CSE5010 Wireless Network and Mobile Computing Fall2022

Lab3

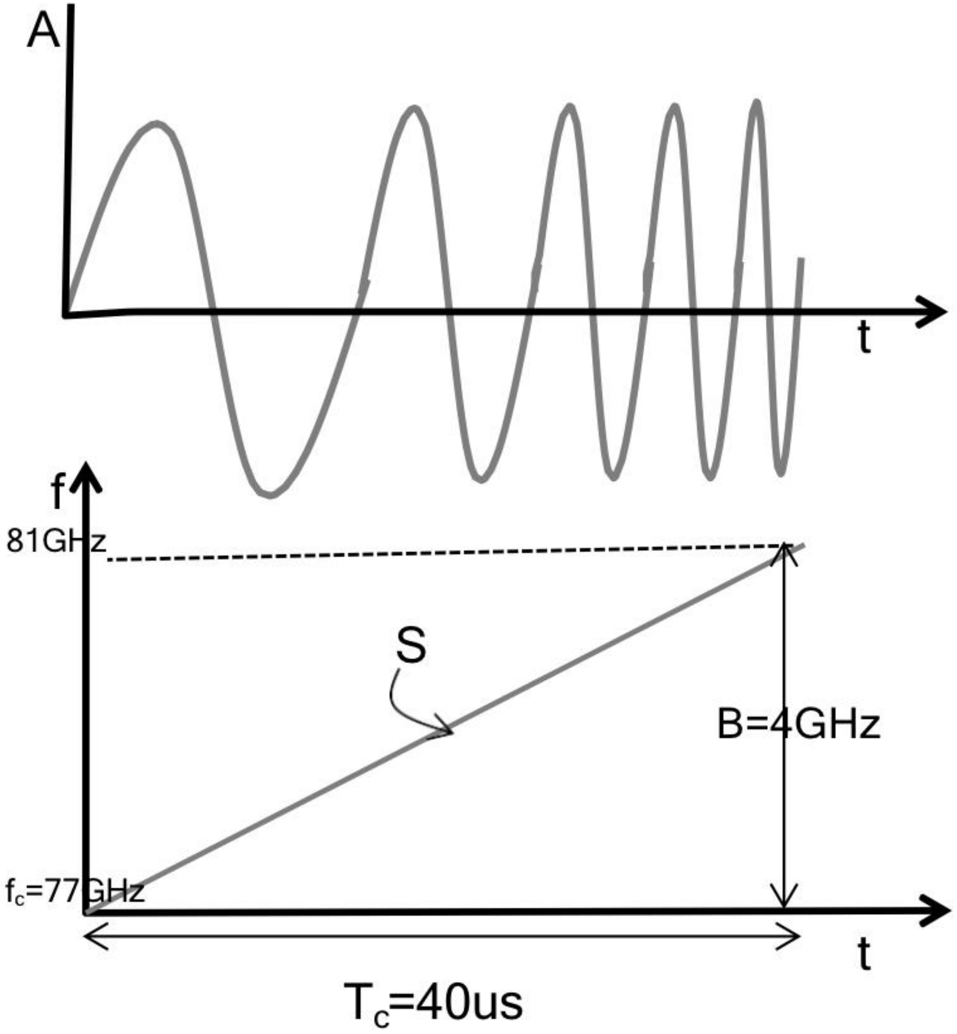
FMCW

&

Phase-based Distance Tracking Theory

FREQUENCY-MODULATED-CONTINUOUS-WAVE (FMCW)

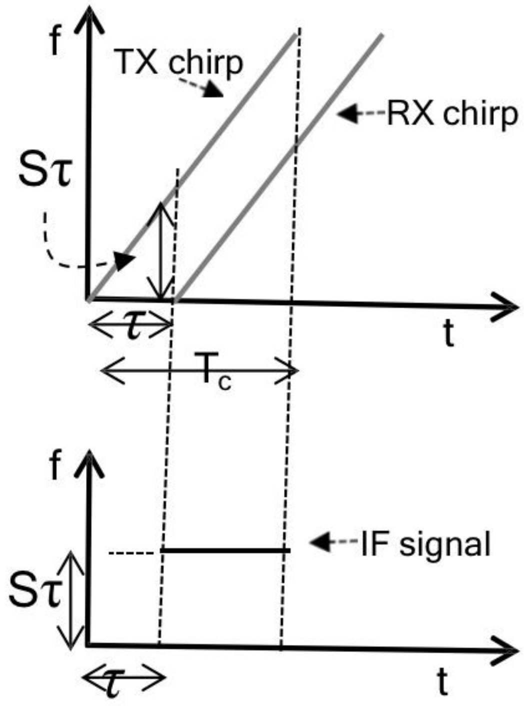
What is a chirp?



An FMCW radar transmits a signal called a "chirp". A chirp is a sinusoid whose frequency increases linearly with time, as shown in the Amplitude vs time (or 'A-t' plot) here.

- A frequency vs time plot (or 'f-t plot') is a convenient way to represent a chirp.
- A chirp is characterized by a start frequency (f_c), Bandwidth(B) and duration (T_c).
- The Slope (S) of the chirp defines the rate at which the chirp ramps up. In this example the chirp is sweeping a bandwidth of 4GHz in 40us which corresponds to a Slope of 100MHz/us

The IF signal



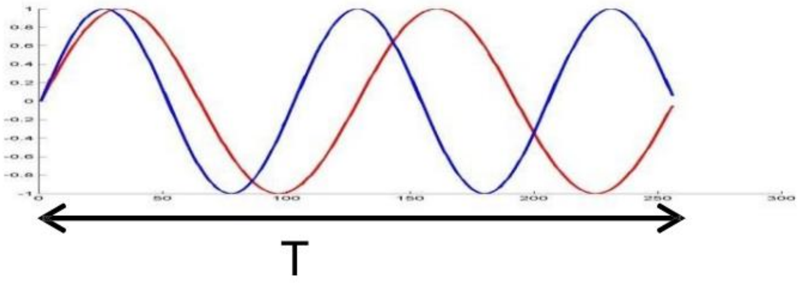
- The top figure shows the TX-signal and the RX-signal that is reflected from an object. Note that the RX-signal is just a delayed version of the TX signal. (τ denotes the round-trip time between the radar and the object. Also S denotes the slope of the chirp)
- Recall that the frequency of the signal at the mixers output is the difference of the instantaneous frequency of the TX-chirp and RX-chirp. As shown in the figure below, this is a straight line.
- Hence: A single object in front of the radar produces an IF signal that is a constant frequency tone.
  - The frequency of this tone is $S\tau = S2d/c$; [Since $\tau = 2d/c$, where d is distance of the object and c is the speed of light]

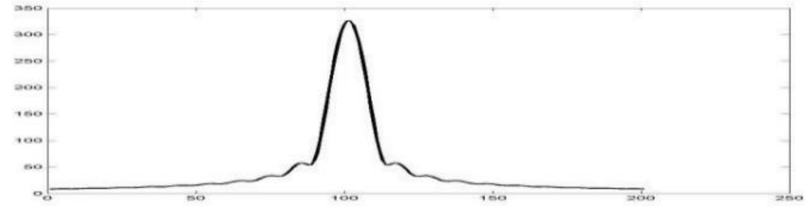
Note that τ is typically a small fraction of the total chirp time =>non-overlapping segment of the TX chirp is usually negligible. E.g. For a radar with a max distance of 300m and Tc=40us. τ/Tc =5%

A single object in front of the radar produces an IF signal with a constant frequency of S2d/c

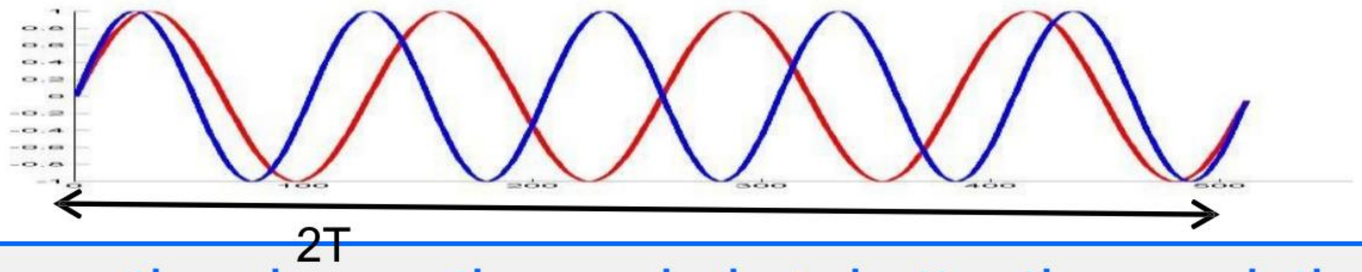
Fourier Transforms: A quick review

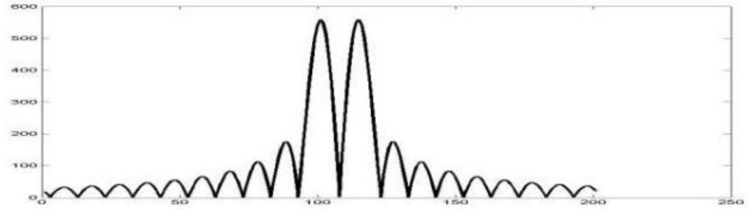
Within the observation window T below, the red tone completes 2 cycles, while the blue tone completes 2.5 cycles.   The difference of 0.5 cycles is not sufficient to resolve the tones in the frequency spectrum.





Doubling the observation window results in a difference of 1 cycle => the tones are resolved in the frequency spectrum

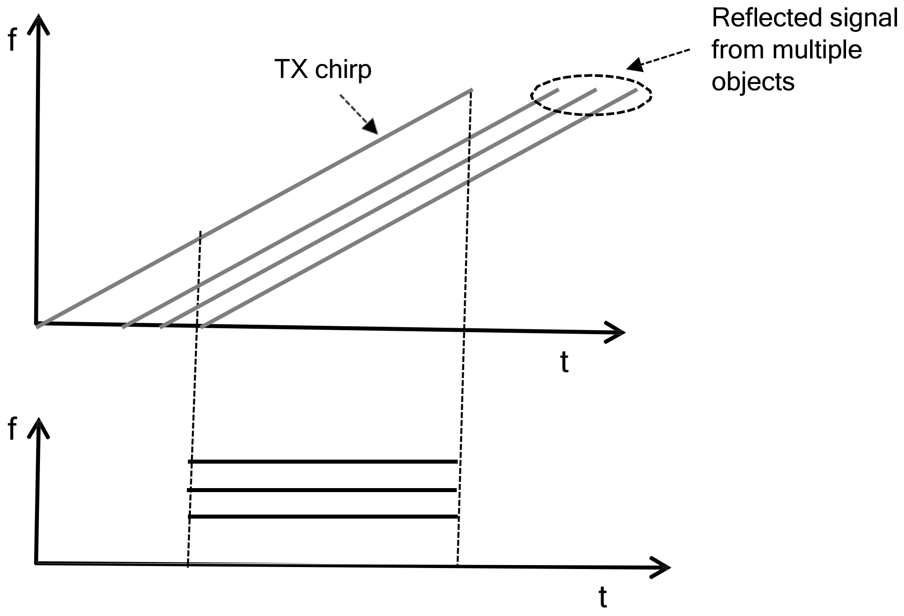




Longer the observation period => better the resolution. In general, an observation window of T can separate frequency components that are separated by more than 1/T Hz

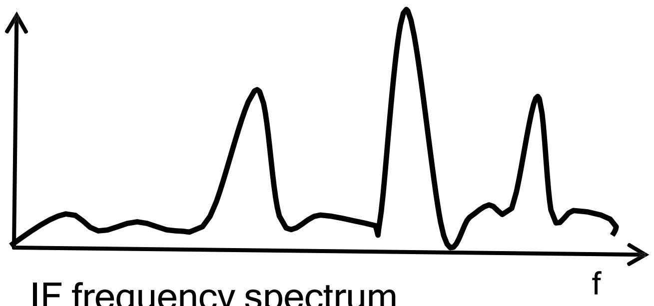
Multiple objects in front of the radar

Multiple objects in front of the radar=> multiple reflected chirps at the RX antenna



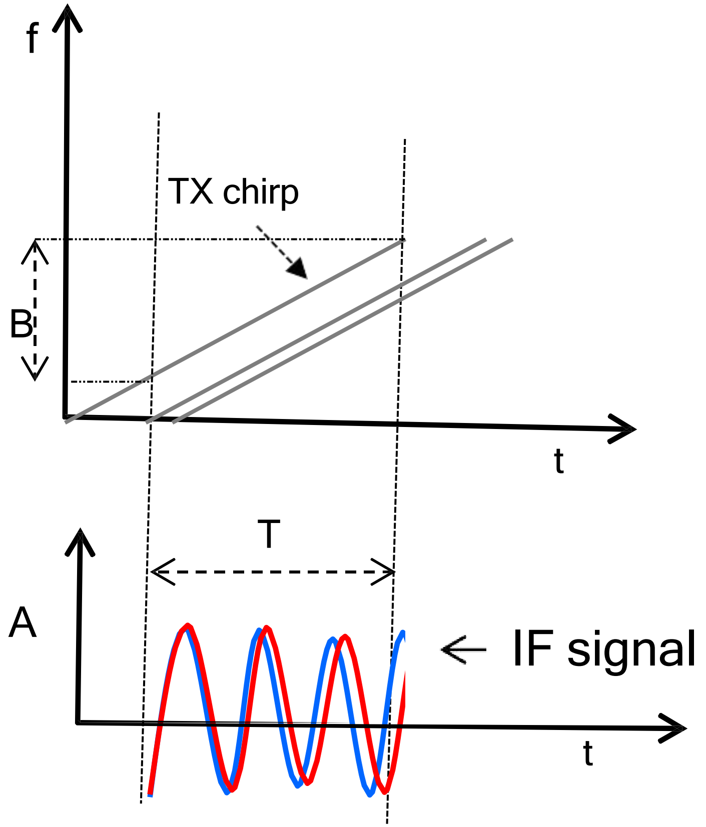
Multiple tones in the IF signal

A frequency spectrum of the IF signal will reveal multiple tones, the frequency of each being proportional to the range of each object from the radar

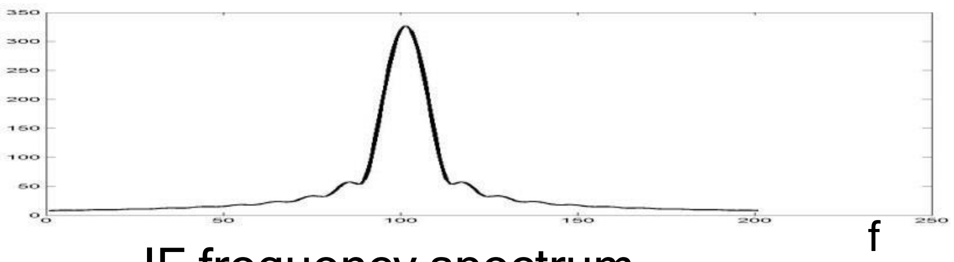


IF frequency spectrum

Range Resolution in a radar

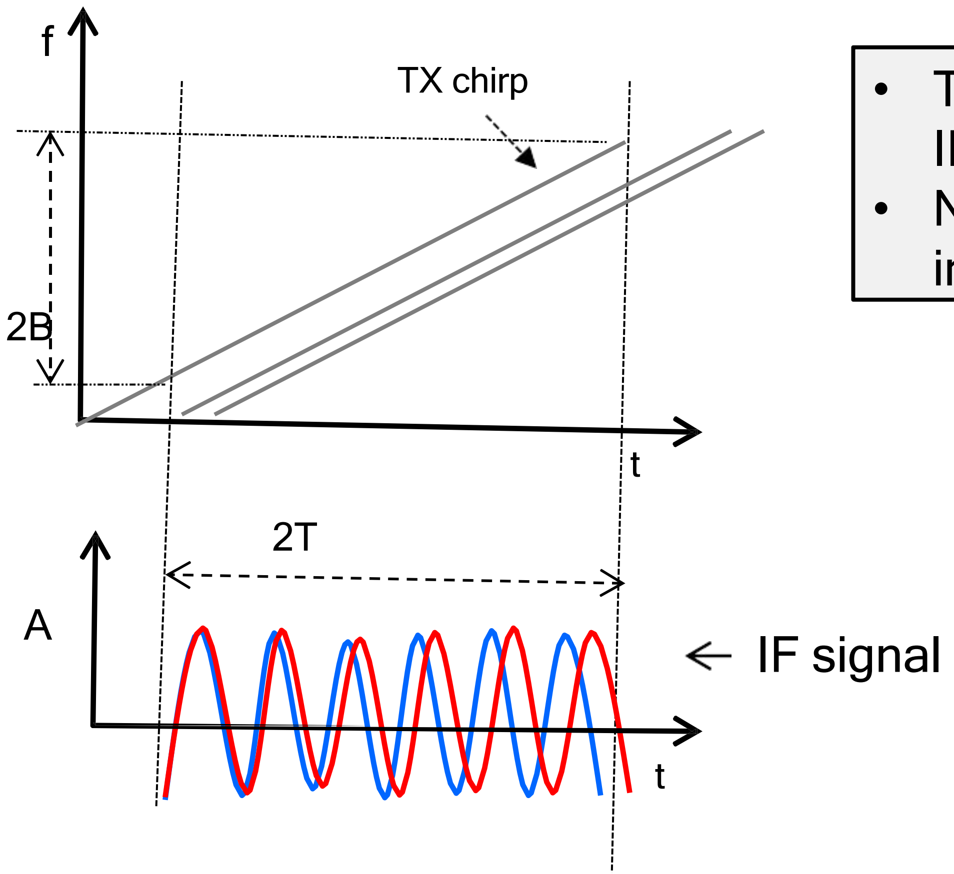


- Range Resolution refers to the ability to resolve two closely spaced objects.
- In this slide, the two objects are too close that they show up as a single peak in the frequency spectrum

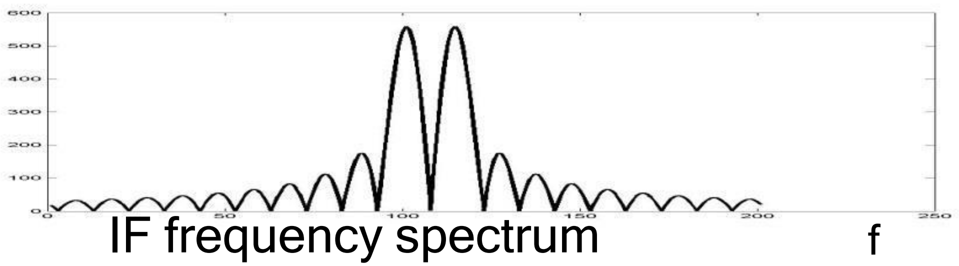


IF frequency spectrum

Range Resolution in a radar



- The two objects can be resolved by increasing the length of the IF signal.
- Note that this also proportionally increases the bandwidth. Thus intuitively: Greater the Bandwidth => better the resolution



Range Resolution in a radar

- Recall that
  - An object at a distance d results in an IF tone of frequency S2d/c
  - Two tones can be resolved in frequency as long as the frequency difference Δf > 1/T
- Can you use the above to derive an equation for the range resolution of the radar?
  - On what parameters does the range resolution depend ? Chirp Duration, Bandwidth, Slope?

For two objects separated by a distance Δd, the difference in their IF frequencies is given by $\Delta f = \frac{S2\Delta d}{c}$

Since the observation interval is T_c, this means that

$$\Delta f > \frac{1}{T_c} \Rightarrow \frac{S2\Delta d}{c} > \frac{1}{T_c} \Rightarrow \Delta d > \frac{c}{2ST_c} \Rightarrow \frac{c}{2B} \quad \text{(since } B=ST_c\text{)}$$

The Range Resolution (d_{res}) depends only on the Bandwidth swept by the chirp

$$d_{res} = \frac{c}{2B}$$

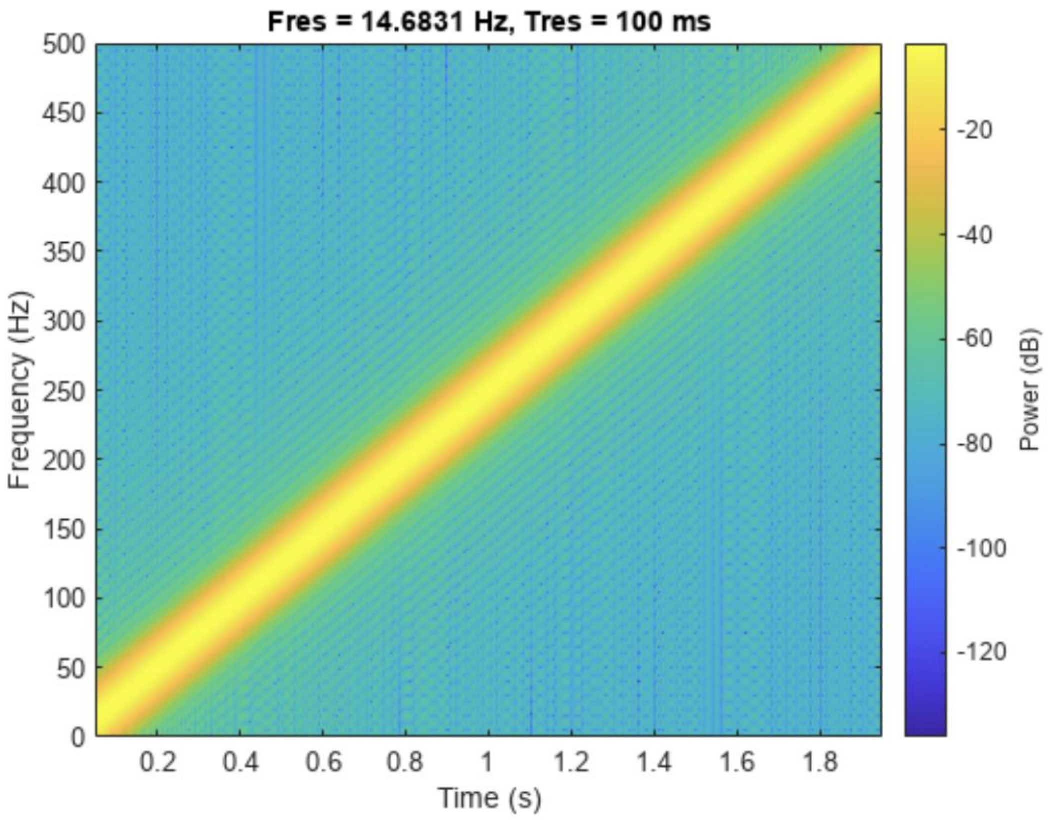
Chirp in MATLAB

Generate a chirp with linear instantaneous frequency deviation. The chirp is sampled at 1 kHz for 2 seconds. The instantaneous frequency is 0 at t = 0 and crosses 250 Hz at t = 1 second.

y = chirp(t,f0,t1,f1) generates samples of a linear swept-frequency cosine signal at the time instances defined in array t. The instantaneous frequency at time 0 is f0 and the instantaneous frequency at time t1 is f1.

```
t = 0:1/1e3:2;
y = chirp(t,0,1,250);
```

```
pspectrum(y,1e3,'spectrogram','TimeResolution',0.1, ...
    'OverlapPercent',99,'Leakage',0.85)
```

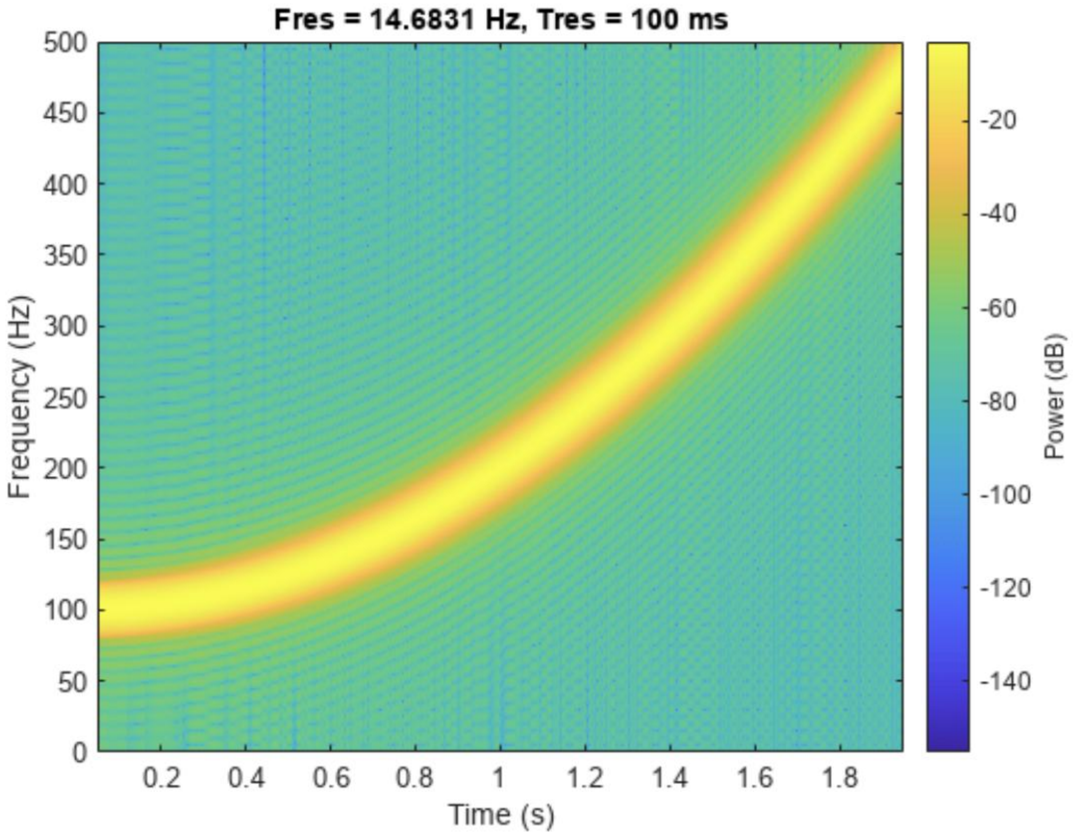


Chirp in MATLAB

Generate a chirp with quadratic instantaneous frequency deviation. The chirp is sampled at 1 kHz for 2 seconds. The instantaneous frequency is 100 Hz at t = 0 and crosses 200 Hz at t = 1 second.

```
t = 0:1/1e3:2;
y = chirp(t,100,1,200,'quadratic');
```

```
pspectrum(y,1e3,'spectrogram','TimeResolution',0.1, ...
    'OverlapPercent',99,'Leakage',0.85)
```



Homework 1

Generate an FMCW signal which can be heard by a human being.

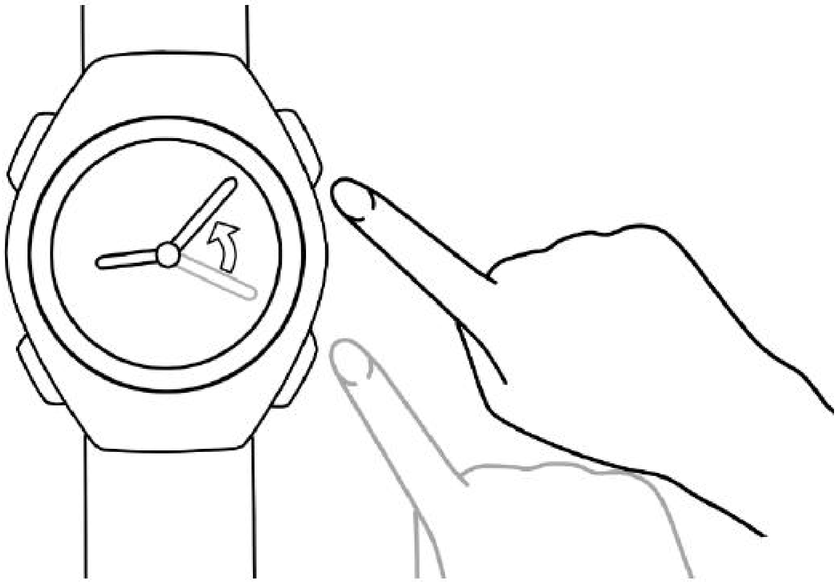
Show its spectrum and save the figure. (pspecturm)

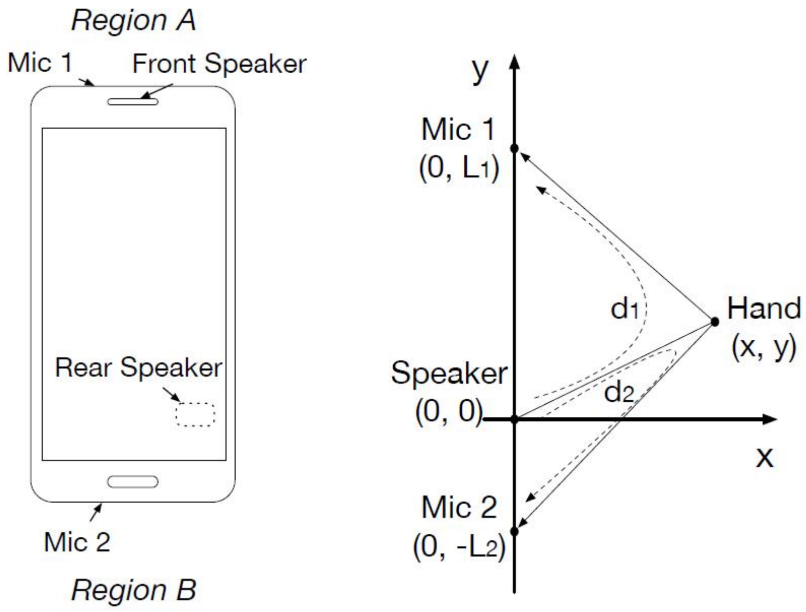
Play it, how does it sounds like?

Hand in your codes , figure and wav file in bb system.

PHASE-BASED DISTANCE TRACKING THEORY

Device-free Gesture Tracking



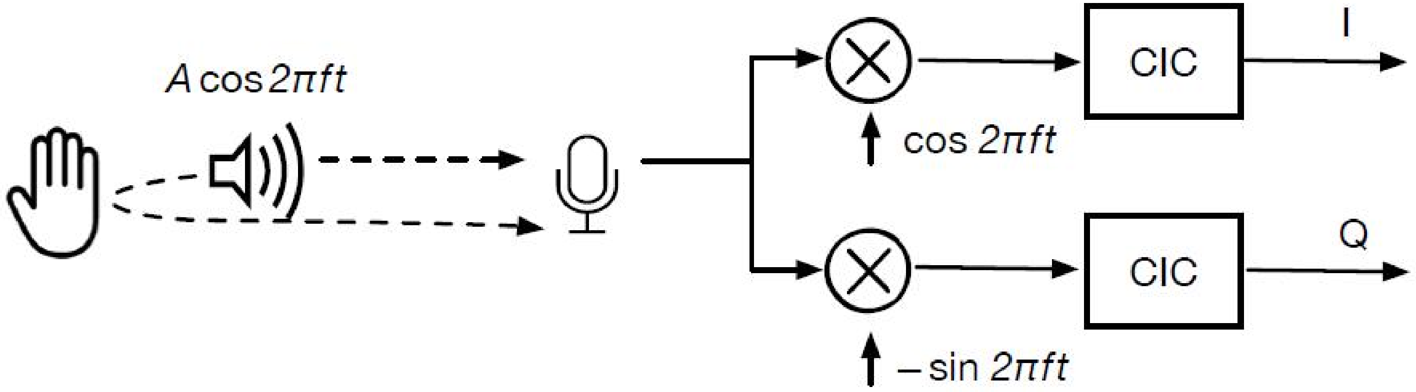


(a) Layout of Samsung S5 (b) Geometric abstraction

Figure 9: Two dimensional tracking

Acoustic signals are transmitted by speaker, reflected by hand/finger, received by microphone.
Use the link below to see a demo:
https://www.youtube.com/watch?v=gs8wMrOSY80

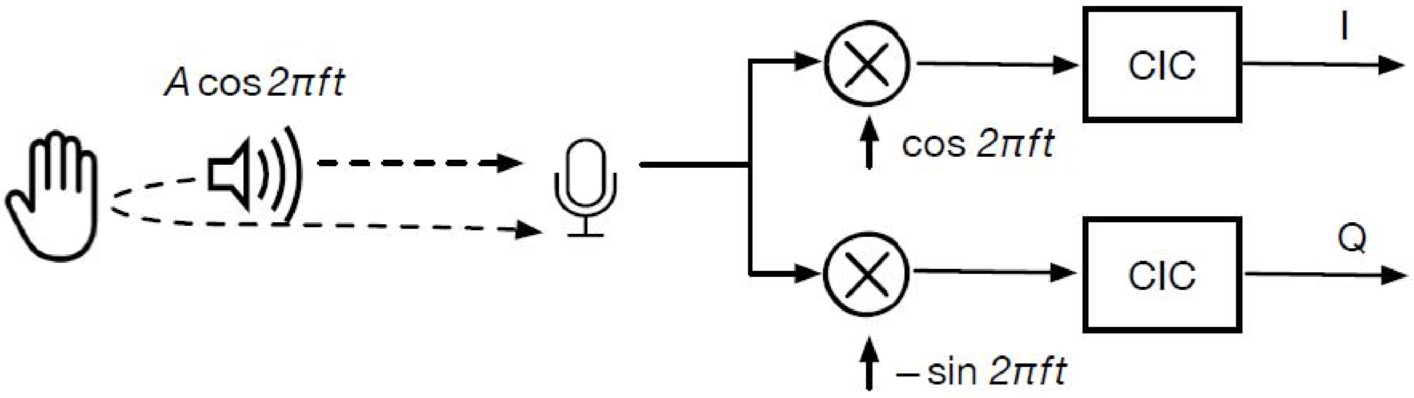
Phase-based Distance Tracking



coherent detector structure

- The sound reflected by a human hand is coherent to the sound emitted by the mobile device. They have a constant phase difference and the same frequency.
- We use a coherent detector to convert the received sound signal into a complex-valued baseband signal.

Phase-based Distance Tracking



coherent detector structure

Transmitted signal:

$$A\cos(2\pi f t)$$

Received signal (after reflection via path p):

$$2A_p^{'}\cos\left(2\pi f t - 2\pi f \frac{d_p(t)}{c} - \theta_p\right)$$

$2A_p^{'}$ – amplitude of the received signal

$d_p(t)$ – propagation distance of path p

c – sound speed

θ_p – phase caused by the hardware delay and phase inversion due to reflection

Phase-based Distance Tracking

- Received signal (after reflection via path p):

$$2A_p^{'} cos(2\pi f\,t - 2\pi f\,\frac{d_p(t)}{c} - \theta_p)$$

- Multiply this received signal with $cos(2\pi f\,t)$:

$$\cos\alpha\cos\beta = \frac{1}{2}[\cos(\alpha+\beta) + \cos(\alpha-\beta)]$$

$$2A_p^{'} cos(2\pi f\,t - 2\pi f\,\frac{d_p(t)}{c} - \theta_p) \times cos(2\pi f\,t)$$

$$= A_p^{'}(\underbrace{cos(-2\pi f\,\frac{d_p(t)}{c} - \theta_p)}_{\text{low-frequency part}} + \underbrace{cos(4\pi f\,t - 2\pi f\,\frac{d_p(t)}{c} - \theta_p)}_{\text{high-frequency part}})$$

- A low-pass filter is then applied, the result is the In-phase signal:

$$I_p(t) = A_p^{'} cos(-2\pi f\,\frac{d_p(t)}{c} - \theta_p)$$

Quadrature signal are derived by multiply the received signal with $-sin(2\pi f\,t)$:

$$Q_p(t) = A_p^{'} sin(-2\pi f\,\frac{d_p(t)}{c} - \theta_p)$$

$$\cos\alpha\sin\beta = \frac{1}{2}[\sin(\alpha+\beta) - \sin(\alpha-\beta)]$$

Phase-based Distance Tracking

$$I_p(t) = A_p^{'} cos(-2\pi f \frac{d_p(t)}{c} - \theta_p)$$
$$Q_p(t) = A_p^{'} sin(-2\pi f \frac{d_p(t)}{c} - \theta_p)$$

- Combining these two components as real and imaginary part of a complex signal, we have the complex baseband as follows:

$$B_p(t) = A_p^{'} e^{-j(2\pi f \frac{d_p(t)}{c} - \theta_p)}$$

- The phase of it is:

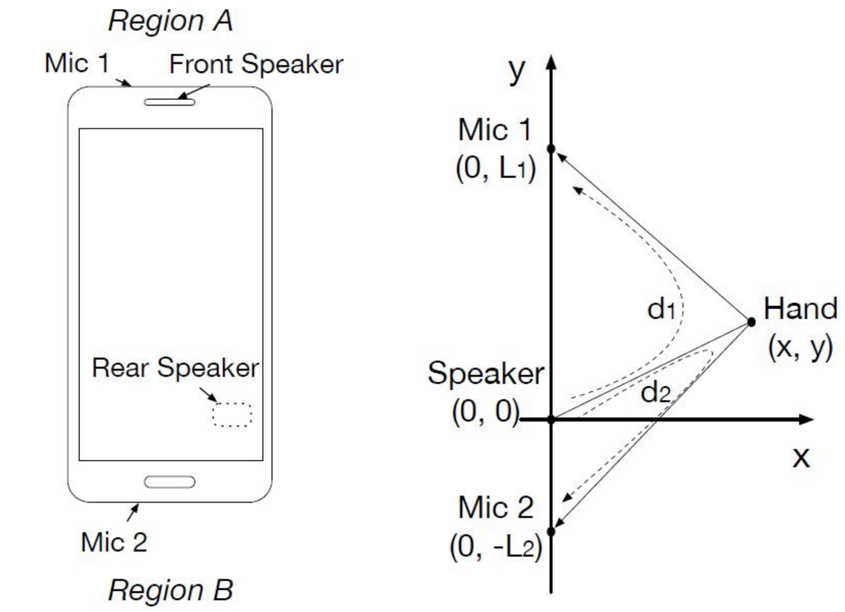
$$\phi_p(t) =- 2\pi f \frac{d_p(t)}{c} - \theta_p$$

- Note that when $d_p(t)$ changes by the amount of sound wavelength $\lambda = \frac{c}{f}$, the phase changes by 2π.

Key Steps

1. Derive In-phase signal I.
   a) Multiply received signal with $cos(2\pi f t)$
   b) Apply a low-pass filter
2. Derive Quadrature signal Q.
   a) Multiply received signal with $-sin(2\pi f t)$
   b) Apply a low-pass filter
3. Calculate the phase using I and Q.
4. Convert phase change to distance change.

Homwork 2



(a) Layout of Samsung S5 (b) Geometric abstraction

Figure 9: Two dimensional tracking

Suppose L1=2, L2=1, (x,y)=(2,1)

Simulate the steps of phase-based distance tracking.

Get the final IQ signals and combining these two components as real and imaginary part of a complex signal.

lowpass, highpass, complex

Hand in your codes , figure and wav file in bb system.

Reference

- S. Rao. Introduction to mmwave sensing: Fmcw radars. Texas Instruments (TI) mmWave Training Series, 2017
- Wang, W., Liu, A. X., & Sun, K. (2016, October). Device-free gesture tracking using acoustic signals. In *Proceedings of the 22nd Annual International Conference on Mobile Computing and Networking* (pp. 82-94).