# Exercise 1

Simon A. Broda

1. (a) Open the file `maunaloa.wf1`; this is a famous data set used in machine learning. Make a time series plot.

   (b) Estimate a linear trend by regressing the `co2` series on an intercept and the variable `time`.

   (c) Plot the data, together with the estimated linear trend.

   (d) Produce a forecast for 2005M1, first manually using the fitted model

   $$\widehat{Y}_t = \widehat{\beta}_0 + \widehat{\beta}_1 t,$$

   then using Eviews.

   (e) Repeat Questions 1b through 1d, but using a quadratic trend.

   (f) Repeat Questions 1b through 1d, but using an exponential trend.

2. (a) Compute the 3rd order moving average of the `co2` series for 1964M6 by hand.

   (b) Estimate the trend with a 12 month moving average (12 months are necessary to cover a full cycle). Then plot the resulting trend estimate and the data together in a time series plot.

3. (a) Estimate a model with a linear trend and 12 monthly dummies (and no intercept) for the `co2` series. Then, produce an (in-sample) forecast for 2004M12, both by hand and using EViews. Also create an actual-fitted-residual plot.

   (b) Same, but include an intercept and remove the last dummy.

# Exercise 2

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1. (a) Open the file `simulations.xlsx`. The sheet "White Noise" simulates $T = 1000$ observations from a (Gaussian) white noise process; i.e., 1000 uncorrelated mean-zero normals. By repeatedly pressing `F9`, you can draw new random numbers. Describe your observations.

   (b) Similarly, the sheet "Random Walk" simulates $T = 1000$ observations from a (Gaussian) random walk. Describe your observations.

2. (a) Open the file `sp500.wp1`. Generate a new series `logsp500` containing the log prices, and a series `r` containing the continuously compounded returns. Make a time series plot for each, and a histogram of the returns. Describe your findings.

   (b) Use the skewness and kurtosis given in the histogram to manually conduct a Jarque-Bera test.

   (c) Generate a correlogram of the returns and interpret it.

   (d) Test whether the first 10 autocorrelations are jointly significant at the 5% level.

   (e) Generate a correlogram of the log prices and interpret it.

3. (a) Show that for the random walk $Y_t = Y_{t-1} + U_t$, where $U_t$ is white noise and $Y_0$ some constant,

$$Y_t = Y_0 + U_1 + U_2 + \cdots + U_t = Y_0 + \sum_{s=1}^{t} U_t.$$

   (b) Building on the result from the previous question, show that

$$\begin{aligned} \mathbb{E}[Y_t] &= Y_0, \quad \text{and} \\ \operatorname{var}(Y_t) &= \sigma^2 t. \end{aligned}$$

# Exercise 3

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1. (a) Open the file `simulations.xlsx`. The sheet "AR(1)" simulates $T = 1000$ observations from an AR(1) process. Play around with $\alpha$ and $-1 < \phi_1 < 1$ and describe your observations.
   (b) Also try setting $\phi_1 = 1$ and describe the effect of $\alpha$.
   (c) The file `simulated_data.wf1` contains three series simulated using the same spreadsheet, `simulation.xlsx`, one each for an AR(1), an MA(1), and an ARMA(1, 1) process. The AR and ARMA processes use $\phi_1 = 0.7$, and the MA and ARMA processes use $\theta_1 = 0.7$. Describe your observations.
2. (a) Use the Box-Jenkins approach to model year-on-year real GDP growth in the file `realgdpch.wf1`.
   (b) Produce a forecast for 2022Q3 and 2022Q4, both manually and using EViews.
3. (a) Obtain the mean and variance of a random walk with drift.
   (b) Show that the random walk with drift is integrated of order 1.
   (c) Derive the expression for the variance of a stationary AR(1) given in the slides.
   (d) Find the mean, variance, and ACF of an MA(1).
   (e) **Optional**: Find the ACF of a stationary AR(1).

# Exercise 4

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1. Open the file `simulations.xlsx`. Use the sheets "AR(1)" (with $\phi_1$ set to 1) to simulate a random walk with drift, and the sheet "Linear Trend" to simulate a trend-stationary process. Play with the parameters and describe your observations.

2. (a) The file `tbill.wf1` contains monthly data for the 3-month T-Bill rate. Plot them, study the correlogram, and conduct a unit root test.

   (b) Model the first difference of the T-Bill rate as an ARMA process, hence modelling the T-Bill rate as an ARIMA process.

   (c) Forecast the T-Bill rate for 2022M11 and 2022M12 based on the model you found in the previous question.

3. (a) The file `ibm_capm.wf` contains data for the S&P500, IBM stock, and the 3-month T-Bill rate. Use it to estimate the CAPM $\beta$ of IBM, by regressing the excess returns of IBM on the excess returns of the market.

   (b) Use the Durbin-Watson test to test for first-order autocorrelation in the residuals.

   (c) Use the Breusch-Pagan test to test for autocorrelation up to order 5 in the residuals.

   (d) Re-estimate the regression using HAC standard errors.

4. (a) Show that for both

$$\begin{aligned} Y_{1,t} &= \delta t + U_{1,t} \quad \text{and} \\ Y_{2,t} &= \delta + Y_{2,t-1} + U_{2,t}. \end{aligned}$$

   we have $\mathbb{E}[\Delta Y_{i,t}] = 0$.

   (b) Derive the ADF regression for an AR(2) process.

# Exercise 5

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1. (a) In the file `sp500`, construct the returns, produce a correlogram of the squared residuals, and interpret it.
   (b) Perform an ARCH-LM test by regressing the returns on an intercept.
   (c) Compute the historical volatility and plot it.
   (d) Compute the EWMA volatility and plot it.
   (e) Find a suitable GARCH/TGARCH/EGARCH model. Start with a GARCH(1, 1) or an ARCH(6) model, and determine whether it needs to be adjusted.
   (f) Make a plot of the volatility estimates that your model generates, and of the NIC.
   (g) Forecast the volatility for $T + 1$.
2. (a) Show that
$$\widehat{\sigma}^2_{t+1,EWMA} = \lambda\widehat{\sigma}^2_{t,EWMA} + (1-\lambda)r_t^2, \qquad 0 < \lambda < 1.$$
   (b) Show that in the GARCH(1, 1) model,
$$\widehat{\sigma}^2_{t+1} = \widehat{\sigma}^2 + \hat{\alpha}(\hat{u}_t^2 - \widehat{\sigma}^2) + \hat{\beta}(\widehat{\sigma}^2_t - \widehat{\sigma}^2),$$
   with $\widehat{\sigma}^2 = \hat{\omega}/(1-\hat{\alpha}-\hat{\beta})$.
   (c) Show that in the GARCH(1, 1) model,
$$\widehat{\sigma}^2_{t+s} = \widehat{\sigma}^2 + (\hat{\alpha}+\hat{\beta})^{s-1}(\widehat{\sigma}^2_{t+1} - \widehat{\sigma}^2).$$

# Exercise 6

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1. (a) In the file `sp500.wf1`, compute the historical 1% VaR using `@quantile`. Note that this uses the entire sample, rather than the last $m$ returns.

   (b) Determine the Normal VaR, both using EViews and manually, based on the mean return of 0.000390 and the volatility 0.010946.

   (c) Determine the VaR based on a GARCH(1, 1) model with Normal innovations, and with standardized $t$ innovations.

   (d) Produce a manual VaR forecast for 10/27/2022 based on the GARCH model with $t$ innovations, using $\sigma_t = 0.014875$.

   (e) Make a plot with your VaR estimates overlaid on the negative log returns.

   (f) Test your VaR forecasts for correct unconditional coverage, independence, and correct conditional coverage.

# Exercise 7

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1. Until 1971, as part of the Bretton-Woods system of fixed exchange rates, the US dollar was convertible to gold, i.e., it was possible for foreign central banks to redeem US dollars for gold at a fixed rate of 35$ per troy ounce, so that the price of gold was fixed. In 1971, US president Nixon unilaterally cancelled the direct convertibility, ultimately ending the Bretton-Woods agreement. Gold became a floating asset, and its price increased sharply; in other words, the US$ was massively devalued. In this exercise, we will analyze the hypothesis that the increasing price (in US$) of oil is not a consequence of an increased demand for (or a reduced supply of) oil, but rather of a continued devaluation of the US$. We have at our disposal monthly data from April 1968 to January 2017 (586 observations) on the following variables:

   - GOLD, the spot price of one troy ounce of gold in US$;
   - OIL, the spot price of one barrel of WTI crude oil in US$.

   (a) Assuming that GOLD is integrated of order one, explain why the hypothesis that the relative price of oil (in troy ounces of gold per barrel) is stationary implies cointegration between log(OIL) and log(GOLD).

   (b) Using the file `oil_gold_2017.wf1`, analyze whether this cointegrating relationship can be found in the data, based on the Engle-Granger procedure.

   (c) Same, but using the Johansen procedure.

2. Consider the model

$$\begin{aligned} Y_t &= \beta_1 + \beta_2 X_t + U_{1,t} \\ X_t &= X_{t-1} + U_{2,t} \end{aligned}$$

   where $\beta_2 \neq 0$, $U_{1,t}, U_{2,t} \overset{\text{iid}}{\sim} (0, \sigma^2)$ independently of each other.

   (a) Is $X_t$ stationary?

   (b) Is $Y_t$ stationary?

   (c) Are $X_t$ and $Y_t$ cointegrated? If yes, what is the cointegrating vector?

   (d) Derive the bivariate VECM for $Y_t$ and $X_t$.