Monetary Policy with Near-Rational Expectations in Open Economies[*]

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Abstract

We investigate robustly optimal monetary policy in an open economy where private agents' expectations are boundedly rational. The theoretical framework incorporates policymakers' concerns regarding potentially distorted private expectations into a standard small open economy New Keynesian environment. The model predicts that following a cost-push shock, the optimal monetary policy calls for a slower response of domestic inflation and greater reaction in the initial response of the nominal exchange rate, as the central bank's concerns about distorted expectations increase. We develop an algorithm to implement Bayesian inference using macroeconomic time series on Canada and Mexico and estimate the degree of distorted expectations from the rational expectations (RE) benchmark. Mexico exhibits a significant deviation from RE, whereas Canada shows a small deviation. The model with distorted expectations substantially outperforms the RE model for Mexico. It successfully predicts the historical path of the monetary policy rate and the high persistence of the inflation rate, demonstrating that robustly optimal monetary policy causes inertia in the inflation rate.

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1 Introduction

There is growing consensus on widespread bounded rationality in economic agents' decision-making from various survey data and laboratory experiments. Investigating the macroeconomic consequences of boundedly rational agents has become a popular research area in macroeconomic theory. One of the critical developments in the research area is introduced in Hansen and Sargent (2008), which relaxed the assumption of a unique probability model for decision-makers. They formulate the departure of the model from the rational expectations (RE) benchmark as an unknown probabilistic feature of uncertainty, motivated by the concept of *ambiguity* in Gilboa and Schmeidler (1989). Under the potential threat of a malevolent probabilistic environment under uncertainty about the model, decision-makers employ a robust control – the minimax optimization procedure – to minimize the worst-case outcome.

Woodford (2010) extended the idea of Hansen and Sargent (2008) to design Ramsey-optimal monetary policy in a standard closed-economy New Keynesian model, but private agents exhibit near-rational expectations (NRE). The NRE is potentially distorted from the RE with a given radius of relative entropy. Woodford (2010) finds that when the central bank's concerns regarding the worst-case outcome (caused by the NRE) increase, the robustly optimal monetary policy calls for a more conservative policy in the sense that the policy becomes more history-dependent than that in the RE benchmark.

In this paper, we study the robustly optimal monetary policy in an open economy context. The model environment is a standard small open economy New Keynesian model as in Gali and Monacelli (2005). However, the private agents exhibit potentially distorted expectations (NRE), and the central bank is the Ramsey planner as in Woodford (2010). We confirm that as the central bank has more serious concerns about NRE, the optimal monetary policy in the open economy also becomes more history-dependent. When a cost-push shock - which creates non-trivial monetary policy trade-offs between domestic inflation and output gap - hits the economy, the optimal monetary policy demands a more sluggish reaction of domestic inflation and a more aggressive initial response of the nominal exchange rate, as the central bank's concerns about NRE increase.

In addition, we estimate the RE and NRE models both using time series data on inflation rates and nominal devaluation rates for Canada and Mexico. We construct an algorithm to estimate the robust Ramsey equilibrium via the likelihood-based Bayesian method. We find that there are stark differences between the estimated NRE models in the two countries. The NRE model shows minor deviations from the RE benchmark in Canada, whereas the NRE model in Mexico shows a substantial departure from its RE benchmark. More interestingly, the NRE model in Mexico successfully predicts the actual path of the monetary policy rate in the data, while the RE model completely fails to predict it. The central insight from the result is that the NRE model for Mexico successfully matches the high persistence of the inflation rate, whereas the RE model does not. In the NRE model, the substantial deviation from the RE model motivates the central bank to adopt a more history-dependent monetary policy, generating strong inertia in the inflation dynamics.

This paper contributes to two strands of the literature. First, this paper studies robustly optimal monetary policy in an open economy context. There is a vast body of work that investigates monetary policy in a New Keynesian environment subject to Hansen and Sargent (2008)'s type of ambiguity, in both a closed-economy (see, for example, Leitemo and Soderstrom (2008b), Levine and Pearlman (2010), Dennis (2010), and Gerke and Hammermann (2016), and many others) and an open-economy case (see, for example, Dennis et al. (2009) and Leitemo and Soderstrom (2008a)). However, most of the works assume that the central bank lacks the commitment device and monetary policy cannot be Ramsey optimal per se. Hansen and Sargent (2012) and Kwon and Miao (2017) studied Ramsey policy problems with commitment under various types of ambiguity, but their applications are closed-economy cases. Up to our knowledge, there is a lack of knowledge about the features of Ramsey optimal monetary policy in an open economy when it is subject to Woodford (2010)'s type of ambiguity. This paper attempts to fill this gap first.

Second, this paper measures the deviation from the RE benchmark by estimating the degree of NRE. Several papers estimate the deviation in various environments of boundedly rational agents. Ilut and Schneider (2014) also applied the Bayesian method to estimate the household's ambiguity aversion, whereas the monetary policy follows the standard Taylor rule. Bhandari et al. (2019) estimated private agents' time-varying subjective beliefs using the survey data. Gust et al. (2020) applied the Bayesian method to estimate the averaged planning horizon of a New Keynesian environment with agents of limited foresight. The information and policy structure of our paper essentially differs from those in these works. Our private agents are not averse to ambiguity, but the central bank is averse to a class of private agents' NRE when it designs the Ramsey-optimal monetary policy. To the best of our knowledge, our paper is also the first attempt to estimate the degree of NRE in Woodford (2010)'s type of ambiguity. The optimal monetary policy is assumed to have a conditionally linear form, making the Ramsey economy summarized as a system of linear difference equations. Thus, we can apply standard techniques for estimating linear state-space models. However, the estimation of the system of the robust-Ramsey equilibrium is computationally demanding. The policy needs to be computed for every draw in the Markov Chain in the estimation procedure. This paper constructs and executes the estimation algorithm.

The remainder of the paper is organized as follows. Section 2 describes a small open economy model under near-rational private agents' expectations and robust-Ramsey optimal monetary policy with commitment. Section 3 investigates normative features of the robust-Ramsey monetary policy. Section 4 estimates the models using the Bayesian method and discusses key results. Section 5 concludes the paper.

2 Model

In this section, we incorporate distorted expectations of private agents into the canonical small open economy New Keynesian model of Gali and Monacelli (2005). As is common in the New Keynesian literature, we further introduce a cost-push shock into the model and consider Ramsey-optimal monetary policy responses. We begin by describing the belief structure of the private agents in the model.

2.1 Uncertainty and Beliefs

The uncertainty of the economy is defined as the set of vectors of an exogenous stochastic disturbance process $\{\boldsymbol{\varepsilon}^t\}_{t=0}^{\infty}$. The probability triple $(\boldsymbol{\Omega}, \boldsymbol{\mathcal{F}}, \boldsymbol{\mathcal{P}})$, where $\boldsymbol{\Omega}$ is the sample space, $\boldsymbol{\mathcal{F}}$ is the σ-Field of the sample space, and $\boldsymbol{\mathcal{P}}$ is the probability measure, defines the probability space of the exogenous process.

The RE operator is denoted by $\mathbb{E}[\cdot]$, which is induced by measure $\boldsymbol{\mathcal{P}}$. This is the *rational* measure for the probability of states of the economy. The private agents' expectations, on the other hand, are not necessarily rational in the sense that their expectation operator is induced by a potentially different measure $\hat{\boldsymbol{\mathcal{P}}}$. However, we impose the restriction that the measure $\hat{\boldsymbol{\mathcal{P}}}$ is absolutely continuous with respect to the measure $\boldsymbol{\mathcal{P}}$. Let $p(\boldsymbol{\varepsilon})$ denote the unconditional probability density of $\boldsymbol{\varepsilon}$ from measure $\boldsymbol{\mathcal{P}}$, where $\boldsymbol{\varepsilon}$ is a dummy random vector with the same dimension as the number of entries of $\boldsymbol{\varepsilon}_t$. Let $\hat{p}\left(\boldsymbol{\varepsilon}|\boldsymbol{\varepsilon}^t\right)$ denote the one-step-ahead probability density for $\boldsymbol{\varepsilon}_{t+1}$, induced by measure $\hat{\boldsymbol{\mathcal{P}}}$, conditioned on date-$t$ information. The likelihood ratio between the two densities is

$$m_{t+1} = \frac{\hat{p}\left(\boldsymbol{\varepsilon}|\boldsymbol{\varepsilon}^t\right)}{p(\boldsymbol{\varepsilon})},$$

and m_{t+1} is nonnegative and

$$\mathbb{E}\left[m_{t+1}|\boldsymbol{\varepsilon}^t\right] = 1. \tag{1}$$

Following Hansen and Sargent (2008), we set $\mathcal{M}_0 = 1$ and recursively construct $\{\mathcal{M}_t\}$ such that

$$\mathcal{M}_{t+1} = m_{t+1}\mathcal{M}_t,$$

which implies

$$\mathcal{M}_{t+j} = \prod_{k=1}^{j} m_{t+k},$$

and it is a martingale process, which satisfies

$$\mathbb{E}[\mathcal{M}_{t+j}|\boldsymbol{\varepsilon}^t] = \mathcal{M}_t.$$

The Radon-Nikodym theorem indicates that private agents' expectation $\hat{\mathbb{E}}[\cdot]$ of a random variable X_{t+j} induced by measure $\hat{\boldsymbol{\mathcal{P}}}$ conditioned on date-t information can be expressed as the expectation induced by the measure $\boldsymbol{\mathcal{P}}$ for the augmented random variable X_{t+j}:

$$\hat{\mathbb{E}}[X_{t+j}|\boldsymbol{\varepsilon}^t] = \mathbb{E}\left[\frac{\mathcal{M}_{t+j}}{\mathcal{M}_t}X_{t+j}\Big|\boldsymbol{\varepsilon}^t\right],$$

where $\frac{\mathcal{M}_{t+j}}{\mathcal{M}_t}$ represents the Radon-Nikodym derivatives, which completely summarize the belief distortions. Then one-step-ahead expectation of the random variable X_{t+1} induced by measure $\hat{\boldsymbol{\mathcal{P}}}$ is expressed as follows:

$$\hat{\mathbb{E}}[X_{t+1}|\boldsymbol{\varepsilon}^t] = \mathbb{E}[m_{t+1}X_{t+1}|\boldsymbol{\varepsilon}^t].$$

Henceforth, we simply express an expectation based on date-t information as the expectation with subscript t, i.e., $\hat{\mathbb{E}}_t X_{t+1} \equiv \hat{\mathbb{E}}[X_{t+1}|\boldsymbol{\varepsilon}^t]$ and $\mathbb{E}_t X_{t+1} \equiv \mathbb{E}[X_{t+1}|\boldsymbol{\varepsilon}^t]$.

We borrow the concept of relative entropy (also known as Kullback-Leibler divergence) from Hansen and Sargent (2008) to measure the distance between two probability measures $\hat{\boldsymbol{\mathcal{P}}}$ and $\boldsymbol{\mathcal{P}}$. The distance of one-period-ahead distorted beliefs from the rational belief is summarized by the following relative entropy:

$$\mathcal{R}_t \equiv \mathbb{E}_t m_{t+1} \ln m_{t+1},$$

which is always nonnegative by Gibb's inequality.

In the next subsections, we will use the operator $\hat{\mathbb{E}}[\cdot]$ to represent the private sector's distorted expectations.

2.2 A Small Open Economy Environment

Now, we consider a small open economy New Keynesian environment developed in Gali and Monacelli (2005) and assume that the agents' expectations are potentially distorted and a cost-push shock exists. The aggregate demand of the economy can be represented as the following equation:

$$x_t = \hat{\mathbb{E}}_t x_{t+1} - \frac{1}{\sigma_\alpha}\left(i_t - \hat{\mathbb{E}}_t \pi_{H,t+1} - \bar{r}r_t\right), \tag{2}$$

where x_t is the domestic output gap defined by actual output minus the natural level of output under flexible prices and $\pi_{H,t}$ is the net inflation rate for the domestically produced good. The variable i_t is the monetary policy rate determined by the policymaker. The parameter σ_α is defined as:

$$\sigma_\alpha \equiv \frac{\sigma}{1+\alpha(\omega-1)},$$

where $\sigma > 0$ governs the elasticity of intertemporal substitution in the private agents' utility functions, $\alpha \in [0,1]$ governs the degree of home bias, and ω governs the effect of changes in the *terms of trade* on output. The variable $\bar{rr}_t$ is the natural interest rate:

$$\bar{rr}_t = \rho + \Lambda_{r,a} a_t + \Lambda_{r,y^*} y_t^*, \tag{3}$$

where a_t denotes the domestic productivity shock and y_t^* is the world output shock. Parameter $\rho = -\log\beta$, and $\Lambda_{r,a}$ and Λ_{r,y^*} are coefficients that are functions of the structural parameters. A detailed description of the two coefficients is in Appendix B.

The aggregate supply of the economy is represented as the following equation:

$$\pi_{H,t} = \beta \hat{\mathbb{E}}_t \pi_{H,t+1} + \kappa x_t + u_t, \tag{4}$$

where $\beta \in (0,1)$ refers to the subjective discount factor. The parameter $\kappa > 0$ is defined as follows:

$$\kappa \equiv \frac{(1-\zeta)(1-\zeta\beta)(\sigma_\alpha+\varphi)}{\zeta},$$

where u_t is the cost-push shock that shifts the aggregate supply curve, ζ is the Calvo-Yun parameter for nominal price rigidity, and φ is the inverse of the Frisch-elasticity of labor supply.

The three shocks described above are assumed to follow the following first-order Markov processes:

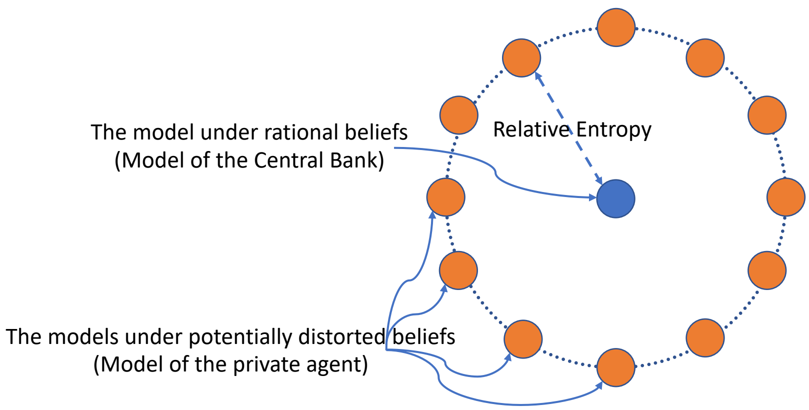
$$u_{t+1} = \rho_u u_t + \sigma_u \epsilon_{t+1}^u, \tag{5}$$

$$a_{t+1} = \rho_a a_t + \sigma_a \epsilon_{t+1}^a, \tag{6}$$

$$y_{t+1}^* = \rho_{y^*} y_t^* + \sigma_{y^*} \epsilon_{t+1}^y, \tag{7}$$

where parameter $\rho_i, i \in \{a, u, y^*\}$ governs the persistence, $\sigma_i, i \in \{a, u, y^*\}$ governs the standard deviation of each stochastic process, and $\varepsilon_{t+1}^i, i \in \{a, u, y^*\}$ follows i.i.d standard normal distribution.

Figure 1: The Entropy Diagram



2.3 A Robust Ramsey Monetary Policy under Near-Rational Expectations of Private Agents

As the Ramsey planner of the economy, the central bank conducts optimal monetary policy using the nominal interest rate i_t as the policy instrument. However, the optimal policy problem differs from the standard Ramsey-optimal policy problem since the belief structures of the central bank and private agents are heterogenous. As is assumed in Woodford (2010), we focus on the case in which, unlike the private agents' potentially distorted beliefs, the central bank has the rational belief. Figure 1 illustrates the environment featuring a central bank and private agents with heterogeneous belief structures. In the figure, each ball refers to a model with distinct beliefs. The blue ball refers to the central bank's model, and the orange balls refer to the set of private agents' models. The set of private agents' models with potentially distorted expectations are around the central bank's model with RE, with the distance reflecting the relative entropy. The private agents have no concerns about their models when they make decisions. The central bank, on the other hand, is concerned about the private agents' potentially distorted expectations when it designs the optimal monetary policy. However, the central bank does not have further information on the private agents' expectations beyond the fact that they are potentially distorted. The decision-making environment under heterogenous beliefs is consistent with the "Type-III" ambiguity aversion or concerns for robustness in Hansen and Sargent (2012).

Throughout this paper, we use the term 'concerns about distorted expectations' when we refer to the central bank's concerns regarding the potentially distorted expectations of private agents.

Under the belief structure, the central bank seeks to minimize the private agents' welfare loss with a *paternalistic* objective (the welfare loss under rational belief) and with ambiguity aversion, which is introduced in Woodford (2010) and Adam and Woodford (2012). The lifetime welfare loss function can be written as follows:

$$\mathbb{L} = \underbrace{\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \frac{1}{2} \left(\pi_{H,t}^2 + \lambda_x \left(x_t - \bar{x} \right)^2 \right)}_{\text{the term of paternalistic welfare loss}} - \underbrace{\theta \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t m_{t+1} \ln m_{t+1}}_{\text{the term of concerns for robustness}} ,$$

where the first term in parentheses on the right-hand side represents the (discounted) lifetime welfare loss of private agents under RE and the second term represents the concerns about distorted expectations. The parameter $\lambda_x > 0$ captures the weight of the welfare loss from the deviation of the output gap from its target value, $\bar{x}$, and $\theta \in (0, \infty)$ is related to central bank's concerns about distorted expectations. The welfare loss is a convex function with respect to $\pi_{H,t}, x_t$, but it is a concave function with respect to m_{t+1}.

Given its concerns about distorted expectations, the central bank attempts to minimize the welfare loss under the *worst-case* outcomes caused by the potentially distorted belief m_{t+1}. Following the framework of Woodford (2010), we assume that a hypothetically malevolent nature chooses $\{m_{t+1}\}$ to maximize the welfare loss, and given this, the policymaker chooses the best responses of $\{\pi_t, x_t, i_t\}$ to minimize the loss. The minimax problem can be summarized as follows:

$$\min_{\{\pi_{H,t}, x_t, i_t\}} \max_{\{m_{t+1}\}} \mathbb{L}, \tag{8}$$

subject to equations (1), (2), (3), (4), (5), (6), and (7). Note that when $\theta \to \infty$, the optimal behavior for malevolent nature to maximize the welfare loss is to choose $m_{t+1} = 1$, which is the rational belief. When θ is a small number, on the other hand, the optimal choice of m_{t+1} causes a substantial departure from the rational belief. In this sense, θ^{-1} can be interpreted as the degree of concerns about distorted expectations.

As a Ramsey planner, the central bank commits to its past policy promises, which implies a history-dependent policy. The Ramsey problem consists of choosing a sequence of potentially time-varying $\{\pi_{H,t}, x_t\}_{t=0}^{\infty}$, which generally creates nonlinearity in the system. Instead, we employ the conditionally linear and self-consistent commitment policy developed in Woodford (2010) to maintain the linear system under commitment. Under conditionally linear commitment, solving the problem reduces to choosing a sequence of $\{\pi_{H,t}, x_t\}_{t=1}^{\infty}$ while taking the initial commitment $\{\pi_{H,0}, x_0\}$ as given. We suppose that the initial commitment takes linear form:

$$\begin{bmatrix} \pi_{H,0} \\ x_0 \end{bmatrix} = \mathbf{\Phi}_{-1} + \mathbf{\Gamma}_{-1} \boldsymbol{\varepsilon}_0,$$

where $\mathbf{\Phi}_{-1}$ and $\mathbf{\Gamma}_{-1}$ complete an initial commitment. Then, we focus on the following conditionally linear rule:

$$\begin{bmatrix} \pi_{H,t+1} \\ x_{t+1} \end{bmatrix} = \mathbf{\Phi}_t + \mathbf{\Gamma}_t \boldsymbol{\varepsilon}_{t+1},\ t \geq 0, \tag{9}$$

where $\mathbf{\Phi}_t$ is stochastic and $\mathbf{\Gamma}_t$ is deterministic. For any initial commitment $\{\mathbf{\Phi}_{-1}, \mathbf{\Gamma}_{-1}\}$, the central bank chooses $\{\mathbf{\Phi}_t^0, \mathbf{\Gamma}_t\}_{t=0}^{\infty}$. The initial commitment is self-consistent if

$$\mathbf{\Phi}_{-1} \equiv^d \mathbf{\Phi}_t \sim \mathbf{\Phi}, \quad \forall t \geq 0, \tag{10}$$

$$\mathbf{\Gamma}_{-1} = \mathbf{\Gamma}_t = \mathbf{\Gamma}, \quad \forall t \geq 0, \tag{11}$$

which means that $\mathbf{\Phi}_{-1}$ and $\{\mathbf{\Phi}_t\}_{t=0}^{\infty}$ follow the same unconditional distribution, and $\mathbf{\Gamma}_{-1}$ and $\{\mathbf{\Gamma}_t\}_{t=0}^{\infty}$ are the same deterministic matrices.

To compute the robust Ramsey equilibrium under the commitment device, we follow the solution method in Kwon and Miao (2019), which generalizes the approach in Woodford (2010). The system of equations for the linearized equilibrium can be rewritten in the following form:

$$\begin{bmatrix} \boldsymbol{X}_{t+1} \\ \hat{\mathbb{E}}_t \boldsymbol{Y}_{t+1} \end{bmatrix} = \boldsymbol{A} \begin{bmatrix} \boldsymbol{X}_t \\ \boldsymbol{Y}_t \end{bmatrix} + \boldsymbol{B} i_t + \boldsymbol{C} \boldsymbol{\varepsilon}_{t+1} \tag{12}$$

where $\boldsymbol{X}_t \equiv [1, u_t, a_t, y_t^*]'$ is a vector of exogenous variables, $\boldsymbol{Y}_t \equiv [\pi_{H,t}, x_t]'$ is a vector of domestic inflation and the output gap which are non-predetermined variables, i_t is the nominal interest rate, which is the Ramsey policy instrument, and $\boldsymbol{\varepsilon}_{t+1} \equiv [\varepsilon_{t+1}^u, \varepsilon_{t+1}^a, \varepsilon_{t+1}^y]'$ is a vector of shocks to the exogenous variables.

The minimax problem in (8) can be rewritten as the problem of choosing $\{\mathbf{\Phi}_t, \mathbf{\Gamma}_t\}_{t\geq 0}$, $\{m_t\}_{t\geq 1}$, $\{i_t\}_{t\geq 0}$ for given initial commitment $(\mathbf{\Phi}_{-1}, \mathbf{\Gamma}_{-1})$. The Lagrangian of the problem is given as follows:

$$\mathcal{L} = \mathbb{E}_{-1} \sum_{t=0}^{\infty} \beta^t \Bigg\{ \frac{1}{2} \left(\pi_{H,t}^2 + \lambda_x (x_t - \bar{x})^2 \right) - \theta m_{t+1} \ln m_{t+1} + \phi_t (\mathbb{E}_t m_{t+1} - 1)$$

$$+ [\boldsymbol{\mu}_{x,t+1}', \boldsymbol{\mu}_{y,t}'] \left(\begin{bmatrix} \boldsymbol{X}_{t+1} \\ \mathbb{E}_t m_{t+1} \boldsymbol{Y}_{t+1} \end{bmatrix} - \boldsymbol{A} \begin{bmatrix} \boldsymbol{X}_t \\ \boldsymbol{Y}_t \end{bmatrix} - \boldsymbol{B} i_t \right) - \boldsymbol{C} \boldsymbol{\varepsilon}_{t+1} \Bigg\},$$

where $\beta^t \phi_t$ is the Lagrange multiplier for the constraint (1) and $\beta^t [\boldsymbol{\mu}_{x,t+1}, \boldsymbol{\mu}_{y,t}]$ are vectors of the Lagrange multiplier for the system of equations (2), (3), (4), (5), (6), and (7).

Solving this problem consists of the following steps. First, the hypothetical malevolent nature chooses m_{t+1} to maximize the loss, which gives the solution for the worst-case belief m_{t+1}. Next, the policymaker chooses $\{\boldsymbol{X}_t, \boldsymbol{\Phi}_t, \boldsymbol{\Gamma}_t, i_t\}$ after substituting for the chosen solution of m_{t+1} in the Lagrangian. Since we are interested in self-consistent policy, we additionally impose $\boldsymbol{\Gamma}_t = \boldsymbol{\Gamma}$. To obtain the solution for Γ, we start with a guess for $\boldsymbol{\Gamma}$ and solve the first-order conditions except for the first-order condition for i_t. The solution is then used to obtain a new value of $\boldsymbol{\Gamma}$ from the first-order condition for i_t. This process will be repeated until we obtain convergence in the value of $\boldsymbol{\Gamma}$. We solve the resulting system of linear differential equations using Klein (2000)'s method. The entire procedure to solve the model is described in Appendix A.

The solution of the system takes the following state-space form of the law of motion of the state variables:

$$\begin{bmatrix} \boldsymbol{\varepsilon}_{t+1} \\ \boldsymbol{X}_{t+1} \\ \boldsymbol{\Phi}_t \end{bmatrix} = \boldsymbol{H} \begin{bmatrix} \boldsymbol{\varepsilon}_t \\ \boldsymbol{X}_t \\ \boldsymbol{\Phi}_{t-1} \end{bmatrix} + \begin{bmatrix} \boldsymbol{I} \\ \boldsymbol{C}_x \\ \boldsymbol{0} \end{bmatrix} \boldsymbol{\varepsilon}_{t+1}, \tag{13}$$

and the policy rules for the non-predetermined variables:

$$\begin{bmatrix} i_t \\ \boldsymbol{\mu}_{y,t} \\ \boldsymbol{\mu}_{x,t} \\ \mathbb{E}_t \boldsymbol{\mu}_{x,t+1} \end{bmatrix} = \boldsymbol{G} \begin{bmatrix} \boldsymbol{\varepsilon}_t \\ \boldsymbol{X}_t \\ \boldsymbol{\Phi}_{t-1} \end{bmatrix}. \tag{14}$$

3 Normative Analyses of Optimal Monetary Policy

In this section, we investigate normative features of the robust Ramsey-optimal monetary policies. Specifically, we investigate the optimal policy coefficient of the domestic inflation and the output gap in equation (11), and impulse response functions of macroeconomic indicators to exogenous shocks, subject to various degrees of concern about distorted expectations and various structural parameters. Thereafter, we will frequently use the acronym 'RE' when we refer to the model under $\theta^{-1} = 0$.

We found that regardless of any $\theta \geq 0$, the Ramsey optimal policy calls for zero domestic inflation and output gap in response to domestic productivity shock a_t and world output shock y_t^*, i.e., $\Gamma_{\pi,a} = \Gamma_{x,a} = \Gamma_{\pi,y^*} = \Gamma_{x,y^*} = 0$. The reason is because the natural interest rate $\bar{rr}_t$ in (3) compromises the two shocks. The shock in the natural interest rate shifts the aggregate demand equation (2). Still, it can be perfectly stabilized by the Ramsey optimal policy regardless of the potentially distorted expectation. The insight is that there are no trade-offs between stabilizing domestic inflation and the output gap. Thus both can be simultaneously stabilized, which is also known as the divine coincidence. From the observation, we will only focus on the policy coefficients and dynamics of variables in response to cost-push shock for the next subsections. The Ramsey optimal dynamics in response to domestic productivity and world output shocks are exhibited in Appendix C.

3.1 Features of the Policy Coefficients

Figure 2 shows the Ramsey optimal policy coefficients under various standard deviations of the cost-push shock $\sigma_u \in [0, 0.05]$, persistence of the shock $\rho_u \in [0, 1)$, and degrees of concern about distorted expectations $\theta^{-1} \in \{0, 100, 500\}$. For the other parameters, we set $\sigma = 1$, $\omega = 1$, $\varphi = 3$, $\beta = 0.99$, $\theta = 0.75$, and $\alpha = 0.4$ following benchmark calibration in Gali and Monacelli (2005).

The upper panels exhibits the optimal policy coefficients for domestic inflation, $\Gamma_{\pi,u}$. When $\theta^{-1} = 0$, the private expectations are fully rational. In that case, for any given level of persistence ρ_u, the Ramsey policy coefficient $\Gamma_{\pi,u}$ is an increasing linear function of the standard deviation of the cost-push shock, σ_u, which implies that the certainty equivalence principle holds in the optimal monetary policy.

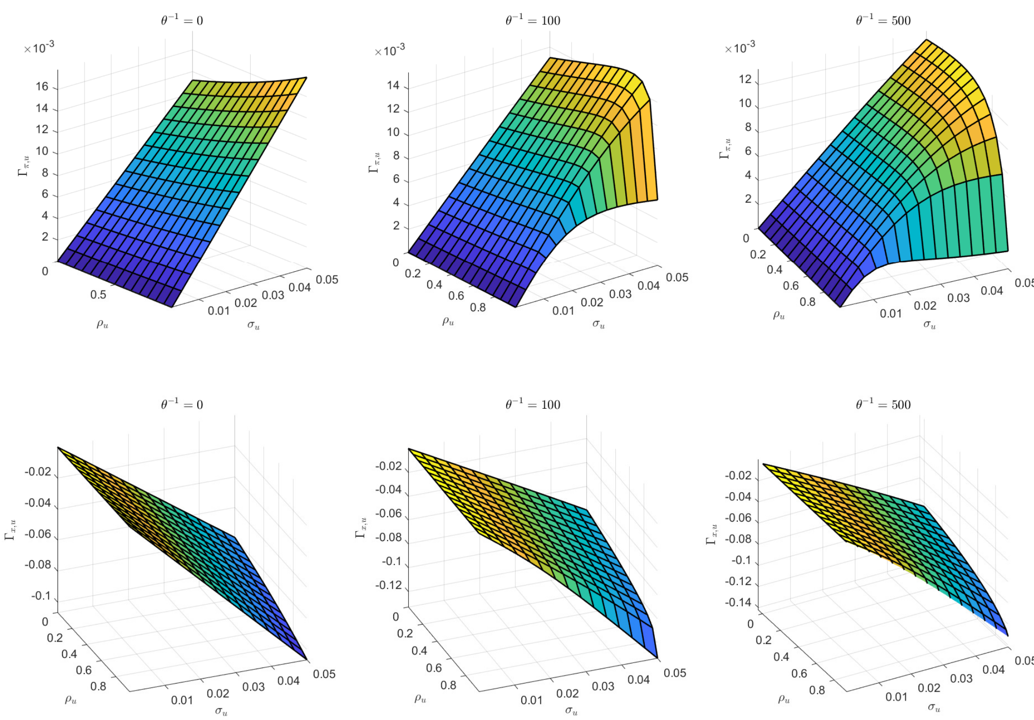
If $\theta^{-1} > 0$, however, the central bank is concerned about distorted expectations since the private expectations now potentially deviate from RE. As discussed in Woodford (2010), its concern causes a breakdown of the certainty equivalence principle in the optimal monetary policy. For example, when $\theta^{-1} = 100$, $\Gamma_{\pi,u}$ is an increasing but concave function of σ_u. This implies that as σ_u increases, the central bank becomes more conservative in responding to $\Gamma_{\pi,u}$ because of its concerns about distorted expectations. Furthermore, as the persistence of the cost-push shock ρ_u increases, the decrease in the rate of the increase in $\Gamma_{\pi,u}$ to σ_u increases, which implies that the central bank becomes increasingly conservative in responding $\Gamma_{\pi,u}$ when the persistence of the shock increases. Remarkably, when $\theta^{-1} = 500$, $\Gamma_{\pi,u}$ begins to *decrease* after a certain point σ_u when ρ_u is sufficiently high. The central bank now decides to decrease its response to $\Gamma_{\pi,u}$ given its serious concerns about distorted expectations.

The lower panels in Figure 2 exhibits the $\Gamma_{x,u}$ under various σ_u and ρ_u. The aggregate supply equation (4) implies that given $x_{H,t}, \hat{E}_t\pi_{H,t+1}$, and u_t, the equilibrium output gap x_t must be determined as follows:

$$x_t = \frac{\pi_{H,t} - \beta\hat{\mathbb{E}}_t\pi_{H,t+1} - u_t}{\kappa}, \tag{15}$$

which indicate that the optimal policy coefficient for the response of the output gap $\Gamma_{x,u}$ depends on the gap between the current domestic inflation and the distorted expectation for the future inflation, $\pi_{H,t} - \beta\mathbb{E}_t\pi_{H,t+1}$, the cost-push shock, u_t, and the slope of the aggregate supply curve κ. Like in the case of $\Gamma_{\pi,u}$, when $\theta^{-1} = 0$, the equilibrium is RE. There is the certainty equivalence of the Ramsey policy coefficients. At any level of the persistence ρ_u, $\Gamma_{x,u}$ is a linear function of σ_u. And it is a decreasing function of σ_u, since an increase of σ_u increases the size of u_t when

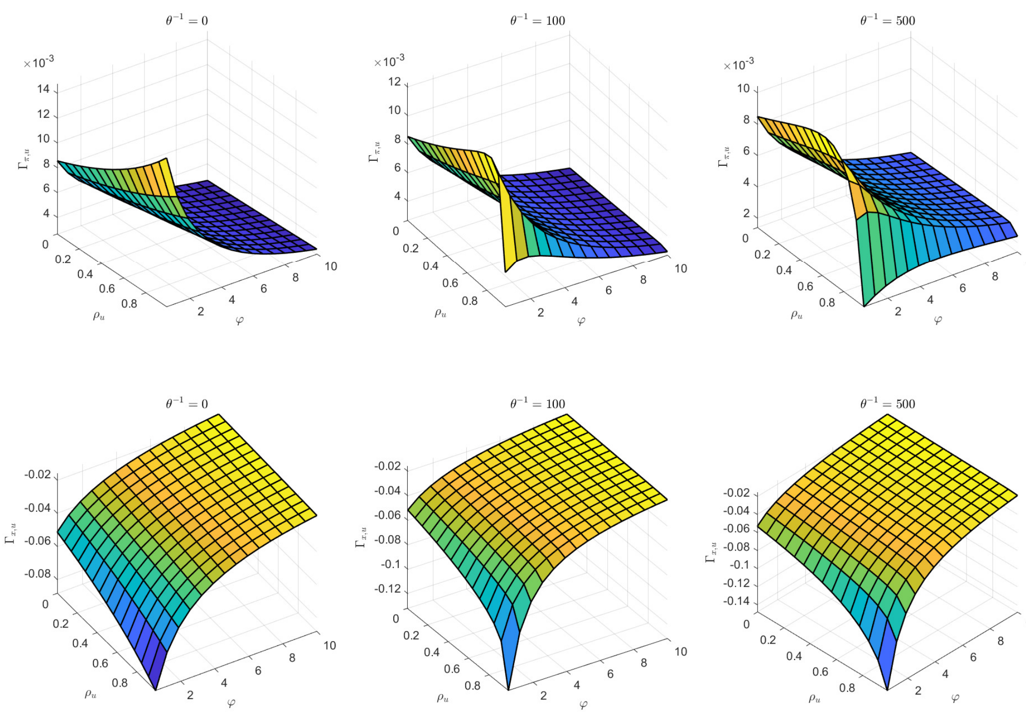
Figure 2: Robust Ramsey-Optimal Policy Coefficients with Various σ_u and ρ_u



the shock occurs, and it negatively affects x_t by the Phillips curve relation (15). When $\theta^{-1} > 0$, the concerns about distorted expectations comes in. The $\Gamma_{x,u}$ is not a linear function of σ_u, and as ρ_u gets higher, the $\Gamma_{x,u}$ decreases further than it in the case of the certainty equivalence. The mechanism is as follows. As we observed in the upper panels, as σ_u and ρ_u increase, the motivations for the conservative response of $\Gamma\pi, u$ gets stronger. The motivation is to give a signal to private agents that the central bank doesn't have plans to change domestic inflation that much so that to maintain the gap $\pi_{H,t} - \beta\hat{\mathbb{E}}_t\pi_{H,t+1}$ to a small and stable level. As ρ_u and σ_u get higher and higher, the central bank wants to get $\pi_{H,t} - \beta\hat{\mathbb{E}}_t\pi_{H,t+1}$ smaller and smaller. With the equation (15), these result in the decrease of $\Gamma_{x,u}$ further than the case of RE. When $\theta^{-1} = 500$, that pattern gets quantitatively stronger.

The upper panels in Figure 3 show the $\Gamma_{\pi,u}$ under various φ and its interaction with ρ_u. The standard deviation of the cost-push shock is fixed at $\sigma_u = 0.02$. When φ increases, the slope of the aggregate supply curve becomes steeper, which implies that the effect of monetary policy becomes weaker. Thus at all degrees of concern about distorted expectations θ^{-1} and all levels of persistence of the cost-push shock ρ_u, the policy coefficient $\Gamma_{\pi,u}$ exhibits a low response when φ is high. When φ is small, i.e., the aggregate supply curve is flatter, the policy coefficient becomes strikingly different along with the concerns about distorted expectations. If $\theta^{-1} = 0$, $\Gamma_{\pi,u}$ increases

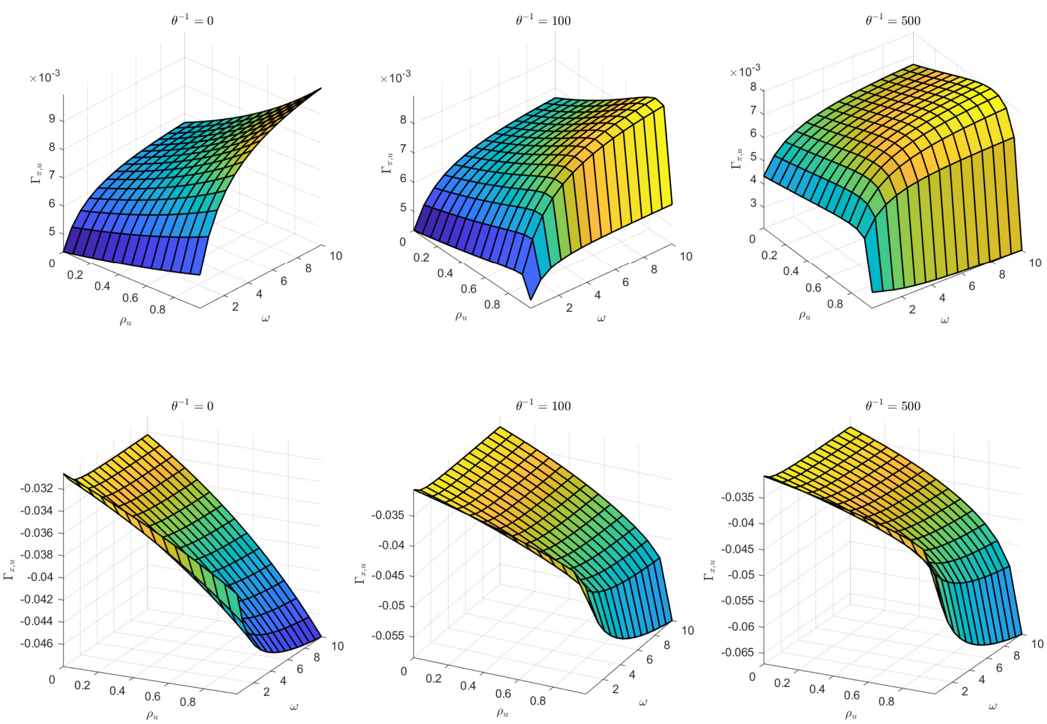
Figure 3: Robust Ramsey-Optimal Policy Coefficients with Various φ and ρ_u



when ϕ decreases. Moreover, $\Gamma_{\pi,u}$ increases when ρ_u increases. This implies that when the private expectations are fully rational, the optimal monetary policy is to conduct aggressive monetary policy (higher $\Gamma_{\pi,u}$) when the aggregate supply curve is flatter and when the cost-push shock is more persistent. If $\theta^{-1} = 100$, however, $\Gamma_{\pi,u}$ will decrease when φ becomes sufficiently small and ρ_u becomes sufficiently high. The pattern of the decrease in $\Gamma_{\pi,u}$ becomes quantitatively stronger when $\theta^{-1} = 500$. The results have the following implication. Suppose that the cost-push shock is substantially persistent, the aggregate supply curve is substantially flat, and the central bank has deep concerns about distorted expectations. In that case, the central bank decides to restrain its actions and conducts conservative monetary policy.

The lower panels in Figure 3 shows $\Gamma_{x,u}$ under various φ and its interaction with ρ_u. As we observed in the upper panels, the higher φ yields higher κ, which increases the denominator of the right hand side of equation . Thus, $\Gamma_{x,u}$ uniformly increases toward 0 when φ increases, for given any ρ_u and θ^{-1}. When $\theta^{-1} = 0$, $\Gamma_{x,u}$ decreases as ρ_u increases, for any given φ. The reason is because the RE gap $\pi_{H,t} - \beta \mathbb{E}_t \pi_{H,t+1}$ gets smaller when the persistence of the shock gets higher, since the expected path of domestic inflation also has higher serial correlation. When $\theta^{-1} = 100$, the gap $\pi_{H,t} - \beta \hat{\mathbb{E}}_t \pi_{H,t+1}$ gets more smaller because of the conservative response of the $\Gamma_{\pi,u}$ caused by the concerns about distorted expectations. Thus $\Gamma_{x,u}$ decreases more than the case of RE, when

Figure 4: Robust Ramsey-Optimal Policy Coefficients with Various ω and ρ_u

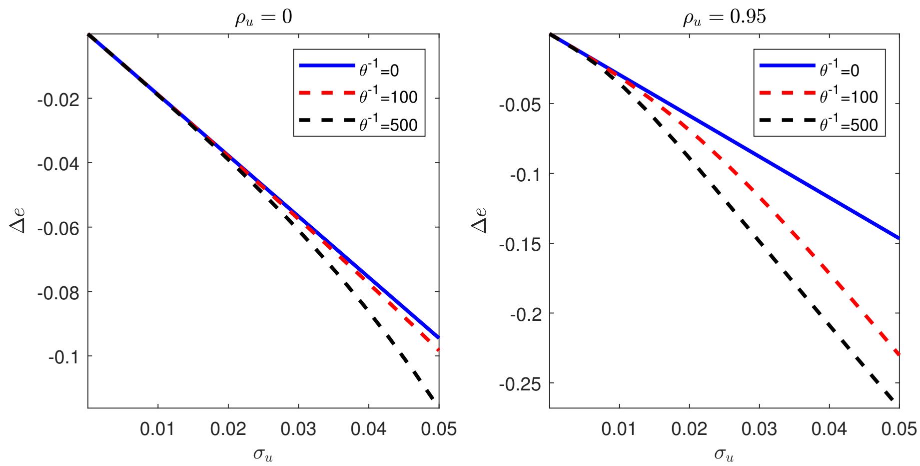


ρ_u increases. The pattern becomes quantitatively stronger when $\theta^{-1} = 500$.

The upper panels in Figure 4 shows $\Gamma_{\pi,u}$ under various ω and its interaction with ρ_u. Here, we fix $\sigma_u = 0.02$ and $\varphi = 1$. The parameter ω governs the slope of the aggregate demand curve $-\frac{1}{\sigma_\alpha}$, since $\sigma_\alpha \equiv \frac{\sigma}{1+\alpha(\omega-1)}$. A higher ω implies a steeper slope of the aggregate demand curve. Note that ω also affects the aggregate supply curve. The slope κ_α becomes flattered as ω increases. Thus, an increase in ω strengthens the propagation effect of the monetary policy. If $\theta^{-1} = 0$, i.e., the private expectations are fully rational, $\Gamma_{\pi,u}$ increases when ω increases. If $\theta^{-1} = 100$, the increase in $\Gamma_{\pi,u}$ becomes sluggish when ρ_u is sufficiently high. When $\theta^{-1} = 500$, $\Gamma_{\pi,u}$ actually decreases as ω increases if ρ_u is sufficiently high. The intuition is the following. A steeper aggregate demand curve caused by a high ω strengthens the effect of monetary policy. As there are concerns about distorted expectations, the central bank decides to conduct conservative monetary policy, i.e., adopt a smaller $\Gamma_{\pi,u}$ than the one under RE.

The lower panels in Figure 4 shows $\Gamma_{x,u}$ under various ω and its interaction with ρ_u. As we observed in the upper panels, the higher ω causes flatter κ. Thus, $\Gamma_{x,u}$ uniformly decreases when ω increases, for given any ρ_u and θ^{-1}. When $\theta^{-1} = 0$, at any fixed ω, the $\Gamma_{x,u}$ decreases as ρ_u increases. Similar to the previous case, the reason is the gap $\pi_{H,t} - \beta\mathbb{E}_t\pi_{H,t+1}$ gets smaller with the higher persistence of the shock. When $\theta^{-1} = 100$, the declining patterns get stronger by the

Figure 5: Initial Responses of the Devaluation Rate to the Cost-Push Shock



concerns for the robustness. When ρ_u gets higher, the decrease of $\Gamma_{x,u}$ becomes more drastic than the case of RE, since the gap $\pi_{H,t} - \beta\hat{\mathbb{E}}_t\pi_{H,t+1}$ gets more smaller by the concerns for the robustness. The case of $\theta^{-1} = 500$ strengthens the pattern.

How does the optimal monetary policy affect the movement of the exchange rate? First, let s_t be the log of the terms of trade. In the model, the log difference in the terms of trade Δs_t is determined by the equation

$$\Delta s_t = \sigma_\alpha \Delta(x_t + \tilde{y}_t - y_t^*), \tag{16}$$

where $\tilde{y}_t$ is the natural level of domestic output, which is a function of exogenous processes a_t and y_t^*. Let $e_t \equiv \int_0^1 e_{i,t}$ be the log of the effective nominal exchange rate, where $e_{i,t}$ is the log of the bilateral nominal exchange rate between the domestic country and country i (i.e, the price of the currency of country i in terms of the domestic currency). The nominal devaluation rate Δe_t is then determined by the equation

$$\Delta e_t = \pi_{H,t} + \Delta s_t - \pi_t^*, \tag{17}$$

where π_t^* is the exogenous inflation rate from the rest of the world. A positive (negative) Δe_t means nominal depreciation (appreciation) of the domestic currency. From the solution of the model, we can also obtain the solution for the devaluation rate in the Ramsey equilibrium.

Figure 5 shows impulse response functions of the devaluation rate in the initial period when the cost-push shock occurs. The left panel is the case in which the shock is i.i.d; $\rho_u = 0$. When $\theta^{-1} = 0$ (blue solid line), the responses are from the model with RE. There is more initial appreciation (negative response in Δe) as σ_u increases. When $\theta^{-1} = 100$ (red dashed line), the initial devaluations become larger than those under RE. When $\theta^{-1} = 500$ (blue dashed line), the initial response of the devaluation rate accelerates when σ_u exceeds 0.02. Thus, the model shows a far larger initial devaluation than those in the former two models.

The gaps of the initial response of devaluation rates between the models with different degrees of distorted expectations become more dramatic when the shock is more persistent. The right panel in the same figure is when the shock is persistent; $\rho_u = 0.95$. All three models with different θ^{-1} values show larger initial devaluation rates than those with the i.i.d shock. However, the increase in the initial devaluation rates in the model under RE is still relatively proportional as σ_u increases. In contrast, the rise in the initial devaluation rates in the models with distorted expectations becomes exponential as σ_u increases.

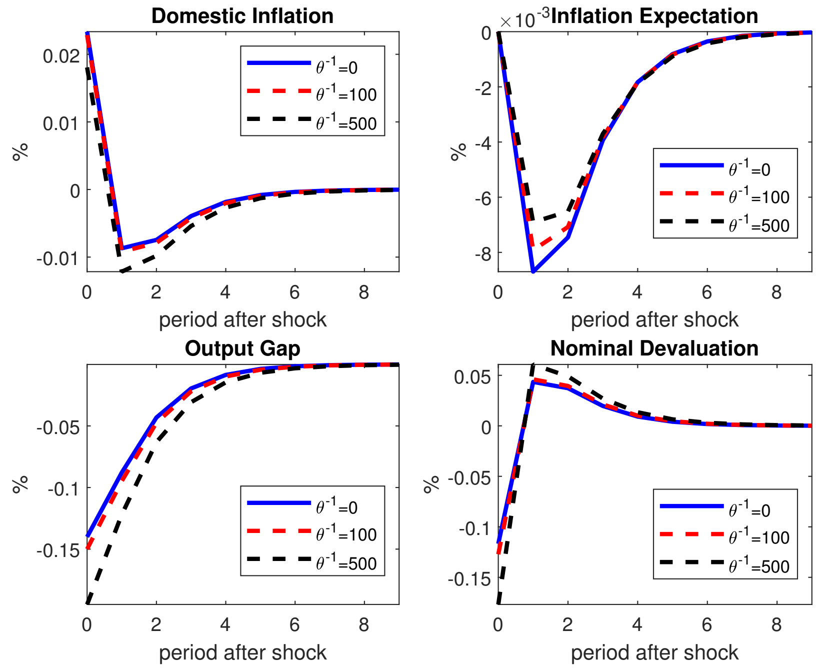
The results indicate that a more conservative monetary policy for domestic inflation causes a more volatile initial exchange rate response. The mechanism is as follows. The cost-push shock creates tradeoffs between domestic inflation and the output gap in all models. When $\theta^{-1} > 0$, the initial response of the output gap becomes larger than that under RE as a result of the sluggish response of domestic inflation. Since the nominal devaluation is determined by equation (17), the movements of domestic inflation and the change in the terms of trade both matter. Equation (16) indicates that the terms of trade is a linear function of the output gap. The magnitude of the response of the output gap outweighs the response of domestic inflation for a broad range of the parameters, and the magnitude increases as θ^{-1} increases. Consequently, the nominal devaluation shows a larger initial response (overreaction) than that under RE when $\theta^{-1} > 0$.

3.2 Ramsey-Optimal Dynamics

Now, we investigate the Ramsey-optimal dynamics of the key macroeconomic indicators when the cost-push shock hits the economy. Figures 6 shows the dynamics of domestic inflation, private expectations for domestic inflation, the output gap, and the nominal devaluation rate in response to a one-standard-deviation cost-push shock. In the figure, we set a moderate persistence of the cost-push shock, $\rho_u = 0.2$ and a very large standard deviation, $\sigma_u = 0.05$. In addition, we set $\varphi = 0.5$ and $\omega = 1$.

Note that the impulse response functions of the indicators exhibit optimal expected paths after the occurrence of the shock, which are induced by the central bank's intervention. When the private expectations are RE, $\theta^{-1} = 0$ (blue solid line), the expectations are also model consistent. Thus, the dynamics of next-period domestic inflation and the inflation expectations are consistent. The impulse response functions show optimal inflation-output gap tradeoffs to stabilize private agents' rational expectations. If $\theta^{-1} > 0$, on the other hand, the private expectations become worst-case expectations, which are not consistent with the actual path of domestic inflation. The impulse response functions show robustly optimal dynamics induced by the central bank, which seeks to stabilize private agents' worst-case expectations. In the figure, we observe that when $\theta^{-1} > 0$, domestic inflation not only jumps less than that under RE but also maintains a lower level than under RE over the dynamics. This implies that when $\theta^{-1} > 0$, the economy ends up with a lower domestic price level than under RE. This result is consistent with the prediction in the closed-

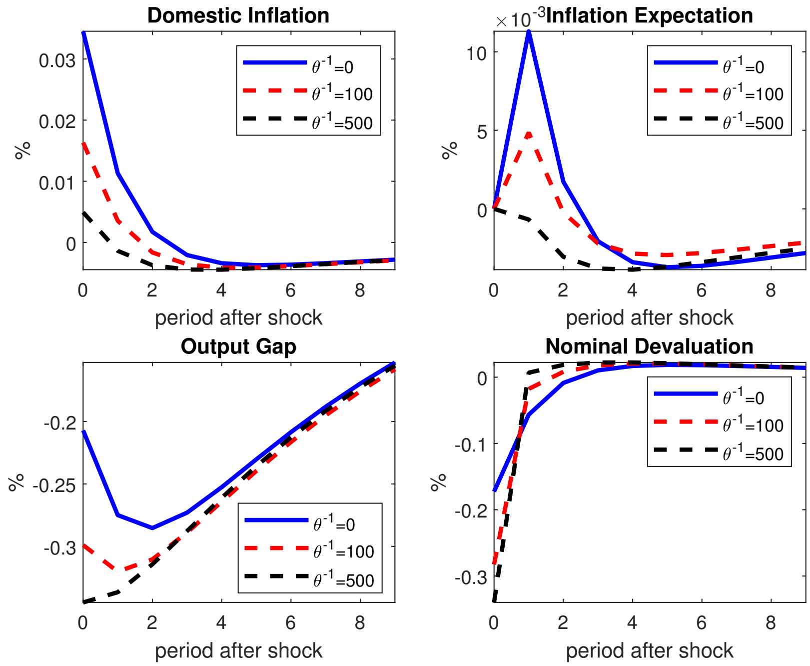
Figure 6: Dynamic Responses of Variables to the Cost-Push Shock (1)



economy environment in Woodford (2010). The nominal devaluation rate, on the other hand, behaves in a different way. When $\theta^{-1} > 0$, the nominal devaluation rate initially jumps below that under RE and jumps up in the following period more than that under RE. Then the nominal devaluation rates maintain higher levels than those under RE over the dynamics. The dynamics of the nominal devaluation rates are closely linked to the initial response and the slopes of the responses of the output gap over time.

Figure 7 shows the impulse response functions of the same indicators, but the persistence of the cost-push shock is now $\rho_u = 0.9$. When private expectations are RE, $\theta^{-1} = 0$, the optimal monetary policy calls for increasing the domestic inflation rate and then smoothing it gradually in response to the highly persistent shock. When $\theta^{-1} > 0$, on the other hand, the initial responses of domestic inflation and the following dynamics become milder than those under RE. The nominal devaluation rates initially jump down and then jump up, and the magnitudes of the jumps are larger than those under RE. Note that compared to Figure 6, the quantitative gaps between the responses when $\theta^{-1} > 0$ and those under RE here are considerably larger. As discussed in Figures 2–4, the key underlying mechanism of the results is that the higher persistence of the shock strengthens the central bank's motivation to conduct more conservative monetary policy when it is concerned about distorted expectations.

Figure 7: Dynamic Responses of Variables to the Cost-Push Shock (2)



4 Bayesian Inference

In the previous section, the analysis is based on models with a specific value of θ^{-1} and other structural parameters. The crucial questions are then what the actual data indicate about the degree of θ^{-1} and the other parameters and how the models predict actual monetary policies. To answer the questions, we estimate the models in this section. We consider two classes of models, one is the RE model with $\theta^{-1} = 0$ and the other is the model with distorted expectations, in which θ^{-1} is an estimable parameter. We will use the acronym 'NRE' when we refer to the model in which θ^{-1} is an estimable parameter.

We use time-series data for Canada and Mexico for the estimation. Canada is a typical developed small open economy, and Mexico is a familiar less-developed counterpart. All models are estimated by using time-series data and the likelihood-based Bayesian method.

4.1 Data

We use quarterly data on the CPI inflation rate, nominal devaluation rate, and US CPI inflation rate as observables for our empirical analysis. To construct the observables, we use quarterly CPI price indices and nominal exchange rate (vis-á-vis the US dollar). All price indices are seasonally adjusted using program package X13-ARIMA-SEATS. The time spans of the CPI inflation rate and nominal devaluation rate are Q2:1991–Q4:2008 for Canada and Q1:1991–Q4:2008 for Mexico. The US CPI inflation rate is used for the overlapping periods for each country.

4.2 The Linear State Space System

We reproduce the system governing the solution to the robust Ramsey equilibrium. We denote the vector of state variables as $\boldsymbol{\zeta}_t \equiv [\epsilon_t^u, \epsilon_t^a, \epsilon_t^y, 1, u_t, a_t, y_t^*, \Phi_{\pi,t-1}, \Phi_{x,t-1}]'$ and denote $\boldsymbol{S}^{\varepsilon} \equiv [\boldsymbol{I}, \boldsymbol{C}_x', \boldsymbol{0}]'$, where $\boldsymbol{I}$ and $\boldsymbol{0}$ are dimensions of identity and zero matrices, respectively. Equation (13) can be rewritten as

$$\boldsymbol{\zeta}_{t+1} = \boldsymbol{H}\boldsymbol{\zeta}_t + \boldsymbol{S}^{\varepsilon}\boldsymbol{\varepsilon}_{t+1},$$

where $\boldsymbol{H}$ and $\boldsymbol{S}^{\varepsilon}$ are matrices. Similarly, we denote the set of non-predetermined variables as $\boldsymbol{\chi}_t \equiv [i_t, \mu_{\pi,t}, \mu_{x,t}, \mu_{1,t}, \mu_{u,t}, \mu_{a,t}, \mu_{y^*,t}, \mu_{1,t+1},\ \mu_{u,t+1}, \mu_{a,t+1}, \mu_{y^*,t+1}]'$, so that equation (14) can be rewritten as

$$\boldsymbol{\chi}_t = \boldsymbol{G}\boldsymbol{\zeta}_t,$$

where $\boldsymbol{G}$ is a matrix.

The time-series observables are now used to identify some parameters of interest in the system of equations that represent the Ramsey-optimal solution of the central bank. The equations for aggregate demand and aggregate supply, (2) and (4), fully describe the equilibrium determination of domestic inflation and the output gap, subject to distorted beliefs and stochastic shocks. There are some practical limitations in measuring output gaps for these countries. However, we can indirectly map the data to the two model variables by using observables on domestic CPI inflation rates, devaluation rates, and US CPI inflation rates. The CPI inflation π_t in the model is determined as

$$\pi_t = \pi_{H,t} + \alpha\Delta s_t, , \tag{18}$$

and the nominal devaluation rate Δe_t is determined by equation (17), where we use US series as proxies for the series for the rest of the world π_t^*. The process of $\tilde{y}_t^*$ is exogenously given in the model with calibrated parameters ρ_{y^*} and σ_{y^*}. Thus, using observations on π_t and $\Delta\tilde{e}_t$ such that

$$\Delta\tilde{e}_t \equiv \Delta e_t + \pi_t^* \tag{19}$$

with combining equations (18) and (17) yields indirect observations for $\pi_{H,t}$ and Δx_t for the model counterparts.

Since the set of variables needed to express π_t and $\Delta\tilde{e}$ as solutions to the robust Ramsey equilibrium also features their lagged counterparts, the vector $[\boldsymbol{\zeta}_t', \boldsymbol{\zeta}_{t-1}']'$ should be used as the state variable in the system of equations. Given the observables, the measurement equation relates the

Table 1: Calibrated Parameters

Parameter	Canada	Mexico
σ	1	
ζ	0.75	
ρ_{y^*}	0.92	
$\sigma_{y^*} \times 100$	0.55	
β	0.99	0.975
α	0.4	0.248

observables with the state variables (accounting for measurement errors) and is given as:

$$\begin{bmatrix} \pi_t \\ \Delta\tilde{e}_t \end{bmatrix} = \tilde{\boldsymbol{H}} \begin{bmatrix} \boldsymbol{\zeta}_t \\ \boldsymbol{\zeta}_{t-1} \end{bmatrix} + \begin{bmatrix} \sigma_\pi^{me} & 0 \\ 0 & \sigma_{\Delta\tilde{e}}^{me} \end{bmatrix} \cdot \begin{bmatrix} \epsilon_t^{me,\pi} \\ \epsilon_t^{me,\Delta\tilde{e}} \end{bmatrix}, \tag{20}$$

where $[\epsilon_t^{me,\pi}, \epsilon_t^{me,\Delta\tilde{e}}]'$ are orthogonal standard normal random variables and $[\sigma_\pi^{me}, \sigma_{\Delta\tilde{e}}^{me}]'$ are standard deviations of the measurement errors on observables for π_t and $\Delta\tilde{e}$, respectively.

The transition equation for the evolution of the state variables is given as:

$$\begin{bmatrix} \boldsymbol{\zeta}_{t+1} \\ \boldsymbol{\zeta}_t \end{bmatrix} = \tilde{\boldsymbol{G}} \begin{bmatrix} \boldsymbol{\zeta}_t \\ \boldsymbol{\zeta}_{t-1} \end{bmatrix} + \boldsymbol{\nu}_{t+1}, \tag{21}$$

thus equations (20) and (21) complete the description of the linear state space system. The details for $\boldsymbol{H}, \boldsymbol{S}^\varepsilon, \boldsymbol{G}, \tilde{\boldsymbol{H}}, \tilde{\boldsymbol{G}}$, and $\boldsymbol{\nu}_{t+1}$ are presented in Appendix B.

4.3 Calibrated Parameters

Some of parameters of the models have appeared in the existing literature and are, therefore, calibrated. The values of these parameters directly follow the literature or are based on the data. The set of parameters that are calibrated govern the equilibrium dynamics (equations (2) to (7)). These parameters can further be classified into two subgroups: the structural parameters that have an economic interpretation (σ, ζ, β, and α) and the parameters that govern the transition process of shocks (ρ_{y^*}, and σ_{y^*}). The values of these parameters are listed in Table 1.

The parameter σ captures the curvature of the utility function of the household and is assumed to be 1, resulting in a log utility function, for all the countries as in Gali and Monacelli (2005). The Calvo-Yun parameter of price stickiness, ζ, is assumed to be 0.75 for all countries, signifying an average period of 1 year between price adjustments. For rest of the world output, we use the output series of the US and estimate an AR(1) regression. This yields a persistence, ρ_{y^*}, of 0.9261 and a standard deviation, σ_{y^*}, of 0.0055. The discount factor, β, is calibrated separately for each

Table 2: Prior Distributions of Parameters for Estimation

Parameter	Support	Distribution	Para (1)	Para (2)	[mean, std]
θ^{-1}	$[0, 10^{10}]$	Uniform	0	10^{10}	$[5 \cdot 10^9, 2.88 \cdot 10^4]$
ω	$\mathbb{R}^+$	Gamma	2.0	2.0	[4.0, 2.82]
φ	$\mathbb{R}^+$	Gamma	2.0	2.0	[4.0, 2.82]
ρ_u	$[0, 1)$	Beta	2.0	3.0	[0.4, 0.2]
ρ_a	$[0, 1)$	Beta	2.0	3.0	[0.4, 0.2]
$\sigma_u \times 100$	$\mathbb{R}^+$	Gamma	0.25	2.0	[0.5, 1.0]
$\sigma_a \times 100$	$\mathbb{R}^+$	Gamma	0.25	2.0	[0.5, 1.0]

country. The value of β is chosen to match the average interest rate, over the sample period, in the corresponding country. For example, Canada and Mexico have an annual interest rate of 4% and 10% on an average. Thus, the value of β for Canada is 0.99, whereas it is 0.975 for Mexico. Accordingly, the values of $\rho = -\ln \beta$ for Canada and Mexico are 0.01 and 0.026, respectively. The home-bias parameter, α, is also calibrated separately for each country. It is calibrated to match the average import-GDP ratio over the time series. Thus, α is 0.4 for Canada and 0.248 for Mexico, which signifies that imports represent 40% of GDP for Canada and 25% of GDP for Mexico.

4.4 Estimated Parameters

We estimate the remaining 7 structural parameters $\mathbf{\Theta} \equiv [\theta^{-1}, \omega, \varphi, \rho_u, \sigma_u, \rho_a, \sigma_a]'$ and 2 nonstructural parameters $[\sigma_\pi^{me}, \sigma_{\Delta e}^{me}]'$ using the likelihood-based Bayesian method.

Table 2 provides the details of the prior distribution of the parameters. To account for the possibility of being in the vicinity of RE, the minimum value of θ^{-1} is 0 while the maximum value of θ^{-1} is 10^{10}, a sizable departure from RE. The prior distribution of θ^{-1} is assumed to be uniform between these lower and upper bounds. The parameter ω governs the aggregate supply curve's slope, and Gali and Monacelli (2005) focuses on a special case of $\omega = 1$. However, we allow for a broad range of ω. The prior distribution of ω follows a Gamma (2,2) distribution, resulting in a mean of 4 and a standard deviation of 2.82. The Frisch elasticity of labor supply in the model is given as φ^{-1}. Given that many microeconomic estimates of the Frisch elasticity lie between 0.3 and 0.5, while many macroeconomists use an estimate between 2 and 4, we allow for a wide range of values for φ. Thus, we assume that φ follows a Gamma (2,2) distribution. The persistence of shock processes is assumed to follow a Beta (2,3) distribution, with a support of $[0, 1]$, which allows for a mean of 0.4 and a standard deviation of 0.2. Last, the one hundred times of the standard deviations of the shock processes are assumed to follow a Gamma (0.25,2) distribution, with a support of $(0, \infty)$. This allows for a mean and standard deviation of 0.5 and 1.0, respectively. Finally, for the standard deviation parameters of the measurement errors of the observables, we impose uniform distributions with the maximum supports to be 25% of the standard deviation of each observable.

We obtain draws from the posterior distribution of the estimated parameters $\boldsymbol{\Theta}$ conditional on the matrix of observables $\boldsymbol{O} \equiv [\pi_t, \Delta\tilde{e}_t]$. The posterior distribution, denoted as $P(\boldsymbol{\Theta}|\boldsymbol{O})$, is the product of the likelihood function of $\boldsymbol{O}$ and the prior distribution of $\boldsymbol{\Theta}$, which is denoted as $L(\boldsymbol{O}|\boldsymbol{\Theta})P(\boldsymbol{\Theta})$. The likelihood function $L(\boldsymbol{O}|\boldsymbol{\Theta})$ is evaluated numerically using the linear state space system (20)-(21) and Kalman filter. To evaluate the posterior distribution $P(\boldsymbol{\Theta}|\boldsymbol{O})$, we use a random-walk Metropolis-Hastings sampler described in Herbst and Schorfheide (2016). We use the last 1 million draws from the 5 million MCMC chains for the posterior analysis with an acceptance rate of 25 percent. On a standard desktop computer, the estimation of the RE models will take less than five hours for each country. The estimation of the NRE models, on the other hand, requires approximately five days for each country.

4.5 Posterior Distributions: RE vs. NRE

Table 3 shows the posterior distributions of the parameters for Canada and Mexico, in both the RE and NRE models. In the case of Canada, the RE and NRE models exhibit small overall differences. The posterior median of θ^{-1} in the NRE model is 7.56, with a 90% confidence interval of $[1.61, 17.6]$. This means that the NRE model exhibits only a slight departure from the RE benchmark. The posterior medians for ω in the RE and NRE models are 3.53 and 3.34, the posterior medians for φ in the two models are 1.30 and 0.70, and the 90% confidence intervals for the two parameters have substantial overlaps between the two models. The similarities between the two parameters result in similar slopes for the aggregate supply and aggregate demand curves. Using the posterior medians with other calibrated parameters, the slopes of the aggregate demand curve (2), σ_α^{-1}, for the RE and NRE models are 2.01 and 1.94, respectively. Moreover, the slope of the aggregate supply curve (4), $\kappa \equiv \frac{(1-\zeta)(1-\zeta\beta)(\sigma_\alpha+\varphi)}{\zeta}$, for the RE and NRE models is 0.15 and 0.10, respectively. The small posterior median of θ^{-1} and the similarity of the equilibrium relation of aggregate demand and supply entail similar estimates for the exogenous processes. The posterior medians of the persistence of the cost-push shock ρ_u of the RE and NRE models are 0.91 and 0.89, and the posterior medians of the standard deviation of the cost-push shock σ_u for the two models are 0.029 and 0.022. The posterior medians of the persistence of the domestic productivity shock ρ_a for the two models are 0.92 and 0.88, respectively, and the posterior medians of the standard deviation of the shock σ_a for the two models are 0.004 and 0.008, respectively. The 90% confidence intervals for the parameters are similar across the two models.

In the case of Mexico, on the other hand, there are substantial differences in the estimation results between the RE and NRE models. The posterior median of θ^{-1} in the NRE model is 504.2, with a 90% confidence interval of $[342.0, 733.9]$, which is substantially larger than that of the NRE model for Canada. The high value of θ^{-1} means that the NRE model presents a sharp departure from the RE benchmark, and which also affects the estimation of the other structural parameters. The posterior medians for ω and φ in the RE model are 5.15 and 0.76. On the other hand, the

Table 3: Posterior Distribution of Estimated Parameters

	Canada				Mexico			
	RE		NRE		RE		NRE	
Parameters	Median	[5%, 95%]	Median	[5%, 95%]	Median	[5%, 95%]	Median	[5%, 95%]
θ^{-1}	NA	NA	7.56	[1.61 , 17.6]	NA	NA	504.2	[342.0. 733.9]
ω	3.53	[2.96, 4.07]	3.34	[2.74, 4.15]	5.15	[3.92, 6.92]	7.08	[6.36, 8.18]
φ	1.30	[0.41, 3.02]	0.70	[0.11, 2.07]	0.76	[0.17, 2.01]	1.10	[0.68, 2.07]
ρ_u	0.91	[0.84, 0.96]	0.89	[0.80, 0.95]	0.93	[0.86, 0.97]	0.08	[0.02, 0.19]
ρ_a	0.92	[0.85, 0.97]	0.88	[0.67, 0.96]	0.97	[0.94, 0.99]	0.70	[0.20, 0.92]
σ_u	0.029	[0.018, 0.050]	0.022	[0.015, 0.038]	0.036	[0.024, 0.060]	0.065	[0.053, 0.091]
σ_a	0.004	[0.001, 0.007]	0.008	[0.004, 0.012]	0.106	[0.081, 0.143]	0.019	[0.00, 0.035]

posterior medians for ω and φ in the NRE model are 7.08 and 1.10. The intersections of the 90% confidence intervals of the same parameters between the two models are far narrower than in the case of Canada. The slopes of aggregate demand σ_α^{-1} for the two models become 2.03 and 2.51, and the slopes of aggregate supply κ in the RE and NRE models become 0.11 and 0.13, There are stark differences in parameters of the exogenous processes between the two models. For the parameters of the cost-push shock, the posterior medians of ρ_u and σ_u in the RE model are 0.93 and 0.036. In the NRE model, on the other hand, ρ_u and σ_u are 0.08 and 0.70. For the domestic productivity parameters, the posterior medians of ρ_a and σ_a in the RE model are 0.97 and 0.106, whereas they are 0.65 and 0.019 in the NRE model. In addition, there are few intersections between the 90% confidence intervals for the same parameters of the two models.

4.6 Predictive Power of the NRE Model for Monetary Policy Rates

In the estimation, we use inflation rates and nominal devaluation rates as observables. Based on estimated and calibrated parameters, the monetary policy rate in the model is computed as the Ramsey-optimal solution in equation (14). How well do the estimated models predict the actual monetary policy rate in the data? To answer this question, we compare monetary policy rates predicted from the estimated models and in the data.

The left panel in Figure 8 compares actual monetary policy rate data (black dotted line) and model predictions from the RE (solid blue line) and NRE (red dashed line) models for Canada. The 'predicted' policy rates are Ramsey-optimal monetary policy rates generated by the models using estimated parameters and the historical processes of shocks extracted from the Kalman smoother. Since the interest rates are not targeted in estimation, the volatilities of the three series are quite different. The model-generated monetary policy rates show higher short-term volatilities than the actual data. However, all three series have similar 'ups' and 'downs'. To clearly see the comovements among the three series, we compare the trends of the series by applying the Hodrick-Prescott filter, as depicted in the right panel of Figure 8. The model-generated interest rates show strong comovements with the actual data. In terms of the distance from the actual data, the

Figure 8: Monetary Policy Rates, Data and Model Predictions: Canada

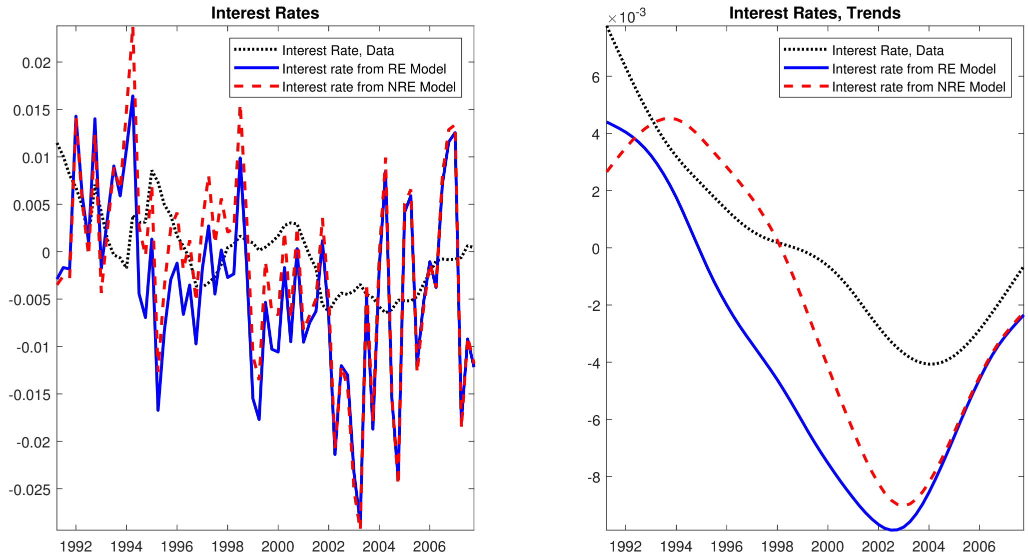
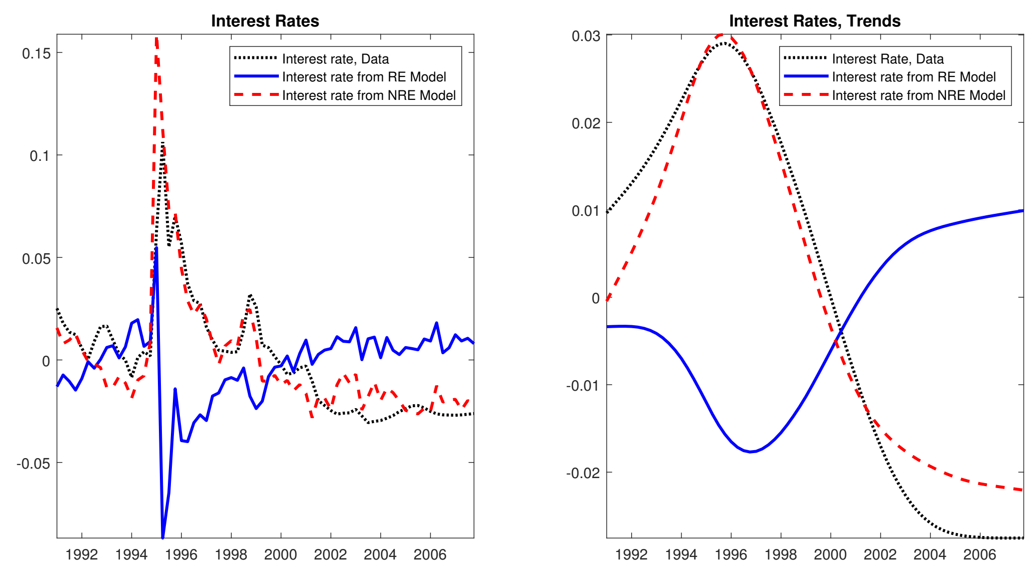


Figure 9: Monetary Policy Rates, Data and Model Predictions: Mexico



interest rate generated by the NRE model performs slightly better than that of the RE model. The small differences between the interest rates generated by the RE and NRE models derive from the observation that the posterior distributions of the two models are close, associated with $\theta^{-1} = 7.56$, not far from zero.

In the case of Mexico, on the other hand, the NRE model outperforms the RE model in pre-

Table 4: Second Moments of Observables

	Canada			Mexico		
Moments	Data	RE	NRE	Data	RE	NRE
$\sigma(\pi_t)$	0.30	0.33	0.33	2.47	2.62	1.90
$\sigma(\Delta\tilde{e}_t)$	3.16	3.18	3.18	8.70	8.58	8.38
$\rho(\pi_t, \Delta\tilde{e}_t)$	-0.19	-0.25	-0.30	0.34	0.25	0.14
$\rho(\pi_t, \pi_{t-1})$	0.67	0.04	0.14	0.88	0.23	0.77
$\rho(\pi_t, \pi_{t-2})$	0.15	-0.01	0.04	0.67	0.09	0.44
$\rho(\pi_t, \pi_{t-4})$	-0.09	-0.04	-0.04	0.44	-0.02	0.12
$\rho(\Delta\tilde{e}_t, \Delta\tilde{e}_{t-1})$	0.30	0.35	0.38	0.08	0.11	-0.16
$\rho(\Delta\tilde{e}_t, \Delta\tilde{e}_{t-2})$	0.03	0.10	0.13	-0.02	0.04	-0.10
$\rho(\Delta\tilde{e}_t, \Delta\tilde{e}_{t-4})$	-0.07	-0.05	-0.05	0.00	-0.01	-0.03
Log MDD		436.34	417.67		213.31	263.40

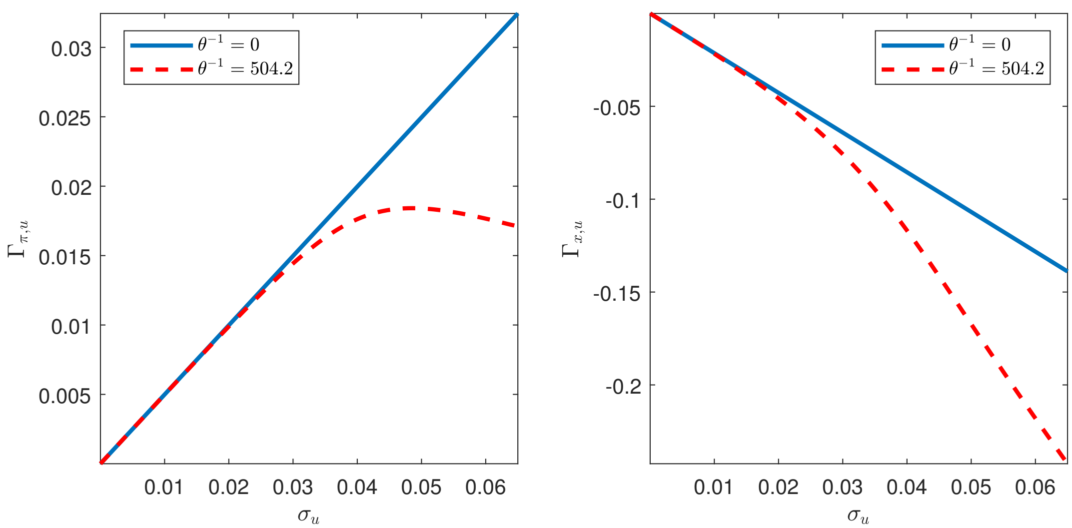
Notes. The log marginal data densities are computed using Geweke (1999)'s harmonic mean estimator.

dicting the actual path of the monetary policy rate. The left panel of Figure 9 exhibits the three series for Mexico. Throughout the sample period, the monetary policy rate generated by the NRE model almost replicates the path of the actual data. In particular, the policy rate from the NRE model predicts the interest rate spikes during the Mexican peso crisis (1994–1996). On the other hand, in the RE model, the monetary policy rate moves in the opposite direction as in the actual data throughout nearly all of the sample period. The right panel of Figure 9 shows that trends in monetary policy rates between the data and the NRE model are very similar, whereas the RE model shows nearly the opposite dynamics.

Given that no moments of the monetary policy rate are targeted, the finding that the monetary policy rate in the NRE model in Mexico closely matches that in the actual data is striking. How is the result related to the posterior median $\theta^{-1} = 504.2$ in Mexico? Why is the performance of the RE model so poor? Table 4 shows the second moments of observables used in the estimation: the CPI inflation rate π_t^* and the sum of nominal devaluation rates and US inflation rate $\Delta\tilde{e}_t$. The table provides the insight that the persistence of inflation becomes a key identifying moment for the parameter that governs degree of distorted expectations, θ^{-1}. In the case of Canada, the performance of the RE and NRE models in matching moments of observations is qualitatively similar. The two models successfully match the standard deviations of observations and their cross-correlation. Both models are unsuccessful in matching high and positive autocorrelations of π_t for the first order, although the NRE model marginally improves the moments. The two models match the autocorrelations of $\Delta\tilde{e}_t$ well.

In the case of Mexico, on the other hand, there are essential differences in second moments between the two models. The RE model performs relatively better in matching standard deviations of observations and their cross-correlations, although the NRE model also matches them reasonably

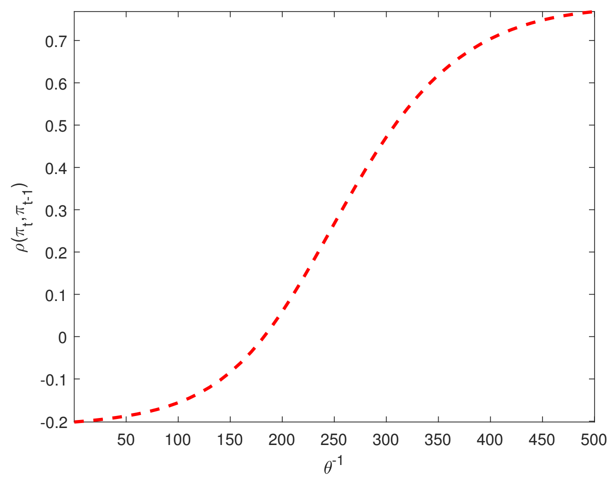
Figure 10: Policy Coefficients $\Gamma_{\pi,u}$ and $\Gamma_{x,u}$ in the NRE Model for Mexico with Various σ_u



well. In terms of matching autocorrelations of π_t, on the other hand, the NRE model remarkably outperforms the RE model. In the data, there is strong persistence of the inflation rate up to higher orders. The first-, second-, and fourth-order autocorrelations are 0.88, 0.67, and 0.44, respectively, all of which are much higher than the moments in Canada. The RE model fails to match the moments. The RE model predicts 0.23, 0.09, and -0.02 for the three moments. The NRE model predicts 0.77, 0.44, and 0.12 for the three moments, which are much closer to the moments from the data. In terms of matching autocorrelations of $\Delta\tilde{e}_t$, the two models both match them fairly well in the sense that the predicted autocorrelations are all close to zero, which are also shown in the data. The last row in Table 5 shows the log marginal data densities (MDD) for the models. In the case of Canada, the MDD of the NRE model is smaller than that of the RE model. In the case of Mexico, the MDD of the NRE model increases by 25 percent relative to the RE model. This implies that the NRE model, which has one more estimable parameter θ^{-1}, does not necessarily always yield better performance, and the higher MDD of the NRE model for Mexico comes from its success in explaining inflation persistence.

How does the NRE model succeed in explaining inflation persistence in Mexico? The critical insight is the conservative (more history-dependant than RE) monetary policy caused by the concerns about distorted expectations, as discussed in Section 3. The posterior median $\theta^{-1} = 504.2$ in Mexico indeed produces a sluggish response of domestic inflation. Figure 10 shows the Ramsey policy coefficients of domestic inflation $\Gamma_{\pi,u}$ and the output gap $\Gamma_{x,u}$ in response to the volatility of the cost-push shock σ_u. Except for σ_u and θ^{-1}, the parameters are set using the calibrated and estimated parameters in the NRE model for Mexico. When $\theta^{-1} = 0$, $\Gamma_{\pi,u}$ increases linearly with the increase in σ_u, which is clearly the certainty equivalence principle. When $\theta^{-1} = 504.2$, which

Figure 11: First-Order Autocorrelation of π_t, Conditional on θ^{-1}



is the posterior median, $\Gamma_{\pi,u}$ increases less than proportionally with σ_u. Furthermore, $\Gamma_{\pi,u}$ begins to decrease when σ_u is larger than approximately 0.04. Note that the posterior median of σ_u is 0.065. The conservative monetary policy makes the domestic inflation exhibit a sluggish response to the shock, which yields inflation persistence. The Ramsey policy coefficients for output gap $\Gamma_{x,u}$ is determined from (15). And as it is also discussed in Section 3, when $\theta^{-1} = 504.2$, $\Gamma_{x,u}$ with declines more aggressively than the case under RE as σ_u increases.

Remark that we use CPI inflation as an observable in the estimation. Combining (16) and (18) gives the following relation between CPI inflation, domestic inflation, the change of output gap, and the world output gap:

$$\pi_t = \pi_{H,t} + \alpha\sigma_\alpha(\Delta x_t + \tilde{y}_t - y_t^*). \tag{22}$$

With our calibrated and estimated parameters in Mexico, $\alpha\sigma_\alpha = 0.098$, which implies that the business cycle dynamics of the CPI inflation in (22) is mainly driven by the the dynamics of $\pi_{H,t}$. Figure 11 shows the patterns of the first-order autocorrelation of the NRE model for Mexico when θ^{-1} varies from 0 to 504.2. The graph has an S-shape, showing that the inflation persistence uniformly increases as θ^{-1} increases.

5 Conclusion

In this paper, we study robustly optimal monetary policy in a small open economy where private agents' expectations are potentially distorted. As the central bank's concerns about distorted expectations increase, it conducts a more conservative monetary policy. The policy yields a sluggish domestic inflation response and a greater overreaction in the initial exchange rate response. The estimated models indicate that there are small deviations from rational expectations in Canada, whereas there are substantial deviations in Mexico. The estimated models of the distorted expectations predict the actual path of the monetary policy rate well in Mexico. The key mechanism is that the model of the distorted expectations can successfully match inflation persistence.

Measuring a model's degree of deviation from the rational expectations benchmark is increasingly important in macroeconomic modeling. We highlight a way of measuring the deviation using time series and the Bayesian method. Although the scale of the benchmark model is highly parsimonious, considering an expectation wedge provides us with a sharp improvement in predictive power. We believe that the extension of the analysis to more prosperous and more complex environments could enable us to better understand how distorted expectations relate to other shocks, wedges, and optimal policy designs. We leave such work to future research.

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Monetary Policy with Near-Rational Expectations in Open Economies

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Online Appendix

A Solving the LQ System with Conditionally Linear Commitment

We apply the solution method in Kwon and Miao (2019) which generalizes the method in Woodford (2010). Solving the problem is a two step process. In the first step, the hypothetical malevolent nature chooses m_{t+1} to maximize the welfare loss. The first-order condition of the Lagrangian with respect to m_{t+1} is:

$$\theta(1+\ln m_{t+1}) - \phi_t - \boldsymbol{\mu}'_{y,t}\boldsymbol{Y}_{t+1} = 0,$$

which can be written in terms of $\boldsymbol{\varepsilon}_{t+1}$ by using the relation $\boldsymbol{y}_{t+1} = \boldsymbol{\Phi}_t + \boldsymbol{\Gamma}_t\boldsymbol{\varepsilon}_{t+1}, t \geq 0$. This relation along with constraint (1) yields the following expression:

$$m_{t+1} = \exp\left(-\frac{1}{2}\theta^{-2}\boldsymbol{\mu}'_{y,t}\boldsymbol{\Gamma}_t\boldsymbol{\Gamma}'_t\boldsymbol{\mu}_{y,t} + \theta^{-1}\boldsymbol{\mu}'_{y,t}\boldsymbol{\Gamma}_t\boldsymbol{\varepsilon}_{t+1}\right).$$

Using the solution for the worst-case belief m_{t+1}, we can find the conditional expectations of y and m as:[1]

$$\begin{aligned}
\mathbb{E}_t m_{t+1} y_{t+1} &= \boldsymbol{\Phi}_t + \theta^{-1}\boldsymbol{\Gamma}_t\boldsymbol{\Gamma}'_t\boldsymbol{\mu}_{y,t}, \\
\mathbb{E}_t m_{t+1}\ln m_{t+1} &= \frac{1}{2}\theta^{-2}\boldsymbol{\mu}'_{y,t}\boldsymbol{\Gamma}_t\boldsymbol{\Gamma}'_t\boldsymbol{\mu}_{y,t}.
\end{aligned}$$

In the second step of solving the problem, we can reconstruct the central bank's welfare loss function in matrix form as follows:

$$L(\boldsymbol{X}_t, \boldsymbol{\Phi}_{t-1} + \boldsymbol{\Gamma}_{t-1}\boldsymbol{\varepsilon}_t, i_t) = \frac{1}{2}\begin{bmatrix} \boldsymbol{X}_t \\ \boldsymbol{\Phi}_{t-1} + \boldsymbol{\Gamma}_{t-1}\boldsymbol{\varepsilon}_t \end{bmatrix}' \boldsymbol{Q} \begin{bmatrix} \boldsymbol{X}_t \\ \boldsymbol{\Phi}_{t-1} + \boldsymbol{\Gamma}_{t-1}\boldsymbol{\varepsilon}_t \end{bmatrix}$$

$$+\frac{1}{2}i_t\boldsymbol{R}i_t + \begin{bmatrix} \boldsymbol{X}_t \\ \boldsymbol{\Phi}_{t-1} + \boldsymbol{\Gamma}_{t-1}\boldsymbol{\varepsilon}_t \end{bmatrix}' \boldsymbol{S}i_t,$$

[1] First, we use the pdf under central bank's belief: $\ln f = -\frac{n_\varepsilon}{2}\ln(2\pi) - \frac{1}{2}\varepsilon'_{t+1}\varepsilon_{t+1}$. The probability density function under the worst-case belief can then be written as $\hat{f} = mf \implies \ln\hat{f} = \ln m + \ln f$, which results in a worst-case distribution that is normal with mean $\theta^{-1}\Gamma'_t\mu_{y,t}$ and standard deviation I. Now, the conditional (on time-t) mean $\mathbb{E}_t m_{t+1}y_{t+1} = E_t(m_{t+1}\Phi_t + m_{t+1}\Gamma_t\varepsilon_{t+1}) = \Phi_t + \mathbb{E}_t m_{t+1}\Gamma_t\varepsilon_{t+1} = \Phi_t + \hat{\mathbb{E}}_t\Gamma_t\varepsilon_{t+1} = \Phi_t + \Gamma_t\hat{\mathbb{E}}_t\varepsilon_{t+1} = \Phi_t + \theta^{-1}\Gamma_t\Gamma'_t\mu_{y,t}$. Similarly, we can find $\mathbb{E}_t(m_{t+1}\ln m_{t+1})$.

where the matrix $\begin{bmatrix} \boldsymbol{Q} & \boldsymbol{S} \\ \boldsymbol{S}' & \boldsymbol{R} \end{bmatrix}$ is symmetric and positive definite.

The central bank chooses $\{\boldsymbol{X}_t, \boldsymbol{\Phi}_t, \boldsymbol{\Gamma}_t, i_t\}$ after substituting for the chosen value of m_{t+1} in the Lagrangian. The new Lagrangian is:

$$\mathcal{L} = \mathbb{E}_{-1} \sum_{t=0}^{\infty} \beta^t \Bigg\{ L(\boldsymbol{X}_t, \boldsymbol{\Phi}_{t-1} + \boldsymbol{\Gamma}_{t-1}\boldsymbol{\varepsilon}_t, i_t) - \frac{1}{2\theta} \boldsymbol{\mu}'_{y,t} \boldsymbol{\Gamma}_t \boldsymbol{\Gamma}'_t \boldsymbol{\mu}_{y,t} +$$

$$\begin{bmatrix} \boldsymbol{\mu}_{x,t+1} \\ \boldsymbol{\mu}_{y,t} \end{bmatrix}' \left(\begin{bmatrix} \boldsymbol{X}_{t+1} \\ \boldsymbol{\Phi}_t + \frac{1}{\theta}\boldsymbol{\Gamma}_t\boldsymbol{\Gamma}'_t\boldsymbol{\mu}_{y,t} \end{bmatrix} - \boldsymbol{A} \begin{bmatrix} \boldsymbol{X}_t \\ \boldsymbol{\Phi}_{t-1} + \boldsymbol{\Gamma}_{t-1}\boldsymbol{\varepsilon}_t \end{bmatrix} - \boldsymbol{B} i_t \right) - \boldsymbol{C}\boldsymbol{\varepsilon}_{t+1} \Bigg\}.$$

The first-order necessary conditions with respect to $\{\boldsymbol{X}_t, \boldsymbol{\Phi}_t, \boldsymbol{\Gamma}_t, i_t\}$ are

$$\begin{aligned}
0 &= -\boldsymbol{Q}_{xx}\boldsymbol{X}_t - \boldsymbol{Q}_{xy}(\boldsymbol{\Phi}_{t-1} + \boldsymbol{\Gamma}_{t-1}\boldsymbol{\varepsilon}_t) - \boldsymbol{S}_x i_t - \beta^{-1}\boldsymbol{\mu}_{xt} + \boldsymbol{A}'_{xx}\mathbb{E}_t\boldsymbol{\mu}_{xt+1} + \boldsymbol{A}'_{yx}\boldsymbol{\mu}_{yt}, \\
0 &= -\boldsymbol{R} i_t - \boldsymbol{S}'_x\boldsymbol{X}_t - \boldsymbol{S}'_y(\boldsymbol{\Phi}_{t-1} + \boldsymbol{\Gamma}_{t-1}\boldsymbol{\varepsilon}_t) + \boldsymbol{B}'_x\mathbb{E}_t\boldsymbol{\mu}_{xt+1} + \boldsymbol{B}'_y\boldsymbol{\mu}_{yt}, \\
0 &= -\left(\boldsymbol{Q}_{yy}\boldsymbol{\Phi}_t + \boldsymbol{Q}'_{xy}\mathbb{E}_t\boldsymbol{X}_{t+1}\right) - \boldsymbol{S}_y\mathbb{E}_t u_{t+1} - \beta^{-1}\boldsymbol{\mu}_{yt} + \mathbb{E}_t\left[\boldsymbol{A}'_{xy}\boldsymbol{\mu}_{xt+2} + \boldsymbol{A}'_{yy}\boldsymbol{\mu}_{yt+1}\right], \\
0 &= -\beta\boldsymbol{Q}_{yy}\mathbb{E}_t\left[\boldsymbol{\Phi}_t\boldsymbol{\varepsilon}'_{t+1} + \boldsymbol{\Gamma}_t\boldsymbol{\varepsilon}_{t+1}\boldsymbol{\varepsilon}'_{t+1}\right] - \beta\boldsymbol{Q}'_{xy}\mathbb{E}_t\boldsymbol{X}_{t+1}\boldsymbol{\varepsilon}'_{t+1} \\
&\quad -\beta\boldsymbol{S}_y\mathbb{E}_t i_{t+1}\boldsymbol{\varepsilon}'_{t+1} - \frac{1}{\theta}\boldsymbol{\mu}_{yt}\boldsymbol{\mu}'_{yt}\boldsymbol{\Gamma}_t + \beta\mathbb{E}_t\left[\left(\boldsymbol{A}'_{xy}\boldsymbol{\mu}_{xt+2} + \boldsymbol{A}'_{yy}\boldsymbol{\mu}_{yt+1}\right)\boldsymbol{\varepsilon}'_{t+1}\right],
\end{aligned}$$

where matrices are partitioned as $\boldsymbol{A} \equiv \begin{bmatrix} \boldsymbol{A}_{xx} & \boldsymbol{A}_{xy} \\ \boldsymbol{A}_{yx} & \boldsymbol{A}_{yy} \end{bmatrix}$, $\boldsymbol{B} \equiv [\boldsymbol{B}_x, \boldsymbol{B}_y]'$, and $\boldsymbol{C} \equiv [\boldsymbol{C}_x, \boldsymbol{C}_y]'$, $\boldsymbol{Q} \equiv \begin{bmatrix} \boldsymbol{Q}_{xx} & \boldsymbol{Q}_{xy} \\ \boldsymbol{Q}'_{xy} & \boldsymbol{Q}_{yy} \end{bmatrix}$, and $\boldsymbol{S} \equiv \begin{bmatrix} \boldsymbol{S}_x \\ \boldsymbol{S}_y \end{bmatrix}$. In addition, the first-order necessary conditions with respect to $\{\boldsymbol{\mu}_{x,t+1}, \boldsymbol{\mu}_{y,t}\}$ yield the law of motion of the constraints:

$$\begin{aligned}
0 &= \boldsymbol{X}_{t+1} - \boldsymbol{A}_{xx}\boldsymbol{X}_t - \boldsymbol{A}_{xy}(\boldsymbol{\Phi}_{t-1} + \boldsymbol{\Gamma}_{t-1}\boldsymbol{\varepsilon}_t) - \boldsymbol{B}_x i_t - \boldsymbol{C}_x\boldsymbol{\varepsilon}_{t+1}, \\
0 &= \boldsymbol{\Phi}_t + \frac{1}{\theta}\boldsymbol{\Gamma}_t\boldsymbol{\Gamma}'_t\boldsymbol{\mu}_{y,t} - \boldsymbol{A}_{yx}\boldsymbol{X}_t - \boldsymbol{A}_{yy}(\boldsymbol{\Phi}_{t-1} + \boldsymbol{\Gamma}_{t-1}\boldsymbol{\varepsilon}_t) - \boldsymbol{B}_y i_t,
\end{aligned}$$

Adding two more equations, $\mathbb{E}_t(\boldsymbol{\varepsilon}_{t+1}) = \boldsymbol{0}$ and $\mathbb{E}_t\boldsymbol{\mu}_{xt+2} = \boldsymbol{E}_t\boldsymbol{\mu}_{xt+1}$, to the aforementioned set of 5 equations, we solve:

$$\boldsymbol{J}\begin{bmatrix}\mathbb{E}_t\boldsymbol{\varepsilon}_{t+1}\\ \mathbb{E}_t\boldsymbol{X}_{t+1}\\ \boldsymbol{\Phi}_t\\ \mathbb{E}_t i_{t+1}\\ \mathbb{E}_t\boldsymbol{\mu}_{y,t+1}\\ \mathbb{E}_t\boldsymbol{\mu}_{x,t+1}\\ \mathbb{E}_t\boldsymbol{\mu}_{x,t+2}\end{bmatrix}=\boldsymbol{F}\begin{bmatrix}\boldsymbol{\varepsilon}_t\\ \boldsymbol{X}_t\\ \boldsymbol{\Phi}_{t-1}\\ i_t\\ \boldsymbol{\mu}_{y,t}\\ \boldsymbol{\mu}_{x,t}\\ \mathbb{E}_t\boldsymbol{\mu}_{x,t+1}\end{bmatrix}$$

where $\boldsymbol{J}$ and $\boldsymbol{F}$ are matrices that contain parts of $\boldsymbol{A}$, $\boldsymbol{B}$, $\boldsymbol{C}$, $\boldsymbol{Q}$, $\boldsymbol{R}$, $\boldsymbol{S}$, $\boldsymbol{\Gamma}$, and β. Here, $\boldsymbol{\varepsilon}_t$, $\boldsymbol{X}_t$ and $\boldsymbol{\Phi}_{t-1}$ are predetermined state variables, whereas i_t, $\boldsymbol{\mu}_{y,t}$, $\boldsymbol{\mu}_{x,t}$ and $\mathbb{E}_t\boldsymbol{\mu}_{x,t+1}$ are non-predetermined variables.

The solution to the aforementioned system therefore takes the following state-space form:

$$\begin{bmatrix}\boldsymbol{\varepsilon}_{t+1}\\ \boldsymbol{X}_{t+1}\\ \boldsymbol{\Phi}_t\end{bmatrix}=\boldsymbol{H}\begin{bmatrix}\boldsymbol{\varepsilon}_t\\ \boldsymbol{X}_t\\ \boldsymbol{\Phi}_{t-1}\end{bmatrix}+\begin{bmatrix}\boldsymbol{I}\\ \boldsymbol{C}_x\\ \boldsymbol{0}\end{bmatrix}\boldsymbol{\varepsilon}_{t+1}$$

$$\begin{bmatrix}i_t\\ \boldsymbol{\mu}_{y,t}\\ \boldsymbol{\mu}_{x,t}\\ E_t\boldsymbol{\mu}_{x,t+1}\end{bmatrix}=\boldsymbol{G}\begin{bmatrix}\boldsymbol{\varepsilon}_t\\ \boldsymbol{X}_t\\ \boldsymbol{\Phi}_{t-1}\end{bmatrix}$$

where the first equation depicts the law of motion of the state variables and the second equations corresponds to the policy rules for the non-predetermined variables.

For convenience, the partitions of the matrices $\boldsymbol{A}$, $\boldsymbol{B}$, $\boldsymbol{C}$, $\boldsymbol{Q}$, $\boldsymbol{R}$, and $\boldsymbol{S}$ (which constitute the matrices $\boldsymbol{J}$ and $\boldsymbol{F}$) are produced below:

$$\boldsymbol{A}_{xx}=\begin{bmatrix}1&0&0&0\\0&\rho_u&0&0\\0&0&\rho_a&0\\0&0&0&\rho_{y^*}\end{bmatrix};\boldsymbol{A}_{xy}=\begin{bmatrix}0&0\\0&0\\0&0\\0&0\end{bmatrix}$$

$$\boldsymbol{A}_{yx}=\begin{bmatrix}0&-\beta^{-1}&0&0\\-\rho\sigma_\alpha^{-1}&\beta^{-1}\sigma_\alpha^{-1}&\Gamma(1-\rho_a)&\alpha(\Theta+\Psi)(1-\rho_{y^*})\end{bmatrix};\boldsymbol{A}_{yy}=\begin{bmatrix}\beta^{-1}&-\beta^{-1}\kappa_\alpha\\-\beta^{-1}\sigma_\alpha^{-1}&1+\beta^{-1}\sigma_\alpha^{-1}\kappa_\alpha\end{bmatrix}$$

$$\boldsymbol{B}_x = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}; \boldsymbol{B}_y = \begin{bmatrix} 0 \\ \sigma_\alpha^{-1} \end{bmatrix}; \boldsymbol{C}_x = \begin{bmatrix} 0 & 0 & 0 \\ \sigma_u & 0 & 0 \\ 0 & \sigma_a & 0 \\ 0 & 0 & \sigma_{y^*} \end{bmatrix}; \boldsymbol{C}_y = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\boldsymbol{Q}_{xx} = \begin{bmatrix} \lambda_x (x^*)^2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}; \boldsymbol{Q}_{xy} = \begin{bmatrix} 0 & -\lambda_x x^* \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}; \boldsymbol{Q}_{yx} = \boldsymbol{Q}_{xy}^T; \boldsymbol{Q}_{yy} = \begin{bmatrix} 1 & 0 \\ 0 & \lambda_x \end{bmatrix}$$

$$\boldsymbol{R} = [0]; \boldsymbol{S}_x = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}; \boldsymbol{S}_y = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

B Description of The Linear State Space System for Estimation

The linear state space system, governing the solution to the robust Ramsey equilibrium is given as:

$$\boldsymbol{\zeta}_{t+1} = \boldsymbol{H}\boldsymbol{\zeta}_t + \boldsymbol{S}^{\varepsilon}\boldsymbol{\varepsilon}_{t+1},$$

$$\boldsymbol{\chi}_t = \boldsymbol{G}\boldsymbol{\zeta}_t,$$

where $\boldsymbol{\zeta}_t \equiv [\epsilon_t^u, \epsilon_t^a, \epsilon_t^y, 1, u_t, a_t, y_t^*, \Phi_{\pi,t-1}, \Phi_{x,t-1}]'$, $\boldsymbol{\chi}_t \equiv [i_t, \mu_{\pi,t}, \mu_{x,t}, \mu_{1,t}, \mu_{u,t}, \mu_{a,t}, \mu_{y^*,t}, \mu_{1,t+1}, \mu_{u,t+1}, \mu_{a,t+1}, \mu_{y^*,t+1}]'$, $\boldsymbol{S}^{\varepsilon} \equiv [\boldsymbol{I}, \boldsymbol{C}_x', \boldsymbol{0}]'$, and $\boldsymbol{\varepsilon}_{t+1} \equiv [\epsilon_t^u, \epsilon_t^a, \epsilon_t^y]$. The state variables in this system are used to construct the law of motion of the observables in the estimation process.

There are two variables that are used as observables in the estimation. The CPI inflation, π_t, is the first observable. In the model, it is determined as:

$$\begin{aligned}
\pi_t &= \pi_{H,t} + \alpha \Delta s_t \\
&= (\Gamma_{\pi u,t-1} + \alpha\sigma_\alpha \Gamma_{xu,t-1})\,\epsilon_t^u - \alpha\sigma_\alpha \Gamma_{xu,t-2}\epsilon_{t-1}^u + (\Gamma_{\pi a,t-1} + \alpha\sigma_\alpha \Gamma_{xa,t-1})\,\epsilon_t^a - \alpha\sigma_\alpha \Gamma_{xa,t-2}\epsilon_{t-1}^a \\
&+ (\Gamma_{\pi y^*,t-1} + \alpha\sigma_\alpha \Gamma_{xy^*,t-1})\,\epsilon_t^{y^*} - \alpha\sigma_\alpha \Gamma_{xy^*,t-2}\epsilon_{t-1}^{y^*} + \Phi_{\pi,t-1} + \alpha\sigma_\alpha \Phi_{x,t-1} - \alpha\sigma_\alpha \Phi_{x,t-2} \\
&+ \alpha\sigma_\alpha \left[\frac{1+\phi}{\sigma_\alpha + \phi}(a_t - a_{t-1}) - \frac{\sigma_\alpha + \phi + \alpha(\omega - 1)\sigma_\alpha}{\sigma_\alpha + \phi}(y_t^* - y_{t-1}^*) \right]
\end{aligned}$$

The second observable is the devaluation of the domestic economy after adjusting for US CPI inflation, $\Delta\tilde{e}_t$. In the model, it is determined as:

$$
\begin{aligned}
\Delta\tilde{e}_t &= \Delta e_t + \pi_t^* \\
&= \pi_{H,t} + \Delta s_t \\
&= \left(\Gamma_{\pi u,t-1} + \sigma_\alpha \Gamma_{xu,t-1}\right)\epsilon_t^u - \sigma_\alpha \Gamma_{xu,t-2}\epsilon_{t-1}^u + \left(\Gamma_{\pi a,t-1} + \sigma_\alpha \Gamma_{xa,t-1}\right)\epsilon_t^a - \sigma_\alpha \Gamma_{xa,t-2}\epsilon_{t-1}^a \\
&+ \left(\Gamma_{\pi y^*,t-1} + \sigma_\alpha \Gamma_{xy^*,t-1}\right)\epsilon_t^{y^*} - \sigma_\alpha \Gamma_{xy^*,t-2}\epsilon_{t-1}^{y^*} + \Phi_{\pi,t-1} + \sigma_\alpha \Phi_{x,t-1} - \sigma_\alpha \Phi_{x,t-2} \\
&+ \sigma_\alpha \left[\frac{1+\phi}{\sigma_\alpha+\phi}(a_t - a_{t-1}) - \frac{\sigma_\alpha + \phi + \alpha(\omega-1)\sigma_\alpha}{\sigma_\alpha+\phi}(y_t^* - y_{t-1}^*)\right]
\end{aligned}
$$

The two observables, therefore, constitute the measurement equation which can be written in terms of the state variables from the robust Ramsey equilibrium, $\boldsymbol{\zeta}_t$, and their lagged counterpart, $\boldsymbol{\zeta}_{t-1}$. After accounting for measurement errors, the measurement equation is given as:

$$
\begin{bmatrix} \pi_t \\ \Delta\tilde{e}_t \end{bmatrix} = \tilde{\boldsymbol{H}} \cdot \begin{bmatrix} \boldsymbol{\zeta}_t \\ \boldsymbol{\zeta}_{t-1} \end{bmatrix} + \begin{bmatrix} \sigma_\pi^{me} & 0 \\ 0 & \sigma_{\Delta\tilde{e}}^{me} \end{bmatrix} \cdot \begin{bmatrix} \epsilon_t^{me,\pi} \\ \epsilon_t^{me,\Delta\tilde{e}} \end{bmatrix}
$$

The corresponding transition equation, to be used in the estimation, is given as:

$$
\begin{bmatrix} \boldsymbol{\zeta}_{t+1} \\ \boldsymbol{\zeta}_t \end{bmatrix} = \tilde{\boldsymbol{G}} \begin{bmatrix} \boldsymbol{\zeta}_t \\ \boldsymbol{\zeta}_{t-1} \end{bmatrix} + \boldsymbol{\nu}_{t+1}
$$

where the matrices $\tilde{\boldsymbol{H}}$ and $\tilde{\boldsymbol{G}}$ is given as:

$$\tilde{\boldsymbol{H}} = \begin{bmatrix} \Gamma_{\pi u} + \alpha\sigma_\alpha\Gamma_{xu} & \Gamma_{\pi u} + \sigma_\alpha\Gamma_{xu} \\ \Gamma_{\pi a} + \alpha\sigma_\alpha\Gamma_{xa} & \Gamma_{\pi a} + \sigma_\alpha\Gamma_{xa} \\ \Gamma_{\pi y^*} + \alpha\sigma_\alpha\Gamma_{xy^*} & \Gamma_{\pi y^*} + \sigma_\alpha\Gamma_{xy^*} \\ 0 & 0 \\ 0 & 0 \\ \alpha\sigma_\alpha\frac{1+\phi}{\sigma_\alpha+\phi} & \sigma_\alpha\frac{1+\phi}{\sigma_\alpha+\phi} \\ -\alpha\sigma_\alpha\frac{\sigma_\alpha+\phi+(\omega-1)\alpha\sigma_\alpha}{\sigma_\alpha+\phi} & -\sigma_\alpha\frac{\sigma_\alpha+\phi+(\omega-1)\alpha\sigma_\alpha}{\sigma_\alpha+\phi} \\ 1 & 1 \\ \alpha\sigma_\alpha & \sigma_\alpha \\ -\alpha\sigma_\alpha\Gamma_{xu} & -\sigma_\alpha\Gamma_{xu} \\ -\alpha\sigma_\alpha\Gamma_{xa} & -\sigma_\alpha\Gamma_{xa} \\ -\alpha\sigma_\alpha\Gamma_{xy^*} & -\sigma_\alpha\Gamma_{xy^*} \\ 0 & 0 \\ 0 & 0 \\ -\alpha\sigma_\alpha\frac{1+\phi}{\sigma_\alpha+\phi} & -\sigma_\alpha\frac{1+\phi}{\sigma_\alpha+\phi} \\ \alpha\sigma_\alpha\frac{\sigma_\alpha+\phi+(\omega-1)\alpha\sigma_\alpha}{\sigma_\alpha+\phi} & \sigma_\alpha\frac{\sigma_\alpha+\phi+(\omega-1)\alpha\sigma_\alpha}{\sigma_\alpha+\phi} \\ 0 & 0 \\ -\alpha\sigma_\alpha & -\sigma_\alpha \end{bmatrix}^T$$

$$\tilde{\boldsymbol{G}} = \begin{bmatrix} \boldsymbol{H} & \boldsymbol{0} \\ \boldsymbol{I} & \boldsymbol{0} \end{bmatrix}$$

and the vector $\boldsymbol{\nu}_{t+1}$ is:

$$\boldsymbol{\nu}_{t+1} = \begin{bmatrix} \boldsymbol{S}^\varepsilon \\ \boldsymbol{0} \end{bmatrix} \begin{bmatrix} \epsilon_{t+1}^u \\ \epsilon_{t+1}^a \\ \epsilon_{t+1}^y \end{bmatrix}$$

Table B.5: Parameters

Parameter	Definition of the parameter	Parameter value
(1) Parameters that govern the distorted SOE-NK equilibrium dynamics		
(1.1) Primary structural parameters		
β	Discount factor	C-Specific (0.99 for CAN)
ρ	Governs the natural rate of interest, $\rho = \beta^{-1} - 1$	C-Specific (0.01 for CAN)
σ	Governs the curvature of the utility function	1
α	Governs the degree of home bias or openness of the economy	C-Specific (0.20 for CAN)
η	Measures the substitutability between domestic and foreign goods	NA
ϵ	Measures the substitutability between different home varieties	6
γ	Measures the substitutability between goods produced in different foreign countries	NA
θ	Calvo-Yun parameter	0.75
φ	Governs the Frisch elasticity of labor supply	C-Specific (Estimated)
(1.2) Secondary structural parameters		
ω	Governs the effect of changes in the terms of trade on output, $\omega = \sigma\gamma + (1-\alpha)(\sigma\eta - 1)$	C-Specific (Estimated)
Θ	Related to ω, $\Theta = (\omega - 1)$	C-Specific (Comes from ω)
σ_α	Captures the same thing as ω, $\sigma_\alpha = \sigma/(1-\alpha+\alpha\omega)$	C-Specific (Comes from ω and α)
λ	Governs the parameter that defines the slope of the Phillips curve, $\lambda = (1-\beta\theta)(1-\theta)/\theta$	C-Specific (0.0858 for CAN)
κ_α	Slope of Phillips curve, $\kappa_\alpha = \lambda(\sigma_\alpha + \varphi)$	C-Specific (Comes from ω, λ and φ)
$\Lambda_{r,a}$	Governs the effect of the domestic TFP on the natural interest rate: $\frac{1+\varphi}{\sigma_\alpha+\varphi}$	C-Specific (Comes from ω and φ)
Λ_{r,y^*}	Governs the effect of the world output on the natural interest rate: $-\frac{\alpha\Theta\sigma_\alpha}{\sigma_\alpha+\varphi}$	C-Specific (Comes from ω and φ)
(1.3) Parameters that govern the shock processes		
ρ_u	Persistence of cost-push shock process	C-Specific (Estimated)
ρ_a	Persistence of TFP shock process	C-Specific (Estimated)
ρ_{y^*}	Persistence of world-output shock process	0.9261 (Using US output process)
σ_u	Standard deviation of cost-push shock process	C-Specific (Estimated)
σ_a	Standard deviation of TFP shock process	C-Specific (Estimated)
σ_{y^*}	Standard deviation of world-output shock process	0.0055 (Using US output process)
(2) Parameters that govern the central bank's problem		
λ_x	Governs the relative weight that the policymaker assigns to output stabilization, $\lambda_x = (1+\varphi)\lambda/\epsilon$	C-Specific (Comes from ϵ, λ and φ)
x^*	Governs the output gap	0
θ	Governs the robustness concern of the policymaker	C-Specific (Estimated)
(3) Parameters that govern measurement error		
σ_π^{me}	Standard deviation in the measurement error of CPI inflation	C-Specific (Estimated)
$\sigma_{\Delta e}^{me}$	Standard deviation in the measurement error of nominal devaluation plus world inflation	C-Specific (Estimated)

C Ramsey Optimal Dynamics under Natural Rate Shocks

Figure C.12: Dynamic Responses of Variables to the Domestic Productivity Shock, $\rho_a = 0$

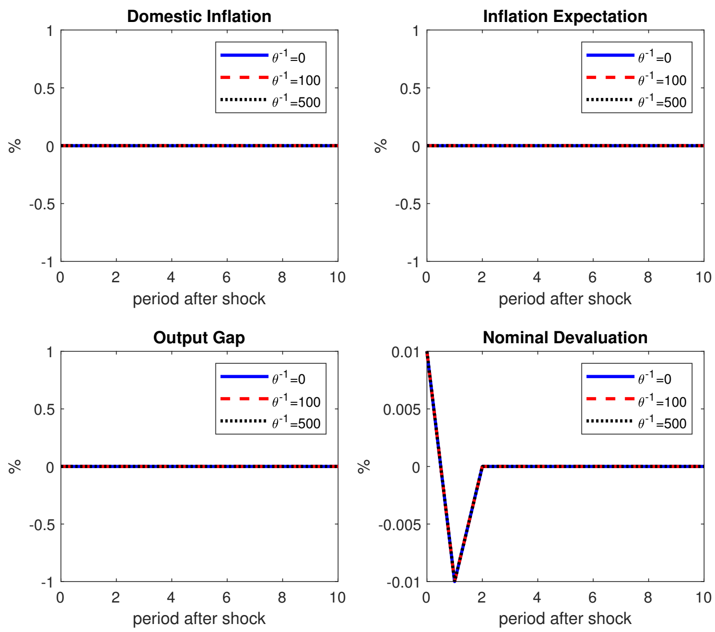


Figure C.13: Dynamic Responses of Variables to the Domestic Productivity Shock, $\rho_a = 0.9$

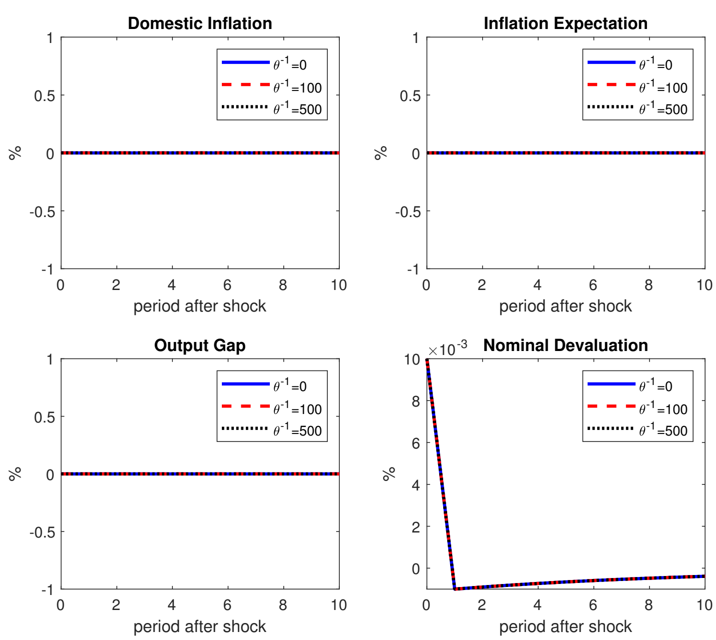


Figure C.14: Dynamic Responses of Variables to the World Output Shock, $\rho_{y^*} = 0$

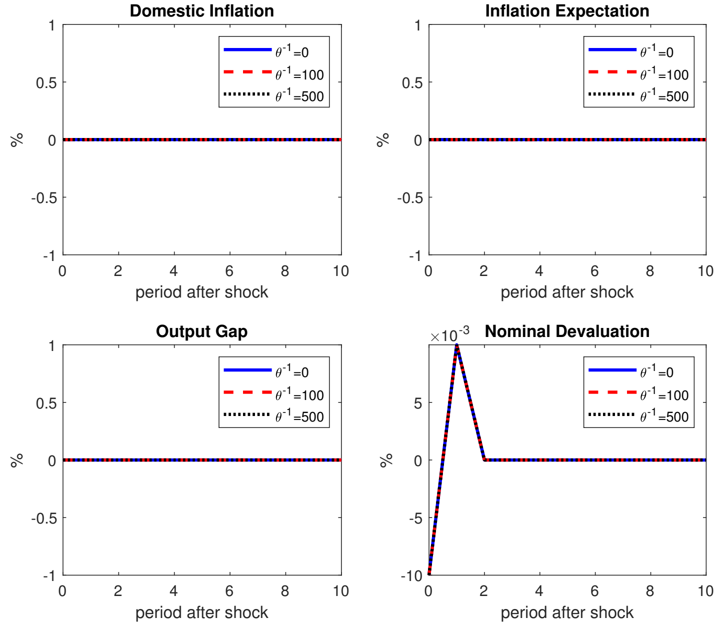
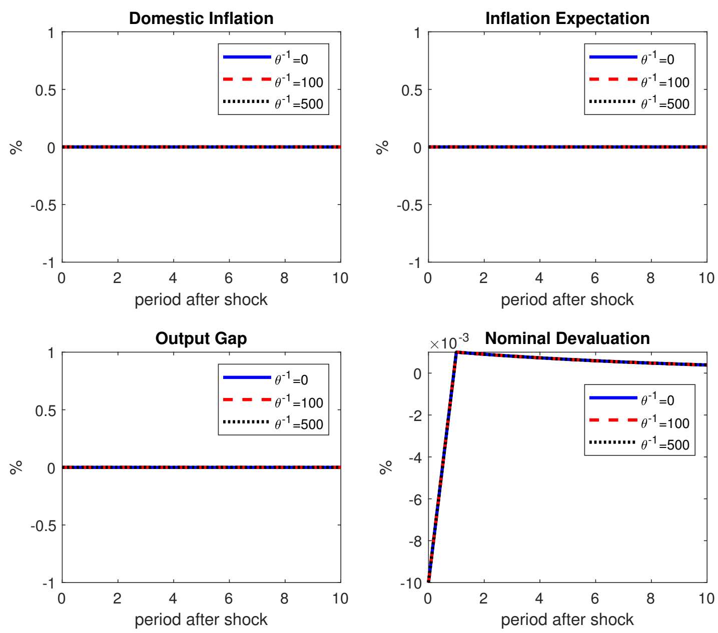


Figure C.15: Dynamic Responses of Variables to the World Output Shock $\rho_{y^*} = 0.9$



D Full Description of The Linear State Space System for Estimation

Figure D.16: Log Posteriors, Accepted Chains

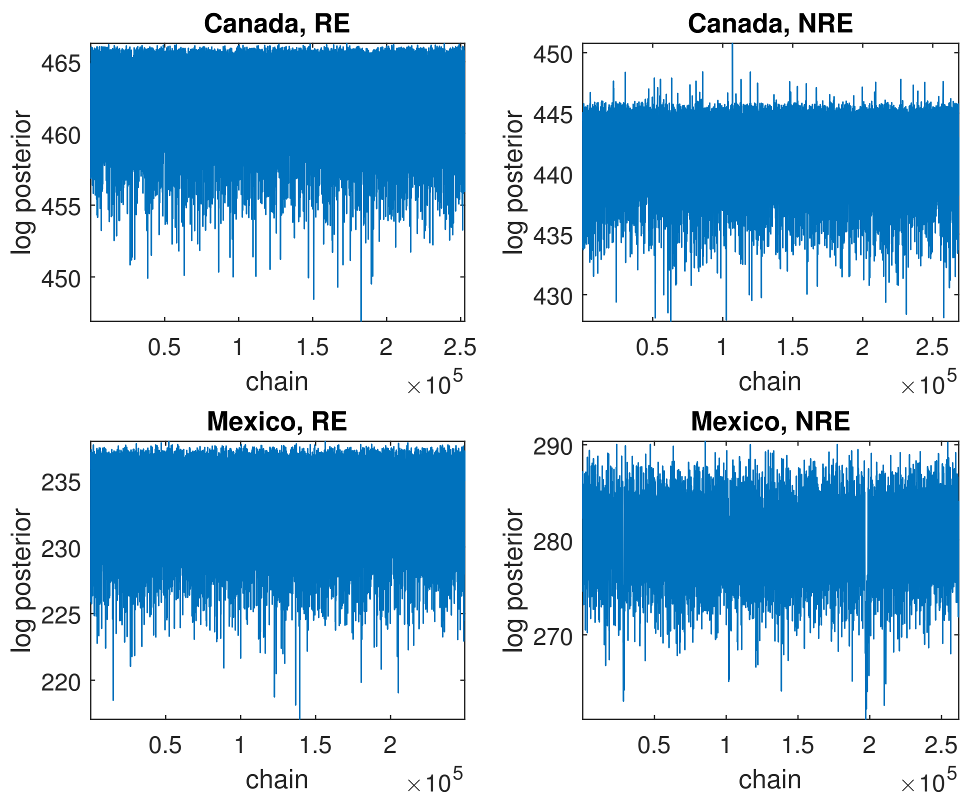


Figure D.17: Prior (red) and Posterior Distributions of the Structural Parameters in the RE Model, Canada

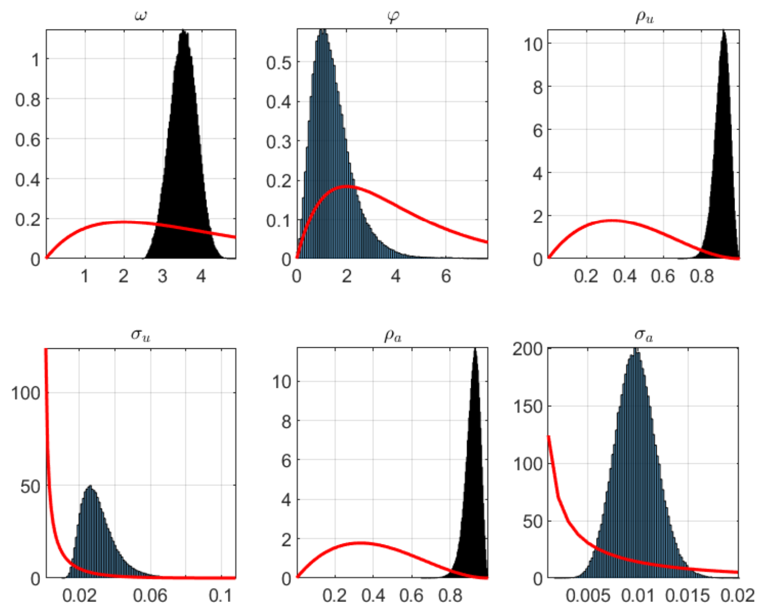


Figure D.18: Prior (red) and Posterior Distributions of the Structural Parameters in the RE Model, Mexico

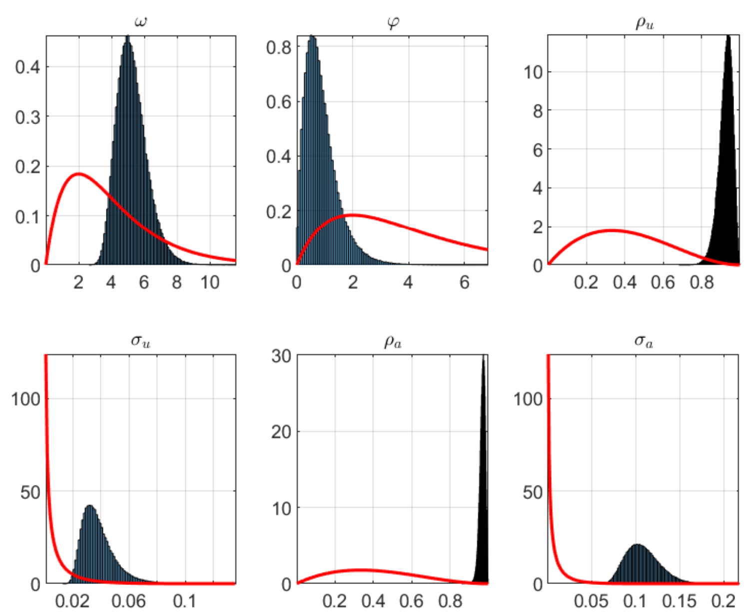


Figure D.19: Prior (red) and Posterior Distributions of θ^{-1} in the NRE Model, Canada

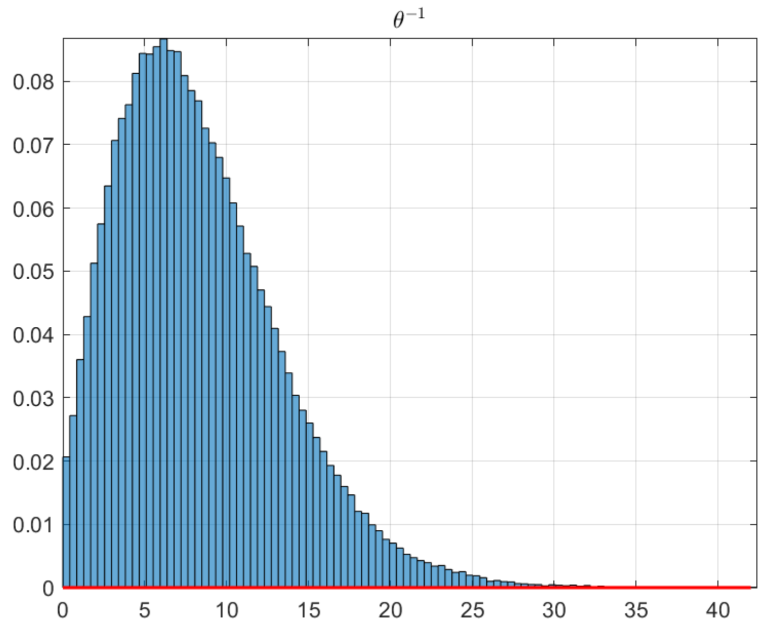


Figure D.20: Prior (red) and Posterior Distributions of Other Structural Parameters in the NRE Model, Canada

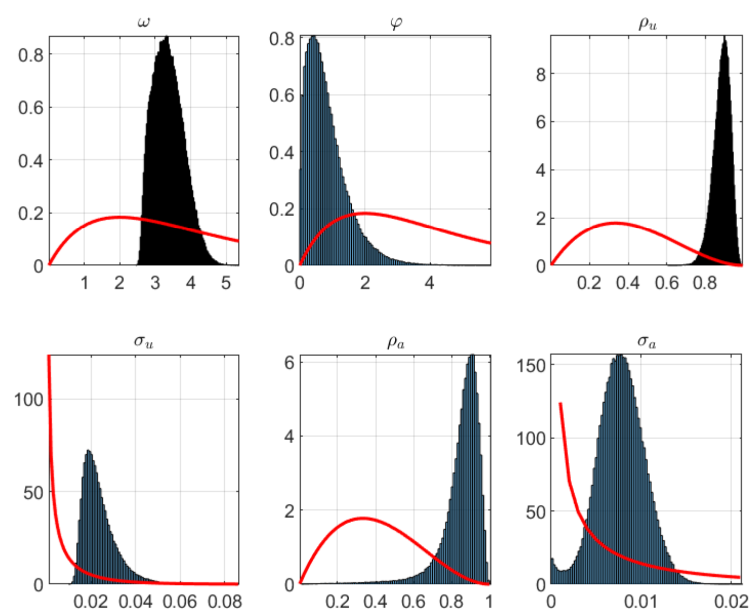


Figure D.21: Prior (red) and Posterior Distributions of θ^{-1} in the NRE Model, Mexico

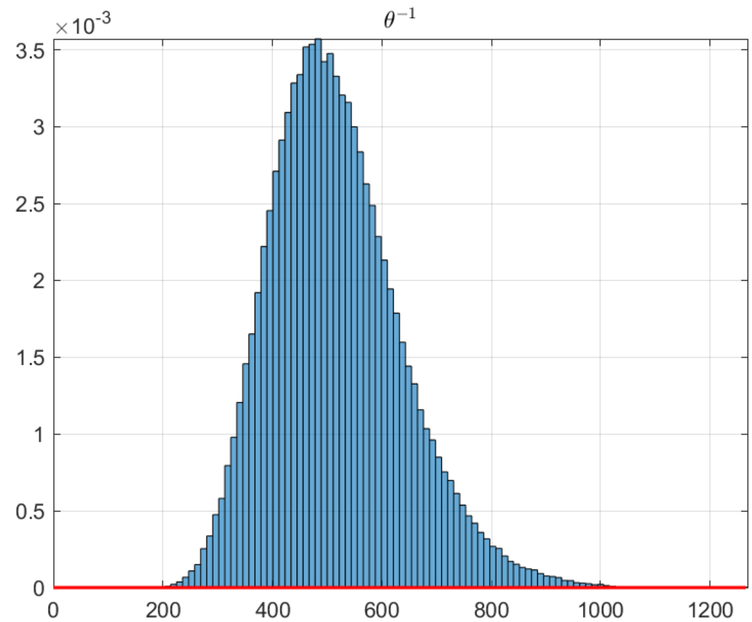


Figure D.22: Prior (red) and Posterior Distributions of Other Structural Parameters in the NRE Model, Mexico