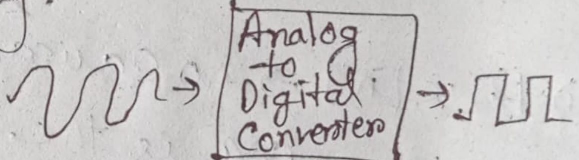
2021.

1.a. Define Digital signal processing.

Digital signal processing (DSP) is the analysis and modification of digital signals to improve their quality extract information or transform their various application.
It includes like filtering, transforming and analyzing signal etc.

* Discuss the advantages and limitations of digital signal processing.

Advantages of DSP:



(i) Noise: It includes digital signal which has a less probablity of getting mixed with unwanted signal, so noise is low.

(ii) Data storage: It is used to store digital data in a simple way.

(iii) Encryption: ~~It is used to~~ Digital signals are involved in simple encryption.

(iv) Detection and correction;

(v) Data transmission

Limitations of DSP:

(i) **Complexity**: use more components, increasing system complexity.

(ii) **Power**: Consumes more power than analog systems.

(iii) **Cost**: Digital signal processors are very expensive.

(iv) **Bandwidth**: Requires more bandwidth for data transmission

(v) **Sampling and Quantization Errors**: Can cause error signal.

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required to avoid aliasing when digitizing a continuous-time signal. defined as twice the highest frequency component.

(b) **Nyquist Rate**: Nyquist Rate is the minimum sampling rate, ~~needed to accurately represent a continuous signal in its discrete form~~.

Nyquist Rate, $f_N = 2 f_m \rightarrow$ max frequency.

$$x_a(t) = 3\cos 50\pi t + 10\sin 300\pi t - \cos 100\pi t$$

$f_1 = 25$ Hz $f_2 = 150$ Hz $f_3 = 50$ Hz

$\omega = 2\pi f$

$f = \frac{\omega}{2\pi}$

$F_{max} = 150$ Hz

$F_N = 2F_{max} = 2 \times 150$
$= 300$ Hz

(Ans)

যদি ω হয়,

$f_1 = \frac{25\omega}{\pi}$ $f_2 = \frac{150\omega}{\pi}$

$f_3 = \frac{50\omega}{\pi}$ $F_{max} = \frac{150\omega}{\pi}$

$F_N = 2F_{max} = \frac{300\omega}{\pi}$

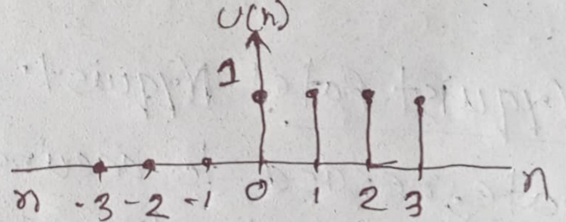
c)

Unit sample sequence $\delta(n)$	Unit step sequence $u(n)$
$\delta[n] = 1$ at $n=0$, 0 elsewhere	$u[n] = 1$ for $n \geq 0$, 0 for $n<0$
~~Nature~~ Impulse (single spike)	Step (constant, $n=0$)
A single point at $n=0$	A flat line starting from $n=0$
$\sum \delta(n) = 0$	$\sum u(n) = \infty$ (diverges).
Used to test system response at a point	Used to analyze system behaviour over time

(*) Signal $x(-n)$ and $x(-n+2)$ where $x(n)$ is a unit step sequence.

discrete unit step:

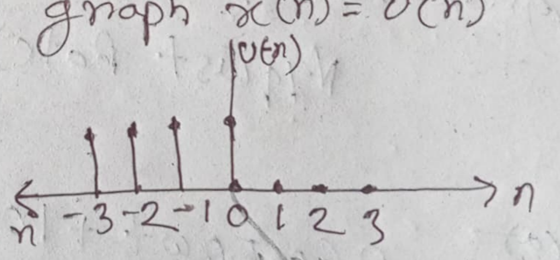
$$u(n) = \begin{cases} 1 & n \geq 0 \\ 0 & n < 0 \end{cases}$$



graph $x(n) = u(n)$

(*) Graph $x(-n)$

$$u(n) = \begin{cases} 1 & n \leq 0 \\ 0 & n > 0 \end{cases}$$

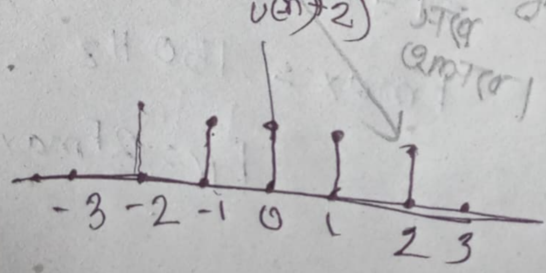


$x(-n+2)$ shift left by 2

$= x(-(n-2)) = x(2-n)$

~~n = -1 0 1 2 3~~

~~x(-n+2) = 1~~



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Q2.1
1.a

The impulse response $h(n)$ of a system characterizes the system's behaviour by defining its output when the input is a unit impulse; it is crucial because it allows the output for any input signal to be determined through convolution with $h(n)$.

$$y(n) = \sum_{K=-\alpha}^{\alpha} x(K)\, h(n-K)$$

$h(n) = 0$ for $n < 0$

$$h(n) = \sum_{K=0}^{\alpha} x(K)\, a^{n-K}$$

$x(0) = 1$ $\quad K = 0,1,2,3$
$x(1) = 0$
$x(2) = 1$
$x(3) = 1$

$x(n) = \{1, 0, 1, 1\}$ we write this

$$y(n) = 1 \cdot a^{n-0} + 0 \cdot a^{n-1} + 1 \cdot a^{n-2} + 1 \cdot a^{n-3}$$

$$y(n) = a^{n} + a^{n-2} + a^{n-3}$$

(Answer)

2.b)

Convolution	Cross-Correlation
(i) $y(n) = x(n) * h(n)$ $= \sum x(K)\, h(n-K)$	(i) $R_{xy}(l) = \sum x(K)\, y(K+l)$ or $\sum x(n)\, x(n-l)$
(ii) Measures output of LTI system.	(ii) Measures similarity between two signals
(iii) one signal is time-reversed and shifted.	(iii) only shifted, not reversed
(iv) Signal filtering, system response	(iv) Pattern matching, signal alignment

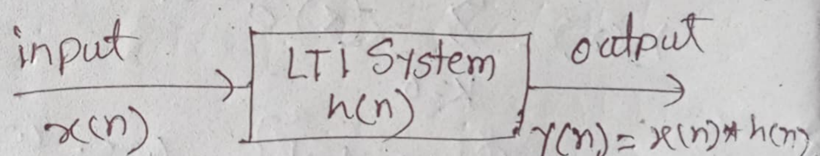
Defination:

Cross-Correlation: Measures the similarity between two signals $x(n)$ and $y(n)$ by shifting one signal across the other without flipping (reversing) any sequence.

defined as -

$$\gamma_{xy}(l) = \sum_{n=\alpha}^{\alpha} x(n)\, y(n-l)$$

Convolution: Calculates the output of a system with an input $x(n)$ and impulse response $h(n)$ by flipping $h(n)$ and then shifting it across $x(n)$.



$$y(n) = x(n) * h(n)$$

$$= \sum_{k=-\alpha}^{\alpha} x(k)\, h(n-k)$$

Given, $x(n) = \{1, 2, 3, 1\}$

$$R_{xx}(l) = \sum_{n} x(n) \cdot x(n-l)$$

lag, $l = 0$,
0 হলে

$$R_{xx}(0) = x(0)^2 + x(1)^2 + x(2)^2 + x(3)^2$$

$$= 1^2 + 2^2 + 3^2 + 1^2$$

$$= 15$$

যখন diff 1 হলে,

$l = 1$, $\cancel{R_{xx}(1) = x(0)\,x(1) + x(1)\,x(1) + x(2)\,x(1) + x(3)\,x(2)}$

$\cancel{= 1\times 2 + 2\times 2 + 3\times 2 + 1\times 2}$

$$R_{xx}(1) = x(0)\,x(1) + x(2)\,x(1) + x(3)\,x(2)$$

$$= 1\times 2 + 3\times 2 + 1\times 3$$

$$= 11$$

diff 2 ঘরে,

$l = 2$,

$\boxed{\text{lag অনুসারে diff}}$

$R_{xx}(2) = x(0)x(2) + x(1)x(3)$

$= 1 \times 3 + 2 \times 1$

$= 5$

$l = 3$ $\quad R_{xx}(3) = x(0)x(3)$

$= 1 \times 1 = 1$

$R_{xx}(l) = 0, \; l > 3$

$l = -1$

$R_{xx}(-1) = x(1)x(0) + x(2)x(1) + x(3)x(2)$

$= 2 + 6 + 3 = 11$

$l = -2$, $R_{xx}(-2) = x(2)x(0) + x(3)x(1)$

$= 3 + 2 = 5$

$l = -3$, $R_{xx}(-3) = x(3)x(0)$

$= 1 \times 1 = 1$

$R_{xx}(l) = 0, \; l < -3$

$R_{xx}(l) = \{1, 5, 11, \underset{\uparrow}{15}, 11, 5, 1\}$

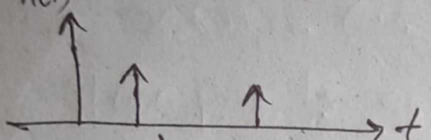
Correlation is max value at lag 0

— * —

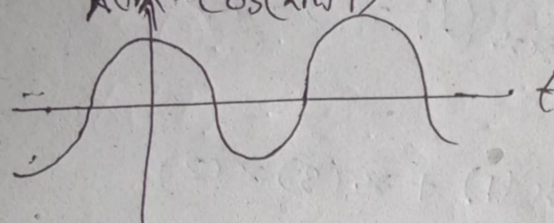
c) Show that convolution in one domain (time domain) equals point-wise multiplication in other domain (frequency domain).

$$y(t) = h(t) * x(t)$$

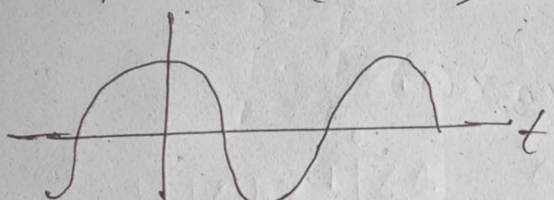
$$x(t) \longrightarrow \boxed{h(t)} \rightarrow y(t)$$



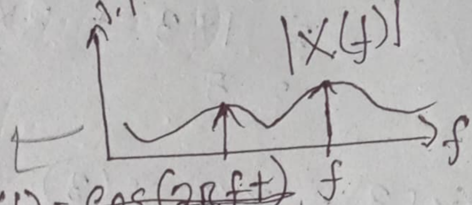
$x(t) = \cos(2\pi f t)$



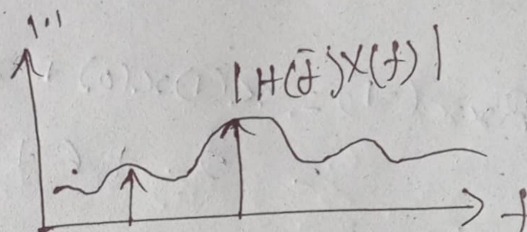
$$y(t) = \cos(2\pi f t - \theta)$$



$$Y(f) = H(f)X(f)$$



~~$x(t) = \cos(2\pi f t)$~~



$$\cos(2\pi f t - \theta) = \cos(2\pi f t)\cos\theta + \sin(2\pi f t)\sin\theta$$

$$\cos(2\pi f t - \theta_1) + \cos(2\pi f t - \theta_2) = A\cos(2\pi f t + B)$$

$A \uparrow$ $|H(f)|$, $B \uparrow$ $\angle H(f)$

Application of Furien Transform:

i) Image Compression
ii) " Analysis
iii) " Filtering
iv) " Reconstruction

Properties -

(i) Linearity property
(ii) Frequency shifting "
(iii) Time reversal property
(iv) Time shifting property

4.a why do we use furier Transform?

Fourier transform is one of the important concepts used in image processing, which helps to decompose the image into the sine and cosine components.

Furier transform is used in the transition of signal from the time spectrum to the frequency domain.

4c

Given,

$x(n) = 0.5^n U(n) + 0.8^n U(-n-1)$

Two parts separated by,

$x_1(n) = (0.5)^n U(n)$

$X_1(Z) = \sum_{n=-\alpha}^{\alpha} (0.5)^n U(n) Z^{-n}$

$= \sum_{n=0}^{\alpha} (0.5)^n Z^{-n}$

$= \sum_{n=0}^{\alpha} (0.5 Z^{-1})^n$

if $|0.5Z^{-1}| < 1$ or $|Z| > 0.5$, this power series

Convergens to $\frac{1}{1-0.5Z^{-1}}$; ROC : $|Z| > 0.5$

$x_2(n) = (0.8)^n U(-n-1)$

$X_2(Z) = \sum_{n=-\alpha}^{\alpha} (0.8)^n U(-n-1) Z^{-n}$

$= \sum_{n=-\alpha}^{-1} (0.8)^n Z^{-n}$

Substitute l for $-n$

$X_2(Z) = \sum_{l=1}^{\alpha} (0.8)^{-l} Z^{l}$

$= \sum_{l=1}^{\alpha} (0.8^{-1} Z)^{l}$

if $|0.8^{-1} Z| < 1$ or, $|Z| < 0.8$ the power series

Convergens to $\frac{0.8^{-1} Z}{1-0.8^{-1} Z}$; ROC : $|Z| < 0.8$

Now,

$$X(z) = X_1(z) + X_2(z)$$

$$= \frac{1}{1-0.5z^{-1}} + \frac{0.8z^{-1}}{1-0.8^{-1}z}$$

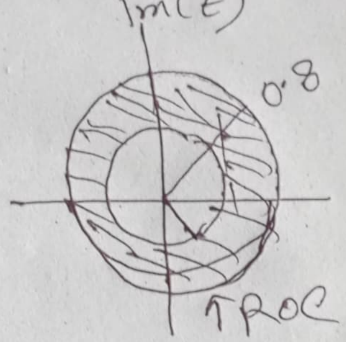
$$= \frac{1}{1-0.5z^{-1}} - \frac{1}{1-0.8z^{-1}} = \frac{-0.8z^{-1}+0.5z^{-1}}{1-0.8z^{-1}-0.5z^{-1}+(0.5\times0.8)z^{-1}z^{-1}}$$

$$= \frac{0.5-0.8}{-z+0.8+0.5-(0.5)(0.8)z^{-1}}$$

$-z$ দ্বারা গুণ

$(-z)$ দ্বারা গুণ

Since $|0.8|>|0.5|$ the ROC is $|0.5|<|z|<|0.8|$



4.b

Define z-transform:

$$X(z) = \sum_{n=-\alpha}^{\alpha} x(n) z^{-n}$$

z is complex variable, the equation sometimes called the direct z-transform because it transform the time-domain signal x(n) into its complex-plane representation X(Z).

$$X(z) \equiv Z\{x(n)\}$$