Development of a Kalman filter based GPS satellite clock time-offset prediction algorithm

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***Abstract*— An enhanced deterministic model along with a stochastic model for describing clock noise is used to compute predictions of the time-offset of individual GPS satellites from the IGS rapid timescale. These are determined with significantly lower prediction uncertainties than may currently be obtained using the IGS ultra-rapid predictions. At prediction length of one day IGS prediction errors are commonly of the order of several ns for all GPS satellite clocks. In comparison the new techniques offers to limit prediction errors at the order of 1 ns for prediction lengths of one day in the newer generation Block IIR and IIF satellites. The factors contributing to the uncertainties in the IGS predictions are discussed. The application of a Kalman filter based prediction algorithm is shown to produce close to optimal predictions.**

I. INTRODUCTION

Accuracy of individual GPS satellite clock time-offset estimates from GPS system time is fundamental to the use of GPS technology for positioning, navigation and timing applications. Estimates of time differences between pairs of GPS satellite clocks are also valuable.

Clock prediction techniques for precise oscillators typically combine a model of deterministic properties such as time-offset, normalized frequency offset and linear frequency drift with a model describing the stochastic processes driving the phase and frequency of the oscillator [1]. Such a technique, commonly used for ground-based precise clocks, is applied here to space-borne clocks with some differences. For example, in the case of GPS satellite clocks, it has been shown in [2] that signals with periods of 6 and 12 hours must be modeled explicitly into a GPS space clock predictor to produce results suitable for high accuracy applications. One such method to account for periodic signals in a GPS satellite clock is presented in [3]. Real-time estimation of GPS satellite clocks has been investigated in [4, 5, 6]. Kalman filter based techniques for clock prediction have been adopted successfully [7].

In this paper a Kalman filter based method is described that has been developed to produce predictions of individual GPS satellite clock time-offsets relative to the IGS rapid timescale and predictions of the time differences between individual pairs of GPS clocks. We examine the predictability of individual GPS satellite clocks and also of the time differences between them. Section II describes features of the IGS clock predictions that could be limiting their quality. In section III the new prediction algorithm is discussed. Section IV and V show results of the technique in predicting individual clocks and the time difference between pairs of clocks respectively. Section VI discusses the potential impact of improved satellite clock predictions for position computation using the precise point positioning (PPP) strategy. Section VII concludes with a discussion.

II. CHARACTERISTICS OF THE IGS PREDICTIONS

The International GNSS Service (IGS) [8] provides GPS users with individual GPS satellite clock time-offset predictions with prediction lengths varying from 15 minutes to 1 day. These prediction sets are provided once every 6 hours as part of the IGS ultra-rapid clock products. A detailed performance analysis of this product has been carried out and an alternative method has been developed and shown here that aims to improve satellite clock prediction performance.

The study of the IGS satellite clock predictions during the period July 2011 to February 2012 revealed several characteristics:

- IGS prediction files are released every 6 hours with each new file providing satellite clock time-offset predictions relative to a new IGS ultra-rapid timescale and belonging to a new prediction run with its unique set of initial time-offset and normalized frequency offsets.
- Signals with periods of 12, 6, 4 and 3 hours are observed in many of the GPS satellite clocks.
- There is significant linear frequency drift in the rubidium satellite clocks.
- Large discrete changes in clock frequency occur in many satellite clocks. These are usually unique events, however these do occur with regularity in a few satellite clocks.

Some of these observed characteristics are shown in this paper for GPS PRN 16 which, in the time period considered in the analysis, corresponds to GPS satellite vehicle number (SVN) 56, a Block IIR satellite with a rubidium as the active onboard frequency standard. At the time of writing on 1 May 2012, there are 19 Block IIR satellites with active rubidium clocks out of 32 total GPS satellites in orbit.

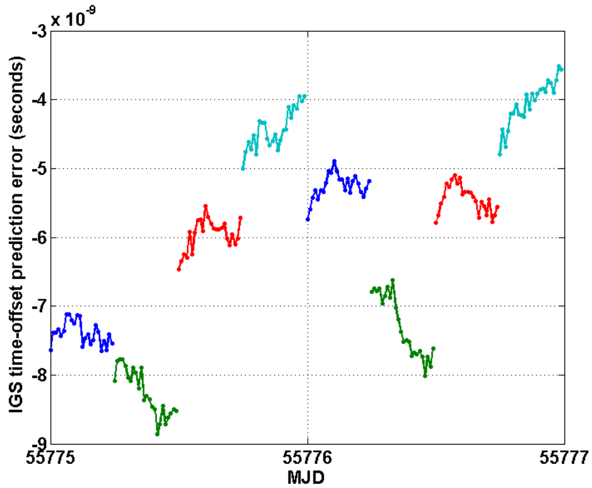

Figure 1 IGS prediction data sets.

Figure 1 shows the IGS ultra-rapid predictions of the clock on satellite PRN 16 using the first six hours of successive IGS ultra-rapid prediction sets. Large discrete changes in the prediction values occur at the boundary of each six-hour data block. This is due in part to a new ultra rapid timescale being used for each block, and in part to the discrete change of the start time of the prediction.

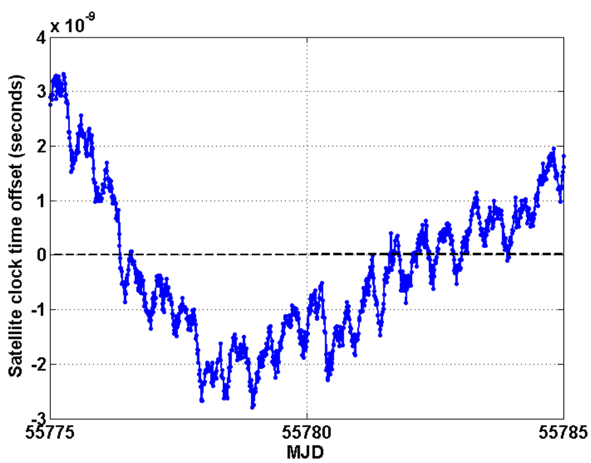

Figure 2 Time-offset from the IGS rapid timescale of GPS satellite PRN 16.

Figure 2 shows periodic signals with periods of multiples of half a sidereal day along with a significant linear frequency drift.

III. THE NEW PREDICTION ALGORITHM

A GPS satellite clock prediction algorithm has been developed. This uses both rapid (IGR) and ultra-rapid (IGU-O) IGS estimated clock data as inputs to a Kalman filter. The algorithm operates five times per day, four times on receipt of new IGU-O data sets and once on receipt of a new IGR clock data set. The stages in the algorithm processing are shown in figure 3.

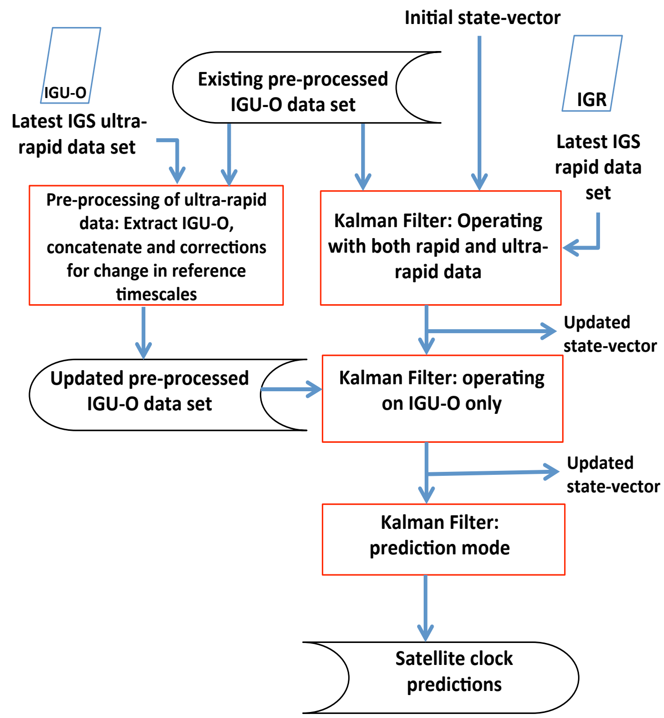

Figure 3 Overview of the new clock prediction algorithm

The ultra-rapid clock data is pre-processed, using data from all available satellites to correct for the resetting of both the time-offset and normalized frequency offset that occurs with each new ultra-rapid data block.

Standard Kalman filter equations [9] are used in this algorithm, with both IGR and IGU-O clock data being input at each epoch. Currently 15 minutes data spacing is used. At epochs where only ultra-rapid clock data is available the algorithm will input only this data set. The algorithm is designed so that estimates made at the most recent epochs will have made use of all available IGR and IGU-O clock data. Satellite clock predictions are obtained by extrapolating the state vector propagation equation and the parameter covariance matrix propagation equation in the absence of any input data.

The state vector, x, consists of the following components

$$x = (R_x, R_y, R_z, U_x, U_y, x_{p1} \ldots x_{p2n}, x_{m1} \ldots x_{mr}) \qquad (1)$$

where R_x, R_y and R_z are the time-offset, normalized frequency offset and linear frequency drift of the satellite clock from the IGS rapid timescale. U_x and U_y are the time and normalized frequency offsets between the rapid and the pre-processed ultra-rapid estimates of the satellite clock offsets. These are caused by errors accumulating in the ultra-rapid pre-processing technique. $x_{p1} \ldots x_{p2n}$ model n periodic clock signals, there are two state vector components per signal. $x_{m1} \ldots x_{mr}$ consists of r state vector components that model noise memory processes in the clock noise using either Markov or integrated Markov processes [7].

To obtain close to optimal performance, the values of the associated clock noise parameters, and initial values of the Kalman filter state vector and parameter covariance matrix, must be carefully determined. This may be achieved using a significant length of historical data. Up to one month of historical data has been used in this study.

The new prediction technique has the potential to offer the following performance improvements in comparison with the predictions of the IGS:

- Ability to cope with the realignment of the IGS ultra-rapid timescales with the release of each new IGU-O set.
- Effective identification and prediction of periodic features.
- Unbiased predictions in the presence of linear frequency drift.
- Improved predictions of individual GPS satellite clocks (in an RMS time-offset prediction error sense).
- Improved predictions of clock differences.

For White Frequency Modulation (WFM), that is the dominant short-term noise, the Prediction Error Deviation (PED) and ADEV are related by PED = ADEV*τ where τ is again both the prediction length and the averaging time. The two curves agree reasonably well at some averaging times. At averaging times of $3\text{x}10^4$ s, $6\text{x}10^4$ s and $3\text{x}10^5$ s the PED/τ values are significantly higher. This is due both to signals of period half a day as well as the clock linear frequency drift being correctly predicted but still contributing to the clock ADEV.

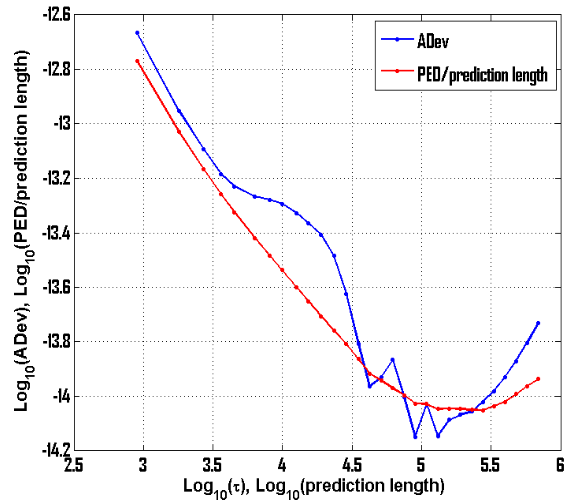

Figure 4 ADEV and prediction error plots for satellite PRN 16

Figure 4 shows plots of $\text{Log}_{10}(\text{ADEV})$, and $\text{Log}_{10}(\text{PED}/\tau)$ against $\text{Log}_{10}(\tau)$, where τ is both the prediction length and the averaging time.

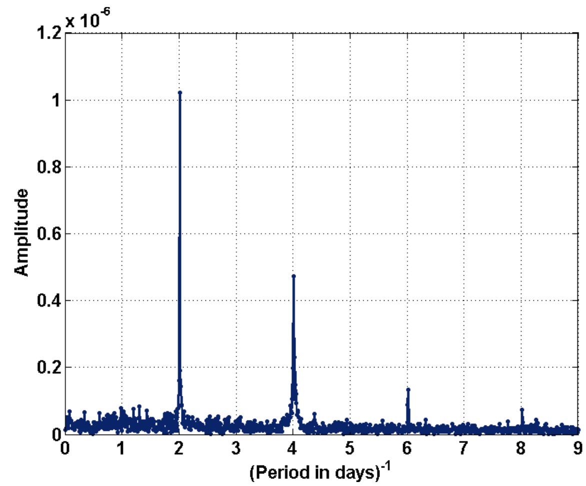

Figure 5 Results of applying a FFT to the Kalman filter residuals

The result of applying a Fast Fourier Transform (FFT) to the Kalman filter residuals is shown in figure 5, in the case where the filter contains no active periodic states. Signals with periods ½, ¼, 1/6, and 1/8 of a sidereal day are very clearly identified. This information is used then to include suitable state vector components in the clock model.

IV. PREDICTION OF INDIVIDUAL SATELLITE CLOCKS

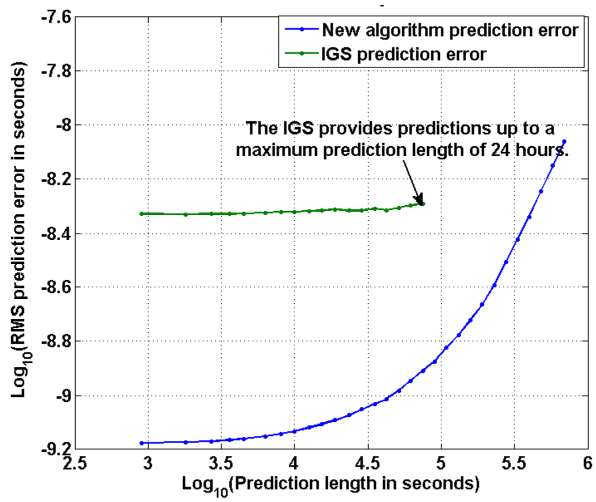

Figure 6 RMS prediction errors obtained using IGS ultra-rapid predictions and the new prediction techniques.

Figure 6 shows the prediction errors resulting from the IGS ultra-rapid predictions and from the new prediction technique on the PRN 16 clock. These results are typical of those obtained from most GPS satellite clocks. The IGS prediction errors are much larger than obtained using the new technique, and are not strongly dependent upon prediction length. In contrast, the new prediction technique produces prediction errors that are strongly dependent upon prediction length. There results are due primarily to the resetting of the IGS ultra-rapid timescale at each new computation of the IGS ultra-rapid predictions.

V. PREDICTION OF DIFFERENCES BETWEEN PAIRS OF CLOCKS

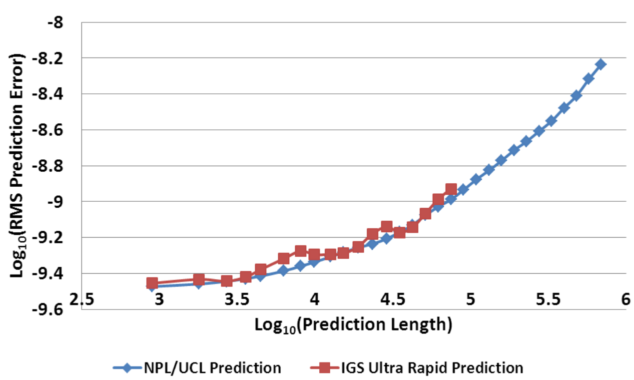

Figure 7 Prediction errors of (PRN16 – PRN15) clock differences

Figure 7 shows plots of RMS prediction error against prediction length on a log-log scale, obtained from (PRN16 – PRN15) clock differences. Results obtained from the new prediction technique and those obtained from the IGS predictions are much more similar to each other than in the case of individual clock predictions. The new prediction technique is noticeably better at prediction lengths of $8x10^3$ s, and $3x10^4$ s, the latter of these may be due to signals of period 1/2 day being better predicted. At prediction length of 1 day the new prediction technique is producing a small but significant improvement. Careful tuning of the Kalman filter, so that the clock models match closely the actual clock noise is required in order to achieve these results.

VI. PPP COMPUTATION USING CLOCK PREDICTIONS

One of the aims of the new satellite clock time-offset prediction algorithm is improved positioning performance. Figure 8 shows the results of a position domain analysis using the precise point positioning (PPP) strategy, as implemented in UCL's PPP software, to illustrate the impact of the predicted satellite clocks on positioning. The IGS station, Herstmonceux (HERT) was considered on 17 August 2011. The position solution using the GPS satellite clock predictions of the IGS product, igu16492_00.sp3, gives standard deviations of 0.590, 0.670 and 0.992 m in easting, northing and height, respectively.

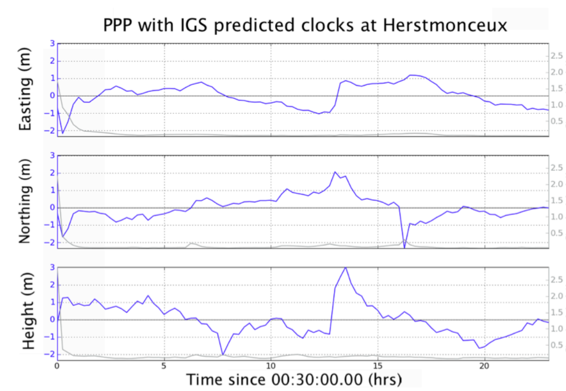

Figure 8 Coordinate time series from PPP with IGS predicted clocks.

Centimetre-level positioning is achieved with the IGS rapid and IGS final clock products (see figure 9). The aim of the this

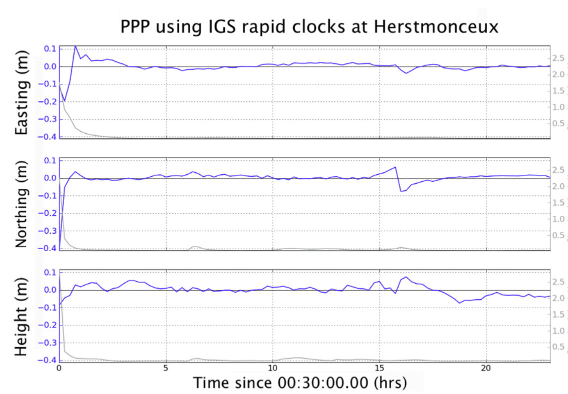

Figure 9 Coordinate time series from PPP with IGS rapid clocks.

new satellite clock prediction algorithm is to bridge the gap between the IGS rapid and the IGS predicted clock product to assist in real-time centimeter level precise point positioning. Figures 8 and 9 show significant differences in the positioning precision. This is at the centimeter level when using the post-processed IGR product, in comparison with decimeter level precision achieved using the clock predictions provided in the IGS predicted product.

VII. DISCUSSION

Typically, satellite clock predictions are computed by a simple linear (or polynomial) extrapolation using previous satellite clock solutions. A new clock prediction technique has been developed in this paper. The resulting enhanced deterministic model (accounting for periodic signals) combined with a stochastic model has been shown to significantly improve individual satellite clock prediction performance. This prediction improvement occurs at all prediction lengths, in comparison to predictions published by the IGS. The plot of prediction error against prediction length for PRN 16 shown in figure 6 is typical for all GPS satellite clocks. This shows prediction errors at the sub nano-second level, up to predictions length greater than $5x10^4$ seconds, whereas the IGS prediction errors are substantially greater at the level of a few nanoseconds.

Future work involves further development of the prediction algorithm to ensure that instances of missing data, outliers and abrupt changes can be dealt with. Development is also ongoing to enable the output of the prediction algorithm to plug in to the UCL PPP software.

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