The Art of (Recursive Bayesian) Persuasion

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Abstract

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Keywords: AI; quantum; bitcoin; Elon Musk

Introduction

Modeling Evidence Effects

World Model: The Stick Task

The stick task consists of a judge, and a set of speakers indexed by I. A sample of N sticks whose lengths are given by

$$\mathcal{S}_N = \{S_1, S_2, ..., S_N\} \tag{1}$$

are drawn i.i.d. from the Uniform$[0,1]$ distribution and is fixed throughout the task.[1]

Each speaker observes the full sample before the task, and at each time step t a speaker i (one per time step, taking it in turn) chooses one stick from the sample to reveal to the judge. The speakers are not permitted to reveal a stick that has previously been shown to the judge. Notationally, the agent chooses action

$$a_t^{(i)} \in \{s_1, s_2, ..., s_N\} \setminus \mathcal{A}_{t-1}, \tag{2}$$

where s_i is the realization of random variable S_i and $\mathcal{A}_{t-1}$ denotes the first $t-1$ actions chosen. For simplicity, we let $\mathcal{A}_0 = \emptyset$.

At each time step t, the judge reasons about the sample mean of the sticks, $\bar{S} = \frac{1}{N}\sum_{n=1}^{N} S_n$.[2] In particular, the judge evaluates his posterior over whether the sample is 'long' or 'short', i.e., whether or not $\bar{S}_N \geq 0.5$. Hence, the relevant posterior for the judge at time step t is

$$p(\bar{S}_N \geq 0.5 \mid \mathcal{A}_t). \tag{3}$$

[1] In practice, we use a discrete approximation to this distribution by modeling the sticks as being drawn uniformly from the set $\{0.025, 0.075, ..., 0.975\}$.

[2] It is assumed that the judge knows N, but only observes the stick values through the speakers' actions.

Each speaker will have an incentive to select evidence that persuades the judge that the sample is either long or short, or the speaker will be indifferent towards the outcome.

The task runs for a total of $T \leq N$ time steps.

Agent Models

In the manner of Rational Speech Act (RSA) models, we model the agents as performing recursive Bayesian inference about one another's beliefs. To capture both evidence effects, we require four layers, which we divide into the 'naïve' layers and the 'pragmatic' layers. The model is implemented in WebPPL, a probabilistic programming language that allows for fast hierarchical inference in low-dimensional domains such as this one.

Naïve Judge and Speaker The first layer, J_0, describes the naïve judge - this judge is naïve in the sense that he does not model the speakers as having any incentives, and instead assumes that the actions are selected uniformly from the available sample. Relabeling without loss of generality, the posterior of J_0 at time t is given by

$$p_{J_0}(\bar{S}_N \geq 0.5 \mid S_1 = s_1, S_2 = s_2, ..., S_t = s_t). \tag{4}$$

The second layer, S_1, describes the naïve speaker, whose choice about which stick to show at time t is represented as a posterior over the available sticks defined in reference to p_{J0}. Importantly, each speaker has a bias β, where in our simulations we model the biases as being in the range $\beta \in \{-10, -5, -2, 0, 2, 5, 10\}$. This bias represents the incentive regarding the judge's inference: a positive (resp. negative) bias entails that the speaker is incentivized to show sticks to the judge that steer the judge towards the belief that the sample is long (resp. short).

To produce this behavior, the speaker samples a stick based on the soft-maximization of biased informativity of that stick, taking into account the previous sticks shown:

$$p_{S_1}(a_t \mid \mathcal{A}_{t-1},\ \mathcal{S}_N) \propto \exp\big(\beta \cdot a_t \cdot p_{J_0}(\bar{S}_N \geq 0.5 \mid \mathcal{A}_{t-1} \cup \{a_t\}\big). \tag{5}$$

Observe that, if $\beta = 0$, this entails that the speaker will sample the sticks uniformly based on the available evidence, which is equivalent to the speaker being indifferent to the outcome.

Pragmatic Judge and Speaker The pragmatic judge, J_1, performs a similar inference as J_0, though with one crucial difference: the judge models each stick as having been sampled by a naïve speaker. To see how this is possible, observe that we can express the action of speaker i at time t by the random variable

$$A_t^{(i)} \sim p_{S_1}(\cdot \mid \mathcal{A}_{t-1},\, \mathcal{S}_N), \tag{6}$$

which is a categorical distribution computed by Equation 9. For ease of notation, we write the set of observations of S_1 speakers

$$\mathcal{A}'_T = \{A_1^{(i_1)} = a_1^{(i_1)}, A_2^{(i_2)} = a_2^{(i_2)}, \ldots, A_T^{(i_T)} = a_T^{(i_T)}\}, \tag{7}$$

and let $\mathcal{A}'_0 = \emptyset$.

The posterior of J_1 can now be computed via Bayes' rule:

$$\begin{aligned} p_{J_1}(\mathcal{S}_N \mid \mathcal{A}'_T) &\propto p(\mathcal{A}'_T \mid \mathcal{S}_N)p(\mathcal{S}_N) \\ &= p(\mathcal{S}_N)\prod_{t=1}^{T} p_{S_1}(A_t^{(i_t)} = a_t^{(i_t)} \mid \mathcal{A}'_{t-1}) \end{aligned}$$

Using the sum rule, we arrive at the relevant posterior, which in the case of discretized stick lengths is given by

$$p_{J_1}(\bar{S}_N \geq 0.5 \mid \mathcal{A}'_T) = \sum_{\mathcal{S}_N :\, \bar{S}_N \geq 0.5} p_{J_1}(\mathcal{S}_N \mid \mathcal{A}'_T). \tag{8}$$

We consider two distinct versions of the pragmatic judge: one in which he knows the biases of each agent in advance, and one in which these biases are drawn, i.i.d., from a categorical prior over the aforementioned range of bias values. This is necessary as the weak and strong evidence effects demand the former and latter versions, respectively.

We further divided the latter case into two by considering two possible priors: a flat prior over the bias range, and a 'V-shaped' prior which down-weights the likelihood of neutrality.[3] We considered these two priors as, although the V-shaped prior better describes the context of the stick task, we wished to show that strong evidence effects were robust to our choice of prior. Factoring in uncertainty over the bias is then a matter of marginalizing over the possible bias settings using the sum rule.

The final layer, the pragmatic speaker S_2, is nearly identical to the naïve speaker S_1, except for the fact that the pragmatic speaker performs soft-maximization based on J_1's judgment, as opposed to J_0's judgment:

$$p_{S_2}(a_t \mid \mathcal{A}_{t-1},\, \mathcal{S}_N) \propto \exp\big(\beta \cdot a_t \cdot p_{J_1}(\bar{S}_N \geq 0.5 \mid \mathcal{A}_{t-1} \cup \{a_t\}\big). \tag{9}$$

This layer is only required to capture the strong evidence effects.

Simulations

Simulation 1: Weak Evidence Effect

Set-Up

[3]Concretely, the V-shaped prior is given by the normalized probability vector $\frac{1}{29}[8,4,2,1,2,4,8]$.

CoGNiTiVe ScIeNcE

Figure 1: This is a figure.

Results

Simulation 2: Strong Evidence Effect for Speakers

Set-Up

CoGNiTiVe ScIeNcE

Figure 2: This is a figure.

Table 1: Sample table title.

Error type	Example
Take smaller	63 - 44 = 21
Always borrow	96 - 42 = 34
0 - N = N	70 - 47 = 37
0 - N = 0	70 - 47 = 30

Results

Simulation 3: Strong Evidence Effect for Judges

Set-Up

CoGNiTiVe ScIeNcE

Figure 3: This is a figure.

Results

Discussion and Future Work

References

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