

Gamma-ray spectroscopy

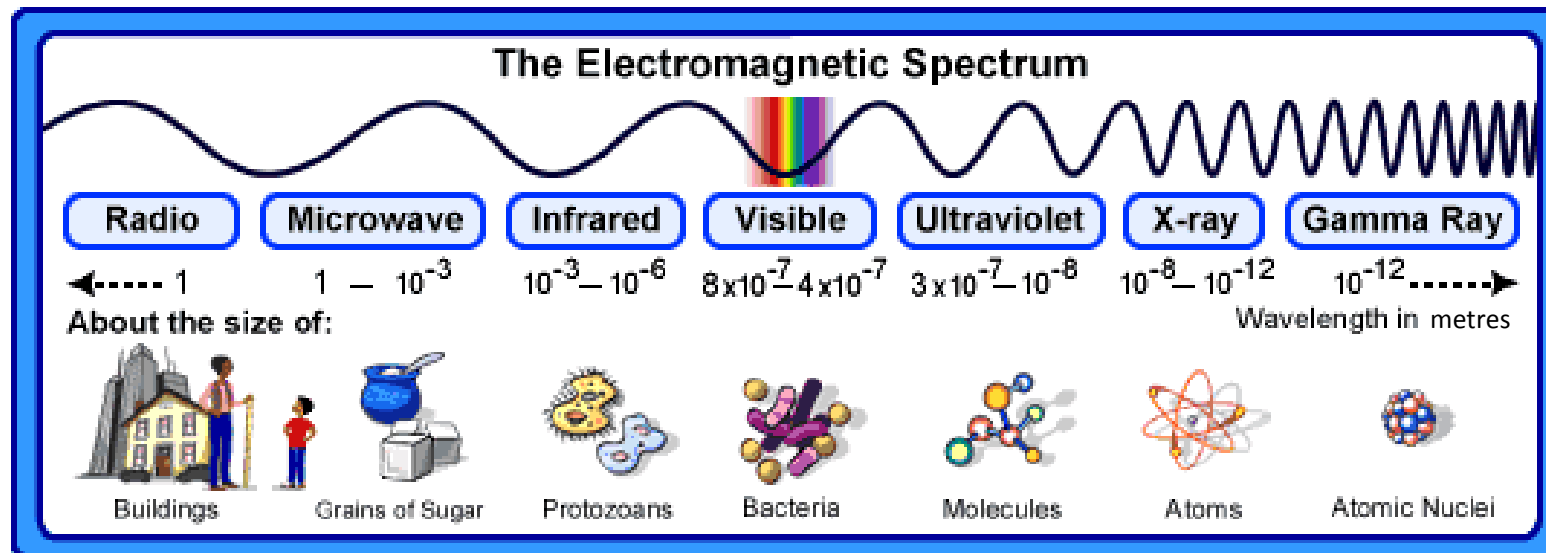
Alison Bruce
University of Brighton

Aim is to give an overview for non-specialists but also some details that specialists might find useful.

Contents:

- Basic gamma-ray properties, observables
- Methods of producing the nuclei of interest (not an exhaustive list)
- Gamma-ray interactions in matter
- Detector types
- Detector arrays
- Measurement techniques:
 - Angular correlation, angular distribution
 - Linear polarisation
 - Lifetime measurements: Doppler Shift Attenuation Method
 - Recoil Distance Method
 - Electronic timing

What is a gamma ray?

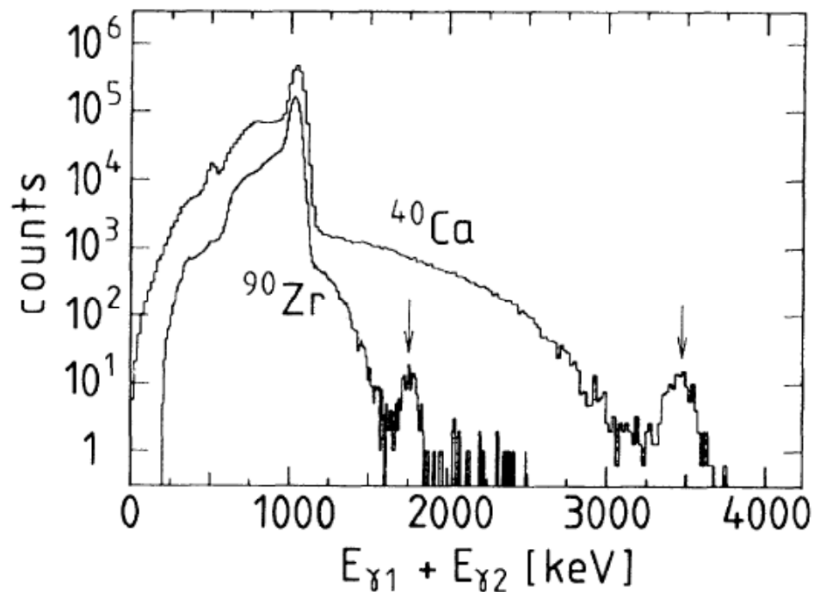


Gamma ray: high frequency / short wavelength electromagnetic radiation.

Useful as a probe of the nucleus as the electromagnetic interaction is well understood and only weakly perturbs the nucleus.

Gamma rays carry spin 1

which leads to interesting cases where a 0^+ state is the lowest excited level:



Sum-energy spectrum in coincidence with protons populating the states of interest.

3.35 MeV 0^+

0 MeV 0^+

^{40}Ca

Schirmer et al., PRL 53 (1984) 1897

See also ^{137}Ba , Pietrella et al., Nature 526 (2015) 406

What is gamma-ray spectroscopy?

We study gamma rays emitted from excited nuclei to obtain information about:

Transition energies and coincidence relationships
Level structure

Transition rates
Lifetimes, quadrupole moment

Angular correlations and linear polarisations
Spin, Parity (also magnetic moments).

Transition branching ratios, mixing ratios
Wavefunctions, transition matrix elements, etc

Why study gamma rays ?

Gamma rays provide a superb probe for nuclear structure!

Relatively easy to detect with good efficiency and resolution.

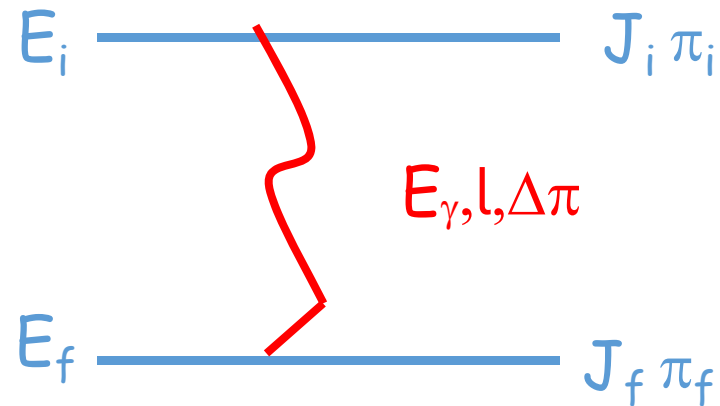
Emitted by almost all low-lying states.

Penetrating enough to escape from target chambers to reach detectors.

Gamma rays arise from EM interactions and allow a probe of structure without large perturbations of the nucleus.

No model dependence in the interaction (EM is well understood).

The basics of the situation:



The energy of the gamma ray is given by $E_i - E_f$

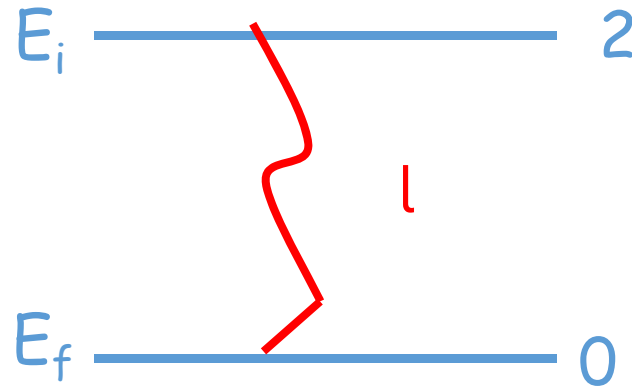
The angular momentum carried away is given by $J_i - J_f$

From conservation of angular momentum:

$$|J_i - J_f| \leq l \leq J_i + J_f$$

where l is the multipolarity of the transition

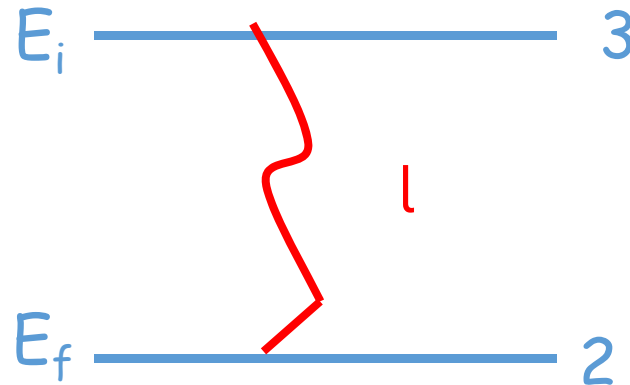
The basics of the situation:



$$|2-0| \leq \ell \leq 2+0$$

Here $\Delta J = 2$ and $\ell = 2$
so we say this is a stretched transition

The basics of the situation:

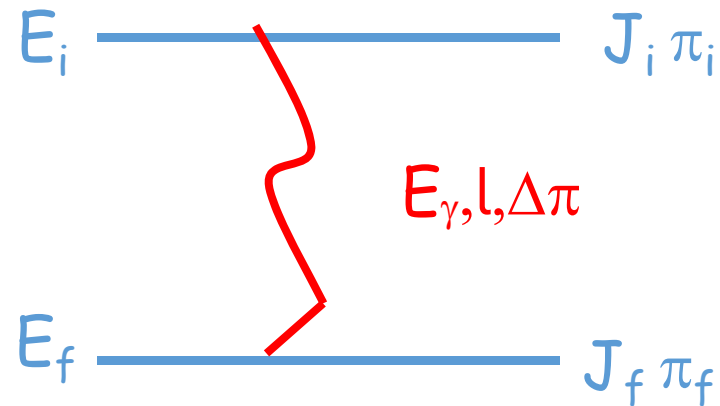


$$|3-2| \leq l \leq 3+2$$

Here $\Delta J = 1$ but $l = 1, 2, 3, 4, 5$

and the transition can be a mix of 5 multipolarities

The basics of the situation:



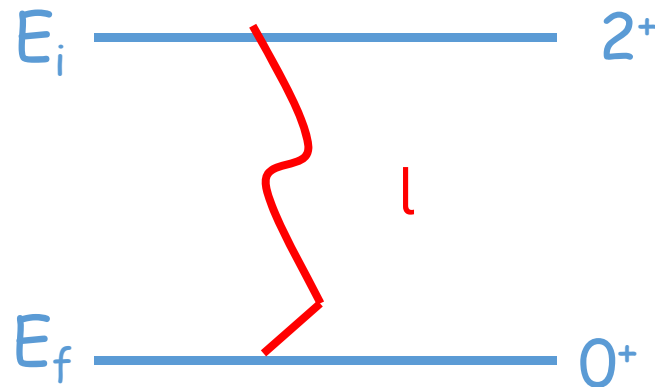
Electromagnetic transitions:

$$\Delta\pi (\text{electric}) = (-1)^\ell$$

$$\Delta\pi (\text{magnetic}) = (-1)^{\ell+1}$$

$\Delta\pi$	YES	E1	M2	E3	M4
	NO	M1	E2	M3	E4

The basics of the situation:



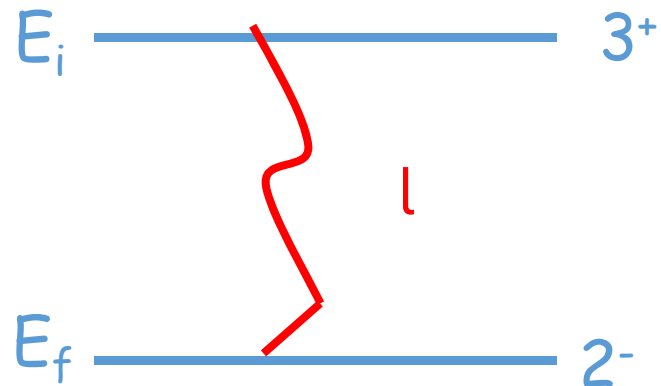
$$|2-0| \leq l \leq 2+0$$

$l = 2$ and no change in parity

$\Delta\pi$					
	NO	M1	E2	M3	E4

pure (stretched) E2

The basics of the situation:

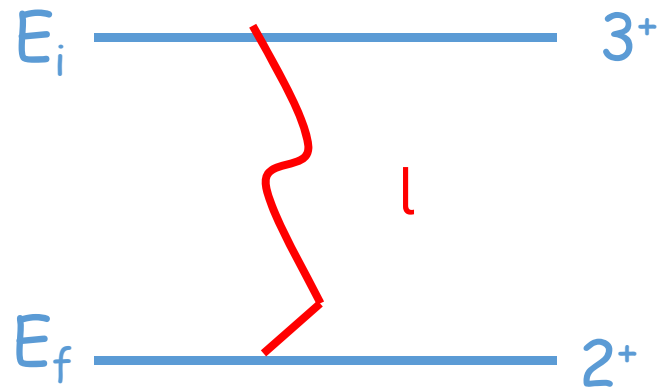


Here $\Delta J = 1$ but $\ell = 1, 2, 3, 4, 5$

$\Delta\pi$	YES	E1	M2	E3	M4

Mixed E1/M2/E3/M4/E5

The basics of the situation:



Here $\Delta J = 1$ but $\ell = 1, 2, 3, 4, 5$

$\Delta\pi$					
	NO	M1	E2	M3	E4

Mixed M1/E2/M3/E4/M5

The basics of the situation:

$3^+ \rightarrow 2^+$: mixed M1/E2/M3/E4/M5

$3^+ \rightarrow 2^-$: mixed E1/M2/E3/M4/E5

In general only the lowest 2 multipoles compete
and (for reasons we will see later)

$l+1$ multipole generally only competes if it is electric so:

$3^+ \rightarrow 2^+$: mixed M1/E2

$3^+ \rightarrow 2^-$: almost pure E1 (v. little M2 admixture)

The basics of the situation:

Amount of mixing is given by the mixing ratio δ

$$\text{given by } \delta = \frac{\langle \psi_f \| M(l+1) \| \psi_i \rangle}{\langle \psi_f \| M(l) \| \psi_i \rangle}$$

$3^+ \rightarrow 2^+$: M1/E2 admixture

$$\%E2 = \frac{\delta^2}{1+\delta^2}$$

$$\%M1 = \frac{1}{1+\delta^2}$$

Not quite so basic:

Transition rate Γ [s^{-1}] given by:

$$\Gamma_{f \rightarrow i}(\sigma\lambda) = \frac{2(l+1)}{\epsilon_0 l [(2l+1)!!]^2} \left(\frac{E_\gamma}{c}\right)^{2l+1} \langle \psi_f | M(\sigma\lambda) | \psi_i \rangle^2$$

Note the l dependence on the energy, the double factorial (what is that?)

$$\langle \psi_f | M(\sigma\lambda) | \psi_i \rangle^2$$

this is the bit that has the physics in it,
it's the matrix element between the initial
and final wavefunctions squared and is
related to the $B(\sigma\lambda)$ by

$$B(\sigma\lambda) = \frac{1}{2J_i+1} \sum_{M_i M_f} \langle \psi_f | M(\sigma\lambda) | \psi_i \rangle^2$$

Not quite so basic:

Typical transition rates [s^{-1}]:

$$T(E1) = 1.59 \times 10^{15} (E_\gamma)^3 B(E1)$$

$$T(E2) = 1.22 \times 10^9 (E_\gamma)^5 B(E2)$$

$$T(M1) = 1.76 \times 10^{13} (E_\gamma)^3 B(M1)$$

$$T(M2) = 1.35 \times 10^7 (E_\gamma)^5 B(M2)$$

E_γ in MeV

$B(E\lambda)$ in $e^2\text{fm}^{2\lambda}$

$B(M\lambda)$ in $\left(\frac{e\hbar}{2Mc}\right)^2\text{fm}^{2\lambda-2}$

Note:

- Electric transitions faster than magnetic
- Higher multipolarity \Rightarrow slower rate
- Transition probability proportional to transition energy \Rightarrow low-energy transitions are hard to observe and other processes e.g. internal conversion start to compete.

Not quite so basic:

Single particle estimates (also called Weisskopf estimates)

$$T(E1) = 1.02 \times 10^{14} (E_\gamma)^3 A^{2/3}$$

$$T(E2) = 7.26 \times 10^7 (E_\gamma)^5 A^{4/3}$$

$$T(M1) = 3.18 \times 10^{13} (E_\gamma)^3$$

$$T(M2) = 2.26 \times 10^7 (E_\gamma)^5 A^{2/3}$$

Measuring level lifetimes gives us transition rates which can be interpreted as single particle (or collective) and hence give an indication of the type of motion

Methods of producing the nuclei of interest

Out-of-beam spectroscopy:

Nucleus is stopped

Not many gamma rays emitted (Gamma-ray multiplicity low)

e.g. decay from a fission source, from stopped radioactive ion beams (ISOL or fragmentation), or de-excitation of isomeric states

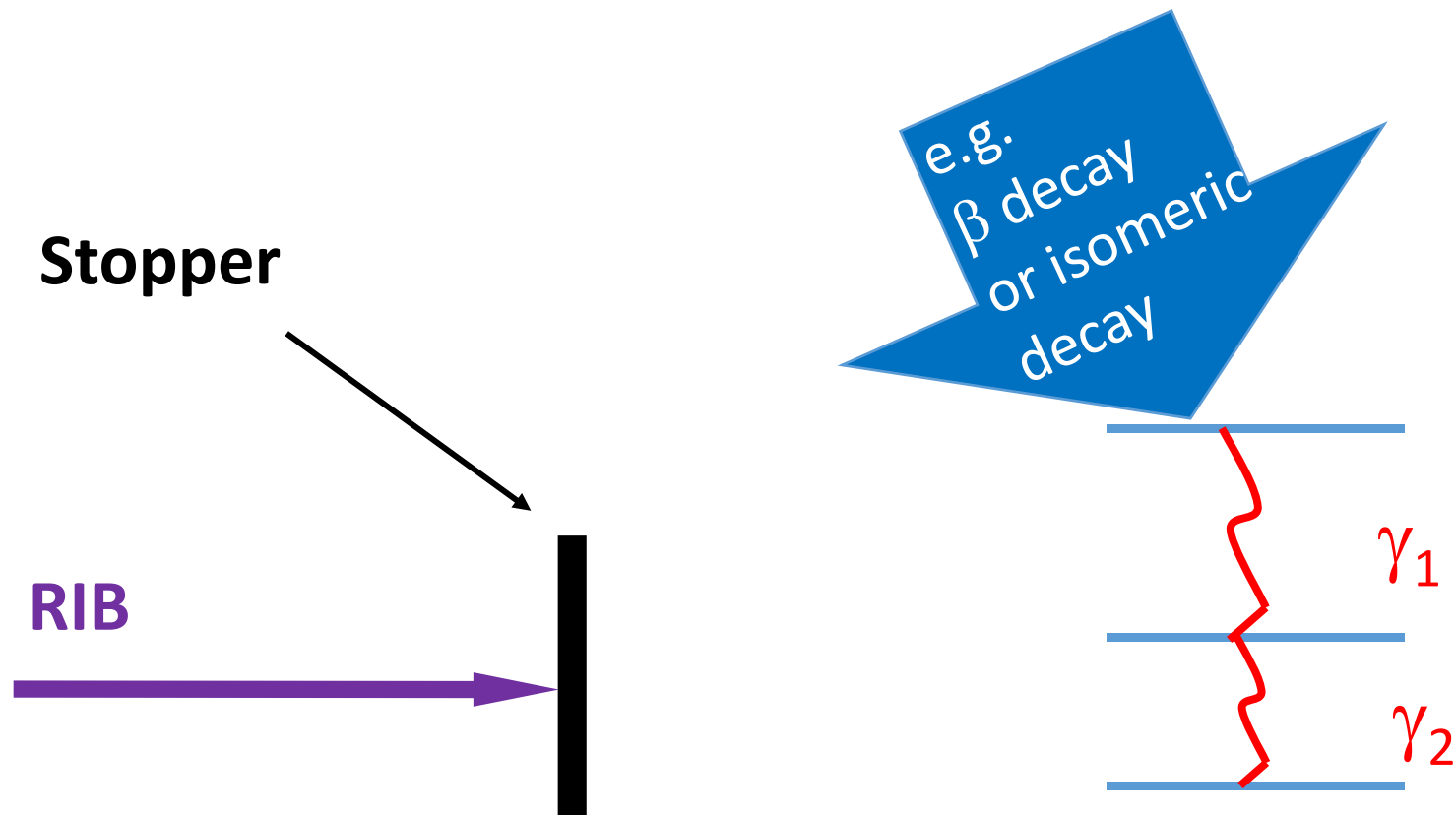
In-beam spectroscopy:

Nucleus is moving

Lots of gamma rays emitted (Gamma-ray multiplicity high)

e.g. compound nucleus reaction, Coulomb excitation

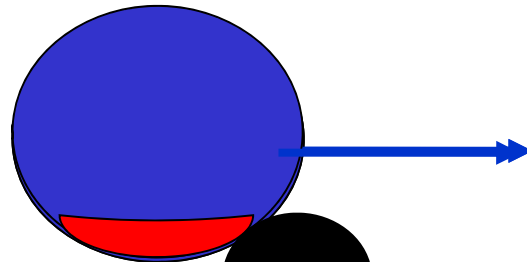
Out-of-beam spectroscopy



Get first information on lifetimes, decay modes, Q-values and scheme of excited levels

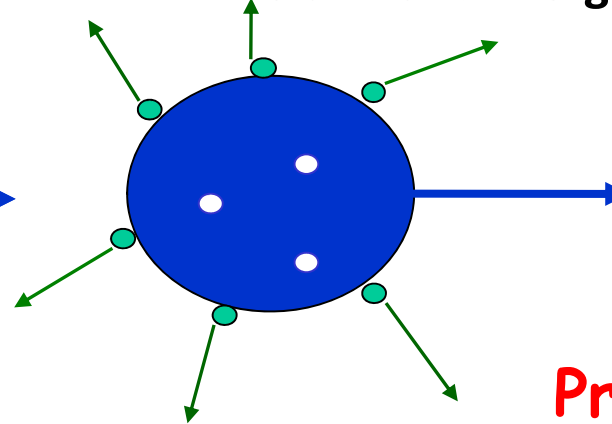
Two Reaction Processes

Beam at Relativistic Energy
 $\sim 0.5-1 \text{ GeV/A}$

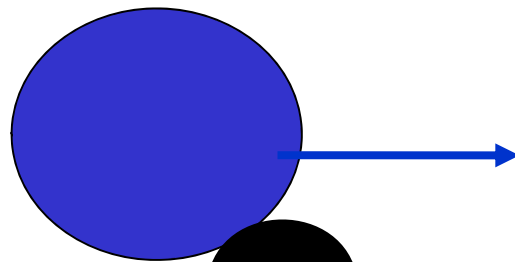


Target Nucleus

Reaction Products still travelling at
Relativistic Energies

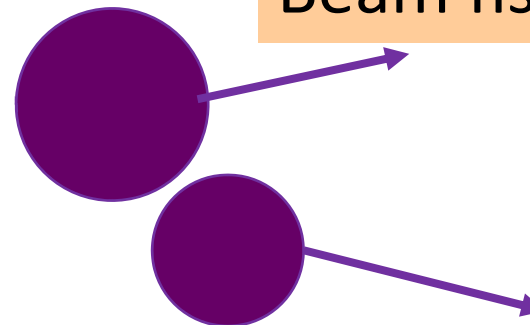


**Projectile
fragmentation**



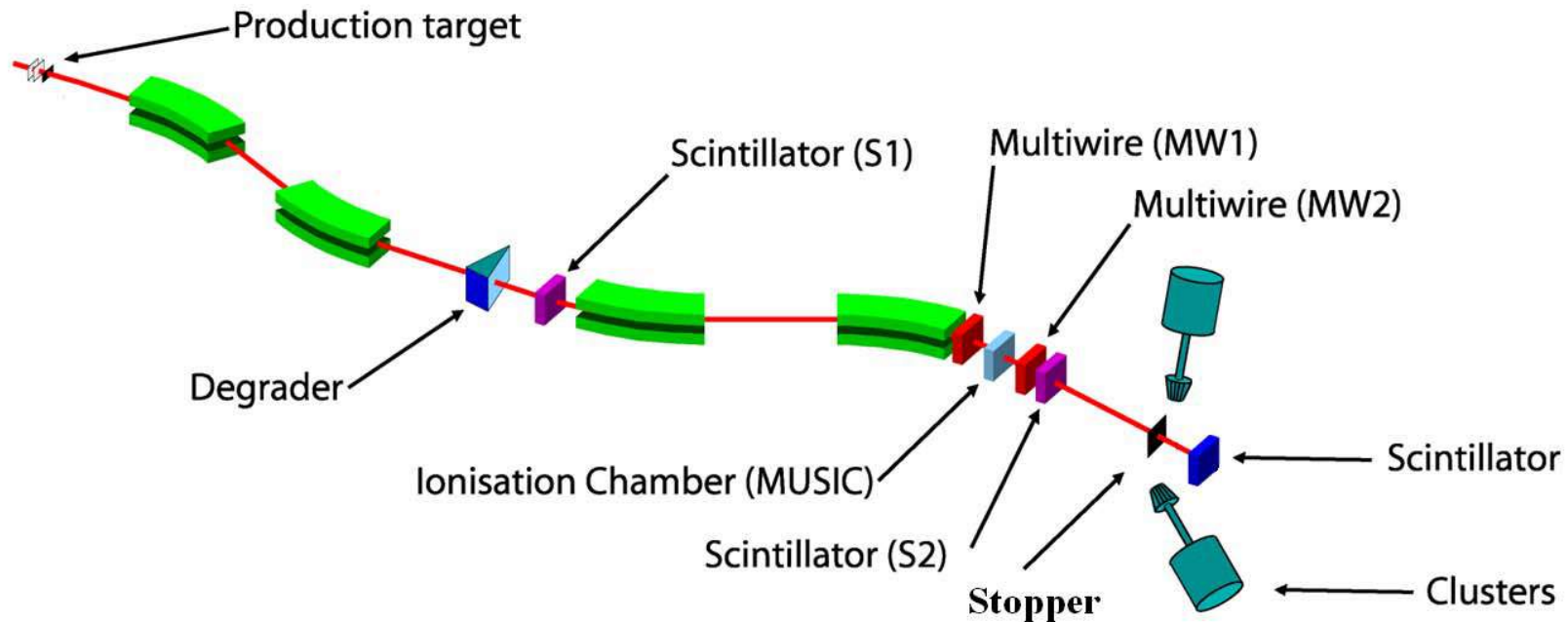
Target Nucleus

Beam fissions



**Projectile
fission**

Ion-by-ion identification with e.g the FRS:



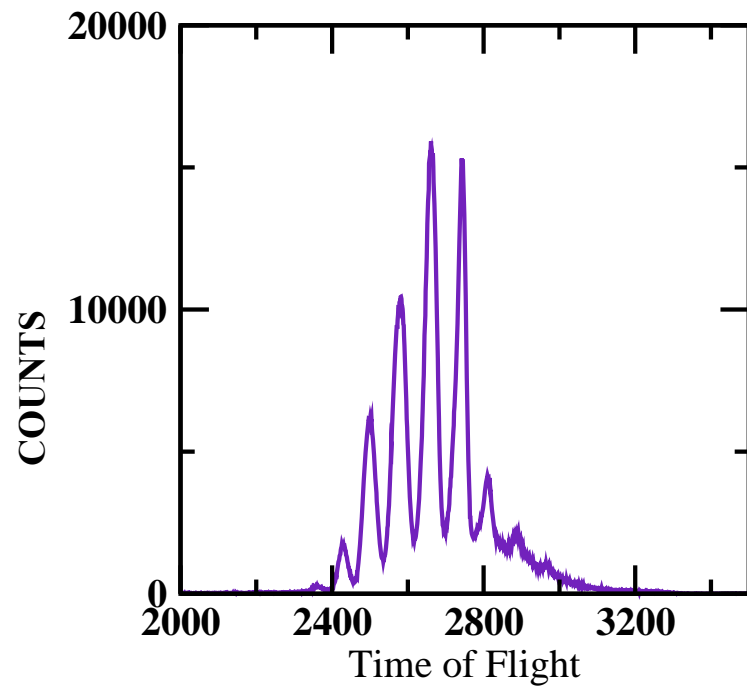
$$\frac{A}{Z} \propto \frac{B\rho}{\beta\gamma}$$

FRS = Fragment Recoil Spectrometer at GSI

Cocktail of fragments
Chemically independent

H.Geissel et al., NIM B70 (1992) 286.

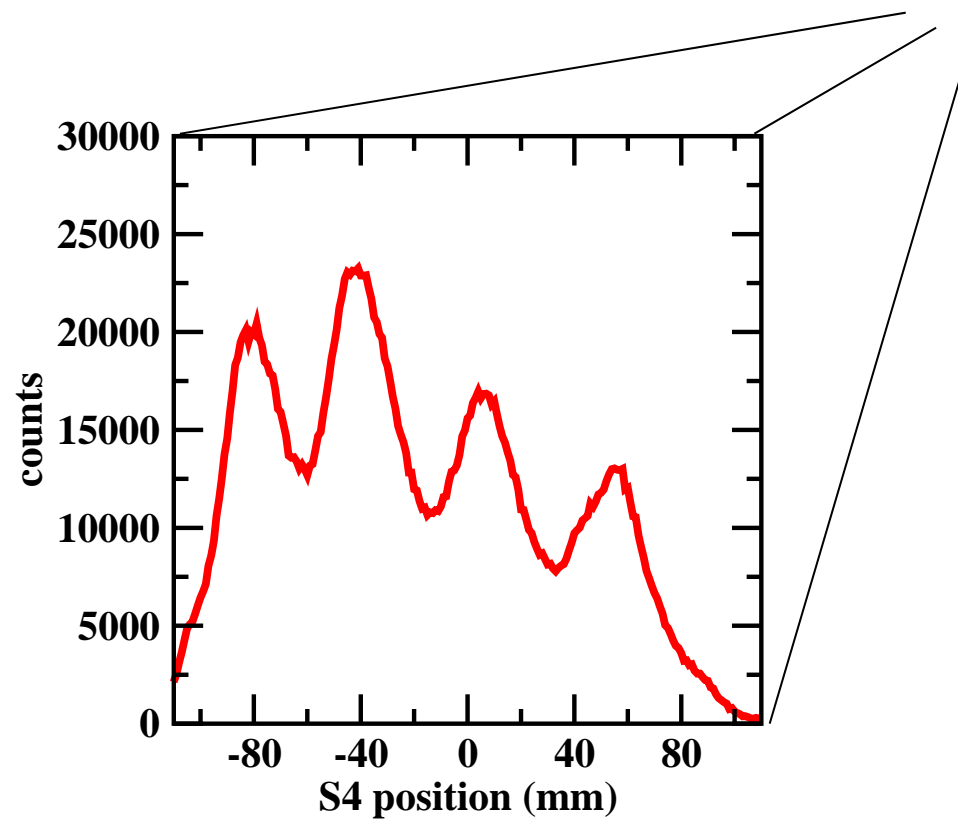
Ion-by-ion identification with e.g the FRS:



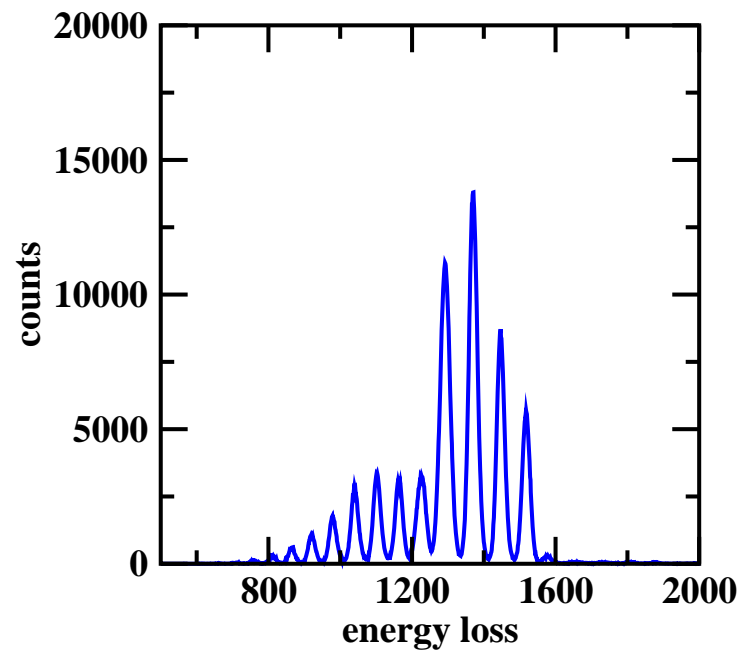
Time of flight

- > β

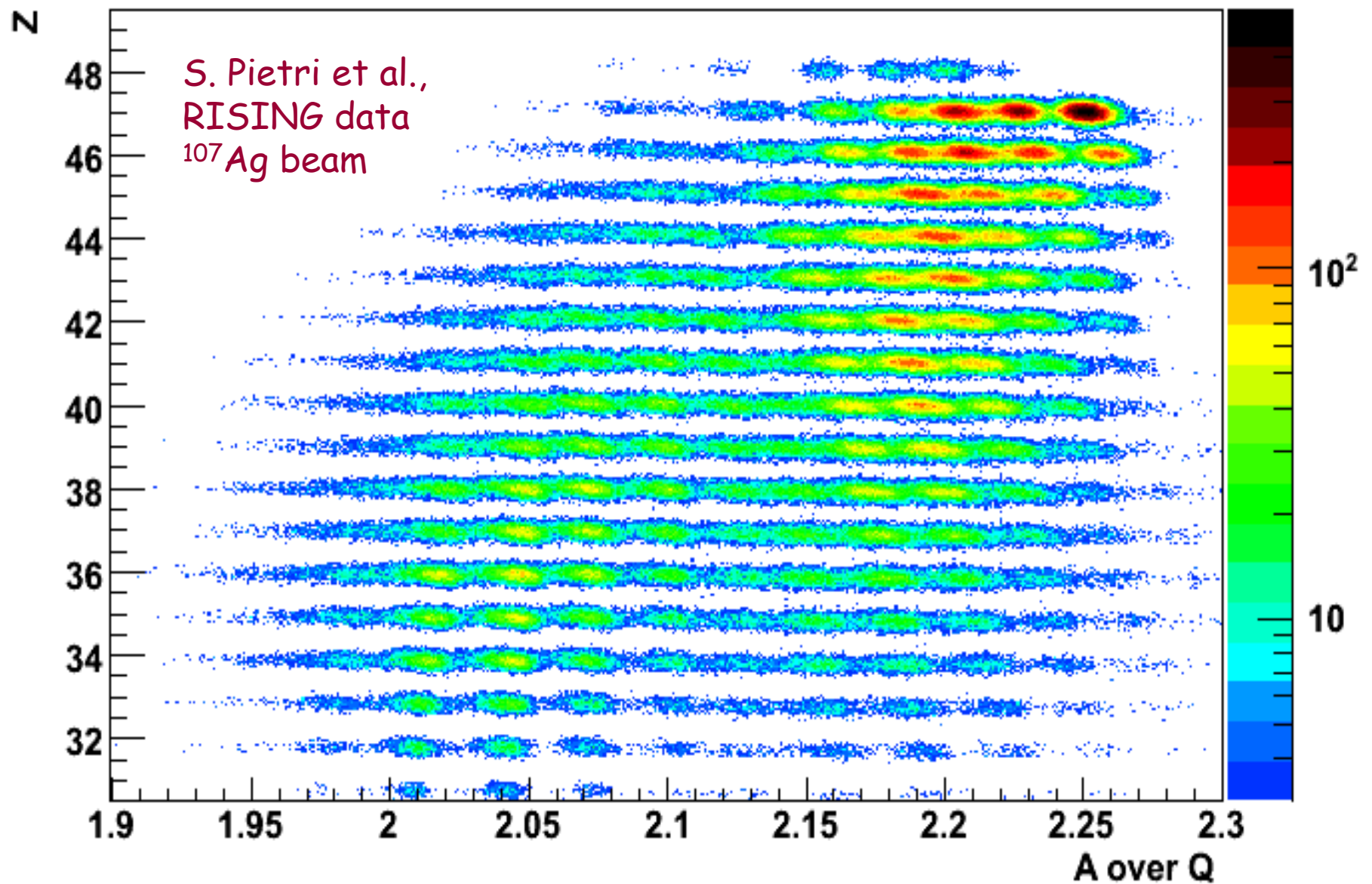
Ion-by-ion identification with e.g the FRS:



Ion-by-ion identification with e.g. the FRS:

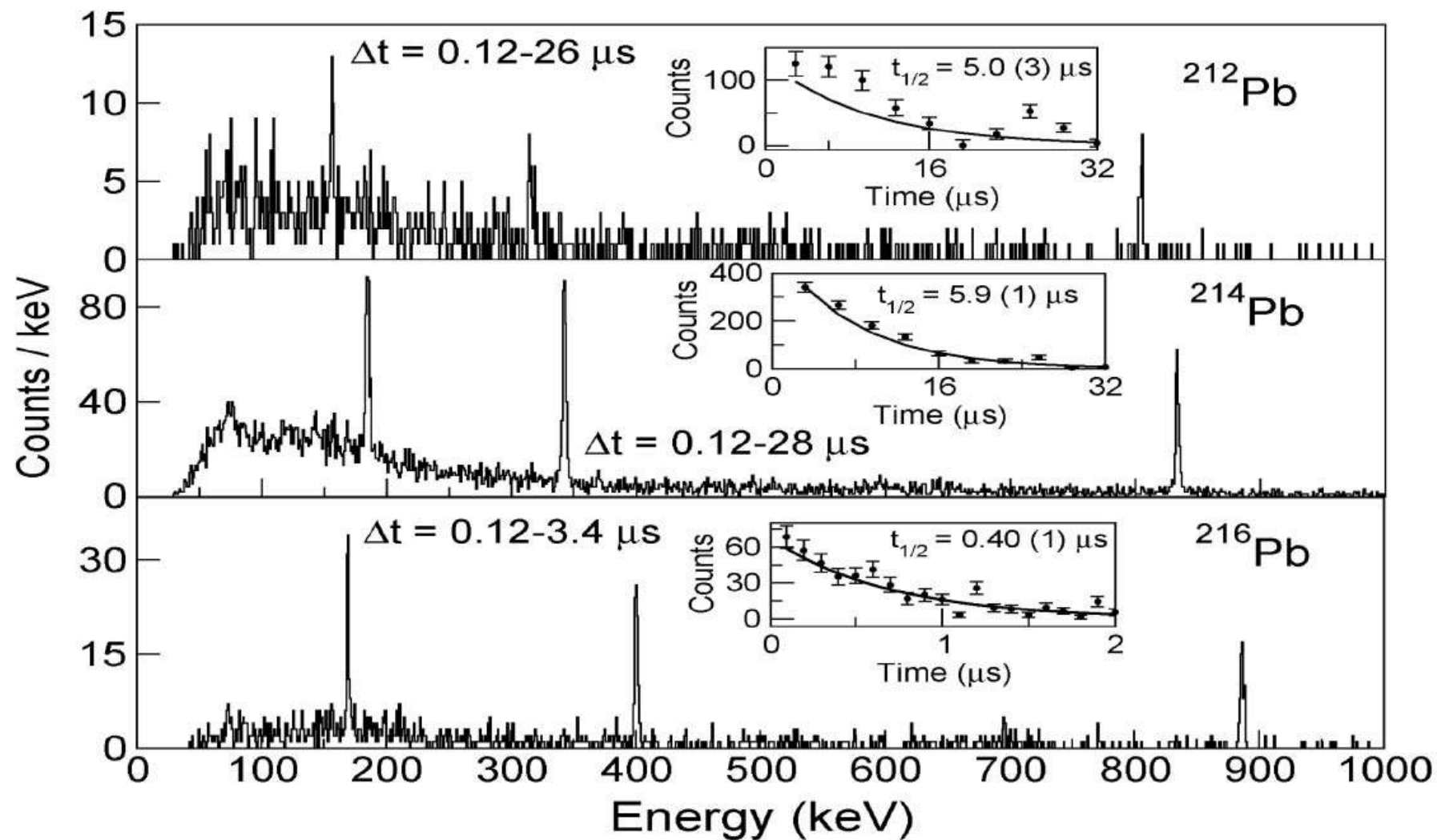


Isotope identification



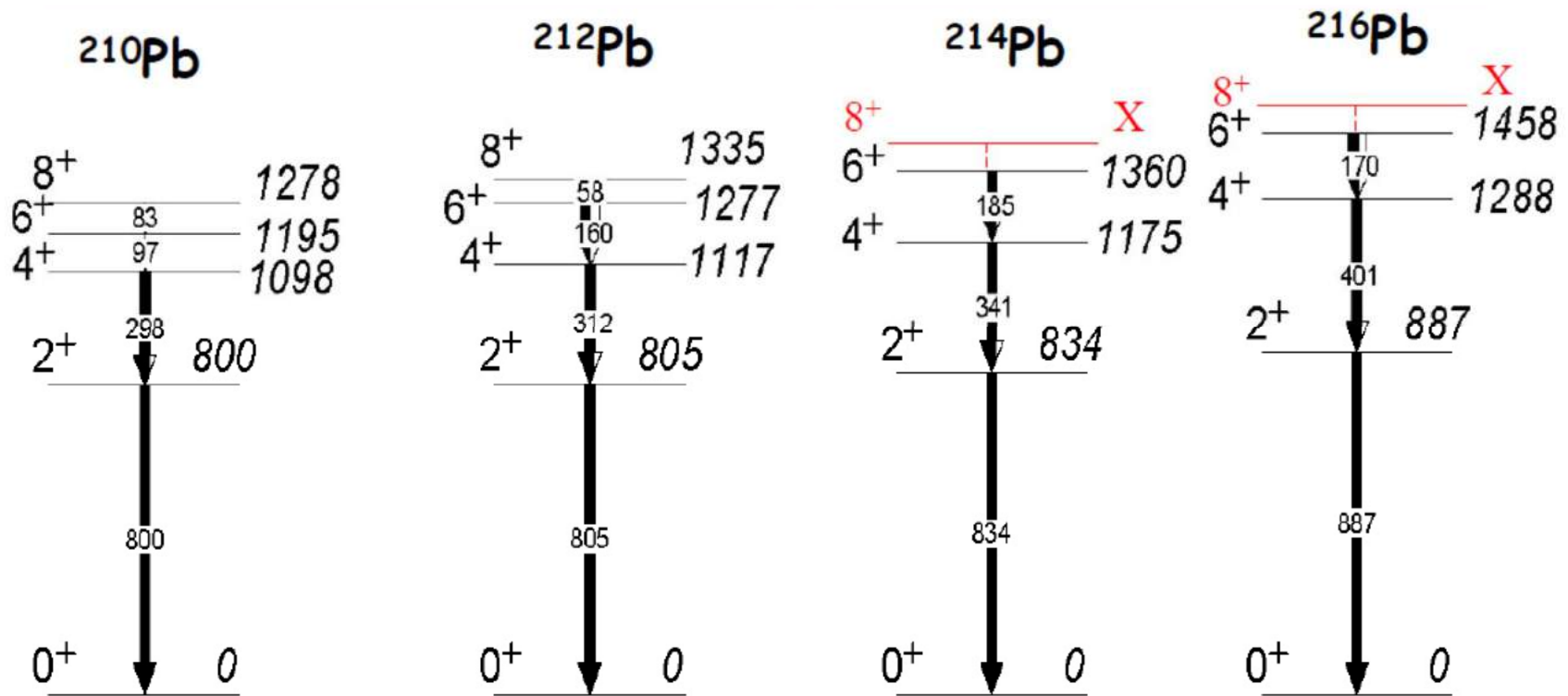
$^{212,214,216}\text{Pb}$: 8^+ isomers:

A. Gottardo, J.J. Valiente Dobon, G. Benzoni et al., PRL 109 (2012) 162502



Energy levels well described in seniority scheme

A. Gottardo, J.J. Valiente Dobon, G. Benzoni et al., PRL 109 (2012) 162502



In-beam spectroscopy

Low-energy in-beam spectroscopy (up to 10 MeV per nucleon)

e.g.

Fusion evaporation.

Multi-nucleon transfer reactions

Coulomb excitation

Single nucleon transfer reactions

Intermediate-energy in-beam spectroscopy (50 -100 MeV per nucleon)

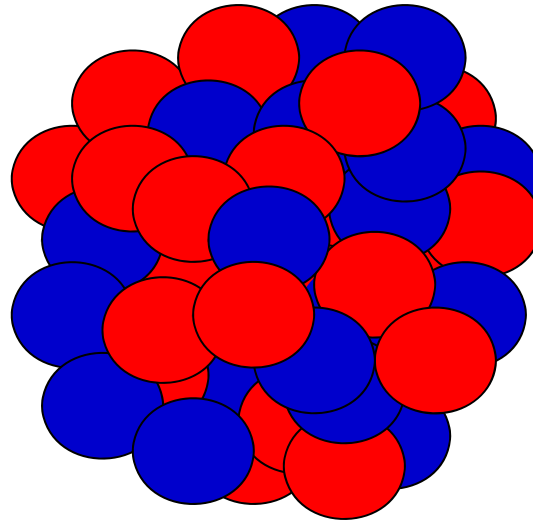
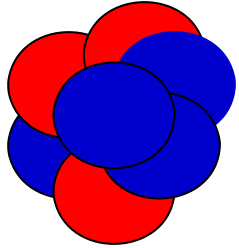
e.g.

Coulomb excitation

Spallation

Knockout

E.g. fusion evaporation:



p

n

First particles are emitted..

p

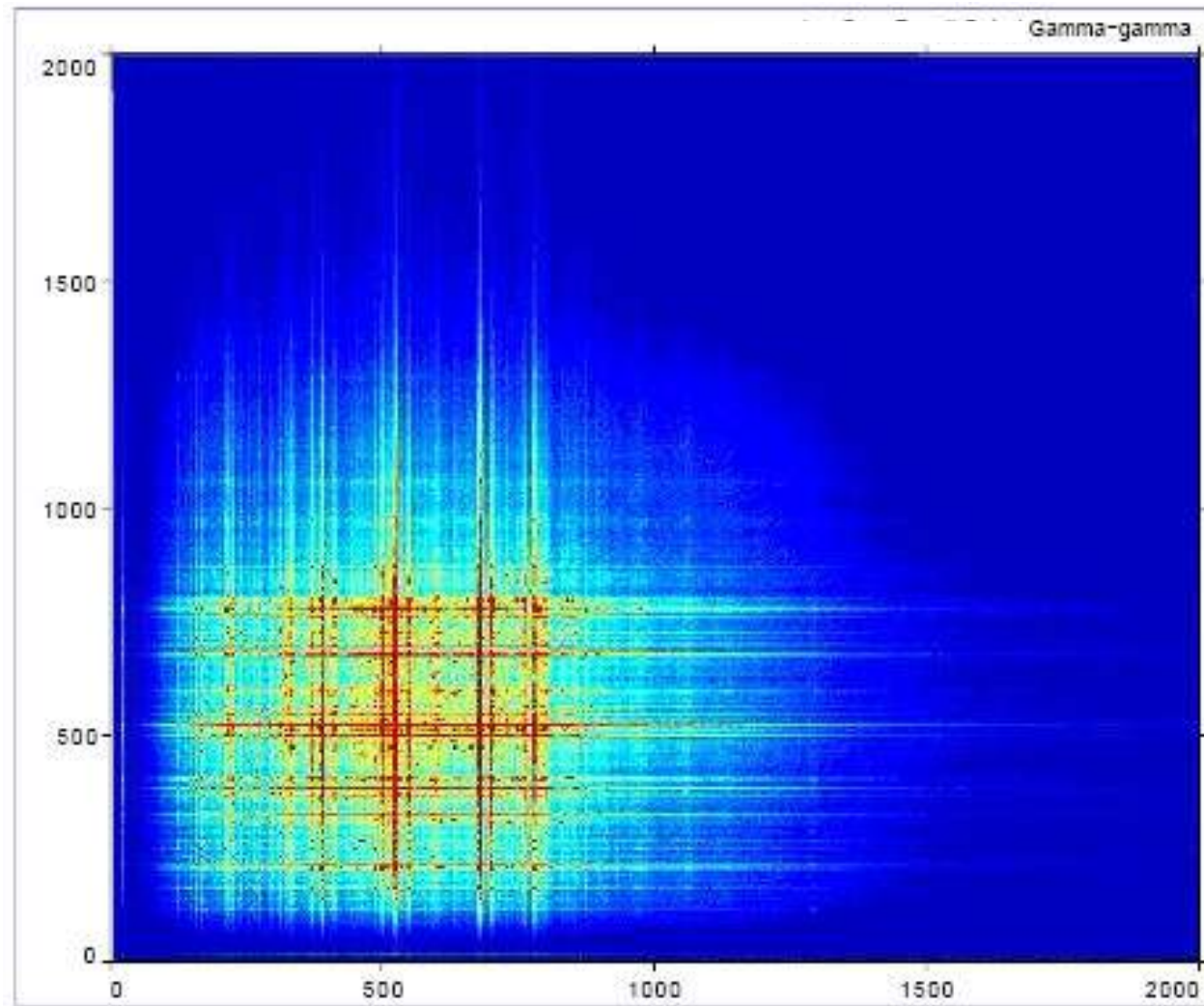
Then gamma rays

Level Schemes



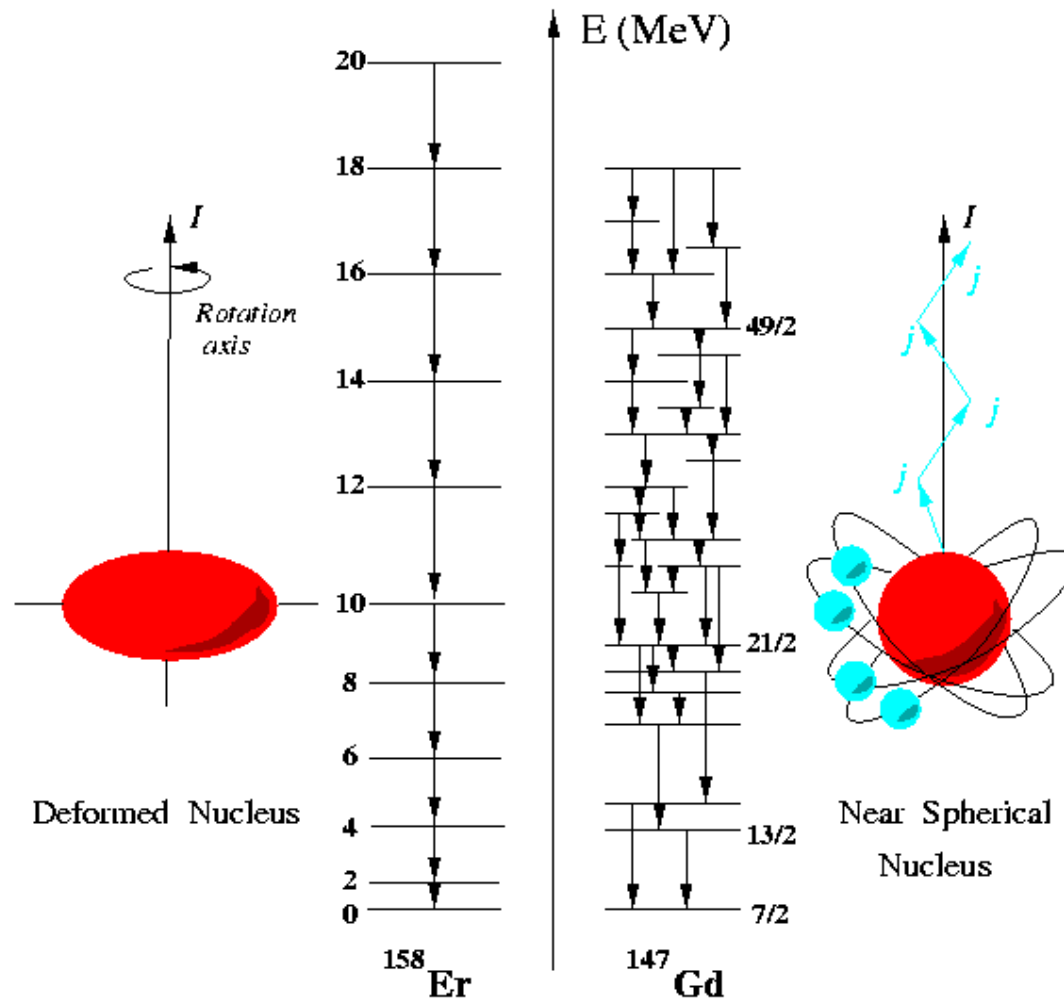
Gamma-gamma coincidence matrices

Usually
set a
time window
~ 500 ns



From Dave Cullen's web page

Gamma-ray patterns reveal nuclear structure



- Collective rotation (left) of a deformed nucleus leads to regular band structures
- Single-particle generation of spin (right) in a spherical nucleus leads to an irregular level structure

Suggestions for tutorial discussion:

1. How can the 0^+ first excited state in ^{40}Ca decay to the ground state by 2 gamma rays if there are no intermediate levels?
2. Calculate E1, E2, M1, M2 transition rates for a single particle, 500 keV transition in an $A=100$ nucleus. What would the corresponding level lifetimes be?
3. What is the meaning of the phrase '**Energy levels well described in seniority scheme**' when describing the Pb isomers? What particles are involved and in what orbits? (Pb, $Z=82$)
4. Estimate the angular momentum (in units \hbar) brought into the compound nucleus ^{156}Dy from the fusion of a ^{48}Ca beam on a ^{108}Pd target at a beam energy of $E_{\text{beam}} = 206 \text{ MeV}$.

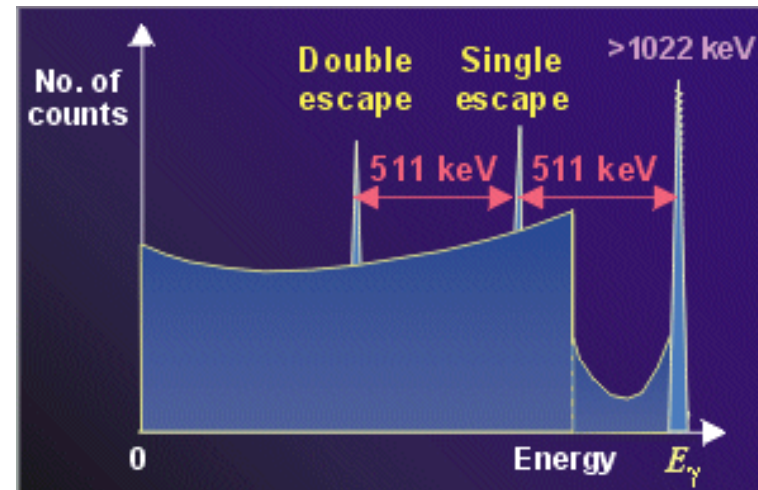
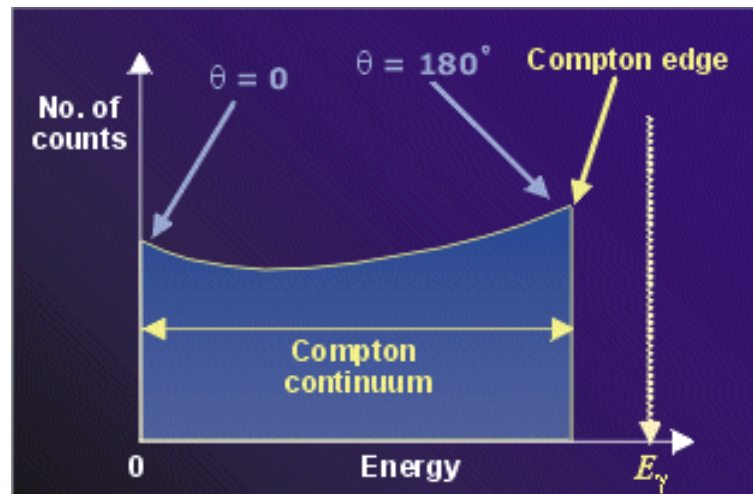
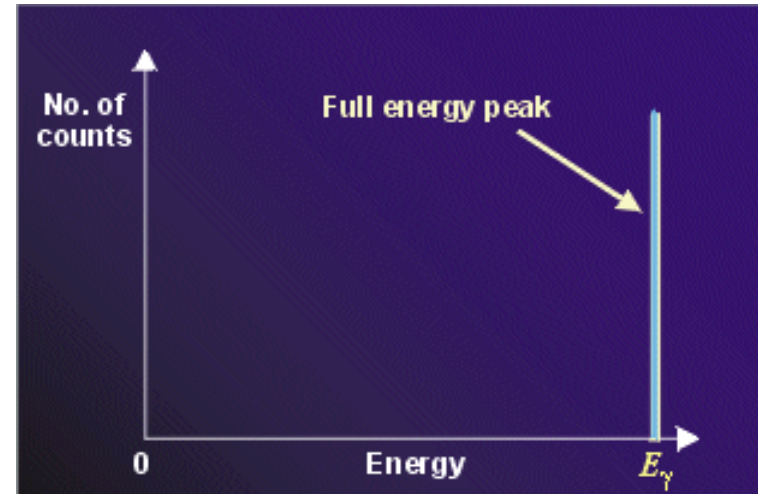
Gamma-ray interactions in matter

Gamma rays interact with matter via three main reaction mechanisms:

Photoelectric absorption

Compton scattering

Pair production

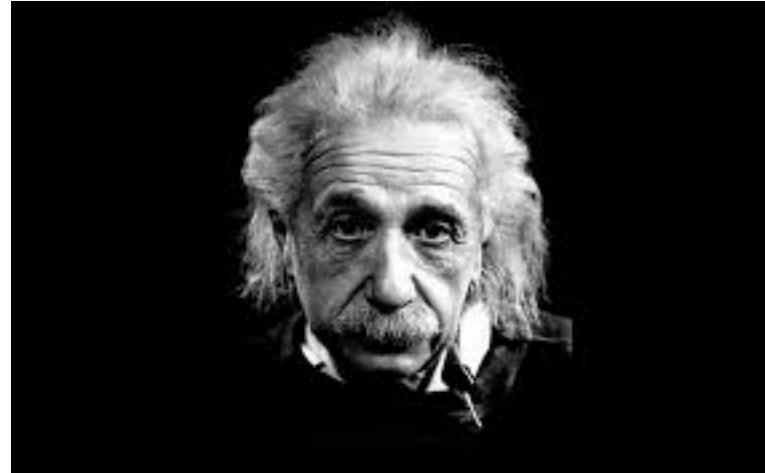


Pictures from University of Liverpool website

Photoelectric absorption

Einstein won the **Nobel Prize for Physics** for the discovery of the photoelectric effect.

In this mechanism a γ -ray interacts with a bound atomic electron.



The photon completely disappears and is replaced by an energetic photoelectron. The energy of the photoelectron can be written

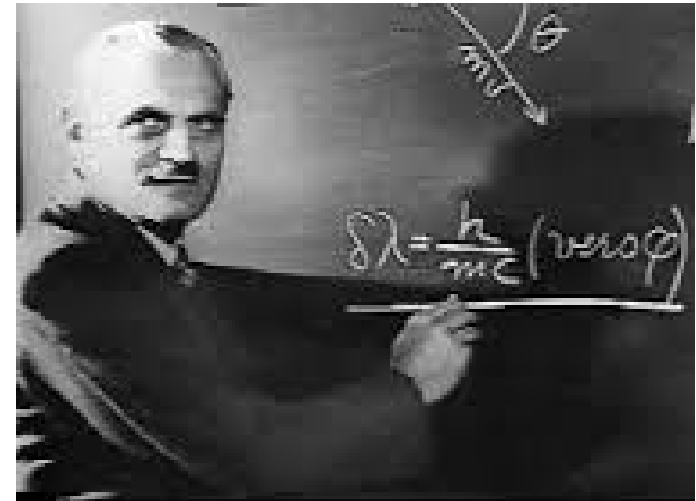
$$E_e = E_\gamma - E_B.$$

The energy of the incident gamma-ray photon minus that of the binding energy of the electron ($E_B = 12\text{eV}$ in germanium).

Compton scattering

Compton won the **Nobel Prize for Physics** for the discovery of the Compton effect.

In this mechanism a γ -ray interacts with a loosely bound atomic electron.



The incoming γ -ray is scattered through an angle θ with respect to its original direction.

The photon transfers a proportion of its energy to a recoil electron. The expression that relates the energy of the scattered photon to the energy of the incident photon is

$$E_s = \frac{E_\gamma}{1 + \frac{E_\gamma}{m_e c^2} (1 - \cos\theta)}$$

Pair production

Nobody won the **Nobel Prize for Physics** for the discovery of the pair production effect (as far as I know).

If the energy of the γ -ray exceeds twice the rest mass energy of an electron (1.022 MeV), then the process of pair production is possible.

The incoming γ -ray disappears in the Coulomb field of the nucleus and is replaced by an electron-positron pair which has kinetic energy

$$E_{\gamma} - 1.022 \text{ MeV}.$$

The positron is slowed down and eventually annihilates in the medium. Two annihilation photons are emitted back to back and these may or may not escape from the detector. Hence three peaks can be observed.

MR. NOBODY

Roger Hargreaves



Other interactions in a real detector

Thomson Scattering

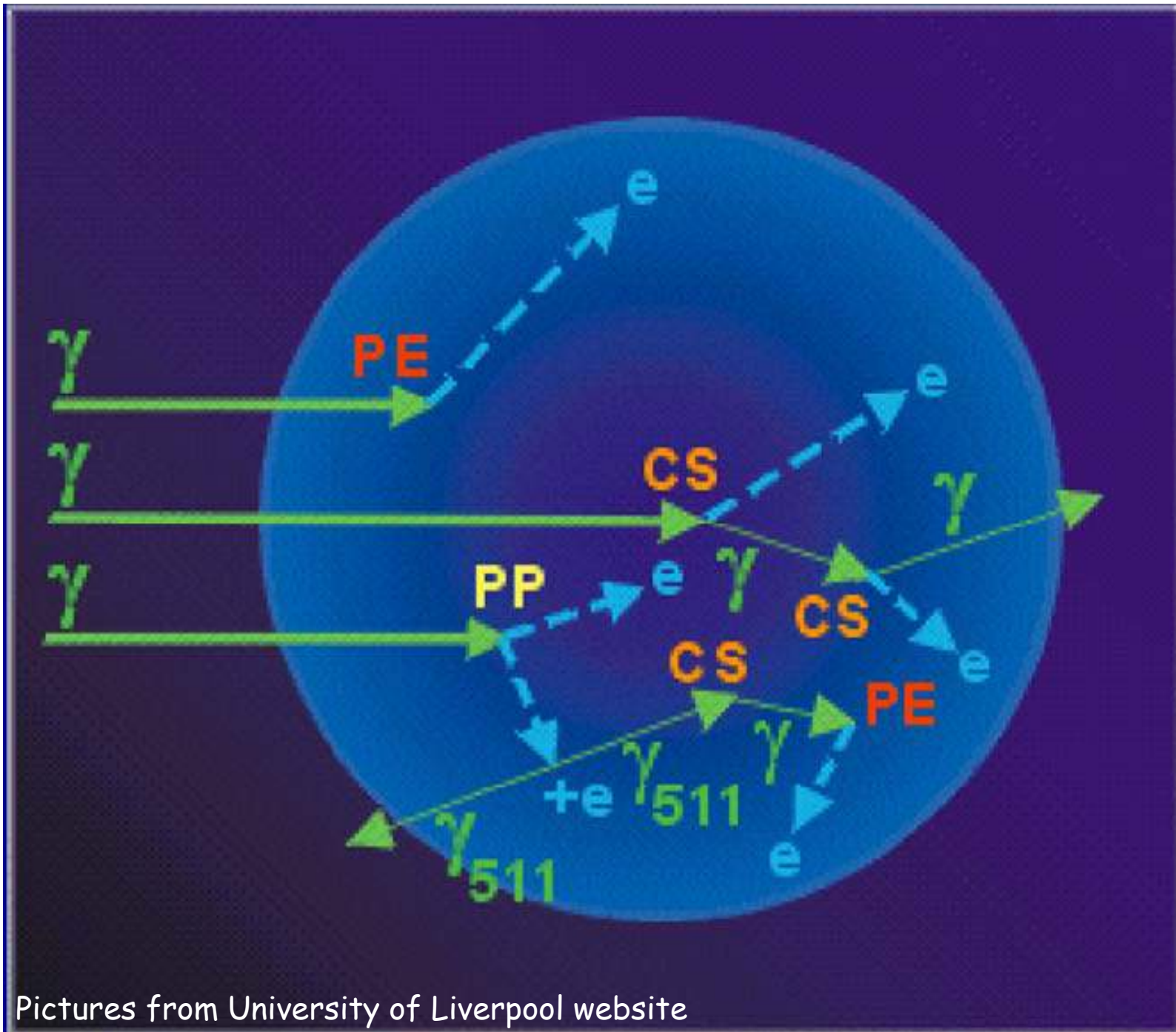
Low-energy coherent scattering off free electrons. Not important in the energy range concerned with most nuclear structure studies.

Nuclear Thomson Scattering

Low energy coherent scattering off nucleus.
Small effect.

Dellbrück Scattering

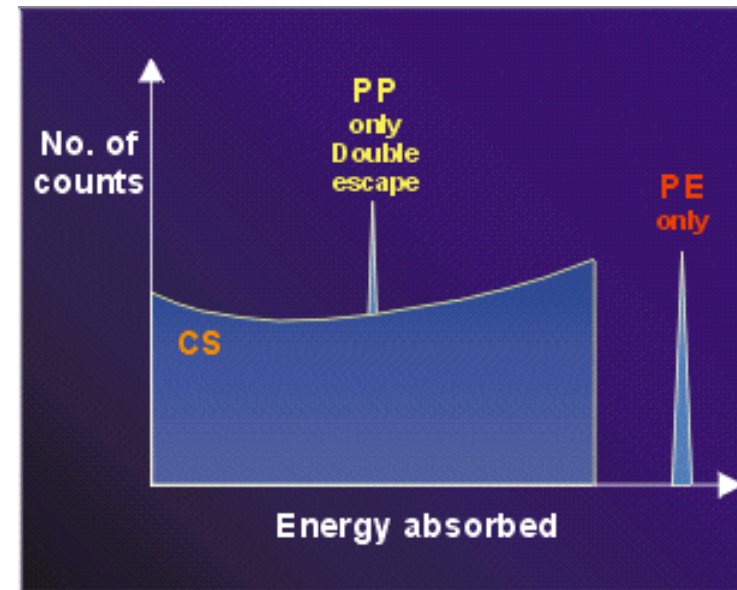
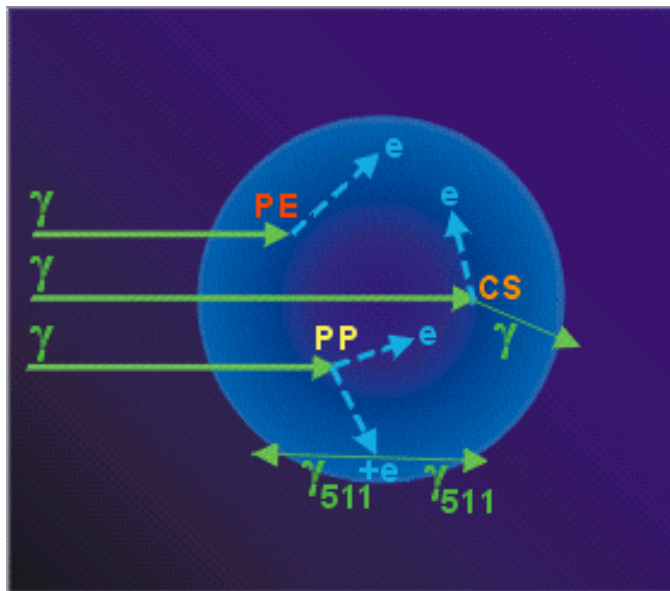
Scattering in the Coulomb field of the nucleus.
Important at $E_\gamma > 3 \text{ MeV}$.



Pictures from University of Liverpool website

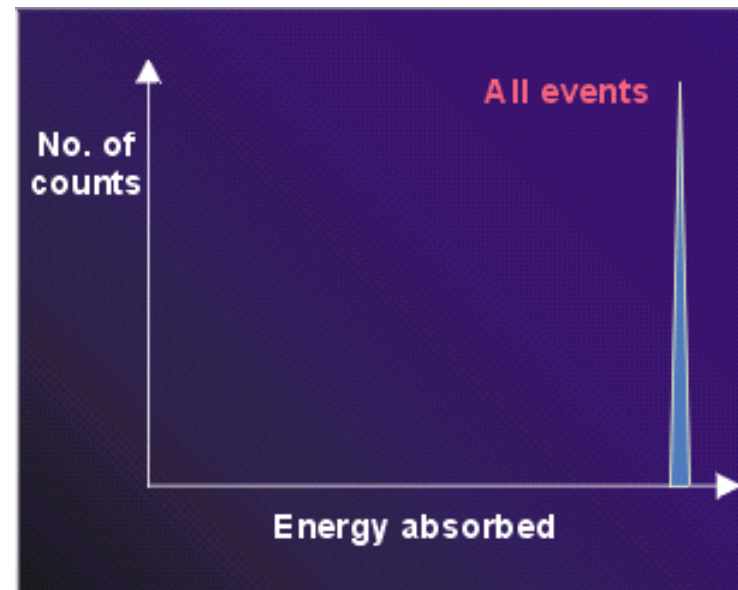
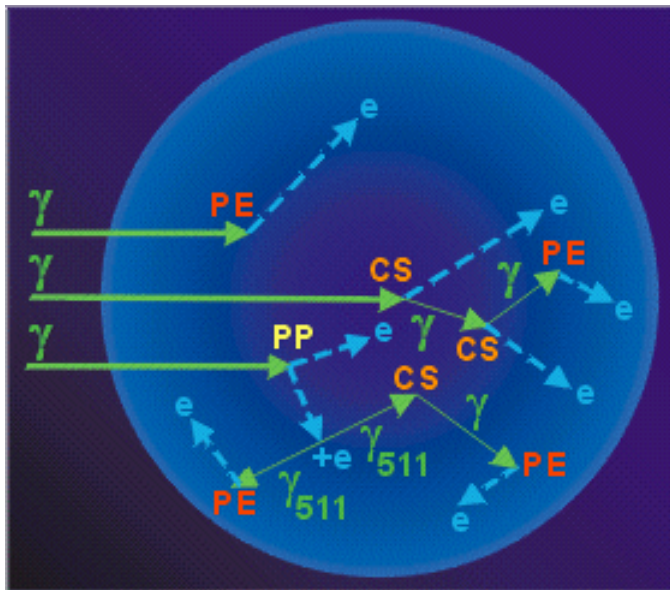
Interactions in a small detector

A **small detector** is one in which only one interaction can take place. Only the photoelectric effect will produce full energy absorption. Compton scattering events will produce the Compton continuum. Pair production will give rise to the double escape peak due to both gamma-rays escaping.



Interactions in a large detector

A **large detector** is one in which there will be complete absorption of the gamma ray and a single gamma-ray peak, referred to as the full energy peak will be observed.



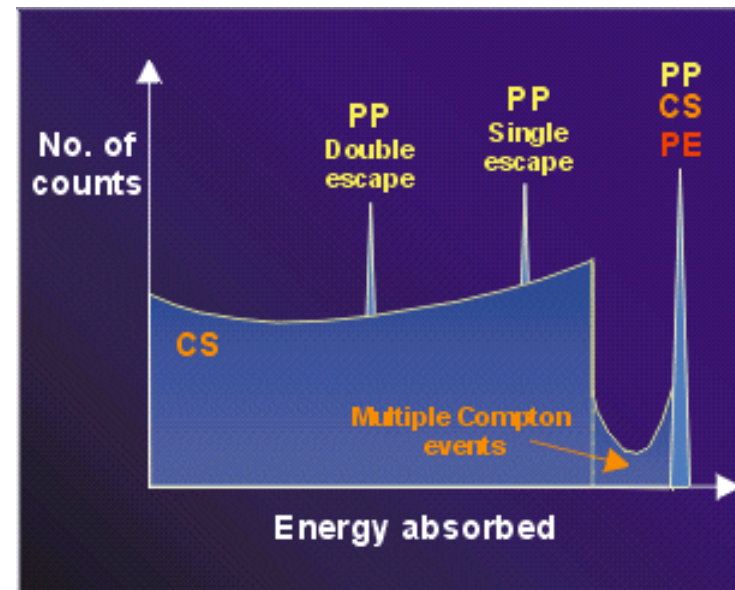
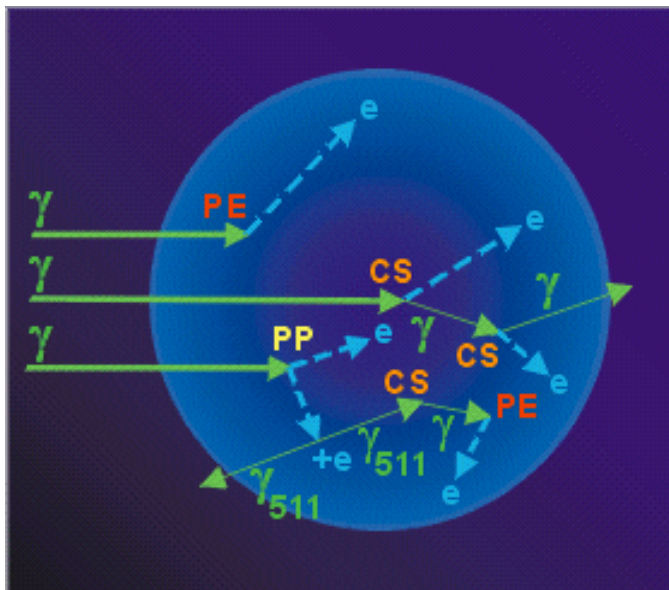
Pictures from University of Liverpool website

Interactions in a real detector

Within a **real detector**, the interaction outcome is not as simple to predict as e.g.

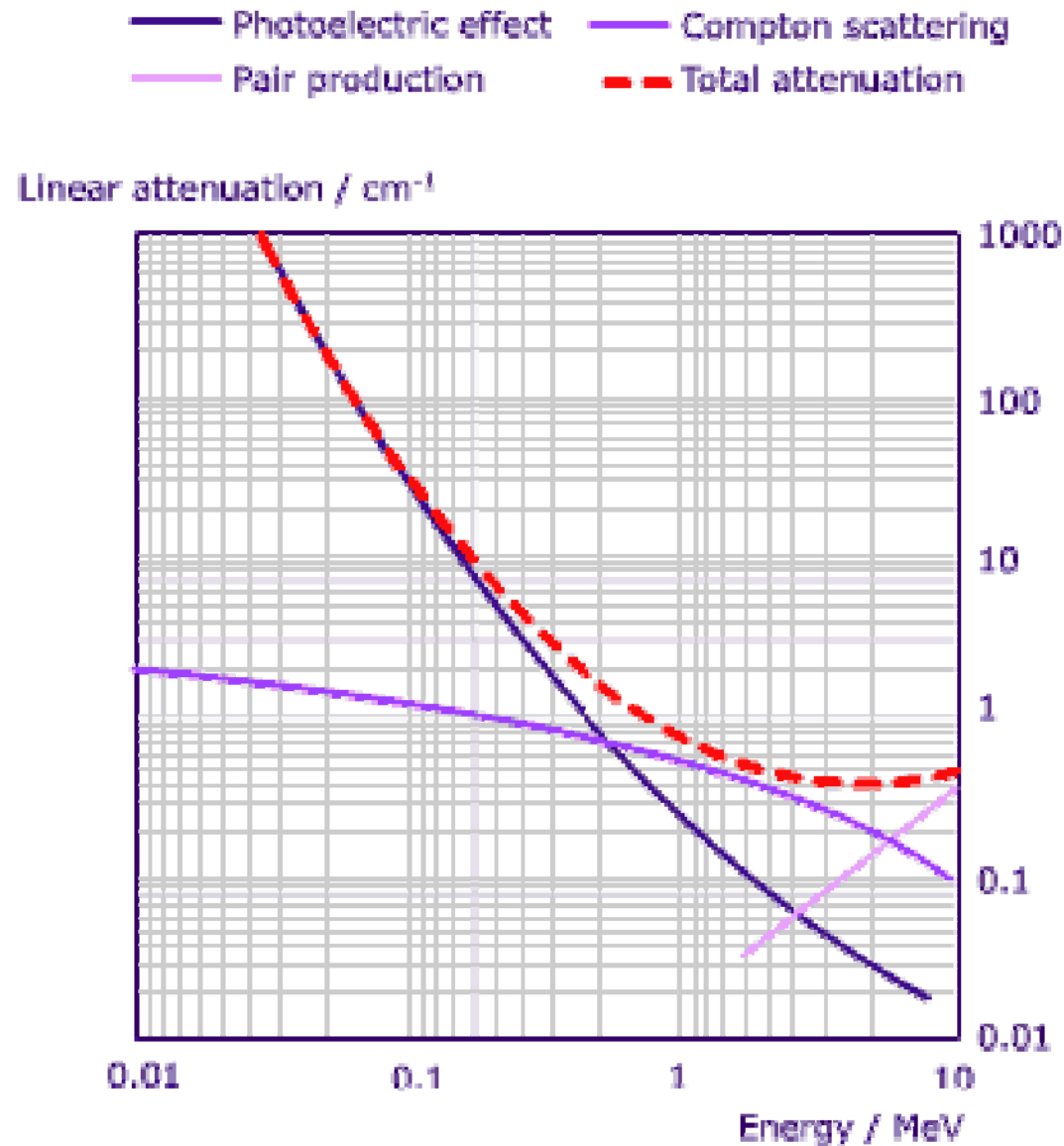
one **Compton scattering** could be followed by another before the gamma-ray photon escapes from the detector.

in the case of **pair production**, both, one or neither of the annihilation photons could escape from the detector. Hence all three peaks may be observed.



Pictures from University of Liverpool website

Energy dependence of the interactions



Typically we are interested in transitions of energy $60 \text{ keV} < E_\gamma < 10 \text{ MeV}$.

Detector types

Two main types of material are used:

Solid state semiconductor detectors e.g. Ge, CZT

Electron-hole pairs are collected as charge

knock-on effect \Rightarrow an avalanche arrives at the electrode

lots of electrons \Rightarrow good energy resolution

cooled to liquid N₂ temperature (77K) to reduce noise

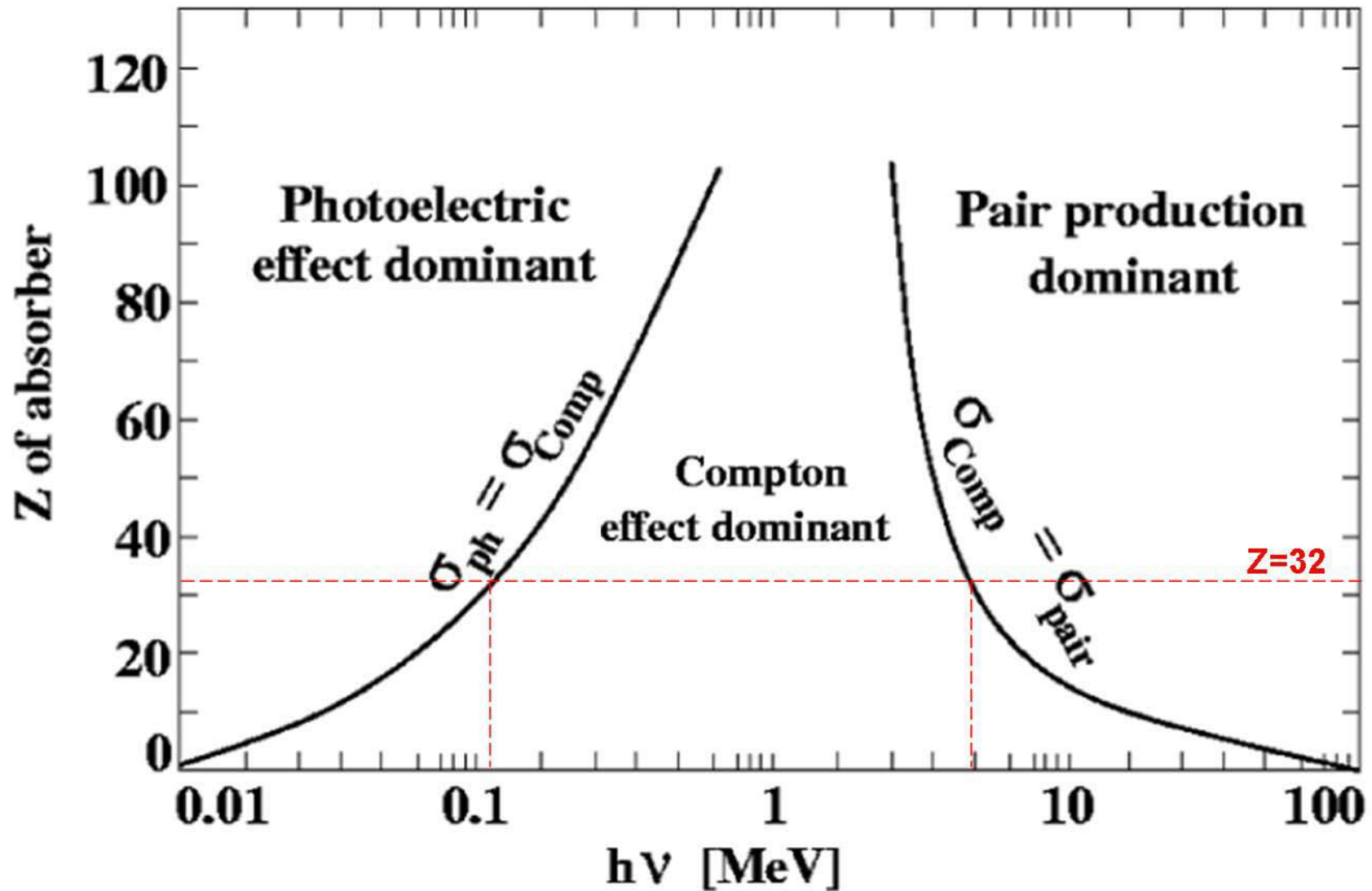
Advantage: good energy resolution ($\sim 0.15\%$ (FWHM) at 1.3 MeV)

Disadvantages: relatively low efficiency

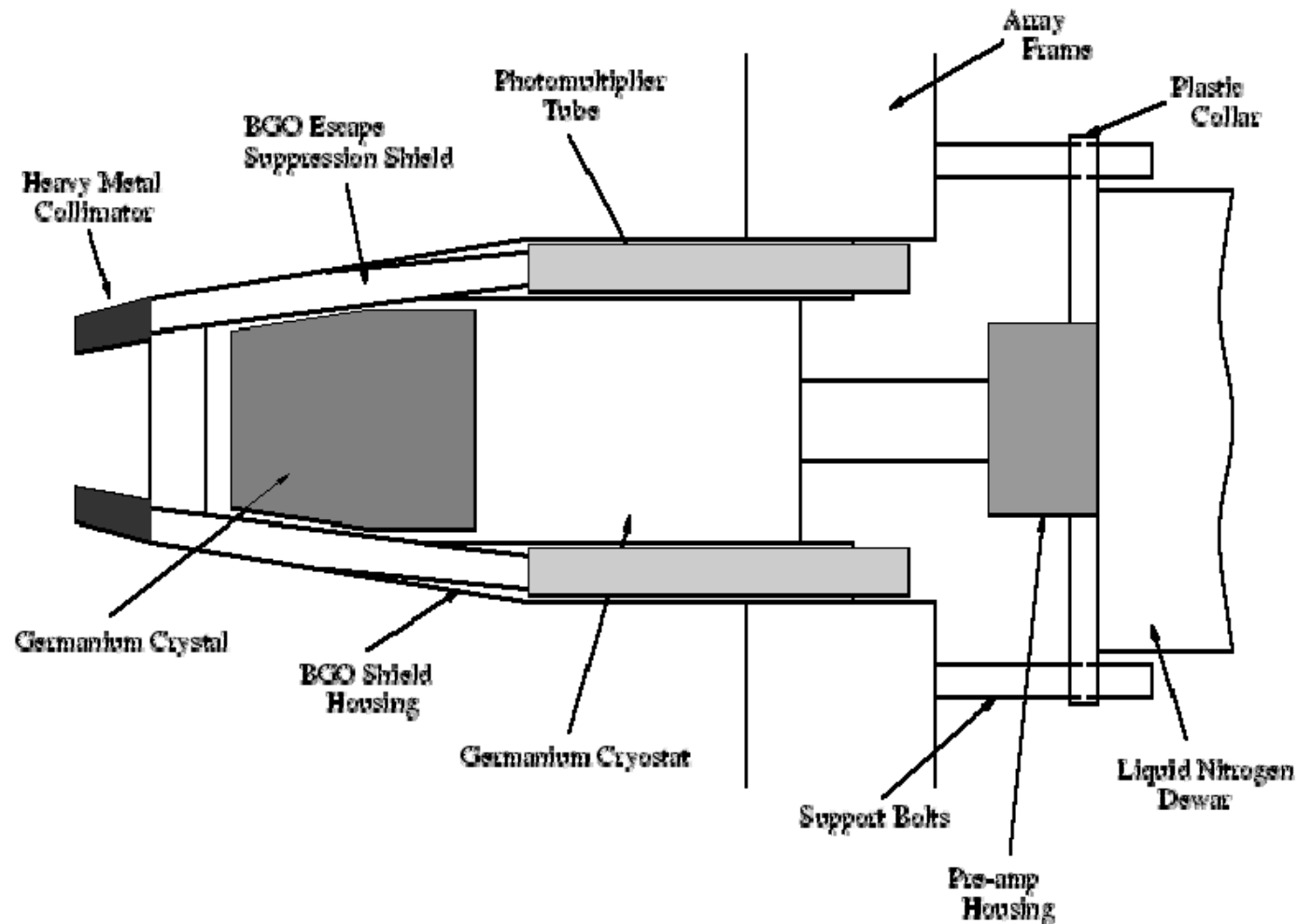
cryogenic operation

limit to the size of crystal/detector

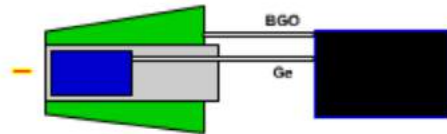
Z dependence of interaction probabilities



Escape suppressed HP Ge detector



Large Gamma Arrays based on Compton Suppressed Spectrometers



EUROBALL

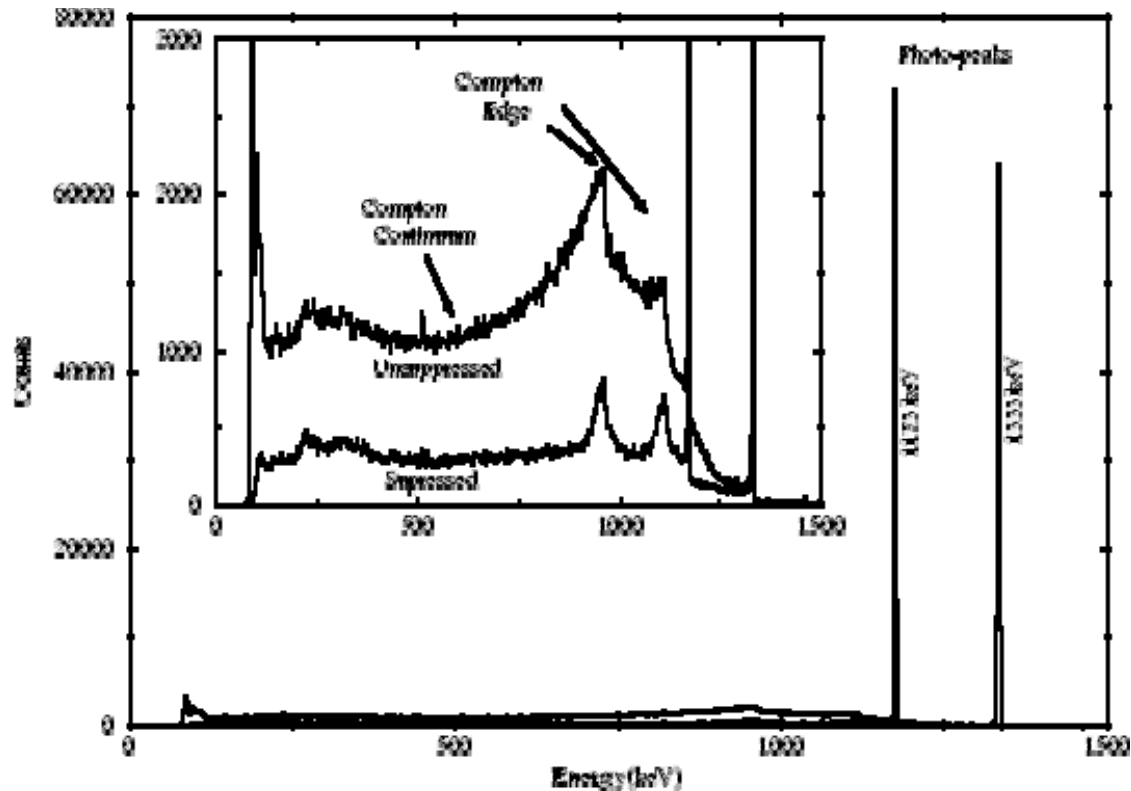


GAMMASPHERE

$$\varepsilon \sim 10 - 5 \%$$

($M_{\gamma}=1 - M_{\gamma}=30$)

Compton suppression increases peak to total



Bare detector: P/T ~20% at ~ 1 MeV i.e only 20% of single events have the full energy measured
for γ - γ only 4% of events are full energy, γ - γ - γ - γ 0.16%

Compton suppression: P/T ~ 60%

Detector types

Two main types of material are used:

Scintillation detectors e.g. NaI, BGO, LaBr₃(Ce)

Recoiling electrons excite atoms, which then de-excite by emitting visible light.

Light is collected in photomultiplier tubes (PMT) where it generates a pulse proportional to the light collected.

Advantage: good timing resolution

can be made relatively large e.g. NaI detectors 14" ϕ x 10"

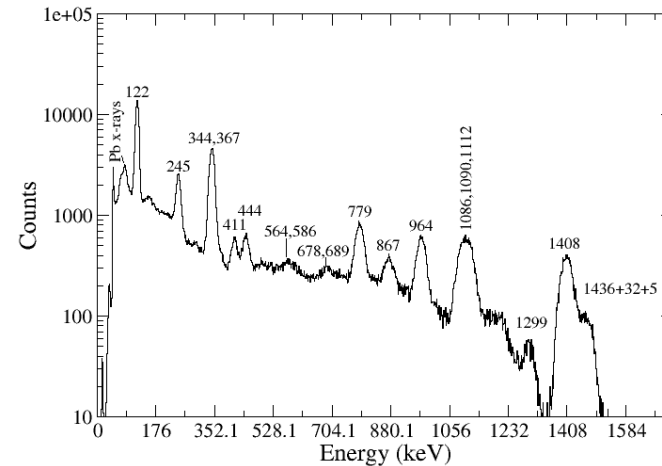
no need for cryogenics

Disadvantages: poor energy resolution

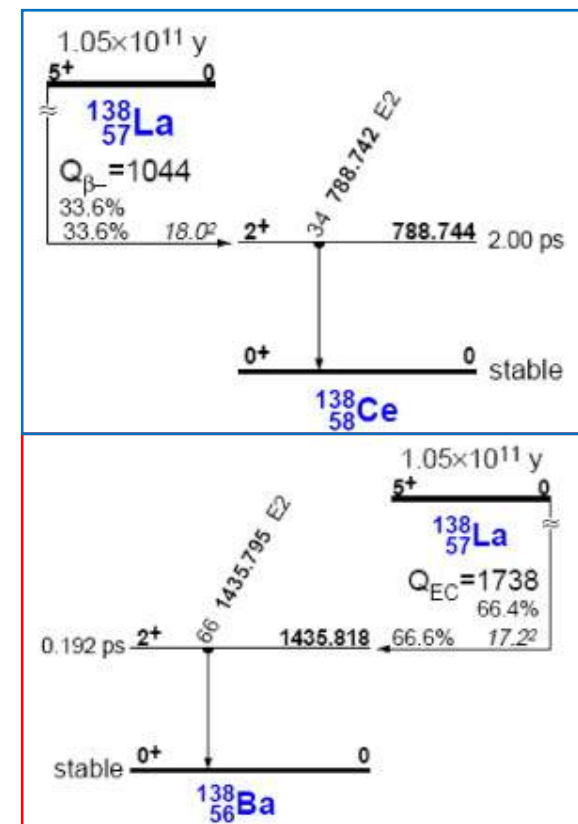
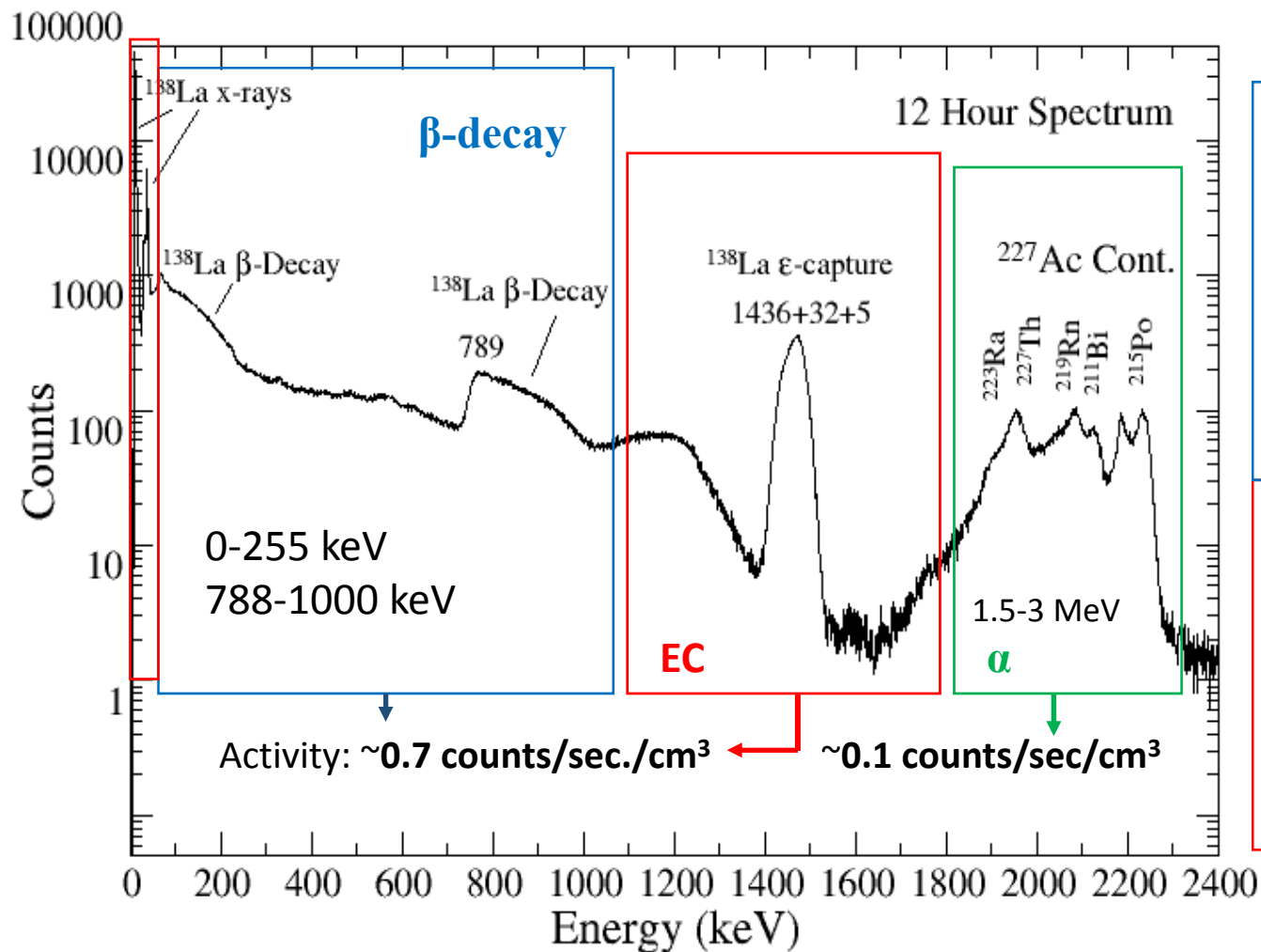
Scintillation detectors

LaBr₃(Ce)

- ▶ LaBr₃(Ce) timing properties:
 - ~ 25 ns decay time
 - Timing Resolution FWHM of 130-150 ps with ⁶⁰Co for a Ø1"x1" crystal.
- ▶ High energy resolution, 3 % FWHM at 662 keV.
- ▶ Peak Emission wavelength in Blue/UV part of EM spectrum (380 nm), compatible with PMTs.

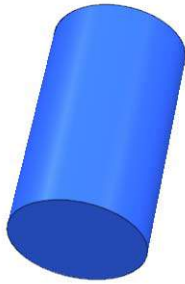


Detector Characterisation



J. McIntyre et al., NIM A 652, 1, 2011, 201-204

Timing resolution of cylindrical crystals



ø1"x1"

FWHM 200 ps

FWHM 150 ps



ø1.5"x1.5"

360 ps

180 ps



ø2"x2"

450 ps at 511 keV

300 ps at 1332 keV

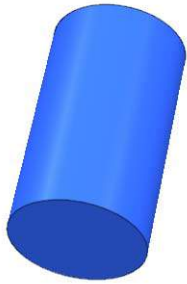
N. Mărginean et al. *Eur. Phys. J A* 46, 329-336, 2010.

I. Deloncle et al. *J. Phys.:Conf. Series* 205, 012044, 2010.

M. Moszynski et al. *Nucl. Instr. Methods A* 567, 2007.

L.M. Fraile et al. ISOLDE Workshop, Fast timing results at ISOLDE,
<http://indico.cern.ch/getFile.py/access?contribId=36&sessionId=8&resId=0&materialId=slides&confId=67060>,
November, 2009.

Timing resolution of cylindrical crystals



ø1"x1"

FWHM 200 ps

FWHM 150 ps



ø1.5"x1.5"

360 ps

180 ps



ø2"x2"

450 ps at 511 keV

300 ps at 1332 keV

Trade off between resolution and efficiency

Timing Precision

Trade off between good timing resolution (small detectors)

lots of statistics (large detectors)

Timing precision defined as:

$$TP = \frac{T(FWHM)}{\sqrt{N}}$$

Detector requirements for in-beam spectroscopy

Gamma rays emitted by moving reaction product:

Good energy resolution (as lots of gamma rays emitted)

High granularity (to uniquely define the γ -ray angle and because lots of gamma rays are emitted - high multiplicity)

High photopeak detection efficiency (to see the weakest channels)

Good peak to total ratio (so that coincidence gates can be clean)

Energy resolution

The major factors affecting the final energy resolution (FWHM) at a particular energy are as follows:

$$\Delta E_{\gamma}^{\text{final}} = \left(\Delta E_{\text{Int}}^2 + \Delta \theta_{\text{D}}^2 + \Delta \theta_{\text{N}}^2 + \Delta V^2 \right)^{1/2}$$

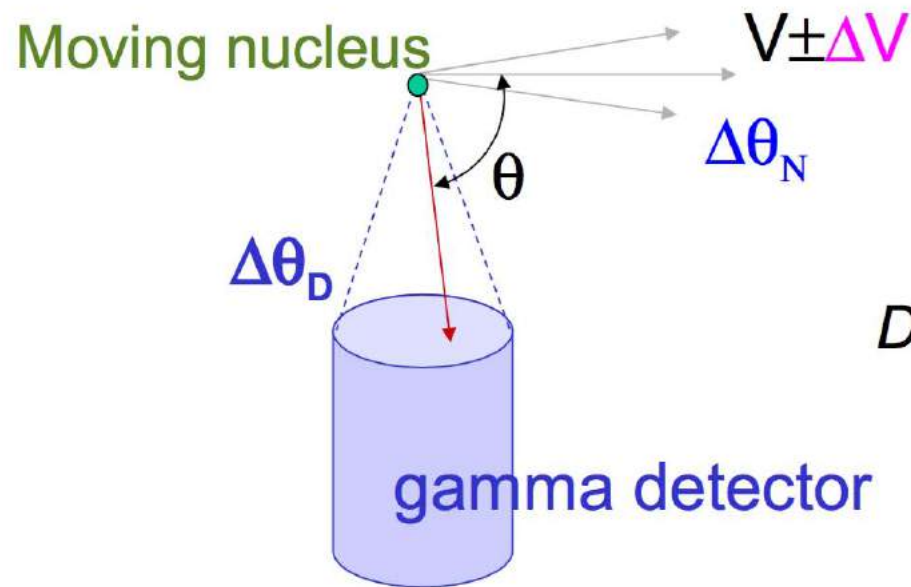
ΔE_{Int} - The intrinsic resolution of the detector system.
This includes contributions from the detector itself and the electronic components used to process the signal.

$\Delta \theta_{\text{D}}$ - The Doppler broadening arising from the opening angle of the detectors.

$\Delta \theta_{\text{N}}$ - The Doppler broadening arising from the angular spread of recoils in the target.

ΔV - The Doppler broadening arising from the velocity (energy) variation of the recoils across the target.

Doppler broadening



Doppler shift

$$E_\gamma = E_\gamma^0 \frac{\sqrt{1 - \frac{V^2}{c^2}}}{1 - \frac{V}{c} \cos \theta}$$

Broadening of detected γ -ray energy arises from:

- Spread in recoil velocity ΔV
- Distribution in the direction of recoil $\Delta\theta_N$
- Detector opening angle $\Delta\theta_D$

Minimising Doppler broadening

There are two ways in which gamma-ray spectroscopists can mitigate the effects of Doppler broadening.

- Reduce the detector opening angle (lower $\Delta\theta_D$)
 - Detector granularity
- Minimise the target thickness (lower spread in recoil velocity ΔV)

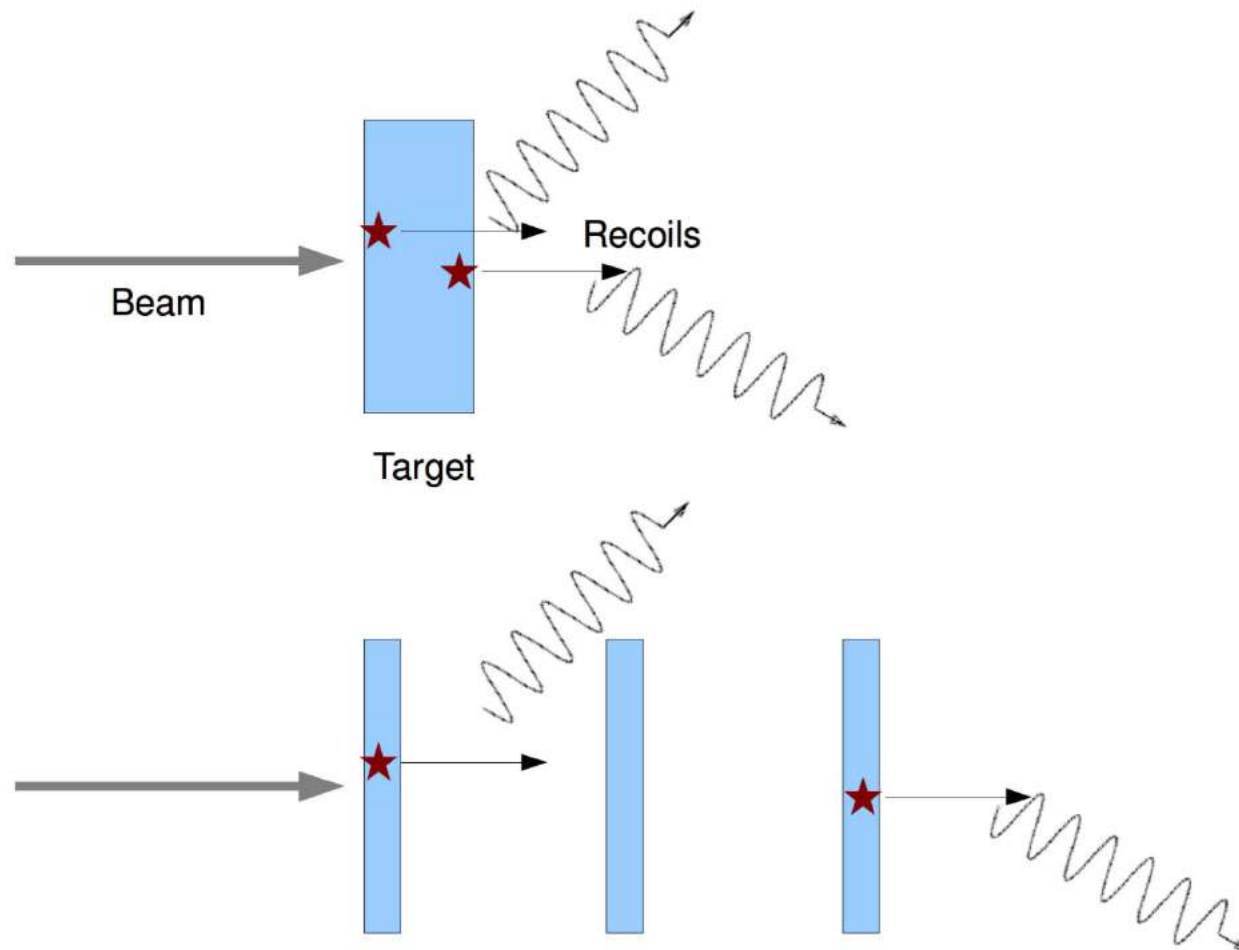
Experimentalists often choose to use two or three stacked targets rather than a single thick target e.g. $2 \times 0.5 \text{ mg/cm}^2$ rather than a single 1 mg/cm^2 .

This works for normal kinematics where the spread in velocities ΔV arises from the slowing of the recoil and not the beam.

Minimising Doppler broadening

Target thickness

Light beam, heavy recoil



Successful Compton suppression arrays:



Spectroscopy of ^{158}Er

~1980 yrast states to spin ~30, naked Ge arrays



~1980-1982 TESSA

Escape suppressed array at NBI

Non-yrast bands to spins in mid 20's

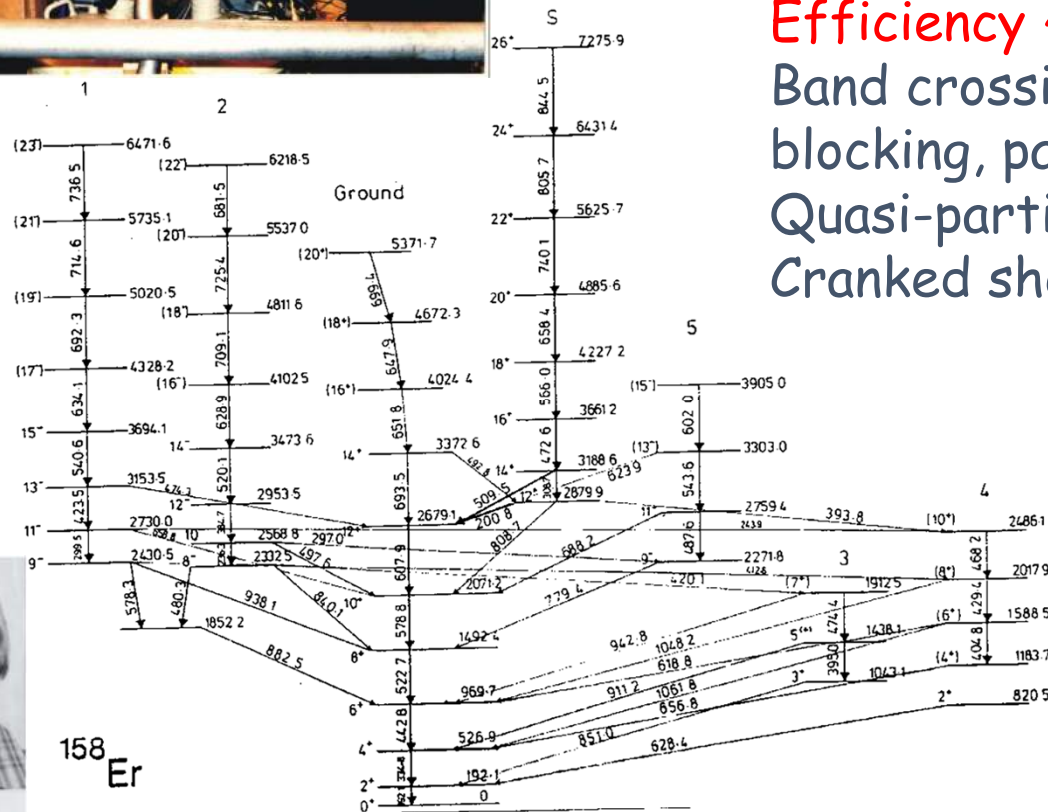
Intensity few %

Efficiency ~0.2%

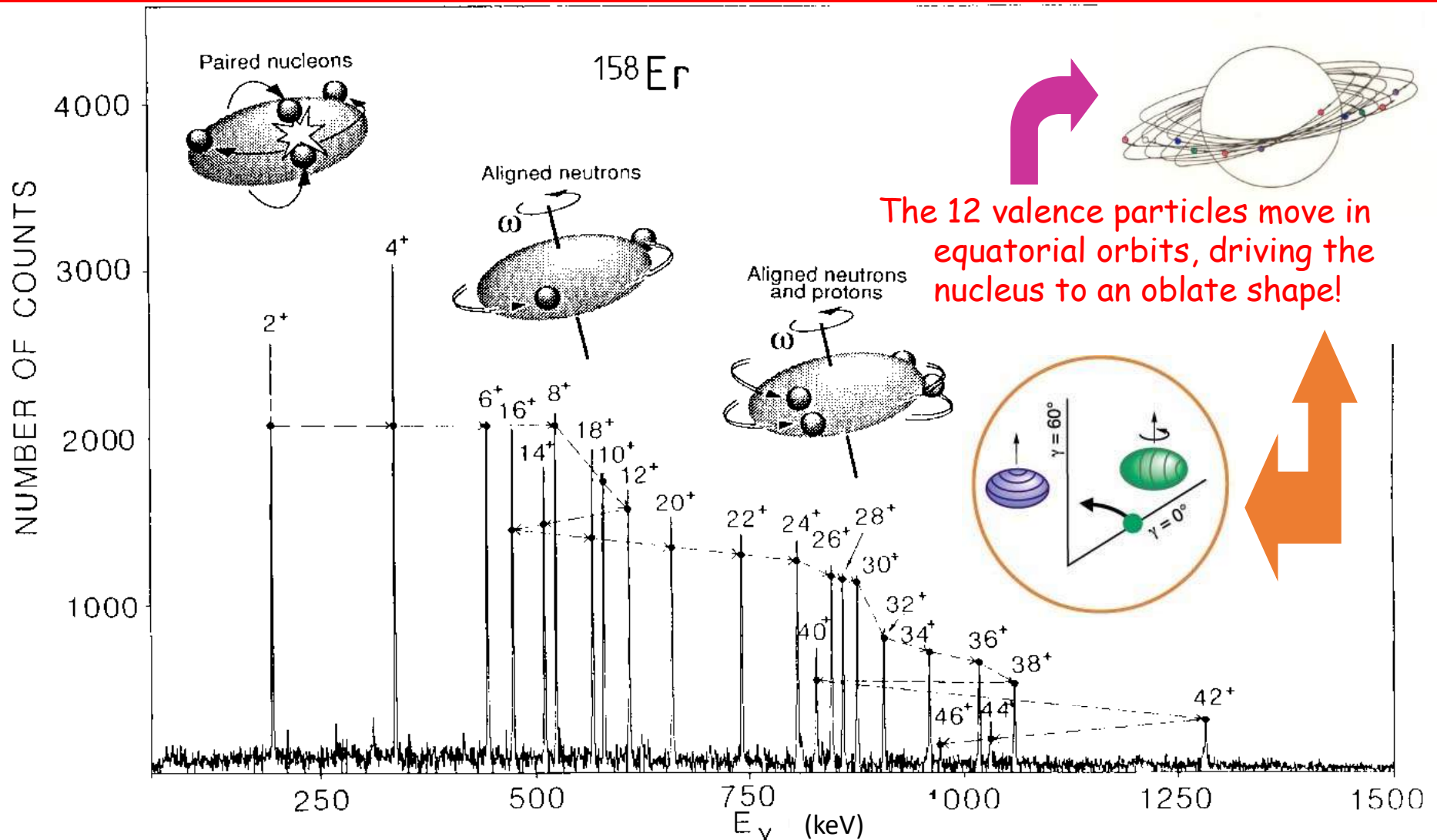
Band crossing systematics,
blocking, pairing reduction

Quasi-particle configurations

Cranked shell model



^{158}Er expt. Daresbury - Sharpey-Schafer, Riley, Simpson et al. mid 1980's



Simpson et al., *Phys. Rev. Lett.* (1984) - prolate-oblate shape change

P.O. Tjøm et al., *PRL* 55 (1985) 2405 -lifetime measurements

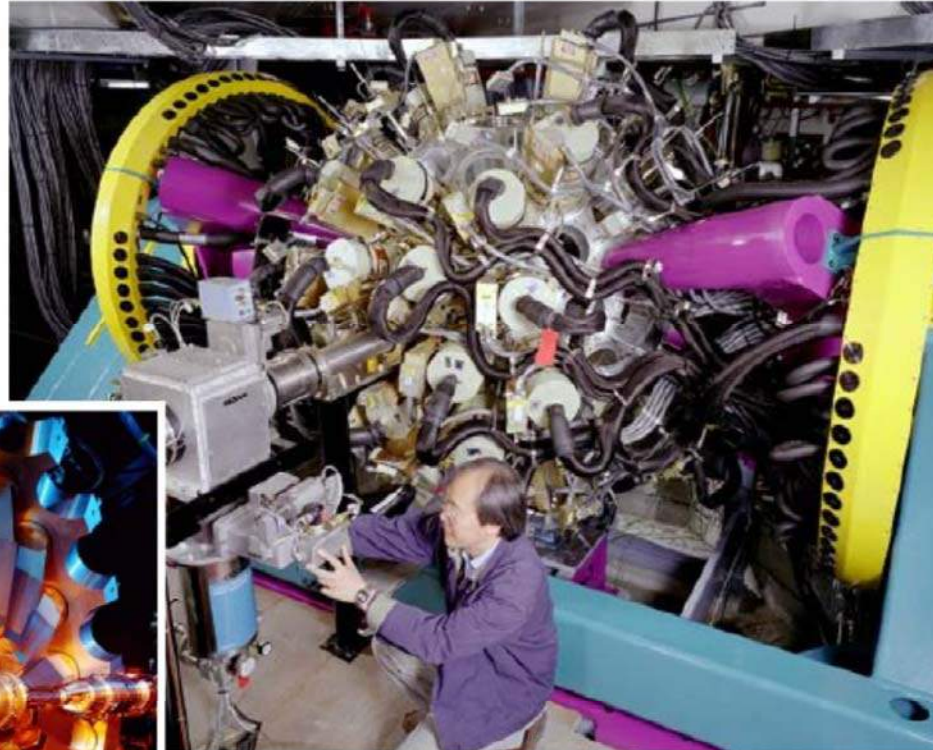
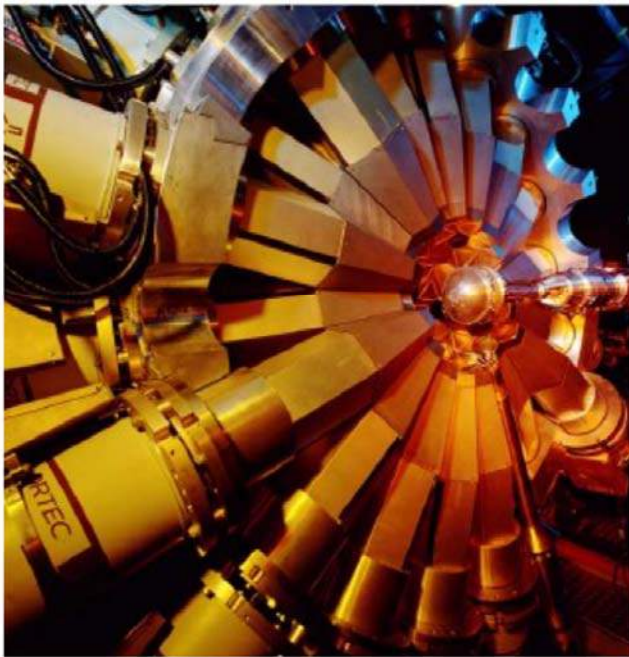
T. Bengtsson and I. Ragnarsson, *Physica Scripta* T5 (1983) 165

J. Dudek, W. Nazarewicz *Phys. Rev C* 32 (1985) 298

Ragnarsson, Xing, Bengtsson and Riley, *Phys. Scripta* 34 (1986) 651

Successful Compton suppression arrays:

Gammasphere

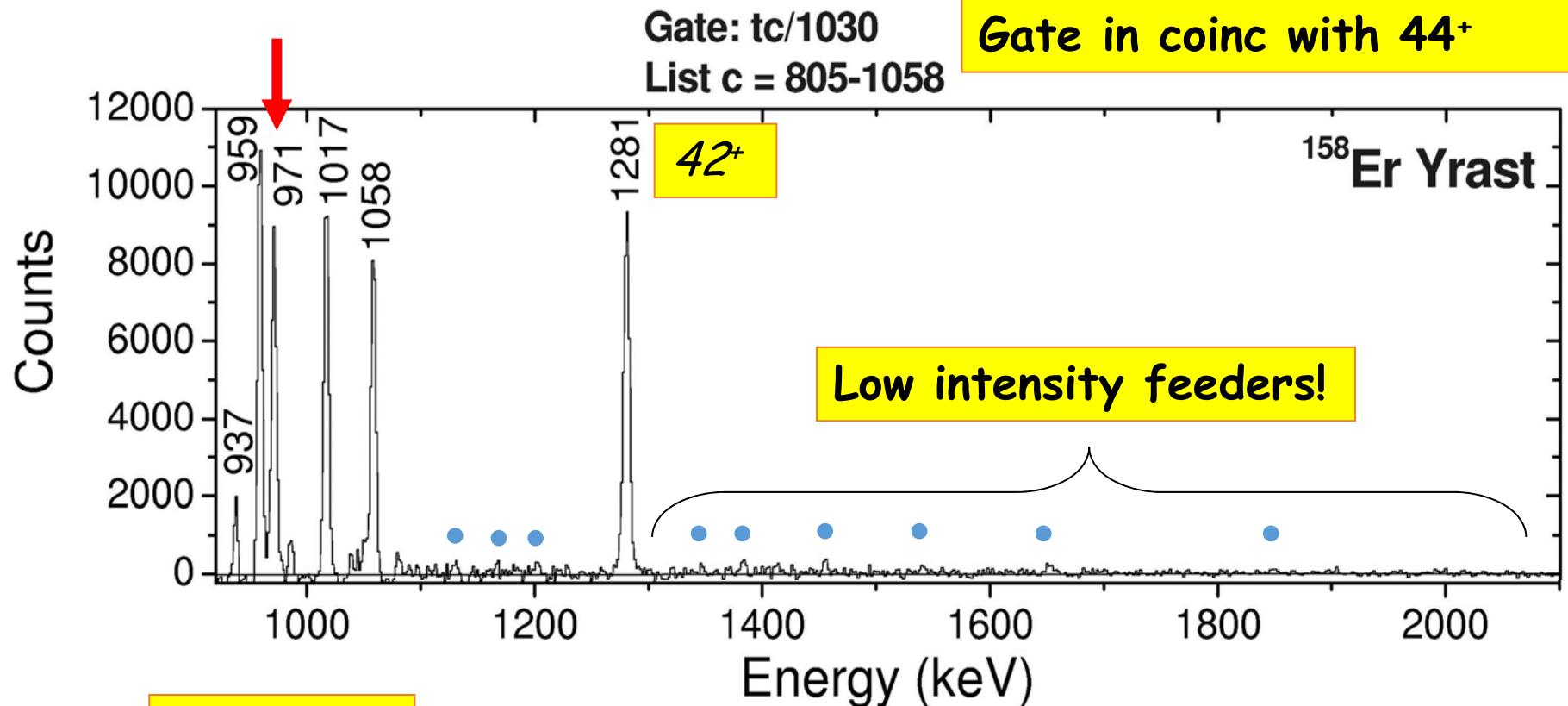


110 Compton-suppressed Ge detectors,
each with 70% efficiency.
Total efficiency = 9% at 1.3 MeV.

What about ^{158}Er above 46^+ ?

No wonder we could not see it before!

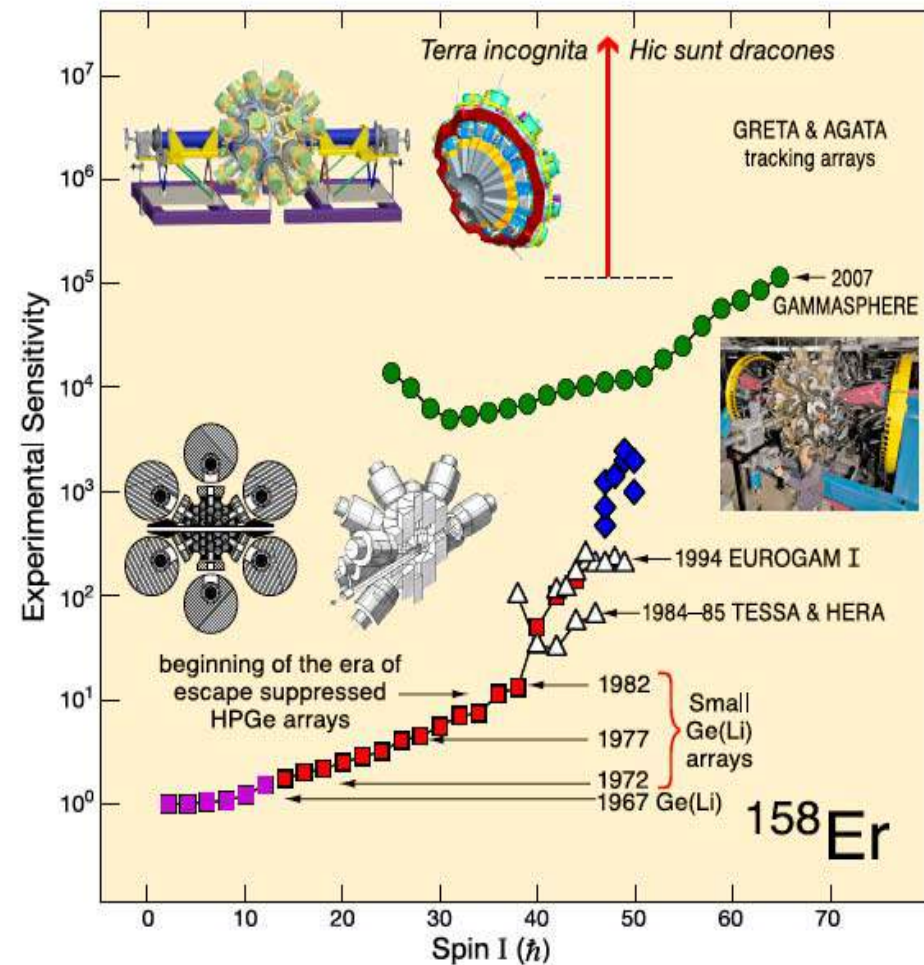
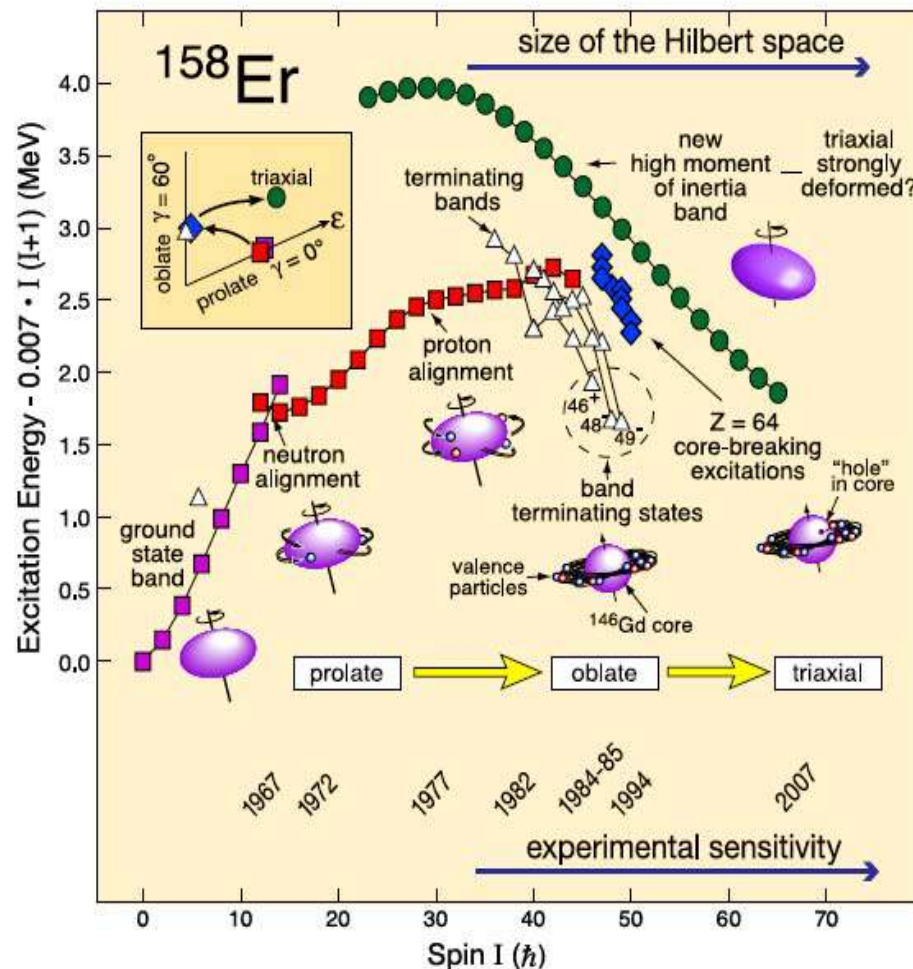
$46^+ = 1\%$ of $2^+ \rightarrow 0^+$



$\varepsilon = 9.5\%$

Evolution of Gamma-Ray Spectroscopy

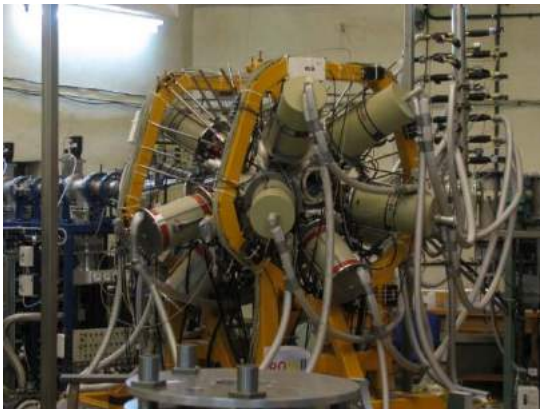
New Detector Systems \longleftrightarrow New Physics



Mixed arrays

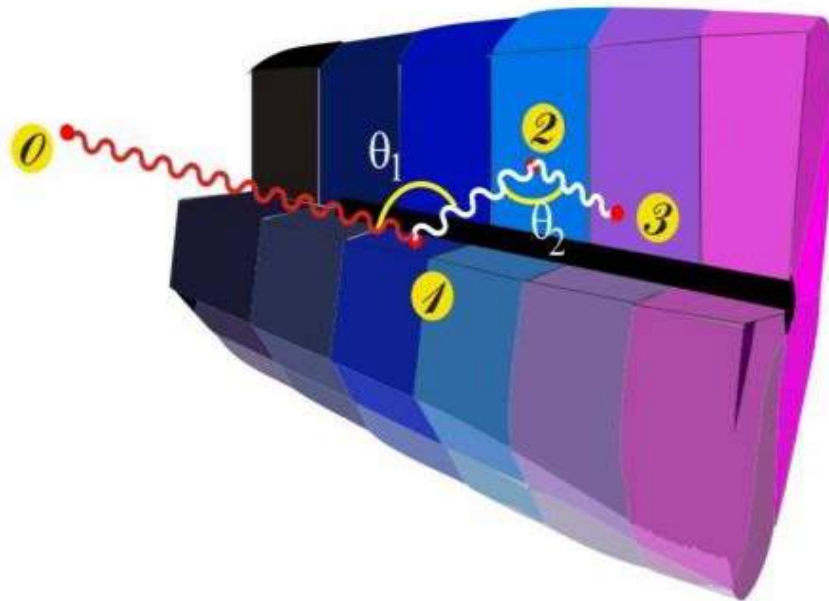
More recently, $\text{LaBr}_3(\text{Ce})$ detectors have been added to arrays of Compton suppressed detectors because of their good timing properties.

E.G. ROSPHERE at Bucharest



- 15 HPGe detectors (A/C):
 - 10 x HPGe detectors @ 37°
 - 1 x HPGe detector @ 64°
 - 4 x HPGe detectors @ 90°
- 11 $\text{LaBr}_3(\text{Ce})$:
 - $\varnothing 2'' \times 2''$ @ 90 and 64° (three) (Cylindrical)
 - $\varnothing 1.5'' \times 1.5''$ @ 90 (six) (Cylindrical)
 - $\varnothing 1'' \times 1.5''$ @ 64° (two) (Conical)

Next generation : tracking

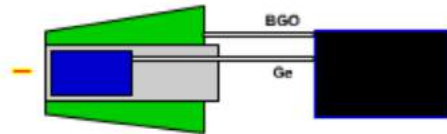


Detectors are segmented
longitudinally and radially.

Pinpoint the position of the first
interaction to get the angle of the
incident gamma ray (very important
if have a high v/c).

Idea of γ -ray tracking

Large Gamma Arrays based on
Compton Suppressed Spectrometers



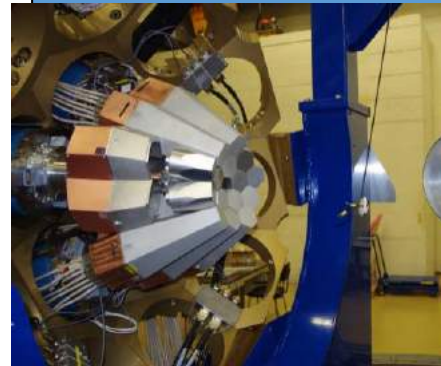
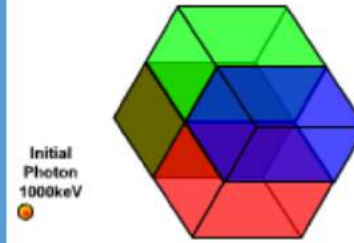
EUROBALL



GAMMASPHERE

$\epsilon \sim 10 - 5 \%$
($M_\gamma=1 - M_\gamma=30$)

Tracking Arrays based on
Position Sensitive Ge Detectors



AGATA

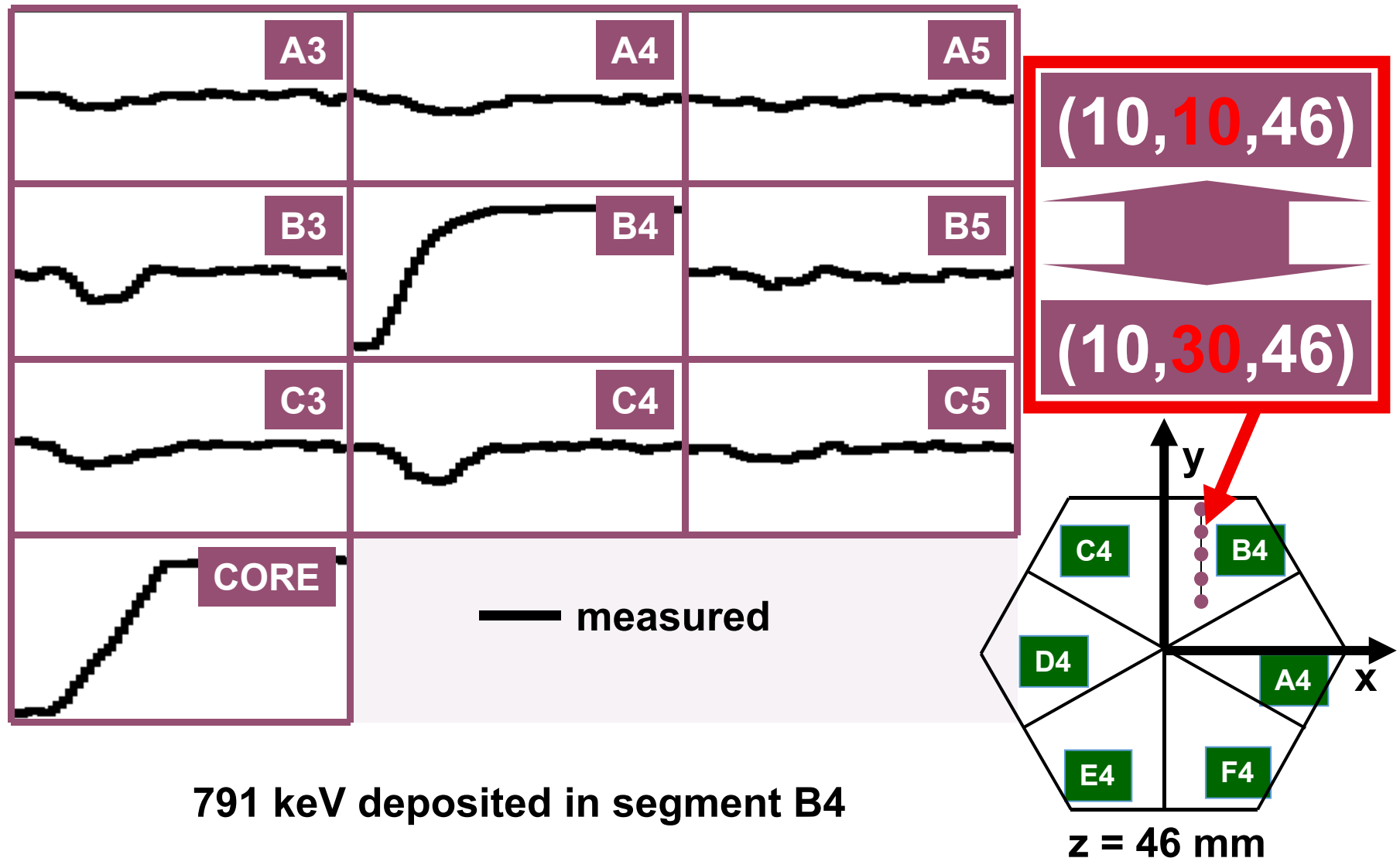


GRETINA/GRETA

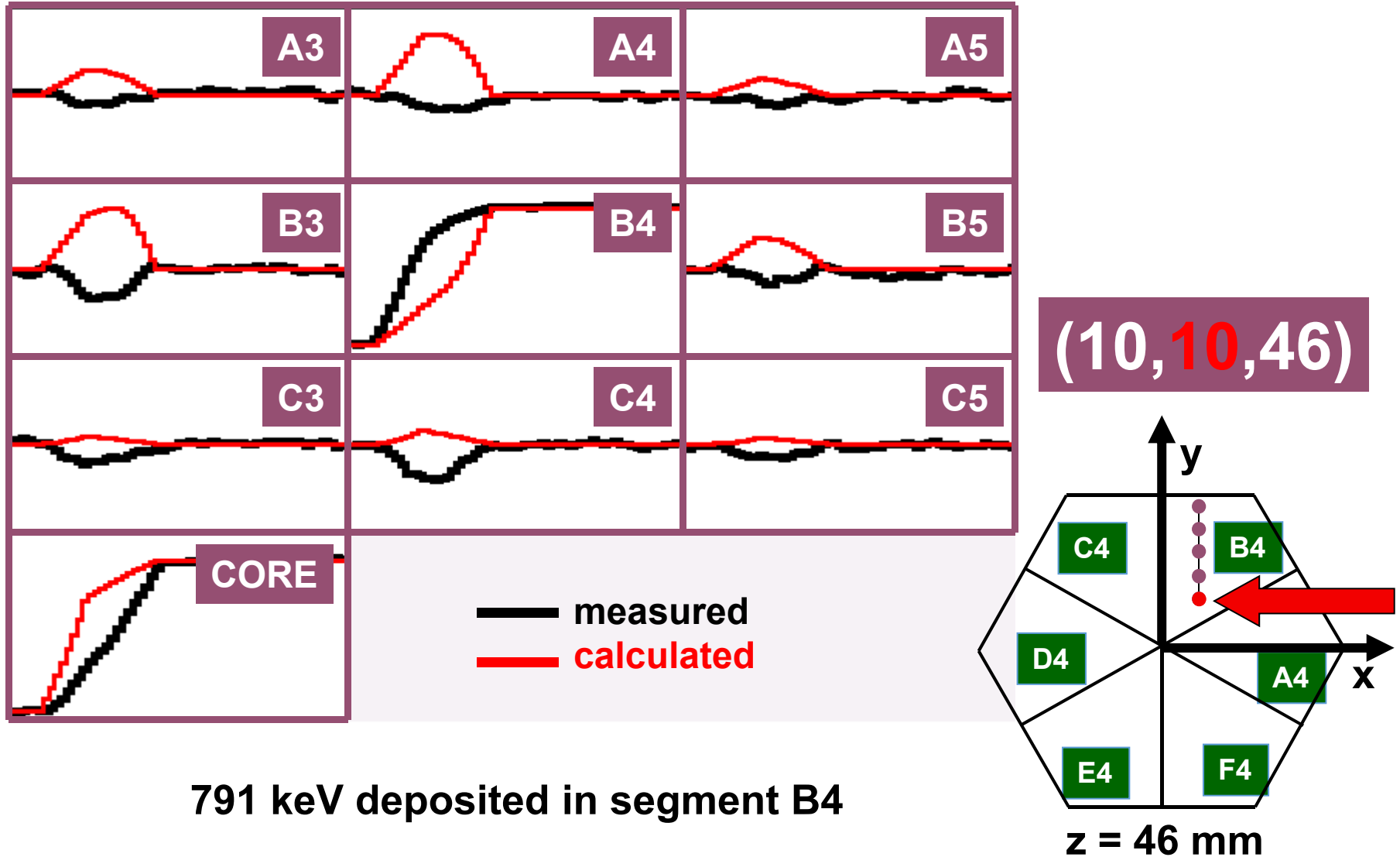
$\epsilon \sim 40 - 20 \%$
($M_\gamma=1 - M_\gamma=30$)

Huge increase in sensitivity

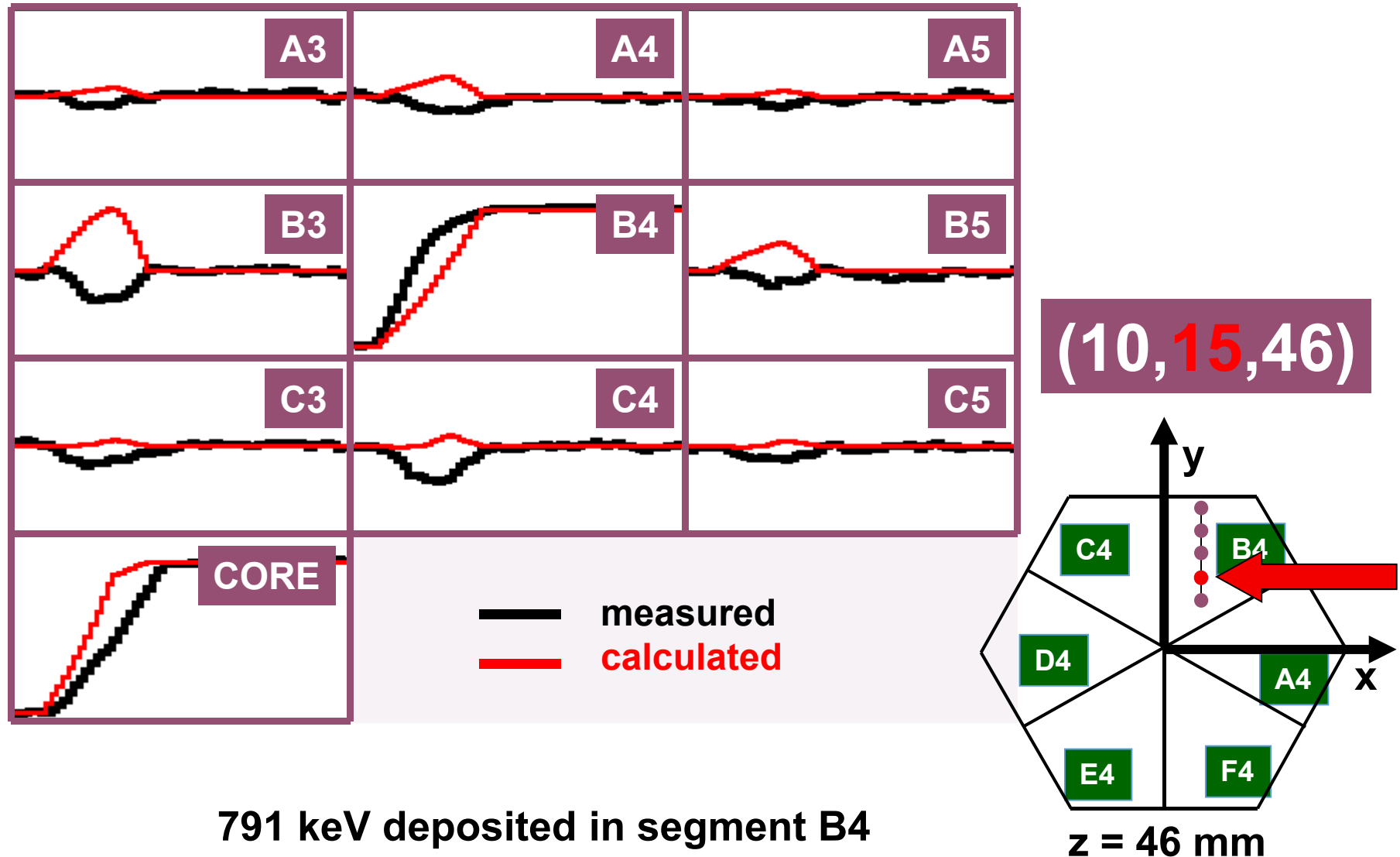
Pulse Shape Analysis concept



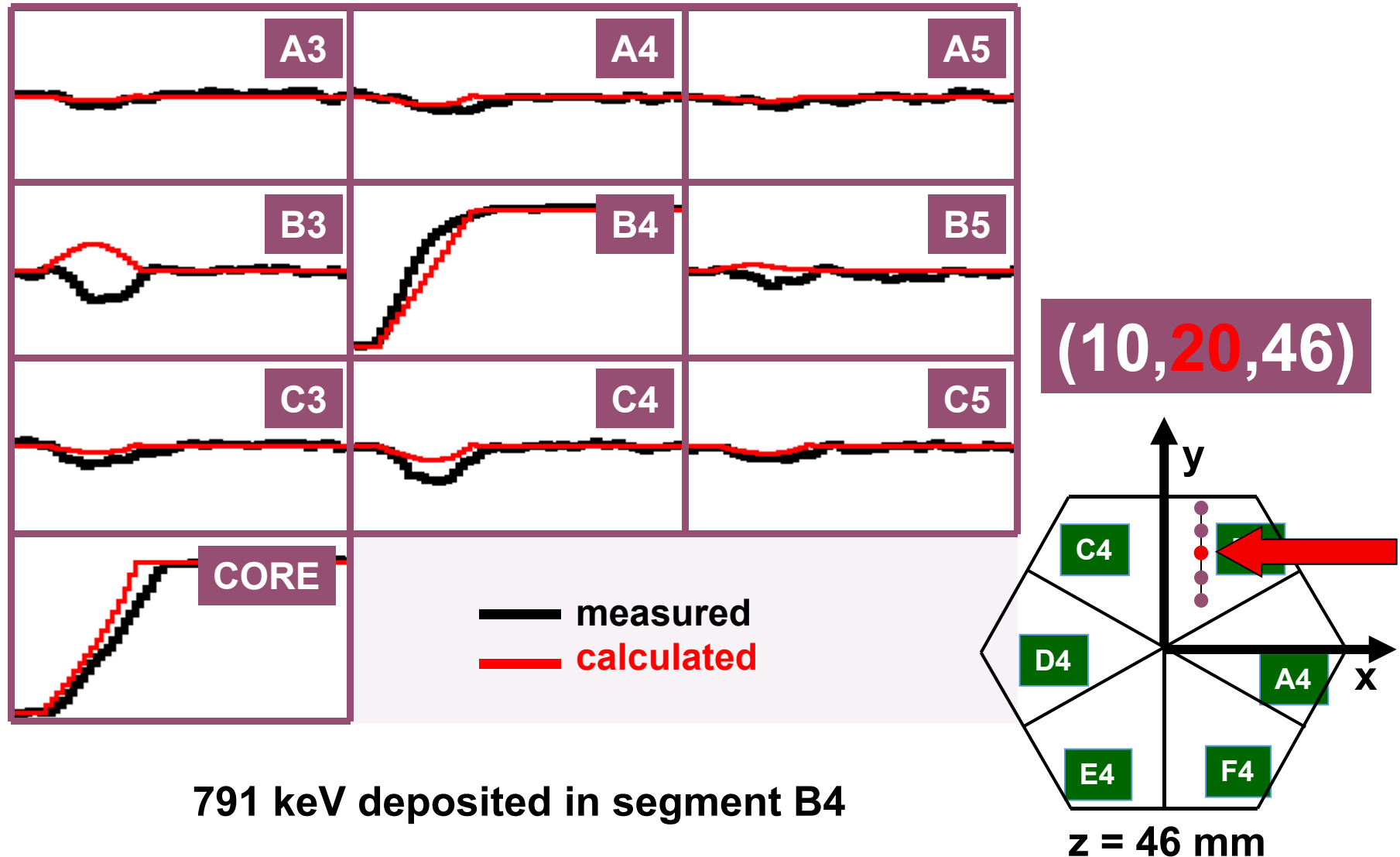
Pulse Shape Analysis concept



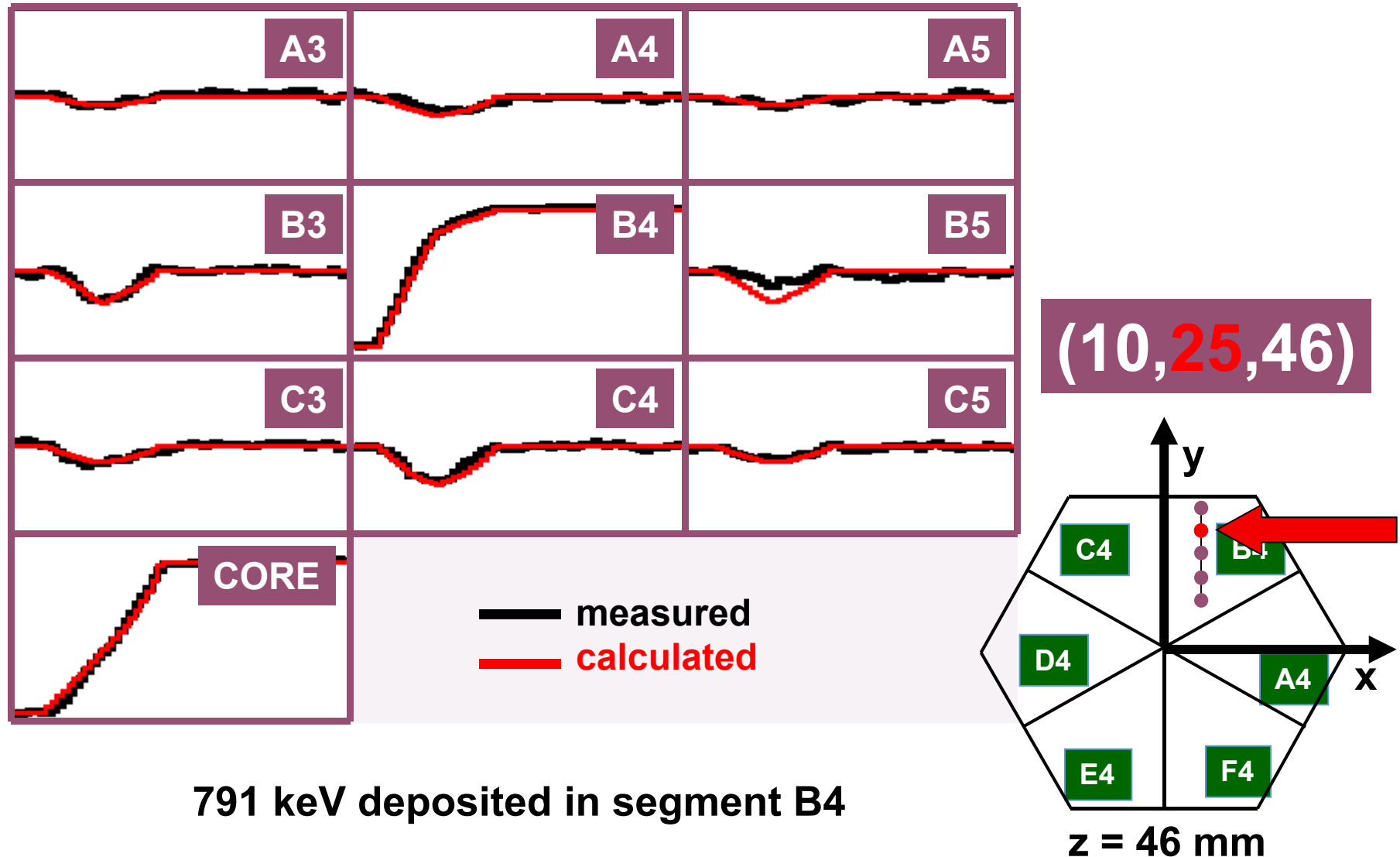
Pulse Shape Analysis concept



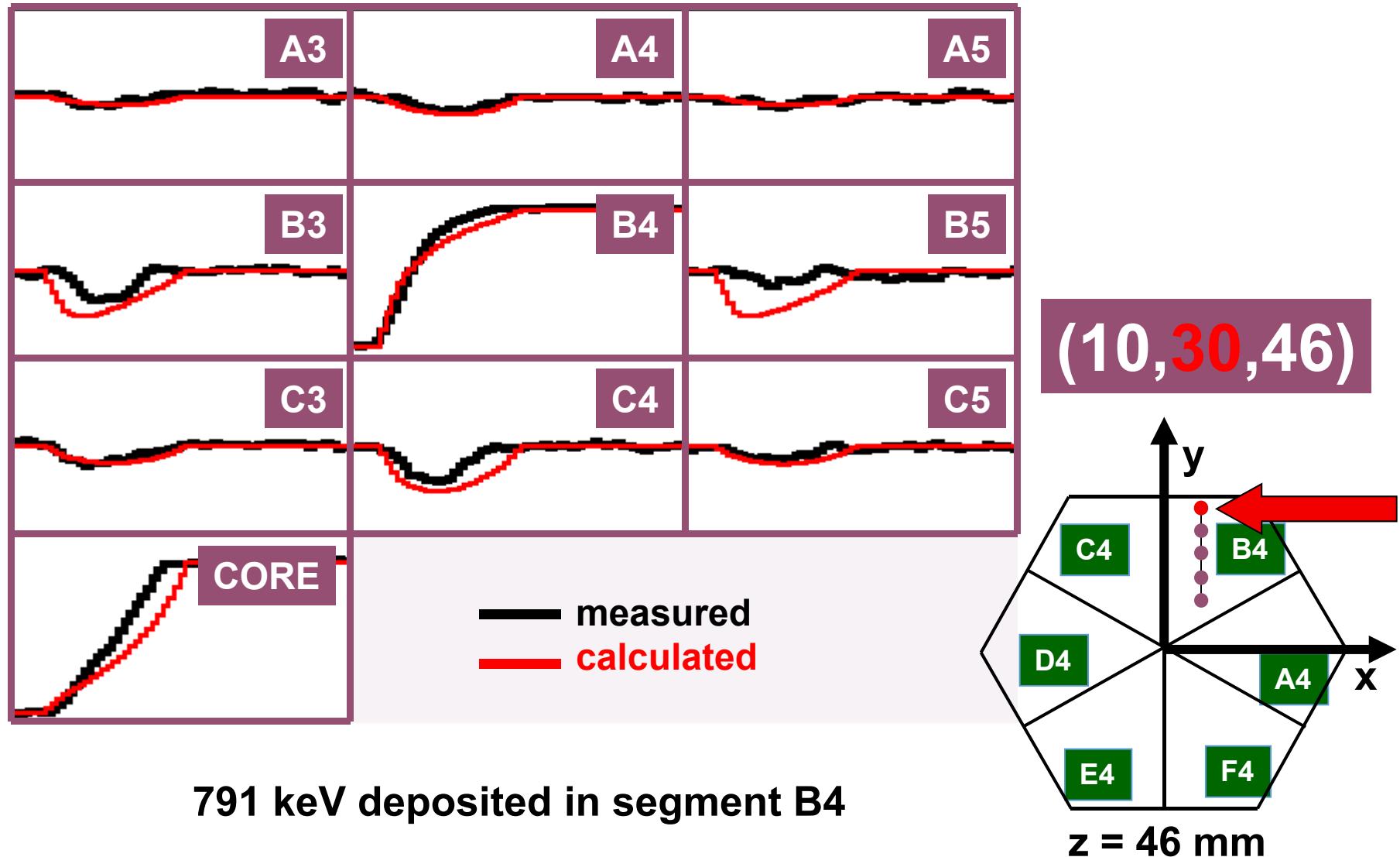
Pulse Shape Analysis concept



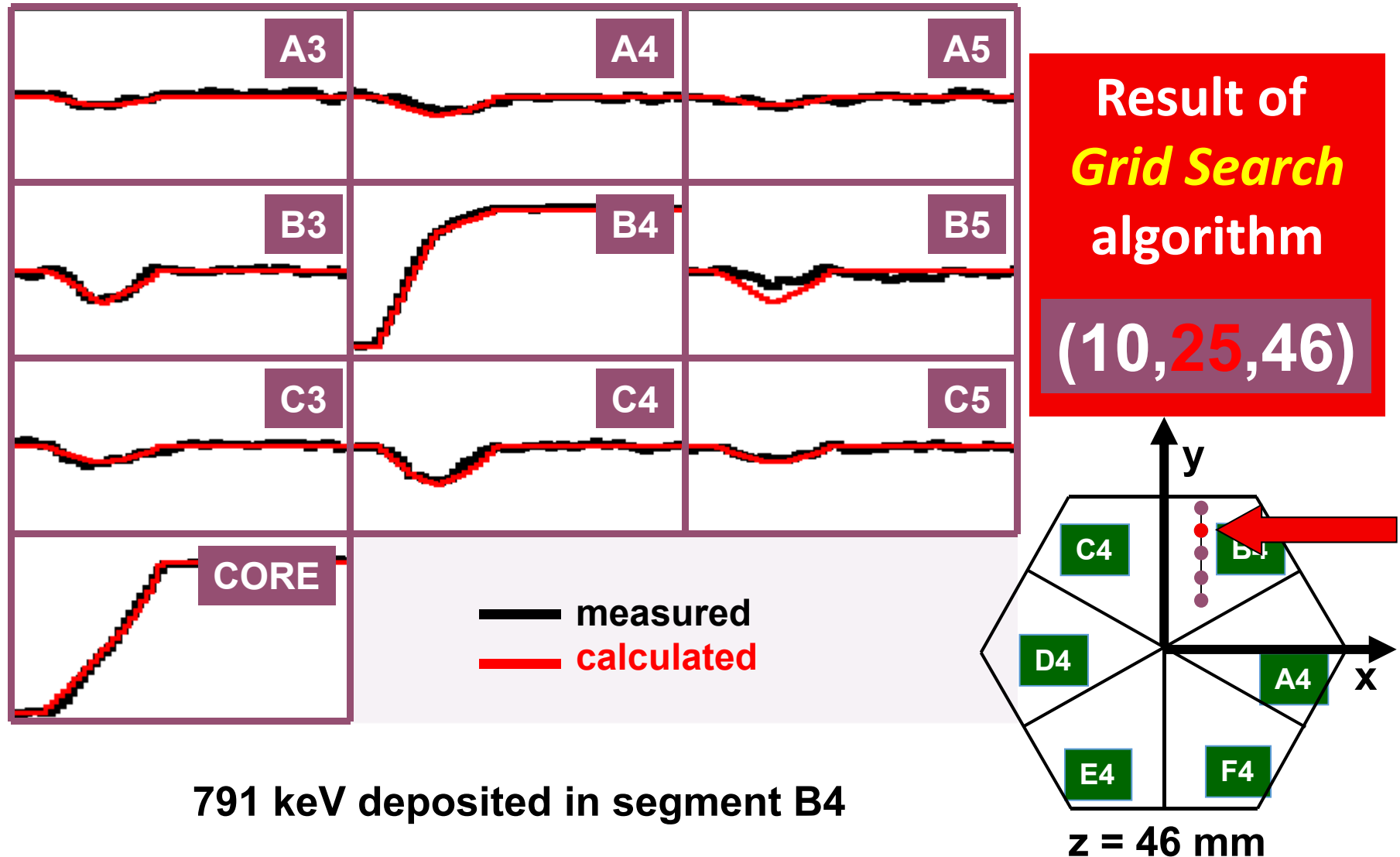
Pulse Shape Analysis concept



Pulse Shape Analysis concept



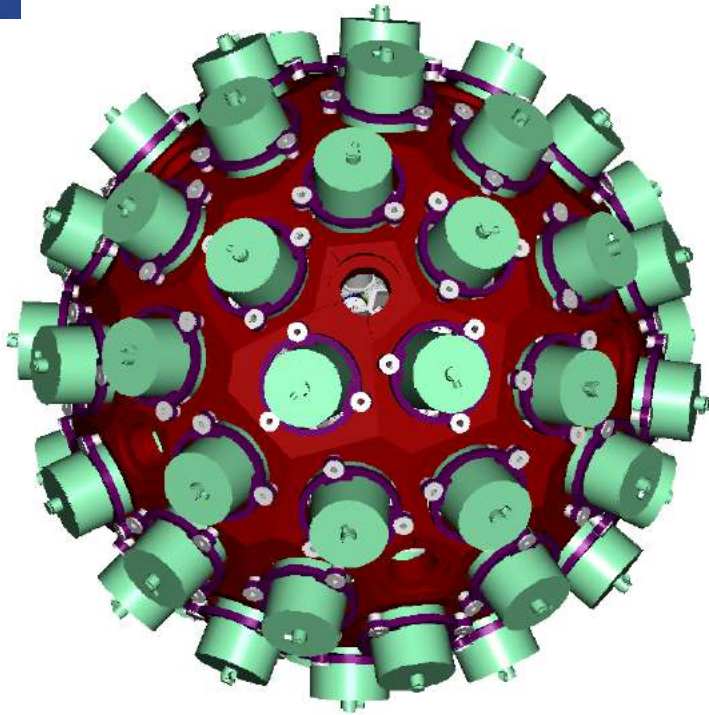
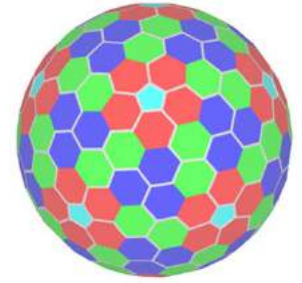
Pulse Shape Analysis concept



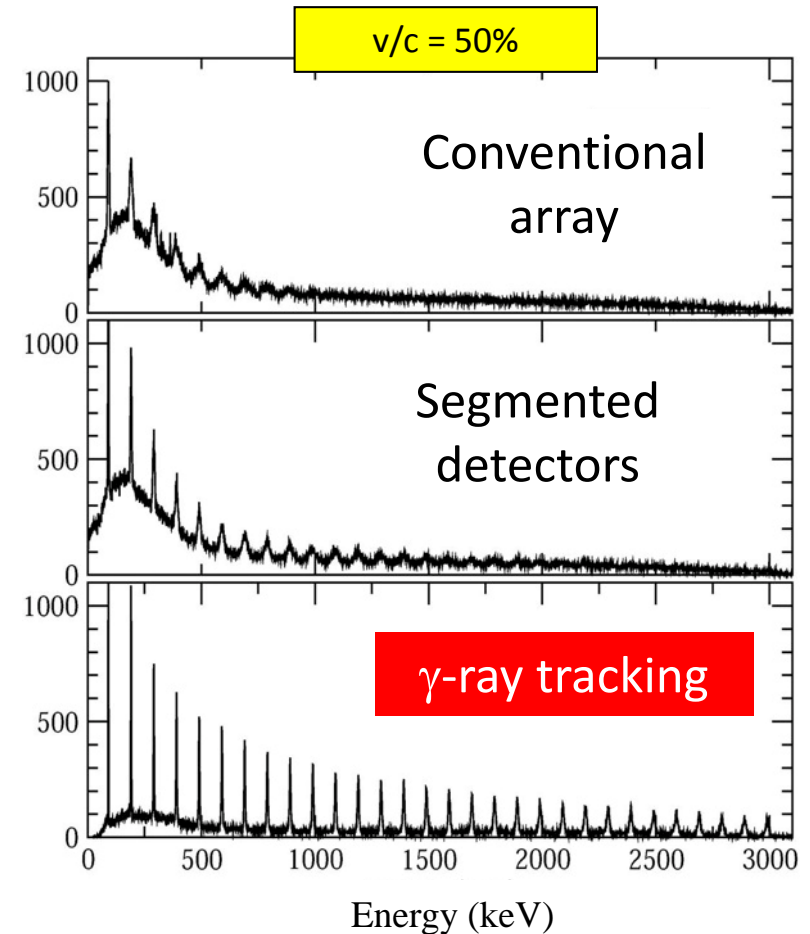


AGATA

(Advanced GAMMA Tracking Array)

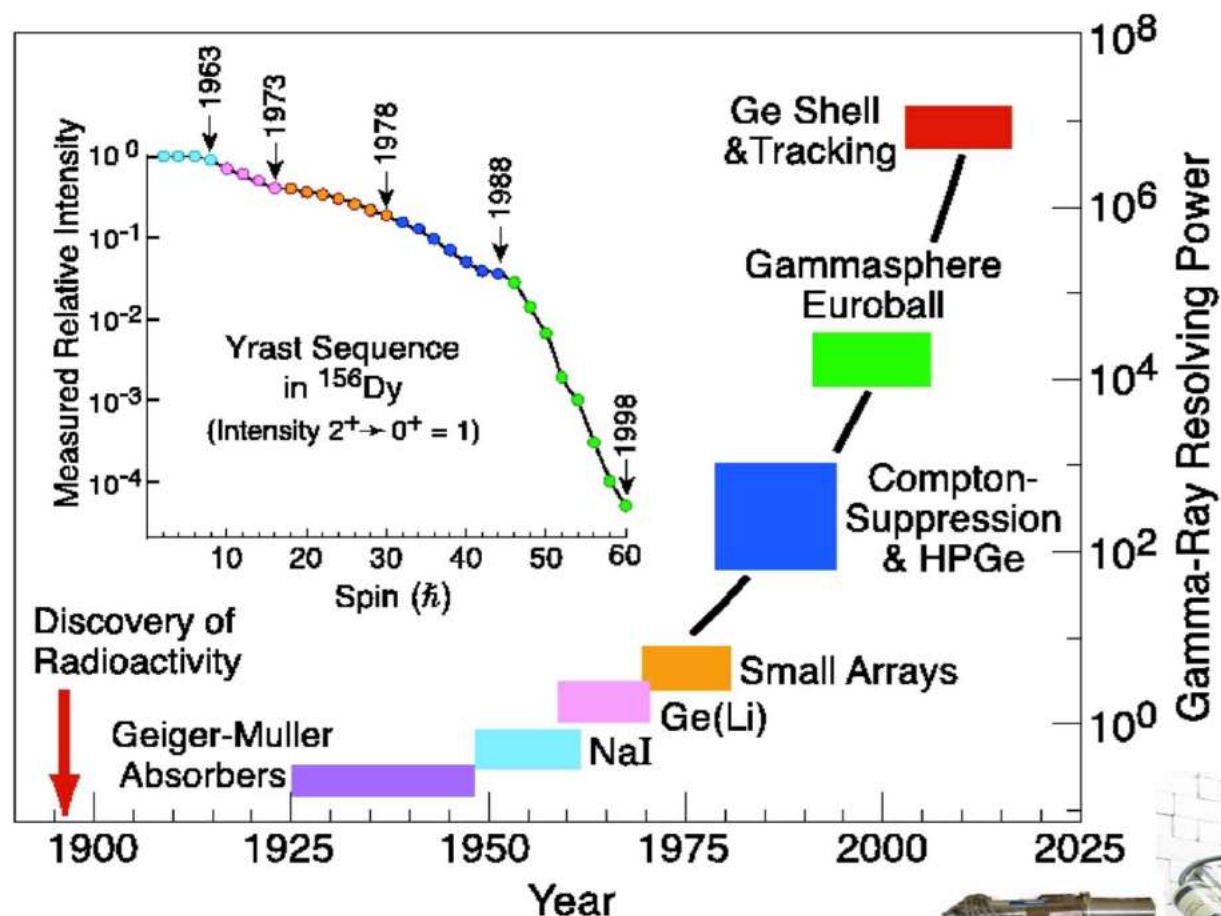


The innovative use of detectors (pulse shape analysis, γ -ray tracking, digital DAQ) will result in high efficiency ($\sim 40\%$) and excellent energy resolution, making AGATA the ideal instrument for spectroscopic studies of weak channels.



The effective energy resolution is maintained also at "extreme" v/c values

Gamma-Ray Detection Evolution



Detector requirements for out-of-beam spectroscopy

Gamma rays emitted by a stopped source:

Doppler broadening is not an issue..detectors can be very close but if you do this you lose angle definition which is required for angular measurements.

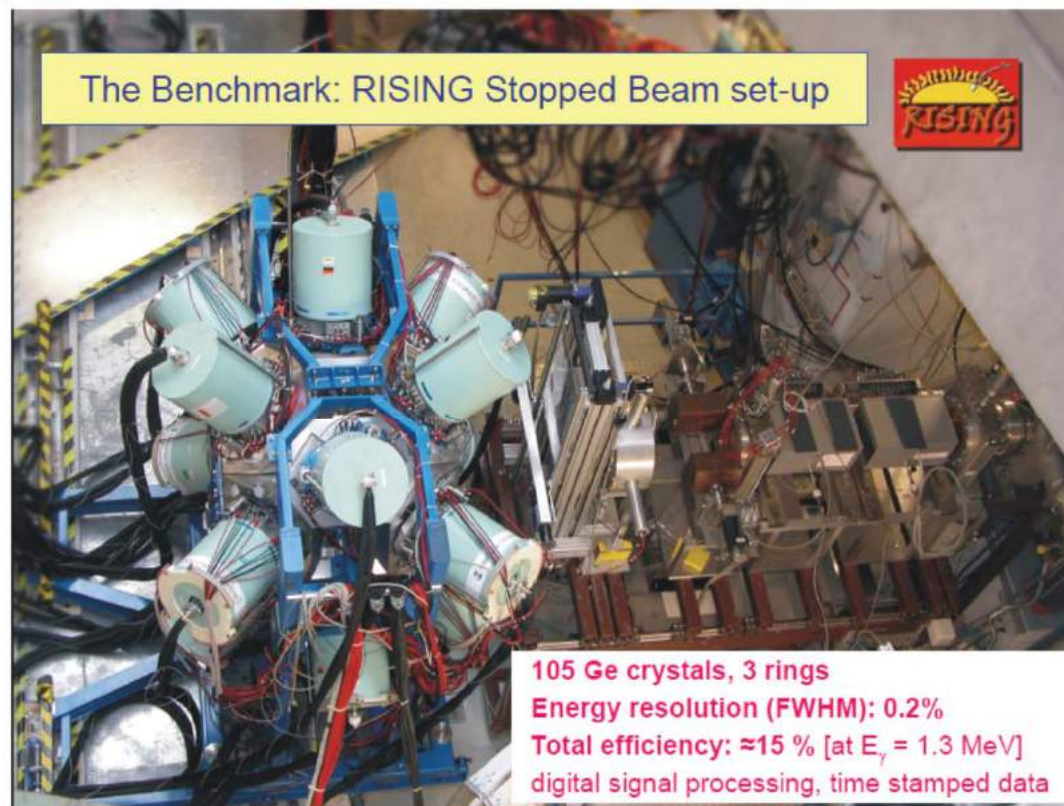
Few gamma rays means you can have fewer big detectors but see note above re detector angle

High photopeak detection efficiency (to see the weakest channels)

Good peak to total ratio (so that coincidence gates can be clean)

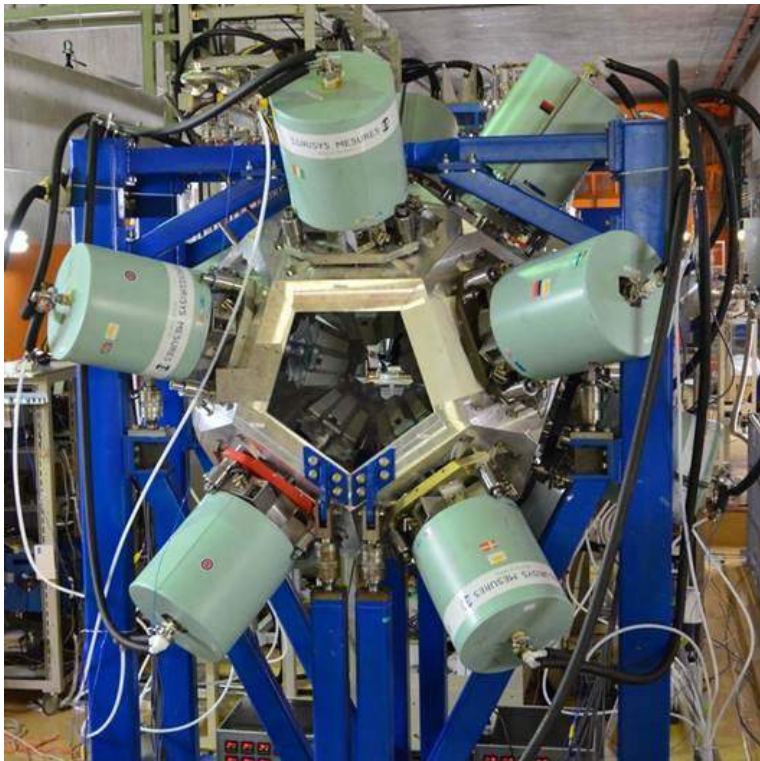
Detector requirements for out-of-beam spectroscopy

In recent years, experiments have still tended to be performed with arrays e.g. the use of the RISING array of Euroball clusters for experiments at the FRS at GSI.

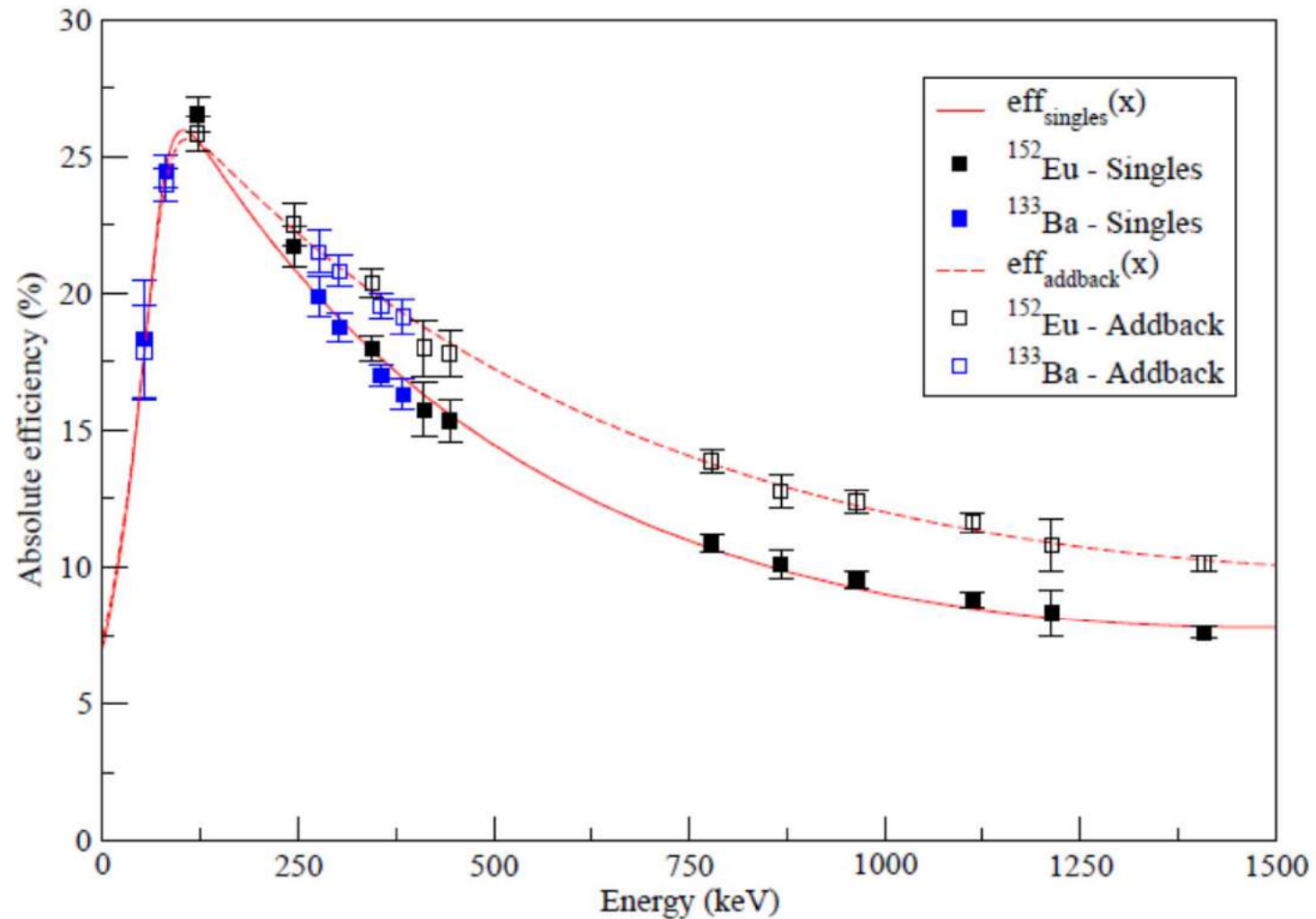


Detector requirements for out-of-beam spectroscopy

Now also using mixed arrays e.g. the use of the EURICA array of 12 Euroball clusters with 18 $\text{LaBr}_3(\text{Ce})$ detectors at BigRIPS at RIKEN.



Typical efficiency curve



Efficiency of the 12 EURICA Cluster detectors for singles (solid symbols) and add back (open symbols)

Suggestions for tutorial discussion:

1. Explain the origin of the part of the spectrum labelled "multiple Compton events" in the spectrum for the 'real detector'
2. Why are Compton suppression shields made from scintillator material?
3. Why is the timing resolution worse for bigger scintillator detectors?
4. Why does the efficiency curve have this shape?

Spins and parities

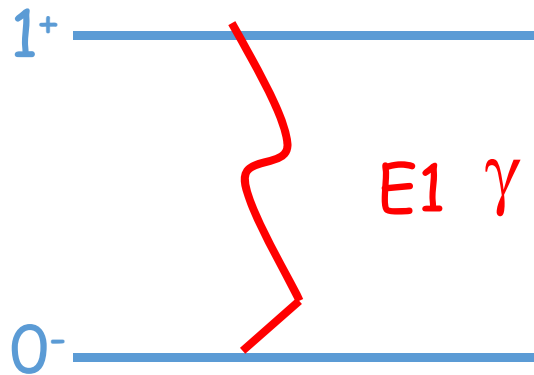
Two distinct types of measurements:

Angular correlation: can be done with a non-aligned source but need γ - γ coincidence information.

Angular distribution: need an aligned source but can be done with singles data.

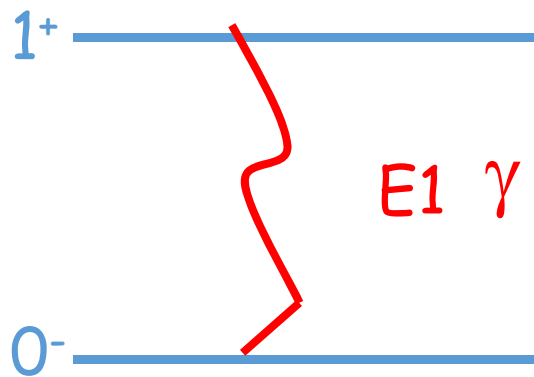
...note that these cannot measure parity but you can usually infer something about the transition.

The basics of the situation:



Imagine the situation of an E1 decay between two states, the initial one has a J^π value of 1^+ and the final one a J^π of 0^- .

The basics of the situation:

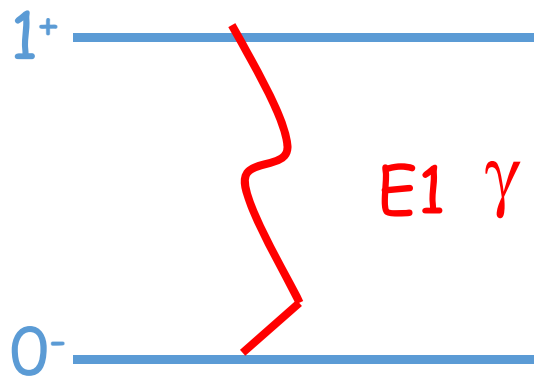


The $J^\pi = 1^+$ state has 3 substates with m values of ± 1 and 0 .

The $J^\pi = 0^-$ state has only 1 substate, with $m=0$.

When the substates of the $J^\pi = 1^+$ state decay, the γ rays emitted have different angular patterns.

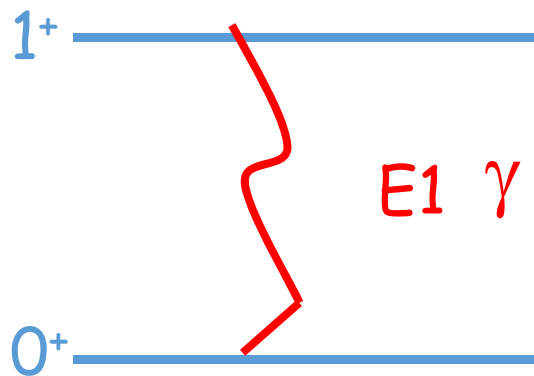
The basics of the situation:



$J^\pi = 1^+, m=0$ decays to
 $J^\pi = 0^-, m=0$ with a
 $\sin^2\theta$ distribution.

$J^\pi = 1^+, m=\pm 1$ decays to
 $J^\pi = 0^-, m=0$ with a
 $\frac{1}{2}(1+\cos^2\theta)$ distribution.

The basics of the situation:

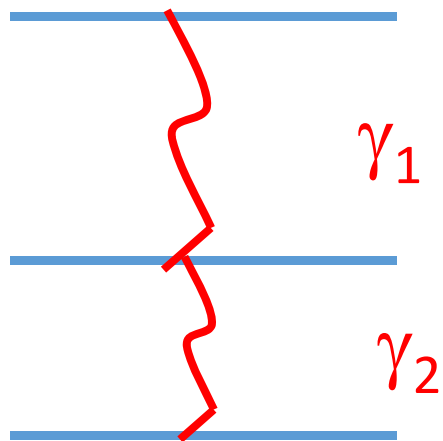


$J^\pi = 1^+, m=0$ decays to
 $J^\pi = 0^+, m=0$ with a
 $\sin^2\theta$ distribution.

$J^\pi = 1^+, m=\pm 1$ decays to
 $J^\pi = 0^+, m=0$ with a
 $\frac{1}{2}(1+\cos^2\theta)$ distribution.

So the total distribution is $\frac{1}{2}(1+\cos^2\theta) + \sin^2\theta + \frac{1}{2}(1+\cos^2\theta)$
 $= 1+\cos^2\theta + \sin^2\theta$
 $= 2$...flat, no angular dependence

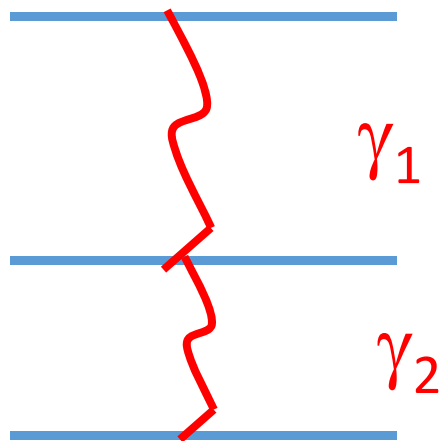
Angular correlation - non-oriented source



Let's imagine we have two γ -rays which follow immediately after each other in the level scheme.

If we measure γ_1 or γ_2 in singles then the distribution will be **isotropic** (same intensity at all angles)... there is no preferred direction of emission.

Angular correlation - non-oriented source



Now imagine that we measure γ_1 or γ_2 in coincidence. We say that measuring γ_1 causes the intermediate state to be aligned. We define the z direction as the direction of γ_1 .

The angular distribution of the emission of γ_2 then depends on the spin/parities of the states involved and on the multipolarity of the transition.

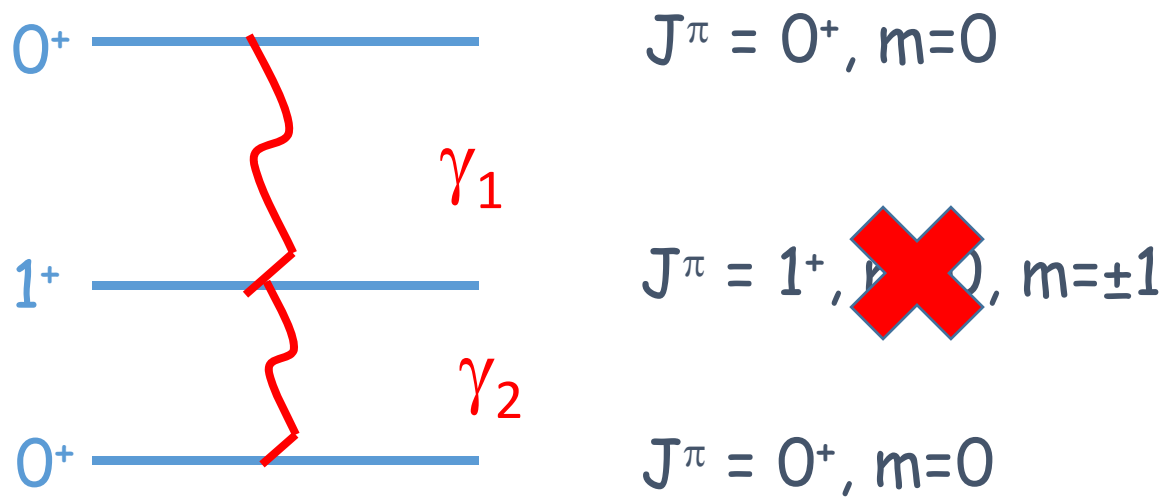
A simple example:

$$0^+ \text{ ————— } J^\pi = 0^+, m=0$$

$$1^+ \text{ ————— } J^\pi = 1^+, m=0, m=\pm 1$$

$$0^+ \text{ ————— } J^\pi = 0^+, m=0$$

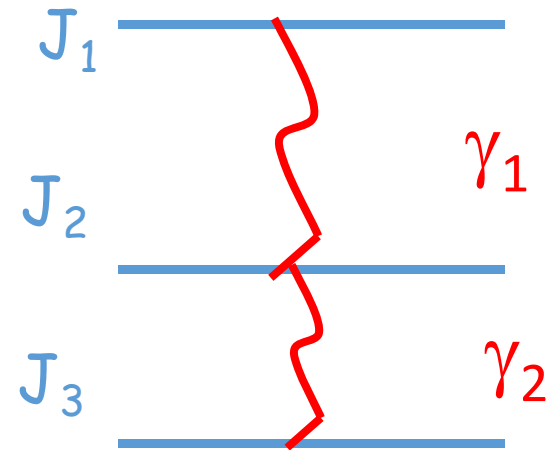
A simple example:



Hence for γ_2 we only see the $m = \pm 1$ to $m=0$ part of the distribution i.e we see that the intensity measured as a function of angle (w.r.t γ_1) follows a $1+\cos^2\theta$ distribution.

General formula

In general,
The gamma-ray intensity
varies as:



$$W(\theta) = \sum_{k_{\text{even}}} A_k(\gamma_1) A_k(\gamma_2) Q_k(\gamma_1) Q_k(\gamma_2) P_k(\cos\theta)$$

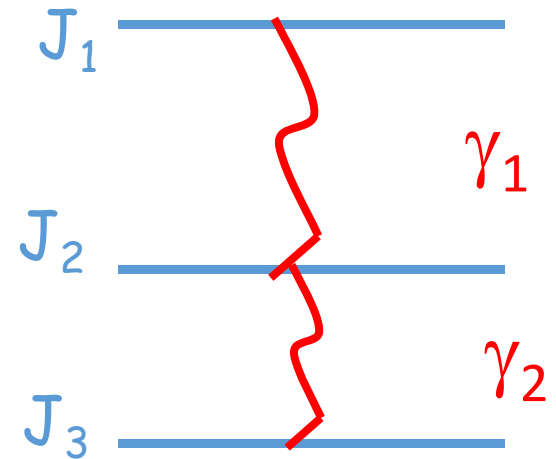
where

θ is the relative angle between the two γ -rays

Q_k accounts for the fact that we do not have point detectors

A_k depends on the details of the transition and the spins of the levels

General formula



$$A_k(\gamma_1) = \frac{F_k(J_2J_1ll) - 2\delta F_k(J_2J_1ll + 1) + \delta^2 F_k(J_2J_1l + 1l + 1)}{1 + \delta^2}$$

$$A_k(\gamma_2) = \frac{F_k(J_2J_3LL) + 2\delta F_k(J_2J_3LL + 1) + \delta^2 F_k(J_2J_3L + 1L + 1)}{1 + \delta^2}$$

The F_k coefficients contain angular momentum coupling information3j, 6j symbols.

Legendre polynomials

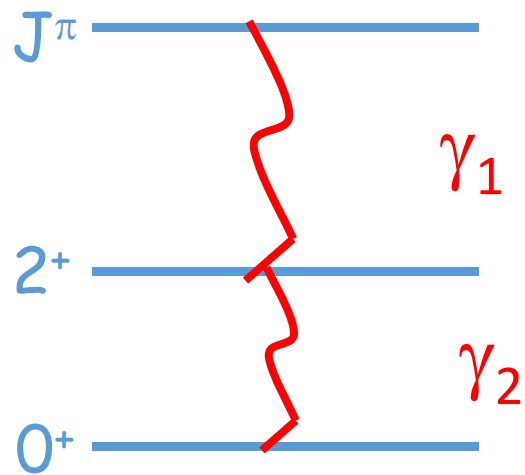
$$P_2(\cos\theta) = \frac{1}{2} (3\cos^2\theta - 1)$$

$$P_4(\cos\theta) = \frac{1}{8} (35\cos^4\theta - 30\cos^2\theta + 3)$$

$$P_6(\cos\theta) = \frac{1}{16} (231\cos^6\theta - 315\cos^4\theta + 105\cos^2\theta - 5)$$

Note the dependence on \cos^2 .

A specific case:



$\delta=0$ as this is a pure E2

$$A_k(\gamma_2) = F_k(2022)$$

$$= -0.5976$$

A specific case:

$^{195}\text{Pt}(n,\gamma)^{196}\text{Pt}$ reaction

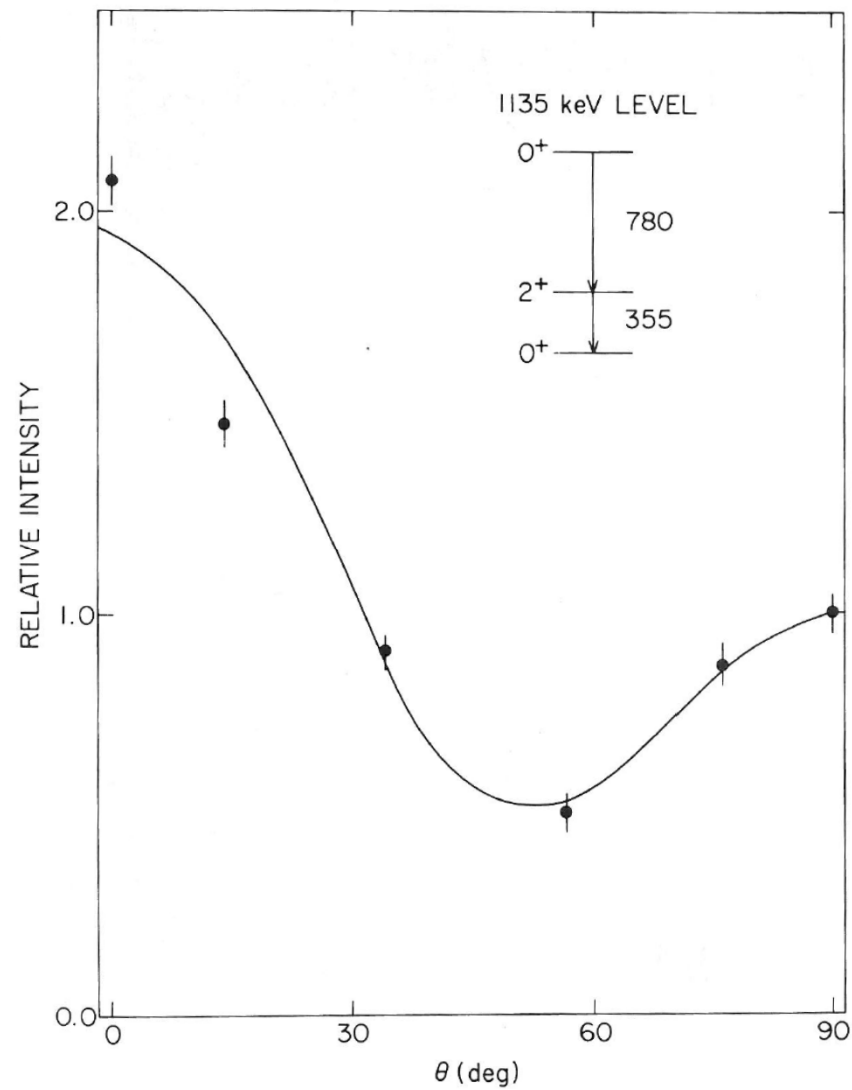
slow-neutron capture so
definitely no alignment
brought into the system

v simple system, 2 detectors
1 moving
1 fixed

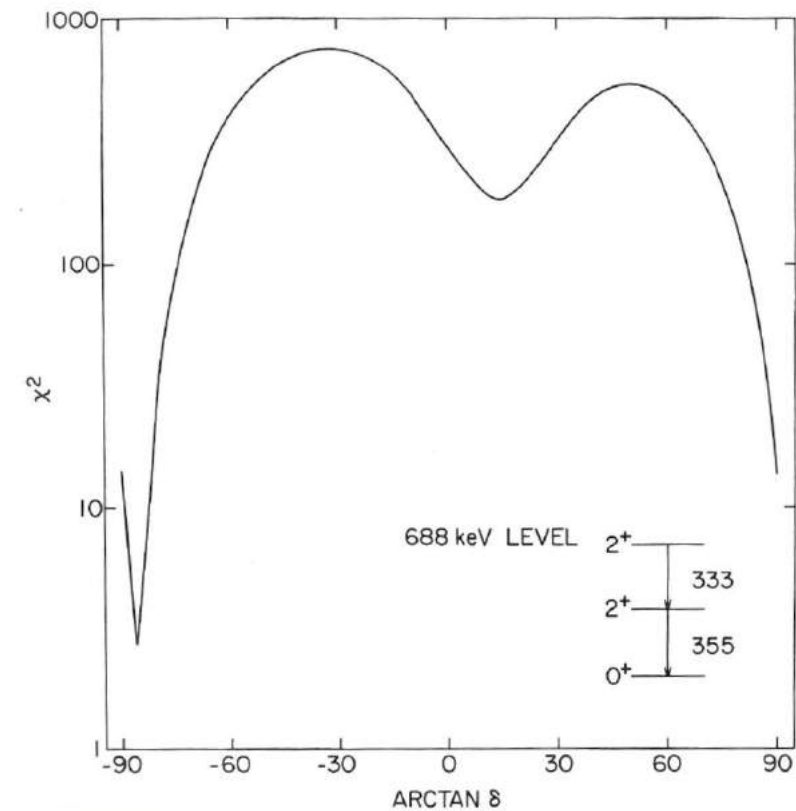
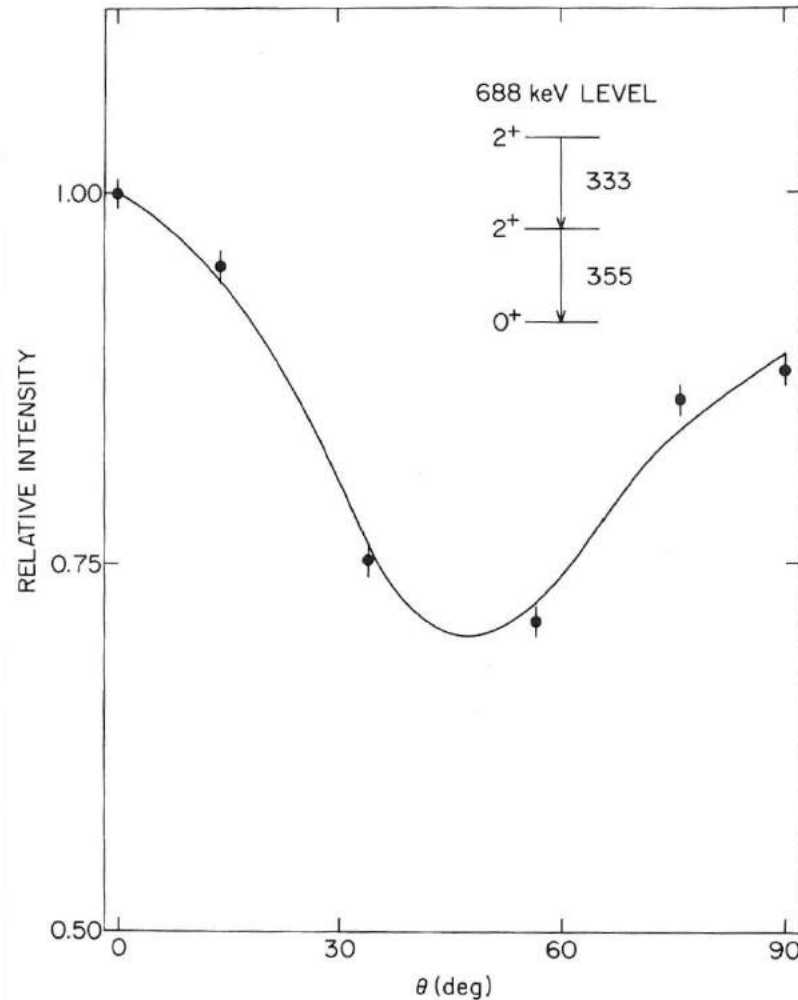
circa. 1983



A specific case : $^{195}\text{Pt}(n,\gamma)^{196}\text{Pt}$



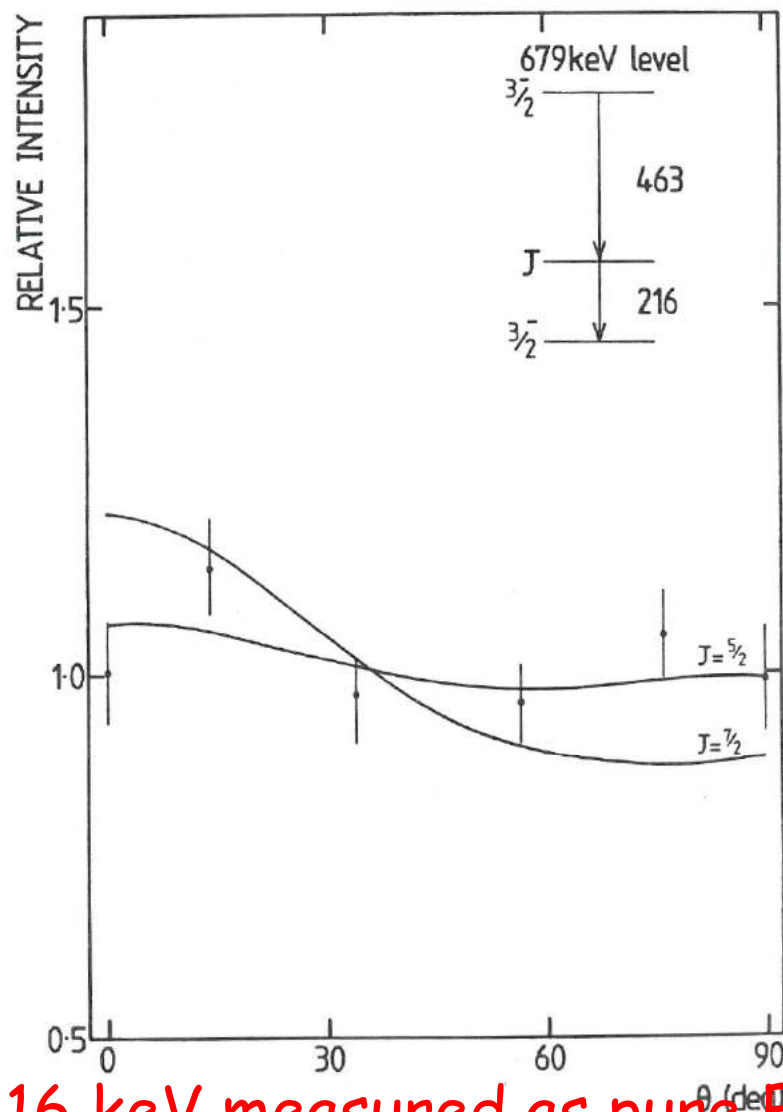
A specific example - extract mixing ratios



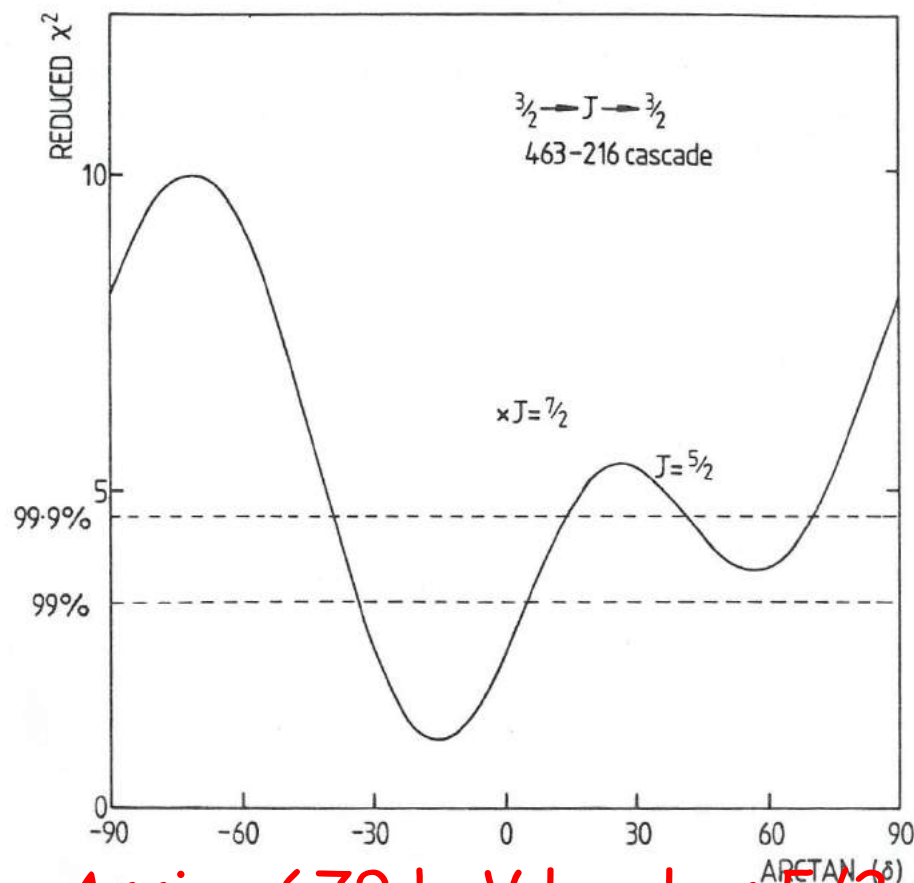
$\arctan \delta \sim -86$
 $\delta = -14.3 (40)$

A specific example - assign spins

$^{188}\text{Os}(n,\gamma)^{189}\text{Os}$



216 keV measured as pure E2



Assign 679 keV level as $\frac{5}{2}$
and measure $\text{arctan } \delta \sim -20$
 $\delta = -0.36$ 12% quadrupole
88% dipole

Angular correlations with arrays

Many arrays are designed symmetrically so the range of possible angles is reduced.

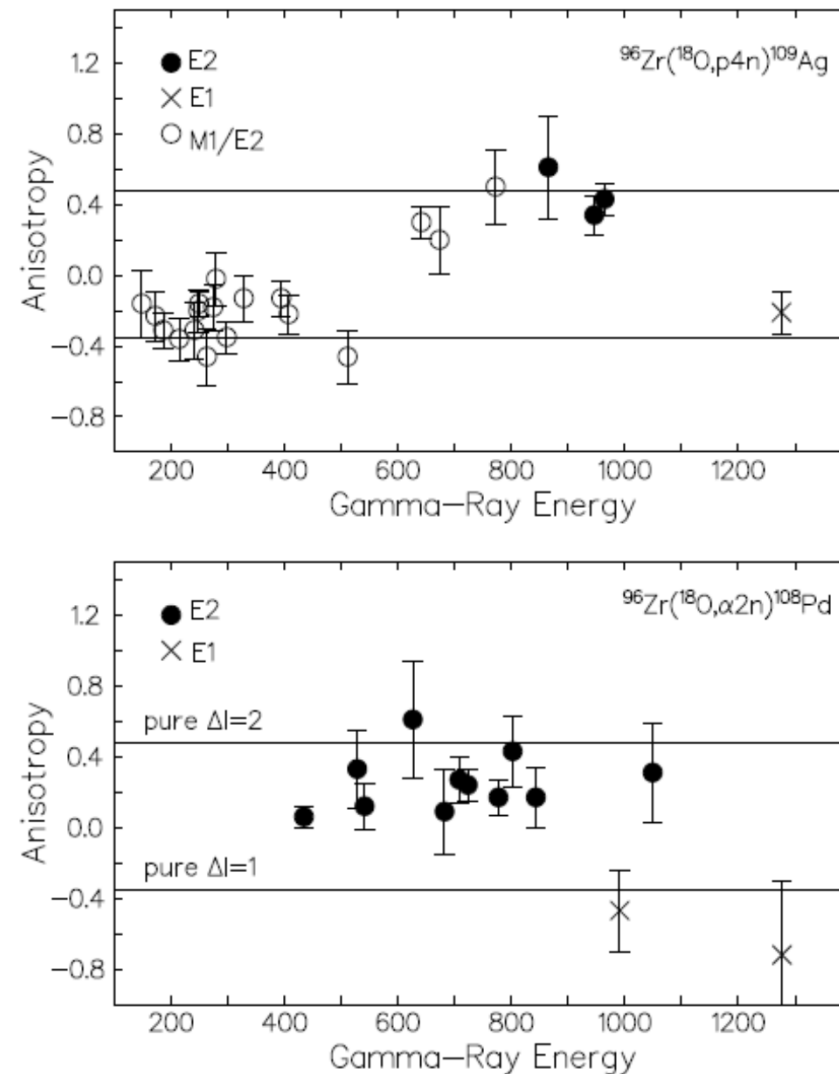
In these cases it is most common to measure a 'DCO' ratio. e.g. in the simplest case, if you have an array with detectors at 35° and 90° :

Gate on 90° detector, measure coincident intensities in:

- other 90° detectors
- 35° detectors

Take the ratio and compare with calculation...can usually separate quadrupoles from dipoles but cannot measure mixing ratios.

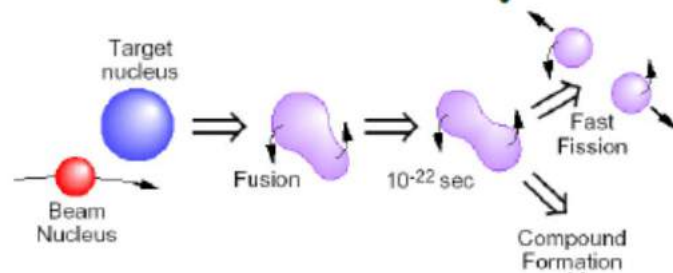
Angular correlations with arrays



K.R.Pohl et al., Phys Rev C53 (1996) 2682

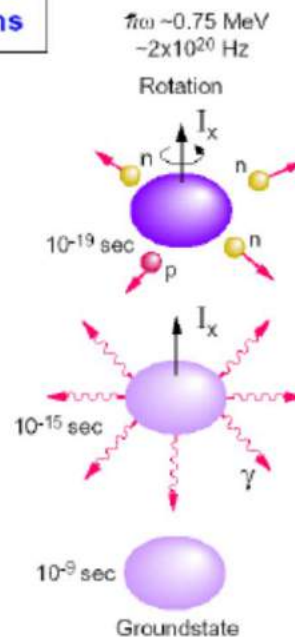
Angular distribution

Fusion Evaporation Reactions



**In-Beam Gamma Spectroscopy:
Fusion-Evaporation Reactions**

- Large cross section; ~ 1 barn
- Ideal for populating states with very high angular momentum (as high as $\sim 70 \hbar$)
- Large gamma-ray multiplicity
 - need high-granularity high-efficiency high-resolution gamma detector array
- No Coulomb barrier for neutrons
 - tends towards proton-rich nuclei, hard to make neutron-rich



How to Make High Spin Nuclei

		Time Scale	Number of Rotations
1. Preformation	^{35}Cl Beam + ^{105}Pd Target	$< 0\text{s}$	0
2. Fusion		10^{-22}s	< 1
3. Particle Emission		10^{-19}s	10-100
4. γ -ray Emission		$10^{-17}-10^{-10}\text{s}$	10^5-10^{10}
5. Ground State	^{136}Pm	10^{-9}s	10^{11}

Angular distribution

In heavy-ion fusion-evaporation reactions, the compound nuclei have their spin aligned in a plane perpendicular to the beam axis:

$$\underline{\ell} = \underline{r} \times \underline{p}$$

Depending on the number and type of particles ‘boiled off’ before a γ ray is emitted, transitions are emitted from **oriented** nuclei and therefore their intensity shows an angular dependence.

Angular distribution

$$W(\theta) = A_0 + A_2 B_2 P_2(\cos\theta) + A_4 B_4 P_4(\cos\theta) + A_6 B_6 P_6(\cos\theta) + \dots$$

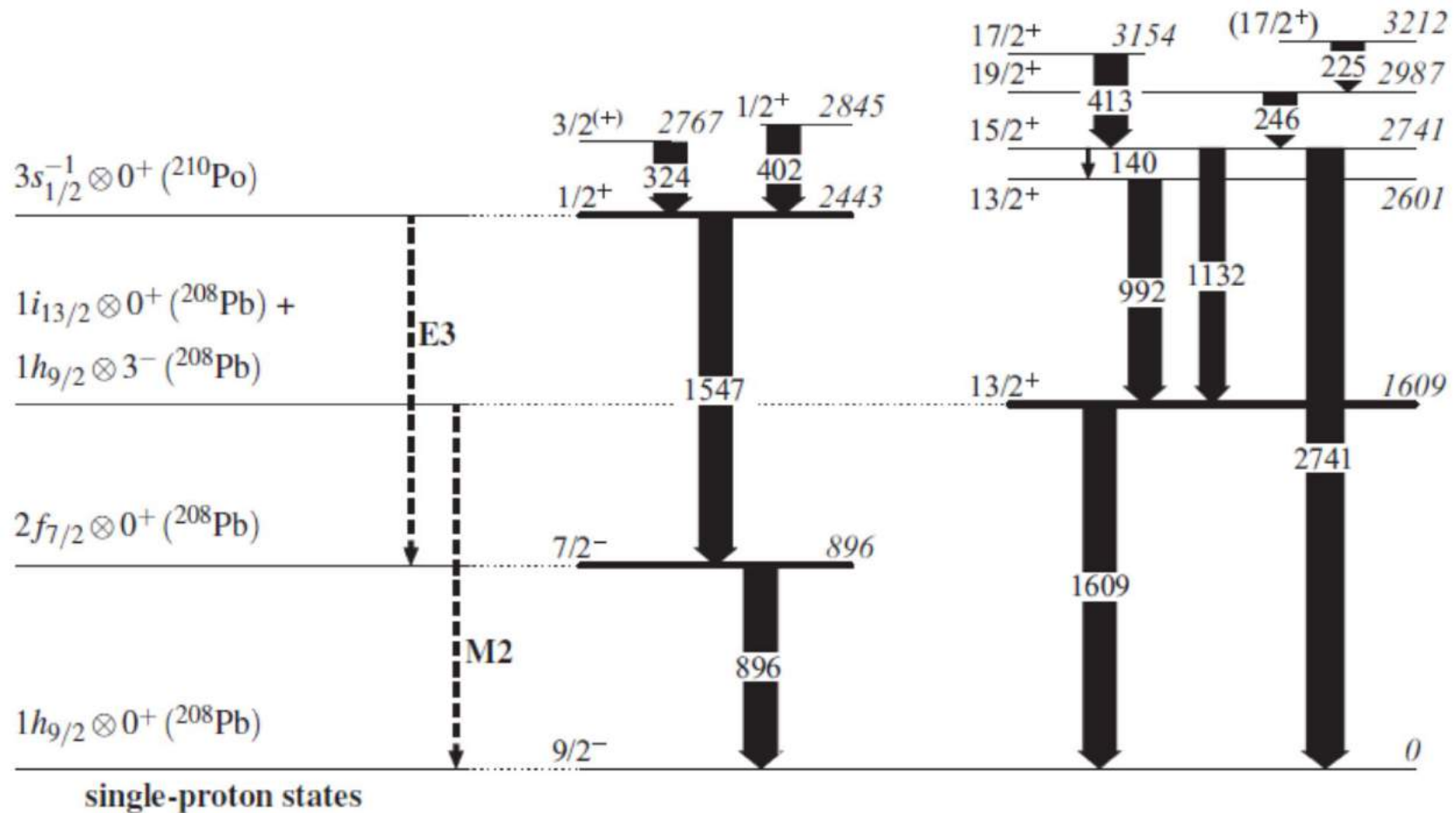
$$= A_0 \left(1 + \frac{A_2}{A_0} B_2 P_2(\cos\theta) + \frac{A_4}{A_0} B_4 P_4(\cos\theta) + \frac{A_6}{A_0} B_6 P_6(\cos\theta) + \dots \right)$$

$$= A_0 (1 + a_2 B_2 P_2(\cos\theta) + a_4 B_4 P_4(\cos\theta) + a_6 B_6 P_6(\cos\theta) + \dots)$$

Where A_k and P_k are as before and B_k contains information about the alignment of the state

(in principle there should be a Q_k in here too but let's forget for now)

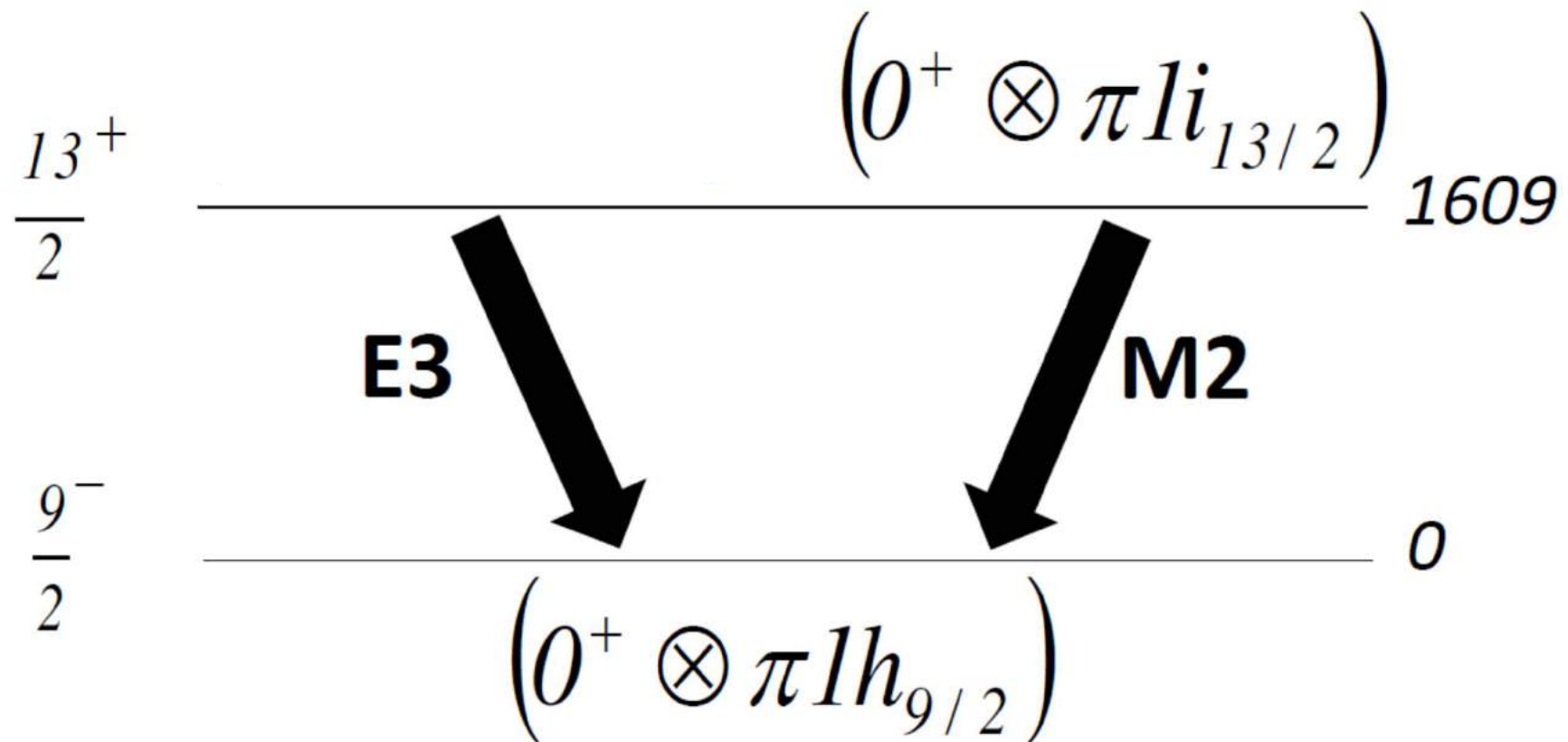
Angular distribution: worked example ^{209}Bi



Measure mixing ratio of 1609 keV transition

Angular distribution: worked example ^{209}Bi

What's the physics?



Angular distribution: worked example ^{209}Bi

^{209}Bi was populated in the $^{208}\text{Pb}(^7\text{Li}, 2n\alpha\gamma)^{209}\text{Bi}$

reaction at a ^7Li beam energy of 32 MeV...

...it's a sort of compound nucleus reaction but

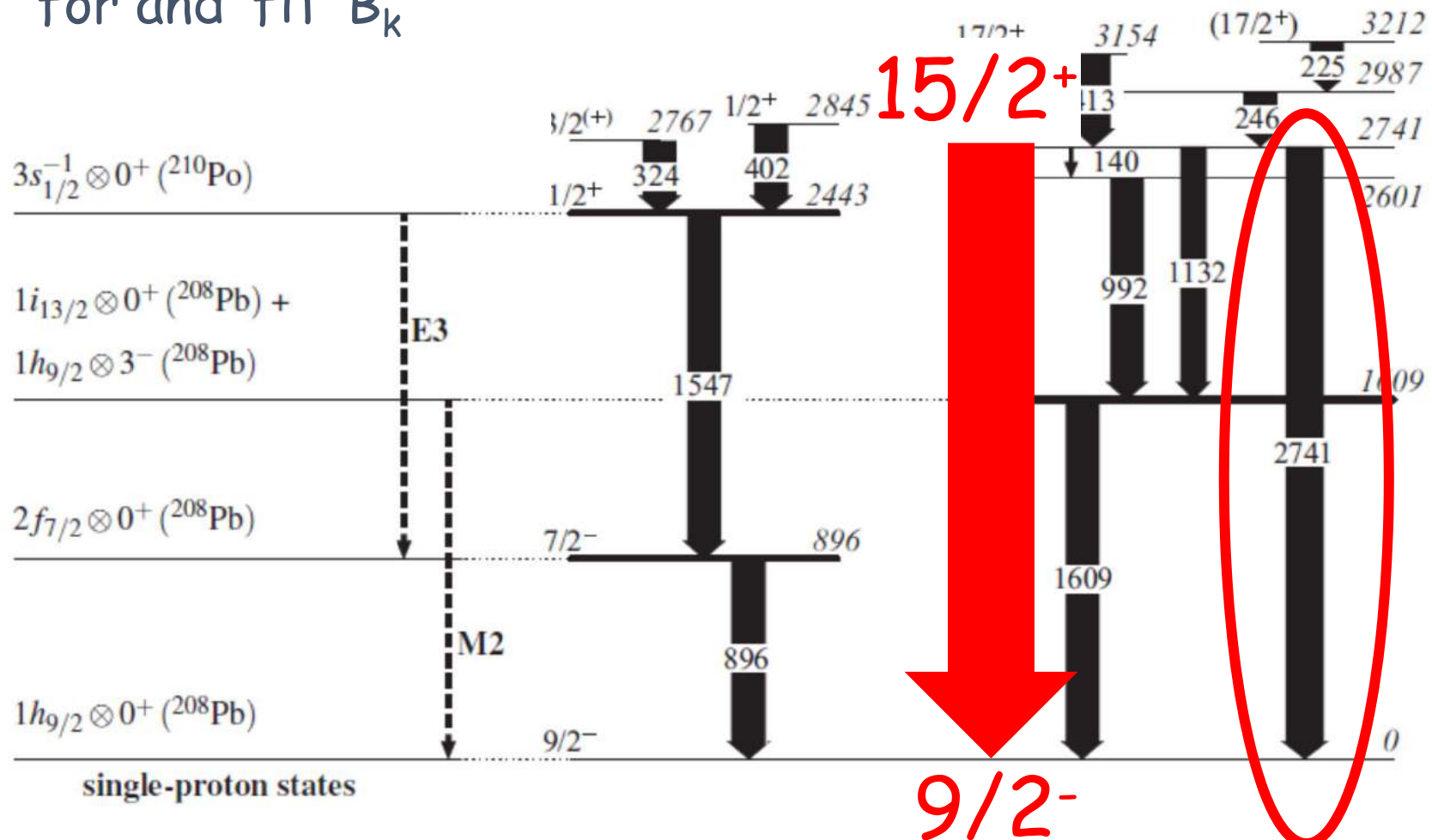
^7Li breaks up into ^4He and ^3H so is the ^4He just a

spectator?

Anyway, the question is how much alignment is in the initial states in ^{209}Bi ??

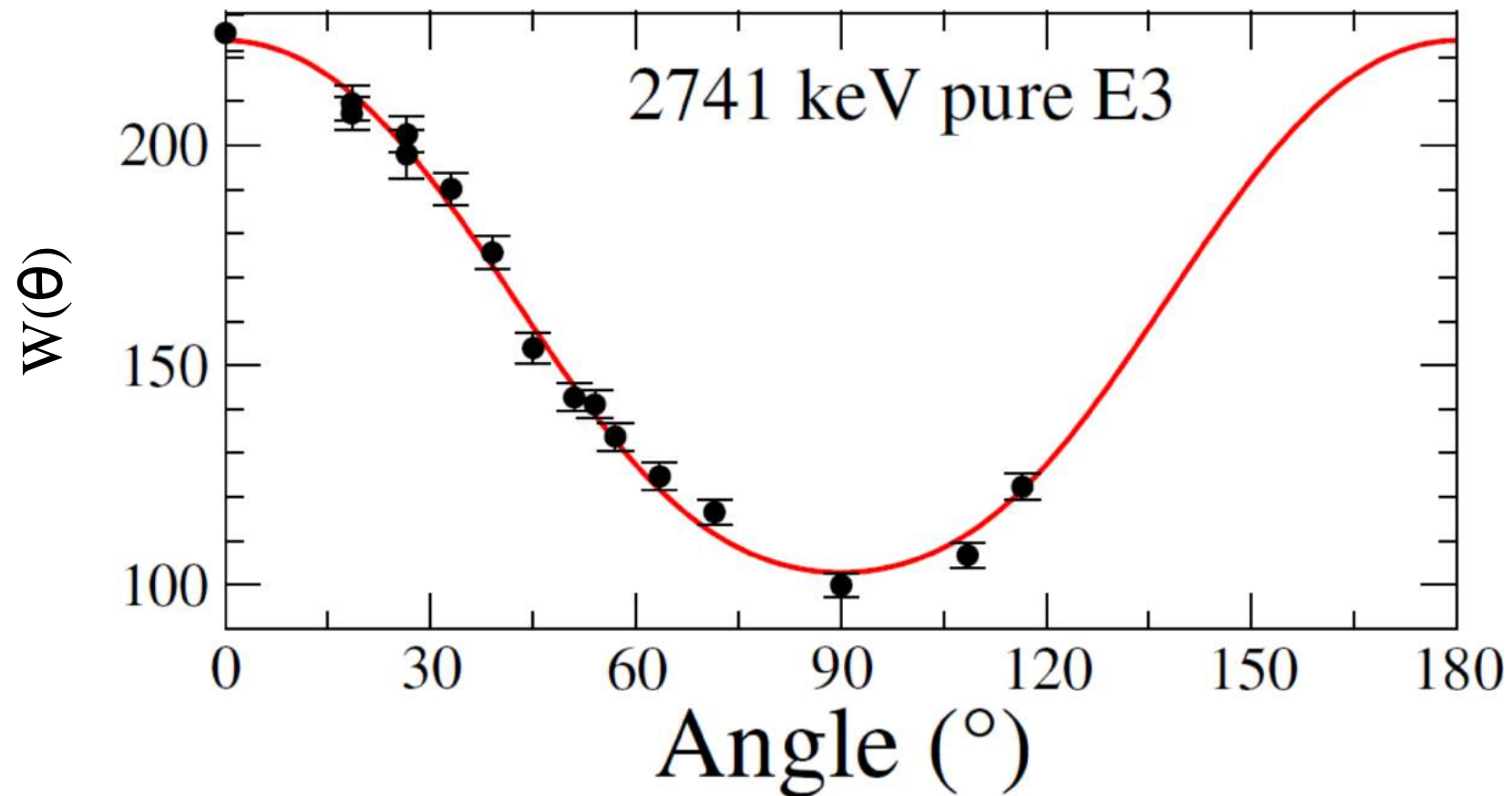
Angular distribution: worked example ^{209}Bi

Answer...look at something we know the A_k values for and 'fit' B_k



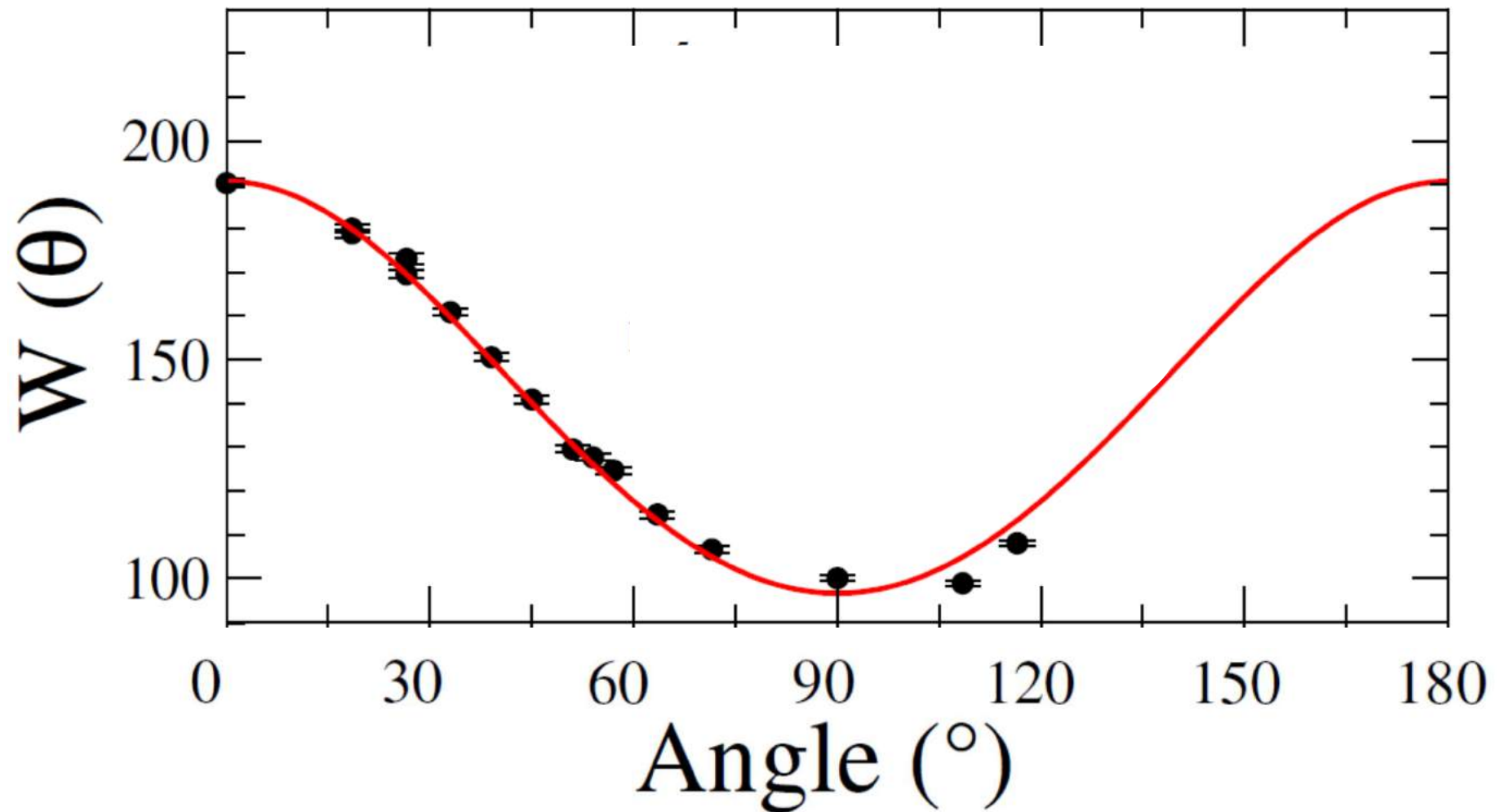
Angular distribution: worked example ^{209}Bi

Answer...look at something we know the A_k values for and 'fit' B_k



Angular distribution: worked example ^{209}Bi

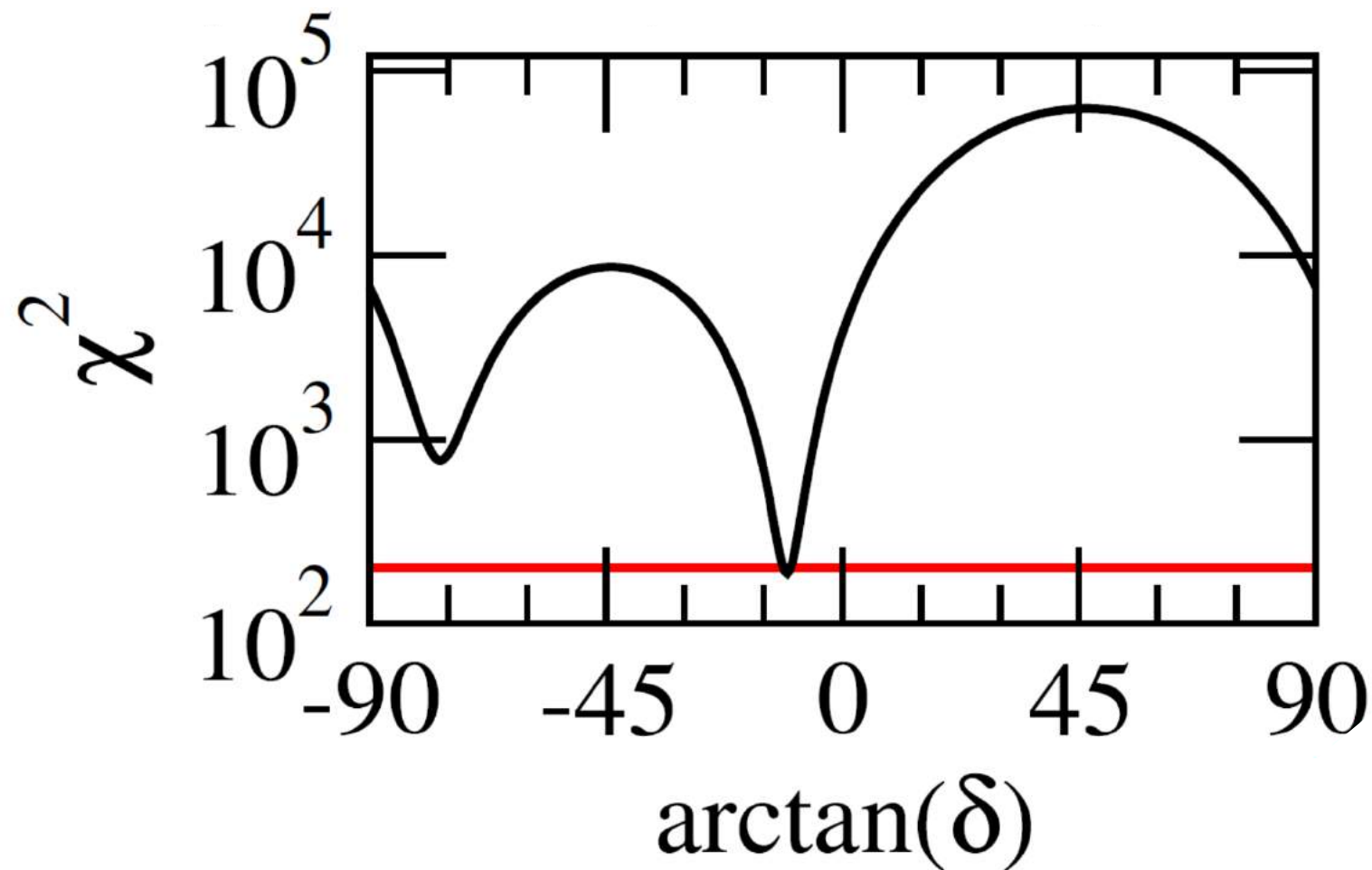
1609 keV transition



Angular distribution: worked example ^{209}Bi

1609 keV transition

min at $\arctan(\delta) = -10.54$



Angular distribution: worked example ^{209}Bi

1609 keV transition:

$$\arctan(\delta) = -10.54$$

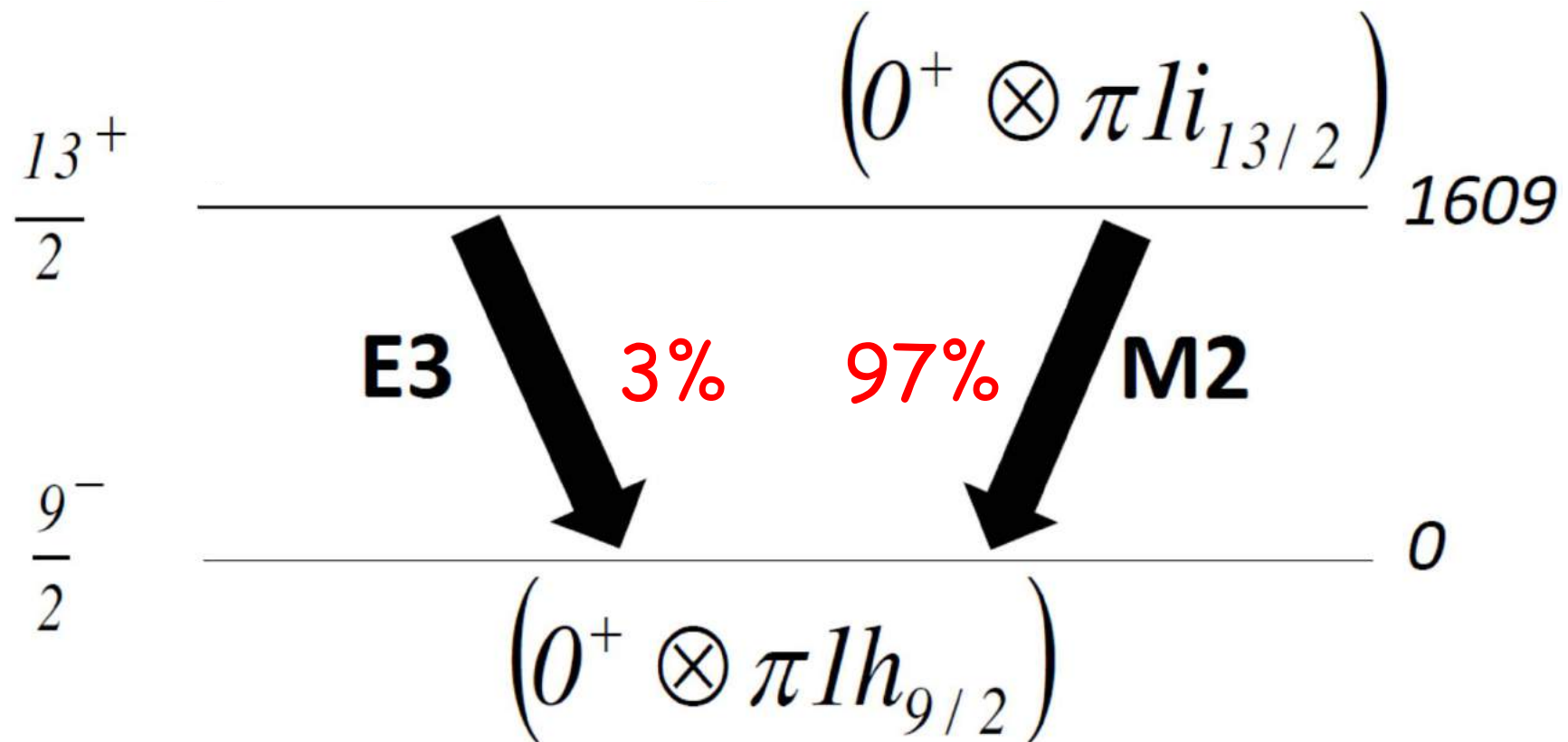
$$\delta \text{ (E3/M2)} = -0.184(13)$$

$$\%E3 = \frac{\delta^2}{1+\delta^2} = 3\%$$

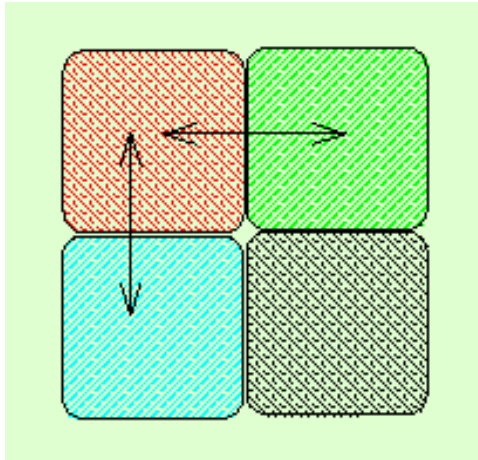
$$\%M2 = \frac{1}{1+\delta^2} = 97\%$$

Angular distribution: worked example ^{209}Bi

What's the physics?



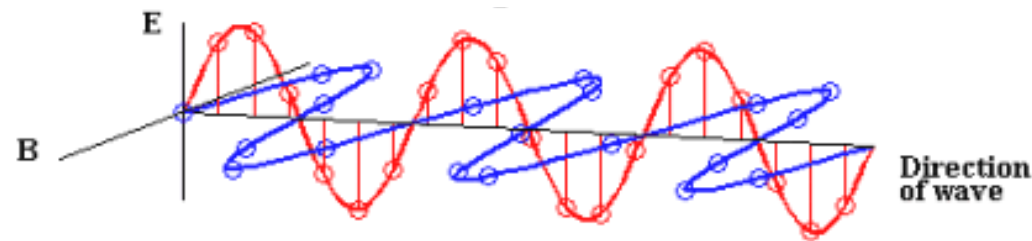
Linear polarisation



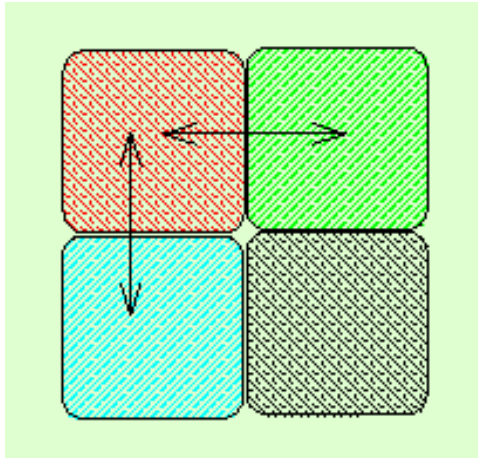
γ ray travelling into the slide

A segmented detector can be used to measure the **linear polarisation** which can be used to distinguish between magnetic (M) and electric (E) character of radiation of the same multipolarity.

The **Compton** scattering cross section is larger in the direction perpendicular to the electric field vector of the radiation.



Linear polarisation



γ ray travelling into the slide

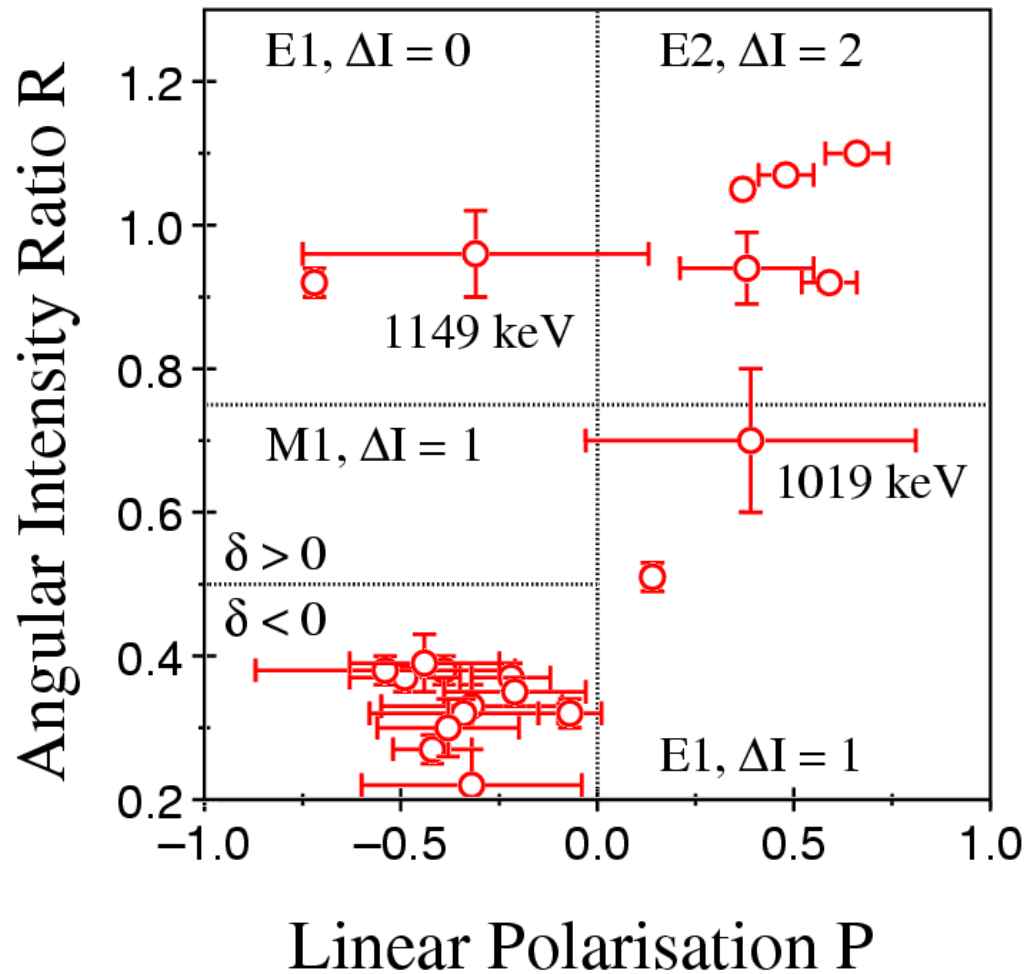
Define experimental asymmetry as:

$$A = \frac{N_{90} - N_0}{N_{90} + N_0}$$

where N_{90} and N_0 are the intensities of scattered photons perpendicular and parallel to the reaction plane.

The experimental linear polarisation $P = A/Q$ where Q is the polarisation sensitivity of the dectector.

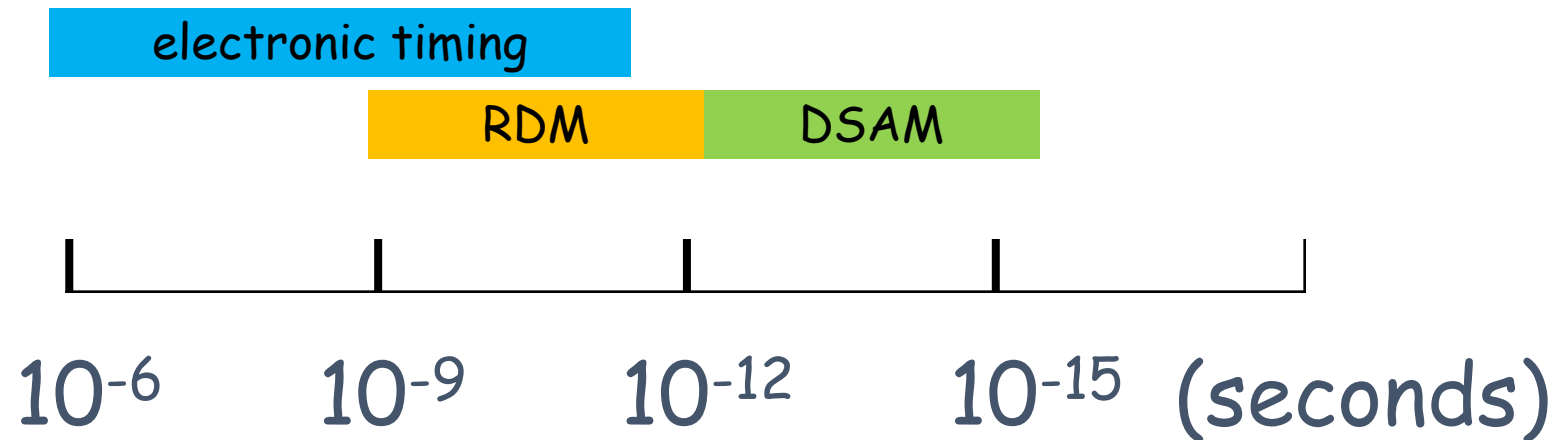
Linear polarisation



Plot P against the angular distribution information to uniquely define the multipolarity.

Data from Eurogam

Measuring level lifetimes



DSAM - Doppler shift attenuation method

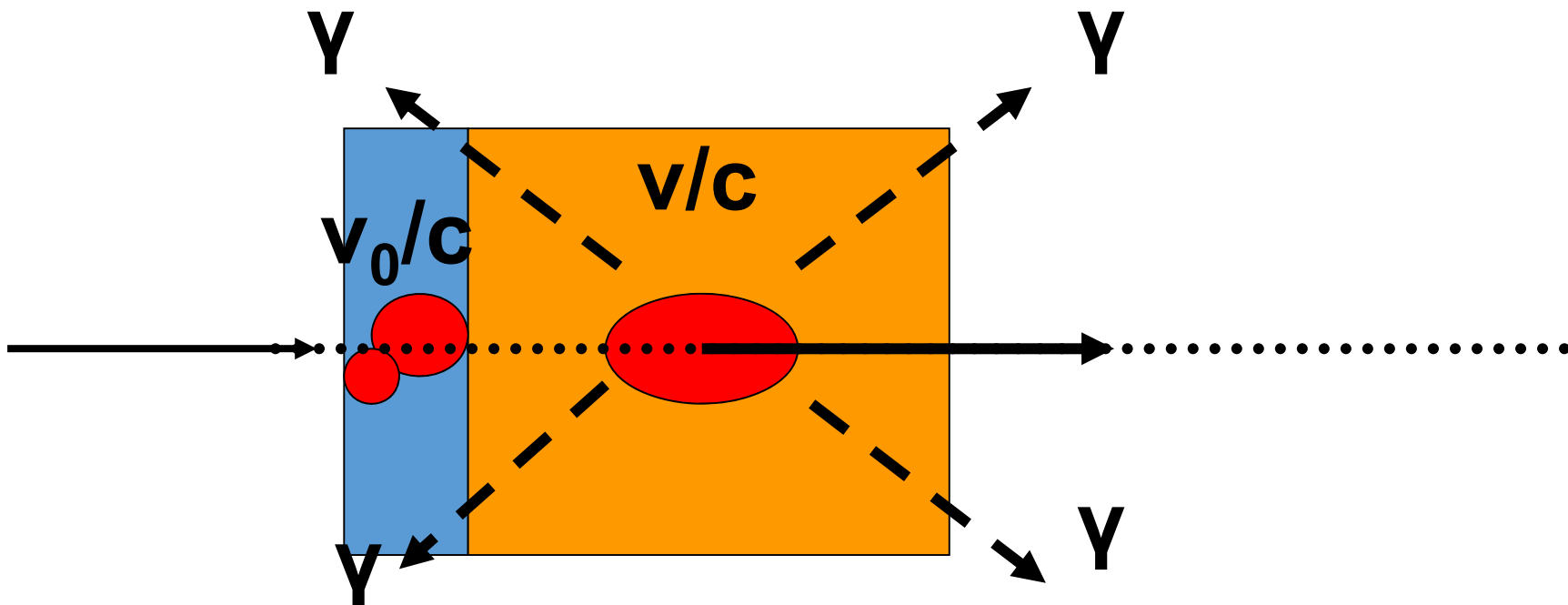
RDM - Recoil distance method

Electronic timing - Using arrays of fast scintillation detectors

Doppler shift attenuation method

- Measure lifetimes in the range $10^{-15} < \tau < 10^{-12}$ s.
- Stopping time in metal foil is comparable to lifetimes of excited states.

$$E_{\text{shifted}} = E_{\text{true}} \left(1 + \frac{v}{c} \cos \theta \right)$$



Centroid shift method

$$E_f = E_{\text{true}} \left(1 + \frac{v}{c} \cos \theta_f \right)$$

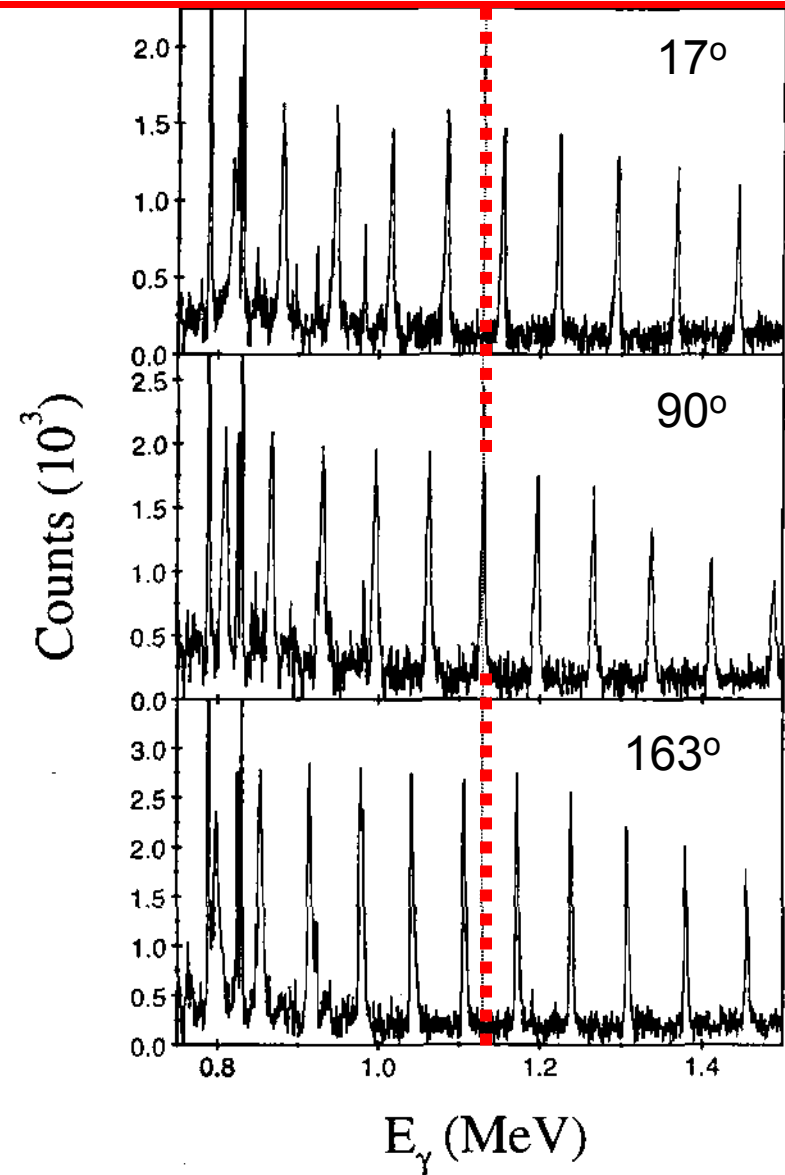
$$E_b = E_{\text{true}} \left(1 + \frac{v}{c} \cos \theta_b \right)$$

If $\theta = \theta_{\text{forward}} = 180 - \theta_{\text{backward}}$

$$E_f - E_b = 2E_{\text{true}} \frac{\bar{v}}{c} \cos \theta$$

$$F(\tau) = \frac{\bar{v}}{v_0} \quad \text{and} \quad \beta = \frac{v_0}{c}$$

$$F(\tau) = \frac{E_f - E_b}{2E_{\text{true}} \beta \cos \theta}$$



Recoil distance method

Have a space between the target and the stopper.

Measuring the v/c you can work out whether the gamma-ray is emitted before/after the stopper

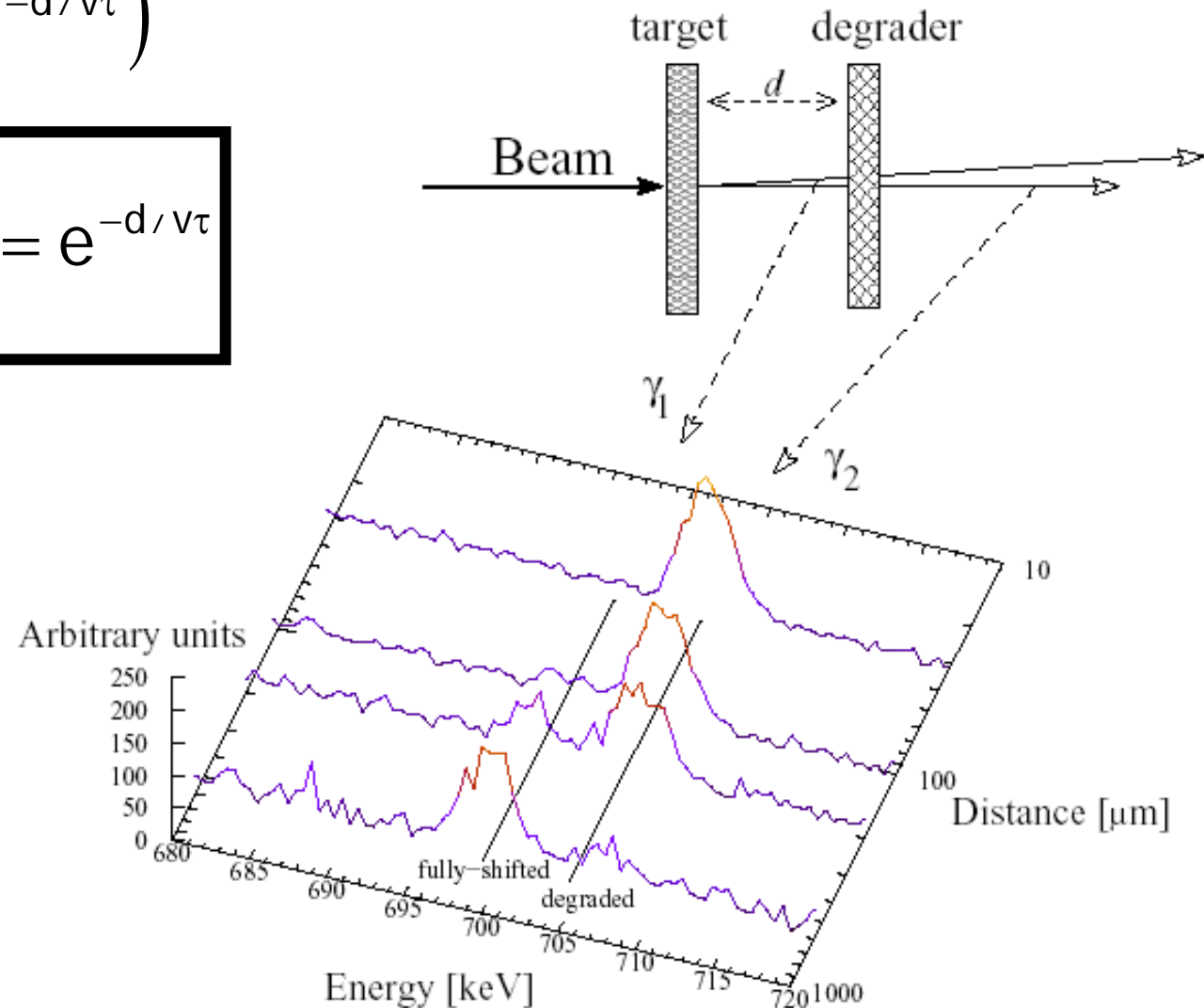
Vary the target/stopper distance to get a measurement of the lifetime of the level

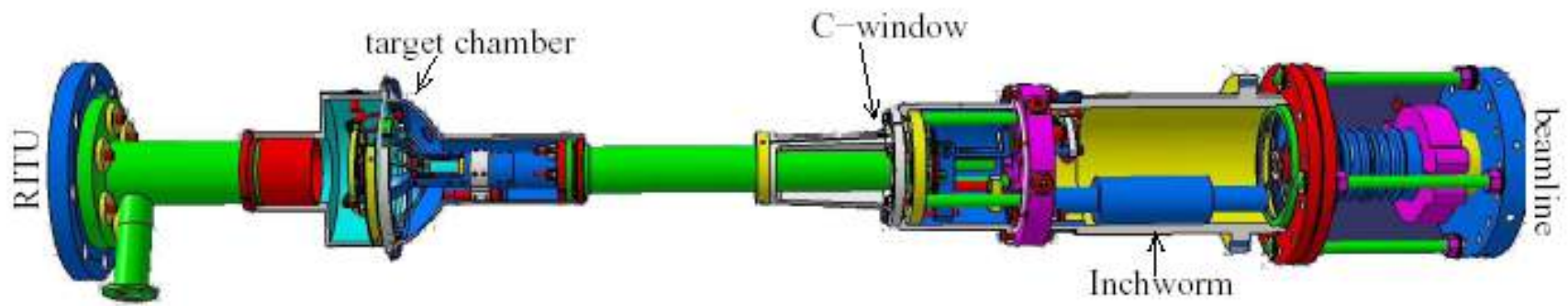
Recoil distance method

$$I_{\text{degraded}} = I e^{-d/v\tau}$$

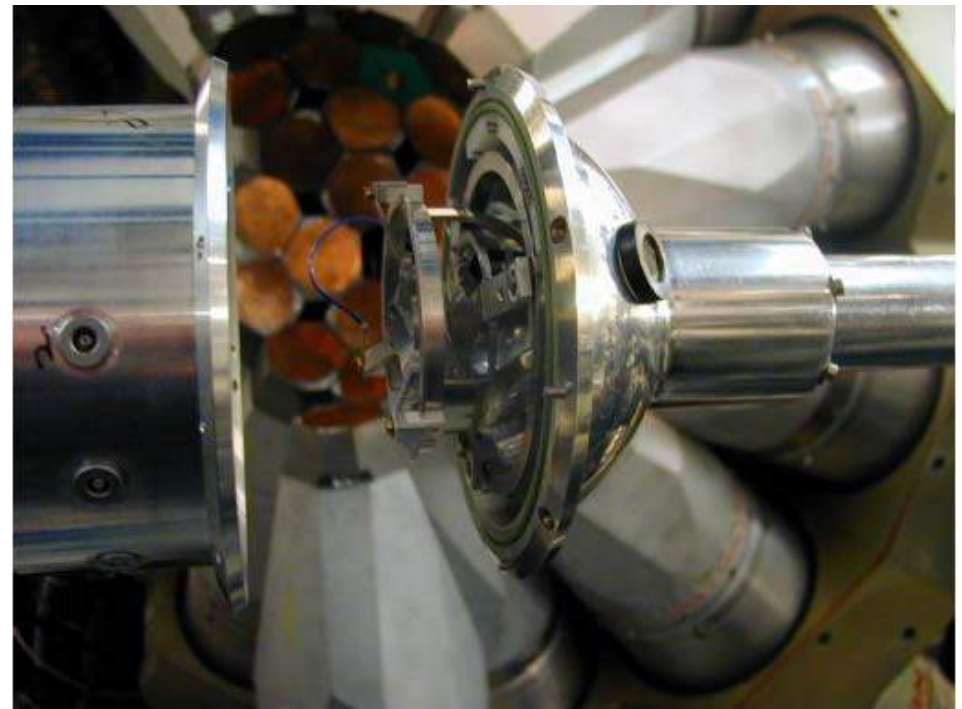
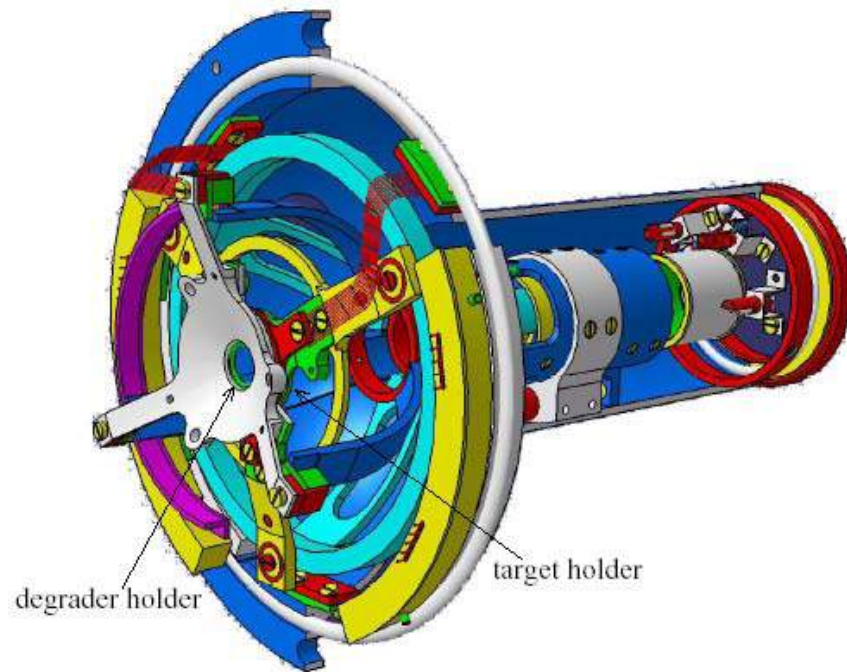
$$I_{\text{shifted}} = I(1 - e^{-d/v\tau})$$

$$\frac{I_{\text{degraded}}}{I_{\text{degraded}} + I_{\text{shifted}}} = e^{-d/v\tau}$$

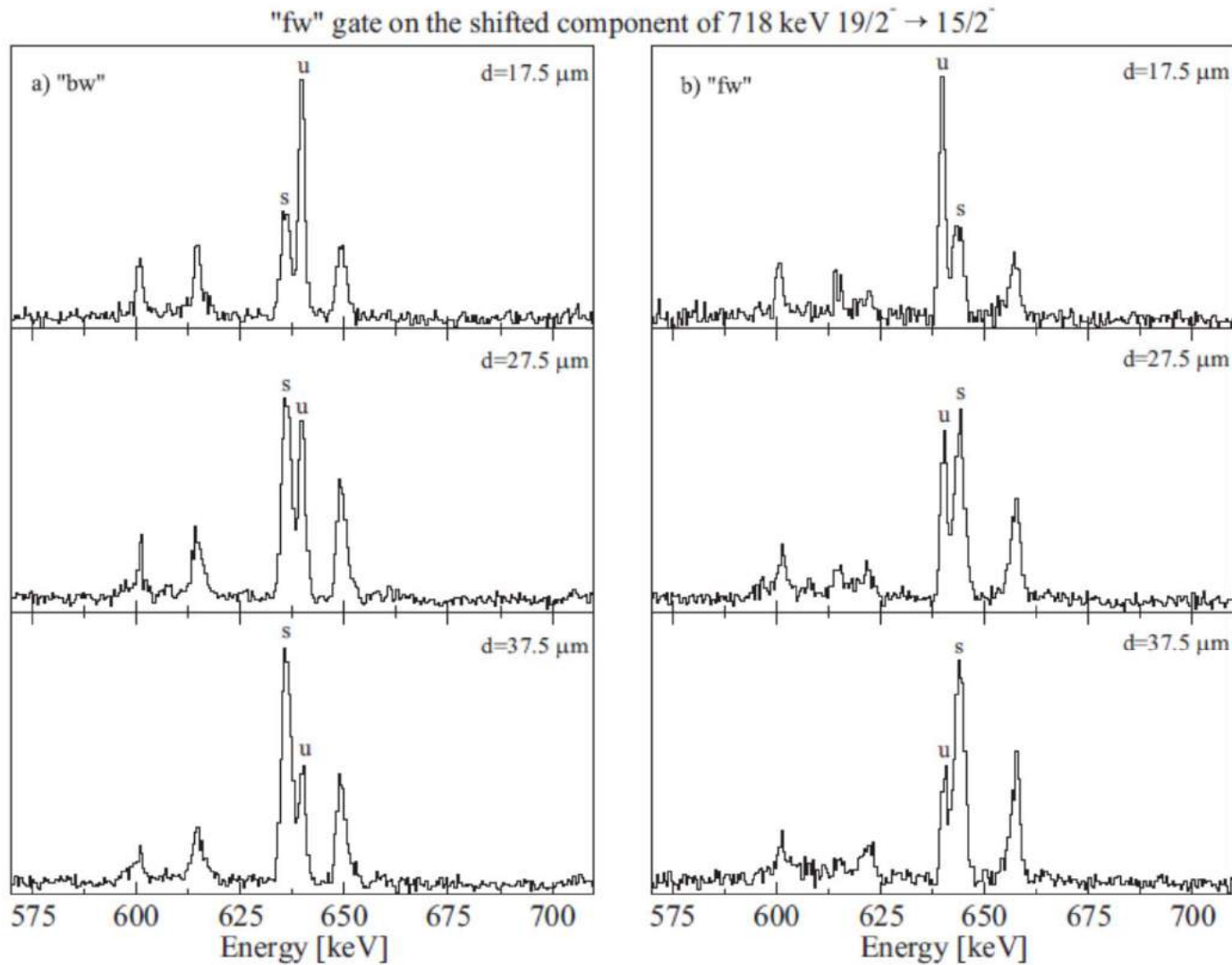




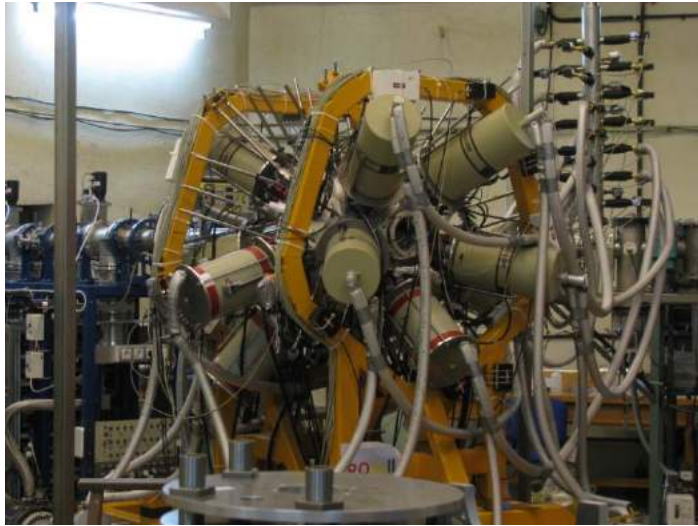
The Koln Plunger Device



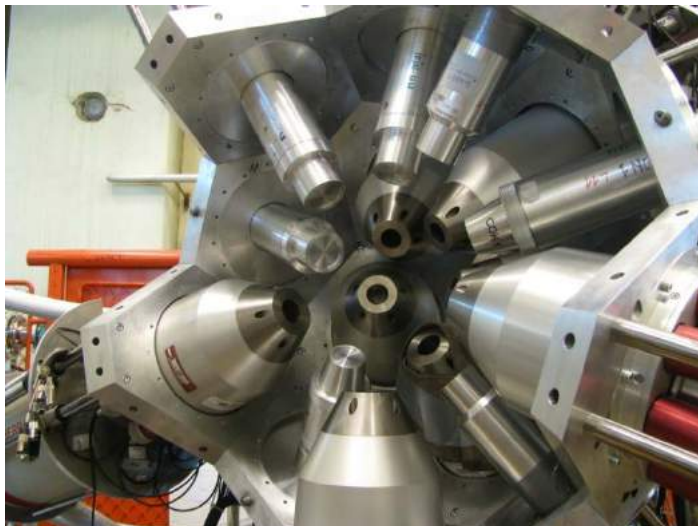
Lifetime of the $15/2^-$ state in ^{119}Te



Direct measurement using ROSPHERE at Bucharest:



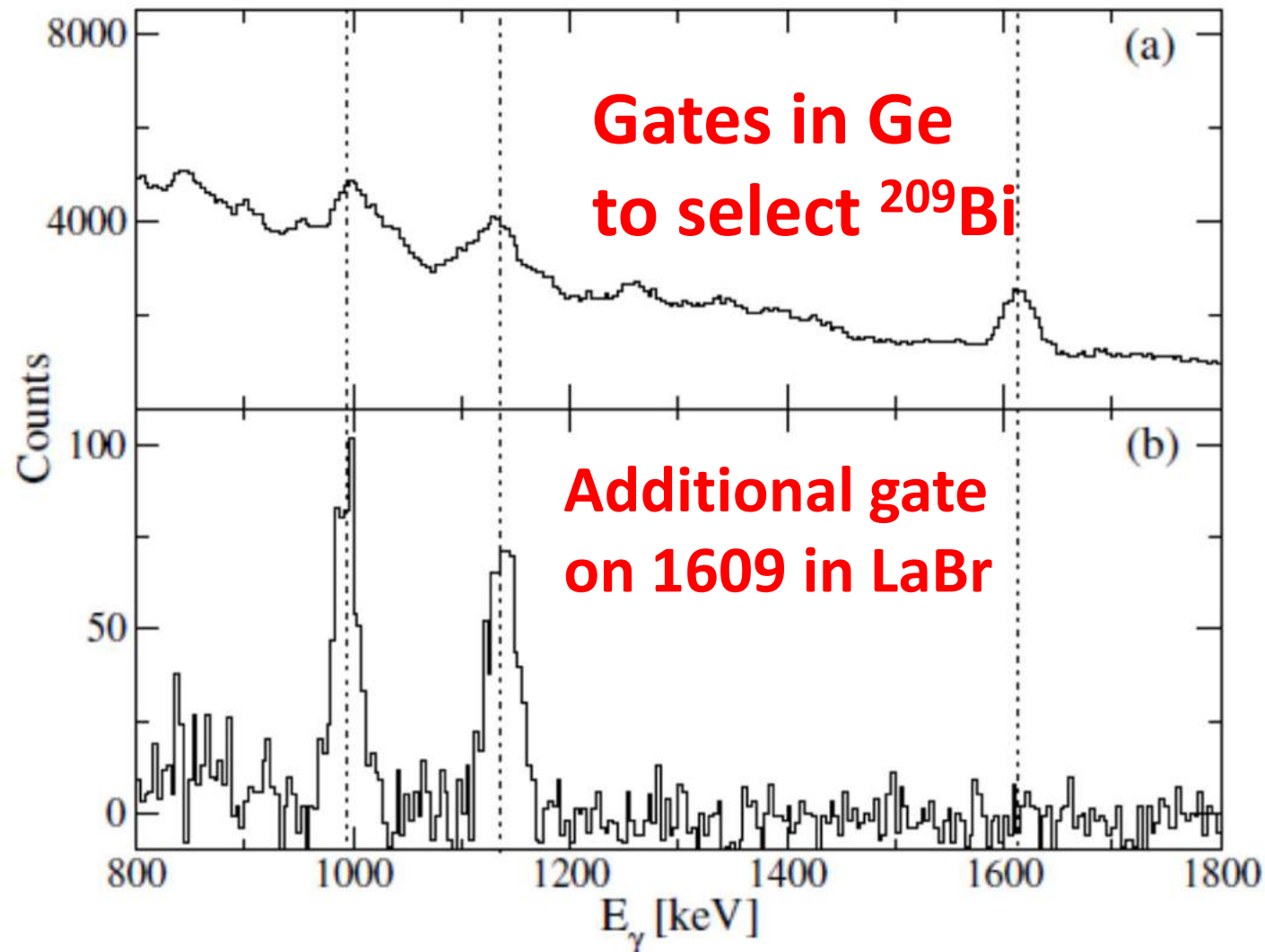
- 15 HPGe detectors (A/C):
 - 10 x HPGe detectors @ 37°
 - 1 x HPGe detector @ 64°
 - 4 x HPGe detectors @ 90°



- 11 LaBr₃(Ce):
 - ø2"x2" @ 90 and 64° (three) (Cylindrical)
 - ø1.5"x1.5" @ 90 (six) (Cylindrical)
 - ø1"x1.5" @ 64° (two) (Conical)

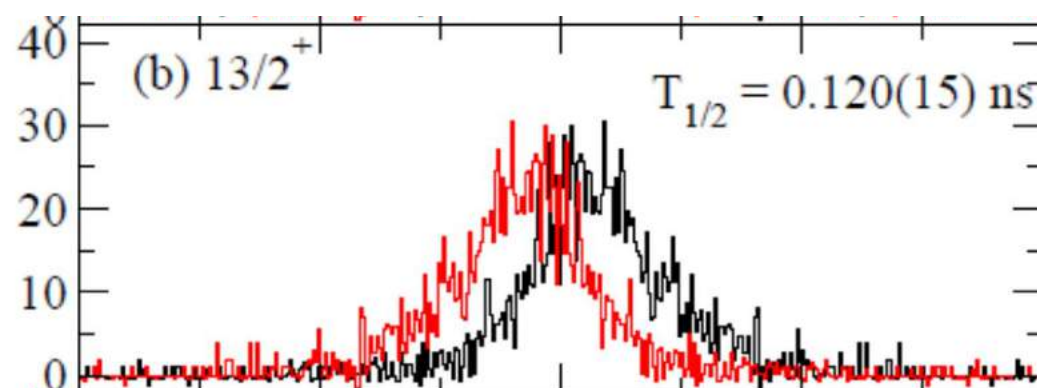
ROSPHERE at Bucharest: example of ^{209}Bi

$^{208}\text{Pb}(^7\text{Li}, 2n\alpha\gamma)^{209}\text{Bi}$ at beam energy of 32 MeV

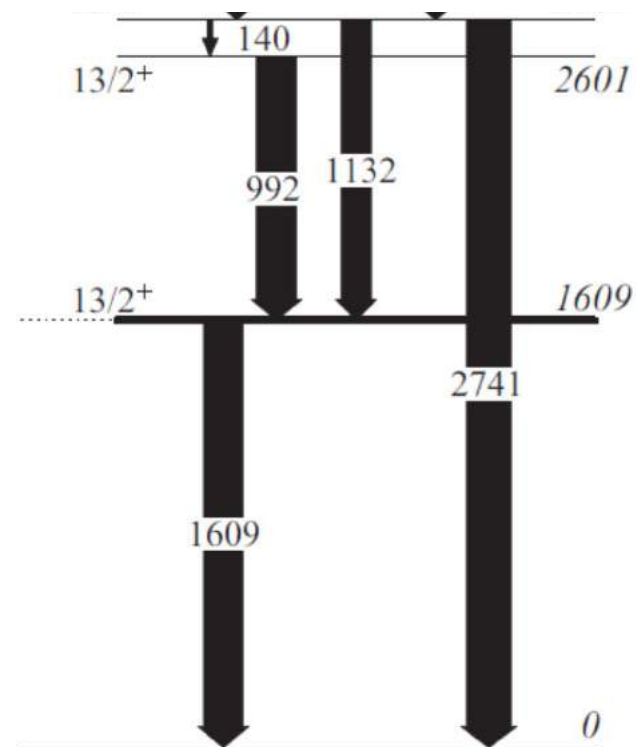


ROSPHERE at Bucharest: example of ^{209}Bi

Red = start on 1609, stop on 992 or 1132



Black = start on 992 or 1132, stop on 1609

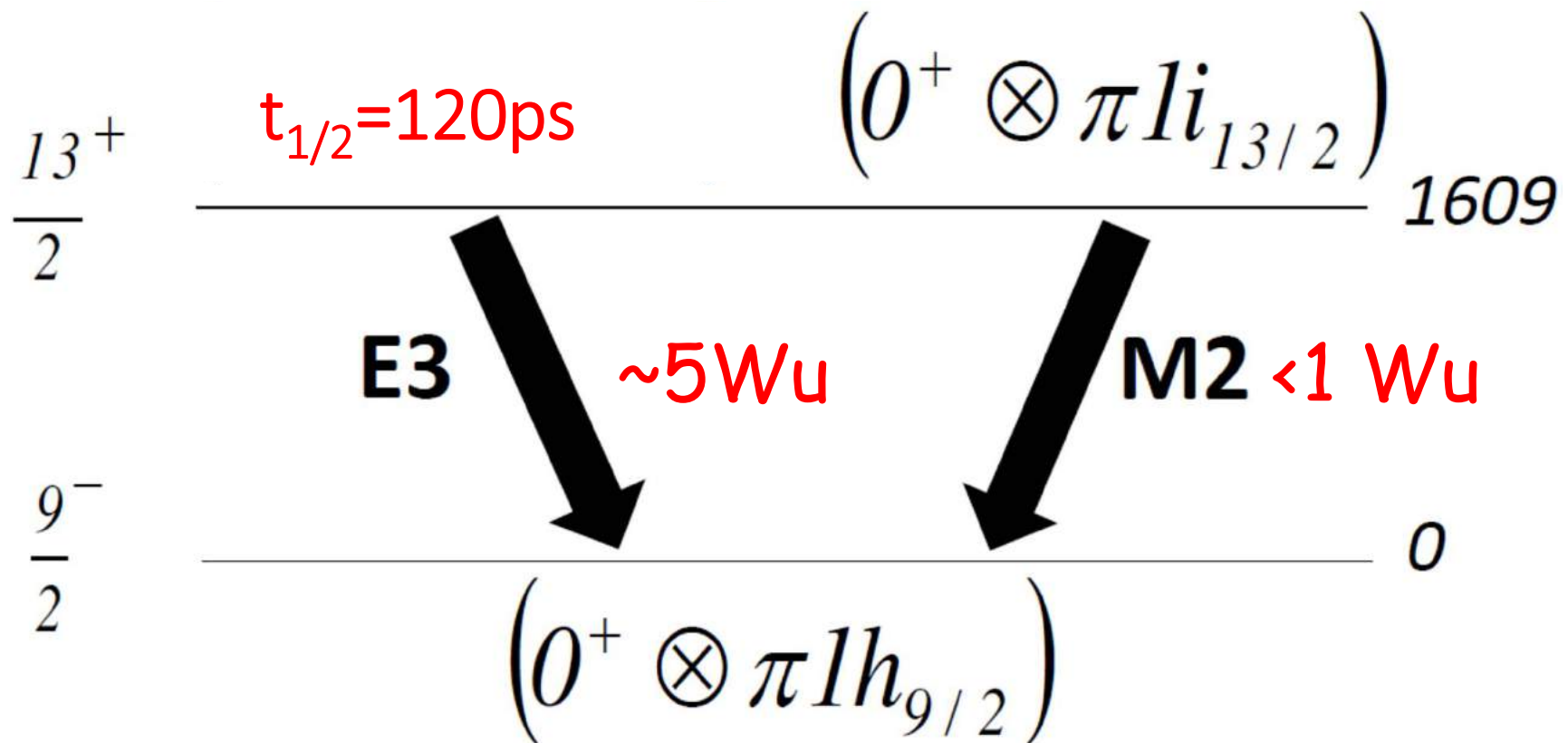


Difference in the centroids is 2τ (give or take some correction factors)

O.J.Roberts, C.R.Nita et al.,
Phys Rev C93 (2016) 014309

Angular distribution: worked example ^{209}Bi

What's the physics?



From first lecture:

Typical transition rates:

$$T(E1) = 1.59 \times 10^{15} (E_\gamma)^3 B(E1)$$

$$T(E2) = 1.22 \times 10^9 (E_\gamma)^5 B(E2)$$

$$T(M1) = 1.76 \times 10^{13} (E_\gamma)^3 B(M1)$$

$$T(M2) = 1.35 \times 10^7 (E_\gamma)^5 B(M2)$$

E_γ in MeV

$B(E1)$ in $e^2 \text{fm}^{2\lambda}$

$B(M1)$ in $\left(\frac{e\hbar}{2Mc}\right)^2 \text{fm}^{2\lambda-2}$

$$\frac{1}{\tau} = T(\sigma\lambda)$$

If E2:

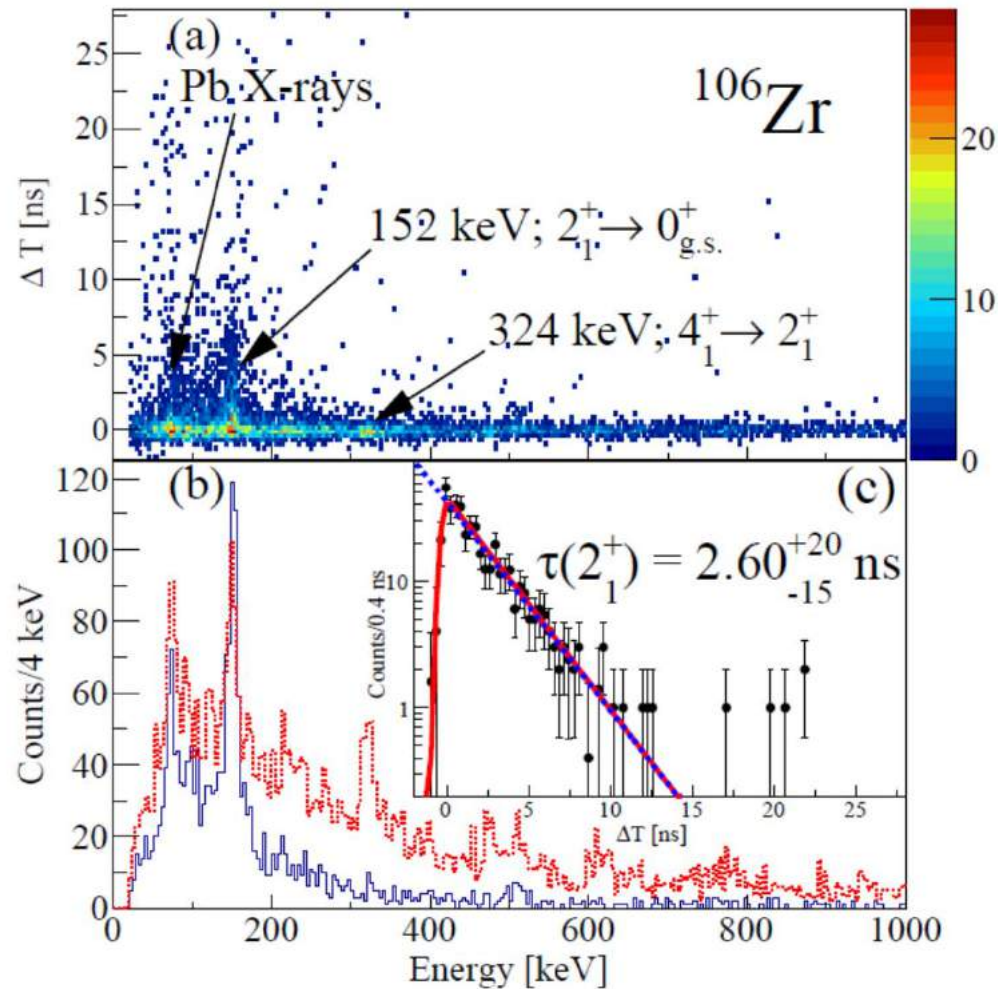
$$T(E2) = 1.22 \times 10^9 (E_\gamma)^5 B(E2)$$

The transition quadrupole moment Q_0 is obtained from:

$$Q_0^2 = \frac{16\pi B(E2)}{5 \langle J_i K 2 0 | J_f K \rangle^2}$$

So measuring a lifetime gives us a transition quadrupole moment (note that we cannot get the sign of Q_0)

Using the EURICA/FATIMA array at RIKEN



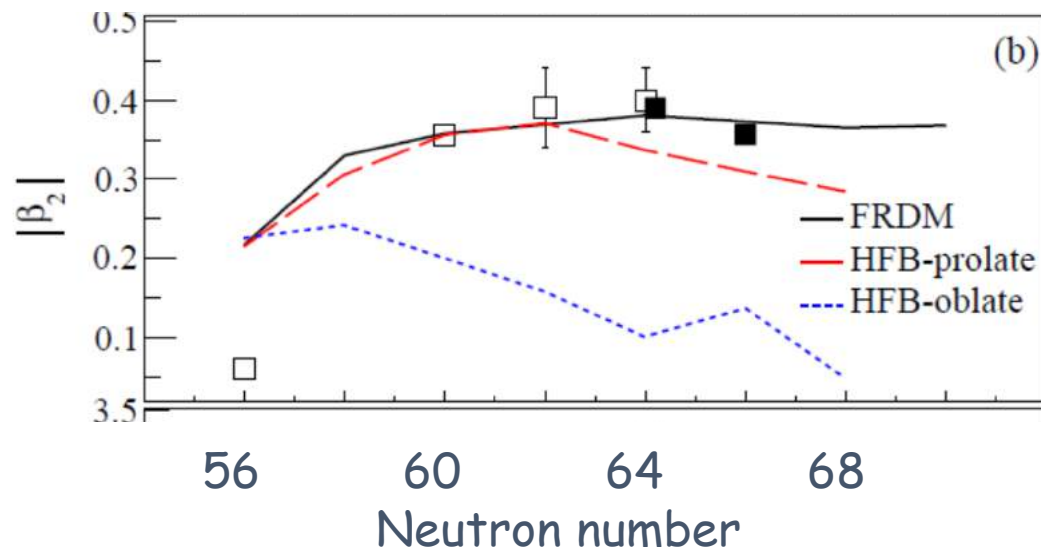
Browne et al. PLB750 (2015) 448

Quadrupole moment relates to shape

Assuming a quadrupoloid shape and that the deformation is the same for both states, the deformation β_2 can be obtained from:

$$Q_0 = \frac{3}{\sqrt{5\pi}} Z R^2 \beta_2 \left(1 + \frac{2}{7} \sqrt{\frac{5}{\pi}} \beta_2 + \frac{1}{14\pi} \beta_2^2 + \dots \right)$$

Neutron-rich Zr nuclei



Formula:
Löbner, Vetter and Honig,
Nucl. Data Tab A7 (1970) 495.

Data:
Browne et al. PLB750 (2015) 448

Suggestions for tutorial discussion:

1. In the angular correlation formula, on what might Q_k depend?
2. How does the deformation parameter β_2 relate to ε_2 and δ (which you may see elsewhere)
3. What methods are there to measure the sign of the quadrupole moment?



Acknowledgements
(for slides and
animations)

Dr Dave Joss

Prof John Simpson

Useful points of reference for angular correlation and distribution

Chapters 12,14 and 15 in 'The electromagnetic interaction in nuclear spectroscopy' edited by W.D.Hamilton

Q factors: Camp and van Lehn, NIM 76 (1969) 192

Alignment in compound nuclear reactions:
Butler and Twin, NIM 190 (1981) 283

Errors on δ from $\arctan \delta$ plots:
James, Twin and Butler, NIM 115 (1974) 105