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Article in ARABIAN JOURNAL FOR SCIENCE AND ENGINEERING · May 2017

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Arab J Sci Eng (2017) 42:4079–4098
DOI 10.1007/s13369-017-2573-4

RESEARCH ARTICLE - CIVIL ENGINEERING

Which Fuzzy Ranking Method is Best for Maximizing Fuzzy Net Present Value?

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Received: 14 September 2016 / Accepted: 8 May 2017 / Published online: 23 May 2017

Abstract In this paper, ten fuzzy ranking methods are used to find the importance of the activities with respect to the fuzzy net present value (FNPV) of the project. Each method gives a rank for each activity's cash flow. If these ranks are used as priorities for scheduling activities, thus each method will give different FNPV. In literature, there is a lack of integrated procedure to calculate FNPV in practice, for instance for large projects and when direct cost is given in crisp value. Thus, this paper has twofold objectives. The first objective is to develop an algorithm to maximize FNPV of the project using cash flow weight technique. The second objective is to discover which fuzzy ranking method is best for maximizing FNPV. In the formulation of the algorithm, two scenarios of neglecting and considering inflation rate are adopted in dealing with FNPV. The procedure is applied to an example to show how the algorithm performs. Three case studies are adopted to generalize the results. The main contribution of the paper is that Chu and Tsao (Comput Math Appl 43:111–117, 2002) method is the best method for maximizing FNPV for the adopted scenarios and then Thorani et al. (Int J Contemp Math Sci 7(12):555–573, 2012). The worst methods are Chen and Chen (Appl Intell 26:1–11, 2007, Expert Syst Appl 36:6833–6842, 2009) in case of neglecting and considering inflation, respectively. These methods are consistently the worst depending on a global sensitivity analysis. Among the main findings of the research is extracting an equation for calculating cash flow for large projects.

Keywords Resource scheduling · Fuzzy cash flow · Fuzzy net present value · Fuzzy ranking methods

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1 Introduction

One of the most important goals for project is to get a maximum cash flow from the project. That is why project scheduling with the objective function net present value (NPV) of the project (maximized) is becoming more and more popular. There are several different algorithms proposed in the literature which maximize NPV of the project. These algorithms could be categorized into two groups. The first group assumes crisp values of cash flows. The second group takes the risk of planned cash flows into account by expressing it as random or fuzzy variables. In the current research we will choose fuzzy logic to model uncertain cash flows to enable us to deal with the uncertainty.

Net present value is maximized when positive net cash flow is as close as possible to the start of the project. In constrained resource allocation (resource scheduling) problem, the activities of a project are rearranged such that it achieves the constraint on the resource units. The risk of planned cash flows could be taken into account by expressing it as fuzzy variables. Fuzzy ranking methods are used to find the importance of the activities with respect to the FNPV of the project. Each method gives a rank for each activity's cash flow. If we use these ranks as priorities for scheduling activities, thus each method will give different cash flow pattern and intern different NPV. Accordingly, the effect of ranking methods on NPV should be assessed. On the other hand, there is a lack of integrated procedures to calculate FNPV in a more practical way for large projects. These issues are coupled with other aspects such as; if for instance, the contractor has estimated direct cost in crisp values, how to fuzzify it to take the risk of increasing the cost.

The above situation lends the authors to suggest studying this topic. The paper has twofold objectives. The first objective is to develop an algorithm to maximize FNPV of the project using cash flow weight technique. The second objective is to discover which fuzzy ranking method is best for maximizing FNPV. However, attaining these objectives is of great importance for construction contractors in both adopting integrated procedures to calculate FNPV and applying the method which maximize it. This is because maximizing FNPV leads to high value of profit and improving cash flow pattern for construction projects. Improving these aspects represents the real value of this study.

The rest of the paper is organized as follows. Previous research papers that deal with maximizing net present value of the project in the crisp and probabilistic case are reviewed in the next section. The studies which have fuzzy cash flows are also presented. In Sect. 3, cash flow weight algorithm is described. Fuzzy logic and operation of the fuzzy procedures are presented in Sect. 4. Fuzzy ranking methods are described in Sect. 5. Formulation of the developed algorithm is presented in Sect. 6. The next step is to apply the developed algorithm to an example project to show how the algorithm performs step by step in Sect. 7. To generalize the results three case studies are presented in Sect. 8. Analysis and discussion of the results are conducted in Sect. 9. To examine the consistency of the results, a fuzzy global sensitivity analysis of FNPV is carried out in Sect. 10. The conclusions are drawn in the last section.

2 Literature Review

Elmaghraby and Herroelen [1] presented an elementary approach that maintains the essential simplicity of the problem of maximizing NPV of the project. Yang et al. [2] described an integer programming algorithm for determining scheduled start and finish times for the activities of a project subjected to resource limitations during each period of the schedule duration to maximize the NPV of the project. Sung and Lim [3] proposed a solution procedure for a general resource-constrained project scheduling problem with the objective of NPV maximization. A two-phase heuristic solution algorithm was exploited and tested with various numerical problems for its effectiveness and efficiency. Etgar et al. [4] formulated an optimization model for the problem of scheduling a project to maximize its NPV when net cash flow magnitudes are dependent on the time of their occurrence. In their work, Baroum and Patterson [5] developed heuristic procedures for obtaining improved solutions to the maximization of NPV in a network problem. Pinder and Marucheck [6] evaluated the performance of seventeen scheduling heuristics separately on maximization of project NPV and minimization of project duration. Neumann and Zimmermann [7] presented exact heuristic procedures for resource leveling and NPV problems. A heuristic procedure for several different types of objective functions was also prescribed. Vanhoucke et al. [8] developed an exact branch and bound procedure for the unconstrained project scheduling problem with discounted cash flows. The net cash flows were assumed to be dependent on the completion times of the corresponding activities to maximize NPV of the project subjected to precedence constraints and a fixed deadline. Mika et al. [9] presented a multi-mode resource-constrained project scheduling problem with discounted cash flows. Waligora [10] adopted a discrete–continuous project scheduling problem with positive discounted cash flows for the maximization of the NPV. Hartmann and Briskorn [11] summarized and classified publications on various variants and generalizations of the resource-constrained project scheduling problem. Among the most popular extensions are multiple modes, generalized time lags, and objectives based on the NPV. Tantisuvanichkul [12] proposed a new rule called modified cumulative cash flow (m-CCF) with improved performance from the existing one. The m-CCF is embedded in serial and parallel schedule generation schemes and is extended by implementing in a forward and backward strategy. The main findings of the research discovered that the m-CCF is beneficial to utilize forward–backward solution for scheduling improvement and selecting the schedule with the largest NPV among those available.

On the other hand, probability theory on project scheduling was adopted in some works to maximize NPV of a project. As an example, Yang et al. [13] evaluated nine stochastic scheduling rules for maximizing NPV of a project with probabilistic cash flow. Eight of these rules are extensions of corresponding single pass heuristic scheduling rules, while the ninth rule uses the process of simulated annealing. Sobel et al. [14] formulated an algorithm to identify an optimal adaptive policy to schedule a project with stochastic activity's durations. The objective is to maximize the expected present value of the project cash flow. In their work, Creemers et al. [15] adopted a continuous-time Markov decision chain on project scheduling with NPV objective and exponential activity's durations. Wiesemann et al. [16] proposed a model for maximizing the expected NPV of a project under uncertainty. The activity durations and cash flows are described by a discrete set of alternative scenarios with associated occurrence probabilities. Shaffie and Jaaman [17] proposed the employment of Monte Carlo method in the NPV model in order to achieve reliable cash flow estimation. Monte Carlo provides the risk analysis which can be adopted by investors in making capital budgeting decisions.

Another group of studies on resource-constrained project scheduling to maximize NPV which posses fuzzy cash flows has been found in the literature as follows. Examples of these studies are as follows: Kuchta [18] proposed fuzzy equivalents for all the methods of evaluating and comparing investment projects which are usually presented in the literature and used in the practice. These fuzzy equivalents evaluate projects whose cash flows and/or duration are given in the form of fuzzy numbers. Chui and Park [19] proposed an engineering economic decision model in which the uncertain cash flows and discount rates are specified as triangular fuzzy numbers.

Melik [20] presented a realistic, reliable and cost-schedule integrated cash flow modeling technique by using fuzzy set theory. The idea behind using this technique is to include the uncertainties in project cost and schedule resulting from complex and ambiguous nature of construction works. In their work, Ucal and Kuchta [21] presented two different heuristic methods for project scheduling to maximize FNPV of a project. In their application, they found that the schedules resulting from cash flow weight and discounted cash flow weight heuristics are different which make differences on project's FNPV. They advocated that the ranking method chosen for the ranking could change FNPV and realization time of the project. Accordingly, in the current research, the authors will adopt different fuzzy ranking methods in the developed algorithm to identify which one is best for maximizing FNPV. Ho and Liao [22] proposed a fuzzy binomial approach that can be used in project valuation under uncertainty. This approach revealed the value of flexibilities embedded in the project. Also, they provided a method to compute the mean value of a project's fuzzy expanded NPV that represents the entire value of project. Lu [23] developed a FNPV method by taking vague cash flow and imprecise required rate of return into account for evaluating the value of the build–operate–transfer (BOT) sport facilities. He extended the classical NPV method by constructing an easy-to-understand and more realistic FNPV method without losing the essence of original capital budgeting decision. Tsao [24] adopted the fuzzy concept and concentrated on developing practical novel computational algorithms for calculating the net present values of capital investments. The net present value was calculated in an environment, which is subject to uncertainty from randomness of outcomes and vagueness of estimation. Novel equations for calculating fuzzy variance and standard deviation of FNPV were developed. In their work, Nosratpoura et al. [25] proposed a model that considers all expenditures and revenues in terms of triangular fuzzy numbers and based on fuzzy theory. The model calculates the fuzzy NPV. Sarı and Kuchta [26] proposed a fuzzy global sensitivity analysis method for FNPV to determine the influences of the factors on the worth of an investment project. The global importance values of fuzzy cash flows for FNPV of a project were formulated by defining cash flows and interest rates as fuzzy numbers and the variance of the fuzzy numbers by their crisp equivalences. Kumar and Bajaj [27] proposed a new model to calculate NPV under intuitionistic fuzzy (IF) environment. They also developed two different methods for project scheduling to maximize IFNPV of investment project. Further, they concluded that resulting NPV from these methods are different which cause impact on investment project. Dinagar and Kamalanathan [28] proposed a new ranking procedure called "shortest distance of the point of intersection of legs of trapezium." They proposed two different heuristic methods for project scheduling to maximize FNPV of a project, these are: fuzzy cash flow weight algorithm and fuzzy discounted cash flow algorithm. In their work, they applied generalized trapezoidal fuzzy numbers to the network scheduling in maximizing FNPV for which [21] had worked on triangular fuzzy numbers.

3 Cash Flow Weight Algorithm

Ucal and Kuchta [21] assumed that if a project with activities ($A_i, i = 1, 2, \ldots, N$), the resources required by activities denoted by r_{ik} ($i = 1, 2, \ldots, N$ and $k = 1, 2, \ldots, m$), the durations of the activities denoted by di ($i = 1, 2, \ldots, N$), net cash flows of activities occur at the beginning or end of the related activity and it's value is independent of the starting or ending moment of the activity, the sum of all the cash flows from different activities starting or finishing in moment j will be denoted as CF_j ($j = 1, 2, \ldots, T^{\mathrm{H}}$) where T^{H} denotes time horizon), then present value (PV) of a single future payment occurred in the end of nth year from now is given in (1) where F stands for amount of the payment and r denotes the interest rate (cost of capital).

$$\mathrm{PV} = \frac{F}{(1+r)^n} \tag{1}$$

The goal is to find a schedule with a maximal NPV which is the sum of all discounted cash flows formulated as in Eq. 2:

$$\mathrm{NPV} = \sum_{j=0}^{n} \frac{\mathrm{CF}_j}{(1+r)^j} \tag{2}$$

where CF_j denotes cash flow

Cash flow weight (CFW) heuristic is a heuristic which dynamically selects a high priority activity from available activities for the assignment of resources. In the considered heuristic procedure, the priority of an activity is linked to the cash flows linked to every activity and all the activities which follow it. The priority is measured by means of cash flow weighting [5].

The idea of cash flow weight (CFW) heuristic procedures is selecting a high priority activity from a list of available activities for the assignment of resource [5]. This is because, the higher NPV is achieved by advancing positive cash flows as close to the start of the project as possible, while delaying negative payment as far back (to the right) as possible, while not moving any activities on the critical path and still satisfying precedence and resource constraint. It must be noted if resource conflicts arise, the activity that is holding back a greater sum of cash inflows is scheduled first or receives higher priority in the assignment of resources.

However, cash flow weight procedure consists of three steps. In the first step, the cash flow weights of each activity are determined and all activities are included to the list of available activities in an order of $i\,(i = 1, 2, \ldots, N)$ without taking into account the predecessors. In the second step, the activity with the highest CFW is selected from the list of available activities. The activity with the lowest number is assigned first if a tie is exist. In order to assign the selected activity as soon as possible, the predecessors of the selected activity are assigned respectively in the increasing order of their indices $i\ (i = 1, 2, \ldots, N)$ and as soon as possible with respect to the resources available. The available resources are updated after assignment of the selected activity. In the third step, if there is any unassigned activity, then second step is repeated, otherwise the project schedule is completed [5].

4 Fuzzy Logic and Operation of the Fuzzy Procedures

Fuzzy set theory was first founded by [29]. It has become an important tool for modeling the uncertainty. There is a general definition of fuzzy numbers but usually their simplest form, triangular fuzzy numbers (TFN) are preferred to simplify the calculations. A TFN has linear membership (possibility) functions both on the left and right sides. The membership function of TFN is given by Eq. 3.

$$\mu(x) = \begin{cases} \frac{x-M_{\mathrm{l}}}{M_{\mathrm{m}}}, & M_{\mathrm{l}} \le x \le M_{\mathrm{m}} \\ \frac{M_{\mathrm{r}}-x}{M_{\mathrm{r}}-M_{\mathrm{m}}}, & if \quad M_{\mathrm{m}} \le x \le M_{\mathrm{r}} \\ 0, & \text{otherwise} \end{cases} \tag{3}$$

where M_{l} and M_{r} represents the smallest and greatest values of fuzzy number M. M_{m} is the mean or the mode of the fuzzy number. The difference interval $(M_{\mathrm{l}},\ M_{\mathrm{r}})$ is called support of the fuzzy number M. Algebraic operations for TFNs are given by Eqs. 4–10 where all the fuzzy numbers are positive (here it is assumed to mean $M_{\mathrm{l}}, \geq 0$, $N_{\mathrm{l}} \geq 0$ [30]:

$$(M_{\mathrm{l}}, M_{\mathrm{m}}, M_{\mathrm{r}}) + (N_{\mathrm{l}}, N_{\mathrm{m}}, N_{\mathrm{r}}) \cong (M_{\mathrm{l}} + N_{\mathrm{l}}, M_{\mathrm{m}} + N_{\mathrm{m}}, M_{\mathrm{r}} + N_{\mathrm{r}}) \tag{4}$$

$$(M_{\mathrm{l}}, M_{\mathrm{m}}, M_{\mathrm{r}}) - (N_{\mathrm{l}}, N_{\mathrm{m}}, N_{\mathrm{r}}) \cong (M_{\mathrm{l}} - N_{\mathrm{r}}, M_{\mathrm{m}} - N_{\mathrm{m}}, M_{\mathrm{r}} - N_{\mathrm{l}}) \tag{5}$$

$$(M_{\mathrm{l}}, M_{\mathrm{m}}, M_{\mathrm{r}}) \times (N_{\mathrm{l}}, N_{\mathrm{m}}, N_{\mathrm{r}}) \cong (M_{\mathrm{l}}N_{\mathrm{l}}, M_{\mathrm{m}}N_{\mathrm{m}}, M_{\mathrm{r}}N_{\mathrm{r}}) \tag{6}$$

$$(M_{\mathrm{l}}, M_{\mathrm{m}}, M_{\mathrm{r}}) \div (N_{\mathrm{l}}, N_{\mathrm{m}}, N_{\mathrm{r}}) \cong \left(\frac{M_{\mathrm{l}}}{M_{\mathrm{r}}}, \frac{M_{\mathrm{m}}}{M_{\mathrm{m}}}, \frac{M_{\mathrm{r}}}{M_{\mathrm{l}}}\right) \tag{7}$$

$$\lambda \times (M_{\mathrm{l}}, M_{\mathrm{m}}, M_{\mathrm{r}}) \cong \begin{cases} (\lambda M_{\mathrm{l}}, \lambda M_{\mathrm{m}}, \lambda M_{\mathrm{r}}), \ \lambda \ge 0 \\ \qquad\qquad if \quad , \forall \lambda \in R \\ (\lambda M_{\mathrm{r}}, \lambda M_{\mathrm{m}}, \lambda M_{\mathrm{l}}), \ \lambda \le 0 \end{cases} \tag{8}$$

$$\lambda \div (M_{\mathrm{l}}, M_{\mathrm{m}}, M_{\mathrm{r}}) \cong \begin{cases} \left(\frac{\lambda}{M_{\mathrm{r}}}, \frac{\lambda}{M_{\mathrm{m}}}, \frac{\lambda}{M_{\mathrm{l}}}\right), \ \lambda \ge 0 \\ \qquad\qquad if \quad , \forall \lambda \in R \\ \left(\frac{\lambda}{M_{\mathrm{l}}}, \frac{\lambda}{M_{\mathrm{m}}}, \frac{\lambda}{M_{\mathrm{r}}}\right), \ \lambda \le 0 \end{cases} \tag{9}$$

$$(M_{\mathrm{l}}, M_{\mathrm{m}}, M_{\mathrm{r}})^{\lambda} \cong \begin{cases} \left(M_{\mathrm{l}}^{\lambda}, M_{\mathrm{m}}^{\lambda}, M_{\mathrm{r}}^{\lambda}\right), \quad \lambda \ge 0 \\ \qquad\qquad if \quad , \forall \lambda \in R \\ \left(\frac{1}{M_{\mathrm{r}}{}^{\lambda}}, \frac{1}{M_{\mathrm{m}}{}^{\lambda}}, \frac{1}{M_{\mathrm{l}}{}^{\lambda}}\right), \ \lambda \le 0 \end{cases} \tag{10}$$

A negative fuzzy number is a positive fuzzy number multiplied by -1.

For the purpose of the current research, triangular fuzzy numbers are only applied for their simplicity. Also, to focus the scope of the research, only normal numbers are dealt with because their membership function is equal to 1 [31].

5 Fuzzy Ranking Methods

Ranking fuzzy numbers is usually used in decision making, data analysis, artificial intelligence, economic systems and operation research [32]. In a fuzzy environment, ranking is a very important decision making procedure. In order to rank fuzzy numbers, one fuzzy number needs to be evaluated and compared to the other, but this may not be easy. Since fuzzy numbers are represented by possibility distributions, they can overlap with each other and, thus, it is difficult to determine clearly whether one fuzzy number is larger or smaller than other [32]. In recent years, many methods have been proposed for ranking different types of fuzzy numbers and can be classified into four major classes: preference relation, fuzzy mean and spread, fuzzy scoring, and linguistic expression, but each method appears to have advantages as well as disadvantages

[30]. One of the most commonly used methods under the class of fuzzy scoring is the centroid point method. Therefore, in this research, the centroid point methods in ranking fuzzy numbers are only applied.

Yager [33] was the first researcher to propose a centroid index ranking method to calculate the value of fuzzy number A. Yager [33] made no assumption on the normality and on the convexity of the fuzzy number. Chen and Chen [34] proposed an approach for ranking generalized trapezoidal fuzzy numbers based on centroid point and standard deviation to overcome the drawbacks of [33] approach.

Cheng [35] used a centroid-based distance approach to rank fuzzy numbers. For a trapezoidal fuzzy number $A = (a, b, c, d; w)$, the larger the value of ranking ($R(A)$), the better the ranking will be of A. For trapezoidal fuzzy numbers Eqs. 11, 12 and 13 can be used.

$$\bar{x}_A = \frac{w(d^2 - 2c^2 + 2b^2 - a^2 + dc - ab) + 3(c^2 - b^2)}{3w(d - c + b - a) + 6(c - b)} \tag{11}$$

$$\bar{y}_A = \frac{w}{3}\left[1 + \frac{(b + c) - (a + d)(1 - w)}{(b + c - a - d) + 2(a + d)w}\right] \tag{12}$$

$$R\left(\bar{A}\right) = \sqrt{\bar{x}_A^2 + \bar{y}_A^2} \tag{13}$$

In the case of a triangular fuzzy number $A = (a, b, d; w)$, $c = b$ and for normal fuzzy number $w = 1$. Therefore, for a triangular normal fuzzy number A, Eqs. 14 and 15 can be used instead of Eqs. 11 and 12, respectively.

$$\bar{x}_A = \frac{\left(d^2 - a^2 + db - ab\right)}{3\,(d - a)} \tag{14}$$

$$\bar{y}_A = \frac{1}{3}\left[1 + \frac{2b}{2b + a + d}\right] \tag{15}$$

Chu and Tsao [36] proposed a new ranking index function (S_A).

$$S_A = \bar{x}_A \times \bar{y}_A \tag{16}$$

The larger the value is of S_A, the better the ranking will be of A. For triangular normal fuzzy number, $\bar{x}_A$ and $\bar{y}_A$ are as given by Eqs. 14 and 15, respectively.

Chen and Chen [34] proposed an approach for ranking generalized trapezoidal fuzzy numbers. The ranking value for a generalized trapezoidal fuzzy number $A = (a_{1,} a_2,\ a_3,\ a_4;\ w)$ is defined as

$$\text{Rank}\,(A) = x + (-y)^{S_A}\,(y + 0.5)^{1-w} \tag{17}$$

where

$$y = \frac{w}{6}\left(\frac{a_3 - a_2}{a_4 - a_1} + 2\right)\ for\ a_1 \neq a_4;\ w/2 \tag{18}$$

$$x = \frac{y\,(a_3 + a_2) + (a_4 + a_1)\,(w - y)}{2w} \tag{19}$$

$$S_A = \sqrt{\frac{\sum_{j=1}^{4}\left(a_j - \bar{a}\right)^2}{3}} \tag{20}$$

$$\bar{a} = \frac{a_1 + a_2 + a_3 + a_4}{4} \tag{21}$$

The larger the value of rank(A), the better the ranking of A.

In the case of a triangular normal fuzzy number $A = (a, b, c; w)$, $a_2 = a_3 = b$ and $w = 1$. Equations 22, 23, 24, and 25 are used instead of Eqs. 18, 19, 20, and 21, respectively.

$$y = \frac{1}{3} \tag{22}$$

$$x = \frac{2by + (c + a)\,(1 - y)}{2} \tag{23}$$

$$S_A = \sqrt{\frac{\sum_{j=1}^{3}(a_j - \bar{a})^2}{2}} \tag{24}$$

$$\bar{a} = \frac{a + b + c}{3} \tag{25}$$

Wang et al. [37] presented the following formulas for the trapezoidal fuzzy numbers

$$\tilde{x}_0\left(\tilde{A}\right) = \frac{1}{3}\left[a + b + C + d - \frac{dc - ab}{(d + c) - (a + b)}\right] \tag{26}$$

$$\tilde{y}_0\left(\tilde{A}\right) = w \cdot \frac{1}{3}\left[1 + \frac{c - b}{(d + c) - (a + b)}\right] \tag{27}$$

And the ranking function is

$$R\left(\tilde{A}\right) = \sqrt{\bar{x}_0^2 + \bar{y}_0^2} \tag{28}$$

The larger the value of $R\left(\tilde{A}\right)$ the better the ranking of A.

In the case of a triangular normal fuzzy number $A = (a, b, d; w)$, $c = b$ and $w = 1$, Eqs. 29 and 30 are applied instead of 26 and 27, respectively.

$$\tilde{x}_0\left(\tilde{A}\right) = \frac{1}{3}(a + b + d) \tag{29}$$

$$\tilde{y}_0\left(\tilde{A}\right) = \frac{1}{3} \tag{30}$$

Chen and Chen [38] again indicated the shortcomings of existing centroid methods, for Chu and Tsao's [36], Cheng's [35] approaches. He developed Eq. 31 for calculation of Score (A_i) of a fuzzy number.

$$\text{Score}\,(A_i) = \sqrt{(x_{Ai} - x_{Ai\min})^2 + \left(y_{Ai}^s\right)^2}, \tag{31}$$

where (x_{Ai}) is as defined in Chen and Chen [34] and represented by Eq. 23. y^s_{Ai} given by Eq. 32 and y_{Ai} represented by Eq. 22.

$$y^s_{Ai} = \frac{w_{Ai}}{2} - (y_{Ai} \times s_{Ai}) \tag{32}$$

where

$$S_{Ai} = \sqrt{\frac{\sum_{j=1}^{4} \left(a_{ji} - \bar{a}_i\right)^2}{3}} \tag{33}$$

$$\bar{a}_i = \frac{a_{1i} + a_{2i} + a_{3i} + a_{4i}}{4} \tag{34}$$

The higher the value of Score (Ai), the better the ranking of the fuzzy number Ai.

In the case of a triangular normal fuzzy number $A = (a, b, c; w)$ where $a_2 = a_3 = b$ and $w = 1$, y_{Ai}, x_{Ai}, S_{Ai} and $\bar{a}_i$ are represented by Eqs. 35, 36, 37, and 38, respectively.

$$y_{Ai} = \frac{1}{3} \tag{35}$$

$$x_{Ai} = \frac{2by + (c + a)(1 - y)}{2} \tag{36}$$

$$S_{Ai} = \sqrt{\frac{\sum_{j=1}^{3} (a_j - \bar{a})^2}{2}} \tag{37}$$

$$\bar{a}_i = \frac{a + b + c}{3} \tag{38}$$

Chen and Chen [39] found that the approaches proposed by Chen and Chen [33,38] still have shortcomings. Thus, Chen and Chen [39] proposed an approach for ranking generalized fuzzy numbers with different heights and different spreads. The score value of each standardized generalized fuzzy number $Ai = (a_{1i}, a_{2i}, a_{3i}, a_{4i}; w_{Ai})$ is defined as

$$\text{Score}\,(A) = \frac{(\bar{a}_{Ai} \times w_{Ai})}{(1 + s_{Ai})} \tag{39}$$

where

$$s_{Ai} = \sqrt{\frac{\sum_{j=1}^{4} \left(a_{ij} - \bar{a}_{Ai}\right)^2}{3}} \tag{40}$$

$$\bar{a}_{Ai} = \frac{a_{i1} + a_{i2} + a_{i3} + a_{i4}}{4} \tag{41}$$

The larger the value of Score(A_i), the better the ranking of A_i.

In the case of a triangular normal fuzzy number $A = (a, b, c; w)$, where $a_2 = a_3 = b$ and $w = 1$ Eqs. 40 and 41 become as given by Eqs. 42 and 43 respectively.

$$s_{Ai} = \sqrt{\frac{\sum_{j=1}^{3} (a_j - \bar{a}_{Ai})^2}{2}} \tag{42}$$

$$\bar{a}_{Ai} = \frac{a + b + c}{3} \tag{43}$$

Dat et al. [40] proposed a new ranking method as follows: Suppose $A1, A2, \ldots, An$ are fuzzy numbers. First, the centroid point of all fuzzy numbers are calculated using Wang et al.[37]. Thus Eqs. 29 and 30 of Wang et al. [37] are used first for calculating $\bar{x}_A$ and $\bar{y}_A$, and then the distance between the centroid point, $A_i = (\bar{x}_{Ai}, \bar{y}_{Ai})$, $i = 1, 2, \ldots, n$ and the minimum point $G = (x_{min}, y_{min})$ is as proposed in Eq. 44

$$D(A_i, G) = \sqrt{(\bar{x}_{Ai} - x_{\min})^2 + (\bar{y}_{Ai} - \frac{w}{3} y_{\min})^2} \tag{44}$$

Thus, if A_i, A_j are two fuzzy numbers, then their ranking order is defined as follows:
(1) $A_i \prec A_j$, if $D(A_i, G) \prec D(A_j, G)$; (2) $A_i \succ A_j$, if $D(A_i, G) \succ D(A_j, G)$, and (3) $A_i \sim A_j$, if $D(A_i, G) = D(A_j, G)$.

In their work, Allahviranloo and Saneifard [41] assumed that if there are n fuzzy numbers $A_1, A_2, \ldots, An$, the ranking method for these numbers $A_1, A_2, \ldots, An$ is presented as follows:

Step 1: Use formulas given by Wang et al. [37] to calculate the centroid point $(\bar{x}_{A_j}, \bar{y}_{A_j})$ of each fuzzy number A_j, where $1 \le j \le n$.
Step 2: Calculate the maximum crisp value $\tau_{\max}$ of all fuzzy numbers A_j, where $1 \le j \le n$.
Step 3: Use the point $(\bar{x}_{A_j}, \bar{y}_{A_j})$ to calculate the ranking value $Dist\,(A_j)$ of fuzzy numbers A_j, where $1 \le j \le n$, as in this equation

$$\begin{aligned} Dist(Aj) &= \sqrt{(\bar{x}_{A_j} - \tau_{\max})^2 + (\bar{y}_{A_j} - 0)^2} \\ &= \sqrt{(\bar{x}_{A_j} - \tau_{\max})^2 + (\bar{y}_{A_j})^2} \end{aligned} \tag{45}$$

$Dist(A_1) \prec Dist(A_2)$ if and only if $A_1 \succ A_2$; (2) $Dist(A_1) \succ Dist(A_2)$ if and only if $A_1 \prec A_2$ and (3) $\text{Dist}(A_1) = \text{Dist}(A_2)$ if and only if $A_1 \sim A_2$.

Thorani et al. [31] gave that the generalized trapezoidal fuzzy number $A = (a, b, c, d; w)$ is defined as:

$$I_{\tilde{A}}(\bar{x}_0, \bar{y}_0) = \left[\frac{\alpha\left(\frac{a+2b}{3}\right) + \beta\left(\frac{b+C}{2}\right) + \gamma\left(\frac{2C+d}{3}\right)}{\alpha + \beta + \gamma}, \frac{\alpha\left(\frac{w}{3}\right) + \beta\left(\frac{w}{2}\right) + \gamma\left(\frac{w}{3}\right)}{\alpha + \beta + \gamma}\right) \tag{46}$$

where

$$\alpha = \frac{\sqrt{(c - 3b + 2d)^2 + w^2}}{6} \tag{47}$$

$$\beta = \frac{\sqrt{(2c + d - a - 2b)^2}}{3} \tag{48}$$

$$\gamma = \frac{\sqrt{(3c-2a-b)^2+w^2}}{6} \tag{49}$$

As a special case, for triangular normal fuzzy number $\tilde{A} = (a, b, d; 1)$,

i.e., $c = b$ the centre of centroids is given by Eq. 50

$$I_{\tilde{A}}(\bar{x}_0, \bar{y}_0) = \left[\frac{x\left(\frac{a+2b}{3}\right) + yb + z\left(\frac{2b+d}{3}\right)}{\alpha+\beta+\gamma}, \frac{x\left(\frac{1}{3}\right) + y\left(\frac{1}{2}\right) + z\left(\frac{1}{3}\right)}{x+y+z}\right) \tag{50}$$

where

$$x = \frac{\sqrt{(2d-2b)^2+1^2}}{6} \tag{51}$$

$$y = \frac{\sqrt{(d-a)^2}}{3} \tag{52}$$

$$z = \frac{\sqrt{(2b-2a)^2+1^2}}{6} \tag{53}$$

The ranking function of fuzzy number is

$$R\left(\tilde{A}\right) = \bar{x}_0 \times \bar{y}_0 \tag{54}$$

The larger the value is of $R\left(\tilde{A}\right)$ the better the ranking will be of A.

Gani and Mohamed [42] gave that the centroid point of a trapezoid is considered it is balancing point. Divide the trapezoid into three plane figures (two triangles and a rectangle). Consider a generalized trapezoidal fuzzy number $A = (a, b, c, d : w)$. The centroid of the first triangle is $G_1 = ((a + 2b)/3, w/3)$, the centroid of the rectangle is $G_2 = ((b + c)/2, w/2)$, and the centroid of the second triangle is $G_3 = ((2c + d)/3, w/3)$. Thus, they concluded that the centroid $G_A(\bar{x}_0, \bar{y}_0)$ of the triangle with vertices G1, G2, and G3 of the generalized trapezoidal fuzzy number $A = (a, b, c, d : w)$ as $G_A(\bar{x}_0, \bar{y}_0) = \left(\frac{2a+7b+7c+2d}{18}\right), \left(\frac{7w}{18}\right)$.

The ranking function of the generalized trapezoidal fuzzy number $A = (a, b, c, d : w)$ which maps the set of all fuzzy numbers to a set of real numbers is defined as in Eq. 55.

$$R(A) = \bar{x}_0 \,.\, \bar{y}_0 = \left(\frac{2a+7b+7c+2d}{18}\right)\left(\frac{7w}{18}\right) \tag{55}$$

The larger the value is of R_A, the better the ranking will be of A.

In the case of a triangular normal fuzzy number $A = (a, b, d; 1)$, $c = b$

$$x = \frac{2a+14b+2d}{18} \tag{56}$$

$$Y = \frac{7}{18} \tag{57}$$

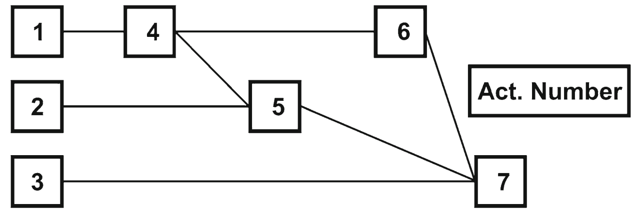


Fig. 1 Assumed logic to show calculating fuzzy cash flow weight

6 Formulation of the Developed Algorithm

The developed algorithm consists of four steps as follows:

Step 1: fuzzy cash flow weight is calculated

Two cases are dealt with in calculating fuzzy cash flow weight. The first case when fuzzy cash flows are given directly and we proceed as follows. If, for example, we have the logic given in Fig. 1, the calculation of cash flow weight for activities 1 and 4 for example are as follows:

$$\mathrm{CFW}_1 = \mathrm{CF}_1 + \mathrm{CF}_4 + \mathrm{CF}_5 + \mathrm{CF}_6 + \mathrm{CF}_7 \tag{58}$$

$$\mathrm{CFW}_4 = \mathrm{CF}_4 + \mathrm{CF}_5 + \mathrm{CF}_6 + \mathrm{CF}_7 \tag{59}$$

For a real project with large number of activities the calculations of cash flow weight will be complicated. Therefore, an equation for calculating cash flow weight is established as follows.

If we modify Eq. 59 to be as given in Eq. 60

$$\mathrm{CFW}_4 = \mathrm{CF}_4 + \mathrm{CF}_5 + \mathrm{CF}_6 + \mathrm{CF}_7 + \mathrm{CF}_7 - \mathrm{CF}_7 \tag{60}$$

Since,

$$\mathrm{CFW}_5 = \mathrm{CF}_5 + \mathrm{CFW}_7 = \mathrm{CF}_5 + \mathrm{CF}_7 \tag{61}$$

$$\mathrm{CFW}_6 = \mathrm{CF}_6 + \mathrm{CFW}_7 = \mathrm{CF}_6 + \mathrm{CF}_7 \tag{62}$$

Substitute from Eqs. 61 and 62 into Eq. 60 result in Eq. 63.

$$\mathrm{CFW}_4 = \mathrm{CF}_4 + \mathrm{CFW}_5 + \mathrm{CFW}_6 - \mathrm{CF7} \tag{63}$$

Activities 5 and 6 are immediate successors to activity 4. Therefore, for any activity i with successor j, we can express about CFW_i with its equivalent value as in Eq. 64. Equation 64 will be used by (Microsoft Excel) to estimate CFW for the case studies presented next.

$$\mathrm{CFW}_i = \mathrm{CF}_i + \sum_{j=1}^{n} \mathrm{CFW}_j - \mathrm{CF}_D \tag{64}$$

where n is the number of successor activities to activity i and $\mathrm{CF_D}$ is duplicated cash flow.

The second case when fuzzy cash flows are not given directly. If the direct cost is estimated and given in crisp value, it could be fuzzified using a random function applied from Excel software. The limits of this function could be determined depending on the reported in [43] for accuracy of direct cost. In [43], for 100% complete tender documents (at tender stage) the direct cost estimate variance ranges from ±5% to ±10. In the current research, the maximum limits will be adopted and a range of variation for direct cost is assumed from 90 to 110%. On the other hand, indirect cost consists of site overhead and head office overhead. [44] reported that site overhead may vary from 5 to 15% of project's direct cost. He added that head office overhead may vary from 2 to 5% of project's direct cost. This is implies that, the range of variation in indirect cost ranges from 7 to 20%. Accordingly, in this research the adopted fuzzy indirect cost is (7, 13.5, 20%) of direct cost.

On the other hand, markup consists of profit margin and risk allowance. The margin of profit includes both the contractor's profit and the cost of finance for his investment in the contract. [45] demonstrated that a typical construction contractor operates with 2% of project's direct cost as a margin of profit and when he uses an effective competitive bidding strategy he can earn 4% of project's direct cost. Therefore, the maximum expected variation in the margin of profit between contractors for a specific project is 2% of it's direct cost. A real assessment of risk allowance for a specific project requires both sensitivity and probability analyses. For the purpose of this study the authors assume that risk allowance ranges from 1 to 3% of project's direct cost. Thus, the range of variation in markup is from 3 to 7%. of project's direct cost will be adopted. Therefore, the proposed fuzzy markup is (3, 5, 7%) of direct cost.

Step 2: fuzzy ranking method is applied

In each method each activity is ranked according to cash flow weight. According to this rank, the priority for each activity is determined, such that the activity with the highest rank will be assigned the highest priority for all ranking methods except Allahviranloo and Saneifard [41] method. In this method the activity with the least distance will be assigned the highest priority.

Step 3: ***A resource scheduling procedure is performed based on activities priorities resulted from step 2 for each ranking method using software program*** (**Microsoft project in our research**).

Step 4: ***Fuzzy net present value is calculated for all the activities in the project according to each schedule resulted from step 3***.

In this research two scenarios for dealing with inflation are adopted. In the first scenario the inflation has been neglected, while in the second one the inflation has been considered. Equations 65 and 66 are applied for calculating FNPV_{S1} and FNPV_{S2} for the first and second scenarios respectively. Best ranking method is then determined according to the highest value of FNPV in each scenario.

$$\mathrm{FNPV}_{S1} = \frac{F}{(1+r)^n} \tag{65}$$

$$\mathrm{FNPV}_{S2} = \frac{F}{(1+m)^n} \tag{66}$$

where F stands for amount of the future payment occurred in the end of nth year from now, r denotes the interest rate, and m denotes inflated interest rate and represented by Eq. 67, where t denotes the inflation rate.

$$m = \tilde{r} + \tilde{t} + \tilde{r} \times \tilde{t} \tag{67}$$

In Egypt the inflation rate was 14% in 2014, and it is anticipated that it is value will be about 7.5% in 2020 [46] as cited at 8/2016. Thus, we adopt a range of variation for inflation from 7.5 to 14%. To express the inflation rate as a fuzzy number $\tilde{t} = (0.075,\ 0.105,\ 0.14)$ will be adopted.

To show how the algorithm performs, an example project is solved step by step in the next section.

7 Example Project

The data of this example were obtained from [21]. The logic shown in Fig. 2 was developed by the authors. In this figure, the fuzzy cash flows occurred at the beginning of each activity. Activities, fuzzy cash flow, immediate predecessors, durations, and resource requirements for each activity are shown in this figure. The project has just one type of resource which is limited to 5 over the project realization time. In this example, an annual fuzzy interest rate $r = (0.08, 0.10, 0.12)$ was used and an annual inflation rate $t = (0.07, 0.105, 0.14)$ was applied.

Step 1: Fuzzy cash flow weight calculation

The calculations of CFW for activities 1 and 4 shown in Fig. 2 are given below as examples:

$$\begin{aligned}
\mathrm{CFW}_1 &= \mathrm{CF}_1 + \mathrm{CF}_4 + \mathrm{CF}_5 + \mathrm{CF}_6 + \mathrm{CF}_7 \\
&= (40, 50, 60) + (-36, -30, -24) + (37, 45, 53) \\
&\quad + (35, 50, 65) + (2, 10, 18) \\
&= (78, 125, 172) \\
\mathrm{CFW}_4 &= \mathrm{CF}_4 + \mathrm{CF}_5 + \mathrm{CF}_6 + \mathrm{CF}_7 \\
&= (-36, -30, -24) + (37, 45, 53) + (35, 50, 65) \\
&\quad + (2, 10, 18) = (38, 75, 112)
\end{aligned}$$

Applying Eqs. 63 or 64 for calculating CFW_4 since CF_7 is duplicated in CFW_5 and CFW_6. Therefore, when we calculate CFW_4 we must subtract CF_7.

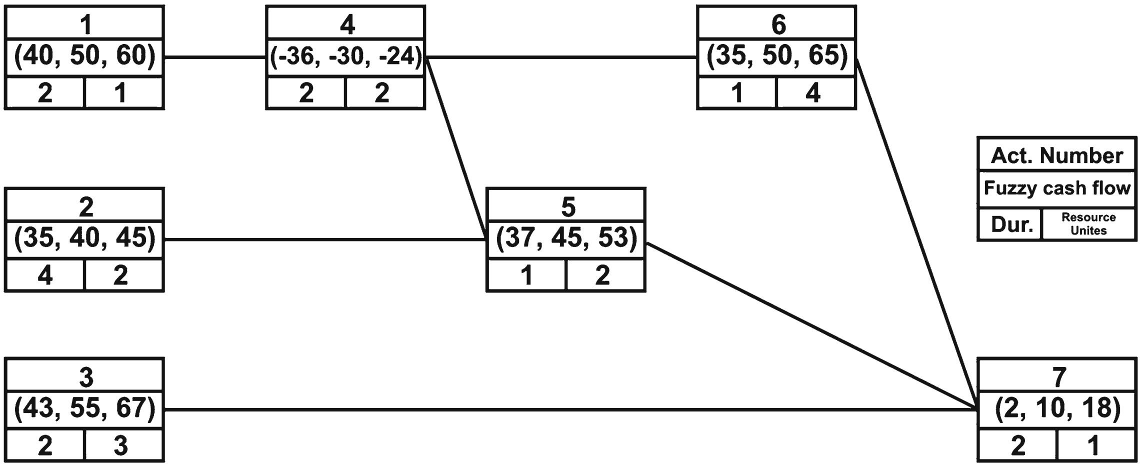


Fig. 2 Logic and data of example project

Table 1 Fuzzy cash flow weights for activities of example project

Activity	Cash			Successor 1	Successor 2	Duplication	Cash flow weight		
	Low	Medium	High				Low	Medium	High
1	40	50	60	4			78	125	172
2	35	40	45	5			74	95	116
3	43	55	67	7			45	65	85
4	−36	−30	−24	5	6	7	38	75	112
5	37	45	53	7			39	55	71
6	35	50	65	7			37	60	83
7	2	10	18				2	10	18

$$\begin{aligned}\mathrm{CFW}_4 &= \mathrm{CF}_4 + \mathrm{CFW}_5 + \mathrm{CFW}_6 - \mathrm{CF7}\\ &= (-36, -30, -24) + (39, 55, 71) + (37, 60, 83)\\ &\quad - (2, 10, 18) = (38, 75, 112)\end{aligned}$$

Using Microsoft Excel, fuzzy cash flow weights were calculated for all activities as shown in Table 1. From this table it can be shown that Eq. 64 is applied for activities have two successors.

Step 2: Fuzzy ranking methods

Ranking for each activity is calculated in the example for each fuzzy ranking method. Calculation the ranking of activity 1 is shown for example for all presented ranking methods. Fuzzy cash flow weight of activity 1 as calculated in step 1 is (78, 125, 172).

1. For Cheng [35], the rank value for cash flow weight is calculated as follows: Apply Eqs. 14, 15 and 13 to calculate $\bar{x}_A$, $\bar{y}_A$ and $R\left(\tilde{A}\right)$, respectively.

$$\begin{aligned}\bar{x}_A &= \frac{\left(d^2 - a^2 + db - ab\right)}{3\,(d-a)}\\ &= \frac{\left(172^2 - 78^2 + 172 * 125 - 78 * 125\right)}{3\,(172-78)} = 125\end{aligned}$$

$$\begin{aligned}\bar{y}_A &= \frac{1}{3}\left[1 + \frac{2b}{2b+a+d}\right]\\ &= \frac{1}{3}\left(1 + \frac{2 * 125}{2 * 125 + 78 + 172}\right) = \frac{1}{2}\end{aligned}$$

$$\begin{aligned}R\left(\tilde{A}\right) &= \sqrt{\bar{x}_0^2 + \bar{y}_0^2} = R\left(\tilde{A}\right)\\ &= \sqrt{125^2 + (1/2)^2} = 125\end{aligned}$$

According to the ranks, the priorities of activities have been established such that the activity with the highest rank is assigned the highest priority (see Table 2).

2. For Chu and Tsao [36], the rank is calculated as follows: Apply Eqs. 14 and 15 to calculate $\bar{x}_A$ and $\bar{y}_A$ as previously in Cheng [35]. These values are 125 and 0.5 as previously determined. Then determine $R\left(\tilde{1}\right)$ using Eq. 16.

$$R\left(\tilde{1}\right) = 125 \times 0.5 = 62.5$$

3. For Chen and Chen [34], Eqs. 22, 23, 25, 24 and 17 are applied for calculating y, x, $\bar{a}$, S and R, respectively, as follows:

Table 2 Ranks and priorities calculated for activities of example project using Cheng [35] method

Activity	1	2	3	4	5	6	7
Rank	125	95.001	65.002	75.002	55.002	60.002	10.012
Priority	7	6	4	5	2	3	1

$$y = \frac{1}{3} \quad x = \frac{2*125*1/3 + (172+78)(1-1/3)}{2*1} = 125$$
$$\bar{a} = \frac{a+b+c}{3} = \frac{78+125+172}{3} = 125$$
$$S = \sqrt{\frac{\sum_{j=1}^{3}(a_j-\bar{a})^2}{2}} = \sqrt{\frac{(78-125)^2+(125-125)^2+(172-125)^2}{2}} = 47$$

$$\text{Rank}(1) = 125 + (1-1/3)^{47}\left(\tfrac{1}{3}+0.5\right)^{(1-1)} = 125$$

4. For Wang et al. [37], Eqs. 29, 30, and 28 are used for calculating x, y and R respectively as follows:

$$x = \frac{1}{3}(a+b+d) = \frac{78+125+172}{3} = 125 \quad y = \frac{1}{3}$$
$$R(1) = \sqrt{125^2+(1/3)^2} = 125$$

5. For Chen and Chen [38], Eqs. 22 and 23 are firstly applied to calculate y and x. Then Eqs. 38, 37, 32 and 31 for determining $\bar{a}_i$, S_{Ai}, y^s_{Ai} and Score (A_i), respectively.

$$y = \frac{1}{3} \quad x = \frac{2by+(c+a)(w-y)}{2w} = \frac{2*125*1/3+(172+78)(1-1/3)}{2*1} = 125$$
$$\bar{a} = \frac{a+b+c}{3} = \frac{78+125+172}{3} = 125$$
$$S = \sqrt{\frac{\sum_{j=1}^{3}(a_j-\bar{a})^2}{2}} = \sqrt{\frac{(78-125)^2+(125-125)^2+(172-125)^2}{2}} = 47$$
$$y^s = 1/2 - (1/3*47) = -15.167$$

It must be noted that x is determined for all activities using Eq. 36. Then, the minimum value is picked $x_{Aimin} = 10$ and is used in Eq. 31.

$$\text{Score}(1) = \sqrt{(125-10)^2+(-15.167)^2} = 115.996$$

6. For Chen and Chen [39], Eqs. 43 and 42 are applied to calculate $\bar{a}_{A1}$ and S_1, for activity number 1 respectively. Then calculate Score (1) using Eq. 39.

$$\bar{a}_{A1} = \frac{a+b+c}{3} = \frac{78+125+172}{3} = 125$$
$$S = \sqrt{\frac{\sum_{j=1}^{3}(a_j-x)^2}{2}} = \sqrt{\frac{(78-125)^2+(125-125)^2+(172-125)^2}{2}} = 47$$
$$\text{Score}(1) = (125*1)/(1+47) = 2.604$$

7. For Dat et al. [40], Eqs. 29 and 30 of Wang et al. [37] are used first for calculating $\bar{x}_1$ and $\bar{y}_1$, and then, Eq. 44 is used to calculate $D(A_i, G)$. As given previously in Chen and Chen [38], $x_{\min} = 10$

$$\bar{x}_1 = \frac{1}{3}(a+b+d) = \frac{78+125+172}{3} = 125$$
$$\bar{y}_1 = \frac{1}{3}$$
$$D(1,G) = \sqrt{(125-10)^2+\left(\frac{1}{3}-\frac{1}{3}*\frac{1}{3}\right)^2} = 115$$

8. For Allahviranloo and Saneifard [41], Eqs. 29 and 30 given by Wang et al. [37] are used first for calculating the centroid point $(\bar{x}_1, \bar{y}_1)$. Then, the maximum crisp value of all fuzzy cash flow weights $\tau_{\max}$ is determined. Finally, use Eq. 45 to calculate *Dist* (A_j)

$$\bar{x}_1 = \frac{1}{3}(a+b+d) = \frac{78+125+172}{3} = 125, \quad \bar{y}_1 = \frac{1}{3}$$
$$\tau_{\max} = 172 \text{(the highest value of cash flow weights for all activities)}$$
$$\text{Dist}(1) = \sqrt{(125-172)^2+\left(\frac{1}{3}\right)^2} = 47.001$$

9. For Thorani et al. [31], Eqs. 51, 52, and 53 are used for calculating x, y, and z, respectively. Then, Eq. 50 is applied to determine $I_{\tilde{1}}\ (\bar{x}_0\ and\ \bar{y}_0)$. Equation 54 is adopted to calculate $R\left(\tilde{1}\right)$.

$$x = \frac{\sqrt{(2*172-2*125)^2+1^2}}{6} = 15.668$$
$$y = \frac{\sqrt{(172-78)^2}}{3} = 31.333$$
$$z = \frac{\sqrt{(2*125-2*78)^2+1^2}}{6} = 15.668$$
$$x_0 = \frac{15.668*\frac{78+2*125}{3}+31.333*125+15.668*\frac{2*125+172}{3}}{15.668+31.333+15.668} = 125$$

Table 3 Priorities for all activities for used ranking methods

Act.	Cash flow weight			Fuzzy ranking method	
	L	M	H	All methods of ranking except Chen and Chen [39]	Chen and Chen [39]
1	78	125	172	7	4
2	74	95	116	6	7
3	45	65	85	4	5
4	38	75	112	5	2
5	39	55	71	2	6
6	37	60	83	3	3
7	2	10	18	1	1

$$y_0 = \frac{15.668 * \frac{1}{3} + 31.333 * \frac{1}{2} + 15.668 * \frac{1}{3}}{15.668 + 31.333 + 15.668} = 0.41666$$
$$R(1) = 125 * 0.41666 = 52.083$$

10. For Gani and Mohamed [42], Eqs. 56 and 57 to calculate X and Y, and then Eq. 55 is adopted to determine $R\left(\tilde{1}\right)$

$$x = \frac{2 * 78 + 14 * 125 + 2 * 172}{18} = 125$$
$$Y = \frac{7 * 1}{18} = 0.389 \qquad R(1) = 125 * 0.389 = 48.611$$

Similarly, the ranks for all other activities adopting previous ranking methods are calculated and the priorities for all activities are established according to these ranks. Table 3 shows the priorities for all activities.

It can be shown that all methods give the same priorities except Chen and Chen [39]. This result could be interpreted as the project is very small; it has just seven activities. For medium- and large-size projects, a large variation between these methods of ranking is anticipated. This issue will be dealt with later in section of case studies.

Step 3: Performing resource scheduling based on activity priority

A resource scheduling based on activities priorities was performed using the priorities established from previous step for each ranking method using a software program (Microsoft Project). Figure 3 shows finish time for each activity for all methods except Chen and Chen [39], while Fig. 4 shows finish time for each activity for Chen and Chen [39] method.

Step 4: Fuzzy net present value calculation

Two scenarios are adopted for calculating FNPV. The first scenario (scenario 1) represents the case of neglecting inflation, while the second one (scenario 2) shows the case of considering inflation. The calculation of FNPV for activity 1 for all methods except Chen and Chen [39] is given below for the two scenarios as an example.

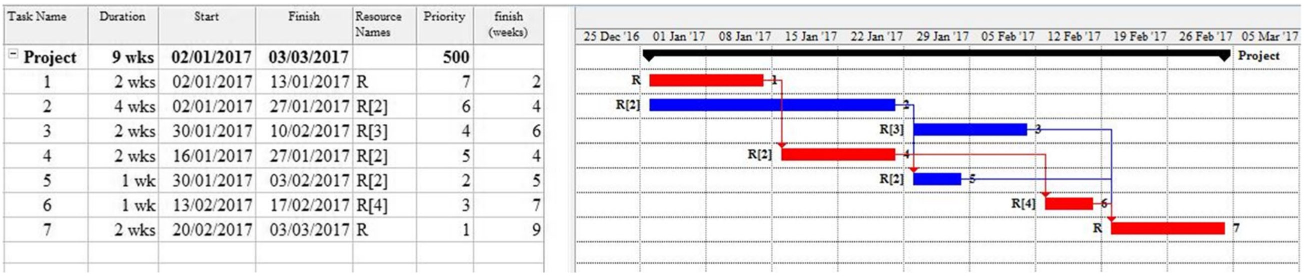
Knowing cash flow of activity 1, $CF_1 = (40, 50, 60)$ and finish time = 2 weeks, fuzzy net present value of activity 1 for scenarios 1 ($FNPV_{S1}$) and 2 ($FNPV_{S2}$) are calculated, respectively, as follows:

$$\begin{aligned}
FNPV_{S1} &= \frac{F}{(1+r)^n} = \frac{(40, 50, 60)}{\left(1 + \frac{(0.08, 0.10, 0.12)}{12}\right)^{2/4}} \\
&= \frac{(40, 50, 60)}{\left(1 + \frac{0.08}{12}, 1 + \frac{0.10}{12}, 1 + \frac{0.12}{12}\right)^{1/2}} \\
&= \frac{(40, 50, 60)}{(1.0067, 1.0083, 1.01)^{1/2}} \\
&= \left(\frac{40}{1.01^{1/2}}, \frac{50}{1.0083^{1/2}}, \frac{60}{1.0067^{1/2}}\right) \\
&= (39.80, 49.79, 59.80)
\end{aligned}$$

$$\begin{aligned}
FNPV_{S2} &= \frac{F}{(1+m)^n} = \frac{(40, 50, 60)}{\left(1 + \frac{(0.215, 0.2155, 0.2768)}{12}\right)^{2/4}} \\
&= \frac{(40, 50, 60)}{\left(1 + \frac{0.215}{12}, 1 + \frac{0.2155}{12}, 1 + \frac{0.2768}{12}\right)^{1/2}} \\
&= \frac{(40, 50, 60)}{(1.0179, 1.018, 1.0231)^{1/2}} \\
&= \left(\frac{40}{1.0231^{1/2}}, \frac{50}{1.018^{1/2}}, \frac{60}{1.0179^{1/2}}\right) \\
&= (39.546, 49.556, 59.470)
\end{aligned}$$

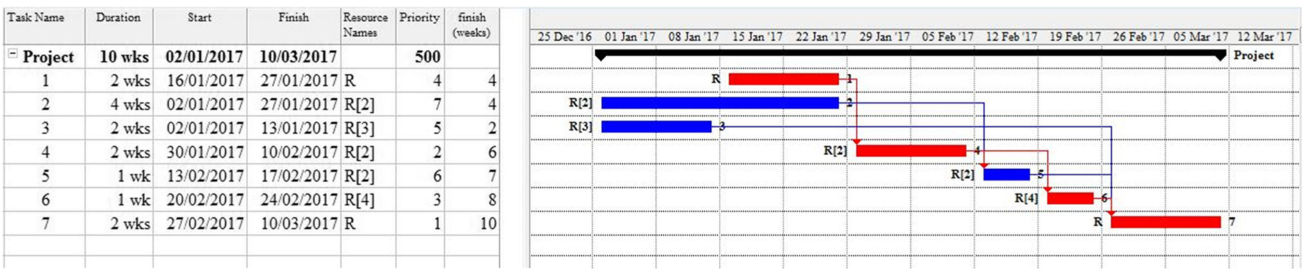
For Chen and Chen [39] and knowing cash flow of activity 1, $CF_1 = (40, 50, 60)$ and finish time = 4 weeks. In this case $FNPV_{S1} = (39.6, 49.59, 59.6)$ and $FNPV_{S2} = (39.1, 49.12, 58.94)$.

The total FNPV for all activities of all cases for the two scenarios are shown in Table 4. However, the same results obtained for scenario 1 or the very small differences resulted in scenario 2 in this example may be due to the small number of activities for this project. Accordingly, three case studies are adopted in the following section to draw a final conclusion about the best ranking method for maximizing FNPV.



Task Name	Duration	Start	Finish	Resource Names	Priority	finish (weeks)
Project	9 wks	02/01/2017	03/03/2017		500	
1	2 wks	02/01/2017	13/01/2017	R	7	2
2	4 wks	02/01/2017	27/01/2017	R[2]	6	4
3	2 wks	30/01/2017	10/02/2017	R[3]	4	6
4	2 wks	16/01/2017	27/01/2017	R[2]	5	4
5	1 wk	30/01/2017	03/02/2017	R[2]	2	5
6	1 wk	13/02/2017	17/02/2017	R[4]	3	7
7	2 wks	20/02/2017	03/03/2017	R	1	9

Fig. 3 Resource scheduling for example problem considering activities priorities (for all methods except Chen and Chen [39])



Task Name	Duration	Start	Finish	Resource Names	Priority	finish (weeks)
Project	10 wks	02/01/2017	10/03/2017		500	
1	2 wks	16/01/2017	27/01/2017	R	4	4
2	4 wks	02/01/2017	27/01/2017	R[2]	7	4
3	2 wks	02/01/2017	13/01/2017	R[3]	5	2
4	2 wks	30/01/2017	10/02/2017	R[2]	2	6
5	1 wk	13/02/2017	17/02/2017	R[2]	6	7
6	1 wk	20/02/2017	24/02/2017	R[4]	3	8
7	2 wks	27/02/2017	10/03/2017	R	1	10

Fig. 4 Resource scheduling for example problem considering activities priorities (for Chen and Chen [39])

Table 4 FNPV, distance, finish time for all activities for used ranking methods for the adopted scenarios

Category	Ranking methods	Scenario	FNPV			Distance	Finish time	Assigned score
			L	M	H			
1	All methods of ranking except Chen and Chen [39]	1	153.95	217.66	218.62	62.92	9	1
		2	151.4	215	277.6	62.92		1
2	Chen and Chen [39]	1	153.95	217.66	218.62	62.77	10	2
		2	151.6	215.2	277.7	62.77		2

The objective of the above example is to show the algorithm's implementation on a small project. However, to clarify the difference between the results of applied ranking methods and to generalize the results of the research, three case studies for actual and large projects are applied.

8 Case Studies

Three case studies are applied in this section. A fuzzy interest rate = (0.08, 0.10, 0.12) is assumed for these case studies. Also, a fuzzy inflation rate (0.075, 0.105, 0.14) is adopted. The data of first case study were obtained from [20], whereas the data of the second and third case studies were obtained by graduation project students who were supervised by the first author in 2014. The direct cost for case studies 1 and 2 was estimated by graduation project students and was revised by the authors.

8.1 Case Study 1

The project of this case study is a warehouse project constructed in Ankara and consists of 18 activities. Some of the activities are assumed to be subcontracted. No resource is assigned to the activities planned to be subcontracted. Table 5 shows the activities, their predecessors, total quantity of work and units which were obtained from [20]. Suitable productivity rates for crews (using standard rates for civil engineering works in Egypt), and suitable number of crews were assumed. Using given quantities, activities' durations were determined (see columns 6–8 in this table). The number of normal labor associated with each crew and accordingly total number of normal labor corresponding to each activity are shown in Table 5 (columns 9 and 10). The number of available resource units (normal labor) for this project is assumed to be limited to 8 over the project realization time. This is because the maximum number of resource units for any activity does not exceed this limit. The labor expenses and incomes for each resource were given in crisp units. Material expenses were given in fuzzy units, while the material incomes were given in crisp units. All expenses and incomes are in terms of dollar value. Table 6 which has been established by the authors, shows activities, resources, expenses, and incomes which are among the data obtained for this case study. It must be noted

Table 5 Data and calculated number of resource units for case study 1

(1) Act. name	(2) Activity description	(3) Predecessor	(4) Quantity	(5) Unit	(6) Produc. unit/day	(7) Crews	(8) Dur.	(9) Labor/crew	(10) Total normal labor
A*	Site preparation*	–					10		
B*	Excavation*	A					9		
C	Formworks of foundation	R	392	m^2	20	3	7	2	6
D	Rebar of foundation	R	34	ton	0.4	4	22	2	8
E	Pouring foundation concrete	C and D	324	m^3	32	1	11	6	6
F	Structural steel erection	E	44.5	ton	10	1	5	5	5
G	Masonry works	F	30	m^3	3	2	5	4	8
H	Insulation	B	680	m^2	100	1	7	5	5
I	Leveling	G	650	m^2	20	4	9	2	8
J	Plastering	G	465	m^2	75	2	4	3	6
K	Floor covering	S	650	m^2	45	3	5	2	6
L	Paint interior	N and K	465	m^2	30	3	6	2	6
M	Paint exterior	N	330	m^2	30	3	4	2	6
N*	Doors and windows*	J					7		
P*	Mechanical works*	F					7		
Q*	Electrical works*	F					7		
R	Pouring concrete for protection	H	680	m^2	25	2	14	4	8
S	Thermal moisture	I	40	m^2	10	2	2	3	6

"*" shows the subcontracted activities

Table 6 Expense, income and cash for each activity for case study 1

Act.	Labor and material resource	Expense			Income			Cash		
		Low	Med	High	Low	Med	High	Low	Med	High
A										
B		25,000	25,000	25,000	28,750	28,750	28,750	3465.3	3512.4	3559.6
C	Formwork	4606	5390	6174	6201.44	6201.44	6201.44	−33.96	760.19	1554.4
D	Rebar	19,210	22,610	26,010	26,001.5	26,001.5	26,001.5	−265.9	3176.6	6619.3
E	Concrete	18,208.8	19,828.8	24,688.8	22,803.12	22,803.12	22,803.12	−2111	2785.9	4443.3
F	Structural steel	44,722.5	46,947.5	56,960	53,989.63	53,989.63	53,989.63	−3505	6595.9	8909.6
G	Masonary	3180	3540	3780	4071	4071	4071	250.69	497.36	864.04
H	Foundation insulation	8132.8	9832.8	10,852.8	11,315.2	11,315.2	11,315.2	350.37	1388.9	3107.5
I	Leveling	1963	2190.5	2288	2522	2522	2522	209.03	310.66	542.3
J	Painting	6919.2	8407.2	10,639.2	9667.35	9667.35	9667.35	−1068	1180.3	2684.1
K	Floor covering	6922.5	9652.5	11,472.5	11,102	11,102	11,102	−480.4	1357.7	4106
L	Interior painting	1329.9	1441.5	2027.4	1655.4	1655.4	1655.4	−388.4	200.22	314.54
M	Exterior painting	1438.8	1854.6	2263.8	2135.1	2135.1	2135.1	−149.8	262.85	682.16
N		20,000	20,000	20,000	23,000	23,000	23,000	2772.3	2809.9	2847.7
P		18,000	18,000	18,000	20,700	20,700	20,700	2495	2528.9	2562.9
Q		12,000	12,000	12,000	13,800	13,800	13,800	1663.4	1686	1708.6
R	Concrete labor Lean concrete material	32,776	35,584.4	41,616	40,915.6	40,915.6	40,915.6	−1106	4993.1	7868.6
S	Insulation of WC	574	654	774	761.6	761.6	761.6	−19.94	101.31	182.56
		249,983.5	267,933.8	299,546.5	308,140.9	308,140.9	308,140.9	5543.5	37,661	56,117

Table 7 FNPV, distance, finish time for different ranking methods for the adopted scenarios (case study 1)

Category	Ranking methods	Scenario	FNPV			Distance	Finish time	Assigned score
			L	M	H			
1	Chen and Chen [34], Chen and Chen [38], Wang et al. [37], Cheng [35], Dat et al. [40], and Allahviranloo and Sanifard [41]	1	5521.4	36,992	55,287	22,687	120	2
		2	5493.6	36,244.8	53,931.2	22,041.3		2
2	Chu and Tsao [36] and Gani and Mohamed [42]	1	5521.4	36,992	55,287	22,687	120	2
		2	5493.6	36,244.8	53,931.2	22,041.3		2
3	Thorani et al. [31]	1	5521.4	36,992	55,287	22,687	120	2
		2	5493.6	36,244.8	53,931.2	22,041.3		2
4	Chen and Chen [39]	1	5521.8	36,990	55,284	22,688	120	1
		2	5494.4	36,241.7	53,924.4	22,044.4		1

that the three values of incomes are the same since their values are obtained in crisp units.

Applying Eq. 64 fuzzy cash flow weight was calculated for each activity. Then, ranking methods were applied. The results for ranking methods revealed that methods (Chen and Chen [34], Chen and Chen [38], Wang et al. [37], Cheng [35], Dat et al. [40], and Allahviranloo and Sanifard [41]) give the same ranking for all activities. The two methods: Chu and Tsao [36] and Gani and Mohamed [42] give the same ranking for all activities. Thorani et al. [31] give different ranking for some activities. Chen and Chen [39] gives different ranking for majority of activities. The ranking methods could be categorized into four categories (see Table 7). A resource scheduling based on activities priorities was performed using Microsoft Project software. According to the resulted finish time in each category, FNPV was calculated for the two adopted scenarios. Table 7 shows the results.

8.2 Case Study 2

This project is a social housing project, model no. 1 in Fashn city at Beni-Suef governorate, Egypt, and consists of 187 activities and has a direct cost of 3,378,483 L.E estimated by the graduation project students. The suitable number of normal labors associated with each crew was assumed. Accordingly total numbers of normal labors corresponding to each activity were calculated. The number of available resource units (normal labor) for this project was assumed to be limited to 5 over the project realization time.

The direct cost for each activity was fuzzified using the random function applied from Excel software with limits ranges from 90 to 110%. The proposed fuzzy indirect cost (7, 13.5, 20%) of direct cost and a proposed fuzzy markup (3, 5, 7%) of direct cost were also adopted as previously explained in the formulation of the developed algorithm. Adopting the same procedures as in case study 1 revealed that methods: Chen and Chen [34], Wang et al. [37], Cheng [35], Dat et al. [40], and Allahviranloo and Sanifard [41]) give the same ranking for all activities. Methods: Chen and Chen [38], Chu and Tsao [36], Thoranie et al. [31] and Gani and Mohamed [42] give different ranking for some activities. Chen and Chen [39] method gives different ranking for majority of activities. The ranking methods could be categorized into six categories (see Table 8). A resource scheduling based on activities' priorities was performed using Microsoft Project software. According to the resulted finish time in each category, FNPV was calculated for the two adopted scenarios (see Table 8).

8.3 Case Study 3

This project is a Water Treatment Plant and it has networks in Mit Ghamr city, at Dakhlia governorate, Egypt. It consists of 15 buildings with 724 activities and has a direct cost of 106,868,658 L.E. estimated by the graduation project students. The number of available resource units (normal labor) for this project was assumed to be limited to 8 over the project realization time.

Similarly, the direct cost for each activity was fuzzified as in case study 2. Also, the same percentages of fuzzy indirect cost and fuzzy markup were applied. Adopting the same procedures as in case study 2 revealed that the ranking methods could be categorized into six categories as in case study 2 (see Table 9). A resource scheduling based on activities' priorities was performed using Microsoft Project software. According to the resulted finish time in each category, FNPV was calculated for the two adopted scenarios. Table 9 presents the results.

Table 8 FNPV, distance, finish time for different ranking methods for the adopted scenarios (case study 2)

Category	Ranking methods	Scenario	FNPV			Distance	Finish time	Assigned score
			L	M	H			
1	Chen and Chen [34], Wang et al. [37], Cheng [35], Dat et al. [40], and Allahviranloo and Sanifard [41]	1	−707,227	126,805	990,233	853,629	451	3
		2	−643,455.3	117,964.6	911,184.7	782,620		4
2	Chen and Chen [38]	1	−707,281	126,756	990,191	853,677	450	1
		2	−643,543.9	117,867.9	911,068.6	782,720.5		2
3	Chen and Chen [39]	1	−704,071	126,499	987,543	853,576	471	6
		2	−636,829.8	117,345	904,523.5	782,836.8		1
4	Chu and Tsao [36]	1	−762,778	135,272.6	1,042,051	853,613	447	5
		2	−644,310.7	118,094.8	912,161.4	782,536.2		6
5	Thorani et al. [31]	1	−707,078	126,786	990,095	853,632	453	2
		2	−643,159.7	117,927.3	910,859.9	782,642.2		3
6	Gani and Mohamed [42]	1	−707,660	126,870	990,643	853,615	447	4
		2	−644,296.8	118,088.8	912,136.8	782,541.7		5

Table 9 FNPV, distance, finish time for different ranking methods for the adopted scenarios (case study 3)

Category	Ranking methods	Scenario	FNPV			Distance	Finish time	Assigned score
			L	M	H			
1	Chen and Chen [34], Wang et al. [37], Cheng [35], Dat et al. [40], and Allahviranloo and Sanifard [41]	1	−17,360,355	3,264,234	26,500,073	22,365,422	2901	4
		2	−11,853,114	2,470,513	18,911,631	15,735,288		4
2	Chen and Chen [38]	1	−17,478,493	3,262,285	26,581,192	22,378,412	2990	2
		2	−12,058,729	2,469,151	19,090,723	15,744,583		2
3	Chen and Chen [39]	1	−15,974,683	3,157,557	25,148,979	22,389,456	3246	1
		2	−10,495,308	2,381,458	17,217,506	15,877,079		1
4	Chu and Tsao [36]	1	−17,366,836	3,265,452	26,507,535	22,364,689	2887	5
		2	−11,860,307	2,472,050	18,922,298	15,733,618		5
5	Thorani et al. [31]	1	−17,374,229	3,266,412	26,514,631	22,364,469	2880	6
		2	−11,872,885	2,473,741	18,936,734	15,732,435		6
6	Gani and Mohamed [42]	1	−17,358,319	3,263,542	26,496,787	22,366,070	2894	3
		2	−11,854,102	2,469,940	18,909,760	15,736,432		3

9 Analysis and Discussion of the Results

In the example project and the three case studies, five ranking methods are among category 1. These methods are: Chen and Chen [34], Wang et al. [37], Cheng [35], Dat et al. [40], and Allahviranloo and Sanifard [41]. Therefore, it has been decided to choose one of them to rank the FNPV results for all methods for the adopted scenarios. Allahviranloo and Sanifard [41] has been chosen, and the distances have been calculated, as shown in Tables 4, 7, 8, 9. Table 10 shows a summary of the results for the example project and the three case studies. In this table Monday, May 15, 2017 at 11:34 am a rank is established for each ranking method in each case study according to the distance calculated, such that the method with the least distance is assigned the highest rank as declared previously in this method and the vice versa. The ranks for each method are then summed for the example project and the three case studies. A final score is then established for each method such that the method with the highest rank receives final score equal to 1. Now we can answer the question about the best fuzzy ranking method for maximizing FNPV. It is clear from Table 10, that Chu and Tsao [36] is the best ranking method for maximizing FNPV depending on the associated final score which equals to 1

Table 10 Results of example project and case studies

No	Fuzzy ranking method	Scenario 1						Scenario 2					
		Assigned rank						Assigned rank					
		E.P.	C.S. (1)	C.S. (2)	C.S. (3)	R.S.	F.S.	E.P.	C.S. (1)	C.S. (2)	C.S. (3)	R.S.	F.S.
1	Chen and Chen [34]	1	2	3	4	10	3	1	2	4	4	11	3
2	Chen and Chen [38]	1	2	1	2	6	4	1	2	2	2	7	4
3	Chen and Chen [39]	2	1	6	1	10	3	2	1	1	1	5	5
4	Wang et al. [37]	1	2	3	4	10	3	1	2	4	4	11	3
5	Cheng [35]	1	2	3	4	10	3	1	2	4	4	11	3
6	Chu and Tsao [36]	1	2	5	5	13	1	1	2	6	5	14	1
7	Dat et al. [40]	1	2	3	4	10	3	1	2	4	4	11	3
8	Allahvi ranloo and Saneifard [41]	1	2	3	4	10	3	1	2	4	4	11	3
9	Thoranie et al. [31]	1	2	2	6	11	2	1	2	3	6	12	2
10	Gani and Mohamed [42]	1	2	4	3	10	3	1	2	5	3	11	3

E.P. Example project, *C.S.* case study, *R.S.* rank summation, *F.S.* final score

Table 11 Results of global sensitivity analysis for example project and case studies

No.	Fuzzy ranking methods	Scenario	Assigned rank													R.S.	F.S.
			E.P.	C.S. (1)	Periods of C.S (2)			Periods of C.S. (3)									
					1	2	3	1	2	3	4	5	6	7	end		
1	Chen and Chen [34]	Scenario 1	1	1	5	5	5	6	5	5	6	6	6	3	2	56	5
2	Chen and Chen [38]		1	1	4	6	4	2	2	2	3	2	2	2	3	34	2
3	Chen and Chen [39]		1	6	1	1	1	1	1	1	2	1	1	1	1	19	1
4	Wang et al. [37]		1	1	5	5	5	6	5	5	6	6	6	3	2	56	5
5	Cheng [35]		1	1	5	5	5	6	5	5	6	6	6	3	2	56	5
6	Chu and Tsao [36]		1	1	2	2	2	5	4	4	5	5	5	5	5	46	3
7	Dat et al. [40]		1	1	5	5	5	6	5	5	6	6	6	3	2	56	5
8	Allahvi ranloo and Saneifard [41]		1	1	5	5	5	6	5	5	6	6	6	3	2	56	5
9	Thoranie et al. [31]		1	1	6	4	6	3	3	3	4	3	3	6	4	47	4
10	Gani and Mohamed [42]		1	1	3	3	3	4	6	6	1	5	4	4	6	47	4
1	Chen and Chen [34]	Scenario 2	1	2	5	4	3	5	5	5	6	6	6	3	5	56	6
2	Chen and Chen [38]		1	2	4	2	4	2	2	2	3	3	2	2	2	31	2
3	Chen and Chen [39]		1	1	1	3	6	5	1	1	2	1	1	1	1	25	1
4	Wang et al. [37]		1	2	5	4	3	5	5	5	6	6	6	3	5	56	6
5	Cheng [35]		1	2	5	4	3	5	5	5	6	6	6	3	5	56	6
6	Chu and Tsao [36]		1	2	2	6	1	4	4	4	5	5	5	5	4	48	4
7	Dat et al. [40]		1	2	5	4	3	5	5	5	6	6	6	3	5	56	6
8	Allahvi ranloo and Saneifard [41]		1	2	5	4	3	5	5	5	6	6	6	3	5	56	6
9	Thoranie et al. [31]		1	2	6	5	5	3	3	3	4	4	4	6	3	49	5
10	Gani and Mohamed [42]		1	2	3	1	2	6	6	6	1	2	3	4	6	43	3

E.P. Example project, *C.S.* case study, *R.S.* rank summation, *F.S.* final score for each of the two adopted scenarios. The second method is Thorani et al. [31] for the two adopted scenarios. The worst methods are Chen and Chen [38] and Chen and Chen [39] in case of neglecting and considering inflation respectively.

10 Global Sensitivity Analysis

Sensitivity analysis can be 'local' or 'global'. Local sensitivity is the sensitivity of model output when only one input factor is changed at a time while all other input factors are at their nominal values. Global sensitivity determines the effect on model output of all the uncertain input factors acting simultaneously over their ranges of uncertainty [47]. In this section, a global sensitivity analysis is performed to assess the effect of net cash flow, interest rate value, and inflation rate value on NPV, since each ranking method results a different cash flow pattern through the life of the project according to the method of ranking. In other words to examine the consistency of the results for other case studies and for other values of interest rate and inflation rate. [26] reported three equations (Eqs. 68, 69, 70) to calculate global importance (GI) of fuzzy cash flows (F_j) on NPV.

$$\mathrm{GI}(F_j)_\mathrm{r} = \frac{(F_{j_\mathrm{r}} - F_{j_\mathrm{l}})^2}{(NPV_\mathrm{r} - \mathrm{NPV}_\mathrm{l})^2(1+i_\mathrm{l})^{2j}} \tag{68}$$

$$\mathrm{GI}(F_j)_\mathrm{m} = \frac{(F_{j_\mathrm{r}} - F_{j_\mathrm{l}})^2}{(\mathrm{NPV}_\mathrm{r} - \mathrm{NPV}_\mathrm{l})^2(1+i_\mathrm{m})^{2j}} \tag{69}$$

$$\mathrm{GI}(F_j)_\mathrm{l} = \frac{(F_{j_\mathrm{r}} - F_{j_\mathrm{l}})^2}{(\mathrm{NPV}_\mathrm{r} - \mathrm{NPV}_\mathrm{l})^2(1+i_\mathrm{r})^{2j}} \tag{70}$$

where $\mathrm{GI}(F_j)_\mathrm{r}$, $\mathrm{GI}(F_j)_\mathrm{m}$, and $\mathrm{GI}(F_j)_\mathrm{l}$ are the upper, most promising and lower values of global importance of a fuzzy cash flow from year j on NPV, respectively. F_{j_r} and F_{j_l} are the upper values and lower values of fuzzy net cash flow at year j, respectively. NPV_r and $\mathrm{NP}V_\mathrm{l}$ are the upper value and the lower values of net present value, respectively. i_l, i_m, and i_r are the lower, most promising and upper values of fuzzy interest rate, respectively.

Fuzzy global importance (GI) values were ranked for each period for each project for all ranking methods using method of Allahviranloo and Sanifard [41]. One period was considered for both the example project and case study 1 due to their short duration. The period for case study 2 is approximately equal to a half of year. The period for case study 3 is equal to year (see Table 11). The ranks assigned for each ranking method were summed. Then, a final score was established such that the method with the least summation of ranks is assigned a final score equal to 1 (see Table 11). It must be noted that high rank summation means a high variability in GI values which reflect that the results may differ for other case studies and the vice versa. Examining the values of final scores shown in Table 11 reveals that, methods of Chen and Chen [39] and Chen and Chen [38] receive the lowest scores; 1 and 2, respectively, for the two scenarios. This means that these methods are consistently the worst as previously presented in section of analysis and discussion of the results. Chu and Taso [36] receives the third priority among five categories for scenario 1 for the consistency of results, whereas it receives the fourth priority among six categories for scenario 2. Thoranie et al. [31] comes after Chu and Taso [36] for the consistency of the results. Therefore, in future more case studies are required to decide if Chu and Taso [36] will still the best method or Thoranie et al. [31].

11 Summary and Conclusions

Cash flow weight technique and ten fuzzy ranking methods are presented. An algorithm for maximizing FNPV was proposed. Two scenarios in dealing with inflation were adopted, these are: neglecting and considering inflation. An example project was solved manually step by step to show how the algorithm performs. Three case studies with large projects are also presented to generalize the results. The main contributions to the body of knowledge depending on example project and case studies are: Chu and Tsao [36] is the best ranking method for maximizing FNPV in case of neglecting and considering inflation. Thoranie et al. [31] method comes after Chu and Tsao [36] for the two adopted scenarios. The worst methods are Chen and Chen [38] and Chen and Chen [39] in case of neglecting and considering inflation respectively. This implies that the result of the best method for maximizing fuzzy net present value in case of neglecting or considering inflation could be generalized in other countries have inflation rate up to 14%, since the inflation has no effect up to 14%.

A global sensitivity analysis was performed to check the consistency of the results. This analysis revealed that: Chen and Chen [38] and Chen and Chen [39] are consistently the worst. Chu and Taso [36] ranking method received the third priority among five categories for scenario 1 for the consistency of the results, whereas it received the fourth priority among six categories for scenario 2. Thoranie et al. [31] comes after Chu and Taso [36] for the consistency of the results. Thus, in future more case studies are required to decide if Chu and Taso [36] will still the best method or Thoranie et al. [31].

The contributions to the practice are: An equation was extracted for calculating cash flow weight to be used for large projects. A random function applied from Excel software was proposed to fuzzify crisp direct cost. It must be noted that the results are limited to triangular normal fuzzy numbers and when the resource scheduling problem is performed for only one type of resources. As a further research the proposed algorithm could be expanded for other types of fuzzy numbers such as trapezoidal and for non normal fuzzy number. Also, more than one type of resources could be applied in resource scheduling problem. Furthermore, the effect of higher percentages of inflation requires study.

References

1. Elmaghraby, S.E.; Herroelen, W.S.: The scheduling of activities to maximize the net present value of projects. Eur. J. Oper. Res. **49**, 35–49 (1990)
2. Yang, K.K.; Talbot, F.B.; Patterson, J.H.: Scheduling a project to maximize its net present value: an integer programming approach. Eur. J. Oper. Res. **64**, 188–198 (1992)
3. Sung, C.S.; Lim, S.K.: A project activity scheduling problem with net present value measure. Int. J. Prod. Econ. **37**, 177–187 (1994)
4. Etgar, R.; Avraham Shtub, A.; LeBlanc, L.J.: Scheduling projects to maximize net present value—the case of time-dependent, contingent cash flows. Eur. J. Oper. Res. **96**, 90–96 (1996)
5. Baroum, S.M.; Patterson, J.H.: The development of cash flow weight procedures for maximizing the net present value of a project. J. Oper. Manag. **14**, 209–227 (1996)
6. Pinder, J.P.; Marucheck, A.S.: Using discounted cash flow heuristics to improve project net present value. J. Oper. Manag. **14**, 229–240 (1996)
7. Neumann, K.; Zimmermann, J.: Procedures for resource leveling and net present value problems in project scheduling with general temporal and resource constraints. Eur. J. Oper. Res. **127**, 425–443 (2000)
8. Vanhoucke, M.; Demeulemeester, E.; Herroelen, W.: Progress payments in project scheduling problems. Eur. J. Oper. Res. **148**, 604–620 (2003)
9. Mika, M.; Waligora, G.; Wezglarz, J.: Simulated annealing and tabu search for multi-mode resource-constrained project scheduling with positive discounted cash flows and different payment models. Eur. J. Oper. Res. **164**, 639–668 (2005)
10. Waligora, G.: Discrete-continuous project scheduling with discounted cash flows—a tabu search approach. Comput. Oper. Res. **35**, 2141–2153 (2008)
11. Hartmann, S.; Briskorn, D.: A survey of variants and extensions of the resource-constrained project scheduling problem. Eur. J. Oper. Res. **207**, 1–14 (2010)
12. Tantisuvanichkul, V.: Optimising net present value using priority rule-based scheduling. Ph.D. Thesis, Faculty of Engineering and Physical Science, University of Manchester (2014)
13. Yang, K.K.; Tay, L.C.; Sum, C.C.: A comparison of stochastic scheduling rules for maximizing project net present value. Eur. J. Oper. Res. **85**, 327–339 (1995)
14. Sobel, M.J.; Szmerekovsky, J.G.; Tilson, V.: Scheduling projects with stochastic activity duration to maximize expected net present value. Eur. J. Oper. Res. **198**, 697–705 (2009)
15. Creemers, S.; Leus, R.; Lambrechta, M.: Scheduling Markovian PERT networks to maximize the net present value. Oper. Res. Lett. **38**, 51–56 (2010)
16. Wiesemann, W.; Kuhn, D.; Rustem, B.: Maximizing the net present value of a project under uncertainty. Eur. J. Oper. Res. **202**, 356–367 (2010)
17. Shaffie, S.S.; Jaaman, S.H.: Monte Carlo on net present value for capital investment in Malaysia. Procedia Soc. Behav. Sci. **219**, 688–693 (2016)
18. Kuchta, D.: Fuzzy capital budgeting. Fuzzy Sets Syst. **111**, 367–385 (2000)
19. Chui, C.Y.; Park, C.S.: Fuzzy cash flow analysis using present worth criterion. Eng. Econ. J. **39**, 113–138 (2007)
20. Melik, S.: Cash flow analysis of construction projects using fuzzy set theory. A M.Sc. Thesis, The Graduate School of Natural and Applied Sciences of Middle East, Technical University (2010)
21. Ucal, I.; Kuchta, D.: Project scheduling to maximize fuzzy net present value. In: Proceedings of the World Congress on Engineering, vol. 2 (2011)
22. Ho, S.H.; Liao, S.H.: A fuzzy real option approach for investment project valuation. J. Expert Syst. Appl. **38**, 15296–15302 (2011)
23. Lu, H.F.: Constructing a fuzzy net present value method to evaluating the BOT sport facilities. Int. J. Math. Comput. Phys. Electr. Comput. Eng **(6)**(6), 637–642 (2012)
24. Tsao, C.T.: Fuzzy net present values for capital investments in an uncertain environment. Comput. Oper. Res. **39**, 1885–1892 (2012)
25. Nosratpoura, M.; Nazerib, A.; Meftahic, H.: Fuzzy net present value for engineering analysis. J. Manag. Sci. Lett. **2**, 2153–2158 (2012)
26. Sarı, I.U.; Kuchta, D.: Fuzzy global sensitivity analysis of fuzzy net present value. J. Control Cybern. **41**(2), 481–496 (2012)
27. Kumar, G.; Bajaj, R.K.: Implementation of intuitionistic fuzzy approach in maximizing net present value. Int. J. Math. Comput. Phys. Electr. Comput. Eng. **8**(7), 1069–1073 (2014)
28. Dinagar, D.S.; Kamalanathan, S.: A note on maximize net present value with new ranking. Int. J. Fuzzy Math. Arch. **7**(1), 63–74 (2015)
29. Zadeh, L.A.: Fuzzy sets. Inf. Control **8**, 338–353 (1965)
30. Chen, S.J.; Hwang, C.L.; Hwang, F.P.: Fuzzy multiple attribute decision making: methods and applications. Springer, Berlin (1992)
31. Thorani, Y.L.P.; Rao, P.P.B.; Shankar, N.R.: Ordering generalized trapezoidal fuzzy numbers. Int. J. Contemp. Math. Sci. **7**(12), 555–573 (2012)
32. Kwang, H.C.; Lee, J.H.: A method for ranking fuzzy numbers and its application to decision making. IEEE Trans. Fuzzy Syst. **7**(6), 677–685 (1999)
33. Yager, R.R.: A procedure for ordering fuzzy subsets of the unit interval. Inf. Sci. **24**, 143–161 (1981)
34. Chen, S.J.; Chen, S.M.: A new method for handling multi-criteria fuzzy decision making problems using FN-IOWA operators. Cybern. Syst. **34**, 109–137 (2003)
35. Cheng, C.H.: A new approach for ranking fuzzy numbers by distance method. Fuzzy Sets Syst. **95**, 307–317 (1998)
36. Chu, T.C.; Tsao, C.T.: Ranking fuzzy numbers with an area between the centroid point and original point. Comput. Math. Appl. **43**, 111–117 (2002)
37. Wang, Y.M.; Yang, J.B.; Xu, D.L.; Chin, K.S.: On the centroid of fuzzy numbers. Fuzzy Sets Syst. **157**, 919–926 (2006)
38. Chen, S.J.; Chen, S.M.: Fuzzy risk analysis based on the ranking of generalized trapezoidal fuzzy numbers. Appl. Intell. **26**, 1–11 (2007)
39. Chen, S.M.; Chen, J.H.: Fuzzy risk analysis based on ranking generalized fuzzy numbers with different heights and different spreads. Expert Syst. Appl. **36**, 6833–6842 (2009)
40. Dat, L.Q.; Yu, F.; Chou, S.Y.: An improved ranking method for fuzzy numbers based on the centroid-index. Int. J. Fuzzy Syst. **14**(3), 413–419 (2012)
41. Allahviranloo, T.; Saneifard, R.: Defuzzification method for ranking fuzzy numbers based on center of gravity. Iran. J. Fuzzy Syst. **9**(6), 57–67 (2012)
42. Gani, A.N.; Mohammed, V.N.: Solution of fuzzy assignment problem by using a new ranking method. Int. J. Fuzzy Math. Arch. **2**, 8–16 (2013)

43. Guide to Cost Predictability in Construction: An analysis of issues affecting the accuracy of construction cost estimates. A report prepared by the Joint Federal Government/Industry Cost Predictability Taskforce, pp. 1–24 (2012)
44. El-Dosouky, A.I.: Principles of Construction Project Management. Mansoura University Press, Mansoura (1993)
45. Park, W.R.: Construction Bidding for Profit. Wiley, New York (1979)
46. http://www.tradingeconomics.com/Egypt/inflation-cpi/forecast
47. Haaker, M.P.R.; Verheijen, P.J.T.: Local and global sensitivity analysis for a reactor design with parameter uncertainty. Chem. Eng. Res. Des. **82**(5), 591–598 (2004)