Solution to Exercise 4

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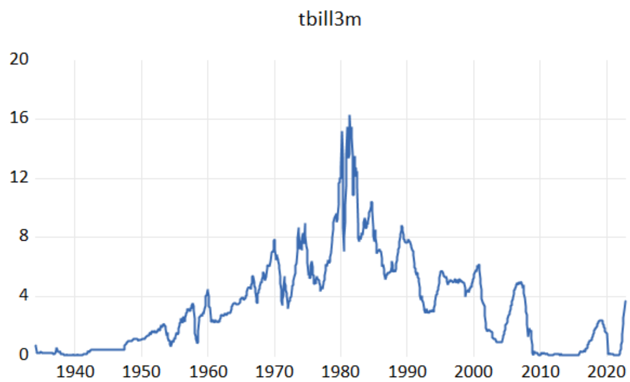
1. If we set α and β_1 to the same value, e.g., 1, then both series trend up by 1 each period. The correlograms look quite similar, but the time series plots don't really, even if we crank up the variance of the trend-stationary series. This is because the latter is stationary after de-trending (subtracting off $\beta_1 \cdot t$, whereas the random walk with drift remains nonstationary if we subtract $\alpha \cdot t$ (it becomes a random walk without drift).

2. (a) The time series plot and the correlogram are shown below.



Date: 11/17/22 Time: 17:42
Sample: 1934M01 2022M10
Included observations: 1066

Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob
		1	0.993	0.993	1054.3	0.000
		2	0.982	-0.329	2085.6	0.000
		3	0.971	0.168	3095.5	0.000
		4	0.961	-0.022	4086.1	0.000
		5	0.952	-0.003	5057.6	0.000
		6	0.941	-0.045	6009.3	0.000
		7	0.933	0.213	6945.6	0.000
		8	0.927	0.004	7870.7	0.000
		9	0.920	-0.142	8782.4	0.000
		10	0.910	-0.108	9675.5	0.000
		11	0.899	0.028	10548.	0.000
		12	0.888	-0.036	11401.	0.000
		13	0.879	0.108	12236.	0.000
		14	0.869	-0.104	13052.	0.000
		15	0.857	-0.069	13848.	0.000
		16	0.846	0.061	14624.	0.000
		17	0.835	-0.113	15381.	0.000
		18	0.824	-0.012	16118.	0.000
		19	0.811	-0.008	16833.	0.000
		20	0.798	0.092	17527.	0.000
		21	0.789	0.106	18205.	0.000
		22	0.782	0.048	18871.	0.000
		23	0.775	0.028	19527.	0.000
		24	0.768	0.009	20172.	0.000
		25	0.762	0.015	20807.	0.000
		26	0.756	0.026	21433.	0.000
		27	0.750	-0.047	22049.	0.000
		28	0.741	-0.008	22652.	0.000
		29	0.733	-0.016	23242.	0.000
		30	0.725	-0.027	23819.	0.000
		31	0.718	0.018	24386.	0.000
		32	0.711	0.055	24943.	0.000
		33	0.706	0.009	25491.	0.000
		34	0.701	0.012	26033.	0.000
		35	0.697	0.019	26569.	0.000
		36	0.693	0.054	27100.	0.000

From the correlogram, it looks like the T-Bill rate is I(1): the autocorrelations are large and decay slowly and approximately linearly. We can confirm this with an ADF test: open the series and click on `View→Unit Root Tests→Standard Unit Root Test...`. The data don't seem to have a trend[1], so we include just an intercept (because the mean is not zero). Choose automatic lag length selection, but switch to the AIC instead of the default BIC. This results in the following output.

[1] One could argue that there is a breaking trend, upwards until around 1982, downwards thereafter, but this kind of test is not available in EViews.

Null Hypothesis: TBILL3M has a unit root
Exogenous: Constant
Lag Length: 21 (Automatic - based on AIC, maxlag=30)

		t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic		-2.206152	0.2043
Test critical values:	1% level	-3.436395	
	5% level	-2.864098	
	10% level	-2.568183	

*MacKinnon (1996) one-sided p-values.

Augmented Dickey-Fuller Test Equation
Dependent Variable: D(TBILL3M)
Method: Least Squares
Date: 11/17/22 Time: 17:41
Sample (adjusted): 1935M11 2022M10
Included observations: 1044 after adjustments

Variable	Coefficient	Std. Error	t-Statistic	Prob.
TBILL3M(-1)	-0.007083	0.003210	-2.206152	0.0276
D(TBILL3M(-1))	0.428522	0.031217	13.72731	0.0000
D(TBILL3M(-2))	-0.192362	0.033870	-5.679419	0.0000
D(TBILL3M(-3))	0.067953	0.034329	1.979469	0.0480
D(TBILL3M(-4))	-0.058873	0.034422	-1.710351	0.0875
D(TBILL3M(-5))	0.129746	0.034477	3.763218	0.0002
D(TBILL3M(-6))	-0.219951	0.034516	-6.372418	0.0000
D(TBILL3M(-7))	0.015916	0.035015	0.454557	0.6495
D(TBILL3M(-8))	0.058508	0.034911	1.675922	0.0941
D(TBILL3M(-9))	0.122711	0.034920	3.514096	0.0005
D(TBILL3M(-10))	-0.031473	0.034841	-0.903338	0.3666
D(TBILL3M(-11))	0.117373	0.034680	3.384416	0.0007
D(TBILL3M(-12))	-0.137393	0.034856	-3.941757	0.0001
D(TBILL3M(-13))	0.060972	0.034933	1.745389	0.0812
D(TBILL3M(-14))	0.093066	0.034949	2.662912	0.0079
D(TBILL3M(-15))	-0.114104	0.035068	-3.253817	0.0012
D(TBILL3M(-16))	0.124406	0.034513	3.604620	0.0003
D(TBILL3M(-17))	-0.014426	0.034526	-0.417819	0.6762
D(TBILL3M(-18))	0.031963	0.034467	0.927367	0.3540
D(TBILL3M(-19))	-0.066898	0.034435	-1.942716	0.0523
D(TBILL3M(-20))	-0.082596	0.033888	-2.437344	0.0150
D(TBILL3M(-21))	-0.053385	0.031389	-1.700730	0.0893
C	0.026842	0.014594	1.839237	0.0662

R-squared	0.290413	Mean dependent var	0.003372
Adjusted R-squared	0.275124	S.D. dependent var	0.362986
S.E. of regression	0.309045	Akaike info criterion	0.511124
Sum squared resid	97.51447	Schwarz criterion	0.620194
Log likelihood	-243.8069	Hannan-Quinn criter.	0.552492
F-statistic	18.99390	Durbin-Watson stat	2.001414
Prob(F-statistic)	0.000000		

The observed test statistic is -2.206, larger than the critical value -2.86, so the test does not reject the null that the data are I(1), as expected. The same conclusion can be drawn from the p-value of 0.2043[2].

(b) We start by inspecting the correlogram of the first difference. This can either be done

[2] Please be aware that in an exam, I might delete the top of the EViews output, and give you only the test regression at the bottom. Note that the p-value of the t-statistic there is wrong; it corresponds to the p-value from a standard regression with stationary variables, i.e., it's from a normal distribution instead of the Dickey-Fuller distribution. You can try this yourself by running the ADF regression manually, by entering `d(TBILL3M) c TBILL3M(-1) d(TBILL3M(-1)) ...TBILL3M(-21)` under `Quick→Estimate Equation...`.

by creating a new series via `genr DTBILL3M = d(TBILL3M)` and inspecting its correlogram, or by opening `TBILL3M` (i.e., the series in levels), and then clicking `View`→ `Correlogram...`, and specifying that you want the correlogram of the first difference.

Date: 11/18/22 Time: 14:36
Sample (adjusted): 1934M02 2022M10
Included observations: 1065 after adjustments

Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob
		1	0.340	0.340	123.72	0.000
		2	-0.059	-0.198	127.44	0.000
		3	-0.064	0.032	131.85	0.000
		4	-0.009	-0.007	131.94	0.000
		5	0.043	0.044	133.93	0.000
		6	-0.166	-0.236	163.56	0.000
		7	-0.155	0.012	189.39	0.000
		8	0.093	0.138	198.64	0.000
		9	0.206	0.107	244.33	0.000
		10	0.080	-0.043	251.20	0.000
		11	-0.013	0.043	251.38	0.000
		12	-0.098	-0.131	261.78	0.000
		13	0.053	0.120	264.77	0.000
		14	0.117	0.050	279.63	0.000
		15	-0.067	-0.063	284.55	0.000
		16	0.006	0.116	284.59	0.000
		17	0.058	-0.002	288.24	0.000
		18	0.082	-0.000	295.45	0.000
		19	-0.033	-0.108	296.66	0.000
		20	-0.218	-0.109	348.57	0.000
		21	-0.162	-0.056	377.15	0.000
		22	-0.042	-0.034	379.06	0.000
		23	-0.014	-0.013	379.28	0.000
		24	-0.033	-0.020	380.50	0.000
		25	-0.031	-0.036	381.52	0.000
		26	0.066	0.053	386.24	0.000
		27	0.129	-0.007	404.52	0.000
		28	0.028	0.021	405.37	0.000
		29	-0.055	0.023	408.72	0.000
		30	-0.059	-0.025	412.48	0.000
		31	-0.069	-0.060	417.68	0.000
		32	-0.038	-0.017	419.30	0.000
		33	-0.065	-0.013	423.99	0.000
		34	-0.074	-0.027	430.10	0.000
		35	-0.042	-0.060	432.07	0.000
		36	-0.027	-0.015	432.89	0.000

The first difference looks stationary. After some specification search[3], we find that an ARMA(5, 5) model is the most adequate, even though it doesn't remove the autocorrelation completely. Estimating this model (either as `d(TBILL3M) c AR(1 to 5) MA(1 to 5)`, or if you created a new variable for the difference above, `DTBILL3M c AR(1 to 5) MA(1 to 5)`) yields the output and correlogram below.

[3] `freeze(mode=overwrite, armatable) tbill3m.autoarma(tform=none, diff=1, select=sic, maxar=10, maxma=10, atable) forec c`

Dependent Variable: D(TBILL3M)
Method: ARMA Maximum Likelihood (OPG - BHHH)
Date: 11/18/22 Time: 14:41
Sample: 1934M02 2022M10
Included observations: 1065
Convergence achieved after 233 iterations
Coefficient covariance computed using outer product of gradients

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.002897	0.013904	0.208373	0.8350
AR(1)	0.922979	0.049452	18.66408	0.0000
AR(2)	-1.062794	0.076221	-13.94354	0.0000
AR(3)	0.002677	0.090163	0.029689	0.9763
AR(4)	0.176621	0.066271	2.665159	0.0078
AR(5)	-0.500188	0.031151	-16.05682	0.0000
MA(1)	-0.511548	0.051091	-10.01258	0.0000
MA(2)	0.662301	0.059644	11.10430	0.0000
MA(3)	0.460269	0.053463	8.609122	0.0000
MA(4)	-0.249555	0.041143	-6.065538	0.0000
MA(5)	0.526464	0.026709	19.71112	0.0000
SIGMASQ	0.099211	0.001604	61.86396	0.0000

R-squared	0.232134	Mean dependent var	0.002817
Adjusted R-squared	0.224113	S.D. dependent var	0.359617
S.E. of regression	0.316767	Akaike info criterion	0.550842
Sum squared resid	105.6592	Schwarz criterion	0.606850
Log likelihood	-281.3231	Hannan-Quinn criter.	0.572063
F-statistic	28.93938	Durbin-Watson stat	1.998371
Prob(F-statistic)	0.000000		

Inverted AR Roots	.65-.65i	.65+.65i	.15-.93i	.15+.93i
	-.66			
Inverted MA Roots	.59-.69i	.59+.69i	.09-.87i	.09+.87i
	-.84			

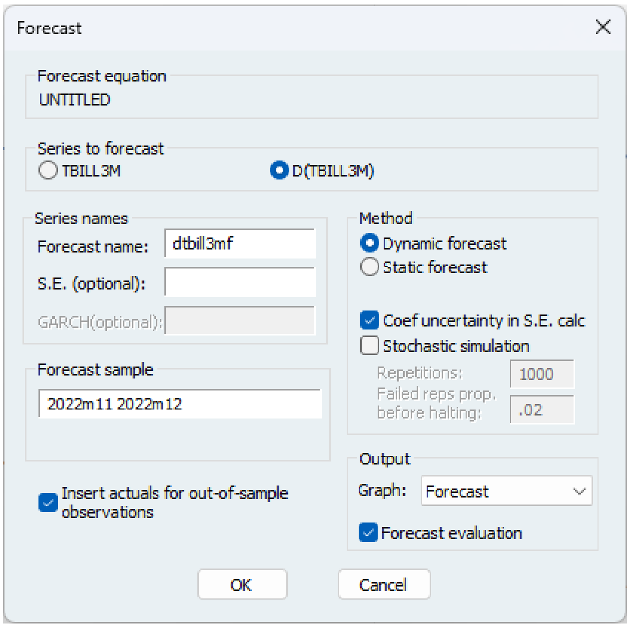
Date: 11/18/22 Time: 14:42
Sample (adjusted): 1934M02 2022M10
Q-statistic probabilities adjusted for 10 ARMA terms

Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob
		1	-0.000	-0.000	7.E-05	
		2	0.011	0.011	0.1211	
		3	-0.005	-0.005	0.1531	
		4	-0.013	-0.013	0.3309	
		5	0.032	0.032	1.4183	
		6	-0.023	-0.023	2.0070	
		7	-0.011	-0.012	2.1431	
		8	0.053	0.054	5.1272	
		9	-0.021	-0.020	5.5833	
		10	-0.041	-0.044	7.3512	
		11	0.116	0.120	21.940	0.000
		12	-0.024	-0.023	22.576	0.000
		13	0.008	-0.001	22.644	0.000
		14	0.121	0.130	38.449	0.000
		15	-0.089	-0.090	47.001	0.000
		16	0.093	0.081	56.292	0.000
		17	-0.037	-0.024	57.798	0.000
		18	-0.015	-0.016	58.046	0.000
		19	-0.023	-0.043	58.645	0.000
		20	-0.108	-0.093	71.399	0.000
		21	-0.064	-0.064	75.860	0.000
		22	-0.026	-0.050	76.599	0.000
		23	-0.042	-0.026	78.567	0.000
		24	-0.005	-0.012	78.592	0.000
		25	-0.028	-0.058	79.464	0.000
		26	0.014	0.049	79.687	0.000
		27	0.064	0.039	84.198	0.000
		28	0.006	0.008	84.242	0.000
		29	-0.012	0.014	84.413	0.000
		30	0.001	-0.024	84.414	0.000
		31	-0.086	-0.050	92.601	0.000
		32	-0.030	-0.033	93.581	0.000
		33	-0.030	-0.010	94.557	0.000
		34	-0.040	-0.024	96.349	0.000
		35	-0.041	-0.049	98.177	0.000
		36	-0.066	-0.041	103.04	0.000

Some autocorrelation remains, but we won't bother to model it. So our final model for Δ TBILL3M is an ARMA(5, 5). This means that the levels TBILL3M follow an ARIMA(5, 1, 5) model.

(c) To forecast an ARIMA model, we first forecast the ARMA model for the first difference. We'll spare us the hassle of doing a manual forecast for this complicated model, and just use EViews for it. Extending the workfile to 2022M12 via `Proc`→`Structure / Resize Current Page...`, we can forecast by clicking `Forecast` on the estimated equation and setting the options as follows[4]:

[4] Note that because I estimated my model using `d(TBILL3M) c AR(1 to 5) MA(1 to 5)`, EViews gives me the option to directly predict the levels, but we'll ignore this here.



The forecasts are $\widehat{\Delta\text{TBILL}}_{2022M11} = 0.1636$ and $\widehat{\Delta\text{TBILL}}_{2022M12} = -0.0328$. The T-Bill rate in 2022M10 was 3.72, so the corresponding forecasts for the levels are

$$\widehat{\text{TBILL}}_{2022M11} = 3.72 + 0.1636 = 3.8836$$

and

$$\widehat{\text{TBILL}}_{2022M12} = 3.8836 - 0.0328 = 3.8508.$$

3. (a) We begin by creating the excess returns Note that the T-Bill rate is quoted in percent and in annualized terms, so we have to convert it to daily log returns first. The commands are

```
genr rf = log(1+dtb3/100)/365
genr r = dlog(ibm)-rf
genr rm = dlog(spx)-rf
```

Running the CAPM regression of `r` on `rm` and an intercept results in the following output.

Dependent Variable: R
Method: Least Squares
Date: 11/18/22 Time: 18:17
Sample (adjusted): 12/29/2015 11/16/2022
Included observations: 1735 after adjustments

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	-0.000279	0.000294	-0.948949	0.3428
RM	0.851947	0.024060	35.40913	0.0000

R-squared	0.419782	Mean dependent var	1.88E-05
Adjusted R-squared	0.419447	S.D. dependent var	0.016051
S.E. of regression	0.012230	Akaike info criterion	-5.968706
Sum squared resid	0.259208	Schwarz criterion	-5.962414
Log likelihood	5179.853	Hannan-Quinn criter.	-5.966379
F-statistic	1253.806	Durbin-Watson stat	1.979212
Prob(F-statistic)	0.000000		

The intercept (usually called α) is insignificant as predicted by the theory, and the CAPM β of IBM is .852.

(b) The DW statistic is 1.9792. This value is between $d_u = 1.78$ and 4, so we don't reject the null of no first order autocorrelation.

(c) In the estimation output, go to `View→Residual Diagnostics→ Serial Correlation LM Test`, and include 5 lags. This results in the output below.

Breusch-Godfrey Serial Correlation LM Test:
Null hypothesis: No serial correlation at up to 5 lags

F-statistic	0.941334	Prob. F(5,1728)	0.4530
Obs*R-squared	4.712901	Prob. Chi-Square(5)	0.4519

Test Equation:
Dependent Variable: RESID
Method: Least Squares
Date: 11/18/22 Time: 18:27
Sample: 12/29/2015 11/16/2022
Included observations: 1735
Presample missing value lagged residuals set to zero.

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	-1.19E-06	0.000294	-0.004058	0.9968
RM	0.001882	0.024087	0.078140	0.9377
RESID(-1)	0.009036	0.024052	0.375695	0.7072
RESID(-2)	-0.029789	0.024061	-1.238089	0.2159
RESID(-3)	-0.020329	0.024062	-0.844837	0.3983
RESID(-4)	-0.016857	0.024059	-0.700651	0.4836
RESID(-5)	-0.033360	0.024081	-1.385292	0.1661

R-squared	0.002716	Mean dependent var	-6.40E-20
Adjusted R-squared	-0.000746	S.D. dependent var	0.012226
S.E. of regression	0.012231	Akaike info criterion	-5.965662
Sum squared resid	0.258504	Schwarz criterion	-5.943639
Log likelihood	5182.212	Hannan-Quinn criter.	-5.957518
F-statistic	0.784445	Durbin-Watson stat	2.000099
Prob(F-statistic)	0.582073		

There are two versions of the test, the F-Test and the LM test. We'll focus on the LM test. The observed test statistic is $T \cdot R^2_{aux} = 1735 \cdot 0.002716 = 4.71$, which doesn't exceed the critical value of 11.07 from the $\chi^2(5)$ distribution, so we don't reject the null of no serial correlation of order 5 or less. The same conclusion can be drawn from the top of the output, but that might get deleted in an exam.

(d) Selecting HAC standard errors in the `Options` tab of the estimation window results in the following.

Dependent Variable: R
Method: Least Squares
Date: 11/18/22 Time: 18:38
Sample (adjusted): 12/29/2015 11/16/2022
Included observations: 1735 after adjustments
HAC standard errors & covariance (Bartlett kernel, Newey-West fixed bandwidth = 8.0000)

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	-0.000279	0.000283	-0.986500	0.3240
RM	0.851947	0.038631	22.05368	0.0000

R-squared	0.419782	Mean dependent var	1.88E-05
Adjusted R-squared	0.419447	S.D. dependent var	0.016051
S.E. of regression	0.012230	Akaike info criterion	-5.968706
Sum squared resid	0.259208	Schwarz criterion	-5.962414
Log likelihood	5179.853	Hannan-Quinn criter.	-5.966379
F-statistic	1253.806	Durbin-Watson stat	1.979212
Prob(F-statistic)	0.000000	Wald F-statistic	486.3648
Prob(Wald F-statistic)	0.000000		

As you can see, the difference isn't that big here, because there was little autocorrelation to begin with. Generally, if there is autocorrelation, then the HAC standard errors will be larger, i.e., the regular standard errors underestimate the variance of the estimates.

4. (a) For $Y_{1,t}$, we have

$$\begin{aligned}\mathbb{E}[\Delta Y_{1,t}] &= \mathbb{E}[Y_{1,t} - Y_{1,t-1}] \\ &= \mathbb{E}[\delta t + U_{1,t} - (\delta(t-1) + U_{1,t-1})] \\ &= \delta + \mathbb{E}[U_{1,t} - U_{1,t-1})] \\ &= \delta.\end{aligned}$$

For $Y_{2,t}$,

$$\begin{aligned}\mathbb{E}[\Delta Y_{2,t}] &= \mathbb{E}[Y_{2,t} - Y_{2,t-1}] \\ &= \mathbb{E}[\delta + Y_{2,t-1} + U_{2,t} - Y_{2,t-1}] \\ &= \mathbb{E}[\delta + U_{2,t}] \\ &= \delta.\end{aligned}$$

(b) Consider the AR(2) process

$$Y_t = \phi_1 Y_{t-1} + \phi_2 Y_{t-2} + U_t.$$

We would like to test the null that $\phi_1 + \phi_2 = 1$ (unit root) vs. $\phi_1 + \phi_2 < 1$ (stationarity). This can be done by rearranging the equation as follows:

$$\begin{aligned}Y_t &= \phi_1 Y_{t-1} + \phi_2 Y_{t-2} + U_t & &| - Y_{t-1} \\ Y_t - Y_{t-1} &= (\phi_1 - 1)Y_{t-1} + \phi_2 Y_{t-2} + U_t & &| \pm \phi_2 Y_{t-1} \\ Y_t - Y_{t-1} &= (\phi_1 - 1)Y_{t-1} + \phi_2 Y_{t-1} - \phi_2 Y_{t-1} + \phi_2 Y_{t-2} + U_t \\ Y_t - Y_{t-1} &= (\phi_1 + \phi_2 - 1)Y_{t-1} - \phi_2 \Delta Y_{t-1} + U_t \\ \Delta Y_t &= \psi Y_{t-1} + \alpha_1 \Delta Y_{t-1} + U_t,\end{aligned}$$

where $\psi := (\phi_1 + \phi_2 - 1)$ and $\alpha_1 := -\phi_2$. Thus testing $\phi_1 + \phi_2 = 1$ vs. $\phi_1 + \phi_2 =< 1$ is equivalent to testing $\psi = 0$ vs. $\psi < 0$ in a regression of ΔY_t onto Y_{t-1}, augmented by one lag of ΔY_{t-1}.