Solution to Exercise 6

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1. (a) We start by generating the log returns as usual. We also create the negative returns for plotting later.

```
genr r = dlog(sp500)
genr negative_returns = -r
```

The historical VaR is then obtained via

```
genr var_hist = -@quantile(r, 0.01)
```

Notice that this uses the whole sample instead of the last 250 days for each t. Doing that would require us to write a loop.

(b) The normal VaR is generated as

```
genr var_norm = -@mean(r)-@stdev(r)*@qnorm(0.01)
```

(c) For the GARCH VaR, we estimate a GARCH(1, 1) as usual, first using normal errors. We then save the predicted variances by clicking `Proc→Make GARCH Variance Series...` and entering `garchn_sig2` as the series name. We then convert the variances into volatilities and compute the VaR, using the commands

```
genr garchn_sig = @sqrt(garchn_sig2)
genr var_garch_n = -c(1)-garchn_sig * @qnorm(0.01)
```

Here, `c(1)` refers to the estimated intercept in the mean equation. We then re-estimate the model using standardized t innovations (just choose it in the GARCH specification dialog). This results in the estimated model below.

Dependent Variable: R
Method: ML ARCH - Student's t distribution (BFGS / Marquardt steps)
Date: 12/01/22 Time: 17:14
Sample (adjusted): 10/30/2012 10/27/2022
Included observations: 2546 after adjustments
Convergence achieved after 27 iterations
Coefficient covariance computed using outer product of gradients
Presample variance: backcast (parameter = 0.7)
GARCH = C(2) + C(3)*RESID(-1)^2 + C(4)*GARCH(-1)

Variable	Coefficient	Std. Error	z-Statistic	Prob.
C	0.000904	0.000126	7.176877	0.0000
		Variance Equation		
C	2.92E-06	6.17E-07	4.743725	0.0000
RESID(-1)^2	0.221296	0.026849	8.242232	0.0000
GARCH(-1)	0.773192	0.023233	33.27982	0.0000
T-DIST. DOF	5.275193	0.568869	9.273121	0.0000

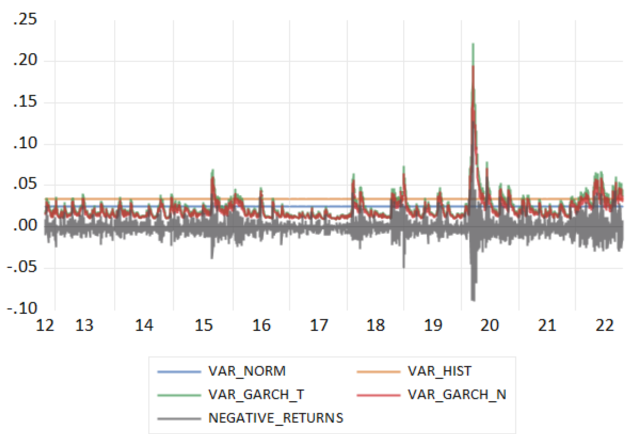
R-squared	-0.002211	Mean dependent var	0.000390
Adjusted R-squared	-0.002211	S.D. dependent var	0.010946
S.E. of regression	0.010958	Akaike info criterion	-6.810961
Sum squared resid	0.305579	Schwarz criterion	-6.799488
Log likelihood	8675.354	Hannan-Quinn criter.	-6.806800
Durbin-Watson stat	2.285926		

The estimated degrees of freedom are 5.28, which corresponds to fairly heavy tails. We then save the GARCH variance series as before, choosing the name `garcht_sig2`. Then we convert the variances into volatilities and compute the VaR, using the commands

```
genr garcht_sig = @sqrt(garcht_sig2)
genr var_garch_t = -c(1)-garcht_sig *
                      @sqrt((c(5)-2)/c(5))*@qtdist(0.01, c(5))
```

Note that the second command should be entered on a single line. Also, `c(5)` corresponds to the estimated degrees of freedom.

(d) The plot below can be generated by opening the four VaR predictions, together with the negative returns, as a group.



2. The four hit series are generated as follows:

```
genr hit_norm = (negative_returns > var_norm)
genr hit_hist = (negative_returns > var_hist)
genr hit_garchn = (negative_returns > var_garch_n)
genr hit_garcht = (negative_returns > var_garch_t)
```

Each of the hit series will be equal to one at time t if the negative return on that day exceeded the respective VaR estimate. We begin by testing, for each hit series, if the proportion of VaR violations is significantly different from zero. For example, for the historical VaR, this works as follows: compute $\hat{\pi} = T_1/T = \frac{1}{T}\sum_{t=1}^{T} I_t$ via

```
scalar p_hist = @mean(hit_hist)
```

This yields $\hat{\pi} = 0.009819$. From this, we can compute the t-statistic

$$t = \frac{\hat{\pi} - p}{\sqrt{\hat{\pi}(1-\hat{\pi})/T}} = \frac{0.009819 - 0.01}{\sqrt{0.009819(1-0.009819)/2547}} = -0.09264.$$

Comparing this with the critical value ± 1.96, we see that $\hat{\pi}$ is not significantly different from p, hence the model has correct unconditional coverage. Note that this is **by construction**, because we defined the the historical VaR as a sample quantile. So by construction, exactly 1% of the negative returns should exceed the VaR. Alteratively, we can just regress $I_t - 0.01$ onto an intercept by entering the regression specification

```
(hit_hist - 0.01) c
```

and test whether the intercept is significant. The output is

Dependent Variable: HIT_HIST-0.01
Method: Least Squares
Date: 12/02/22 Time: 17:59
Sample (adjusted): 10/30/2012 10/27/2022
Included observations: 2546 after adjustments

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	-0.000181	0.001955	-0.092437	0.9264

R-squared	0.000000	Mean dependent var	-0.000181
Adjusted R-squared	0.000000	S.D. dependent var	0.098624
S.E. of regression	0.098624	Akaike info criterion	-1.794608
Sum squared resid	24.75452	Schwarz criterion	-1.792314
Log likelihood	2285.536	Hannan-Quinn criter.	-1.793776
Durbin-Watson stat	1.939040		

We see that the t-statistic obtained this way is almost the same (the two tests are asymptotically equivalent), so we arrive at the same conclusion.

To test for independence, we regress $I_t - 0.01$ on an intercept and I_{t-1}, by entering the regression specification

```
(hit_hist) c hit_hist(-1)
```

The output is

Dependent Variable: HIT_HIST-0.01
Method: Least Squares
Date: 12/02/22 Time: 18:52
Sample (adjusted): 10/31/2012 10/27/2022
Included observations: 2545 after adjustments

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	-0.000476	0.001964	-0.242398	0.8085
HIT_HIST(-1)	0.030476	0.019821	1.537573	0.1243

R-squared	0.000929	Mean dependent var	-0.000177
Adjusted R-squared	0.000536	S.D. dependent var	0.098643
S.E. of regression	0.098617	Akaike info criterion	-1.794362
Sum squared resid	24.73143	Schwarz criterion	-1.789771
Log likelihood	2285.326	Hannan-Quinn criter.	-1.792697
F-statistic	2.364130	Durbin-Watson stat	2.011661
Prob(F-statistic)	0.124278		

Independence can be tested by checking if b_1, the coefficient on the lagged hit, is significant at the 5% level. Here, surprisingly, it isn't, so we don't reject the null of independence of the VaR violations. Finally, we can use an F-test for $H_0 : b_0 = b_1 = 0$ to test the correctness of the conditional coverage. Note that we cannot use the F-statistic provided in the output above, because that is for a test of the null that all coefficients *except the intercept* are significant. Instead, go to `View→Coefficient Diagnostics→Wald Test`, and enter the restrictions

```
c(1)=0, c(2)=0
```

The result is shown below.

Wald Test:
Equation: Untitled

Test Statistic	Value	df	Probability
F-statistic	1.186156	(2, 2543)	0.3056
Chi-square	2.372311	2	0.3054

Null Hypothesis: C(1)=0, C(2)=0
Null Hypothesis Summary:

Normalized Restriction (= 0)	Value	Std. Err.
C(1)	-0.000476	0.001964
C(2)	0.030476	0.019821

Restrictions are linear in coefficients.

The degrees of freedom of the F-test are 2 and $T - 2$. From the table, we find that the critical value is 3, which is not exceeded by the observed value 1.19. Hence, the null of correct conditional coverage is resoundingly rejected. I should note that this is **highly unusual**; typically, the historical VaR will fail the independence and conditional coverage tests, because the fact that it doesn't react to changes in volatility means that violations will cluster during crises, making them dependent.

Repeating the analysis for the other hit series results in the table below.

	Hist	Norm	GARCHn	GARCHt
$\hat{\pi}\quad(\times 100)$	0.98	2.04	2.59	1.65
$t(\pi = 0.01)$	-0.09	3.72	5.06	2.57
$\hat{b}_1$	0.03	0.08	0.00	0.03
$t(b_1 = 0)$	1.54	3.91	0.23	1.60
$F(b_0 = b_1 = 0)$	1.18	14.60	12.80	4.59

For the normal VaR, all 3 tests reject. The GARCH models pass the independence test, while the conditional and unconditional coverage tests reject.