$with(plots):$

$f := \sin(x)$

$$\sin(x) \tag{1}$$

$seq(\ diff(f, x\$n), n = 1..5);$

$$\cos(x), -\sin(x), -\cos(x), \sin(x), \cos(x) \tag{2}$$

> $seq(simplify(subs(x = 0, diff(f, x\$n))), n = 1..5)$

$$1, 0, -1, 0, 1 \tag{3}$$

> $simplify(subs(x = 0, f)) + sum\left(\frac{simplify(subs(x = 0, diff(f, x\$n))) \cdot x^n}{n!}, n = 1..5\right)$

$$x - \frac{1}{6}x^3 + \frac{1}{120}x^5 \tag{4}$$

$tt := taylor(f, x = 0, 6)$

$$x - \frac{1}{6}x^3 + \frac{1}{120}x^5 + O(x^7) \tag{5}$$

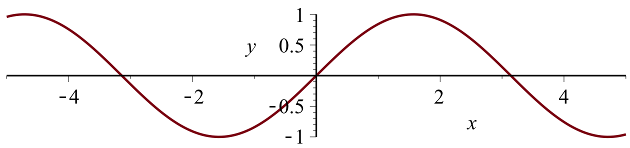
> $tt := taylor(f, x = 0, 14)$

$$tt := x - \frac{1}{6}x^3 + \frac{1}{120}x^5 - \frac{1}{5040}x^7 + \frac{1}{362880}x^9 - \frac{1}{39916800}x^{11} + \frac{1}{6227020800}x^{13} + O(x^{15}) \tag{6}$$

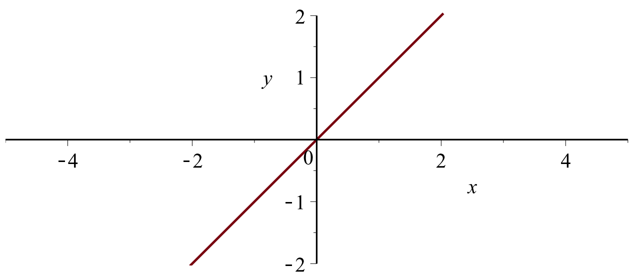
$pp := convert(taylor(f, x = 0, 8), polynom)$

$$x - \frac{1}{6}x^3 + \frac{1}{120}x^5 - \frac{1}{5040}x^7 \tag{7}$$

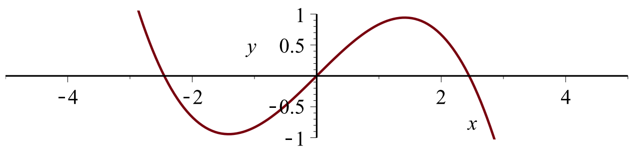
$plot(f, x = -5..5, y = -1..1, scaling = constrained)$



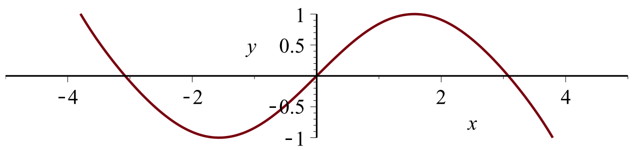
$plot(convert(taylor(f, x = 0, 3), polynom), x = -5..5, y = -2..2, scaling = constrained)$



$$plot(convert(taylor(f, x = 0, 5), polynom), x = -5..5, y = -1..1, scaling = constrained)$$

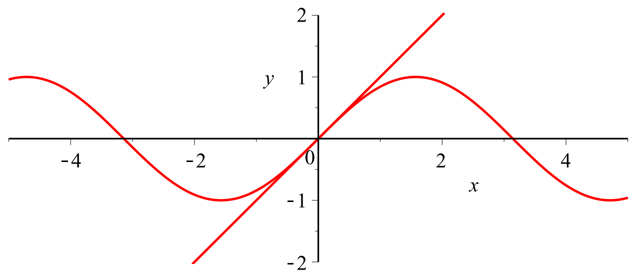


$$plot(convert(taylor(f, x = 0, 9), polynom), x = -5..5, y = -1..1, scaling = constrained)$$



$$animate\left(\text{plot}, \left[\left\{f, sum\left(\frac{simplify(subs(x=0, diff\,(f, x\$n)))\cdot x^n}{n!}, n=0\,..A\right)\right\}, x=-5\,..5, y=-2\,..2,\right.\right.$$
$$\left.\left. scaling=constrained\right], A=1\,..16, frames=16, trace=8\right)$$

$A = 1.$



restart
with(*plots*) :
$f := \exp(x)$

$$e^x \tag{8}$$

seq(*diff* (*f*, *x*\$*n*), *n* = 1 ..5);

$$e^x, e^x, e^x, e^x, e^x \tag{9}$$

> *seq*(*simplify*(*subs*(*x* = 0, *diff* (*f*, *x*\$*n*))), *n* = 1 ..5)

$$1, 1, 1, 1, 1 \tag{10}$$

> $simplify(subs(x=0, f)) + sum\left(\frac{simplify(subs(x=0, diff(f, x\$n)))\cdot x^n}{n!}, n = 1..5\right)$

$$1 + x + \frac{1}{2}\,x^2 + \frac{1}{6}\,x^3 + \frac{1}{24}\,x^4 + \frac{1}{120}\,x^5 \tag{11}$$

$tt := taylor(f, x = 0, 6)$

$$1 + x + \frac{1}{2}\,x^2 + \frac{1}{6}\,x^3 + \frac{1}{24}\,x^4 + \frac{1}{120}\,x^5 + \mathrm{O}(x^6) \tag{12}$$

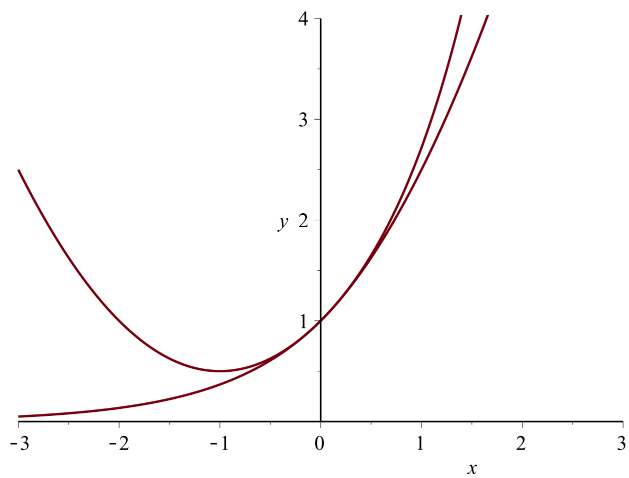
> $tt := taylor(f, x=0, 14)$

$$tt := 1 + x + \frac{1}{2}x^2 + \frac{1}{6}x^3 + \frac{1}{24}x^4 + \frac{1}{120}x^5 + \frac{1}{720}x^6 + \frac{1}{5040}x^7 + \frac{1}{40320}x^8 + \frac{1}{362880}x^9 + \frac{1}{3628800}x^{10} + \frac{1}{39916800}x^{11} + \frac{1}{479001600}x^{12} + \frac{1}{6227020800}x^{13} + \mathrm{O}(x^{14}) \quad \textbf{(13)}$$

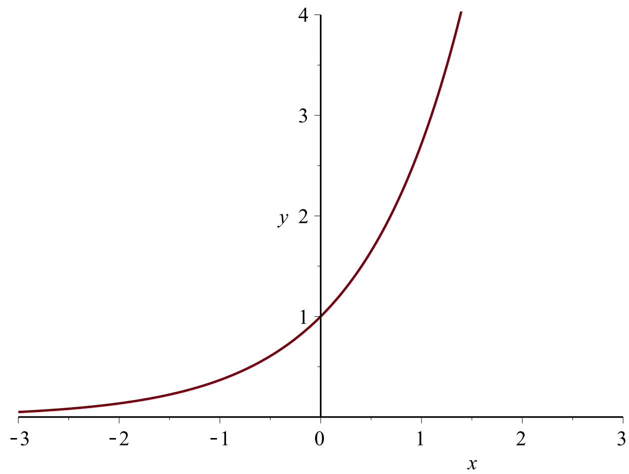
$pp := convert(taylor(f, x=0, 3), polynom)$

$$1 + x + \frac{1}{2}x^2 \quad \textbf{(14)}$$

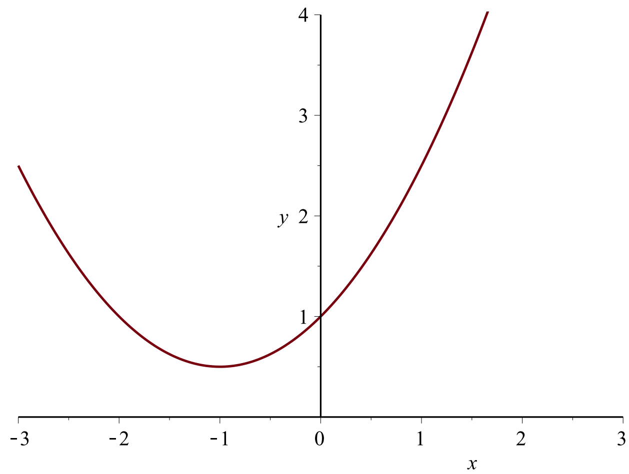
$p1 := plot(f, x=-3..3, y=0..4):$
$p2 := plot(pp, x=-3..3, y=0..4):$
$display([p1, p2], scaling=constrained, color=["red", "green"])$



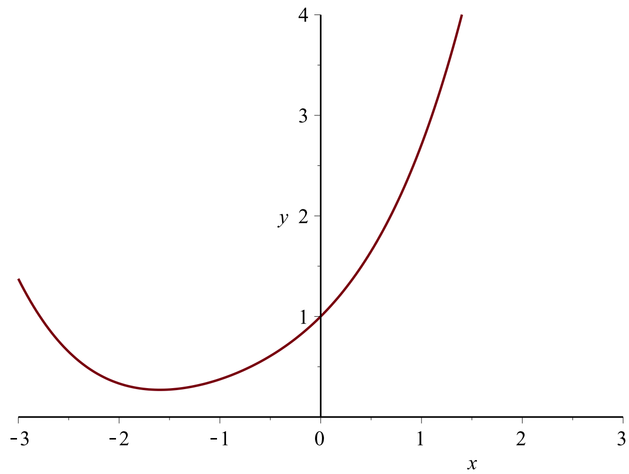
$plot(f, x=-3..3, y=0..4, scaling=constrained)$



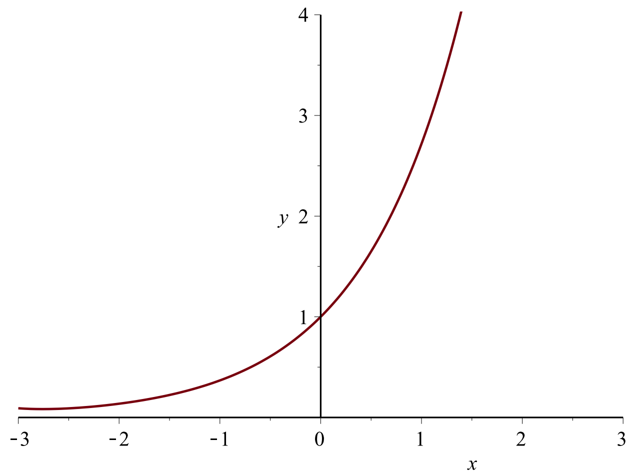
$plot(convert(taylor(f, x = 0, 3), polynom), x = -3 ..3, y = 0 ..4, scaling = constrained)$



$plot(convert(taylor(f, x = 0, 5), polynom), x = -3..3, y = 0..4, scaling = constrained)$



$$plot(convert(taylor(f, x = 0, 9), polynom), x = -3 ..3, y = 0 ..4, scaling = constrained)$$



$$animate\left(\text{plot}, \left[\left\{f, simplify(subs(x=0, f)) + sum\left(\frac{simplify(subs(x=0, diff(f, x\$n)))\cdot x^n}{n!}, n=1..A\right)\right\}, x=-3..3, y=0..4, scaling=constrained\right], A=2..14, frames=7, trace=0\right)$$

Error, numeric exception: division by zero

$restart$

$with(plots):$

$$f := \frac{1}{x}$$

$$\frac{1}{x} \qquad \textbf{(15)}$$

$seq(\ diff(f, x\$n), n=1..5);$

$$-\frac{1}{x^2}, \frac{2}{x^3}, -\frac{6}{x^4}, \frac{24}{x^5}, -\frac{120}{x^6} \qquad \textbf{(16)}$$

> $seq(simplify(subs(x=1, diff(f, x\$n))), n=1..5)$

$$-1, 2, -6, 24, -120 \qquad \textbf{(17)}$$

> $simplify(subs(x=1, f)) + sum\left(\frac{simplify(subs(x=1, diff(f, x\$n)))\cdot(x-1)^n}{n!}, n=1..5\right)$

(18)

$$2-x+(x-1)^2-(x-1)^3+(x-1)^4-(x-1)^5 \quad \textbf{(18)}$$

$tt := taylor(f, x=1, 6)$

$$1-(x-1)+(x-1)^2-(x-1)^3+(x-1)^4-(x-1)^5+\mathrm{O}\left((x-1)^6\right) \quad \textbf{(19)}$$

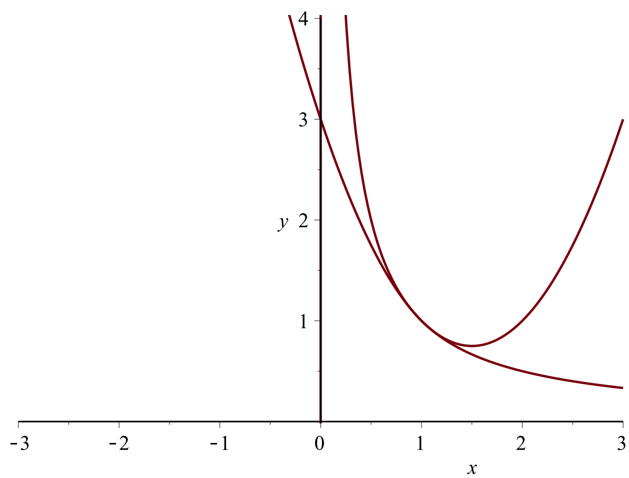
> $tt := taylor(f, x=1, 14)$

$$tt := 1-(x-1)+(x-1)^2-(x-1)^3+(x-1)^4-(x-1)^5+(x-1)^6-(x-1)^7+(x-1)^8-(x-1)^9+(x-1)^{10}-(x-1)^{11}+(x-1)^{12}-(x-1)^{13}+\mathrm{O}\left((x-1)^{14}\right) \quad \textbf{(20)}$$

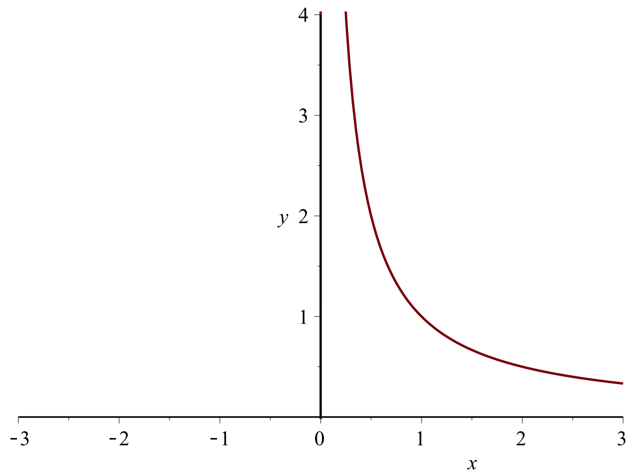
$pp := convert(taylor(f, x=1, 3), polynom)$

$$2-x+(x-1)^2 \quad \textbf{(21)}$$

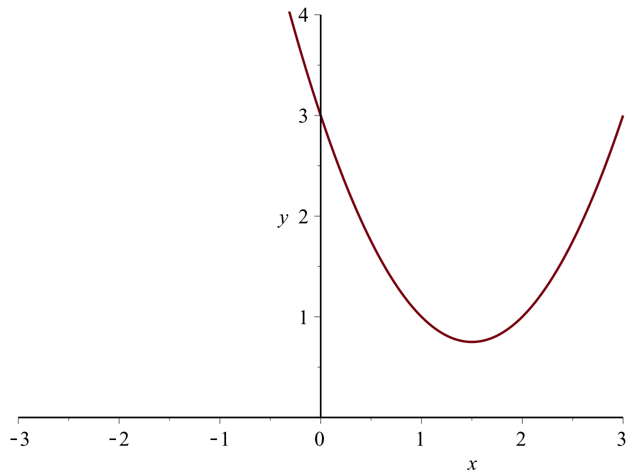
$p1 := plot(f, x=-3..3, y=0..4):$
$p2 := plot(pp, x=-3..3, y=0..4):$
$display([p1, p2], scaling=constrained, color=[\text{"red"}, \text{"green"}])$



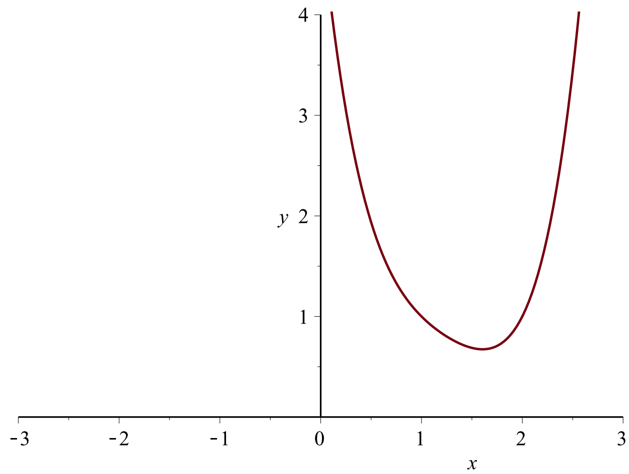
$plot(f, x=-3..3, y=0..4, scaling=constrained)$



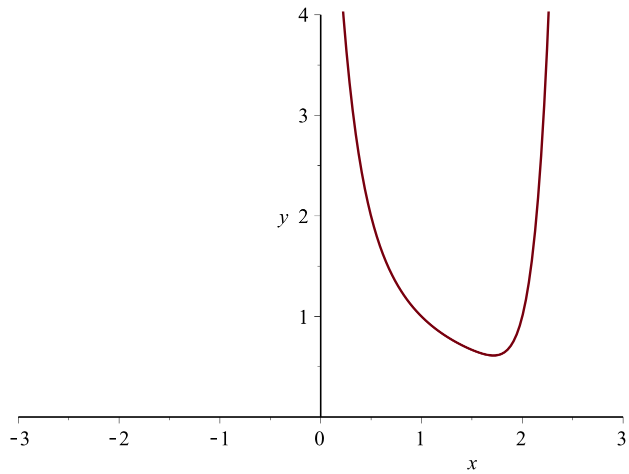
$plot(convert(taylor(f, x=1, 3), polynom), x=-3..3, y=0..4, scaling=constrained)$



$plot(convert(taylor(f, x = 1, 5), polynom), x = -3 ..3, y = 0 ..4, scaling = constrained)$



$$plot(convert(taylor(f, x=1, 9), polynom), x=-3..3, y=0..4, scaling=constrained)$$



$$animate\left(\text{plot}, \left[\left\{f, simplify(subs(x=1, f)) + sum\left(\frac{simplify(subs(x=1, diff(f, x\$n)))\cdot(x-1)^n}{n!}, n = 1..A\right)\right\}, x=-3..3, y=0..4, scaling=constrained\right], A=2..16, frames=8, trace=5\right)$$

$A = 2.$