

## Module 9.3: Time Series Analysis

### Fall Term 2023

**Week 5:**

Volatility Modelling

# Outline in Weeks

- ➊ Introduction; Descriptive Modelling
- ➋ Returns; Autocorrelation; Stationarity
- ➌ ARMA Models
- ➍ Unit Roots; Regressions between Time Series
- ➎ Volatility Modelling
- ➏ Value at Risk
- ➐ Cointegration

# Outline

- 1 Introduction
- 2 Historical, RiskMetrics
- 3 The ARCH and GARCH Models
- 4 Estimation of GARCH Models
- 5 Testing GARCH Models
- 6 Asymmetry and the News Impact Curve
- 7 Volatility Forecasting
- 8 Epilogue

# Goal

- Recall these stylized facts about asset returns:
  - 1 Lack of autocorrelation (efficient market hypothesis)
  - 2 Volatility clustering
  - 3 Distribution has heavy tails
  - 4 Leverage effects
- Goal today: model the last 3 of these, starting with the volatility clustering.

# Volatility

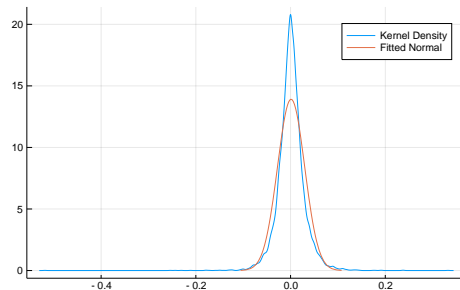
- The *volatility* of an investment is a measure of its *risk*. Usually defined as the standard deviation of the return on the investment.
- Volatility is an important ingredient in:
  - portfolio selection;
  - risk management;
  - option pricing.
- Daily financial returns display *volatility clustering*: periods of high volatility alternate with more tranquil periods.
- In other words: large (in absolute value) returns tend to be followed by large (in absolute value) returns.
- This forms the basis for the *autoregressive-conditional heteroskedasticity* model (ARCH; Engle, 1982) and the *generalized ARCH* model (GARCH; Bollerslev, 1986).

## Example: Daily Returns on Apple Stock

Volatility Clustering



Fat Tails



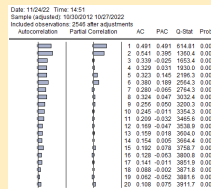
# Reminder: Parameters vs. sample values

- We usually write  $\sigma$  for the standard deviation of, e.g., a normally distributed variable.
- $\sigma$  is a *parameter* and therefore unknown.
- The best we can hope for is to *estimate* it, usually with the *sample standard deviation*  $s$ .
- With stock returns, the standard deviation (or *volatility*) changes over time, due to *volatility clustering*.
- We write  $\sigma_t$  for the volatility in period  $t$ .
- Note that  $\sigma_t$  is *unobserved*. The best we can do is *estimate* it. We'll write  $\hat{\sigma}_t$  for this estimate.
- Today, we'll mostly discuss different methods of estimation.

# Detecting Volatility Clustering (I)

- Since volatility clustering means that large returns tend to be followed by large returns, it is possible to detect it by inspecting the correlogram of the squared returns.

Example: correlogram of squared S&P500 returns.



- Clearly, there is a lot of predictability in squared returns (unlike returns themselves).



## Detecting Volatility Clustering (II)

- Besides relying on the  $Q$ -tests from the correlogram, another formal test is Engle's **ARCH-LM** test (essentially a Breusch-Godfrey test applied to the squared residuals).
- EViews only offers it for residuals, not for a series itself. Hence, we start by regressing the returns on an intercept.
- The ARCH-LM test is based on the auxiliary regression

$$\hat{u}_t^2 = \gamma_0 + \gamma_1 \hat{u}_{t-1}^2 + \dots + \gamma_m \hat{u}_{t-m}^2 + e_t.$$

- The lag length  $m$  is chosen by the user, e.g., 5 for daily data.
- The test statistic is  $T \cdot R_{aux}^2$  and has a  $\chi^2(m)$  distribution under  $H_0 : \gamma_1 = \dots = \gamma_m = 0$  (no volatility clustering).

## Example: ARCH-LM test for the S&P500

### Heteroskedasticity Test: ARCH

F-statistic	302.2045	Prob. F(5,2535)	0.0000
Obs*R-squared	948.9587	Prob. Chi-Square(5)	0.0000

Test Equation:

Dependent Variable: RESID^2

Method: Least Squares

Date: 11/24/22 Time: 15:11

Sample (adjusted): 11/06/2012 10/27/2022

Included observations: 2541 after adjustments

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	3.14E-05	8.68E-06	3.614959	0.0003
RESID^2(-1)	0.299355	0.019651	15.23388	0.0000
RESID^2(-2)	0.396998	0.020528	19.33915	0.0000
RESID^2(-3)	-0.087729	0.021921	-4.001999	0.0001
RESID^2(-4)	-0.015277	0.020529	-0.744168	0.4568
RESID^2(-5)	0.145365	0.019653	7.396390	0.0000

R-squared	0.373459	Mean dependent var	0.000120
Adjusted R-squared	0.372223	S.D. dependent var	0.000528
S.E. of regression	0.000419	Akaike info criterion	-12.71671
Sum squared resid	0.000444	Schwarz criterion	-12.70292
Log likelihood	16162.59	Hannan-Quinn criter.	-12.71171
F-statistic	302.2045	Durbin-Watson stat	2.054495
Prob(F-statistic)	0.000000		

The null of no volatility clustering is clearly rejected ( $p$ -value is zero,  $T \cdot R_{aux}^2 = 948.96$  much larger than critical value 11.07).

# Outline

- 1 Introduction
- 2 Historical, RiskMetrics**
- 3 The ARCH and GARCH Models
- 4 Estimation of GARCH Models
- 5 Testing GARCH Models
- 6 Asymmetry and the News Impact Curve
- 7 Volatility Forecasting
- 8 Epilogue

# Historical Volatility

- A first simple estimator is *historical volatility*, i.e., the sample standard deviation of the most recent  $m$  observations (often  $m = 250$ , one year).
- If  $r_t = \ln P_t - \ln P_{t-1}$  denotes the daily log-return, then

$$\hat{\sigma}_{t+1, HIST}^2 = \frac{1}{m} \sum_{j=0}^{m-1} r_{t-j}^2.$$

(Typically the average return is relatively close to zero). This is an estimate of the squared volatility over day  $t + 1$ , made at the end of day  $t$ .

- Main disadvantages:
  - either noisy (small  $m$ ), or reacts slowly to new information (large  $m$ );
  - “ghosting” feature: large shock leads to higher volatility for exactly  $m$  periods, then drops out.

# RiskMetrics

- Problems with historical volatility are addressed by replacing equally weighted moving average by an *exponentially* weighted moving average (EWMA), also used in JPMorgan's *RiskMetrics* system:

$$\begin{aligned}\hat{\sigma}_{t+1,EWMA}^2 &= (1 - \lambda) \sum_{j=0}^{\infty} \lambda^j r_{t-j}^2 \\ &= \lambda \hat{\sigma}_{t,EWMA}^2 + (1 - \lambda) r_t^2, \quad 0 < \lambda < 1.\end{aligned}$$

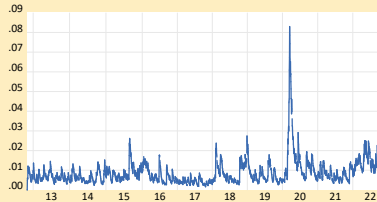
- This means that observations further in the past get a smaller weight.
- In practice we do not have  $r_{t-\infty}$ , but the second equation can be started up by an initial estimate / guess  $\sigma_{0,EWMA}^2$ .
- The larger  $\lambda$ , the stronger the persistence of shocks (large returns).
- For daily data, RiskMetrics recommends  $\lambda = 0.94$ .

## Example: S&P500 volatility, historical and EWMA ( $\lambda = 0.8, 0.94, 0.99$ )

SIGMA\_HIST



SIGMA\_EWMA



SIGMA\_EWMA



SIGMA\_EWMA



# Outline

- 1 Introduction
- 2 Historical, RiskMetrics
- 3 The ARCH and GARCH Models**
- 4 Estimation of GARCH Models
- 5 Testing GARCH Models
- 6 Asymmetry and the News Impact Curve
- 7 Volatility Forecasting
- 8 Epilogue

# The ARCH Model

- The first-order *autoregressive-conditional heteroskedasticity* (ARCH(1)) model, due to Engle (1982), for a return  $r_t$  with mean zero is

$$\sigma_{t+1}^2 = \omega + \alpha r_t^2.$$

- In practice, we need to allow for  $\mathbb{E}[r_{t+1}] = \mu_{t+1} \neq 0$ . Then  $r_{t+1} = \mu_{t+1} + u_{t+1}$ , and the model becomes

$$\sigma_{t+1}^2 = \omega + \alpha u_t^2.$$



# The ARCH Model

- When trying to estimate ARCH models one might find that more lags are needed, leading to ARCH( $q$ ):

$$\sigma_{t+1}^2 = \omega + \alpha_1 u_t^2 + \dots + \alpha_q u_{t-q+1}^2.$$

- *Note:* Variances must be positive, therefore we need to impose  $\omega > 0$ ,  $\alpha_i \geq 0$ ,  $i = 1, \dots, q$ .
- It can be shown that an ARCH( $q$ ) models corresponds to an AR( $q$ ) for the squared returns. Thus, we could determine the order from the correlogram of the squared returns: SPACF should cut off after  $q$  lags.
- In the example above, we might conclude that we need an ARCH(6) model.

# The GARCH Model

- A simpler structure than ARCH( $q$ ) is an ARMA(1,1) for  $r_t^2$  or  $u_t^2$ , which leads to the *generalized ARCH* model of orders (1,1) (GARCH(1,1)), due to Bollerslev (1986):

$$\sigma_{t+1}^2 = \omega + \alpha u_t^2 + \beta \sigma_t^2, \quad \omega > 0, \alpha \geq 0, \beta \geq 0.$$

- *Advantage*: Flexible structure with only 3 parameters to estimate.

# The GARCH Model

- The GARCH(1,1) model is stationary if the unconditional (“average”) variance  $\sigma^2 = \mathbb{E}[\sigma_t^2]$  is positive, constant and finite.
- This requires

$$\begin{aligned}\sigma^2 = \mathbb{E}[\sigma_{t+1}^2] &= \omega + \alpha \mathbb{E}[u_t^2] + \beta \mathbb{E}[\sigma_t^2] \\ &= \omega + \alpha \sigma^2 + \beta \sigma^2.\end{aligned}$$

- Hence, provided that  $\alpha + \beta < 1$  (the *stationarity condition*),

$$\sigma^2 = \frac{\omega}{1 - \alpha - \beta}.$$

- The nonstationary model with  $\alpha + \beta = 1$  is called *integrated GARCH* (IGARCH): infinite variance, no mean-reversion in volatility.
- Notice that an IGARCH with  $r_t = u_t$ ,  $\omega = 0$ ,  $\beta = \lambda$ , and  $\alpha = (1 - \lambda)$  is just the RiskMetrics model.

# The GARCH Model

Some other properties:

- The ACF and PACF of  $r_t^2$  in case of stationary GARCH(1,1) are both exponentially decaying, no cut-off point.
- The *standardized returns*

$$z_{t+1} = \frac{r_{t+1} - \mu_{t+1}}{\sigma_{t+1}}$$

satisfy  $E(z_{t+1}) = 0$  and  $\text{var}(z_{t+1}) = 1$ . Therefore the model may be formulated as

$$\begin{aligned} r_{t+1} &= \mu_{t+1} + u_{t+1} = \mu_{t+1} + \sigma_{t+1} z_{t+1}, \\ \sigma_{t+1}^2 &= \omega + \alpha u_t^2 + \beta \sigma_t^2. \end{aligned}$$

- Often it is assumed that  $z_t$  are i.i.d. as  $N(0, 1)$ .
- Even if  $z_t \sim N(0, 1)$ , it can be shown that varying  $\sigma_t$  implies that  $r_t$  has non-normal distribution, with higher kurtosis.

# The GARCH( $p, q$ ) Model

- The GARCH(1, 1) model can be extended to the GARCH( $p, q$ ) model

$$\sigma_{t+1}^2 = \omega + \alpha_1 u_t^2 + \cdots + \alpha_q u_{t-q+1}^2 + \beta_1 \sigma_t^2 + \cdots + \beta_p \sigma_{t-p+1}^2$$

although in practice, this is rarely necessary.

- The model is stationary if  $\sum_{i=1}^p \beta_i + \sum_{i=1}^q \alpha_i < 1$ , and the unconditional variance is

$$\frac{\omega}{1 - \sum_{i=1}^p \beta_i - \sum_{i=1}^q \alpha_i}.$$

# Outline

- 1 Introduction
- 2 Historical, RiskMetrics
- 3 The ARCH and GARCH Models
- 4 Estimation of GARCH Models**
- 5 Testing GARCH Models
- 6 Asymmetry and the News Impact Curve
- 7 Volatility Forecasting
- 8 Epilogue

# Estimation of GARCH Models

- GARCH cannot be estimated by ordinary least-squares (because  $\sigma_t^2$  is not observed).
- Such models are estimated by *maximum likelihood*: the joint density of the observations  $\{r_1, \dots, r_T\}$  is maximized with respect to the parameters.
- Maximization of  $\log L$  can be done by numerical optimization algorithms. By default, EViews does this under the assumption of normality.
- If we are not sure that the  $z_t$ 's are normally distributed, then we may still use the same estimation technique. This is called *quasi-maximum likelihood estimator*.
- However, we need to construct standard errors via a more robust method (*Bollerslev-Wooldridge standard errors*).

## Example: EViews output, estimated GARCH model for S&P500

Dependent Variable: R  
 Method: ML ARCH - Normal distribution (BFGS / Marquardt steps)  
 Date: 11/24/22 Time: 15:40  
 Sample (adjusted): 10/30/2012 10/27/2022  
 Included observations: 2546 after adjustments  
 Convergence achieved after 18 iterations  
 Coefficient covariance computed using outer product of gradients  
 Presample variance: backcast (parameter = 0.7)  
 GARCH = C(2) + C(3)\*RESID(-1)^2 + C(4)\*GARCH(-1)

Variable	Coefficient	Std. Error	z-Statistic	Prob.
C	0.000799	0.000135	5.911544	0.0000

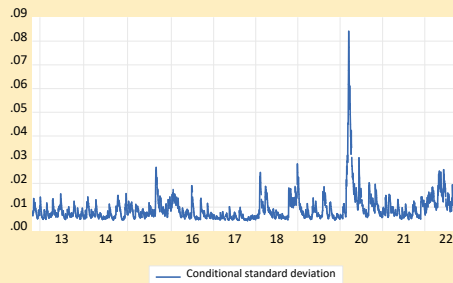
Variance Equation

C	4.50E-06	4.30E-07	10.46617	0.0000
RESID(-1)^2	0.223750	0.016516	13.54710	0.0000
GARCH(-1)	0.741309	0.016652	44.51892	0.0000

R-squared	-0.001398	Mean dependent var	0.000390
Adjusted R-squared	-0.001398	S.D. dependent var	0.010946
S.E. of regression	0.010953	Akaike info criterion	-6.742939
Sum squared resid	0.305331	Schwarz criterion	-6.733760
Log likelihood	8587.761	Hannan-Quinn criter.	-6.739609
Durbin-Watson stat	2.287783		



## Example: Estimated GARCH volatility of S&P500



# Outline

- 1 Introduction
- 2 Historical, RiskMetrics
- 3 The ARCH and GARCH Models
- 4 Estimation of GARCH Models
- 5 Testing GARCH Models**
- 6 Asymmetry and the News Impact Curve
- 7 Volatility Forecasting
- 8 Epilogue

# Testing GARCH Models

- Diagnostic tests are based on the *standardized residuals*  $\hat{z}_t := \hat{u}_t / \hat{\sigma}_t$ . If  $\mu_t$  and  $\sigma_t$  are correctly specified, we should find no autocorrelation in  $\hat{z}_t$  and  $\hat{z}_t^2$ .
- Therefore, the model can be tested using  $Q$ -statistics for  $\hat{z}_t$  or  $\hat{z}_t^2$ .
- Lagrange-Multiplier (LM) test against ARCH, which is obtained by  $T \cdot R^2$  in the regression

$$\hat{z}_t^2 = \gamma_0 + \gamma_1 \hat{z}_{t-1}^2 + \dots + \gamma_m \hat{z}_{t-m}^2 + e_t.$$









































- To test for normality of  $z_t$ , we can use the Jarque-Bera test based on the skewness and kurtosis of  $\hat{z}_t$ .

## Example: Correlogram of standardized residuals for the S&P500

Date: 11/24/22 Time: 15:48

Sample (adjusted): 10/30/2012 10/27/2022

Included observations: 2546 after adjustments

Autocorrelation	Partial Correlation	AC	PAC	Q-Stat	Prob*
		1 -0.032	-0.032	2.6093	0.106
		2 0.010	0.009	2.8792	0.237
		3 -0.007	-0.007	3.0169	0.389
		4 -0.014	-0.014	3.4899	0.479
		5 -0.022	-0.023	4.7188	0.451
		6 -0.022	-0.023	5.9352	0.430
		7 0.027	0.025	7.7406	0.356
		8 -0.028	-0.027	9.8064	0.279
		9 0.008	0.005	9.9642	0.353
		10 -0.009	-0.009	10.169	0.426
		11 0.021	0.020	11.328	0.416
		12 -0.033	-0.032	14.092	0.295
		13 -0.030	-0.033	16.403	0.228
		14 -0.027	-0.030	18.323	0.192
		15 -0.014	-0.014	18.815	0.222
		16 0.000	-0.002	18.815	0.278
		17 0.022	0.021	20.073	0.271
		18 0.021	0.017	21.170	0.271
		19 -0.021	-0.020	22.254	0.272
		20 0.018	0.015	23.127	0.283

\*Probabilities may not be valid for this equation specification.

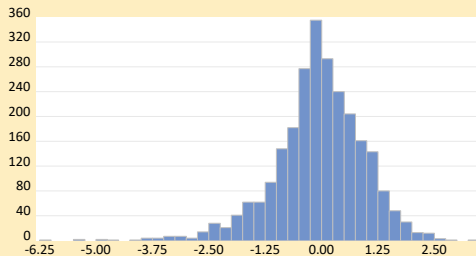
## Example: Correlogram of squared standardized residuals for the S&P500

Date: 11/24/22 Time: 15:48  
Sample (adjusted): 10/30/2012 10/27/2022  
Included observations: 2546 after adjustments

Autocorrelation	Partial Correlation	AC	PAC	Q-Stat	Prob*
		1 0.001	0.001	0.0011	0.973
		2 -0.000	-0.000	0.0015	0.999
		3 -0.003	-0.003	0.0182	0.999
		4 0.024	0.024	1.4430	0.837
		5 -0.028	-0.028	3.4029	0.638
		6 -0.008	-0.008	3.5668	0.735
		7 -0.021	-0.021	4.6583	0.702
		8 -0.017	-0.017	5.3624	0.718
		9 -0.008	-0.006	5.5064	0.788
		10 0.038	0.037	9.1425	0.519
		11 -0.004	-0.004	9.1864	0.605
		12 -0.017	-0.017	9.8904	0.626
		13 0.000	-0.001	9.8906	0.703
		14 -0.018	-0.020	10.676	0.711
		15 -0.002	-0.001	10.686	0.774
		16 0.003	0.004	10.712	0.827
		17 0.002	0.002	10.720	0.871
		18 -0.007	-0.005	10.850	0.901
		19 -0.007	-0.009	10.990	0.924
		20 0.020	0.017	11.993	0.916

\*Probabilities may not be valid for this equation specification.

## Example: Normality test of standardized residuals for the S&P500



Series: Standardized Residuals  
Sample 10/30/2012 10/27/2022  
Observations 2546

Mean	-0.060225
Median	-0.034548
Maximum	3.439897
Minimum	-6.012431
Std. Dev.	0.998517
Skewness	-0.737406
Kurtosis	5.246133

Jarque-Bera	765.9409
Probability	0.000000

# Outline

- 1 Introduction
- 2 Historical, RiskMetrics
- 3 The ARCH and GARCH Models
- 4 Estimation of GARCH Models
- 5 Testing GARCH Models
- 6 Asymmetry and the News Impact Curve**
- 7 Volatility Forecasting
- 8 Epilogue

# Asymmetry and the News Impact Curve

- The *news impact curve* (NIC) is the effect of  $u_t$  on  $\sigma_{t+1}^2$ , keeping  $\sigma_t^2$  and the past fixed.
- For GARCH(1,1), this is the parabola  $NIC(u_t | \sigma_t^2 = \sigma^2) = A + \alpha u_t^2$ , with  $A = \omega + \beta\sigma^2$ . This has a minimum at  $u_t = 0$ , and is symmetric around that minimum.
- For equity, a large negative shock is expected to increase volatility more than a large positive shock, because of *leverage effect*:
  - ↓ value of firm's stock
  - ⇒ ↓ equity value of the firm
  - ⇒ ↑ debt-to-equity ratio
  - ⇒ shareholders (as residual claimants) perceive future cashflows as more risky.
- Multiple extensions exist to deal with this issue. Here we focus on Glosten, Jagannathan and Runkle's GJR-GARCH model.



# GJR-GARCH (or TARARCH, threshold GARCH)

The GJR-GARCH(1,1) model is

$$\sigma_{t+1}^2 = \omega + \alpha u_t^2 + \gamma u_t^2 I_t + \beta \sigma_t^2.$$

where

$$I_t = \begin{cases} 1 & \text{if } u_t < 0 \\ 0 & \text{if } u_t \geq 0 \end{cases},$$

and  $u_t/\sigma_t$  has a symmetric distribution.

Properties:

- NIC is asymmetric if and only if  $\gamma \neq 0$ ; leverage effect if  $\gamma > 0$ ;
- $\sigma_t^2$  is positive if  $\omega > 0$ ,  $\alpha \geq 0$ ,  $\gamma \geq 0$ ,  $\beta \geq 0$ ;
- $u_t^2$  is stationary if  $0 \leq \alpha + \frac{1}{2}\gamma + \beta < 1$ , with unconditional variance  $\sigma^2 = \omega / [1 - \alpha - \frac{1}{2}\gamma - \beta]$ .

# EGARCH

The EGARCH(1,1) model is

$$\log \sigma_{t+1}^2 = \omega + \gamma z_t + \alpha(|z_t| - E|z_t|) + \beta \log \sigma_t^2,$$

with  $z_t = u_t/\sigma_t$  as usual. If  $z_t \sim \text{i.i.d. } N(0, 1)$  then  $E|z_t| = \sqrt{2/\pi}$ .

Properties:

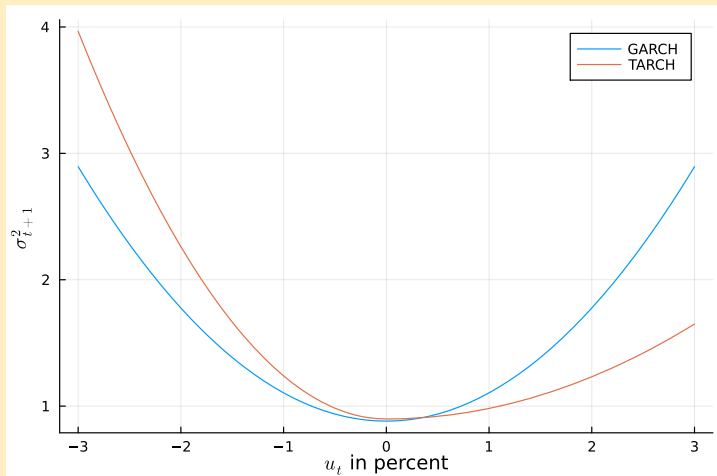
- NIC is asymmetric if and only if  $\gamma \neq 0$ ; leverage effect if  $\gamma < 0$ ;
- $\sigma_t^2$  is positive for all parameter values;
- $\gamma z_t + \alpha(|z_t| - E|z_t|)$  is an i.i.d. mean-zero shock to log-volatility;
- if  $|\beta| < 1$ ,  $\log \sigma_t^2$  is stationary with mean  $\omega/(1 - \beta)$ .

## Example: EViews output, estimated TARCH model for S&P500

Dependent Variable: R  
 Method: ML ARCH - Normal distribution (BFGS / Marquardt steps)  
 Date: 11/24/22 Time: 15:56  
 Sample (adjusted): 10/30/2012 10/27/2022  
 Included observations: 2546 after adjustments  
 Convergence achieved after 26 iterations  
 Coefficient covariance computed using outer product of gradients  
 Presample variance: backcast (parameter = 0.7)  
 GARCH = C(2) + C(3)\*RESID(-1)^2 + C(4)\*RESID(-1)^2\*(RESID(-1)<0) +  
 C(5)\*GARCH(-1)

Variable	Coefficient	Std. Error	z-Statistic	Prob.
C	0.000472	0.000146	3.229615	0.0012
Variance Equation				
C	4.18E-06	3.67E-07	11.39238	0.0000
RESID(-1)^2	0.083481	0.008795	9.491794	0.0000
RESID(-1)^2*(RESID(-1)<0)	0.257376	0.028027	9.183082	0.0000
GARCH(-1)	0.755584	0.014405	52.45111	0.0000
R-squared	-0.000056	Mean dependent var		0.000390
Adjusted R-squared	-0.000056	S.D. dependent var		0.010946
S.E. of regression	0.010946	Akaike info criterion		-6.767193
Sum squared resid	0.304922	Schwarz criterion		-6.755719
Log likelihood	8619.636	Hannan-Quinn criter.		-6.763031
Durbin-Watson stat	2.290853			

## Example: NIC of GARCH and TARCH models for S&P500



# Outline

- 1 Introduction
- 2 Historical, RiskMetrics
- 3 The ARCH and GARCH Models
- 4 Estimation of GARCH Models
- 5 Testing GARCH Models
- 6 Asymmetry and the News Impact Curve
- 7 Volatility Forecasting**
- 8 Epilogue

# Volatility Forecasting

- GARCH models directly provide forecasts of next day's volatility:

$$\hat{\sigma}_{t+1}^2 = \hat{\omega} + \hat{\alpha} \hat{u}_t^2 + \hat{\beta} \hat{\sigma}_t^2.$$

- Multi-period forecasts can be constructed recursively. In principle, one would use

$$\hat{\sigma}_{t+2}^2 = \hat{\omega} + \hat{\alpha} \hat{u}_{t+1}^2 + \hat{\beta} \hat{\sigma}_{t+1}^2,$$

but  $\hat{u}_{t+1}^2$  is unobserved.

- Solution: replace  $\hat{u}_{t+1}^2$  with its estimate,  $\hat{\sigma}_{t+1}^2$ .
- Result:

$$\hat{\sigma}_{t+2}^2 = \hat{\omega} + (\hat{\alpha} + \hat{\beta}) \hat{\sigma}_{t+1}^2,$$

# Outline

- 1 Introduction
- 2 Historical, RiskMetrics
- 3 The ARCH and GARCH Models
- 4 Estimation of GARCH Models
- 5 Testing GARCH Models
- 6 Asymmetry and the News Impact Curve
- 7 Volatility Forecasting
- 8 Epilogue**

# Learning Goals

## Students

- can use appropriate tests to detect volatility clustering,
- are able to estimate, interpret, and forecast the various models (historical volatility, RiskMetrics, (G)ARCH, TARCH, EGARCH), and to apply diagnostic tests to the standardized residuals,
- and understand the concept of leverage, and the NIC.



# Homework

- Exercise 5
- Questions 1 and 3 from Chapter 9 of Brooks (2019)