Schools and Learning Inequalities

A Theoretical Model of Quantitative and Qualitative School Effects

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Roadmap

Do schools contribute to inequalities in learning outcomes?

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5. School *quality* versus *exposure*

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5. School *quality* versus *exposure*
6. Do our results hold in other contexts?

Schools and Inequality: Two conflicting narratives

Reproduction

- Class analyses and cultural capital
  (Bowles and Gintis, 1976; Bourdieu and Passeron, 1977)
- Differences in school resources
  $\rightarrow$ inequality in outcomes
  (Jennings et al., 2015; Rauscher, 2016)
- Sorting behavior of teachers
  (Kalogrides et al., 2013; Hanselman, 2019)

within and between school differences in **quality**

Investing in schools → less inequality

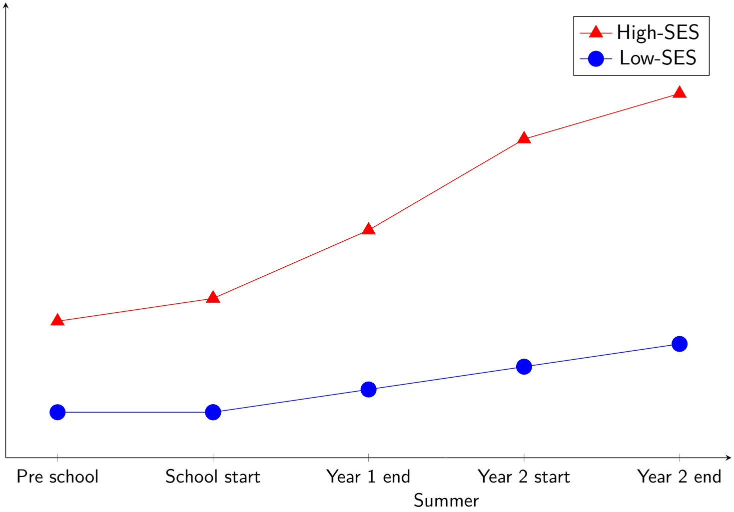
Equalization

- Seasonal comparison studies
  (Alenxander et al., 1992; Downey et al., 2004, 2018)
- Differential exposure approach
  (Passaretta and Skopek, 2021)
- School length/starting age reforms
  (Grätz, 2024)

School **exposures** ineffective because home environments more unequal

Intervening in schools ineffective

Different counterfactuals



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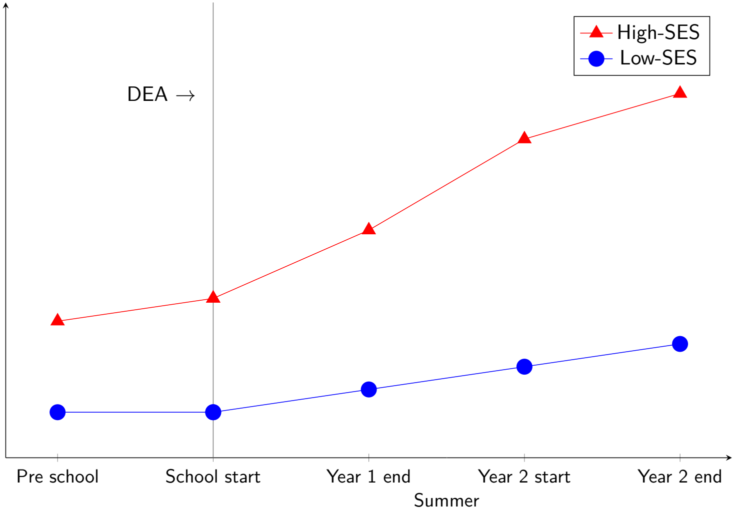

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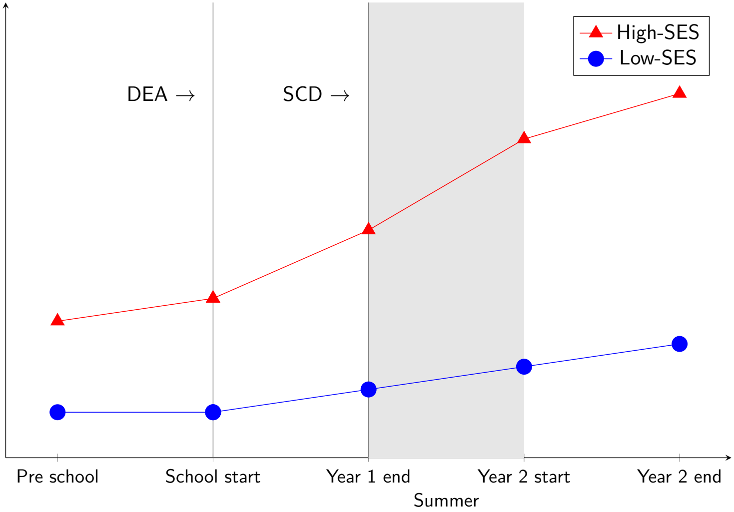

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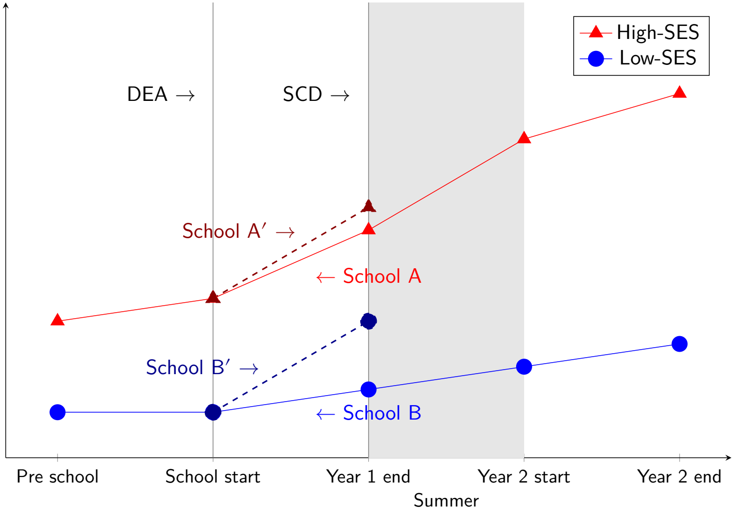

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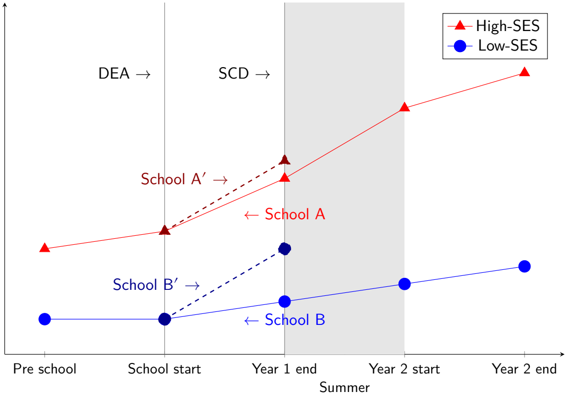

Three **different** counterfactuals

1. DEA: shortening pre-school
2. SCD: Shortening summer break
3. Investing in quality

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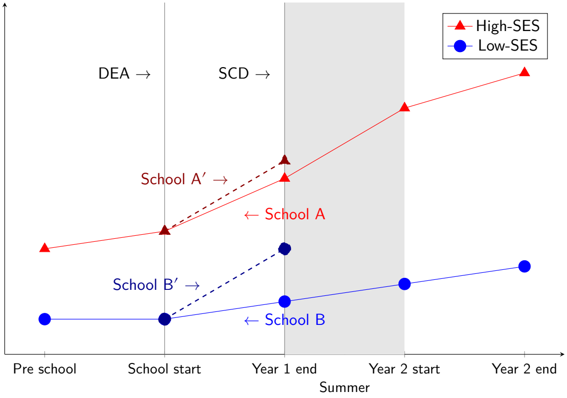

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Quantitative vs *qualitative* effects

Empirical setup

How do schools contribute to learning inequalities?

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- School quality measured as value-added
- Standardized test scores in reading and mathematics
- Detailed information on parents' SES

School quality measure

Standard school value-added model (Koedel et al., 2015)

$$A_{ijt} = \omega A_{ij,t-1} + \mathbf{x}'_{ijt}\beta + \alpha_{jt} + \epsilon_{ijt}$$

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- very high autocorrelation $(\hat{\alpha}_{j,t}, \hat{\alpha}_{j,s>t})$

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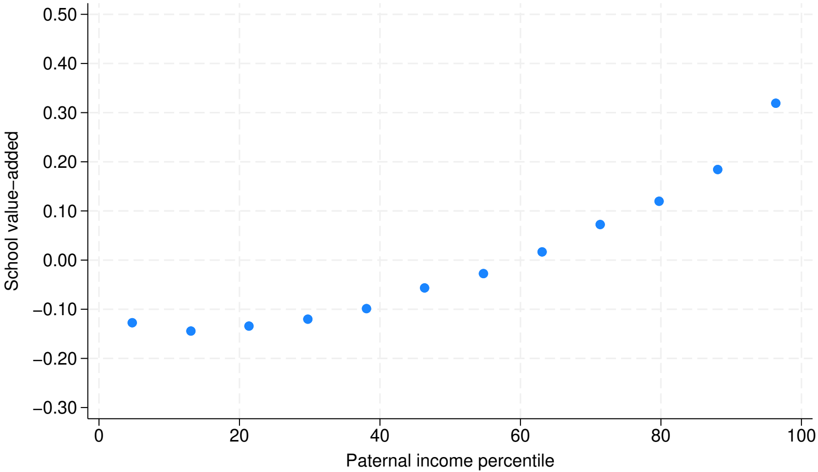
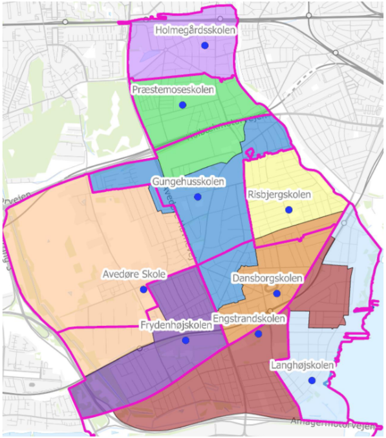


School district changes



- School districts re-drawn every year

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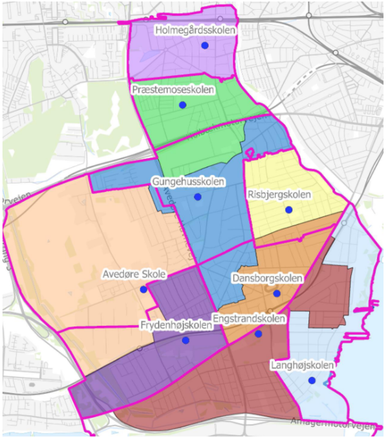

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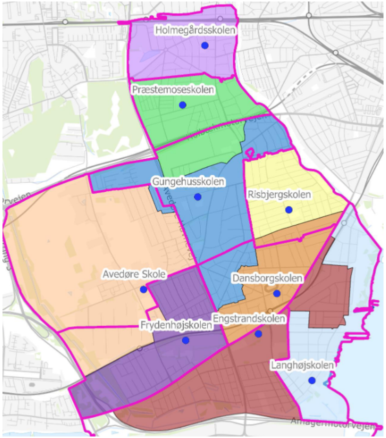

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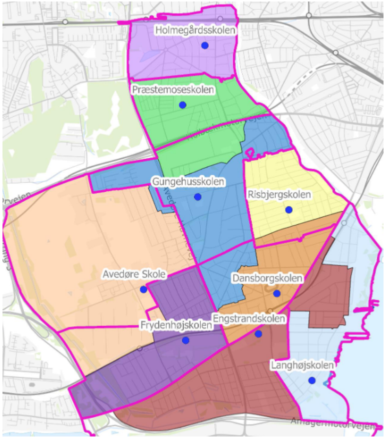

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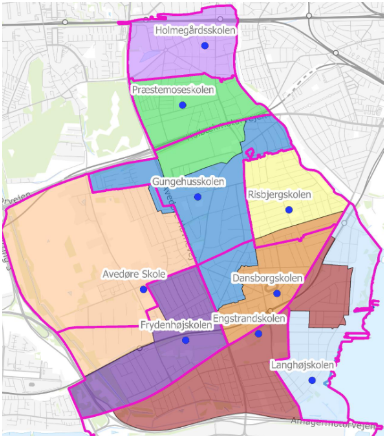
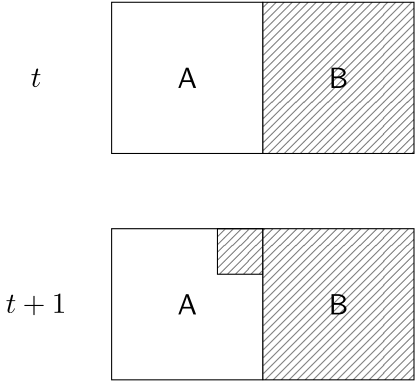


- School districts re-drawn every year
- Exactly 1 school per district
- Determined the year child turns 5
- $\approx 85\%$ attend assigned district school
- Exceptions, outside options, etc. directly observed

School district changes

Year	N(Obs.)	N(Changes)	% Any change	% Positive	% Negative
2009	34,319	358	1.04	0.52	0.53
2010	37,703	218	0.58	0.33	0.25
2011	36,405	452	1.24	0.69	0.55
2012	38,247	609	1.59	0.44	1.15
2013	36,793	201	0.55	0.40	0.15
2014	37,403	429	1.15	0.38	0.76
2015	33,532	574	1.71	1.01	0.70
Total	254,402	2,841	1.12	0.53	0.59

Does school quality impact test scores?



$$y_{ijt} = \rho Q_{jt} + \gamma L_i + \delta(Q_{jt} \times L_i) + \mathbf{x}_i' \beta + (\eta_{j'} \times \lambda_t) + \varepsilon_{ijt}$$

where:

- y_{ijt} student test score in reading (grade 2)
- η_j' old school FE
- Q_{jt} school quality (post-redistricting)
- L_i low-SES dummy (no parent with university degree)

Fixed effects results

	No controls		Controls	
	Coef	(SE)	Coef	(SE)
SVA	0.021	(0.027)	0.026	(0.025)
Low-SES	-0.462^{***}	(0.007)	-0.440^{***}	(0.006)
Low-SES $\times$ SVA	0.037^{***}	(0.007)	0.035^{***}	(0.006)
N (obs.)	254,397			
N (identifying obs.)	2,841			
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Total impact for low-SES $\approx 0.06\sigma$

School quality versus exposure

School quality versus exposure

Let Z_j be a dummy instrument for every school district change $j = 1, \ldots, 81$.

$$Q_{it} = \alpha_0 + \sum_{j=1}^{K} \alpha_j \cdot 1(Z_i = j) + \lambda_{1,t} + e_{it}$$

$$E_{it} = \beta_0 + \sum_{j=1}^{K} \beta_j \cdot 1(Z_i = j) + \lambda_{2,t} + u_{it}$$

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Use to estimate the second stage Eq:

$$y_{it} = \delta_1 \hat{Q}_i + \delta_2 \hat{E}_i + \delta_3 \hat{A}_i + \varphi_t + \epsilon_i$$

School quality versus exposure

	Full Sample	Low-SES	High-SES
	(1)	(2)	(3)
Quality	0.126*	0.103*	0.048
	(0.055)	(0.044)	(0.073)
Exposure	0.036	0.044	−0.010
	(0.096)	(0.081)	(0.089)
Observations	199,690	149,120	50,570

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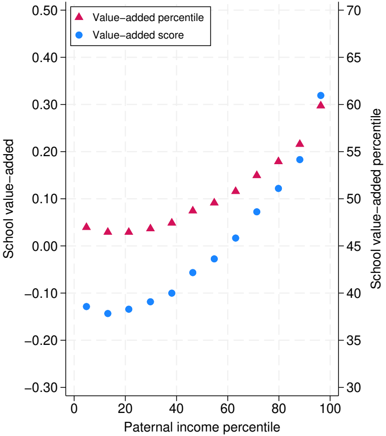
Differential impact for low-SES: $\approx 0.05\sigma$

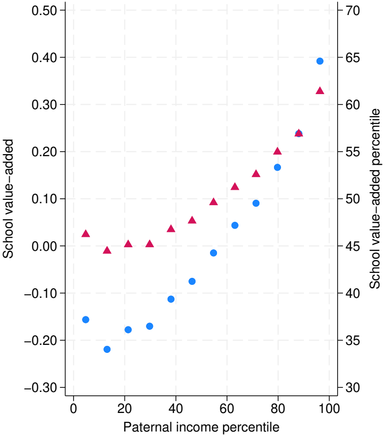
Results for Copenhagen

How would these results look in a more unequal setting?

Results for Copenhagen



(a) Full sample



(b) Copenhagen

Results for Copenhagen

	Full sample		Copenhagen	
	Coef	(SE)	Coef	(SE)
SVA	0.026	(0.025)	0.028	(0.021)
Low-SES	−0.440***	(0.006)	−0.455***	(0.010)
Low-SES × SVA	0.035***	(0.006)	0.044***	(0.009)
N (obs.)	254,397		67,851	
N (identifying obs.)	2,841		1,444	
R^2	0.177		0.189	

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- Total SES gap ≈ 45% SD—similar in US (Reardon, 2016)

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School effectivenss literature:

- 1 year of schooling effect ≈ $^1/_3\sigma$–$^1/_4\sigma$ (Woessmann, 2016)

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School effectivenss literature:

- 1 year of schooling effect $\approx$ $^1/_3\sigma$–$^1/_4\sigma$ (Woessmann, 2016)
- Re-allocating low-SES kids to higher quality schools $\approx$ $^1/_4$ school year

Conclusion

Theoretical contributions

1. Model that makes sense of contradictory findings

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Empirical contributions

4. Large inequalities in quality even in Denmark
5. Quality improvements especially important for low-SES kids
6. Investments in quality have larger returns in unequal settings

Thank You!

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E: Exposure to schooling

Q: Quality of schooling

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$$y = A \cdot Q \cdot E$$

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Simple model of skill formation: $Y = f(E, Q, A)$, where:

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$$y = AQ^{\alpha}E^{\beta}$$

Assumption 1: Marginal diminishing returns

Formal model

Simple model of skill formation: $Y = f(E, Q, A)$, where:

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$$y_L = AQ_L^{\alpha}E^{\beta} \qquad y_H = AQ_H^{\alpha}E^{\beta}$$

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Assumption 2: Selection into school quality (but not exposure)

Assumption 3: Flexibility (differential returns between groups)

Formal model: Quantitative (exposure) effects

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Relative ratio between derivatives:

(Relative gains–inequality of increasing E)

$$\Theta_E = \left(\frac{1}{y_H}\frac{\partial y_H}{\partial E}\right) \Big/ \left(\frac{1}{y_L}\frac{\partial y_L}{\partial E}\right) = \frac{\beta_H}{\beta_L}$$

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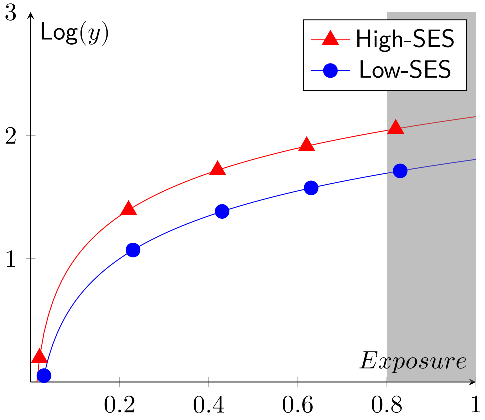
Intuition:

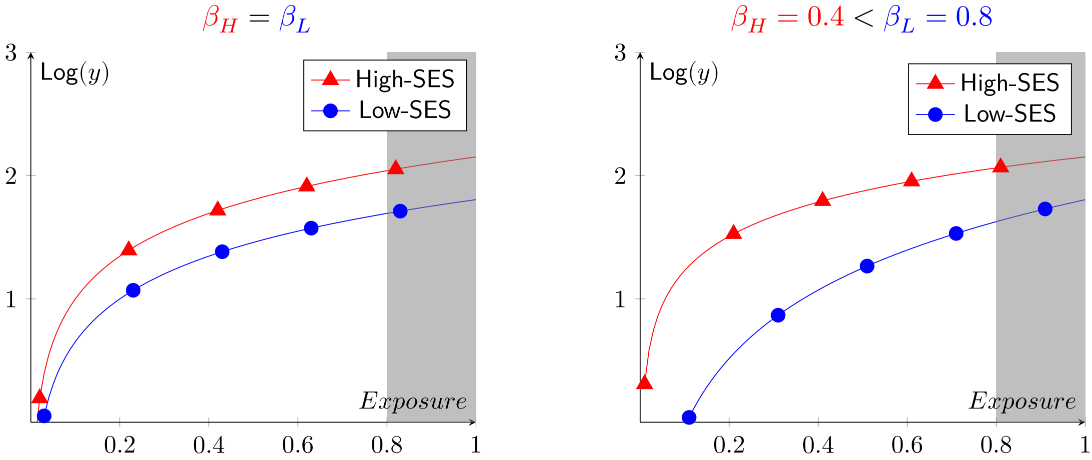
$$\Theta_E = \begin{cases} 1 & \text{if } \beta_H = \beta_L \text{ (Preserves inequality)} \\ < 1 & \text{if } \beta_H < \beta_L \text{ (Reduces inequality)} \\ > 1 & \text{if } \beta_H > \beta_L \text{ (Increases inequality)} \end{cases}$$

Formal model: Quantitative effects

$\beta_H = \beta_L$



Formal model: Quantitative effects

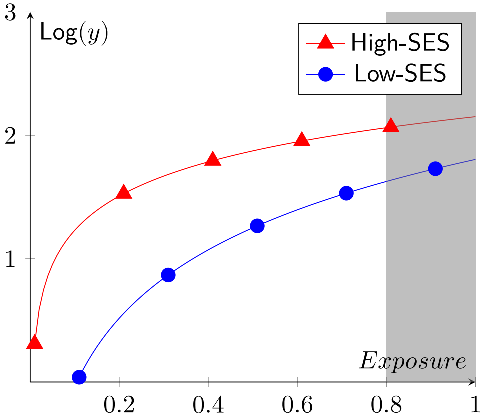


Formal model: Quantitative effects

Takeaways:

- Limited gains to exposure investments even if low-SES children benefit *more* from exposure

$\beta_H = 0.4 < \beta_L = 0.8$

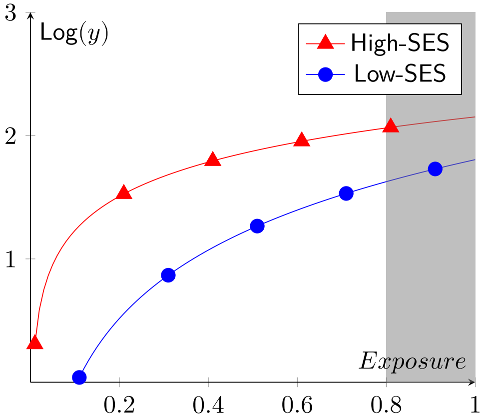


Formal model: Quantitative effects

Takeaways:

- Limited gains to exposure investments even if low-SES children benefit *more* from exposure
- Exposure is hard to scale up!

$\beta_H = 0.4 < \beta_L = 0.8$

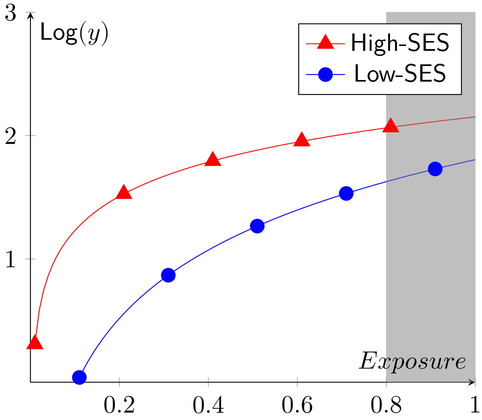


Formal model: Quantitative effects

Takeaways:

- Limited gains to exposure investments even if low-SES children benefit *more* from exposure
- Exposure is hard to scale up!
- Selection into quality ⇒ low-SES children get more low quality schooling vs high-SES children get more high quality schooling

$\beta_H = 0.4 < \beta_L = 0.8$



Formal model: Qualitative effects

High-SES group: $y_H = AQ_H^{\alpha_H} E^{\beta_H}$

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$$\frac{1}{y_H}\frac{\partial Y}{\partial Q} = \frac{A\alpha_H Q_H^{\alpha_H - 1} E^{\beta_H}}{AQ_H^{\alpha_H} E^{\beta_H}}$$

Formal model: Qualitative effects

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Formal model: Qualitative effects

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Low-SES group: $y_L = AQ_L^{\alpha_L} E^{\beta_L}$

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Formal model: Qualitative effects

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$$\frac{1}{y_L}\frac{\partial y_L}{\partial Q} = \frac{A\alpha_L Q_L^{\alpha_L - 1} E^{\beta_L}}{AQ_L^{\alpha_L} E^{\beta_L}}$$

Relative ratio between derivatives:

(Relative gains–inequality of increasing Q)

$$\Theta_Q = \left(\frac{1}{Y_H}\frac{\partial Y_H}{\partial Q}\right) \Big/ \left(\frac{1}{Y_L}\frac{\partial Y_L}{\partial Q}\right) = \frac{\alpha_H}{\alpha_L} \cdot \frac{Q_H}{Q_L}$$

Formal model: Qualitative effects

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Relative ratio between derivatives:

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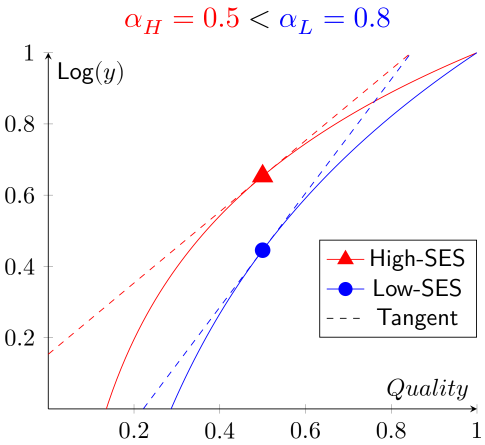
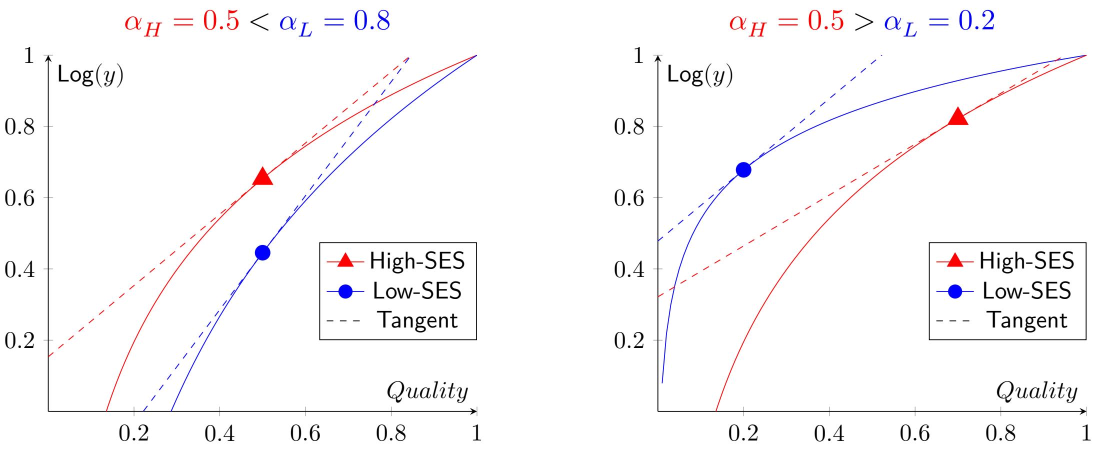
Intuition:

$$\Theta_Q = \begin{cases} 1 & \text{if } \alpha_H = \alpha_L \text{ (Inequality by } Q \text{ only)} \\ < 1 & \text{if } \alpha_H < \alpha_L \text{ (Less inequality at any } Q\text{)} \\ > 1 & \text{if } \alpha_H > \alpha_L \text{ (Less inequality at lower } Q\text{)} \end{cases}$$

Formal model: Qualitative effects



Formal model: Qualitative effects

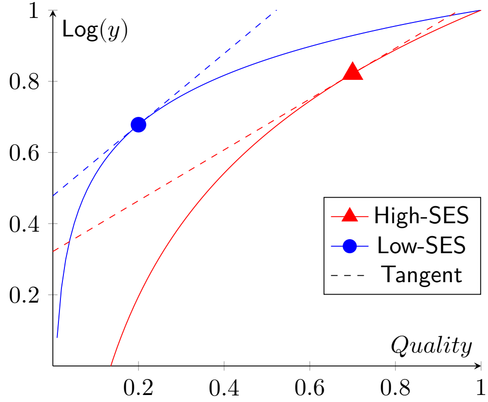


Formal model: Qualitative effects

Takeaways:

- Quality investments more effective at reducing inequality

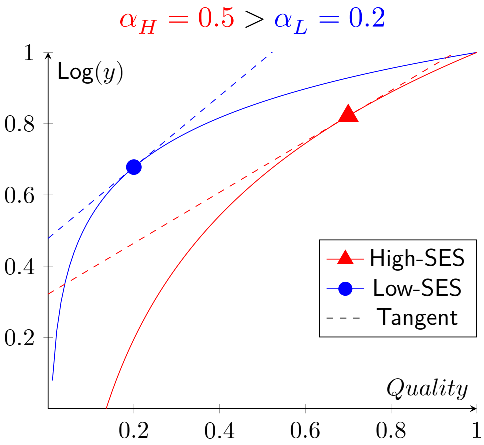
$\alpha_H = 0.5 > \alpha_L = 0.2$



Formal model: Qualitative effects

Takeaways:

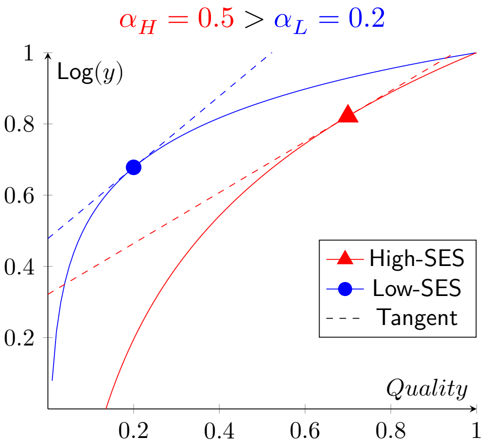
- Quality investments more effective at reducing inequality
- If selection into quality strong, gains even higher among low-SES children



Formal model: Qualitative effects

Takeaways:

- Quality investments more effective at reducing inequality
- If selection into quality strong, gains even higher among low-SES children
- Even if low-SES children have low returns, investing in quality still leads to more equality because of selection



Derivation Θ_Q, Low-SES

We start with the production function for the Low-SES group:

$$y_L = AQ_L^{\alpha_L} E^{\beta_L}$$

Step 1: Compute the Partial Derivative with Respect to Q

$$\frac{\partial y_L}{\partial Q} = \frac{\partial}{\partial Q}\left(AQ_L^{\alpha_L} E^{\beta_L}\right) = A\alpha_L Q_L^{\alpha_L - 1} E^{\beta_L}$$

Step 2: Divide by y_L to Find the Marginal Effect Relative to y_L

$$\frac{1}{y_L}\frac{\partial y_L}{\partial Q} = \frac{A\alpha_L Q_L^{\alpha_L - 1} E^{\beta_L}}{AQ_L^{\alpha_L} E^{\beta_L}} = \frac{\alpha_L}{Q_L}$$

Thus, for the Low-SES group, we have:

$$\frac{1}{y_L}\frac{\partial y_L}{\partial Q} = \frac{\alpha_L}{Q_L}$$

Derivation Θ_Q, High-SES

$$y_H = AQ_H^{\alpha_H} E^{\beta_H}$$

Step 1: Compute the Partial Derivative with Respect to Q

$$\frac{\partial y_H}{\partial Q} = \frac{\partial}{\partial Q}\left(AQ_H^{\alpha_H} E^{\beta_H}\right) = A\alpha_H Q_H^{\alpha_H - 1} E^{\beta_H}$$

Step 2: Divide by y_H to Find the Marginal Effect Relative to y_H

$$\frac{1}{y_H}\frac{\partial y_H}{\partial Q} = \frac{A\alpha_H Q_H^{\alpha_H - 1} E^{\beta_H}}{AQ_H^{\alpha_H} E^{\beta_H}} = \frac{\alpha_H}{Q_H}$$

Thus, for the High-SES group, we have:

$$\frac{1}{y_H}\frac{\partial y_H}{\partial Q} = \frac{\alpha_H}{Q_H}$$

Relative Ratio between Marginal Effects

Now, we form the relative ratio between the derivatives for the High-SES and Low-SES groups:

$$\Theta_Q = \frac{\frac{1}{y_H}\frac{\partial y_H}{\partial Q}}{\frac{1}{y_L}\frac{\partial y_L}{\partial Q}}$$

Substituting the expressions derived for each group:

$$\Theta_Q = \frac{\frac{\alpha_H}{Q_H}}{\frac{\alpha_L}{Q_L}}$$

Simplifying the Ratio

$$\Theta_Q = \frac{\alpha_H}{\alpha_L} \cdot \frac{Q_H}{Q_L}$$

Thus, the relative ratio between the derivatives is:

$$\Theta_Q = \frac{\alpha_H}{\alpha_L} \cdot \frac{Q_H}{Q_L}$$

Theoretical model - assumptions

$$\text{Government problem:} \quad \max_{E,Q} \int Y_1(Q, E, H; \theta) \partial F(\theta) \tag{1}$$

Assume concavity in returns to E and Q:

$$Y_1 = Q^{\alpha_0} E^{\alpha_1} H^{1-\alpha_0-\alpha_1} \theta \tag{2}$$

and a budget constraint:

$$P_E E + P_Q Q \leq G \tag{3}$$

$$\max_{E,Q} \int Q^{\alpha_0} E^{\alpha_1} H^{1-\alpha_0-\alpha_1} \theta \partial F(\theta) \quad \text{s.t.} \quad P_E E + P_Q Q \leq G \tag{4}$$

Theoretical model - Optimal investments

Apply a Lagrange multiplier ($\lambda(\cdot)$) to the problem in (4) and optimize :

$$\max_{E,Q} \int Q^{\alpha_0} E^{\alpha_1} H^{1-\alpha_0-\alpha_1} \theta \partial F(\theta) + \lambda[G - P_E E - P_Q Q] \quad (5)$$

$$\text{w.r.t. to Q:} \quad (6)$$

$$\int \alpha_0 Q^{\alpha_0 - 1} E^{\alpha_1} H^{1-\alpha_0-\alpha_1} \theta \partial F(\theta) - \lambda[P_Q] = 0 \quad (7)$$

$$\text{w.r.t. to E:} \quad (8)$$

$$\int \alpha_1 E^{\alpha_1 - 1} Q^{\alpha_0} H^{1-\alpha_0-\alpha_1} \theta \partial F(\theta) - \lambda(P_E) = 0 \quad (9)$$

So we get:

$$\lambda P_Q = \alpha_0 \, \mathbb{E}_{Q^{-1} Y_1(\theta)} \quad (10)$$

$$\lambda P_E = \alpha_1 \, \mathbb{E}_{E^{-1} Y_1(\theta)} \quad (11)$$

Theoretical model - Optimal investments results 1

1. The optimal investment in E relative to Q is:

$$E = \frac{P_Q \alpha_1}{P_E \alpha_0} Q \tag{12}$$

2. A budget constraint of:

$$
\begin{aligned}
G &= P_Q Q + P_E E && (13)\\
&= P_Q Q + P_E \frac{P_Q \alpha_1}{P_E \alpha_0} Q && (14)\\
&= P_Q Q + \frac{P_Q \alpha_1}{\alpha_0} Q && (15)\\
&= [1 + \frac{\alpha_1}{\alpha_0}] P_Q Q && (16)
\end{aligned}
$$

Theoretical model - Optimal investments results 2

Now re-arrange, to get how much we should spend on quality (here, let Q^* denote the optimal Q):

$$G = [1 + \frac{\alpha_1}{\alpha_0}]P_Q Q \tag{17}$$

$$Q^* = \frac{G}{P_Q}\frac{\alpha_0}{\alpha_0 + \alpha_1} \tag{18}$$

Symmetrically, we can derive the same for the optimal E, E^*:

$$E^* = \frac{G}{P_E}\frac{\alpha_1}{\alpha_0 + \alpha_1} \tag{19}$$

Theoretical model - Experiment 1: Targeted investments

$$\mathbb{E}(Y_1(\theta)) = \mathbb{P}(\theta)Y(\theta) + \mathbb{P}(\theta')Y(\theta') \tag{20}$$

where $\mathbb{P}(\theta)$ is the probability (here, share) of individuals in the population with low initial skills (θ), and $\mathbb{P}(\theta')$ is for high initial skills.

Now consider a policy that increases $Q(\theta)$ $\forall\theta$ holding constant home environment H and exposure to schooling E. This policy satisfies $P_E E + P_Q Q = G$ such that $Q_N = Q + \delta(\theta)$. Here, Q_N denotes the "new" quality level after the investment.

The effect of this policy is given by:

$$Y_1^N(\theta) = \theta(Q + \delta(\theta))^{\alpha_0} E(\theta)^{\alpha_1} H^{1-\alpha_0-\alpha_1} \tag{21}$$

$$Y_1^N(\theta') = \theta'(Q + \delta(\theta'))^{\alpha_0} E(\theta)^{\alpha_1} H^{1-\alpha_0-\alpha_1} \tag{22}$$

$$E(Y_1^N) = \mathbb{P}(\theta) + \mathbb{P}(\theta')Y_1^N(\theta') \tag{23}$$

Theoretical model - Experiment 1: Targeted investments

Now we can ask suppose we want to choose $\mathbb{E}(Y_1^N)$ (the attained skill after the investment) to be larger, how should we choose δ_θ (the amount of additional quality we invest in low-skill individuals) and δ_θ^1 (the amount of additional quality we invest in high-skill individuals):

$$\max_{\{\delta_\theta,\delta_\theta^1\}} \mathbb{E}(Y_1^N) \Leftrightarrow \max_{\{\delta_\theta,\delta_\theta'\}} \mathbb{P}(\theta)Y_1^N(\theta) + \mathbb{P}(\theta')Y_1^N(\theta') \tag{24}$$

The benefit from raising δ_θ for low-skill individuals:

$$\mathbb{P}(\theta)\frac{\partial Y_1^N(\theta)}{\delta_\theta} = P\alpha_0\frac{Y_1^N(\theta)}{Q(\theta)+\delta(\theta)} \tag{25}$$

The benefit from raising $\delta_{\theta'}$ for high skill individuals:

$$(1-\mathbb{P})\alpha_0\frac{Y_1^N(\theta')}{Q(\theta')} + \delta(\theta') \tag{26}$$

Because of concavity of $Y_1(Q)$, then $\forall\theta<\theta'$:

$$\frac{Y_1^N(\theta)}{Q(\theta)+\delta(\theta)} > \frac{Y_1^N(\theta')}{Q(\theta'+\delta(\theta')} \tag{27}$$

Theoretical model - Experiment 2: Indiscriminate investments

I.e., $Q^N = Q + \delta$:

$$\max_{\delta} \mathbb{E}(Y_1) \tag{28}$$

$$\max_{\delta} \mathbb{P}(\theta)Y_1(\theta) + \mathbb{P}(\theta')Y_1(\theta') \tag{29}$$

Then we have:

$$\frac{\mathbb{P}(\theta)Y_1(\theta)}{Q(\theta)+\delta} + \frac{\mathbb{P}(\theta')Y_1(\theta')}{Q(\theta)+\delta'} > 0 \tag{30}$$

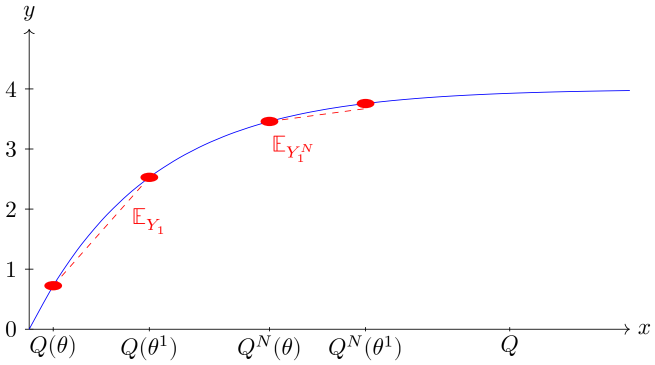
Assuming that

$$\frac{\partial Y}{\partial Q} > 0$$

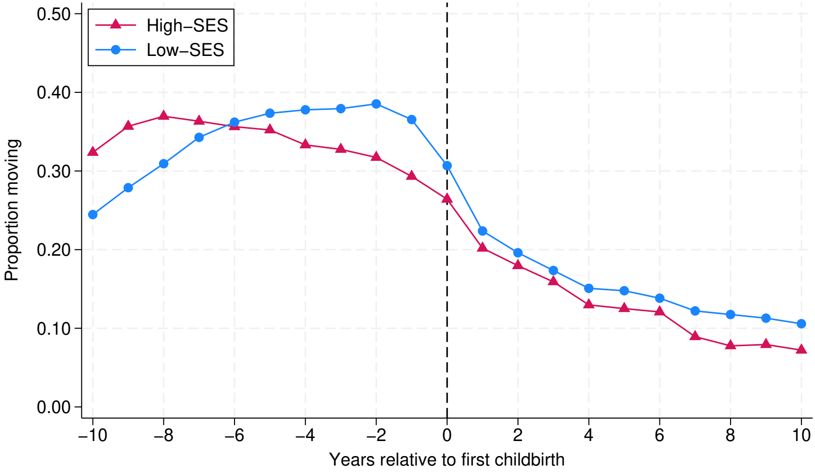
we would rather choose the highest change in quality in order to increase the $\mathbb{E}(Y_1)$, this is because $Y_1(\theta)$ and $Y_1(\theta')$

Theoretical model - Experiment 2: Indiscriminate investments

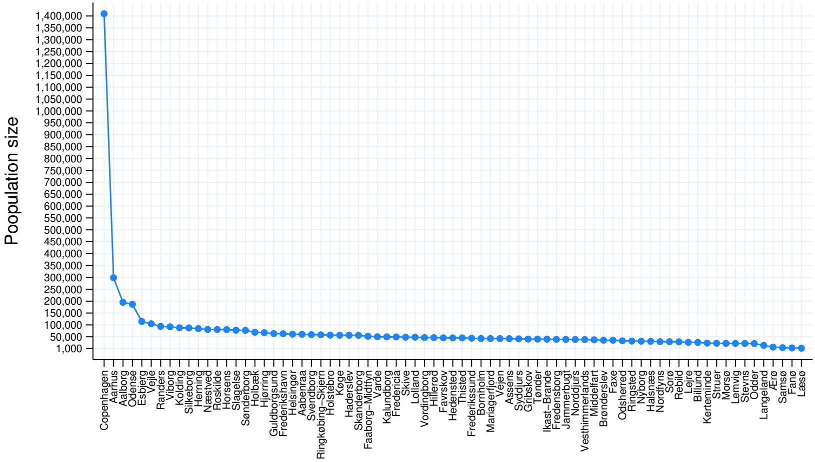
Figure: Illustration



Robustness check: Anticipation of district changes



City sizes


Poopulation size
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1,350,000
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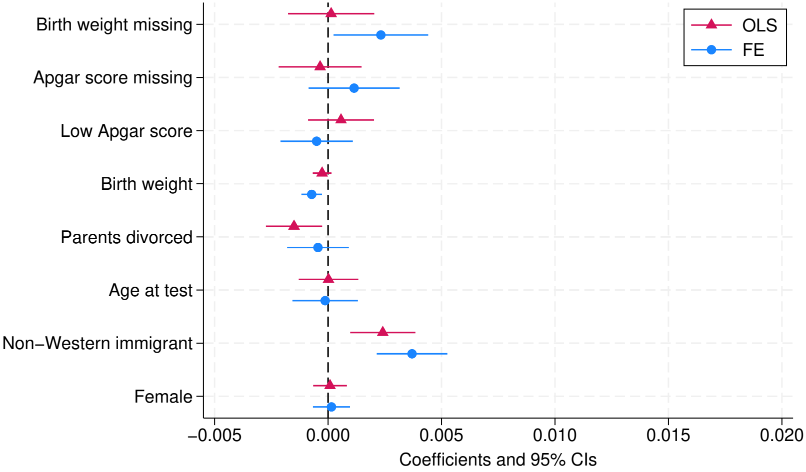
Copenhagen
Aarhus
Aalborg
Odense
Esbjerg
Vejle
Randers
Viborg
Kolding
Silkeborg
Herning
Næstved
Roskilde
Horsens
Slagelse
Sønderborg
Holbæk
Hjørring
Guldborgsund
Frederikshavn
Helsingør
Aabenraa
Svendborg
Ringkøbing–Skjern
Holstebro
Køge
Haderslev
Skanderborg
Faaborg–Midtfyn
Varde
Kalundborg
Fredericia
Skive
Lolland
Vordingborg
Hillerød
Favrskov
Hedensted
Thisted
Frederikssund
Bornholm
Mariagerfjord
Vejen
Assens
Syddjurs
Gribskov
Tønder
Ikast–Brande
Fredensborg
Jammerbugt
Norddjurs
Vesthimmerlands
Middelfart
Brønderslev
Faxe
Odsherred
Ringsted
Nyborg
Halsnæs
Nordfyns
Sorø
Rebild
Lejre
Billund
Kerteminde
Struer
Morsø
Lemvig
Stevns
Odder
Langeland
Ærø
Samsø
Fanø
Læsø

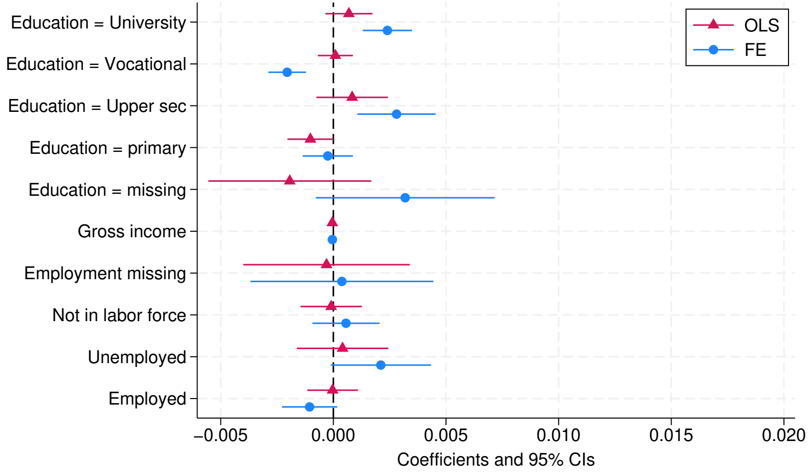
Summary stats by district change

	No District Change		District Change	
	Mean	(SD)	Mean	(SD)
Demograhpic variables				
Female	0.494	(0.500)	0.497	(0.500)
Non-Western immigrant or descendant	0.074	(0.262)	0.097	(0.296)
Age at test	8.088	(0.283)	8.087	(0.283)
Parents divorced	0.100	(0.300)	0.096	(0.295)
Copenhagen	0.264	(0.441)	0.508	(0.500)
Large city[a]	0.375	(0.484)	0.522	(0.500)
Health at birth				
Birth weight (kg)	3.349	(0.899)	3.296	(0.952)
Low Apgar score (< 10)	0.071	(0.256)	0.068	(0.251)
Apgar score missing	0.043	(0.203)	0.048	(0.213)
Birth weight missing	0.040	(0.195)	0.048	(0.214)
Paternal characteristics				
Employed	0.873	(0.333)	0.863	(0.344)
Unemployed	0.035	(0.183)	0.041	(0.199)
Not in labor force	0.082	(0.274)	0.086	(0.280)
Employment missing	0.010	(0.101)	0.011	(0.102)
Gross income (10,000s)[b]	7.145	(7.572)	6.928	(4.838)
Education missing	0.011	(0.103)	0.014	(0.116)
Primary education	0.160	(0.366)	0.157	(0.364)
Upper secondary	0.058	(0.235)	0.073	(0.259)
Vocational	0.604	(0.489)	0.559	(0.497)
University	0.167	(0.373)	0.198	(0.398)

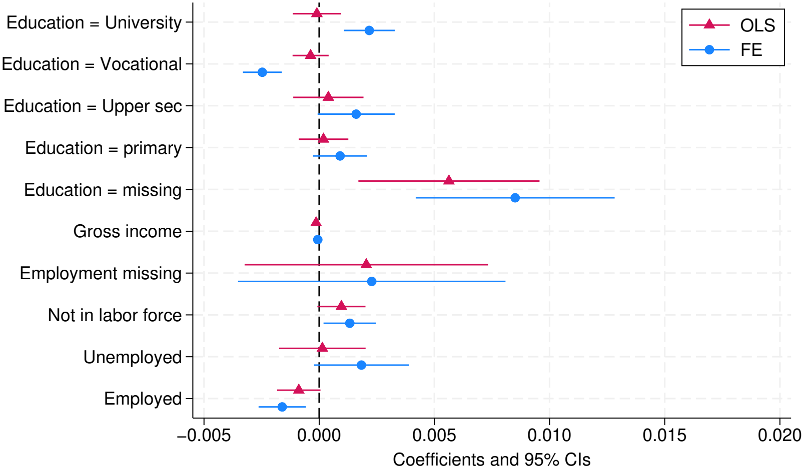
Balance test: Demographics



Balance test: Paternal variables



Balance test: Maternal variables



School district changes by SES

	% Any change		% Positive		% Negative	
Year	Low-SES	High-SES	Low-SES	High-SES	Low-SES	High-SES
2009	1.01	1.18	0.49	0.62	0.52	0.55
2010	0.52	0.76	0.32	0.36	0.21	0.40
2011	1.14	1.58	0.67	0.75	0.47	0.83
2012	1.39	2.20	0.35	0.71	1.04	1.49
2013	0.63	0.28	0.47	0.19	0.17	0.09
2014	0.98	1.63	0.28	0.67	0.70	0.95
2015	1.87	1.30	1.10	0.76	0.76	0.54
Total	1.06	1.29	0.51	0.58	0.55	0.71

FE results: Different SVA measures

	(1) Main results (Jackknife SVA)	(2) Alternative 1 (Chetty et al., 2014)	(3) Alternative 2 (Simple FEs)
SVA	0.026	0.027	0.006
	(0.025)	(0.026)	(0.021)
Low-SES	−0.440***	−0.439***	−0.435***
	(0.006)	(0.006)	(0.006)
Low-SES × SVA	0.035***	0.030***	0.022***
	(0.006)	(0.006)	(0.005)
N(Obs.)	254,397	252,637	254,690
N(Changers)	2,841	2,781	2,879
R^2	0.177	0.177	0.176
Controls	Yes	Yes	Yes

FE results: Different SEs

	(1) Main (pre school)	(2) Pre school $\times$ t	(3) Actual school	(4) Actual school $\times$ t	(5) Bootstrap
SVA	0.026	0.026	0.026	0.026	0.026
	(0.025)	(0.026)	(0.027)	(0.027)	(0.026)
Low-SES	-0.440^{***}	-0.440^{***}	-0.440^{***}	-0.440^{***}	-0.440^{***}
	(0.006)	(0.005)	(0.006)	(0.005)	(0.005)
Low-SES $\times$ SVA	0.035^{***}	0.035^{***}	0.035^{***}	0.035^{***}	0.035^{***}
	(0.006)	(0.005)	(0.006)	(0.005)	(0.003)
N(Obs.)	$254,397$	$254,397$	$254,397$	$254,397$	$254,402$
N(Changers)	$2,841$	$2,841$	$2,841$	$2,841$	$2,841$
R^2	0.177	0.177	0.177	0.177	0.075
Controls	Yes	Yes	Yes	Yes	Yes

FE results: Different SES measures (1)

	(1) Main results	(2) SES index $<$ p90	(3) Ses index $<$ p75	(4) Ses index $<$ p50
SVA	0.026	0.022	0.027	0.042
	(0.025)	(0.027)	(0.026)	(0.026)
Low-SES	-0.440^{***}	-0.334^{***}	-0.403^{***}	-0.364^{***}
	(0.006)	(0.009)	(0.006)	(0.005)
Low-SES $\times$ SVA	0.035^{***}	0.035^{***}	0.032^{***}	0.008
	(0.006)	(0.008)	(0.006)	(0.006)
N(Obs.)	$254,397$	$254,397$	$254,397$	$254,397$
N(Changers)	$2,841$	$2,841$	$2,841$	$2,841$
R^2	0.177	0.156	0.173	0.176
Controls	Yes	Yes	Yes	Yes

FE results: Different SES measures (2)

	(1) Main results	(2) Income < p90	(3) Income < p75	(4) Income < p50	(5) Unemployed
SVA	0.026	0.026	0.036	0.040	0.043
	(0.025)	(0.028)	(0.027)	(0.026)	(0.027)
Low-SES	−0.440***	−0.216***	−0.256***	−0.285***	−0.249***
	(0.006)	(0.008)	(0.006)	(0.005)	(0.005)
Low-SES × SVA	0.035***	0.028***	0.023***	0.015**	0.014**
	(0.006)	(0.008)	(0.006)	(0.005)	(0.005)
N(Obs.)	254,397	254,397	254,397	254,397	254,397
N(Changers)	2,841	2,841	2,841	2,841	2,841
R^2	0.177	0.151	0.158	0.165	0.158
Controls	Yes	Yes	Yes	Yes	Yes

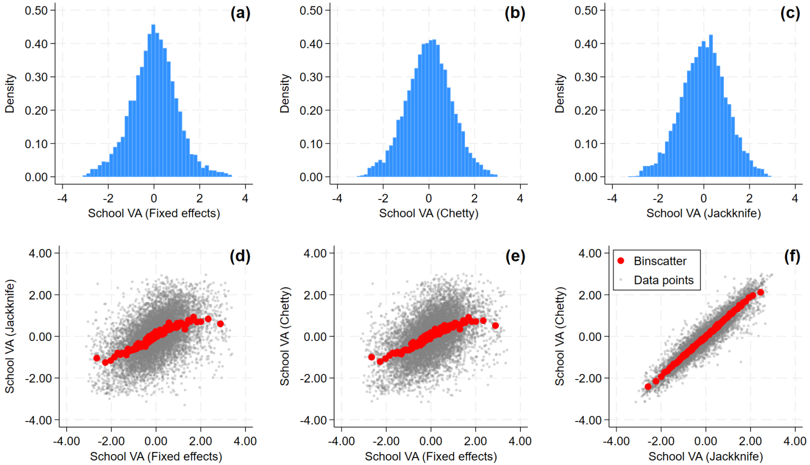
Robustness check compliance: Same school, first borns

	(1) Main results	(2) Firstborns only	(3) Same school
SVA	0.026	0.039	-0.000
	(0.025)	(0.046)	(0.026)
Low-SES	-0.440^{***}	-0.418^{***}	-0.426^{***}
	(0.006)	(0.008)	(0.006)
Low-SES $\times$ SVA	0.035^{***}	0.024^{**}	0.034^{***}
	(0.006)	(0.008)	(0.006)
N(Obs.)	$254,397$	$101,537$	$227,165$
N(Changers)	$2,841$	$1,190$	$2,206$
R^2	0.177	0.201	0.182
Controls	Yes	Yes	Yes

Correlation between SVAs

	SVA Fixed Effects	SVA Jackknife	SVA Chetty
SVA Fixed Effects	1.000		
SVA Jackknife	0.486	1.000	
SVA Chetty	0.467	0.947	1.000

Comparing SVA measures



Autocorrelations in SVA over time

Table: Correlation between SVA in t and lagged SVA measures.

Variables	SVA in t	t -1	t -2	t -3	t -4	t -5	t -6	t -7	t -8
SVA in t	1.000								
$t-1$	0.881	1.000							
$t-2$	0.859	0.895	1.000						
$t-3$	0.831	0.862	0.897	1.000					
$t-4$	0.798	0.824	0.860	0.894	1.000				
$t-5$	0.701	0.791	0.826	0.863	0.892	1.000			
$t-6$	0.606	0.690	0.788	0.829	0.861	0.886	1.000		
$t-7$	0.546	0.597	0.694	0.794	0.833	0.862	0.877	1.000	
$t-8$	0.528	0.550	0.611	0.708	0.810	0.847	0.867	0.858	1.000

Autocorrelation in SVA over time