

Schools and Learning Inequalities

A Theoretical Model of Quantitative
and Qualitative School Effects

Said Hassan Richard Breen

Nuffield College, University of Oxford



Roadmap

Do schools contribute to inequalities in learning outcomes?

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1. What do sociologists mean by this question?

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5. School *quality* versus *exposure*

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4. Natural experiment: school quality → test score gaps
5. School *quality* versus *exposure*
6. Do our results hold in other contexts?

Schools and Inequality: Two conflicting narratives

Reproduction

➤ Class analyses and cultural capital

(Bowles and Gintis, 1976; Bourdieu and Passeron, 1977)

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Investing in schools → less inequality

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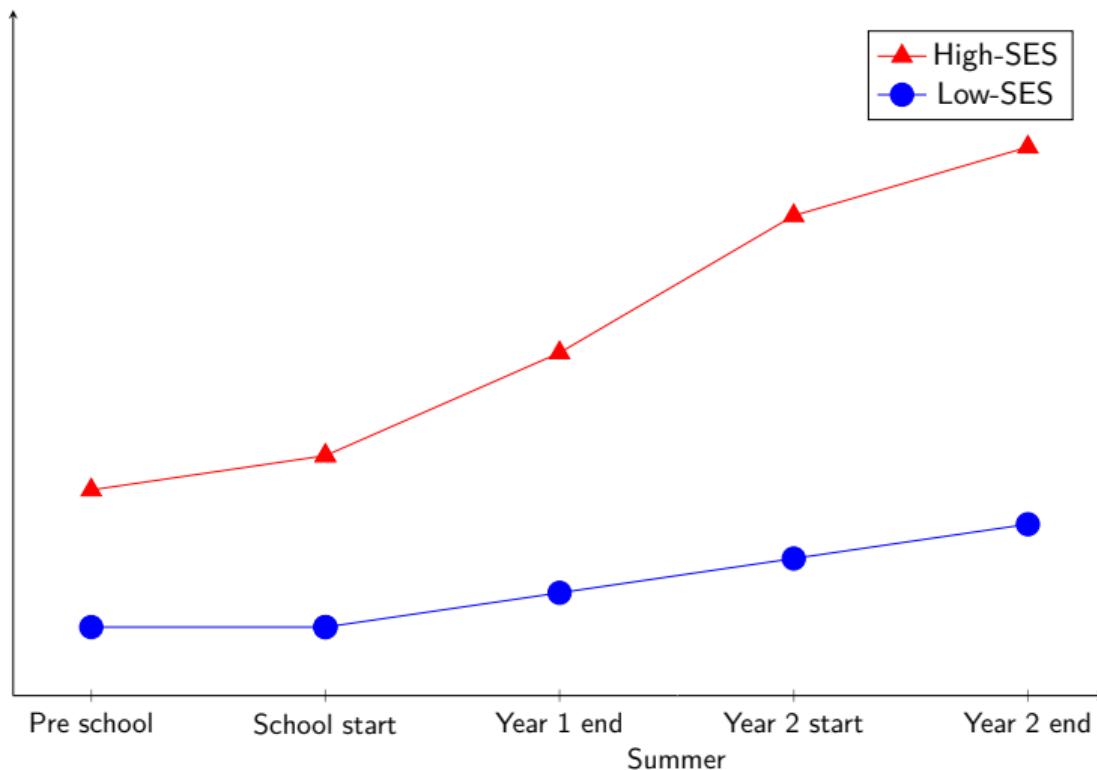
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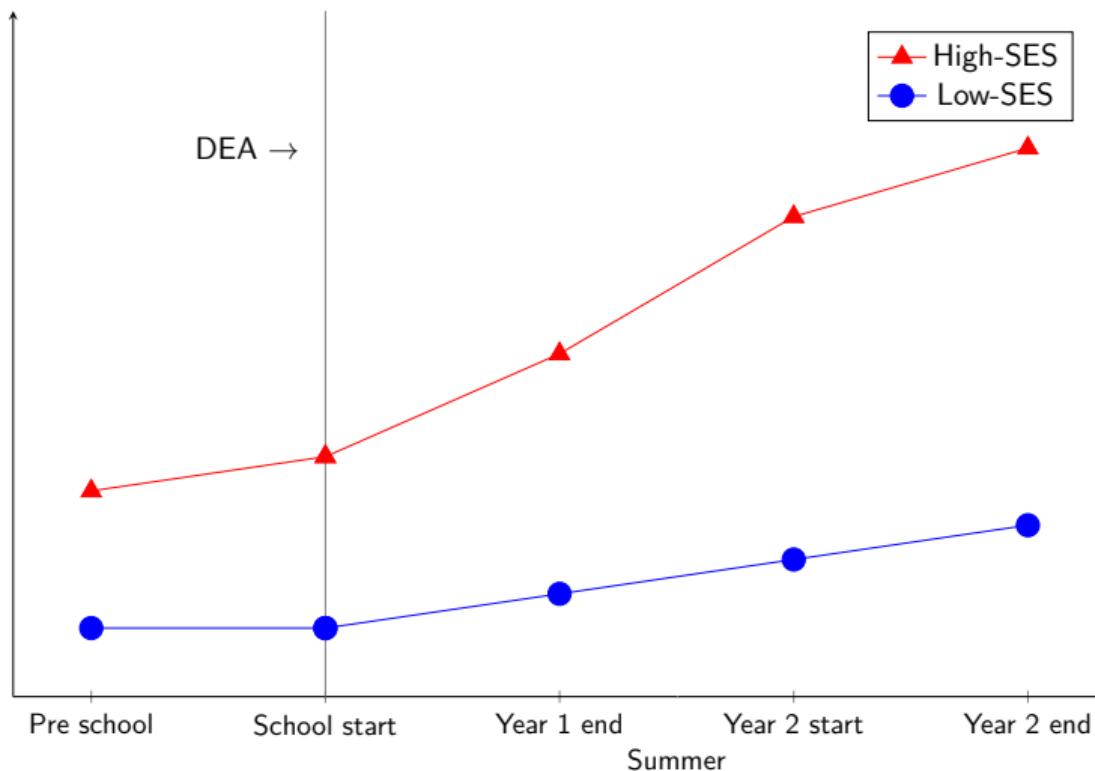
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⌚ School **exposures** ineffective because
home environments more unequal
Intervening in schools ineffective

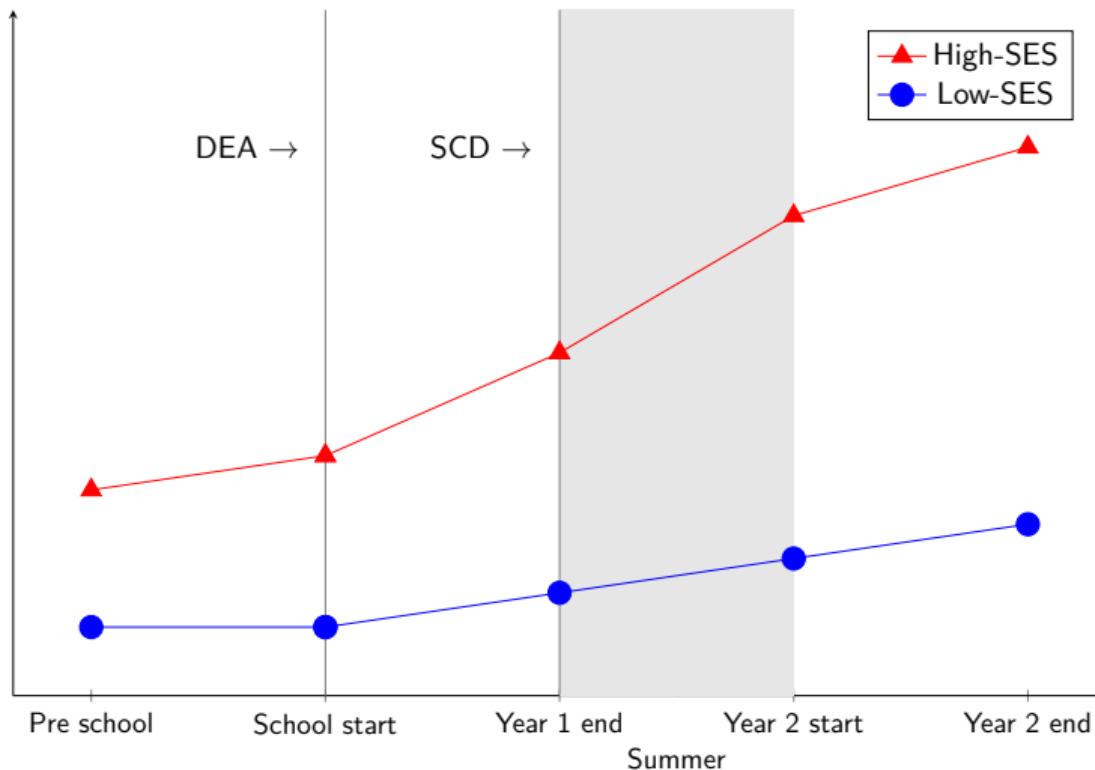
Different counterfactuals



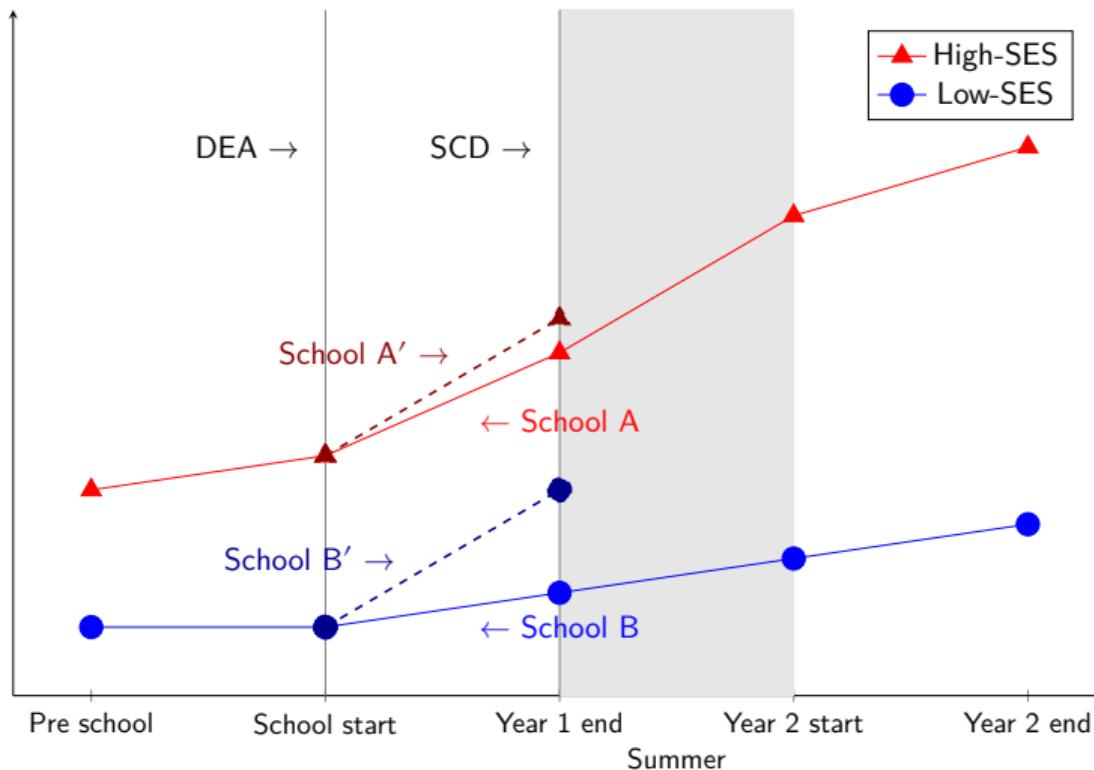
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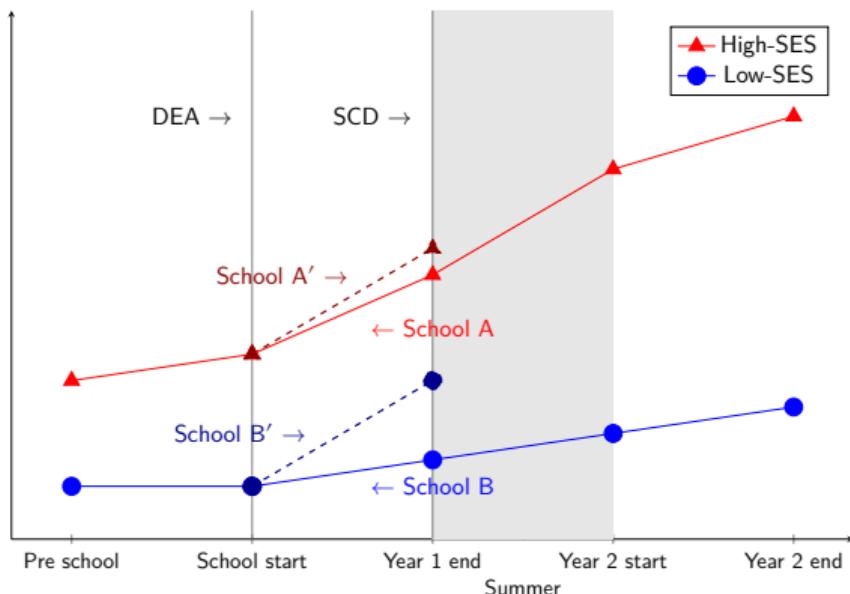
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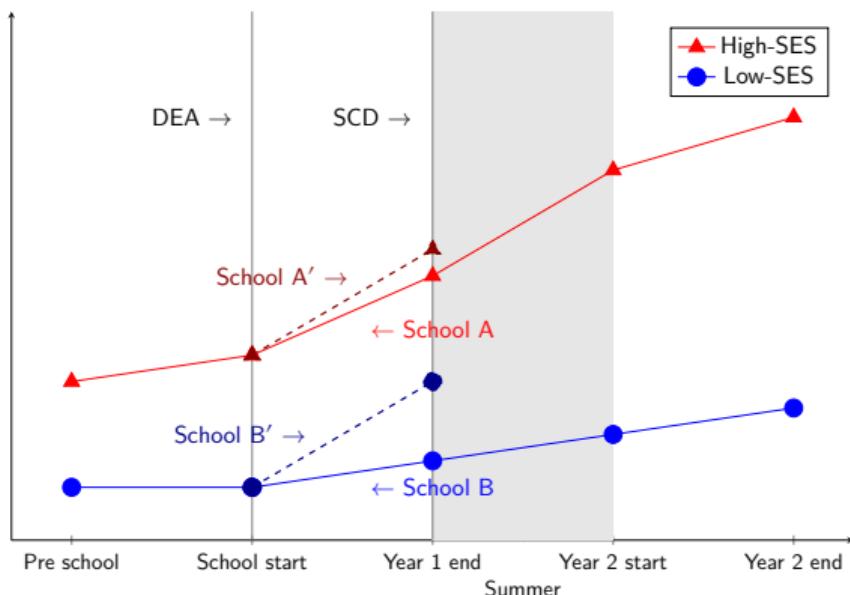
Different counterfactuals



Three **different** counterfactuals

1. DEA: shortening pre-school
2. SCD: Shortening summer break
3. Investing in quality

Different counterfactuals



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2. SCD: Shortening summer break
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Quantitative vs qualitative effects

Empirical setup

How do schools contribute to learning inequalities?

Empirical setup

RQ.1: How would low/high-SES children perform, had they been in lower/higher quality schools?

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- Natural experiment design: unanticipated school district changes
- School quality measured as value-added
- Standardized test scores in reading and mathematics
- Detailed information on parents' SES

School quality measure

Standard school value-added model (Koedel et al., 2015)

$$A_{ijt} = \omega A_{ij,t-1} + \mathbf{x}'_{ijt} \beta + \alpha_{jt} + \epsilon_{ijt}$$

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- net of student characteristics

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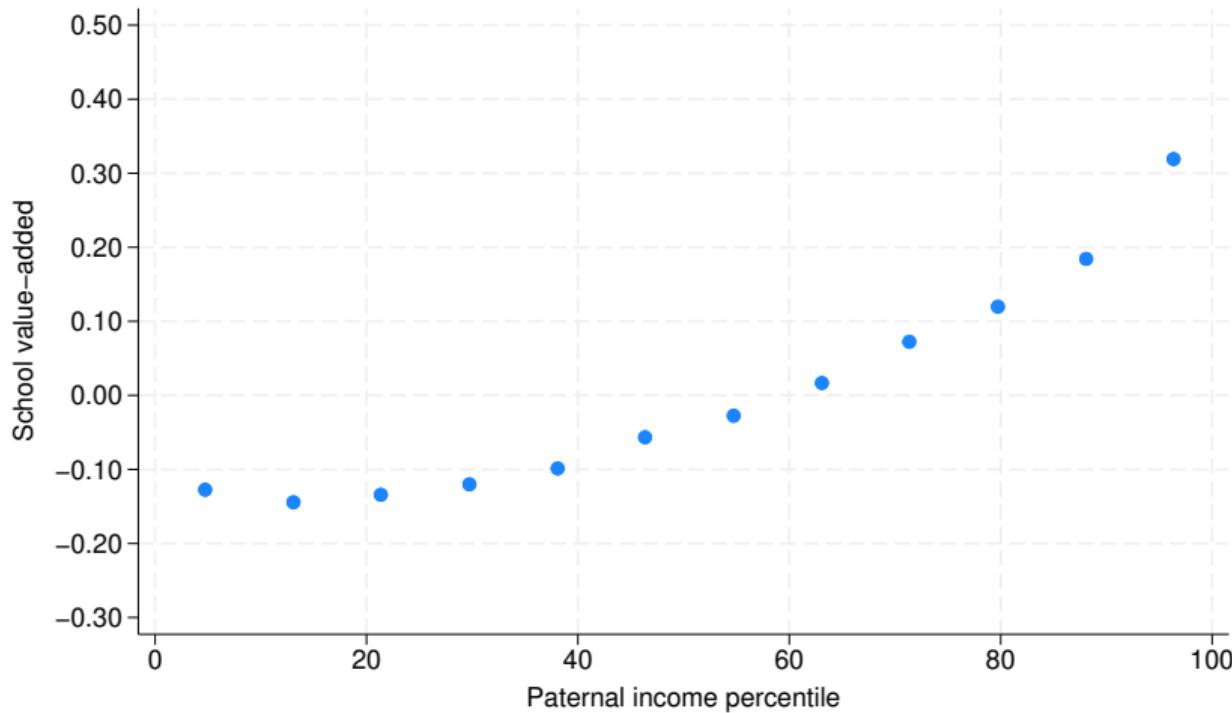
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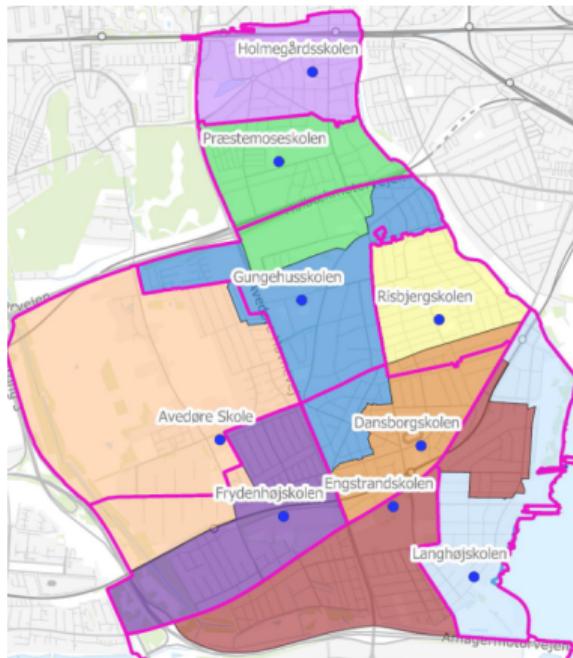
$\hat{\alpha}_{jt}$: contribution of school j to \bar{A}_{jt}

- net of student characteristics
- captures (unobserved) school factors
- very high autocorrelation ($\hat{\alpha}_{j,t}, \hat{\alpha}_{j,s>t}$)

School quality measure

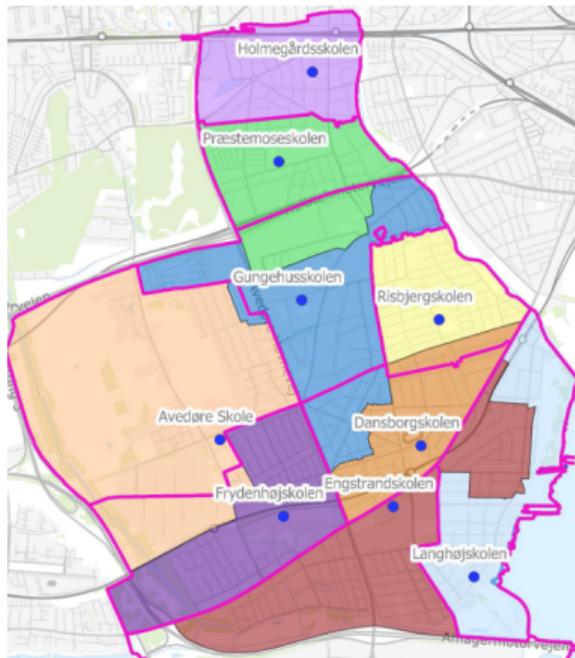


School district changes



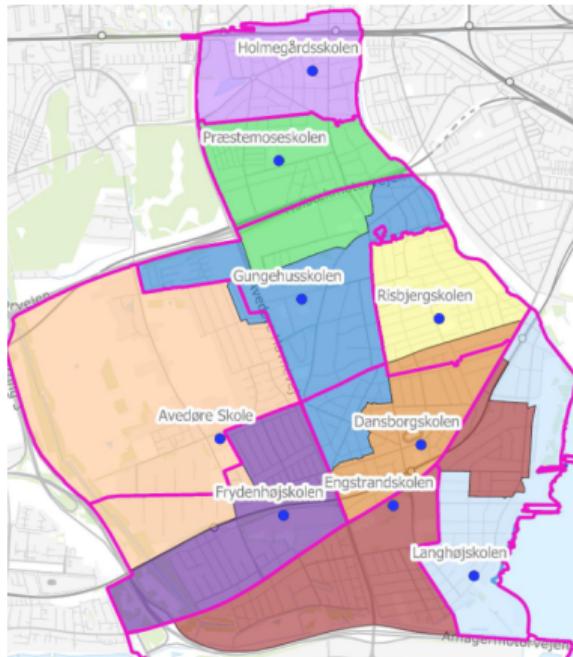
➤ School districts re-drawn every year

School district changes



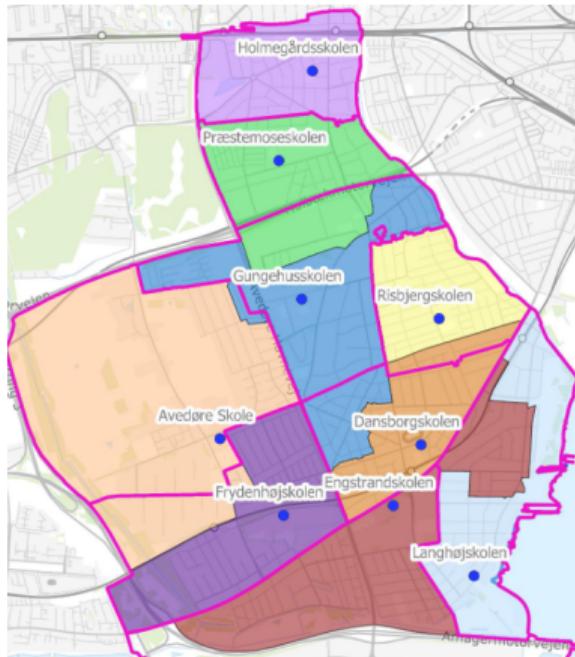
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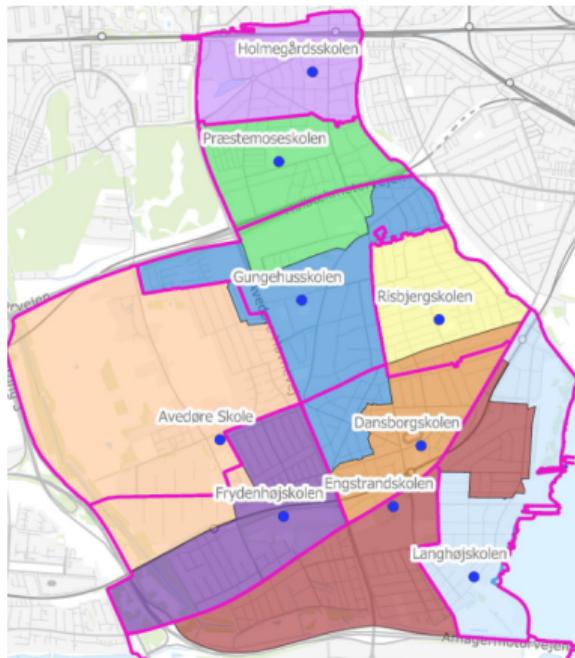
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- $\approx 85\%$ attend assigned district school

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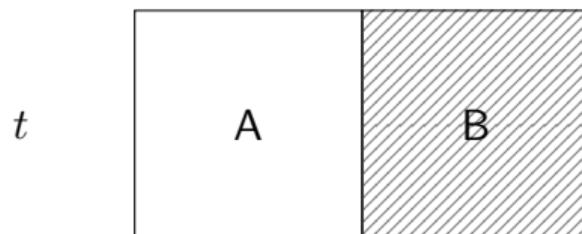


- School districts re-drawn every year
- Exactly 1 school per district
- Determined the year child turns 5
- $\approx 85\%$ attend assigned district school
- Exceptions, outside options, etc. directly observed

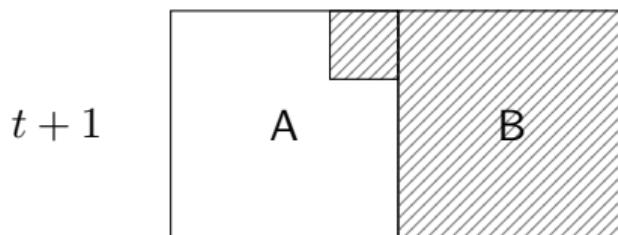
School district changes

Year	N(Obs.)	N(Changes)	% Any change	% Positive	% Negative
2009	34,319	358	1.04	0.52	0.53
2010	37,703	218	0.58	0.33	0.25
2011	36,405	452	1.24	0.69	0.55
2012	38,247	609	1.59	0.44	1.15
2013	36,793	201	0.55	0.40	0.15
2014	37,403	429	1.15	0.38	0.76
2015	33,532	574	1.71	1.01	0.70
Total	254,402	2,841	1.12	0.53	0.59

Does school quality impact test scores?



$$y_{ijt} = \rho Q_{jt} + \gamma L_i + \delta(Q_{jt} \times L_i) + \mathbf{x}'_i \beta + (\eta_{j'} \times \lambda_t) + \varepsilon_{ijt}$$



Does school quality impact test scores?

t



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where:

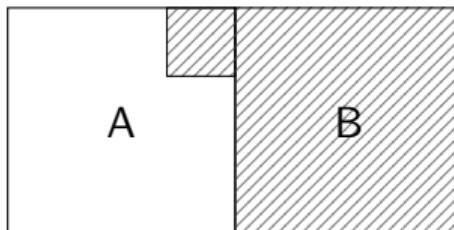
y_{ijt} student test score in reading (grade 2)

η_j' old school FE

Q_{jt} school quality (post-redistricting)

L_i low-SES dummy (no parent with university degree)

t + 1



Fixed effects results

	No controls		Controls	
	Coef	(SE)	Coef	(SE)
SVA	0.021	(0.027)	0.026	(0.025)
Low-SES	-0.462***	(0.007)	-0.440***	(0.006)
Low-SES × SVA	0.037***	(0.007)	0.035***	(0.006)
<i>N</i> (<i>obs.</i>)			254,397	
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Total impact for low-SES $\approx 0.06\sigma$

Motivation
○○○

Empirical setup
○○○○

Results
○○●○○

Conclusion
○○

School quality versus exposure

School quality versus exposure

Let Z_j be a dummy instrument for every school district change $j = 1, \dots, 81$.

$$Q_{it} = \alpha_0 + \sum_{j=1}^K \alpha_j \cdot 1(Z_i = j) + \lambda_{1,t} + e_{it}$$

$$E_{it} = \beta_0 + \sum_{j=1}^K \beta_j \cdot 1(Z_i = j) + \lambda_{2,t} + u_{it}$$

$$A_{it} = \gamma_0 + \sum_{j=1}^K \gamma_j \cdot 1(Z_i = j) + \lambda_{3,t} + v_{it}$$

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$$\text{Age at test} \rightarrow A_{it} = \gamma_0 + \sum_{j=1}^K \gamma_j \cdot 1(Z_i = j) + \lambda_{3,t} + v_{it}$$

Use to estimate the second stage Eq:

$$y_{it} = \delta_1 \hat{Q}_i + \delta_2 \hat{E}_i + \delta_3 \hat{A}_i + \varphi_t + \epsilon_i$$

School quality versus exposure

	Full Sample	Low-SES	High-SES
	(1)	(2)	(3)
Quality	0.126*	0.103*	0.048
	(0.055)	(0.044)	(0.073)
Exposure	0.036	0.044	-0.010
	(0.096)	(0.081)	(0.089)
Observations	199,690	149,120	50,570

School quality versus exposure

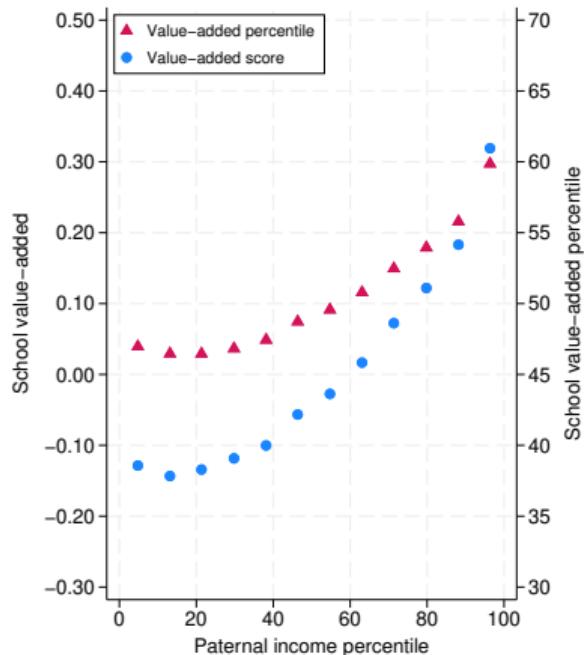
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Differential impact for low-SES: $\approx 0.05\sigma$

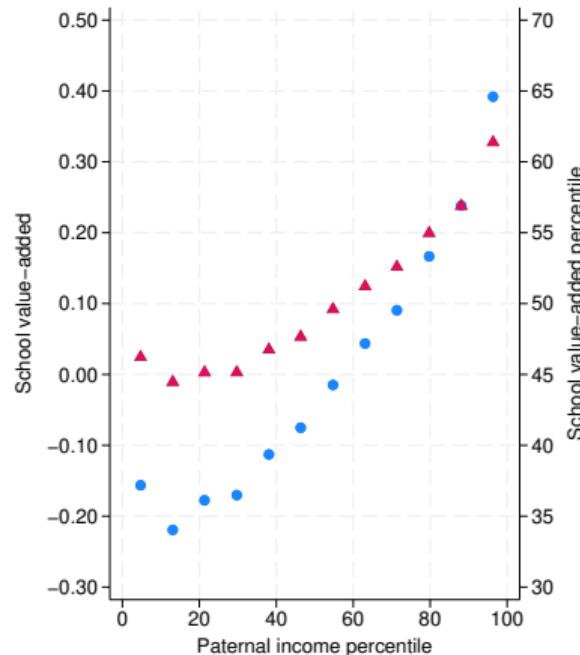
Results for Copenhagen

How would these results look in a more unequal setting?

Results for Copenhagen



(a) Full sample



(b) Copenhagen

Results for Copenhagen

	Full sample		Copenhagen	
	Coef	(SE)	Coef	(SE)
SVA	0.026	(0.025)	0.028	(0.021)
Low-SES	-0.440***	(0.006)	-0.455***	(0.010)
Low-SES × SVA	0.035***	(0.006)	0.044***	(0.009)
<i>N</i> (<i>obs.</i>)	254,397		67,851	
<i>N</i> (<i>identifying obs.</i>)	2,841		1,444	
<i>R</i> ²	0.177		0.189	

Summary

Consistent effect of $\approx 6\text{--}7\%$ SD

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- ▶ Total SES gap $\approx 45\%$ SD—similar in US (Reardon, 2016)

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School effectiveness literature:

- ▶ 1 year of schooling effect $\approx 1/3\sigma\text{--}1/4\sigma$ (Woessmann, 2016)

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Consistent effect of $\approx 6\text{--}7\%$ SD

- Total SES gap $\approx 45\%$ SD—similar in US (Reardon, 2016)
- About $1/6$ of total SES gap

School effectiveness literature:

- 1 year of schooling effect $\approx 1/3\sigma\text{--}1/4\sigma$ (Woessmann, 2016)
- Re-allocating low-SES kids to higher quality schools $\approx 1/4$ school year

Conclusion

Theoretical contributions

1. Model that makes sense of contradictory findings

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2. Explains why improvements in exposure ineffective

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Empirical contributions

4. Large inequalities in quality even in Denmark

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5. Quality improvements especially important for low-SES kids

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Empirical contributions

4. Large inequalities in quality even in Denmark
5. Quality improvements especially important for low-SES kids
6. Investments in quality have larger returns in unequal settings

Thank You!

said.hassan@nuffield.ox.ac.uk

Formal model

Simple model of skill formation: $Y = f(E, Q, A)$,

Formal model

Simple model of skill formation: $Y = f(E, Q, A)$, where:

E: Exposure to schooling

Q: Quality of schooling

A: Non-school factors

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E : Exposure to schooling

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A : Non-school factors

$$y = A \cdot Q \cdot E$$

Formal model

Simple model of skill formation: $Y = f(E, Q, A)$, where:

E : Exposure to schooling

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A : Non-school factors

$$y = AQ^\alpha E^\beta$$

Assumption 1: Marginal diminishing returns

Formal model

Simple model of skill formation: $Y = f(E, Q, A)$, where:

E : Exposure to schooling

Q : Quality of schooling

A : Non-school factors

$$y_L = A Q_L^\alpha E^\beta \quad y_H = A Q_H^\alpha E^\beta$$

Assumption 1: Marginal diminishing returns

Assumption 2: Selection into school quality (but not exposure)

Formal model

Simple model of skill formation: $Y = f(E, Q, A)$, where:

E : Exposure to schooling

Q : Quality of schooling

A : Non-school factors

$$y_L = A Q_L^{\alpha_L} E^{\beta_L} \quad y_H = A Q_H^{\alpha_H} E^{\beta_H}$$

Assumption 1: Marginal diminishing returns

Assumption 2: Selection into school quality (but not exposure)

Assumption 3: Flexibility (differential returns between groups)

Formal model: Quantitative (exposure) effects

High-SES group: $y_H = A Q_H^{\alpha_H} E^{\beta_H}$

Formal model: Quantitative (exposure) effects

High-SES group: $y_H = A Q_H^{\alpha_H} E^{\beta_H}$

$$\frac{1}{y_H} \frac{\partial Y}{\partial E} = \frac{(\beta_H) A Q_H^{\alpha_H} E^{\beta_H - 1}}{A Q_H^{\alpha_H} E^{\beta_H}}$$

Formal model: Quantitative (exposure) effects

High-SES group: $y_H = A Q_H^{\alpha_H} E^{\beta_H}$

$$\frac{1}{y_H} \frac{\partial Y}{\partial E} = \frac{(\beta_H) A Q_H^{\alpha_H} E^{\beta_H - 1}}{A Q_H^{\alpha_H} E^{\beta_H}}$$

Low-SES group: $y_L = A Q_L^{\alpha_L} E^{\beta_L}$

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Relative ratio between derivatives:

(Relative gains–inequality of increasing E)

$$\Theta_E = \left(\frac{1}{y_H} \frac{\partial y_H}{\partial E} \right) \Big/ \left(\frac{1}{y_L} \frac{\partial y_L}{\partial E} \right) = \frac{\beta_H}{\beta_L}$$

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Relative ratio between derivatives:

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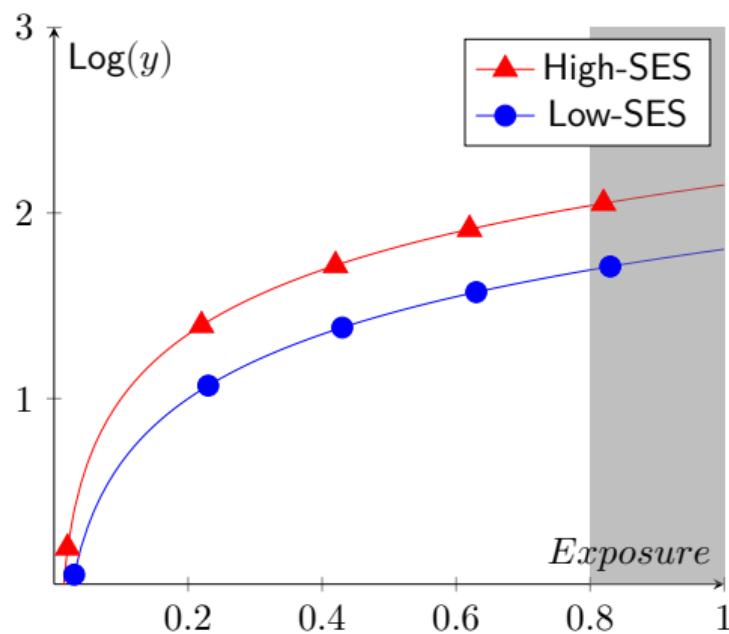
$$\Theta_E = \left(\frac{1}{y_H} \frac{\partial y_H}{\partial E} \right) \Big/ \left(\frac{1}{y_L} \frac{\partial y_L}{\partial E} \right) = \frac{\beta_H}{\beta_L}$$

Intuition:

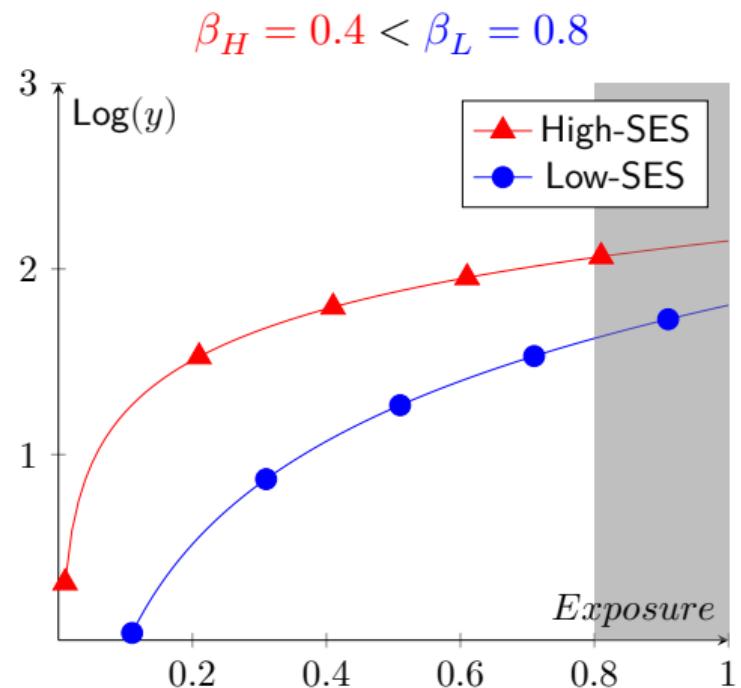
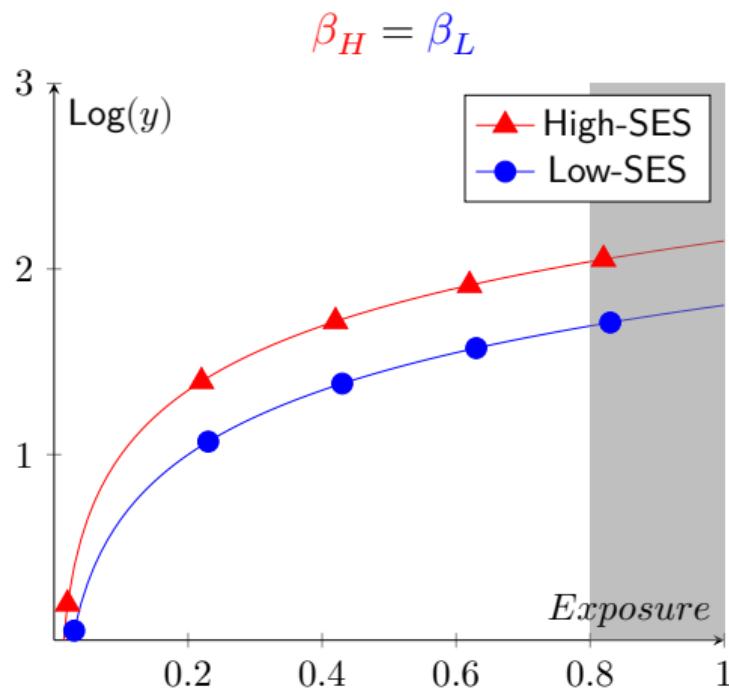
$$\Theta_E = \begin{cases} 1 & \text{if } \beta_H = \beta_L \text{ (Preserves inequality)} \\ < 1 & \text{if } \beta_H < \beta_L \text{ (Reduces inequality)} \\ > 1 & \text{if } \beta_H > \beta_L \text{ (Increases inequality)} \end{cases}$$

Formal model: Quantitative effects

$$\beta_H = \beta_L$$



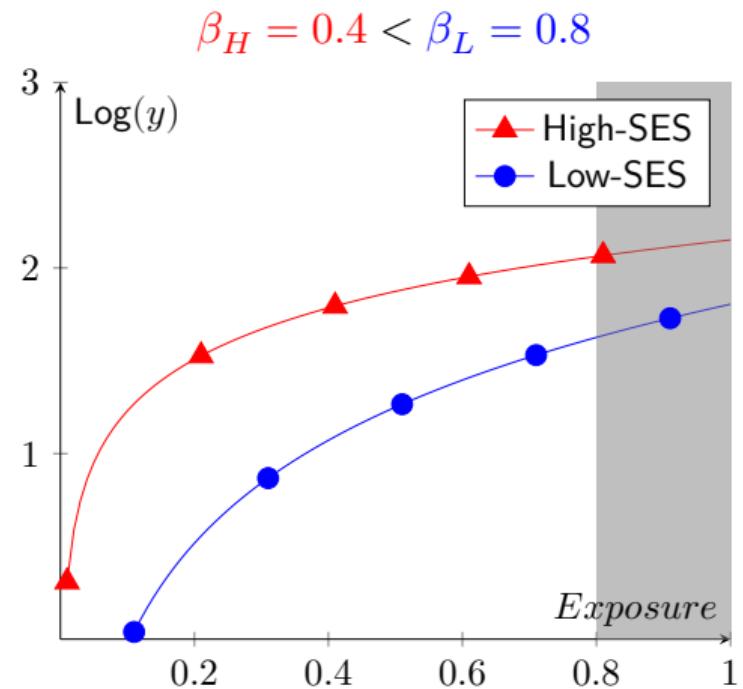
Formal model: Quantitative effects



Formal model: Quantitative effects

💡 Takeaways:

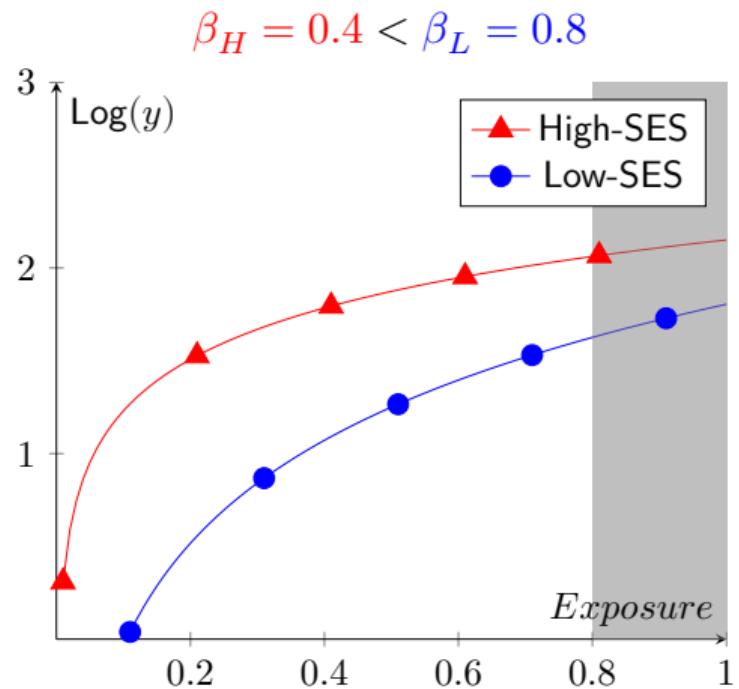
- Limited gains to exposure investments even if low-SES children benefit *more* from exposure



Formal model: Quantitative effects

💡 Takeaways:

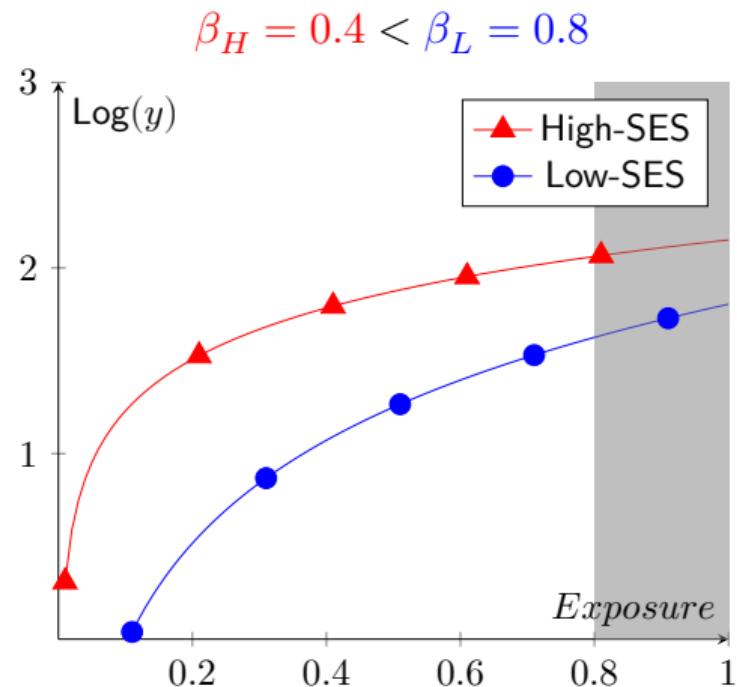
- Limited gains to exposure investments even if low-SES children benefit *more* from exposure
- Exposure is hard to scale up!



Formal model: Quantitative effects

💡 Takeaways:

- Limited gains to exposure investments even if low-SES children benefit *more* from exposure
- Exposure is hard to scale up!
- Selection into quality \Rightarrow low-SES children get more low quality schooling vs high-SES children get more high quality schooling



Formal model: Qualitative effects

High-SES group: $y_H = A Q_H^{\alpha_H} E^{\beta_H}$

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Formal model: Qualitative effects

High-SES group:

$$y_H = A Q_H^{\alpha_H} E^{\beta_H}$$

Relative ratio between derivatives:

(Relative gains–inequality of increasing Q)

$$\frac{1}{y_H} \frac{\partial Y}{\partial Q} = \frac{A \alpha_H Q_H^{\alpha_H - 1} E^{\beta_H}}{A Q_H^{\alpha_H} E^{\beta_H}}$$

$$\Theta_Q = \left(\frac{1}{Y_H} \frac{\partial Y_H}{\partial Q} \right) \Big/ \left(\frac{1}{Y_L} \frac{\partial Y_L}{\partial Q} \right) = \frac{\alpha_H}{\alpha_L} \cdot \frac{Q_H}{Q_L}$$

Low-SES group:

$$y_L = A Q_L^{\alpha_L} E^{\beta_L}$$

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Formal model: Qualitative effects

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Intuition:

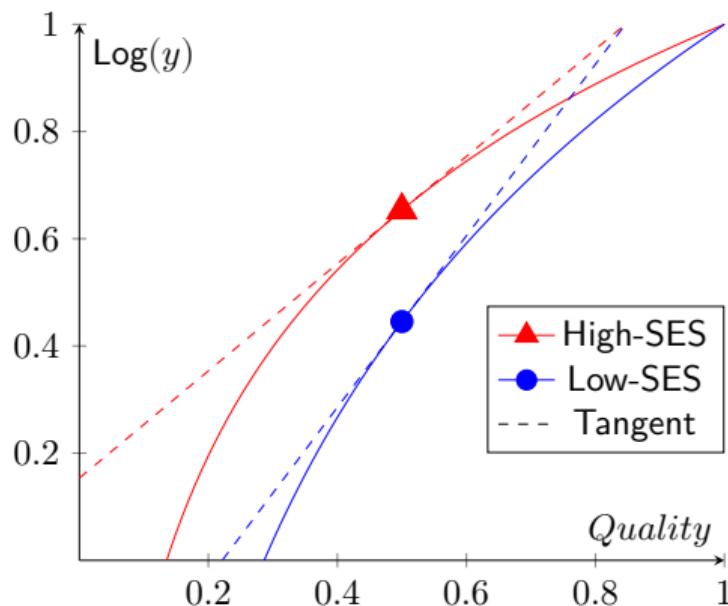
Low-SES group: $y_L = A Q_L^{\alpha_L} E^{\beta_L}$

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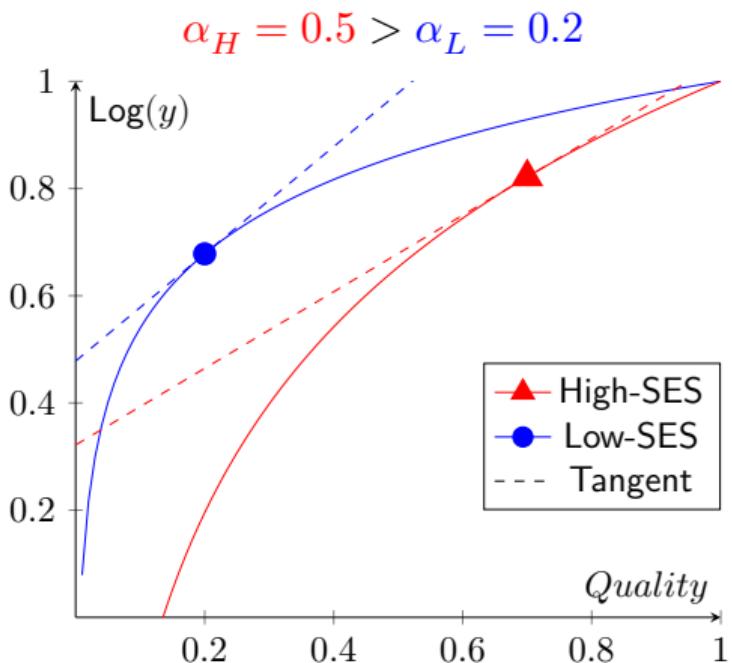
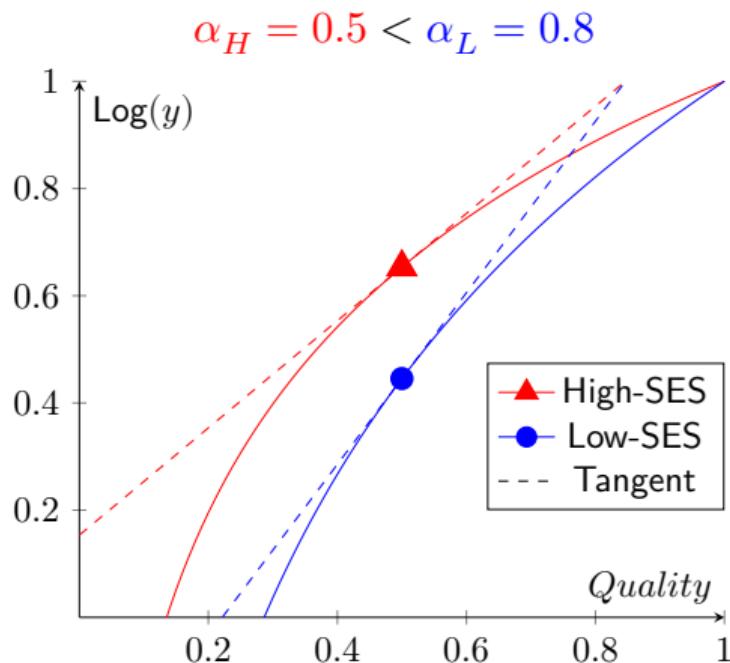
$$\Theta_Q = \begin{cases} 1 & \text{if } \alpha_H = \alpha_L \text{ (Inequality by } Q \text{ only)} \\ < 1 & \text{if } \alpha_H < \alpha_L \text{ (Less inequality at any } Q) \\ > 1 & \text{if } \alpha_H > \alpha_L \text{ (Less inequality at lower } Q) \end{cases}$$

Formal model: Qualitative effects

$$\alpha_H = 0.5 < \alpha_L = 0.8$$



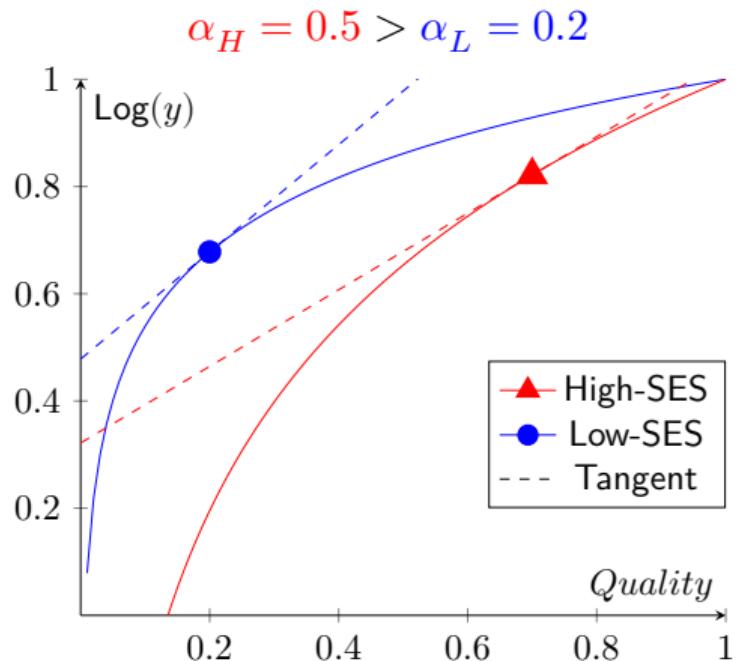
Formal model: Qualitative effects



Formal model: Qualitative effects

💡 Takeaways:

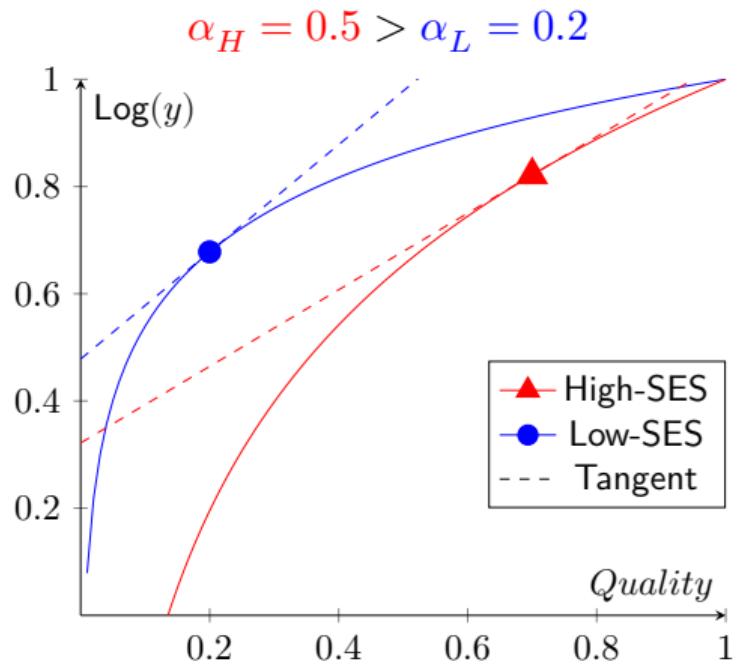
- Quality investments more effective at reducing inequality



Formal model: Qualitative effects

💡 Takeaways:

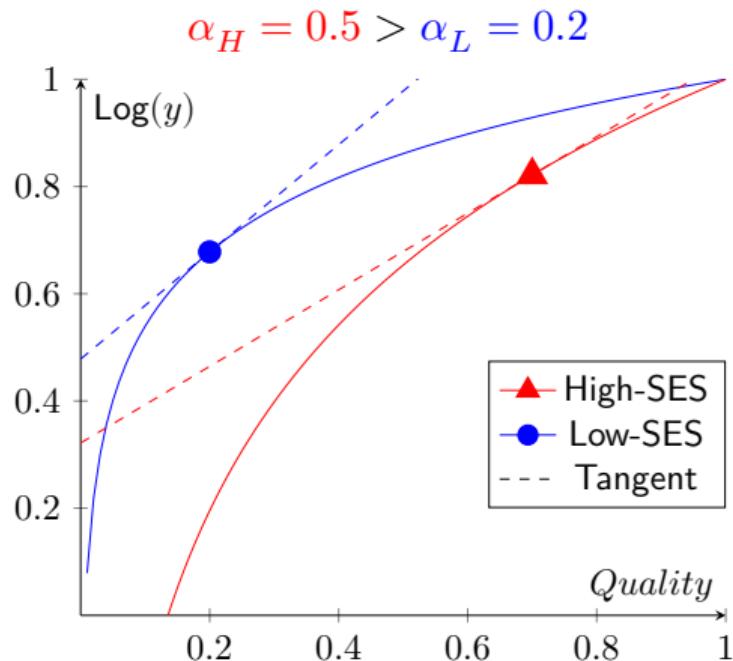
- Quality investments more effective at reducing inequality
- If selection into quality strong, gains even higher among low-SES children



Formal model: Qualitative effects

💡 Takeaways:

- Quality investments more effective at reducing inequality
- If selection into quality strong, gains even higher among low-SES children
- Even if low-SES children have low returns, investing in quality still leads to more equality because of selection



Derivation Θ_Q , Low-SES

We start with the production function for the Low-SES group:

$$y_L = A Q_L^{\alpha_L} E^{\beta_L}$$

Step 1: Compute the Partial Derivative with Respect to Q

$$\frac{\partial y_L}{\partial Q} = \frac{\partial}{\partial Q} (A Q_L^{\alpha_L} E^{\beta_L}) = A \alpha_L Q_L^{\alpha_L - 1} E^{\beta_L}$$

Step 2: Divide by y_L to Find the Marginal Effect Relative to y_L

$$\frac{1}{y_L} \frac{\partial y_L}{\partial Q} = \frac{A \alpha_L Q_L^{\alpha_L - 1} E^{\beta_L}}{A Q_L^{\alpha_L} E^{\beta_L}} = \frac{\alpha_L}{Q_L}$$

Thus, for the Low-SES group, we have:

$$\frac{1}{y_L} \frac{\partial y_L}{\partial Q} = \frac{\alpha_L}{Q_L}$$

Derivation Θ_Q , High-SES

$$y_H = A Q_H^{\alpha_H} E^{\beta_H}$$

Step 1: Compute the Partial Derivative with Respect to Q

$$\frac{\partial y_H}{\partial Q} = \frac{\partial}{\partial Q} (A Q_H^{\alpha_H} E^{\beta_H}) = A \alpha_H Q_H^{\alpha_H - 1} E^{\beta_H}$$

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$$\frac{1}{y_H} \frac{\partial y_H}{\partial Q} = \frac{A \alpha_H Q_H^{\alpha_H - 1} E^{\beta_H}}{A Q_H^{\alpha_H} E^{\beta_H}} = \frac{\alpha_H}{Q_H}$$

Thus, for the High-SES group, we have:

$$\frac{1}{y_H} \frac{\partial y_H}{\partial Q} = \frac{\alpha_H}{Q_H}$$

Relative Ratio between Marginal Effects

Now, we form the relative ratio between the derivatives for the High-SES and Low-SES groups:

$$\Theta_Q = \frac{\frac{1}{y_H} \frac{\partial y_H}{\partial Q}}{\frac{1}{y_L} \frac{\partial y_L}{\partial Q}}$$

Substituting the expressions derived for each group:

$$\Theta_Q = \frac{\frac{\alpha_H}{Q_H}}{\frac{\alpha_L}{Q_L}}$$

Simplifying the Ratio

$$\Theta_Q = \frac{\alpha_H}{\alpha_L} \cdot \frac{Q_H}{Q_L}$$

Thus, the relative ratio between the derivatives is:

$$\Theta_Q = \frac{\alpha_H}{\alpha_L} \cdot \frac{Q_H}{Q_L}$$

Theoretical model - assumptions

$$\text{Government problem: } \max_{E,Q} \int Y_1(Q, E, H; \theta) \partial F(\theta) \quad (1)$$

Assume concavity in returns to E and Q :

$$Y_1 = Q^{\alpha_0} E^{\alpha_1} H^{1-\alpha_0-\alpha_1} \theta \quad (2)$$

and a budget constraint:

$$P_E E + P_Q Q \leq G \quad (3)$$

$$\max_{E,Q} \int Q^{\alpha_0} E^{\alpha_1} H^{1-\alpha_0-\alpha_1} \theta \partial F(\theta) \quad \text{s.t.} \quad P_E E + P_Q Q \leq G \quad (4)$$

Theoretical model - Optimal investments

Apply a Lagrange multiplier ($\lambda(\cdot)$) to the problem in (4) and optimize :

$$\max_{E,Q} \int Q^{\alpha_0} E^{\alpha_1} H^{1-\alpha_0-\alpha_1} \theta \partial F(\theta) + \lambda [G - P_E E - P_Q Q] \quad (5)$$

w.r.t. to Q: (6)

$$\int \alpha_0 Q^{\alpha_0-1} E^{\alpha_1} H^{1-\alpha_0-\alpha_1} \theta \partial F(\theta) - \lambda [P_Q] = 0 \quad (7)$$

w.r.t. to E: (8)

$$\int \alpha_1 E^{\alpha_1-1} Q^{\alpha_0} H^{1-\alpha_0-\alpha_1} \theta \partial F(\theta) - \lambda (P_E) = 0 \quad (9)$$

So we get:

$$\lambda P_Q = \alpha_0 \mathbb{E}_{Q^{-1}Y_1(\theta)} \quad (10)$$

$$\lambda P_E = \alpha_1 \mathbb{E}_{E^{-1}Y_1(\theta)} \quad (11)$$

Theoretical model - Optimal investments results 1

1. The optimal investment in E relative to Q is:

$$E = \frac{P_Q \alpha_1}{P_E \alpha_0} Q \quad (12)$$

2. A budget constraint of:

$$G = P_Q Q + P_E E \quad (13)$$

$$= P_Q Q + P_E \frac{P_Q \alpha_1}{P_E \alpha_0} Q \quad (14)$$

$$= P_Q Q + \frac{P_Q \alpha_1}{\alpha_0} Q \quad (15)$$

$$= [1 + \frac{\alpha_1}{\alpha_0}] P_Q Q \quad (16)$$

Theoretical model - Optimal investments results 2

Now re-arrange, to get how much we should spend on quality (here, let Q^* denote the optimal Q):

$$G = [1 + \frac{\alpha_1}{\alpha_0}] P_Q Q \quad (17)$$

$$Q^* = \frac{G}{P_Q} \frac{\alpha_0}{\alpha_0 + \alpha_1} \quad (18)$$

Symmetrically, we can derive the same for the optimal E , E^* :

$$E^* = \frac{G}{P_E} \frac{\alpha_1}{\alpha_0 + \alpha_1} \quad (19)$$

Theoretical model - Experiment 1: Targeted investments

$$\mathbb{E}(Y_1(\theta)) = \mathbb{P}(\theta)Y(\theta) + \mathbb{P}(\theta')Y(\theta') \quad (20)$$

where $\mathbb{P}(\theta)$ is the probability (here, share) of individuals in the population with low initial skills (θ), and $\mathbb{P}(\theta')$ is for high initial skills.

Now consider a policy that increases $Q(\theta) \forall \theta$ holding constant home environment H and exposure to schooling E . This policy satisfies $P_E E + P_Q Q = G$ such that $Q_N = Q + \delta(\theta)$. Here, Q_N denotes the “new” quality level after the investment.

The effect of this policy is given by:

$$Y_1^N(\theta) = \theta(Q + \delta(\theta))^{\alpha_0} E(\theta)^{\alpha_1} H^{1-\alpha_0-\alpha_1} \quad (21)$$

$$Y_1^N(\theta') = \theta'(Q + \delta(\theta'))^{\alpha_0} E(\theta')^{\alpha_1} H^{1-\alpha_0-\alpha_1} \quad (22)$$

$$E(Y_1^N) = \mathbb{P}(\theta) + \mathbb{P}(\theta')Y_1^N(\theta') \quad (23)$$

Theoretical model - Experiment 1: Targeted investments

Now we can ask suppose we want to choose $\mathbb{E}(Y_1^N)$ (the attained skill after the investment) to be larger, how should we choose δ_θ (the amount of additional quality we invest in low-skill individuals) and δ_θ^1 (the amount of additional quality we invest in high-skill individuals):

$$\max_{\{\delta_\theta, \delta_\theta^1\}} \mathbb{E}(Y_1^N) \Leftrightarrow \max_{\{\delta_\theta, \delta_\theta^1\}} \mathbb{P}(\theta)Y_1^N(\theta) + \mathbb{P}(\theta')Y_1^N(\theta') \quad (24)$$

The benefit from raising δ_θ for low-skill individuals:

$$\mathbb{P}(\theta) \frac{\partial Y_1^N(\theta)}{\partial \delta_\theta} = P\alpha_0 \frac{Y_1^N(\theta)}{Q(\theta) + \delta(\theta)} \quad (25)$$

The benefit from raising $\delta_{\theta'}$ for high skill individuals:

$$(1 - \mathbb{P})\alpha_0 \frac{Y_1^N(\theta')}{Q(\theta')} + \delta(\theta') \quad (26)$$

Because of concavity of $Y_1(Q)$, then $\forall \theta < \theta'$:

$$\frac{Y_1^N(\theta)}{Q(\theta) + \delta(\theta)} > \frac{Y_1^N(\theta')}{Q(\theta') + \delta(\theta')} \quad (27)$$

Theoretical model - Experiment 2: Indiscriminate investments

I.e., $Q^N = Q + \delta$:

$$\max_{\delta} \mathbb{E}(Y_1) \tag{28}$$

$$\max_{\delta} \mathbb{P}(\theta)Y_1(\theta) + \mathbb{P}(\theta')Y_1(\theta') \tag{29}$$

Then we have:

$$\frac{\mathbb{P}(\theta)Y_1(\theta)}{Q(\theta) + \delta} + \frac{\mathbb{P}(\theta')Y_1(\theta')}{Q(\theta) + \delta'} > 0 \tag{30}$$

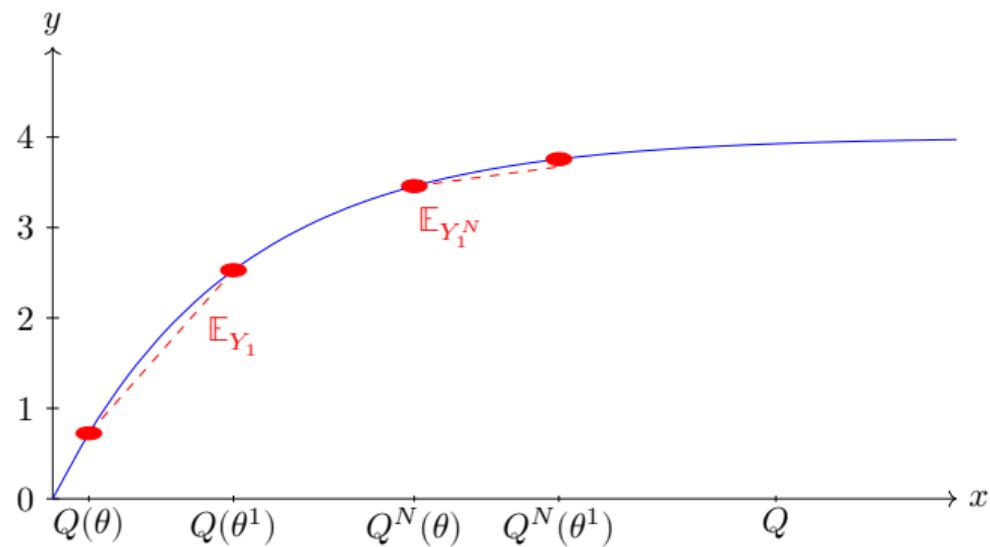
Assuming that

$$\frac{\partial Y}{\partial Q} > 0$$

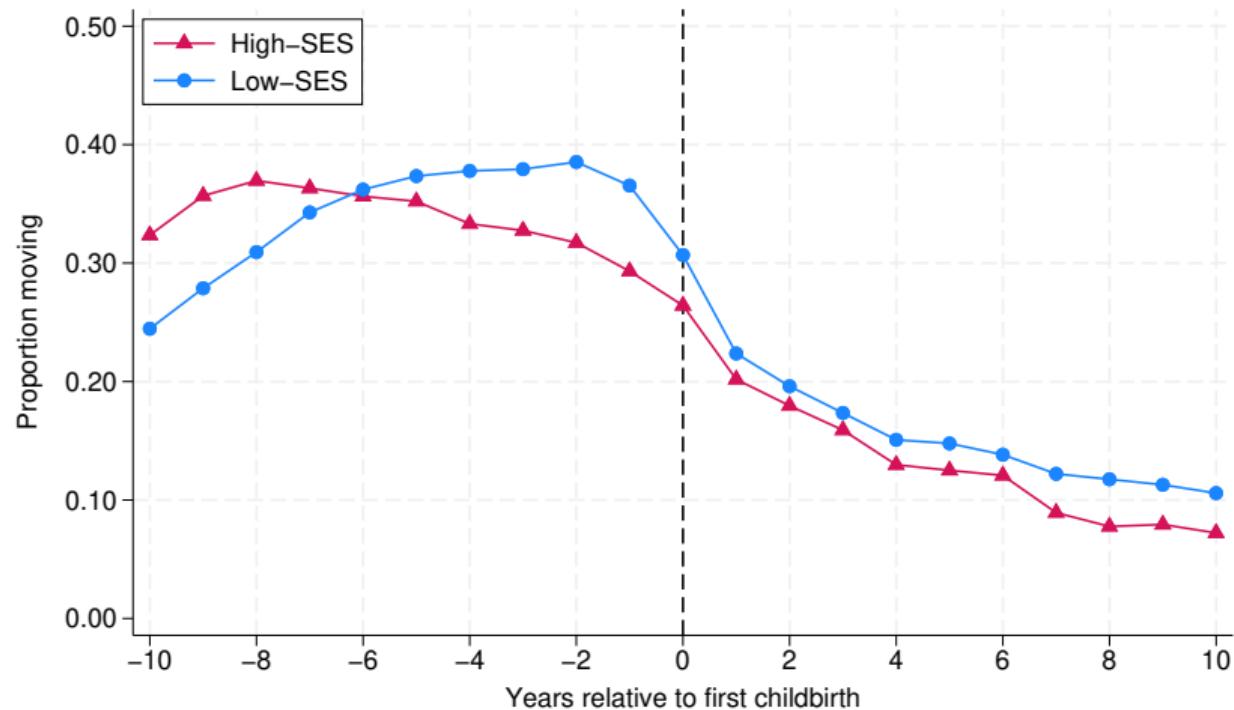
we would rather choose the highest change in quality in order to increase the $\mathbb{E}(Y_1)$, this is because $Y_1(\theta)$ and $Y_1(\theta')$

Theoretical model - Experiment 2: Indiscriminate investments

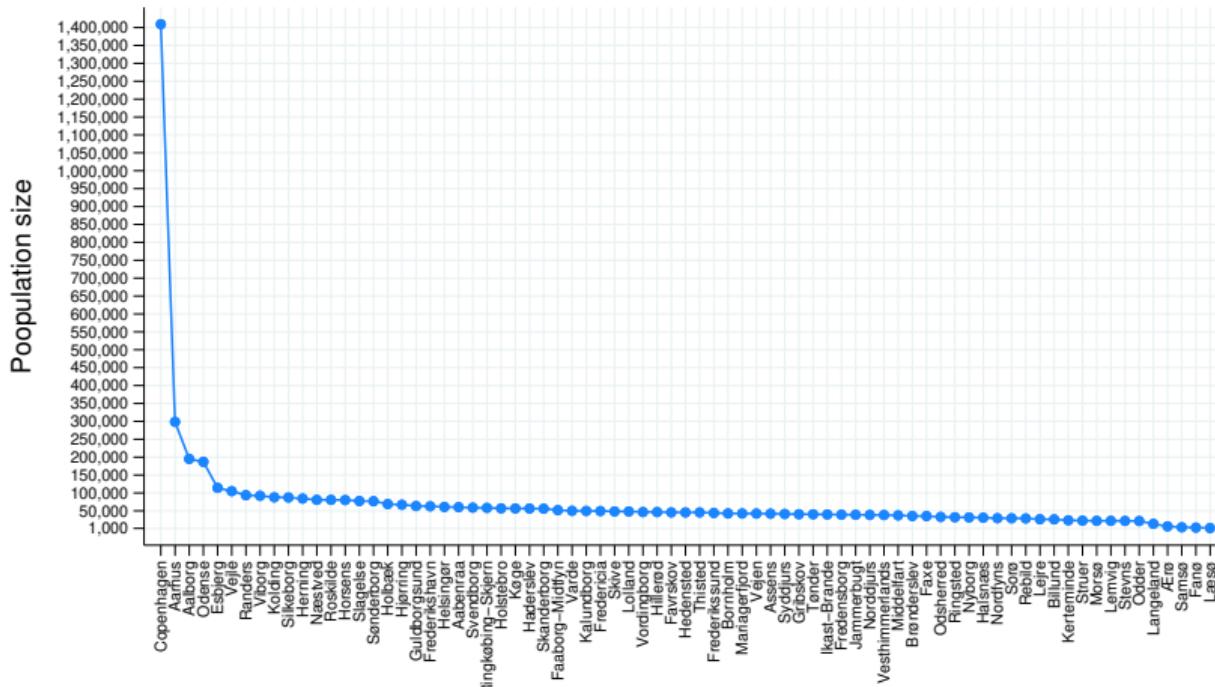
Figure: Illustration



Robustness check: Anticipation of district changes



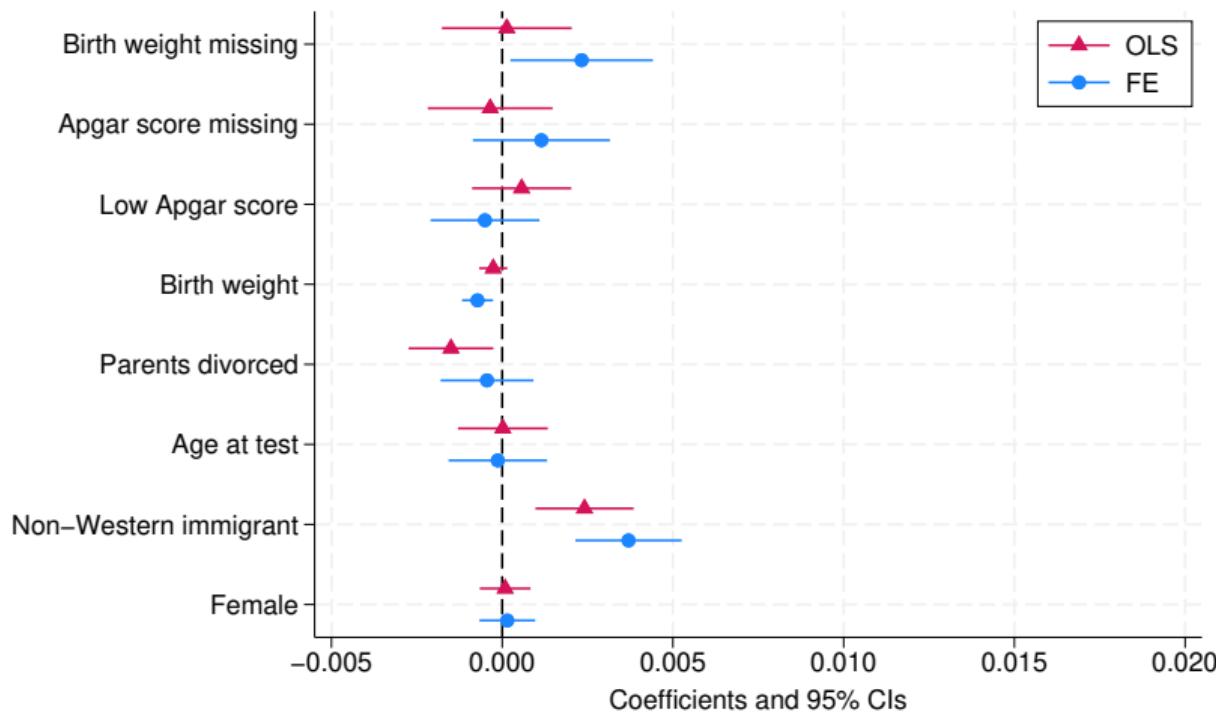
City sizes



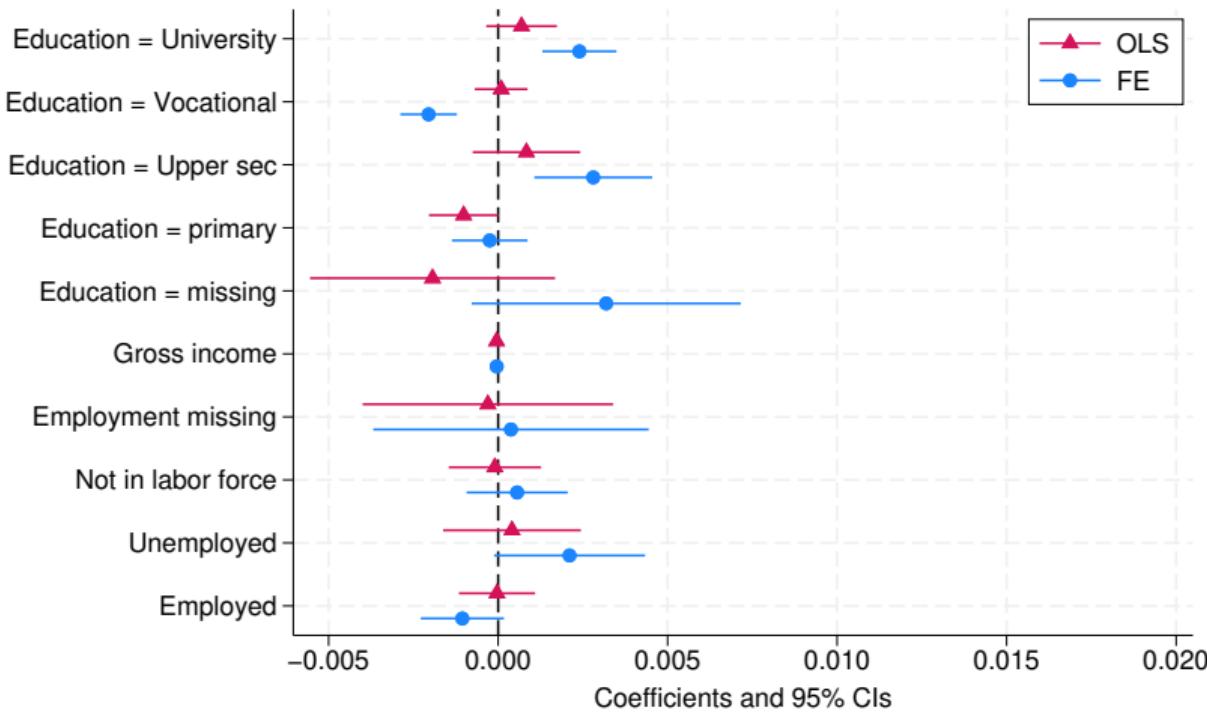
Summary stats by district change

	No District Change	District Change		
	Mean	(SD)	Mean	(SD)
<i>Demographic variables</i>				
Female	0.494	(0.500)	0.497	(0.500)
Non-Western immigrant or descendant	0.074	(0.262)	0.097	(0.296)
Age at test	8.088	(0.283)	8.087	(0.283)
Parents divorced	0.100	(0.300)	0.096	(0.295)
Copenhagen	0.264	(0.441)	0.508	(0.500)
Large city ^a	0.375	(0.484)	0.522	(0.500)
<i>Health at birth</i>				
Birth weight (kg)	3.349	(0.899)	3.296	(0.952)
Low Apgar score (< 10)	0.071	(0.256)	0.068	(0.251)
Apgar score missing	0.043	(0.203)	0.048	(0.213)
Birth weight missing	0.040	(0.195)	0.048	(0.214)
<i>Paternal characteristics</i>				
Employed	0.873	(0.333)	0.863	(0.344)
Unemployed	0.035	(0.183)	0.041	(0.199)
Not in labor force	0.082	(0.274)	0.086	(0.280)
Employment missing	0.010	(0.101)	0.011	(0.102)
Gross income (10,000) ^b	7.145	(7.572)	6.928	(4.838)
Education missing	0.011	(0.103)	0.014	(0.116)
Primary education	0.160	(0.366)	0.157	(0.364)
Upper secondary	0.058	(0.235)	0.073	(0.259)
Vocational	0.604	(0.489)	0.559	(0.497)
University	0.167	(0.373)	0.198	(0.398)

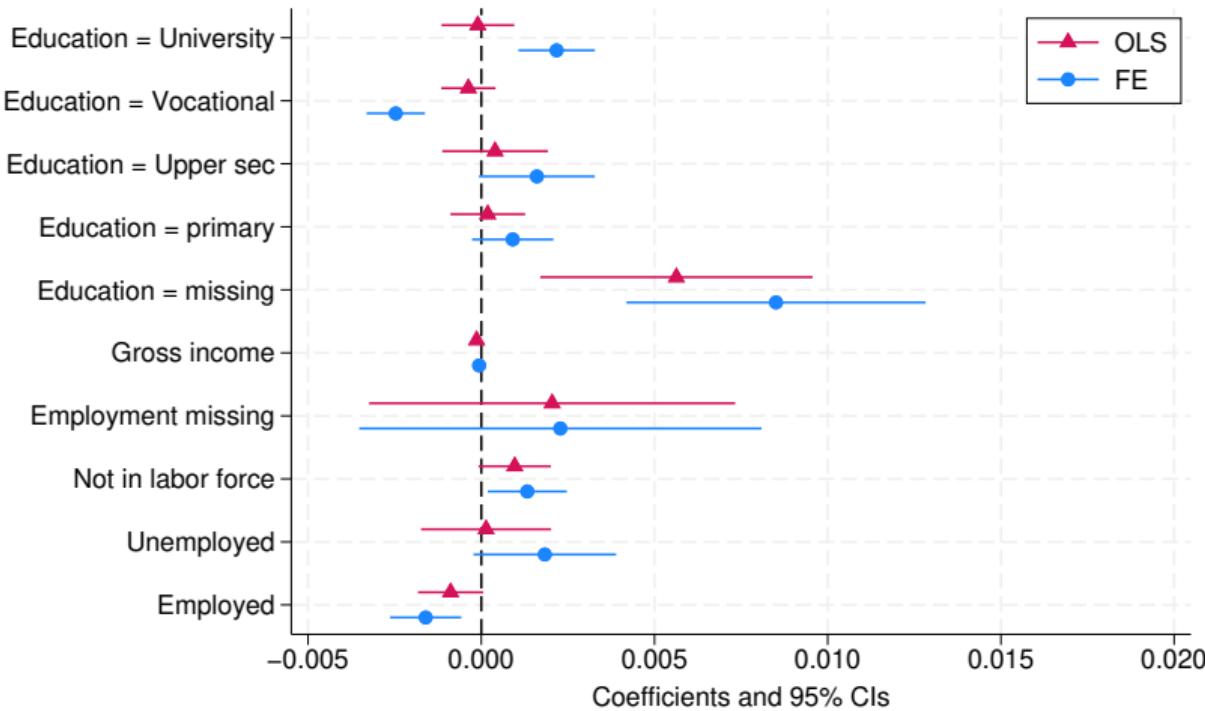
Balance test: Demographics



Balance test: Paternal variables



Balance test: Maternal variables



School district changes by SES

Year	% Any change		% Positive		% Negative	
	Low-SES	High-SES	Low-SES	High-SES	Low-SES	High-SES
2009	1.01	1.18	0.49	0.62	0.52	0.55
2010	0.52	0.76	0.32	0.36	0.21	0.40
2011	1.14	1.58	0.67	0.75	0.47	0.83
2012	1.39	2.20	0.35	0.71	1.04	1.49
2013	0.63	0.28	0.47	0.19	0.17	0.09
2014	0.98	1.63	0.28	0.67	0.70	0.95
2015	1.87	1.30	1.10	0.76	0.76	0.54
Total	1.06	1.29	0.51	0.58	0.55	0.71

FE results: Different SVA measures

	(1) Main results (Jackknife SVA)	(2) Alternative 1 (Chetty et al., 2014)	(3) Alternative 2 (Simple FEs)
SVA	0.026 (0.025)	0.027 (0.026)	0.006 (0.021)
Low-SES	-0.440*** (0.006)	-0.439*** (0.006)	-0.435*** (0.006)
Low-SES × SVA	0.035*** (0.006)	0.030*** (0.006)	0.022*** (0.005)
N(Obs.)	254,397	252,637	254,690
N(Changers)	2,841	2,781	2,879
R ²	0.177	0.177	0.176
Controls	Yes	Yes	Yes

FE results: Different SEs

	(1) Main (pre school)	(2) Pre school $\times t$	(3) Actual school	(4) Actual school $\times t$	(5) Bootstrap
SVA	0.026 (0.025)	0.026 (0.026)	0.026 (0.027)	0.026 (0.027)	0.026 (0.026)
Low-SES	-0.440*** (0.006)	-0.440*** (0.005)	-0.440*** (0.006)	-0.440*** (0.005)	-0.440*** (0.005)
Low-SES \times SVA	0.035*** (0.006)	0.035*** (0.005)	0.035*** (0.006)	0.035*** (0.005)	0.035*** (0.003)
N(Obs.)	254,397	254,397	254,397	254,397	254,402
N(Changers)	2,841	2,841	2,841	2,841	2,841
R ²	0.177	0.177	0.177	0.177	0.075
Controls	Yes	Yes	Yes	Yes	Yes

FE results: Different SES measures (1)

	(1) Main results	(2) SES index < p90	(3) Ses index < p75	(4) Ses index < p50
SVA	0.026 (0.025)	0.022 (0.027)	0.027 (0.026)	0.042 (0.026)
Low-SES	-0.440*** (0.006)	-0.334*** (0.009)	-0.403*** (0.006)	-0.364*** (0.005)
Low-SES × SVA	0.035*** (0.006)	0.035*** (0.008)	0.032*** (0.006)	0.008 (0.006)
N(Obs.)	254,397	254,397	254,397	254,397
N(Changers)	2,841	2,841	2,841	2,841
R ²	0.177	0.156	0.173	0.176
Controls	Yes	Yes	Yes	Yes

FE results: Different SES measures (2)

	(1) Main results	(2) Income < p90	(3) Income < p75	(4) Income < p50	(5) Unemployed
SVA	0.026 (0.025)	0.026 (0.028)	0.036 (0.027)	0.040 (0.026)	0.043 (0.027)
Low-SES	-0.440*** (0.006)	-0.216*** (0.008)	-0.256*** (0.006)	-0.285*** (0.005)	-0.249*** (0.005)
Low-SES × SVA	0.035*** (0.006)	0.028*** (0.008)	0.023*** (0.006)	0.015** (0.005)	0.014** (0.005)
N(Obs.)	254,397	254,397	254,397	254,397	254,397
N(Changers)	2,841	2,841	2,841	2,841	2,841
R ²	0.177	0.151	0.158	0.165	0.158
Controls	Yes	Yes	Yes	Yes	Yes

Robustness check compliance: Same school, first borns

	(1) Main results	(2) Firstborns only	(3) Same school
SVA	0.026 (0.025)	0.039 (0.046)	-0.000 (0.026)
Low-SES	-0.440*** (0.006)	-0.418*** (0.008)	-0.426*** (0.006)
Low-SES × SVA	0.035*** (0.006)	0.024** (0.008)	0.034*** (0.006)
N(Obs.)	254,397	101,537	227,165
N(Changers)	2,841	1,190	2,206
R ²	0.177	0.201	0.182
Controls	Yes	Yes	Yes

Correlation between SVAs

	SVA Fixed Effects	SVA Jackknife	SVA Chetty
SVA Fixed Effects	1.000		
SVA Jackknife	0.486	1.000	
SVA Chetty	0.467	0.947	1.000

Formal Model & Derivations
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Extra results
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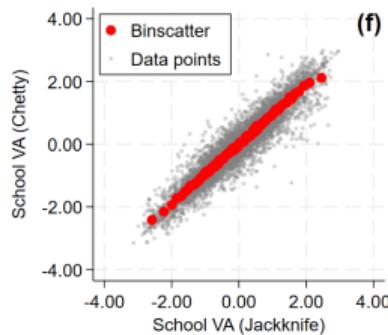
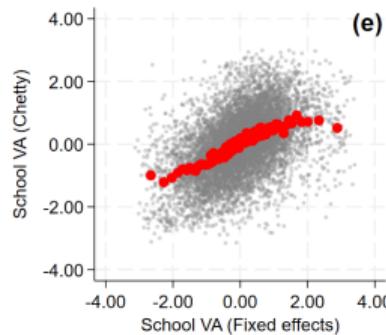
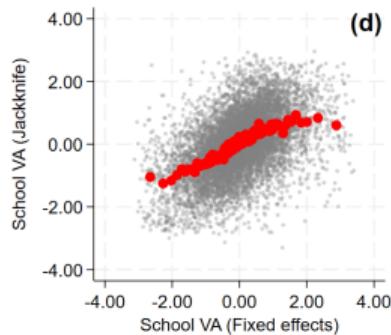
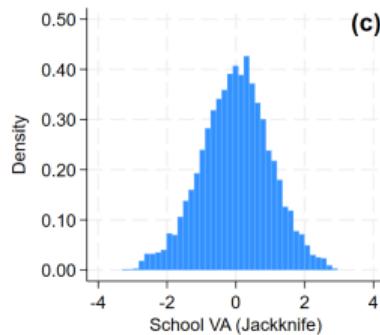
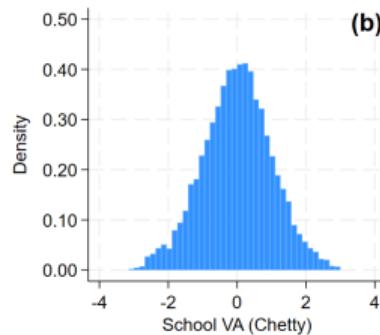
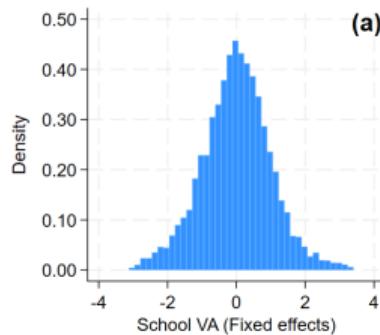
Balancing tests
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Robustness: Measures
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Robustness: Compliance
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Robustness: SVAs
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Comparing SVA measures



Autocorrelations in SVA over time

Table: Correlation between SVA in t and lagged SVA measures.

Variables	SVA in t	$t - 1$	$t - 2$	$t - 3$	$t - 4$	$t - 5$	$t - 6$	$t - 7$	$t - 8$
SVA in t	1.000								
$t - 1$	0.881	1.000							
$t - 2$	0.859	0.895	1.000						
$t - 3$	0.831	0.862	0.897	1.000					
$t - 4$	0.798	0.824	0.860	0.894	1.000				
$t - 5$	0.701	0.791	0.826	0.863	0.892	1.000			
$t - 6$	0.606	0.690	0.788	0.829	0.861	0.886	1.000		
$t - 7$	0.546	0.597	0.694	0.794	0.833	0.862	0.877	1.000	
$t - 8$	0.528	0.550	0.611	0.708	0.810	0.847	0.867	0.858	1.000

Autocorrelation in SVA over time

