Adaptive Filter

2016.8(b)
2019.8(b)

Defination:- An Adaptive filter in DSP (Digital Signal processing) is a digital filter whose parameters are automatically adjusted to adapt to changing characteristics of the input signal or environment.

2016.8(b)

Implementation of the LMS Algorithm:

~~1. Initialize the filter coefficient $W_k(i) = 0$~~

~~2. At each sampling period:~~

~~a) Compute the filter output:~~

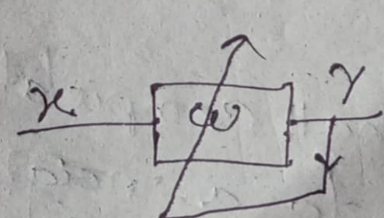
and steepest decent method:

$$\boxed{\xi(n) = E[|e(n)|^2] \rightarrow \text{Mean square error.}}$$

Step-1: Initialization

$w_0 \xrightarrow[\text{optimal value}]{} \omega$

Step-2:- Evaluate the gradient of $\xi(n)$ at current estimate, ω_n



Step-3:- $\omega_{n+1} = \omega_n - \mu \nabla \xi(n)$

$n = 0, 1, \ldots$

Step-4) Repeat step-2:

$$\nabla E(n) = \nabla E\{|e(n)|^2\}$$
$$= E\{\nabla |e(n)|^2\}$$
$$= E\{e(n) \nabla e^*(n)\}$$

$$\nabla e^*(n) = -x^*(n)$$
$$= -E\{e(n)\, x^*(n)\}$$

$$w_{n+1} = w_n - \mu \nabla E(n)$$
$$= w_n + \mu E\{e(n) x^*(n)\}$$

$$E\{e(n) x^*(n)\} = w_n + \mu E[\{d(n) - y(n)\} x^*(n)]$$
$$= w_n + \mu [E[d(n) x^*(n)] - E[y(n) x^*(n)]]$$
$$= w_n + \mu E[d(n) x^*(n)] - E[w_n^T x(n) x^*(n)]$$
$$= w_n + \mu (\underbrace{\hat{\gamma}_{dx}}_{\text{Cross correlation}} - \underbrace{\hat{\gamma}_{xx}}_{\text{auto correlation}} w_n)$$

This is stepest descent method.

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Adaptive filter: (Iterative) Characteristics:-

- Don't need only knowledge of signal statistics before.
- Have small computational complexity
- Converge to a neighborhood of optional solution.

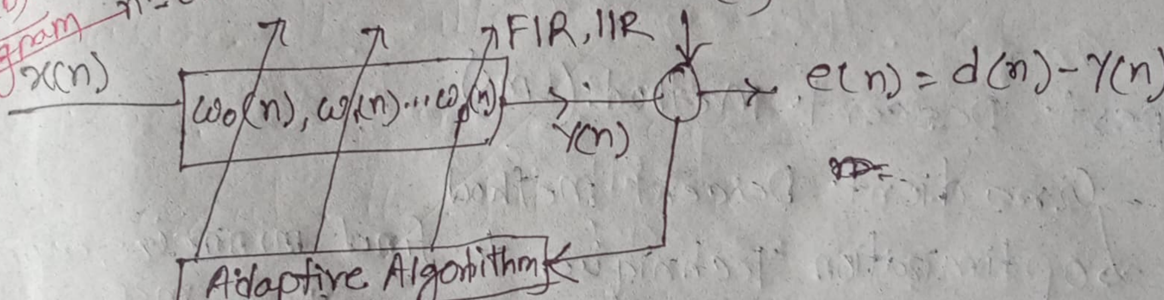
Adaptive filter is good for:

- Real time signal (nonstationary)

Application:

- System identification: (i) Determining the characteristics of an unknown system.
- Signal prediction: (ii) predicts future values of a signal
- noise cancellation: (iii) reducing unwanted noise in audio
- Acoustic Echo cancellation: (iv) Eliminating echoes in telecommunication systems.
- Speech & image coding: (v) Removing noise and artifacts from images

2019, 8(b) adaptive block diagram



$d(n)$ = designed signal

$y(n)$ = output

weight = $w_0(n), \ldots, w_p(n)$

$x(n)$ = input

Two parts:

(I) Filtering: Convolution of $x(n)$, $w(n) \to y(n)$

(II) Adaptive algorithm

error → $e(n)$ feedback

↓ minimize using LMS, RMS

* performance of adapting filter depend as -
  - stablish
  - equality
  - speed of adaptation
  - tracking capabilities

Mean square error: minimize MSE

MSE = $\varepsilon(n) = E\{|e(n)|^2\}$

$= E\{d(n) - y(n)\}^2$

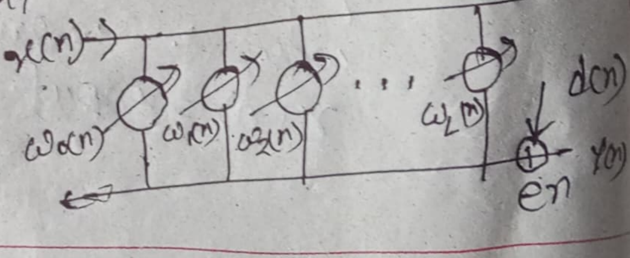
Steepest Descent Method: (LMS)

- Gradient Descent method
  - → optimization Technique to find minimize error MSE

$w(n) = [w_0(n), w_1(n), w_2(n), \ldots, w_L(n)]^T$

$w(n+1) = \Delta w(n) + w(n)$

$\Delta w(n)$ → change in weight



2022

8.a How adaptive filters can be used for noise cancellation.

⇒ Adaptive filters can be used for noise cancellation by dynamically adjusting their coefficients to minimize the difference between a desired signal and the filter output.

- The idea is to subtract the noise from the input signal using a reference noise signal.
- An Adaptive Algorithm (Like-LMS) adjusts the filter so that its output closely matches the noise.
  $\xi(n) = E\{|e(n)|^2\}$
- This output is then subtracted from the noisy signal to obtain the clean desired signal.

example:

In headphones, adaptive filters cancel ambient noise using a microphone that picks up external sounds and inverts them to cancel the noise.

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8.b

Adaptive filter adjust by continuously modifying their coefficients based on the error between the filter output and the desired signal.

The adjustment is required in non-stationary environments such as when the noise or signal characteristics change over time.

Gradient descent method:

1. Compute the ~~error~~ output:

$$y(n) = w^T(n)\,x(n)$$

$x(n)$ → input vector

$w(n)$ → filter weight

2. Compute the error:

$$e(n) = d(n) - y(n)$$

$d(n)$ → desired signal

3. updating the weights:

$$w(n+1) = w(n) + \mu\, e(n)\, x(n)$$

μ → the step-size parameter.

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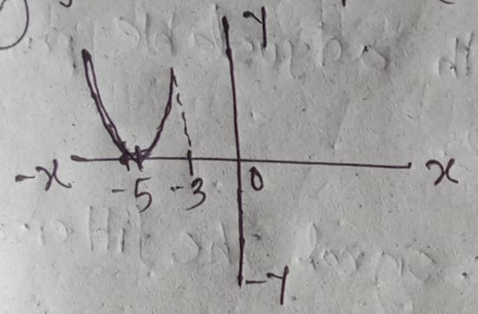
(A) optimization technique to find the minimum value of a function.

⇒ Sag function $y = (x+5)^2$



~~Step~~

(1) choose a starting point,

Let $x = -3$

$y = (x+5)^2$ → Gradient at ⊖ direction

(ii) $\frac{dy}{dx} = 2(x+5)$ Step size, $\mu = 0.01$

(3) $x_0 = -3$, $\frac{dy}{dx} = 2(x+5)$, $\mu = 0.01$

iteration 1:

$x_1 = x_0 - \mu \times \frac{dy}{dx}$

$= (-3) - (0.01) \times 2 \times (-3+5) = -3 - 0.01 \times 2(2)$

$= \cancel{-3.044}\ -3.04$

iterative 2:

$x_2 = x_1 - \mu \times \frac{dy}{dx}$

$= -3.04 - 0.01 \times 2(-3.04+5)$

$= \cancel{-3.044}\ -3.07$

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II adaptive filter এর Block diagram ৭০ define:

(i) Input signal, $x(n)$ ⇒ The signal being processed by the filter.

(ii) Adaptive Filter: A filter with adjustable parameters using FIR or IIR.

(iii) Desired Signal, $d(n)$ ⇒ The signal the filter aims to estimate.

(iv) Error signal, $e(n)$ → difference between the filter's output $y(n)$ and the desired signal $d(n)$.

(v) Adaptive algorithm: based on error signal, usually to minimize it.

(vi) Filter output, $y(n)$ → The output of the adaptive filter.

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