Exercise 5

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1. (a) In the file `sp500`, construct the returns, produce a correlogram of the squared residuals, and interpret it.
   (b) Perform an ARCH-LM test by regressing the returns on an intercept.
   (c) Compute the historical volatility and plot it.
   (d) Compute the EWMA volatility and plot it.
   (e) Find a suitable GARCH/TGARCH/EGARCH model. Start with a GARCH(1, 1) or an ARCH(6) model, and determine whether it needs to be adjusted.
   (f) Make a plot of the volatility estimates that your model generates, and of the NIC.
   (g) Forecast the volatility for $T + 1$.

2. (a) Show that
$$\widehat{\sigma}^2_{t+1,EWMA} = \lambda \widehat{\sigma}^2_{t,EWMA} + (1-\lambda) r_t^2, \qquad 0 < \lambda < 1.$$
   (b) Show that in the GARCH(1, 1) model,
$$\widehat{\sigma}^2_{t+1} = \widehat{\sigma}^2 + \hat{\alpha}(\hat{u}_t^2 - \widehat{\sigma}^2) + \hat{\beta}(\widehat{\sigma}_t^2 - \widehat{\sigma}^2),$$
   with $\widehat{\sigma}^2 = \hat{\omega}/(1 - \hat{\alpha} - \hat{\beta})$.
   (c) Show that in the GARCH(1, 1) model,
$$\widehat{\sigma}^2_{t+s} = \widehat{\sigma}^2 + (\hat{\alpha} + \hat{\beta})^{s-1}(\widehat{\sigma}^2_{t+1} - \widehat{\sigma}^2).$$