT. For example, any giving angle on roulette wheel would have to be 0. But since it is now possible, due to regularity, we need none zero probability at a specific angle. Violates regularity.

Giving infinitestimal value? But probability have to be real number.

Also causing problem on conditionalization:
If B logically entails $-A$, then $P(B|A) = 0$, particularly $P(-E|E) = 0$.
We have

$$P_{new}(.) = P(.|E)$$

$$P_{new}(-E) = P(-E|E) = 0$$

Very first time of conditionalization, we violate regularity.

For mutually exclusive $A_1, A_2, ..., A_n$ we have the dual case of additivity:

$$P(A \vee B) = P(A) + P(B)$$

$$\therefore P(A_1 \vee A_2 \vee ... \vee A_n) = \sum_{i=1}^{n} P(A_i)$$

called finite additivity
For countable additivity that has a infinitely long list.

$$P(A_1 \vee A_2 \vee ... \vee A_\infty) = \sum_{i=1}^{\infty} P(A_i)$$

di Finetti Lottery against countable additivity:
Pick a number at random from a collection, we should have

$$P(1) = P(2) = ... = P(n)$$

For an infinite list of numbers, they all have to have

$$P(1) = P(2) = ... = P(n) = 0$$

However, one of the has to be picked.

$$P(1 \vee 2 \vee 3 \vee ...) = 1$$

There is a contradiction. However, some contradictions are that "what does it mean to draw number at random?"

Review: Dutch Book example

2. Suppose Marco has personal probabilities $Pr(X) = 3/10$, $Pr(Y) = 2/10$, and $Pr(X \vee Y) = 6/10$. Explain how to make a Dutch book against him. Your answer should include all of the following:

   - A list of the bets to be made with Marco.
   - An explanation why Marco will regard these bets as fair.
   - An explanation why these bets will lead to a sure loss for Marco no matter what.

Figure 1: Weisberg P171.2

We have

$$P(X \vee Y) > P(X) + P(Y)$$

General strategy: if need a bet on x, we have decision matrix of a fair bet and fair betting rate: Expected value would be 0, confirmed.

	Payoff
X	$P(X) - 1$
$-X$	$P(X)$

	Payoff
X	$-7/10$
$-X$	$3/10$

	Payoff
Y	$P(Y) - 1 = -8/10$
$-Y$	$P(Y) = 2/10$

In this problem:

Third bet on disjunction: always flip the minus sign in decision matrix

	Payoff
$X \vee Y$	$1 - P(X \vee Y) = 4/10$
$-(X \vee Y)$	$P(-X \vee Y) = -6/10$

Final decision matrix: net payoff.

	Payoff
$X\&Y$	$-11/10$
$X\& -Y$	$-7/10 + 2/10 + 4/10 = -1/10$
$-X\&Y$	$3/10 - 8/10 + 4/10 = -1/10$
$-X\& -Y$	$3/10 + 2/10 - 6/10 = -1/10$

Need general addition rule

$$P(X \vee Y) = P(X) + P(Y) - P(X\&Y)$$