Angular correlation of photon emission following particle emission

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I. INTRODUCTION

The nature of the γ radiation resulting from the decay $B \to C$ following the reaction $A(a,b)B$ is discussed in Sec. 10.7.4 of Satchler [1]. To be clear, here B and C correspond to states in the *same* nucleus. The double differential cross section for detecting b and the photon (γ) is given by Satchler, Eq. 10.126:

$$\frac{\mathrm{d}^2\sigma}{\mathrm{d}\Omega_b \mathrm{d}\Omega_\gamma} = \frac{\mathrm{d}\sigma}{\mathrm{d}\Omega_b}\frac{W}{4\pi}, \tag{1}$$

where we have assumed the branching-ratio factor Γ_γ/Γ in Eq. 10.126 is equal to unity. The angular correlation function W given by Satchler Eqs. 10.127 and 10.130

$$W = \sum_{kq} t_{kq}(I_B)\, R_k\, C^*_{kq}, \tag{2}$$

where $t_{kq}(I_B)$ is the polarization tensor of the nucleus B, R_k are the radiation parameters, and C_{kq} are related to a the spherical harmonics Y_{kq}:

$$C_{kq} = \left[\frac{4\pi}{2k+1}\right]^{1/2} Y_{kq}. \tag{3}$$

Note that we will utilize I_X to denote the intrinsic spin for nucleus X (possibly also designating a particular state). Following the convention used by FRESCO [2], we will choose the z axis to be along the incident beam and assume $\phi_b = 0$. We then have $t_{kq}(I_B) = t_{kq}(\theta_B)$, $\theta_b = \pi - \theta_B$, and $C_{kq} = C_{kq}(\theta_\gamma, \phi_\gamma)$ where θ_B and θ_γ are the usual polar angles and ϕ_γ is the azimuthal angle between the detected b and γ.

II. RADIATION PARAMETERS

According to Satchler [1, Eqs. (10.153)-(10.142), p. 375], the γ radiation parameters given in general by

$$R_k = \sum_{LL'} g_L\, g_{L'}\, R_k(LL'I_B I_C), \tag{4}$$

where

$$R_k(LL'I_BI_C) = (2I_B+1)^{1/2}(2L+1)^{1/2}(2L'+1)^{1/2}(-1)^{I_B-I_C+L-L'+k+1} \times (L'L1-1|k0)\,W(LL'I_BI_B;kI_C). \quad (5)$$

Here, L and L' take on values of the multipolarities of the possible γ-ray transitions, $(LL'1-1|k0)$ is a Clebsch-Gordan coefficient, and $W(LL'I_BI_B;kI_C)$ is a Racah coefficient. The amplitudes g_L describe the relative strength of the multipoles. Note that $R_0 = 1$ and parity considerations require that k only take on even values – but note Eq. (5 is not necessarily zero for k odd (see the footnote in Ref. [1]). It is often the case that only one multipole is present, or that this assumption is a good approximation. In this situation, there is only a single L value and

$$R_k = (2I_B+1)^{1/2}(2L+1)(-1)^{I_B-I_C+k+1}(LL1-1|k0)\,W(LLI_BI_B;kI_C). \quad (6)$$

III. POLARIZATION TENSORS

The polarization tensors $t_{kq}(I_B)$ describe the polarization of the final nucleus B; the precise definition is given in Secs. 10.3.2 and 10.3.3 of Satchler [1]. The polarization tensors can be calculated with FRESCO by noting that the FRESCO scattering amplitudes $f_{m'M':mM}$ are equivalent to Satchler's transition matrix elements $T_{\beta\alpha}$ defined by his Eq. 9.2. Satchler's Eq. 10.32 reads

$$t_{kq}(I_B) = \frac{\mathrm{Tr}[\boldsymbol{T}\boldsymbol{T}^\dagger \tau_{kq}(I_B)]}{\mathrm{Tr}[\boldsymbol{T}\boldsymbol{T}^\dagger]}, \quad (7)$$

where

$$\langle I_B m_B|\tau_{kq}(I_B)|I_B m'_B\rangle = (2k+1)^{1/2}(I_B k m_B q|I_B m'_B). \quad (8)$$

In the notation of Thompson (analogous to Eq. 3.33 of Ref. [2]) this formula becomes

$$t_{kq}(I_B) = \frac{\mathrm{Tr}[\boldsymbol{f}\boldsymbol{f}^\dagger \tau_{kq}(I_B)]}{\mathrm{Tr}[\boldsymbol{f}\boldsymbol{f}^\dagger]} = (2k+1)^{1/2}\,\frac{\displaystyle\sum_{m_b m_B m_a m_A} f_{m_b m_B:m_a m_A}(\theta_b)^* f_{m_b m'_B:m_a m_A}(\theta_b)(I_B k m_B q|I_B m'_B)}{\displaystyle\sum_{m_b m_B m_a m_A} |f_{m_b m_B:m_a m_A}(\theta_b)|^2}, \quad (9)$$

where $m'_B = m_B + q$ is required for the Clebcsh-Gordan coefficient to be non-zero. Also note that $t_{00} = 1$ and that the differential cross section is given by

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega_b} = \frac{1}{(2I_A+1)(2I_a+1)} \sum_{m_b m_B m_a m_A} |f_{m_b m_B:m_a m_A}(\theta_b)|^2. \quad (10)$$

The above equations can be used to calculate the angular correlation using the scattering amplitudes output by FRESCO. In addition the angular distribution of gamma rays can calculated by integrating over all particle emission angles. Note that the scattering amplitude file output by FRESCO is controlled with the LAMPL parameter; typically LAMPL=-2 is needed.

IV. DIFFERENTIAL CROSS SECTION FOR γ-RAY EMISSION

Equation 1 may be integrated over Ω_b to yield the differential cross section for γ-ray emission:

$$\frac{d\sigma}{d\Omega_\gamma} = \sum_k R_k P_k(\cos\theta_\gamma) \int_{-1}^{1} \frac{d\cos\theta_b}{2} \frac{d\sigma}{d\Omega_b} t_{k0}(I_B) \tag{11}$$

$$= \frac{1}{(2I_A+1)(2I_a+1)} \sum_k R_k P_k(\cos\theta_\gamma) \frac{(2k+1)^{1/2}}{2} \tag{12}$$

$$\times \int_{-1}^{1} d\cos\theta_b \sum_{m_b m_B m_a m_A} |f_{m_b m_B : m_a m_A}(\theta_b)|^2 (I_B k m_B 0 | I_B m_B).$$

V. FURTHER DISCUSSION

Considering the definitions in Thompson [2, Eq. (3.31), p. 185], the restriction to $\phi_b = 0$ in the scattering amplitudes may be relaxed by defining

$$\mathcal{M}_{m_b m_B : m_a m_A} = e^{i(m_b + m_B - m_a - m_A)\phi_b} f_{m_b m_B : m_a m_A}. \tag{13}$$

This results in

$$t_{kq}(\theta_b, \phi_b) = e^{iq\phi_b} t_{kq}(\theta_b, 0), \tag{14}$$

as expected for the tensor operator t_{kq}. The calculation of $\frac{d\sigma}{d\Omega_\gamma}$ may be carried out in this coordinate system as well, with the final result being (of course) unchanged.

VI. *R*-MATRIX

In an R-matrix approach, the partial-wave T matrix is calculated from the R-matrix or level matrix. The T matrix can then in principle be used to calculate any experimental observable, with the calculation being independent of the model used to determine the T matrix.

The scattering amplitudes may be constructed from the partial-wave T matrix as follows. According to Lane and Thomas [3, Eq. VIII.2.3, p. 292], the scattering amplitudes connecting non-elastic channels are given in the channel spin basis by

$$A_{bBs'\nu':aAs\nu} = i\frac{\pi^{1/2}}{k_{aA}} \sum_{JM\ell\ell'm'} (2\ell+1)^{1/2} (s\ell\nu 0|JM)(s'\ell'\nu'm'|JM) Y_{\ell'm'}(\Omega_b) T^J_{bBs'\ell':aAs\ell}, \tag{15}$$

where k_{aA} is the center-of-mass wavenumber in the $a+A$ system and $T^J_{bBs'\ell':aAs\ell}$ is the partial-wave T matrix. In the individual particle spin basis, the scattering amplitudes become

$$\begin{aligned}\mathcal{M}_{m_b m_B:m_a m_A} =& i\frac{\pi^{1/2}}{k_{aA}} \sum_{JM\ell\ell'm's\nu s'\nu'} (2\ell+1)^{1/2}(s\ell\nu 0|JM)(s'\ell'\nu'm'|JM) \\ & \times (I_a I_A m_a m_A|s\nu)(I_b I_B m_b m_B|s'\nu') Y_{\ell'm'}(\Omega_b) T^J_{bBs'\ell':aAs\ell},\end{aligned} \tag{16}$$

which suitable for calculating the general particle-γ correlation function. The differential cross section for γ-ray emission corresponding to Eq. (12) is given by

$$\begin{aligned}\frac{d\sigma}{d\Omega_\gamma} =& \frac{1}{(2I_A+1)(2I_a+1)} \sum_k R_k P_k(\cos\theta_\gamma) \frac{(2k+1)^{1/2}}{4\pi} \\ & \times \int_{4\pi} d\Omega_b \sum_{m_b m_B m_a m_A} |\mathcal{M}_{m_b m_B:m_a m_A}|^2 (I_B k m_B 0|I_B m_B).\end{aligned} \tag{17}$$

This equation may be simplified in a manner similar to that used for deriving the formula for $d\sigma/d\Omega_b$ given by Lane and Thomas [3, Sec. VII.2, p. 292]. Two sets of summing indices are introduced, the integration over $d\Omega_b$ is carried out using the orthogonality of the spherical harmonics, and the sums over Clebsch-Gordan coefficients are reduced to Racah coefficients. This procedure results in

$$\begin{aligned}&\int_{4\pi} d\Omega_b \sum_{m_b m_B m_a m_A} |\mathcal{M}_{m_b m_B:m_a m_A}|^2 (I_B k m_B 0|I_B m_B) = \\ &\quad \frac{\pi}{k^2_{aA}} \sum_{J_1 J_2 \ell_1 \ell_2 \ell' s s'_1 s'_2} (-1)^{k+s'_2-s'_1}\,(2J_1+1)(2J_2+1) \\ &\quad \times [(2\ell_1+1)(2I_B+1)(2s'_1+1)(2s'_2+1)]^{1/2} \\ &\quad \times (k\ell_1 00|\ell_2 0)\, W(kI_B s'_2 I_b; I_B s'_1)\, W(ks'_1 J_2 \ell'; s'_2 J_1)\, W(kJ_1\ell_2 s; J_2\ell_1) \\ &\quad \times T^{J_1 *}_{bBs'_1\ell':aAs\ell_1} T^{J_2}_{bBs'_2\ell':aAs\ell_2}.\end{aligned} \tag{18}$$

Note that the factor of $(-1)^k$ in this expression does not actually come into play since k is required to be even.

[1] G. R. Satchler, *Direct Nuclear Reactions* (Clarendon, Oxford, 1983).

[2] Ian J. Thompson, "Coupled reaction channels calculations in nuclear physics," Computer Physics Reports **7**, 167 – 212 (1988).

[3] A. M. Lane and R. G. Thomas, "R-matrix theory of nuclear reactions," Rev. Mod. Phys. **30**, 257–353 (1958).