

Optimization Techniques

Laboratory 4

Variable neighborhood search

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0-1 Knapsack Problem

Assume

N = number of items

$X = \{x_1, x_2, \dots, x_N\}$ = set of items

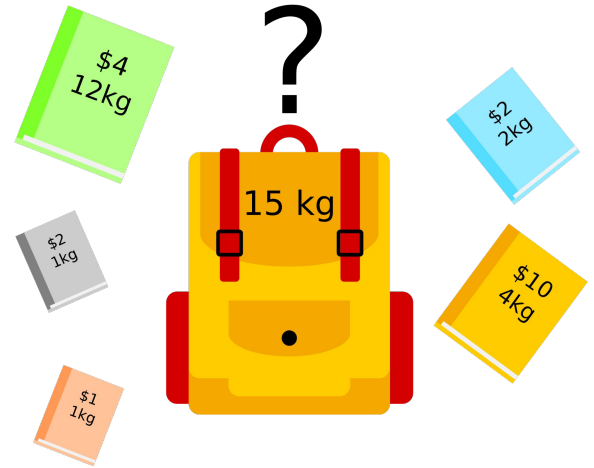
v_i = value of item x_i

w_i = weight of item x_i

W = maximum weight the knapsack can hold

Then the 0/1 knapsack problem can be formulated as follows:

$$\begin{array}{ll}\text{maximize} & \sum_{x_i \in X} v_i x_i \\ \text{subject to} & \sum_{x_i \in X} w_i x_i \leq W \\ & x_i \in \{0, 1\} \forall x_i \in X\end{array}$$



Variable Neighborhood Search

Systematically change the neighbourhood during search

Procedure Basic VNS

```
select  $\{N_k\}$ ,  $k = 1, \dots, k_{\max}$ 
find an initial solution  $x$ 
choose a stopping condition
repeat until stopping condition is met:
     $k \leftarrow 1$ 
    repeat until  $k = k_{\max}$ 
         $x' \leftarrow \text{RandomSolution}(N_k(x))$ 
         $x'' \leftarrow \text{LocalSearch}(x')$ 
        if  $f(x'') < f(x)$  then
             $x \leftarrow x''$ 
             $k \leftarrow 1$ 
        else
             $k \leftarrow k + 1$ 
```

End

Function BestImprovement(x)

```
1 repeat
2    $x' \leftarrow x$ 
3    $x \leftarrow \arg \min_{y \in N(x)} f(y)$ 
   until  $(f(x) \geq f(x'))$ 
return  $x$ 
```

Function FirstImprovement(x)

```
1 repeat
2    $x' \leftarrow x$ ;  $i \leftarrow 0$ 
3   repeat
4      $i \leftarrow i + 1$ 
5      $x \leftarrow \arg \min \{f(x), f(x^i)\}, x^i \in N(x)$ 
   until  $(f(x) < f(x') \text{ or } i = |N(x)|)$ 
until  $(f(x) \geq f(x'))$ 
return  $x$ 
```

Reduces VNS (RVNS)

No local search to save time

Random solutions are generated from increasingly larger neighborhoods of x

- Can converge way faster
- Suitable when local search is expensive

Procedure Reduced VNS

```
select  $\{N_k\}$ ,  $k = 1, \dots, k_{\max}$   
find an initial solution  $x$   
choose a stopping condition  
 $k \leftarrow 1$   
repeat until  $k = k_{\max}$   
   $x' \leftarrow \text{RandomSolution}(N_k(x))$   
  if  $f(x') < f(x)$  then  
     $x \leftarrow x'$   
     $k \leftarrow 1$   
  else  
     $k \leftarrow k + 1$ 
```

End