2022

DSP

Section-B

note by Shawda

5.(a)

Analysis Frequency:

Analysis frequency refers to the specific frequency components at which a signal is analyzed using the DFT. These frequencies are determined based on the number of samples and the sampling rate of the signal.

$$f_K = \frac{K}{N} \cdot f_s \qquad k = 0, 1, \ldots, N-1$$

Ex:

$$x[n] = \cos\left(2\pi \cdot \frac{1}{4} \cdot n\right) \qquad n = 0, 1, 2, 3$$

$N = 4$

$f_s = 1$

$\therefore f_0 = 0\,Hz, \; f_1 = 0.25\,Hz, \; f_2 = 0.5\,Hz, \; f_3 = 0.75\,Hz$

DFT magnitude:

The DFT magnitude represents the amplitude of the frequency components present in the signal.

$$|x[K]| = \sqrt{(Re(X[K]))^2 + (Im(X[K]))^2}$$

Ex:

Signal $x[n] = \{1, 2, 3, 4\}$

$$X[K] = \sum_{n=0}^{N-1} x[n] \cdot e^{-j2\pi kn/N}, \quad k = 0, 1, \ldots N-1$$

$X[0] = 1+2+3+4 = 10$

$X[1] = 1 + 2e^{-j2\pi(1)(1)/4} + 3 \cdot e^{-j2\pi(2)(1)/4} + \cdots$

$\vdots$

$X[3] = 1 + 2 \cdot e^{-j2\pi(1)(3)/4} + 3 \cdot e^{-j2\pi(2)(3)/4} + \cdots$

Then magnitude,

$|X[0]| = \sqrt{(10)^2} = 10$

$\vdots$

$|X[3]| = \sqrt{[Re(X(3))]^2 + [Im(X(3))]^2}$

—x—

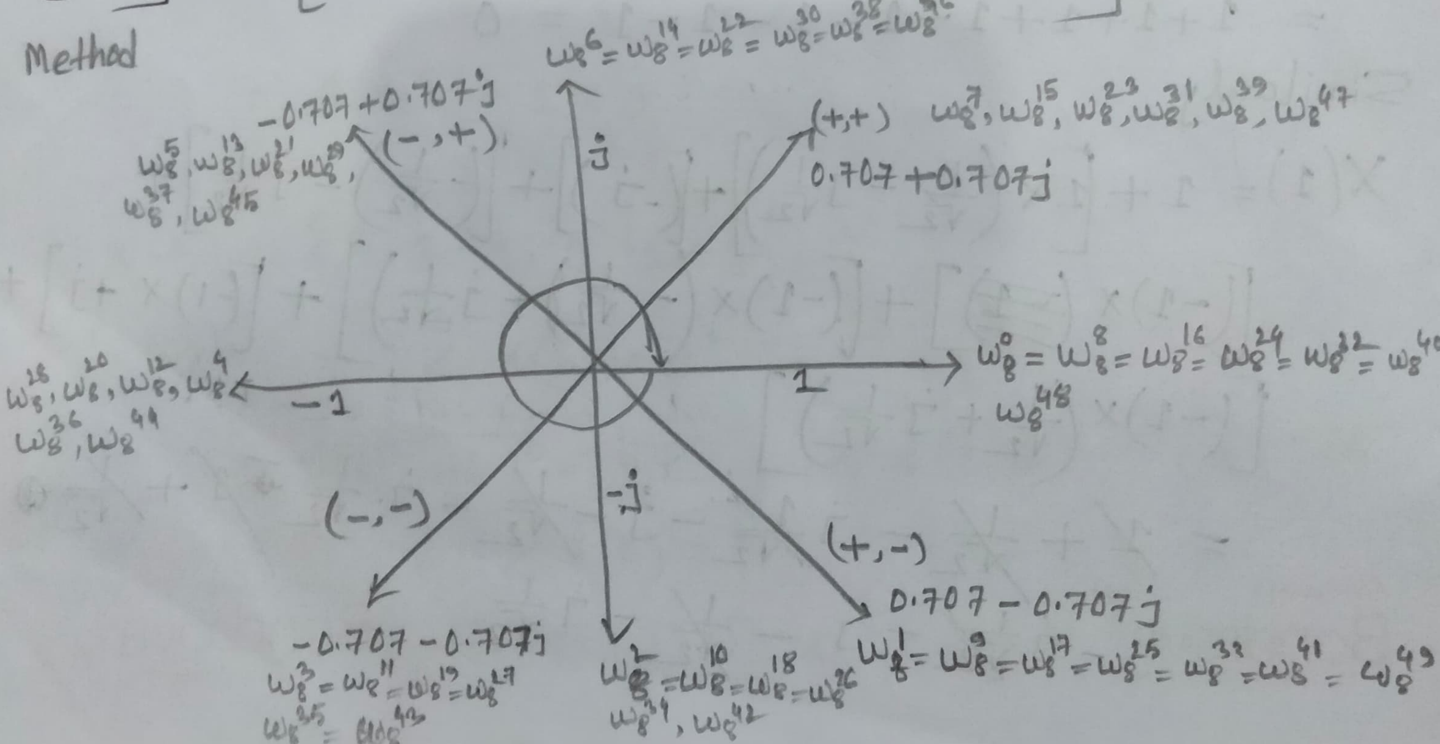
5(c) To choose a window function, must estimate the signal's frequency context to balance frequency resolution and leakage. Signals with closely spaced frequencies need windows with narrow mainlobes. If Leakage is a concern, use windows with low sidelobes. The goal is to highlight true frequencies and suppress false ones. Without frequency estimation, an improper window may distort the spectral analysis.

—x—

5(b) $x(n) = \{1, 1, 1, 1, -1, -1, -1, -1\}$

$$
\underset{X_N}{\begin{bmatrix} X(0) \\ X(1) \\ X(2) \\ X(3) \\ X(4) \\ X(5) \\ X(6) \\ X(7) \end{bmatrix}}
=
\begin{array}{c} \\ 0 \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \\ 7 \end{array}
\begin{array}{c}
\begin{array}{cccccccc} 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 \end{array} \\
\begin{bmatrix}
W_8^0 & W_8^0 & W_8^0 & W_8^0 & W_8^0 & W_8^0 & W_8^0 & W_8^0 \\
W_8^1 & W_8^1 & W_8^2 & W_8^3 & W_8^4 & W_8^5 & W_8^6 & W_8^7 \\
W_8^0 & W_8^2 & W_8^4 & W_8^6 & W_8^8 & W_8^{10} & W_8^{12} & W_8^{14} \\
W_8^0 & W_8^3 & W_8^6 & W_8 & W_8 & W_8 & W_8 & W_8 \\
W_8^0 & W_8^4 & W_8^8 & W_8^{12} & W_8^{16} & W_8^{20} & W_8^{24} & W_8^{28} \\
W_8^0 & W_8^5 & W_8^{10} & W_8^{15} & W_8^{20} & W_8^{25} & W_8^{30} & W_8^{35} \\
W_8^0 & W_8^6 & W_8^{12} & W_8^{18} & W_8^{24} & W_8^{30} & W_8^{36} & W_8^{42} \\
W_8^0 & W_8^7 & W_8^{14} & W_8^{21} & W_8^{28} & W_8^{35} & W_8^{42} & W_8^{49}
\end{bmatrix}
\end{array}
$$

Method



$$
\begin{bmatrix} X(0) \\ X(1) \\ X(2) \\ X(3) \\ X(4) \\ X(5) \\ X(6) \\ X(7) \end{bmatrix}
=
\begin{array}{c|cccccccc}
 & 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 \\
\hline
0 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\
1 & 1 & 0.707-0.707j & -j & -0.707-0.707j & -1 & -0.707+0.707j & j & 0.707+0.707j \\
2 & 1 & -j & -1 & j & 1 & -j & -1 & j \\
3 & 1 & -0.707-0.707j & j & 0.707-0.707j & -1 & 0.707+0.707j & -j & -0.707+0.707j \\
4 & 1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 \\
5 & 1 & -0.707+0.707j & -j & 0.707+0.707j & -1 & 0.707-0.707j & j & -0.707-0.707j \\
6 & 1 & j & -1 & -j & 1 & j & -1 & -j \\
7 & 1 & 0.707+0.707j & j & -0.707+0.707j & -1 & -0.707-0.707j & -j & 0.707-0.707j
\end{array}
\begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \\ - \\ - \\ - \\ - \end{bmatrix}
$$

$$x[0] = 1+1+1+1-1-1-1-1 = 0$$

$$X[1] = (1+\cancel{0.707}-0.707j-j-\cancel{0.707}-0.707j+1+\cancel{0.707}-0.707j-j-\cancel{0.707}-0.707j)$$

$$= 2-2j-4\times 0.707j$$

$$= 2-4.828j$$

$$X[2] = \cancel{1}-\cancel{j}-\cancel{1}+\cancel{j}-\cancel{1}+\cancel{j}+\cancel{1}-\cancel{j} = 0$$

$$X[3] = (1-\cancel{0.707}-0.707j+j+\cancel{0.707}-0.707j+1-\cancel{0.707}-0.707j+j+\cancel{0.707}-0.707j)$$

$$= 2-0.828j$$

$$x(4) = \cancel{1} - \cancel{1} + \cancel{1} - \cancel{1} - \cancel{1} + \cancel{1} - \cancel{1} + \cancel{1}$$
$$= 0$$

$$x(5) = 1 - \cancel{0.707} + 0.707j - j + \cancel{0.707} + 0.707j + 1$$
$$- \cancel{0.707} + 0.707j - j + \cancel{0.707} + 0.707j$$
$$= 2 + 0.828j$$

$$x(6) = 1 + j - 1 - j - 1 - j + 1 + j$$
$$= 0$$

$$x(7) = 1 + \cancel{0.707} + 0.707j + j - \cancel{0.707} + 0.707j + 1 + \cancel{0.707}$$
$$+ 0.707j + j - \cancel{0.707} + 0.707j$$
$$= 2 + 4.828j$$

$$x(k) = \{0,\ 2 - 4.828j,\ 0,\ 2 - 0.828j,\ 0,\ 2 + 0.828j,\ 0,\ 2 + 4.828j\}$$

(Ans)

—x—

6.(a)

Discrete Fourier Transform (DFT)

In mathematics, the discrete Fourier Transform (DFT) is a transformation technique that is used to convert a sequence of discrete datapoints from time domain to frequency domain.

$$\text{DFT: } x(k) = \sum_{n=0}^{N-1} x(n) \cdot e^{-j2\pi kn/N}$$

for $k = 0, 1, 2 \ldots, N-1$

$$\text{or, } x(k) = \sum_{n=0}^{N-1} x(n) \cdot W_N^{kn}$$

Inverse ~~Four~~ Discrete Fourier Transform (IDFT)

The inverse discrete Fourier transform (IDFT) is a mathematical operation that is used to convert a digital signal represented in the frequency domain into the time domain.

$$x(n) = \frac{1}{N} \sum_{k=0}^{N-1} x(k)\, e^{j\frac{2\pi nk}{N}}$$

$$\text{or, } x(n) = \sum_{k=0}^{N-1} X(k)\, W_n^{-nk}$$

(b) Linearity property:

If $x_1[n] \leftrightarrow x_1[k]$ and
$x_2[n] \leftrightarrow x_2[k]$

then,
$a.x_1[n] + b.x_2[n] \leftrightarrow a x_1[k] + b.x_2[k]$

proof:

$$DFT\{x[n]\} = X[k] = \sum_{n=0}^{N-1} x[n]\, e^{-j\frac{2\pi}{N}kn}$$

$$\sum_{n=0}^{N-1} (ax_1[n] + bx_2[n])\, e^{-j.\frac{2\pi}{N}kn} = a \sum x_1[n] e^{-j\frac{2\pi}{N}kn} + b \sum x_2[n] e^{-j\frac{2\pi}{N}kn}$$

$$= aX_1[k] + bX_2[k]$$

Example:

Symmetry property

If $x[n]$ is real, then the DFT exhibits conjugate symmetry

$$X[N-k] = X^*[k] \quad , \text{ for real } x[n]$$

proof:

$$X[k] = \sum_{n=0}^{N-1} x[n] \, e^{-j\frac{2\pi}{N}kn}$$

$$\Rightarrow X^*[k] = \sum_{n=0}^{N-1} x[n] \, e^{j\frac{2\pi}{N}kn} = X[N-k]$$

so, $X[N-k] = X^*[k]$

(c) The DFT is a sampled version of the DTFT at discrete frequency points. DTFT is continuous in frequency, while DFT gives values at N equally spaced points.

DFT assumes the input signal is periodic, unlike DTFT. Both are periodic in frequency with period 2π. DFT is used in practice because it is computationally efficient using the FFT algorithm.

— x —

7. (a)

Transfer function: The transfer function of a filter describes the frequency-domain behavior of a filter, relating the input and output signals in terms of their amplitudes and phases. It essentially shows how a filter affects the signal at different frequencies.

Discuss the realisation of the FIR system using convolution operation -

The FIR can be implemented from the convolution expression directly.

Given the generalized convolution summation as:

$$y[n] = x[n] * h[n] = \sum_{k=-\infty}^{\infty} h[k]\, x[n-k]$$

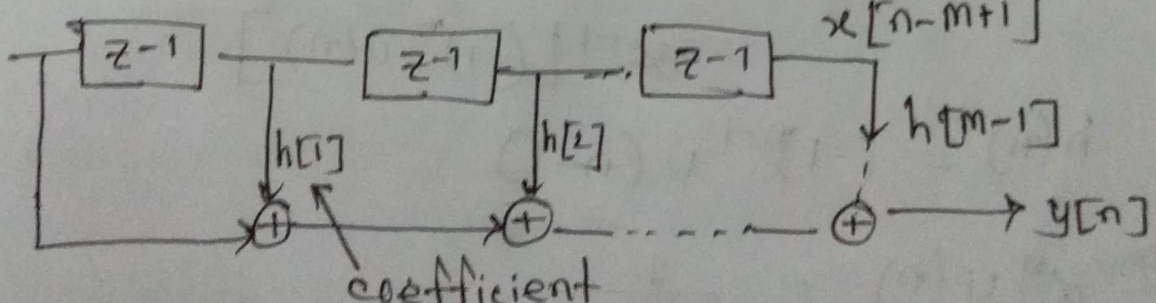
h[n] is the impulse response of the filter and x[n] is our input signal.

Here, is a convolution with a Finite Impulse Response (FIR) which is causal.

$$y[n] = \sum_{k=0}^{M-1} h[k]\, x[n-k] \quad \Big| \quad \text{start, } k=0, \ \text{sample} = M$$

FIR system is easily implementatiod directly from convolution summation.

$$y[n] = \sum_{k=0}^{M-1} h[k]\, x[n-k]$$



— x —

(b) No, Finite Impulse Response (FIR) filters are not recursive. They are also referred to as non-recursive filters. FIR filters have an impulse response that is finite in duration, meaning it settles to zero after a finite number of time samples.

$$y(n) = x(n-1) - y(n-1)$$

z transform both side,

$$Y(z) = X(z) \cdot z^{-1} - Y(z) \cdot z^{-1}$$

$$\Rightarrow Y(z)\left[1 + z^{-1}\right] = X(z) \cdot z^{-1}$$

$$\Rightarrow \frac{Y(z)}{X(z)} = \frac{z^{-1}}{1+z^{-1}}$$

$$= \frac{z^{-1}}{1-(-1)z^{-1}}$$

$$= z^{-1}\left[(-1)^n u(n)\right]$$

$$h(n) = (-1)^{n-1} u(n-1)$$

$$\alpha^n u(n) = \frac{1}{1-\alpha z^{-1}}$$

$$H(z) = h(n)$$

$$x(n) = (-1)^n u(n-1)$$

$$h(n) = \{0, 1, -1, 1, -1 \ldots\}$$ It's IIR filter

2022 same as 2021/7(c)

7(c)

$$y(n) = -0.5y(n-1) + 0.25y(n-2) + x(n) + 0.4x(n-1)$$

or, $y(n) + 0.5y(n-1) - 0.25y(n-2) = x(n) + 0.4x(n-1)$

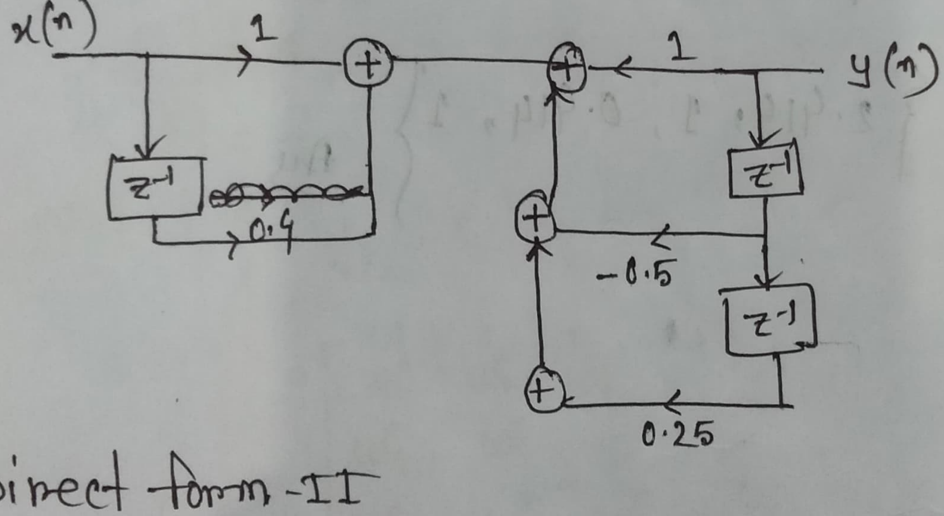
or, $Y(z) + 0.5z^{-1}Y(z) - 0.25z^{-2}Y(z) = X(z) + 0.4z^{-1}X(z)$

or, $Y(z)\left[1 + 0.5z^{-1} - 0.25z^{-2}\right] = X(z)\left[1 + 0.4z^{-1}\right]$.

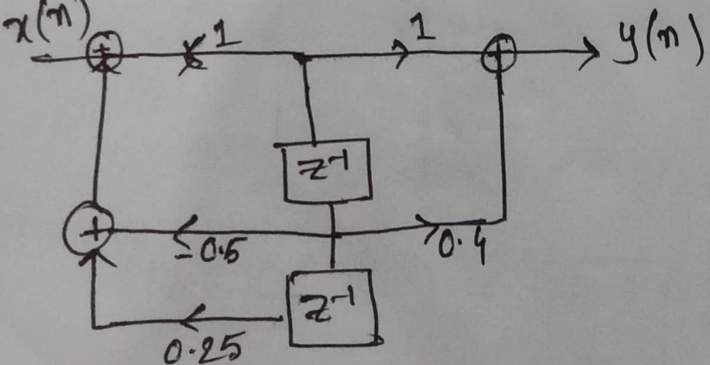
বা, $\dfrac{Y(z)}{X(z)} = \dfrac{1 + 0.4z^{-1}}{1 + 0.5z^{-1} - 0.25z^{-2}}$

বা, $H(z) = \dfrac{1 + 0.4z^{-1}}{1 + 0.5z^{-1} - 0.25z^{-2}}$

Direct form-I



Direct form-II



— x —

2021

5.(a) Same as 6(b)

Time shift properties:

The time-shifting property of discrete-time Fourier transform states that if a signal $x(n)$ is shifted by k in time domain, then its DTFT is multiplied by e^{-Jwk}. Therefore, if

$$x(n) \xleftrightarrow{FT} X(\omega)$$

Then

$$x(n-k) \xleftrightarrow{FT} e^{-J\omega k} X(\omega)$$

where, k is an integer.

Proof: From the definition of DFT, we have,

$$F[x(n)] = X(\omega) = \sum_{n=-\infty}^{\infty} x(n)\, e^{-J\omega n}$$

$$\therefore F[x(n-k)] = \sum_{n=-\infty}^{\infty} x(n-k)\, e^{-J\omega n}$$

substituting $(n-k) = m$ then $n = m+k$, we get

$$F[x(n-k)] = \sum_{m=-\infty}^{\infty} x(m)\, e^{-J\omega(m+k)}$$

$$= \sum_{m=-\infty}^{\infty} x(m)\, e^{-J\omega m}\, e^{-J\omega k}$$

$$= e^{-j\omega k} \sum_{m=-\infty}^{\infty} x(m) e^{-j\omega m}$$

$$\therefore F[x(n-k)] = e^{-j\omega k} X(\omega) ; \text{[Time delay]}$$

Similarly

$$F[x(n+k)] = e^{j\omega k} X(\omega) \text{ [Time advance]}$$

– x –

2021

5.(b) $x(n) = \{1, 1, 0, 0\}$

$$X(m) = \sum_{n=0}^{N-1} x(n)\left[\cos\left(\frac{2\pi nm}{N}\right) - j\sin\left(\frac{2\pi nm}{N}\right)\right]$$

For, $N = 4$

$$X(m) = \sum_{n=0}^{3} x(n)\left[\cos\left(\frac{2\pi nm}{4}\right) - j\sin\left(\frac{2\pi nm}{4}\right)\right]$$

$m = 0$,

$$X(0) = x(0)\left[\cos\left(\frac{2\pi.0.0}{4}\right) - j\sin\left(\frac{2\pi.0.0}{4}\right)\right] + x(1)\left[\cos\left(\frac{2\pi.1.0}{4}\right) - j\sin\left(\frac{2\pi.1.0}{4}\right)\right] + x(2)\left[\cos\left(\frac{2\pi.2.0}{4}\right) - j\sin\left(\frac{2\pi.2.0}{4}\right)\right] + x(3)\left[\cos\left(\frac{2\pi.3.0}{4}\right) - j\sin\left(\frac{2\pi.3.0}{4}\right)\right]$$

$$= 1 + 1 + 0 + 0$$
$$= 2$$

$m = 1$,

$$X(1) = x(0)\left[\cos\left(\frac{2\pi.0.1}{4}\right) - j\sin\left(\frac{2\pi.0.1}{4}\right)\right] + x(1)\left[\cos\left(\frac{2\pi.1.1}{4}\right) - j\sin\left(\frac{2\pi.1.1}{4}\right)\right] + x(2)\left[\cos\left(\frac{2\pi.2.1}{4}\right) - j\sin\left(\frac{2\pi.2.1}{4}\right)\right] + x(3)\left[\cos\left(\frac{2\pi.3.1}{4}\right) - j\sin\left(\frac{2\pi.3.1}{4}\right)\right]$$

$$= 1\times 1 + 1\times(-j) + 0 + 0$$
$$= 1 - j$$

$m=2,$

$$X(2) = x(0)\left[\cos\left(\frac{2\pi.0.2}{4}\right) - j\sin\left(\frac{2\pi.0.2}{4}\right)\right] +$$
$$x(1)\left[\cos\left(\frac{2\pi.1.2}{4}\right) - j\sin\left(\frac{2\pi.1.2}{4}\right)\right] +$$
$$x(2)\left[\cos\left(\frac{2\pi.2.2}{4}\right) - j\sin\left(\frac{2\pi.2.2}{4}\right)\right] +$$
$$x(3)\left[\cos\left(\frac{2\pi.3.2}{4}\right) - j\sin\left(\frac{2\pi.3.2}{4}\right)\right]$$

$$= 1\times 1 + 1\times(-1) + 0 + 0$$
$$= 1 - 1 = 0$$

$m=3,$

$$X(3) = x(0)\left[\cos\left(\frac{2\pi.0.3}{4}\right) - j\sin\left(\frac{2\pi.0.3}{4}\right)\right] +$$
$$x(1)\left[\cos\left(\frac{2\pi.1.3}{4}\right) - j\sin\left(\frac{2\pi.1.3}{4}\right)\right] +$$
$$x(2)\left[\cos\left(\frac{2\pi.2.3}{4}\right) - j\sin\left(\frac{2\pi.2.3}{4}\right)\right] +$$
$$x(3)\left[\cos\left(\frac{2\pi.3.3}{4}\right) - j\sin\left(\frac{2\pi.3.3}{4}\right)\right]$$

$$= 1\times 1 + 1\times j$$
$$= 1 + j$$

$$X(K) = \{2, 1-j, 0, 1+j\}$$

$|X(0)| = 2$, $\angle X(0) = 0^\circ$

$|X(1)| = \sqrt{(1)^2 + (-1)^2} = \sqrt{2}$, $\angle X(1) = \tan^{-1}\left(\frac{-1}{1}\right) = -45^\circ$

$|X(2)| = 0$, $\angle X(2) = 0^\circ$

$|X(3)| = \sqrt{1^2 + 1^2} = \sqrt{2}$, $\angle X(3) = 45^\circ$ Ans

5.(c)

Spectral leakage:

Spectral leakage is a phenomenon that occurs during frequency analysis of signals, specifically when using techniques like the Discrete Fourier Transform or Fast Fourier Transform.

Windowing helps reduce spectral leakage by tapering the signal at its ends before applying the Fourier Transform. This reduces the sharp transitions at the signal boundaries, which are a major cause of leakage. By minimizing discontinuities, window functions like Hamming or Hann smooth the signal edges. This leads to a more accurate representation of frequency components. As a result, energy spreads less into adjacent frequencies.

6. (a) Twiddle factors:

Twiddle factors are a set of values that is used to speed up DFT and IDFT calculations. It represented with the letter W. Twiddle factors are complex constant coefficients used in FFT algorithm.

Twiddle factor help reduce the number of calculations by reusing symmetry properties. They allow efficient reuse of computations in the FFT process. This makes FFT much faster than directly calculating the DFT.

For an N-point DFT, the twiddle factor is defined as:

$$W_N^k = e^{\frac{-j2\pi k}{N}} = \cos\left(\frac{2\pi k}{N}\right) - j\sin\left(\frac{2\pi k}{N}\right)$$

1. $W_0 = e^{\frac{-j \cdot 2\pi \cdot 0}{4}} = 1 + 0j = 1$

2. $W_1 = e^{\frac{-j \cdot 2\pi \cdot 1}{4}} = -j$

3. $w_2 = e^{-j2\pi\frac{2}{4}} = -1+0\cdot j = -1$

4. $w_3 = e^{-j2\pi\frac{3}{4}} = 0+j = j$

5. $w_4 = e^{-j2\pi\frac{4}{4}} = 1+0j = 1$

$\therefore$ the values are

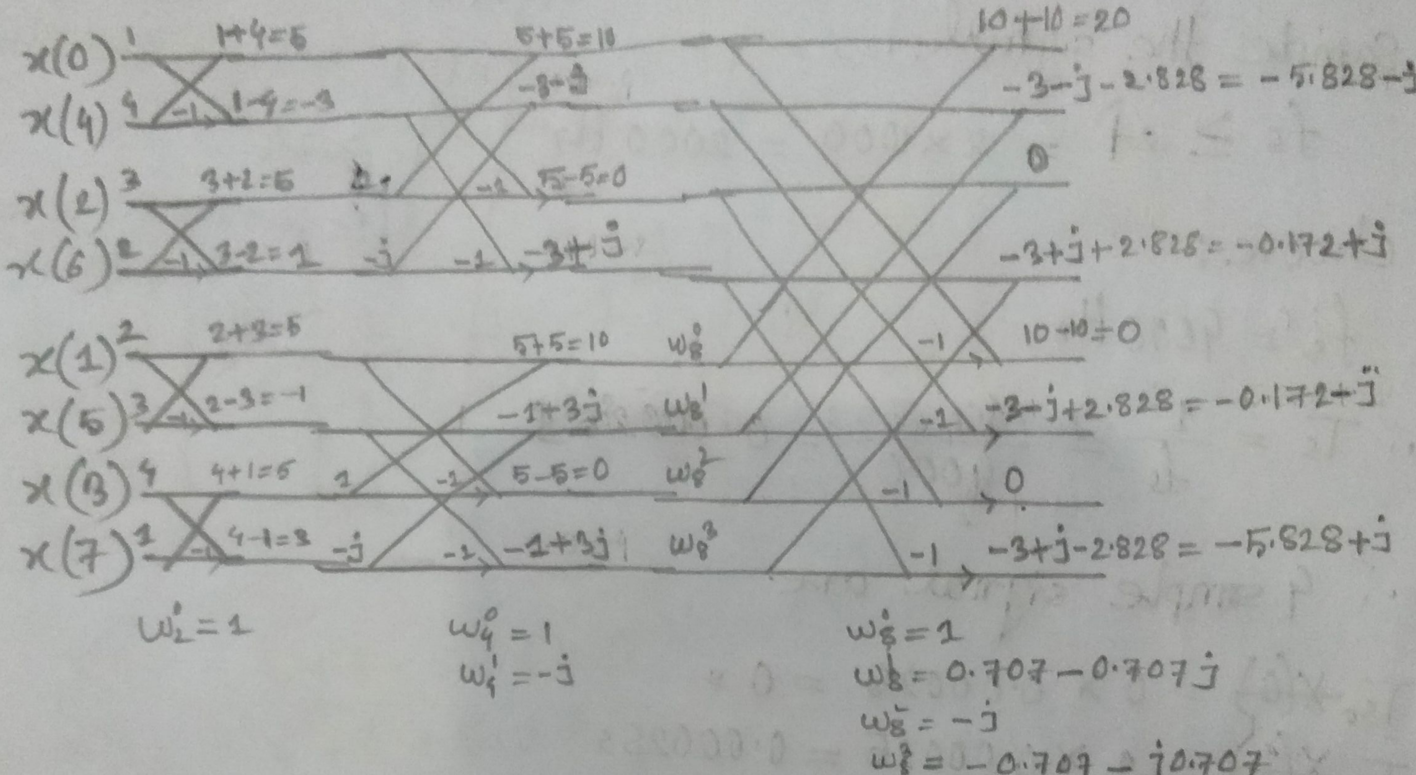
$w_0 = 1$, $w_1 = -j$, $w_2 = -1$, $w_3 = j$

$w_4 = 1$

2021

6(b)

$$x(n) = [\underset{0}{1}, \underset{1}{2}, \underset{2}{3}, \underset{3}{4}, \underset{4}{4}, \underset{5}{3}, \underset{6}{2}, \underset{7}{1}]$$



$(-1-3j) \times (0.707 - 0.0707j)$

$(-1+3j) \times (-0.707 - j0.707)$

$(-1-3j) \times (0.707 - 0.707j)$

$(-1+3j) \times (-0.707 - j0.707)$

— x —

7.(a) same as 2020 7(a)

FIR filter:

$$y[n] = \sum_{k=0}^{N} b_k \cdot x[n-k]$$

Z transform:

$$Y(z) = \sum_{k=0}^{N} b_k \cdot X(z) \cdot z^{-k}$$

$$H(z) = \frac{Y(z)}{X(z)} = \sum_{k=0}^{N} b_k \cdot z^{-k}$$

IIR filter:

$$y[n] = \sum_{k=0}^{M} b_k \cdot x[n-k] - \sum_{k=1}^{N} a_k \cdot y[n-k]$$

z transform

$$Y(z) = \sum_{k=0}^{M} b_k \cdot X(z) \cdot z^{-k} - \sum_{k=1}^{N} a_k \cdot y(z) \cdot z^{-k}$$

$$\frac{Y(z)}{X(z)} = H(z) = \frac{\sum_{k=0}^{M} b_k \cancel{x(z)}\, z^{-k}}{1 + \sum_{k=1}^{N} a_k z^{-k}}$$

8. (a)

—x—

2021

7.(b)

$$y(n) = x(n) + 2x(n-1) + x(n-2)$$
$$h(n) = \delta(n) + 2\delta(n-1) + \delta(n-2)$$

$$H(e^{j\omega}) = 1 + 2 \cdot e^{-j\omega} + e^{-j2\omega}$$

or, $H(e^{j\omega}) = e^{-j\omega}\left(e^{j\omega} + 2 + e^{-j\omega}\right)$ $\left[e^{j\omega} + e^{-j\omega} = 2\cos(\omega)\right]$

$$= e^{-j\omega}\left(2 + 2\cos(\omega)\right)$$
$$= 2e^{-j\omega}\left(1 + \cos\omega\right)$$

$$\left[1 + \cos\omega = 2\cos^2\left(\frac{\omega}{2}\right)\right]$$

$$H(e^{j\omega}) = 4\cos^2\left(\frac{\omega}{2}\right) \cdot e^{-j\omega}$$

Find the magnitude of $H(e^{j\omega})$

$$|H(e^{j\omega})| = \left|4\cos^2\left(\frac{\omega}{2}\right) \cdot e^{-j\omega}\right|$$
$$= 4\left|\cos^2\left(\frac{\omega}{2}\right)\right|$$

$\omega = 0$ then $|H(e^{j\omega})| = 4$

$\omega = \pi$ then $|H(e^{j\omega})| = 0$

$\therefore$ This expression presents a low pass FIR filter.

7(c) same as 2022

— x —

8. (a)

Aliasing effect:

Aliasing is the distortion that occurs when a signal is sampled below its Nyquist rate. It causes different signals to become indistinguishable from each other during reconstruction.

we need an anti aliasing filter to remove high-frequency components before sampling a signal. This prevents aliasing by ensuring the signal is band-limited below the Nyquist frequency. It helps in accurate signal reconstruction without distortion.

—x—

(b)

$$x_a(t) = 10\sin 350\pi t + 25\cos 400\pi t - 15\cos 450\pi t$$

~~we know,~~

$$\cancel{f = \frac{\omega}{2\pi}}$$

or, $x_a(t) = 10\sin(\underbrace{2\times 175\pi t}_{2f\pi t}) + 25\cos(2\times 200\pi t) - 15\cos(2\times 225\pi t)$

$\therefore f_1 = 175$ Hz

$f_2 = 200$ Hz

$f_3 = 225$ Hz

$\therefore$ Highest frequency $= 225$ Hz

Nyquist Sampling Rate $\Rightarrow f_s \geq 2\times$ Highest frequency

$f_s \geq 2\times 225$

$f_s \geq 450$ Hz

$\therefore$ Minimum sampling rate $= 450$ Hz

Maximum magnitude $= |10| + |25| + |15|$
$= 50$

Ans

—x—

(c) Input signal $x(t)$
output $y(t) = x(t)u(t)$

check time invariant:

Apply time shift, $t \to t - t_0$

Input: $x(t - t_0)$
output: $y(t) = x(t - t_0) \cdot u(t)$

compare the output:

$y(t - t_0) = x(t - t_0) \cdot u(t - t_0)$

$\therefore x(t - t_0)u(t) \neq x(t - t_0)u(t - t_0)$

$\therefore$ The system is not time invariant.

Linear:

Input $x_1(t), x_2(t)$
Output $y_1(t), y_2(t)$

$y_1(t) = x_1(t)\, u(t)$

$y_2(t) = x_2(t)\, u(t)$

For any constants a, b:

$a\, y_1(t) + b\, y_2(t) = a_1 x_1(t) u(t) + b x_2(t) u(t)$

$= [a_1 x_1(t) + b x_2(t)] u(t)$

This is same as the output for input

$a_0 x_1(t) + b\{x_2\}(t)$

so, the system is linear.

— x —

2020

5. (a) z transform:

The z-transform is a mathematical tool used to analyze discrete-time signals and systems. It converts a time-domain signal x[n] into a complex frequency-domain representation X(z).

ROC :

ROC (Region of convergence) is the range of values in the complex plane for which the z-transform of a signal converges. It determines the stability and causality of a discrete-time system.

(b)

(i) $x_1(n) = 1, \underset{\uparrow}{2}, 3, 4, 5$

z transform,

$$X_1(z) = 1z + 2 + 3z^{-1} + 4z^{-2} + 5z^{-3}$$

ROC is entire z-plane except possibly $z=0$ and $z=\infty$.

ROC : $0 < |z| < \infty$

(ii) $x_2(n) = \{0, 0, 1, 2, 3, 4, 5, 1\}$

z transform,

$$x_2(z) = 1z^{-1} + 2z^{-2} + 3z^{-3} + 4z^{-4} + 5z^{-5} + z^{-6}$$

Roc is all z except $z = 0$

ROC : $z \neq 0$.

-x-

(c) Fourier series:

The Fourier series is a mathematical tool used to represent a periodic signal as a sum of sin and cos function. It breaks down a complex periodic signal into simpler sinusoidal components of different frequencies, amplitudes and phases.

Some applications of fourier transform:

1. Signal Processing
2. Image processing
3. Communications
4. Vibration and Acoustics
5. Medical Imaging.

—x—

7.(a) Same as 2022 6(a)

(b) Same as 2022 6(b)

~~(c) Same as~~

(c) DFT leakage: DFT leakage occurs when a signals frequency components do not align exactly with the DFT frequency bins.

This causes the energy of a frequency component to spread into adjacent bins, distorting the spectrum and making it harder to identify true frequency components.

(b) **Time Domain:** The Hamming window is a smoothing window function used to reduce spectral leakage in signal processing.

Frequency Domain: In the frequency domain, the Hamming window has a main lobe and side lobes. The main lobe is wider, which reduces frequency resolution. The side lobes are lower in amplitude compared to a rectangular window, which significantly reduces spectral leakage.

—x—

8. (c) Same as 2021 6

Twiddle factor importance in FFT equation:-

Twiddle factors are complex exponential terms used in the FFT algorithm. They are defined as $W_N^k = e^{-j\frac{2\pi k}{N}}$.

They introduce necessary phase shifts to combine smaller DFT results correctly. Their symmetry and periodicity help reduce computation. Twiddle factors are key to achieving the FFT's fast performance.

—x—

DFT

2020

8(b) $x(n) = \{1, 0, 1, 1\}$

4 point DFT —

$$X(m) = \sum_{n=0}^{N-1} x(n)\left[\cos\left(\frac{2\pi n m}{N}\right) - j\sin\left(\frac{2\pi n m}{N}\right)\right]$$

$$\therefore X(m) = \sum_{n=0}^{3} x(n)\left[\cos\left(\frac{2\pi n m}{4}\right) - j\sin\left(\frac{2\pi n m}{4}\right)\right]$$

$m = 0$ then,

$$X(0) = x(0)\left[\cos\left(\frac{2\pi.0.0}{4}\right) - j\sin\left(\frac{2\pi.0.0}{4}\right)\right] + x(1)\left[\cos\left(\frac{2\pi.1.0}{4}\right) - j\sin\left(\frac{2\pi.1.0}{4}\right)\right] + x(2)\left[\cos\left(\frac{2\pi.2.0}{4}\right) - j\sin\left(\frac{2.\pi.2.0}{4}\right)\right] + x(3)\left[\cos\left(\frac{2\pi.3.0}{4}\right) - j\sin\left(\frac{2\pi.3.0}{4}\right)\right]$$

$$= 1\times1 + 0\times1 + 1\times1 + 1\times1$$

$$= 3$$

$m = 1$,

$$X(1) = x(0)\left[\cos\left(\frac{2\pi\,0.1}{4}\right) - j\sin\left(\frac{2\pi.0.1}{4}\right)\right] + x(1)\left[\cos\left(\frac{2\pi.1.1}{4}\right) - j\sin\left(\frac{2\pi.1.1}{4}\right)\right] + x(2)\left[\cos\left(\frac{2\pi.2.1}{4}\right) - j\sin\left(\frac{2\pi.2.1}{4}\right)\right] + x(3)\left[\cos\left(\frac{2\pi.3.1}{4}\right) - j\sin\left(\frac{2\pi.3.1}{4}\right)\right]$$

$$= 1\times1 + 0 + 1\times(-1) + 1\times[0 - j(-1)]$$

$$= 1 - 1 + j = j$$

$m=2,$

$$X(2) = x(0)\left[\cos\left(\frac{2\pi.0.2}{4}\right) - j\sin\left(\frac{2\pi.0.2}{4}\right)\right] +$$
$$x(1)\left[\cos\left(\frac{2\pi.1.2}{4}\right) - j\sin\left(\frac{2\pi.1.2}{4}\right)\right] +$$
$$x(2)\left[\cos\left(\frac{2.\pi.2.2}{4}\right) - j\sin\left(\frac{2\pi.2.2}{4}\right)\right] +$$
$$x(3)\left[\cos\left(\frac{2\pi.2.3}{4}\right) - j\sin\left(\frac{2\pi.2.3}{4}\right)\right]$$

$$= 1\times 1 + 0 + 1\times 1 + 1\times(-1)$$
$$= 1+1-1 = 1$$

$m=3,$

$$X(3) = x(0)\left[\cos\left(\frac{2\pi.0.3}{4}\right) - j\sin\left(\frac{2\pi.0.3}{4}\right)\right] +$$
$$x(1)\left[\cos\left(\frac{2\pi.1.3}{4}\right) - j\sin\left(\frac{2\pi.1.3}{4}\right)\right] +$$
$$x(2)\left[\cos\left(\frac{2\pi.2.3}{4}\right) - j\sin\left(\frac{2\pi.2.3}{4}\right)\right] +$$
$$x(3)\left[\cos\left(\frac{2\pi.3.3}{4}\right) - j\sin\left(\frac{2\pi.3.3}{4}\right)\right]$$

$$= 1\times 1 + 0 + 1\times(-1) + 1\times(-j)$$
$$= \cancel{1} - \cancel{1} - j = -j$$

$$\therefore X(k) = \{3, j, 1, -j\}$$