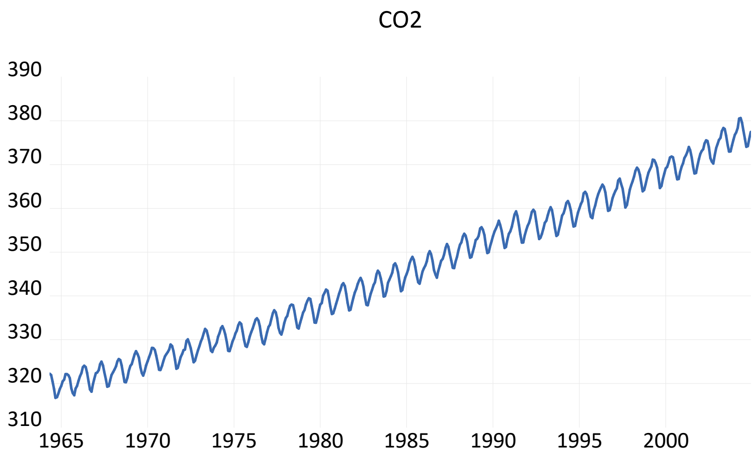
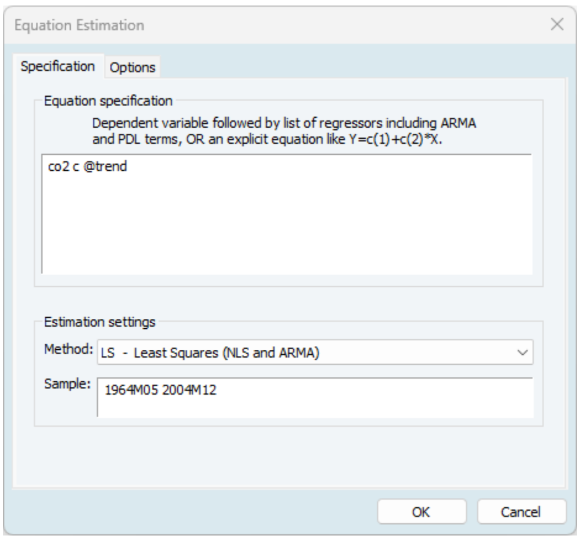
Solution to Exercise 1

Simon A. Broda

1. (a) Double-clicking the `co2` series followed by `View`→`Graph` results in the graph below.



(b) Click on `Quick`→`Estimate Equation` and enter the dependent variable, followed by the independent variables, as follows: `co2 c time`. Here, `c` stands for a constant (intercept) and `time` is the variable `time` from the dataset (take a look at it). Alternatively to `time`, you can also write `@trend`. This is an EViews function that generates the trend automatically (in fact, it's how I created the `time` variable, by writing `genr time = @trend` in the command window up top).



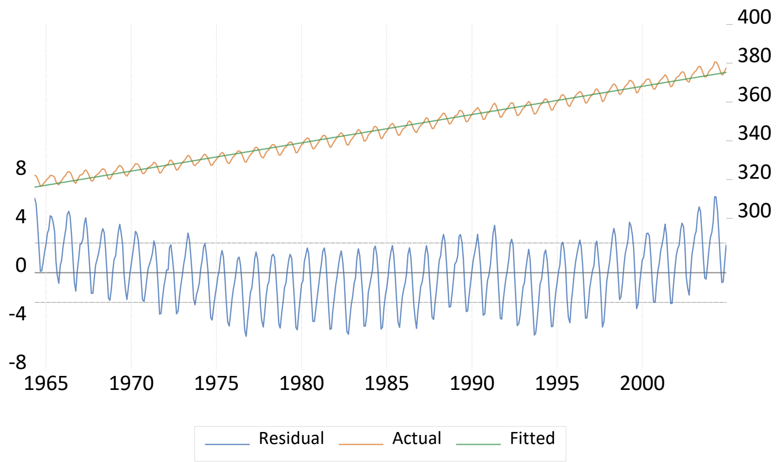
The result is

Dependent Variable: CO2
Method: Least Squares
Date: 10/26/22 Time: 16:19
Sample: 1964M05 2004M12
Included observations: 488

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	316.1023	0.221460	1427.355	0.0000
@TREND	0.121425	0.000787	154.2429	0.0000

R-squared	0.979981	Mean dependent var	345.6694
Adjusted R-squared	0.979940	S.D. dependent var	17.29716
S.E. of regression	2.449868	Akaike info criterion	4.634035
Sum squared resid	2916.901	Schwarz criterion	4.651209
Log likelihood	-1128.705	Hannan-Quinn criter.	4.640781
F-statistic	23790.89	Durbin-Watson stat	0.252267
Prob(F-statistic)	0.000000		

(c) Inside the window with the regression output, go to `View→Actual-Fitted-Residual→Actual-Fitted-Residual Plot`. This results in the follow graph.



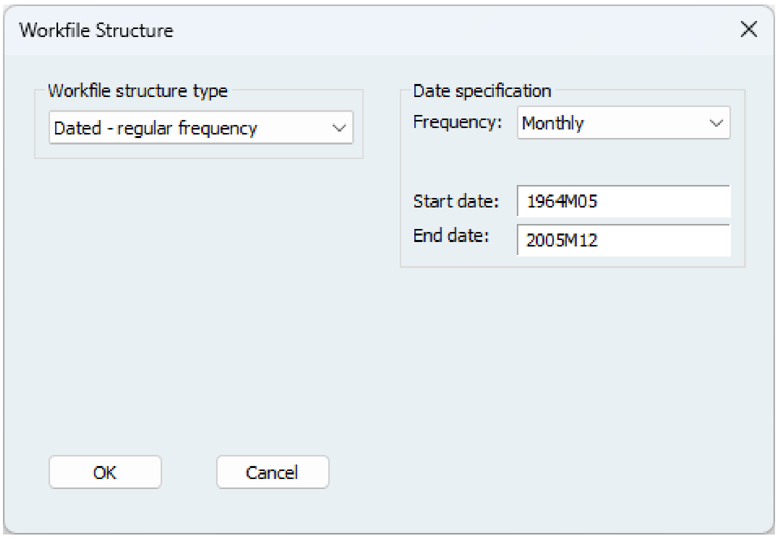
(d) 2005M1 corresponds to $t = 488$. Plugging this into the fitted regression

$$\widehat{Y}_t = 316.1 + 0.121 \cdot t,$$

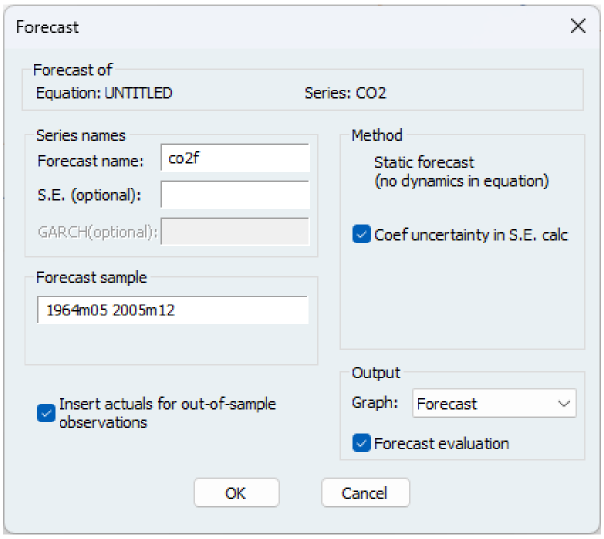
one obtains

$$\widehat{Y_{488}} = 316.1 + 0.121 \cdot 488 = 375.148.$$

Alternatively, inside the workfile pane on the left, go to `Proc→Structure / Resize Current Page...`, and resize the file so that it includes 2005:



Then, inside the window with the estimation output, click on `Proc`→`Forecasts...`, and extend the forecast sample to 2005M12.



Finally, open the `co2f` series and find the value corresponding to 2005M1; it's 375.358. The difference is due to the fact that EViews uses the unrounded estimates.

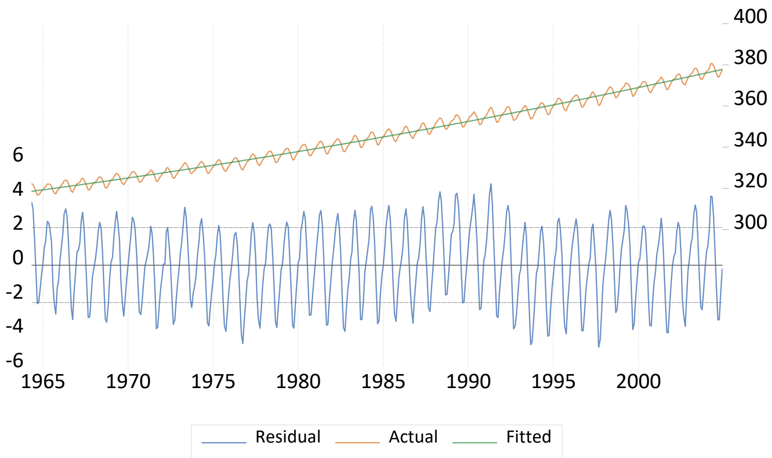
(e) Estimating `co2 c @trend @trend^2` yields

Dependent Variable: CO2
Method: Least Squares
Date: 10/26/22 Time: 18:01
Sample: 1964M05 2004M12
Included observations: 488

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	318.5814	0.295145	1079.407	0.0000
@TREND	0.090819	0.002800	32.44047	0.0000
@TREND^2	6.28E-05	5.57E-06	11.29240	0.0000

R-squared	0.984149	Mean dependent var	345.6694
Adjusted R-squared	0.984083	S.D. dependent var	17.29716
S.E. of regression	2.182234	Akaike info criterion	4.404704
Sum squared resid	2309.640	Schwarz criterion	4.430464
Log likelihood	-1071.748	Hannan-Quinn criter.	4.414822
F-statistic	15055.89	Durbin-Watson stat	0.318438
Prob(F-statistic)	0.000000		

and



Eyeballing, the resulting fit looks better; also, there is less structure in the residuals. The forecast is

$$\widehat{Y_{488}} = 318.58 + 0.09 \cdot 488 + 0.0000628 \cdot 488^2 \approx 377.868.$$

(f) First, we have to take logs of the dependent variable: in the command window up top, write `genr logco2 log(co2)` (`genr` stands for generating a new series; `log` is the natural logarithm). This creates a new series `logco2`. Next, regress this new variable on a constant and the time trend using `logco2 c @trend`. This yields

Dependent Variable: LOGCO2
Method: Least Squares
Date: 10/26/22 Time: 18:12
Sample (adjusted): 1964M05 2004M12
Included observations: 488 after adjustments

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	5.758822	0.000603	9548.688	0.0000
@TREND	0.000351	2.14E-06	163.6211	0.0000

R-squared	0.982170	Mean dependent var	5.844238
Adjusted R-squared	0.982134	S.D. dependent var	0.049914
S.E. of regression	0.006672	Akaike info criterion	-7.177792
Sum squared resid	0.021633	Schwarz criterion	-7.160619
Log likelihood	1753.381	Hannan-Quinn criter.	-7.171046
F-statistic	26771.88	Durbin-Watson stat	0.284591
Prob(F-statistic)	0.000000		

The estimated parameters of our exponential trend

$$F_t = \beta_0 \cdot \beta_1^t$$

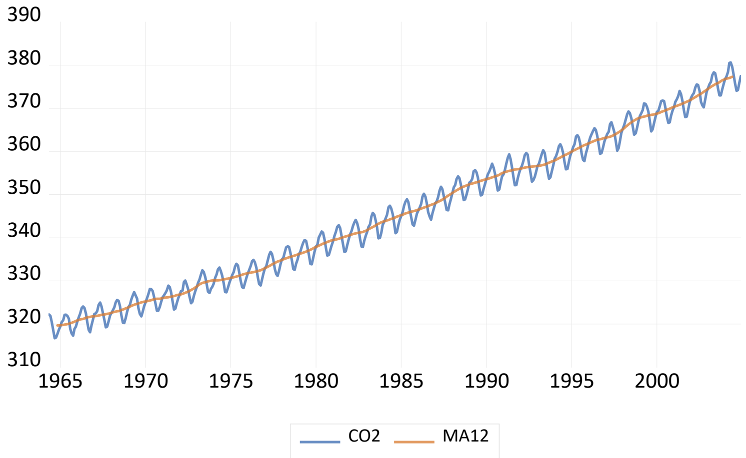
are $\widehat{\beta_0} = \exp(5.758822251222528) = 316.97$ and $\widehat{\beta_1} = \exp(0.0003507827256490508) = 1.0003508442571039$, implying that atmospheric CO2 increases by 0.035% a month, or $1.035^{12} - 1 = 0.42\%$ a year. The forecast for 2005M1 is

$$\widehat{Y_{488}} = 316.97 \cdot 1.0003508442571039^{488} = 376.15.$$

2. (a) We obtain

$$(322.23 + 321.89 + 320.44)/3 = 321.52.$$

(b) Enter `genr ma12 = @movavc(co2, 12)` in the command window (`@movavc` stands for a centered moving average). For the plot, select both series with the mouse (press `CTRL` while clicking to select both), then right-click and select `Open`→`As Group`, and plot as usual. This results in the following plot.



3. (a) Rather than constructing the dummies manually, we can have EViews construct them automatically for us, by entering the equation as `co2 @trend @expand(@month)` (for quarterly data, we would use `co2 @trend @expand(@quarter)`, etc.). This results in the following output:

Dependent Variable: CO2
Method: Least Squares
Date: 10/27/22 Time: 17:10
Sample: 1964M05 2004M12
Included observations: 488

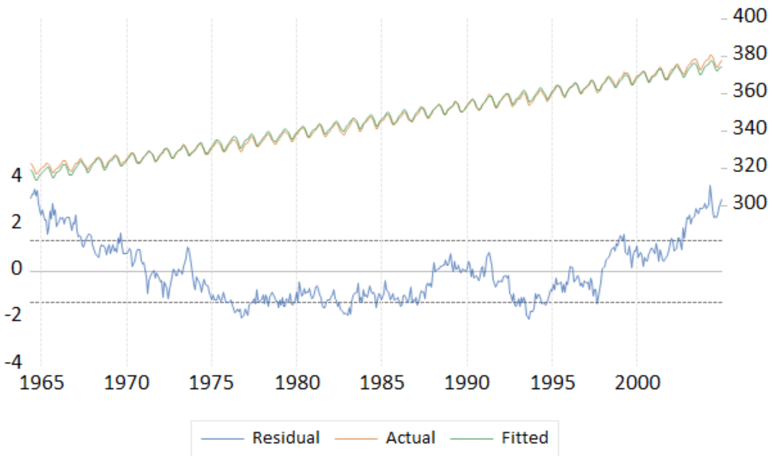
Variable	Coefficient	Std. Error	t-Statistic	Prob.
@TREND	0.121542	0.000422	287.8086	0.0000
@MONTH=1	316.0377	0.231547	1364.898	0.0000
@MONTH=2	316.7015	0.231733	1366.663	0.0000
@MONTH=3	317.4439	0.231921	1368.760	0.0000
@MONTH=4	318.6116	0.232109	1372.683	0.0000
@MONTH=5	319.1081	0.228887	1394.176	0.0000
@MONTH=6	318.4388	0.229074	1390.114	0.0000
@MONTH=7	316.8955	0.229262	1382.243	0.0000
@MONTH=8	314.7891	0.229450	1371.927	0.0000
@MONTH=9	312.9317	0.229640	1362.708	0.0000
@MONTH=10	312.7980	0.229829	1361.001	0.0000
@MONTH=11	314.0347	0.230020	1365.251	0.0000
@MONTH=12	315.2056	0.230211	1369.204	0.0000

R-squared	0.994371	Mean dependent var	345.6694
Adjusted R-squared	0.994228	S.D. dependent var	17.29716
S.E. of regression	1.314072	Akaike info criterion	3.410416
Sum squared resid	820.2228	Schwarz criterion	3.522044
Log likelihood	-819.1416	Hannan-Quinn criter.	3.454264
Durbin-Watson stat	0.052427		

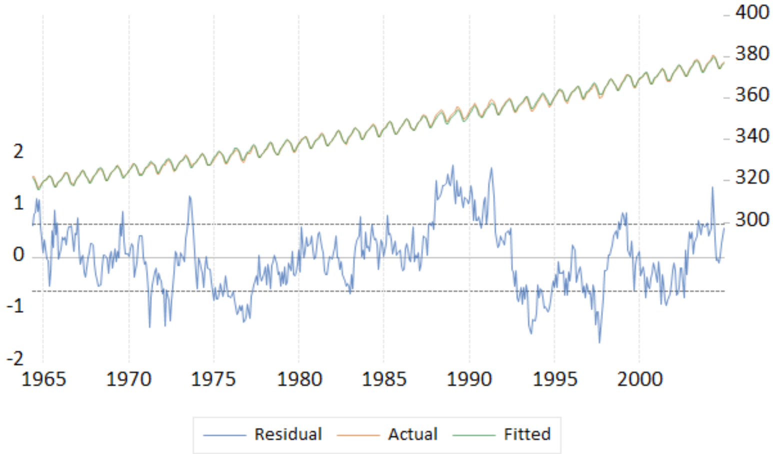
The forecast for 2004M12 is

$$\widehat{Y}_{487} = 0.121542 \cdot 487 + 315.2056 = 374.397,$$

which can also be obtained via `Proc`→`Forecasts...`. The actual-fitted-residual plot is given below.



As we had discovered earlier, there is still structure left in the residuals; a quadratic trend is a better fit. Including a quadratic term (not asked) yields the following:



(b) Now we use the specification `co2 c @trend @expand(@month, @droplast)`. This yields the following output.

Dependent Variable: CO2
Method: Least Squares
Date: 10/27/22 Time: 17:23
Sample: 1964M05 2004M12
Included observations: 488

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	315.2056	0.230211	1369.204	0.0000
@TREND	0.121542	0.000422	287.8086	0.0000
@MONTH=1	0.832127	0.292046	2.849301	0.0046
@MONTH=2	1.495834	0.292043	5.121964	0.0000
@MONTH=3	2.238292	0.292041	7.664308	0.0000
@MONTH=4	3.405999	0.292039	11.66281	0.0000
@MONTH=5	3.902504	0.290245	13.44555	0.0000
@MONTH=6	3.233157	0.290241	11.13956	0.0000
@MONTH=7	1.689907	0.290238	5.822495	0.0000
@MONTH=8	-0.416513	0.290235	-1.435091	0.1519
@MONTH=9	-2.273909	0.290233	-7.834781	0.0000
@MONTH=10	-2.407647	0.290231	-8.295619	0.0000
@MONTH=11	-1.170897	0.290230	-4.034372	0.0001

R-squared	0.994371	Mean dependent var	345.6694
Adjusted R-squared	0.994228	S.D. dependent var	17.29716
S.E. of regression	1.314072	Akaike info criterion	3.410416
Sum squared resid	820.2228	Schwarz criterion	3.522044
Log likelihood	-819.1416	Hannan-Quinn criter.	3.454264
F-statistic	6992.094	Durbin-Watson stat	0.052427
Prob(F-statistic)	0.000000		

Notice how December is now the base case with an average of 315.2, and all other months are measured in relation to it. The forecast for 2004M12 is

$$\widehat{Y}_{487} = 315.2056 + 0.121542 \cdot 487 = 374.397,$$

the same as before.

Solution to Exercise 2

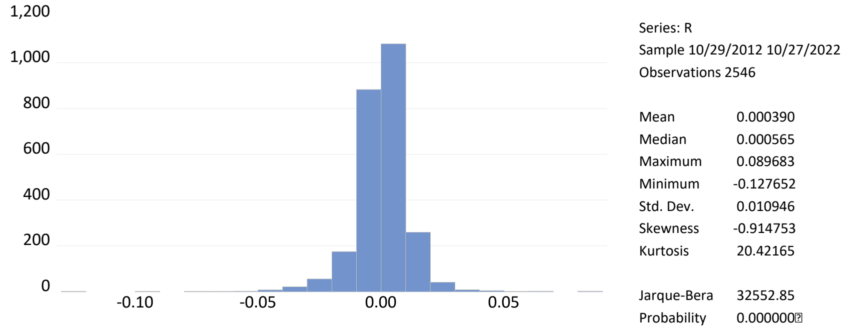
Simon A. Broda

1. (a) The first thing to observe is the difference between the population quantities (parameters) μ and σ, and their estimates $\bar{y}$ and s_y, which are sample quantities. The latter will generally be close to the former because of the law of large numbers, but not the same. The estimates also change every time new random numbers are drawn. The time series plot shows that the observations are randomly scattered around 0. The autocorrelations in the correlogram will mostly be insignificant, i.e., statistically indistinguishable from zero. Formally, the hypothesis being tested is $H_0 : \tau_s = 0$ vs. $H_a : \tau_s \neq 0$. Even if H_0 is true here, there is still a 5% probability that any given autocorrelation will be significant; this is called the type-1 error: the probability of rejecting the null even though it is true. A similar statement concerns the Q-statistics, which test whether the first m autocorrelations are all equal to zero[1]. **Important**: make sure to look at the formulas behind the cells and make sure you understand how they work; specifically, the correlogram, the Q-stats, and their respective critical values. You don't need to understand how the random numbers u_t themselves are generated (they use a trick called inverse transform sampling).

   (b) The time series plot looks very different from that in the other sheet, because a random walk is not mean reverting. Also, the correlogram and the Q-stats are now highly significant, so that we (correctly) reject the null that the data were generated from a white noise process. In fact, the slow and almost linear decay of the correlogram suggests (correctly) that the data were generated by an integrated process. **Important**: make sure you understand how the simulated random walk y_t is constructed, by always taking "yesterday's" value y_{t-1} and adding u_t to it.

2. (a) The command `genr logsp500 = log(sp500)` generates the log prices. From these, the returns can be obtained using the command `genr r = d(logsp500)`; the `d` stands for the first difference Δ. Alternatively, the returns can be obtained from the prices directly using the command `genr r = dlog(sp500)`. The time series plots can be generated like in the exercises for Week 1, resulting in the plots shown in the slides. It is immediately evident that the plot for the returns is very different from that of the (log) prices. This is because the (log) prices are integrated, while returns are stationary. The histogram is obtained by double-clicking the return series, and then clicking `View`→`Descriptive Statistics and Tests`→`Histogram and Stats`. This results in the plot below.

[1]Formally, $Q(m)$ can be used to test $H_0 : \tau_1 = \tau_2 = \ldots = \tau_m = 0$.



(b) The output shows a skewness of $\mathrm{SK} = -0.914753$, and a kurtosis of $\mathrm{K} = 20.42165$, whereas for a normal distribution, one would expect these values to be close to 0 and 3. This shows that the returns are left-skewed and have heavy tails, a very common finding. Inserting these values, together with $T = 2,546$, into the formula

$$\mathrm{JB} = \frac{T}{6}\widehat{\mathrm{SK}}^2 + \frac{T}{24}(\widehat{\mathrm{K}} - 3)^2$$

yields a Jarque-Bera statistic of $\mathrm{JB} = 32,552.84$ (up to rounding error, this value is of course already given in the output). This value is much larger than the 5% critical value of the χ^2_2 distribution, 5.991, so we strongly reject the null that the data were drawn from a normal distribution. We could also have concluded this directly from the p-value given in the output, but that might not be available in an exam...

(c) The correlogram is obtained by double-clicking the return series, and then clicking `View→Correlogram...`. The window that appears allows you to choose the number of lags. Arbitrarily choosing 20 produces the plot below.

Date: 10/28/22 Time: 18:32
Sample (adjusted): 10/30/2012 10/27/2022
Included observations: 2546 after adjustments

Lag	AC	PAC	Q-Stat	Prob
1	-0.146	-0.146	54.007	0.000
2	0.092	0.072	75.647	0.000
3	-0.016	0.008	76.268	0.000
4	-0.052	-0.061	83.165	0.000
5	0.043	0.029	87.853	0.000
6	-0.111	-0.095	119.37	0.000
7	0.155	0.126	180.37	0.000
8	-0.142	-0.098	231.97	0.000
9	0.137	0.096	280.02	0.000
10	-0.063	-0.033	290.04	0.000
11	0.021	0.016	291.19	0.000
12	0.016	-0.003	291.82	0.000
13	-0.086	-0.048	310.65	0.000
14	0.055	-0.007	318.34	0.000
15	-0.080	-0.017	334.82	0.000
16	0.070	0.011	347.37	0.000
17	-0.020	0.026	348.35	0.000
18	0.013	-0.011	348.77	0.000
19	-0.004	-0.009	348.81	0.000
20	-0.024	-0.006	350.23	0.000

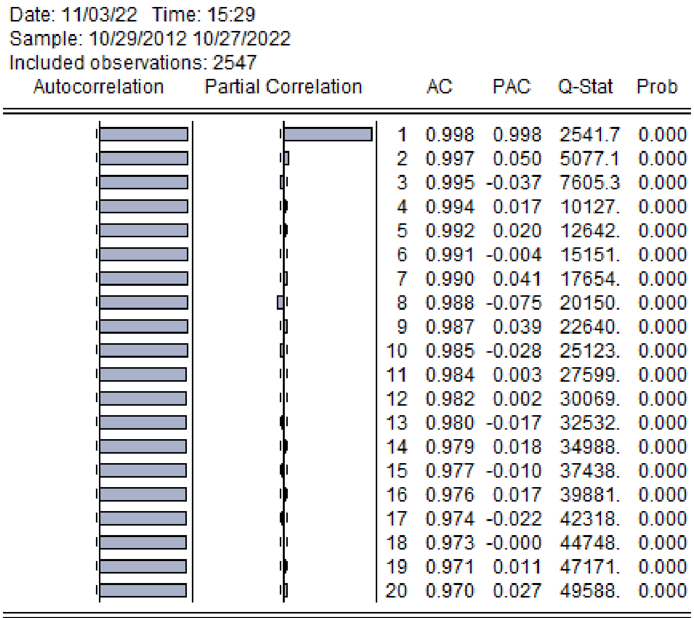
The correlogram doesn't show any indication of non-stationarity, so the data are likely stationary. They don't appear to have been generated by a pure white noise process though, because the autocorrelations and lags 1, 2, 5–10, and 13–17 are significant at the 5% level; their absolute values exceed the little line representing the critical value $1.96/\sqrt{2546} = 0.0388$.

(d) The test statistic for testing $H_0 : \tau_1 = \cdots = \tau_{10} = 0$, vs. the alternative that at least one of them is non-zero, is

$$Q(10) = T(T+2)\sum_{s=1}^{10} \frac{\hat{\tau}_s^2}{T-s} = 290.04 \sim \chi^2_{10}.$$

The observed value is 290.04, much larger than the critical value 18.307. Thus, we reject the null and conclude that at least one of the first 10 autocorrelations is different from zero.

(e) The correlogram of `logsp500` looks as follows.



Date: 11/03/22 Time: 15:29
Sample: 10/29/2012 10/27/2022
Included observations: 2547

	AC	PAC	Q-Stat	Prob
1	0.998	0.998	2541.7	0.000
2	0.997	0.050	5077.1	0.000
3	0.995	-0.037	7605.3	0.000
4	0.994	0.017	10127.	0.000
5	0.992	0.020	12642.	0.000
6	0.991	-0.004	15151.	0.000
7	0.990	0.041	17654.	0.000
8	0.988	-0.075	20150.	0.000
9	0.987	0.039	22640.	0.000
10	0.985	-0.028	25123.	0.000
11	0.984	0.003	27599.	0.000
12	0.982	0.002	30069.	0.000
13	0.980	-0.017	32532.	0.000
14	0.979	0.018	34988.	0.000
15	0.977	-0.010	37438.	0.000
16	0.976	0.017	39881.	0.000
17	0.974	-0.022	42318.	0.000
18	0.973	-0.000	44748.	0.000
19	0.971	0.011	47171.	0.000
20	0.970	0.027	49588.	0.000

The autocorrelations are very high, and appear to decay linearly, not exponentially. This is a clear sign the process that generated the data is integrated. It is not a pure random walk though; for a random walk, we would expect that the sample PACF on the right becomes insignificant after the first lag; here, some partial autocorrelations are significant. This, of course, corresponds to the fact that the returns are not pure white noise, as we determined earlier.

3. (a) By repeatedly plugging in,

$$\begin{aligned} Y_t &= Y_{t-1} + U_t \\ &= Y_{t-2} + U_{t-1} + U_t \\ &= Y_{t-3} + U_{t-2} + U_{t-1} + U_t \\ &\vdots \\ &= Y_0 + \sum_{s=1}^{t} U_s \end{aligned}$$

as claimed.

(b) The result from the previous question implies that

$$\begin{aligned}\mathbb{E}[Y_t] &= \mathbb{E}\left[Y_0 + \sum_{s=1}^{t} U_s\right] \\ &= Y_0 + \sum_{s=1}^{t} \mathbb{E}[U_s] = Y_0.\end{aligned}$$

Here, we have used that the expectation of a sum is the sum of the expectations, together with the fact that Y_0 is assumed to be a constant, and that $\mathbb{E}[U_t] = 0$ because U_t is white noise. For the variance, recall that in general, $\mathrm{var}(X+Y) = \mathrm{var}(X) + \mathrm{var}(Y) + 2\,\mathrm{cov}(X,Y)$. But since the U_t are all independent, we have that $\mathrm{var}(U_s + U_t) = \mathrm{var}(U_s) + \mathrm{var}(U_t) + 0 = 2\sigma^2$. Thus

$$\begin{aligned}\mathrm{var}[Y_t] &= \mathrm{var}\left[Y_0 + \sum_{s=1}^{t} U_s\right] \\ &= \mathrm{var}\left[\sum_{s=1}^{t} U_s\right] \\ &= \sum_{s=1}^{t} \mathrm{var}(U_s) \\ &= \sum_{s=1}^{t} \sigma^2 \\ &= t \cdot \sigma^2.\end{aligned}$$

Solution to Exercise 3

Simon A. Broda

1. (a) We clearly see that unless $|\phi_1|$ approaches 1, the process is stationary; the time series plot looks mean-reverting, and the sample autocorrelations decay exponentially as they should. We also see that $\bar{y}$ is close to $\mathbb{E}[Y_t] = \alpha/(1-\phi_1)$, and that s_y^2 is close to $\mathrm{var}[Y_t] = \sigma^2/(1-\phi_1^2)$.

   (b) If $\phi_1 = 1$, we have a random walk, and α becomes the drift: $\mathbb{E}[Y_t] = Y_0 + \alpha \cdot t$.

   (c) The correlograms of the AR(1), MA(1), and ARMA(1, 1) look respectively as follows.

Date: 11/10/22 Time: 18:49
Sample: 1 1000
Included observations: 1000

Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob
		1	0.684	0.684	469.49	0.000
		2	0.454	-0.026	676.61	0.000
		3	0.285	-0.030	758.29	0.000
		4	0.217	0.075	805.59	0.000
		5	0.172	0.014	835.50	0.000
		6	0.107	-0.054	847.09	0.000
		7	0.055	-0.013	850.13	0.000
		8	-0.007	-0.058	850.18	0.000
		9	-0.051	-0.036	852.83	0.000
		10	-0.088	-0.040	860.62	0.000
		11	-0.127	-0.063	877.09	0.000
		12	-0.113	0.040	889.93	0.000
		13	-0.115	-0.039	903.34	0.000
		14	-0.079	0.044	909.72	0.000
		15	-0.059	0.003	913.28	0.000
		16	-0.013	0.058	913.45	0.000
		17	-0.013	-0.046	913.62	0.000
		18	-0.012	0.001	913.76	0.000
		19	-0.005	0.004	913.78	0.000
		20	0.021	0.030	914.25	0.000

Date: 11/10/22 Time: 18:50
Sample: 1 1000
Included observations: 1000

Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob
		1	0.499	0.499	249.45	0.000
		2	0.062	-0.249	253.29	0.000
		3	0.053	0.197	256.13	0.000
		4	0.041	-0.103	257.81	0.000
		5	-0.001	0.041	257.81	0.000
		6	-0.017	-0.040	258.11	0.000
		7	0.034	0.086	259.26	0.000
		8	0.073	0.010	264.61	0.000
		9	0.065	0.037	268.89	0.000
		10	0.040	-0.009	270.51	0.000
		11	0.049	0.050	272.96	0.000
		12	0.066	0.018	277.42	0.000
		13	0.039	-0.002	278.95	0.000
		14	0.015	0.008	279.20	0.000
		15	0.011	-0.006	279.31	0.000
		16	-0.020	-0.041	279.71	0.000
		17	-0.042	-0.014	281.53	0.000
		18	-0.031	-0.009	282.54	0.000
		19	-0.040	-0.043	284.16	0.000
		20	-0.066	-0.040	288.59	0.000

Date: 11/10/22 Time: 18:51
Sample: 1 1000
Included observations: 1000

Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob
		1	0.833	0.833	695.61	0.000
		2	0.555	-0.452	1004.8	0.000
		3	0.361	0.272	1135.7	0.000
		4	0.242	-0.140	1194.8	0.000
		5	0.181	0.146	1227.9	0.000
		6	0.142	-0.107	1248.3	0.000
		7	0.103	0.050	1259.0	0.000
		8	0.071	-0.027	1264.1	0.000
		9	0.049	0.018	1266.5	0.000
		10	0.039	0.012	1268.0	0.000
		11	0.039	0.009	1269.5	0.000
		12	0.045	0.024	1271.5	0.000
		13	0.061	0.046	1275.3	0.000
		14	0.077	-0.008	1281.3	0.000
		15	0.079	0.000	1287.6	0.000
		16	0.071	0.003	1292.8	0.000
		17	0.056	-0.017	1295.9	0.000
		18	0.039	0.005	1297.5	0.000
		19	0.029	0.001	1298.3	0.000
		20	0.027	0.018	1299.1	0.000

For the AR(1), we see that the SACF decays geometrically, while the SPACF drops to zero (more or less) after 1 lag. For the MA(1), we see that the picture is reversed (the fact that the sign of the SPACF alternates does not play a role, as long as its absolute value decays geometrically). For the ARMA(1, 1), both SACF and SPACF decay geometrically, so it's impossible to determine the order of an ARMA(p, q) process (here, $p = 1$ and $q = 1$) from the correlogram.

2. (a) The correlogram looks as shown below.

Date: 11/10/22 Time: 17:06
Sample (adjusted): 1981Q1 2022Q2
Included observations: 166 after adjustments

Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob
		1	0.695	0.695	81.608	0.000
		2	0.446	-0.072	115.39	0.000
		3	0.228	-0.105	124.27	0.000
		4	-0.048	-0.286	124.66	0.000
		5	-0.051	0.284	125.11	0.000
		6	-0.068	-0.077	125.91	0.000
		7	-0.120	-0.122	128.45	0.000
		8	-0.160	-0.206	132.96	0.000
		9	-0.149	0.220	136.91	0.000
		10	-0.135	-0.048	140.19	0.000
		11	-0.124	-0.102	142.96	0.000
		12	-0.100	-0.136	144.76	0.000
		13	-0.097	0.135	146.48	0.000
		14	-0.070	0.030	147.38	0.000
		15	-0.045	-0.092	147.76	0.000
		16	-0.035	-0.120	147.99	0.000
		17	-0.031	0.072	148.17	0.000
		18	-0.052	-0.003	148.68	0.000
		19	-0.094	-0.161	150.34	0.000
		20	-0.126	-0.127	153.37	0.000

Geometrically decaying ACF, PACF drops to zero after one lag, even though some later values are significant. Still, a simple AR(1) might suffice. We can estimate it by entering `gdp_growth c ar(1)` under `Quick`→ `Estimate Equation...`. The result is

Dependent Variable: GDP_GROWTH
Method: ARMA Maximum Likelihood (OPG - BHHH)
Date: 11/10/22 Time: 17:10
Sample: 1981Q1 2022Q2
Included observations: 166
Convergence achieved after 18 iterations
Coefficient covariance computed using outer product of gradients

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.017264	0.003884	4.444747	0.0000
AR(1)	0.691619	0.042082	16.43513	0.0000
SIGMASQ	0.000206	8.10E-06	25.38758	0.0000

R-squared	0.483435	Mean dependent var	0.017256
Adjusted R-squared	0.477096	S.D. dependent var	0.020010
S.E. of regression	0.014469	Akaike info criterion	-5.611745
Sum squared resid	0.034126	Schwarz criterion	-5.555504
Log likelihood	468.7748	Hannan-Quinn criter.	-5.588916
F-statistic	76.27284	Durbin-Watson stat	1.892005
Prob(F-statistic)	0.000000		

Inverted AR Roots	.69

Everything is significant, and the estimated model is stationary (AR coefficient is less than 1 in absolute value). The residual correlogram (under `View`→`Residual Diagnostics...` looks like this:

Date: 11/10/22 Time: 17:13
Sample (adjusted): 1981Q1 2022Q2
Q-statistic probabilities adjusted for 1 ARMA term

Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob
		1	0.052	0.052	0.4549	
		2	0.042	0.039	0.7559	0.385
		3	0.119	0.115	3.1750	0.204
		4	-0.378	-0.398	27.779	0.000
		5	0.021	0.078	27.854	0.000
		6	0.034	0.052	28.053	0.000
		7	-0.042	0.052	28.357	0.000
		8	-0.091	-0.318	29.819	0.000
		9	-0.043	0.035	30.151	0.000
		10	-0.023	0.047	30.249	0.000
		11	-0.040	0.023	30.543	0.001
		12	0.012	-0.210	30.569	0.001
		13	-0.051	-0.036	31.051	0.002
		14	-0.011	0.073	31.073	0.003
		15	0.011	0.039	31.097	0.005
		16	0.001	-0.137	31.097	0.009
		17	0.030	-0.030	31.266	0.012
		18	0.019	0.094	31.336	0.018
		19	-0.029	-0.012	31.490	0.025
		20	-0.075	-0.215	32.563	0.027

The ACF and PACF at lag 4 are both significant. It's not obvious which model to use for this. One idea is to use an ARMA(1, 4), which we can estimate by entering `gdp_growth c ar(1) ma(1 to 4)` in the estimation window, resulting in the estimation output and residual correlogram below.

Dependent Variable: GDP_GROWTH
Method: ARMA Maximum Likelihood (OPG - BHHH)
Date: 11/10/22 Time: 17:17
Sample: 1981Q1 2022Q2
Included observations: 166
Convergence not achieved after 500 iterations
Coefficient covariance computed using outer product of gradients

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.017535	0.000617	28.43690	0.0000
AR(1)	0.886003	0.051325	17.26249	0.0000
MA(1)	-0.064953	10.31694	-0.006296	0.9950
MA(2)	0.072189	24.96029	0.002892	0.9977
MA(3)	-0.079997	9.575807	-0.008354	0.9933
MA(4)	-0.927237	416.0201	-0.002229	0.9982
SIGMASQ	0.000122	0.003757	0.032452	0.9742

R-squared	0.693653	Mean dependent var	0.017256
Adjusted R-squared	0.682093	S.D. dependent var	0.020010
S.E. of regression	0.011282	Akaike info criterion	-6.019585
Sum squared resid	0.020238	Schwarz criterion	-5.888356
Log likelihood	506.6255	Hannan-Quinn criter.	-5.966318
F-statistic	60.00317	Durbin-Watson stat	2.049757
Prob(F-statistic)	0.000000		

Inverted AR Roots	.89			
Inverted MA Roots	1.00	-.00-1.00i	-.00+1.00i	-.93

Date: 11/10/22 Time: 17:21
Sample (adjusted): 1981Q1 2022Q2
Q-statistic probabilities adjusted for 5 ARMA terms

Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob
		1	-0.026	-0.026	0.1172	
		2	-0.035	-0.036	0.3237	
		3	0.105	0.104	2.2167	
		4	0.038	0.043	2.4661	
		5	-0.016	-0.007	2.5086	
		6	0.052	0.044	2.9845	0.084
		7	0.025	0.019	3.0950	0.213
		8	-0.066	-0.063	3.8743	0.275
		9	-0.056	-0.069	4.4363	0.350
		10	0.027	0.011	4.5672	0.471
		11	-0.016	-0.006	4.6112	0.595
		12	0.005	0.022	4.6154	0.707
		13	-0.067	-0.070	5.4282	0.711
		14	0.006	0.009	5.4353	0.795
		15	0.003	0.007	5.4367	0.860
		16	-0.011	-0.001	5.4595	0.907
		17	-0.001	-0.005	5.4596	0.941
		18	-0.014	-0.020	5.4985	0.963
		19	-0.019	-0.012	5.5697	0.976
		20	-0.044	-0.044	5.9329	0.981

The residual correlogram looks fine now, but note that all the MA coefficients are insignificant. Maybe we can get away with dropping the first 3, leading to the *subset AR model*

$$Y_t = \alpha + \phi_1 Y_{t-1} + U_t + \theta_4 U_{t-4}.$$

This can be estimated by entering `gdp_growth c ar(1) ma(4)` in the estimation window. The results are shown below.

Dependent Variable: GDP_GROWTH
Method: ARMA Maximum Likelihood (OPG - BHHH)
Date: 11/10/22 Time: 17:26
Sample: 1981Q1 2022Q2
Included observations: 166
Convergence achieved after 32 iterations
Coefficient covariance computed using outer product of gradients

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.017502	0.001291	13.55181	0.0000
AR(1)	0.867376	0.024798	34.97700	0.0000
MA(4)	-0.846949	0.054300	-15.59769	0.0000
SIGMASQ	0.000132	5.58E-06	23.64866	0.0000

R-squared	0.668602	Mean dependent var	0.017256
Adjusted R-squared	0.662465	S.D. dependent var	0.020010
S.E. of regression	0.011625	Akaike info criterion	-6.016511
Sum squared resid	0.021893	Schwarz criterion	-5.941524
Log likelihood	503.3704	Hannan-Quinn criter.	-5.986073
F-statistic	108.9462	Durbin-Watson stat	2.106125
Prob(F-statistic)	0.000000		

Inverted AR Roots	.87			
Inverted MA Roots	.96	.00-.96i	.00+.96i	-.96

Date: 11/10/22 Time: 17:27
Sample (adjusted): 1981Q1 2022Q2
Q-statistic probabilities adjusted for 2 ARMA terms

Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob
		1	-0.054	-0.054	0.4897	
		2	0.044	0.041	0.8149	
		3	0.060	0.065	1.4274	0.232
		4	0.043	0.048	1.7454	0.418
		5	-0.039	-0.040	2.0137	0.570
		6	0.114	0.103	4.2740	0.370
		7	-0.023	-0.014	4.3679	0.498
		8	-0.058	-0.068	4.9624	0.549
		9	-0.081	-0.098	6.1174	0.526
		10	0.057	0.047	6.7027	0.569
		11	-0.043	-0.012	7.0308	0.634
		12	0.008	0.004	7.0429	0.721
		13	-0.092	-0.092	8.5734	0.661
		14	0.029	0.029	8.7322	0.726
		15	-0.020	0.013	8.8080	0.787
		16	-0.010	-0.022	8.8249	0.842
		17	-0.024	-0.029	8.9355	0.881
		18	0.007	-0.003	8.9435	0.916
		19	-0.038	-0.006	9.2194	0.933
		20	-0.045	-0.062	9.5998	0.944

The correlogram still looks fine, and all coefficients are now significant. The subset model has a smaller BIC (-5.94 vs. -5.89), so is preferred (better tradeoff between fit and parsimony). The AIC seems to prefer the larger model; this is typical.

Instead of the above manual procedure, we can automate the procedure of finding the model by using autoarma: just paste

```
freeze(armatable) gdp_growth.autoarma(diff=0
, select=sic, maxar=4, maxma=4, atable) forec c
```

into the estimation window (all on one line). This produces the table below.

Model Selection Criteria Table
Dependent Variable: GDP_GROWTH
Date: 11/10/22 Time: 17:43
Sample: 1980Q1 2022Q2
Included observations: 166

Model	LogL	AIC	BIC*	HQ
(0,3)(0,0)	502.567881	-5.994794	-5.901059	-5.956746
(1,4)(0,0)	506.625533	-6.019585	-5.888356	-5.966318
(1,3)(0,0)	503.415183	-5.992954	-5.880473	-5.947297
(2,3)(0,0)	504.232016	-5.990747	-5.859519	-5.937481
(2,4)(0,0)	506.761916	-6.009180	-5.859204	-5.948304
(3,3)(0,0)	504.562277	-5.982678	-5.832703	-5.921802
(0,4)(0,0)	497.181159	-5.917845	-5.805364	-5.872188
(4,3)(0,0)	504.564472	-5.970656	-5.801934	-5.902171
(3,4)(0,0)	504.231623	-5.966646	-5.797924	-5.898161
(4,4)(0,0)	500.953990	-5.915108	-5.727639	-5.839013
(3,2)(0,0)	489.173962	-5.809325	-5.678096	-5.756058
(2,2)(0,0)	484.459400	-5.764571	-5.652090	-5.718914
(4,1)(0,0)	486.270962	-5.774349	-5.643121	-5.721082
(2,1)(0,0)	476.307048	-5.678398	-5.584664	-5.640351
(4,2)(0,0)	482.783828	-5.720287	-5.570312	-5.659411
(4,0)(0,0)	477.086618	-5.675742	-5.563261	-5.630085
(1,2)(0,0)	474.444937	-5.655963	-5.562229	-5.617916
(1,0)(0,0)	468.774819	-5.611745	-5.555504	-5.588916
(2,0)(0,0)	469.185600	-5.604646	-5.529658	-5.574208
(1,1)(0,0)	469.105705	-5.603683	-5.528696	-5.573245
(3,1)(0,0)	474.139810	-5.640239	-5.527757	-5.594582
(3,0)(0,0)	470.120246	-5.603858	-5.510124	-5.565811
(0,1)(0,0)	460.483380	-5.511848	-5.455607	-5.489019
(0,2)(0,0)	460.901021	-5.504832	-5.429844	-5.474394
(0,0)(0,0)	414.274265	-4.967160	-4.929666	-4.951941

We see that the BIC selects a MA(3) model[1], but it has a higher BIC than our subset model, because `autoarma` doesn't consider subset models. Estimating this model via

```
gdp_growth c ma(1 to 3)
```

results in the output below.

[1]The AIC selects an ARMA(1, 4)

Dependent Variable: GDP_GROWTH
Method: ARMA Maximum Likelihood (OPG - BHHH)
Date: 11/10/22 Time: 17:47
Sample: 1981Q1 2022Q2
Included observations: 166
Convergence achieved after 106 iterations
Coefficient covariance computed using outer product of gradients

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.017386	0.004378	3.971259	0.0001
MA(1)	0.887055	0.033045	26.84407	0.0000
MA(2)	0.897620	0.063248	14.19207	0.0000
MA(3)	0.857028	0.038752	22.11553	0.0000
SIGMASQ	0.000134	5.66E-06	23.57846	0.0000

R-squared	0.664468	Mean dependent var	0.017256
Adjusted R-squared	0.656132	S.D. dependent var	0.020010
S.E. of regression	0.011734	Akaike info criterion	-5.994794
Sum squared resid	0.022166	Schwarz criterion	-5.901059
Log likelihood	502.5679	Hannan-Quinn criter.	-5.956746
F-statistic	79.70872	Durbin-Watson stat	2.075441
Prob(F-statistic)	0.000000		

Inverted MA Roots	.02-.96i	.02+.96i	-.92

Date: 11/10/22 Time: 17:49
Sample (adjusted): 1981Q1 2022Q2
Q-statistic probabilities adjusted for 3 ARMA terms

Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob
		1	-0.038	-0.038	0.2491	
		2	-0.060	-0.062	0.8630	
		3	0.026	0.021	0.9778	
		4	-0.032	-0.034	1.1579	0.282
		5	0.000	0.000	1.1579	0.560
		6	0.047	0.042	1.5389	0.673
		7	-0.034	-0.029	1.7367	0.784
		8	-0.107	-0.106	3.7447	0.587
		9	-0.063	-0.079	4.4586	0.615
		10	0.024	0.010	4.5637	0.713
		11	-0.033	-0.038	4.7629	0.783
		12	-0.013	-0.021	4.7950	0.852
		13	-0.070	-0.080	5.6764	0.842
		14	0.008	0.009	5.6875	0.893
		15	0.000	-0.012	5.6875	0.931
		16	-0.015	-0.032	5.7301	0.955
		17	0.009	-0.012	5.7438	0.973
		18	-0.005	-0.011	5.7486	0.984
		19	-0.016	-0.018	5.7945	0.990
		20	-0.044	-0.067	6.1640	0.992

This looks fine too. I prefer to stick with our subset model, because it has a lower BIC.

(b) The estimated parameter c in EViews corresponds to $c = \mathbb{E}[Y_t] = \alpha/(1-\phi_1)$, so we have $\hat{\alpha} = \hat{c}(1-\hat{\phi}_1) = 0.017502 \cdot (1 - 0.867376) = 0.002321$. Thus, our final model is

$$Y_t = 0.002321 + 0.867376 Y_{t-1} + U_t - 0.846949 U_{t-4}.$$

The manual forecast for 2022Q3 is therefore

$$\begin{aligned}\widehat{y}_{t+1} &= 0.002321 + 0.867376 \cdot 0.024037 - 0.846949 \cdot 0.001429 \\ &= 0.021960.\end{aligned}$$

The value for y_{2022Q2}, 0.024037, can be obtained from the spreadsheet view. The value 0.001429 corresponds to $\hat{u}_{2021Q3}$. To find it, go to the estimation output, click on `Proc`→`Make Residual Series...`, and open the resulting series in spreadsheet view.

The manual forecast for 2022Q4 can be constructed analogously. It requires y_{2022Q3}, which we replace with our forecast from the previous question. Hence

$$\begin{aligned}\widehat{y}_{t+2} &= 0.002321 + 0.867376 \cdot 0.021960 - 0.846949 \cdot (-0.003883)\\ &= 0.024657,\end{aligned}$$

where $-0.003883 = \hat{u}_{2021Q4}$. The same forecasts can be obtained using EViews. First, inside the workfile pane on the left, go to `Proc`→ `Structure / Resize Current Page...`, and resize the file so that it includes 2022Q3 and 2022Q4. Next, in the pane with the estimation output, click on `Forecast`. Keep the default of a dynamic forecast, and set the forecast sample to 2022Q3:2022Q4. Your forecast will be saved as a new series `gdp_growthf` You can open it in spreadsheet view and confirm that the forecasts are the same as those obtained above,

3. (a) By repeatedly plugging in,

$$\begin{aligned}Y_t &= \alpha + Y_{t-1} + U_t\\ &= \alpha + (\alpha + Y_{t-2} + U_{t-1}) + U_t\\ &\vdots\\ &= Y_0 + \alpha \cdot t + \sum_{s=1}^{t} U_s,\end{aligned}$$

so that

$$\mathbb{E}[Y_t] = Y_0 + \alpha \cdot t,$$

because white noise has expectation zero. The derivation of the variance is the same as for the case without drift from last week and thus omitted here.

(b) The previous question shows that the random walk with drift is not stationary, because its mean and variance change over time. For it to be I(1), its first difference ΔY_t should be stationary. We immediately se that $\Delta Y_t = Y_t - Y_{t-1} = (\alpha + Y_{t-1} + U_t) - Y_{t-1} = \alpha + U_t$. This is just white noise plus a constant, which is stationary.

(c) Since $\{U_t\}$ is white noise, U_t is uncorrelated with Y_{t-1}, so

$$\begin{aligned}\mathrm{var}(Y_t) &= \mathrm{var}\left(\alpha + \phi_1 Y_{t-1} + U_t\right)\\ &= \phi_1^2 \mathrm{var}(Y_{t-1}) + \mathrm{var}(U_t) + 2\phi_1 \mathrm{cov}(Y_{t-1}, U_t) \qquad = \phi_1^2 \mathrm{var}(Y_t) + \sigma^2,\end{aligned}$$

where the final equality holds because Y_t is stationary, which implies that $\mathrm{var}(Y_t) = \mathrm{var}(Y_{t-1})$. Thus, if and only if $|\phi_1| < 1$,

$$\mathrm{var}(Y_t) = \frac{\sigma^2}{1 - \phi_1^2}.$$

Note that $\mathrm{var}(Y_t) > \mathrm{var}(Y_{t-1})$ if $|\phi_1| \geq 1$, i.e., the variance grows without bounds in that case.

(d) For the MA(1) process
$$Y_t = \alpha + U_t + \theta_1 U_{t-1},$$
we have that
$$\begin{aligned}\mathbb{E}[Y_t] &= \mathbb{E}[\alpha + U_t + \theta_1 U_{t-1}] \\ &= \alpha + \mathbb{E}[U_t] + \theta_1 \mathbb{E}[U_{t-1}] \\ &= \alpha.\end{aligned}$$
For the variance,
$$\begin{aligned}\gamma_0 = \operatorname{var}(Y_t) &= \operatorname{var}(\alpha + U_t + \theta_1 U_{t-1}) \\ &= \operatorname{var}(U_t + \theta_1 U_{t-1}) \\ &= \operatorname{var}(U_t) + \theta_1^2 \operatorname{var}(U_{t-1}) + 2\theta_1 \operatorname{cov}(U_t, U_{t-1}) \\ &= \sigma^2 + \theta_1^2 \sigma^2 + 0 \\ &= \sigma^2(1 + \theta_1^2).\end{aligned}$$
For the first autocovariance,
$$\begin{aligned}\gamma_1 &= \operatorname{cov}(Y_t, Y_{t-1}) \\ &= \operatorname{cov}(\alpha + U_t + \theta_1 U_{t-1}, \alpha + U_{t-1} + \theta_1 U_{t-2}) \qquad (\dagger) \\ &= \operatorname{cov}(\theta_1 U_{t-1}, U_{t-1})\end{aligned}$$
because white noise is uncorrelated. Hence
$$\begin{aligned}\gamma_1 &= \theta_1 \operatorname{cov}(U_{t-1}, U_{t-1}) \\ &= \theta_1 \operatorname{var}(U_{t-1}) \\ &= \theta_1 \sigma^2.\end{aligned}$$
Higher order autocorrelations will be zero, because there will no common U_t terms in (†). Plugging these into the definition of the ACF, we have
$$\tau_1 = \frac{\gamma_1}{\gamma_0} = \frac{\theta_1 \sigma^2}{\sigma^2(1 + \theta_1^2)} = \frac{\theta_1}{1 + \theta_1^2}.$$

(e) **Optional**: The ACF is obtained by repeatedly substituting $Y_{t-i} = \phi_1 Y_{t-i-1} + \alpha + U_{t-i}$:
$$\begin{aligned}Y_t &= \phi_1 Y_{t-1} + \alpha + U_t \\ &= \phi_1^2 Y_{t-2} + \phi_1(\alpha + U_{t-1}) + \alpha + U_t \\ &= \phi_1^3 Y_{t-3} + \phi_1^2(\alpha + U_{t-2}) + \phi_1(\alpha + U_{t-1}) + \alpha + U_t \\ &\vdots \\ &= \phi_1^k Y_{t-k} + \sum_{i=0}^{k-1} \phi_1^i \alpha + \sum_{i=0}^{k-1} \phi_1^i U_{t-i}. \qquad (1)\end{aligned}$$

Therefore,

$$\gamma_k = \mathrm{cov}(Y_t, Y_{t-k}) = \phi_1^k \mathrm{cov}(Y_{t-k}, Y_{t-k}) + \sum_{i=0}^{k-1} \phi_1^i \mathrm{cov}(U_{t-i}, Y_{t-k})$$
$$= \phi_1^k \mathrm{var}(Y_{t-k}),$$

so that

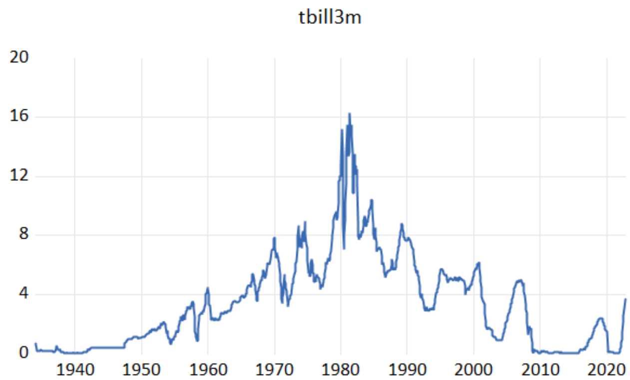
$$\tau_k = \frac{\gamma_k}{\gamma_0} = \phi_1^k.$$

Solution to Exercise 4

Simon A. Broda

1. If we set α and β_1 to the same value, e.g., 1, then both series trend up by 1 each period. The correlograms look quite similar, but the time series plots don't really, even if we crank up the variance of the trend-stationary series. This is because the latter is stationary after de-trending (subtracting off $\beta_1 \cdot t$, whereas the random walk with drift remains nonstationary if we subtract $\alpha \cdot t$ (it becomes a random walk without drift).

2. (a) The time series plot and the correlogram are shown below.



Date: 11/17/22 Time: 17:42
Sample: 1934M01 2022M10
Included observations: 1066

Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob
		1	0.993	0.993	1054.3	0.000
		2	0.982	-0.329	2085.6	0.000
		3	0.971	0.168	3095.5	0.000
		4	0.961	-0.022	4086.1	0.000
		5	0.952	-0.003	5057.6	0.000
		6	0.941	-0.045	6009.3	0.000
		7	0.933	0.213	6945.6	0.000
		8	0.927	0.004	7870.7	0.000
		9	0.920	-0.142	8782.4	0.000
		10	0.910	-0.108	9675.5	0.000
		11	0.899	0.028	10548.	0.000
		12	0.888	-0.036	11401.	0.000
		13	0.879	0.108	12236.	0.000
		14	0.869	-0.104	13052.	0.000
		15	0.857	-0.069	13848.	0.000
		16	0.846	0.061	14624.	0.000
		17	0.835	-0.113	15381.	0.000
		18	0.824	-0.012	16118.	0.000
		19	0.811	-0.008	16833.	0.000
		20	0.798	0.092	17527.	0.000
		21	0.789	0.106	18205.	0.000
		22	0.782	0.048	18871.	0.000
		23	0.775	0.028	19527.	0.000
		24	0.768	0.009	20172.	0.000
		25	0.762	0.015	20807.	0.000
		26	0.756	0.026	21433.	0.000
		27	0.750	-0.047	22049.	0.000
		28	0.741	-0.008	22652.	0.000
		29	0.733	-0.016	23242.	0.000
		30	0.725	-0.027	23819.	0.000
		31	0.718	0.018	24386.	0.000
		32	0.711	0.055	24943.	0.000
		33	0.706	0.009	25491.	0.000
		34	0.701	0.012	26033.	0.000
		35	0.697	0.019	26569.	0.000
		36	0.693	0.054	27100.	0.000

From the correlogram, it looks like the T-Bill rate is I(1): the autocorrelations are large and decay slowly and approximately linearly. We can confirm this with an ADF test: open the series and click on `View→Unit Root Tests→Standard Unit Root Test...`. The data don't seem to have a trend[1], so we include just an intercept (because the mean is not zero). Choose automatic lag length selection, but switch to the AIC instead of the default BIC. This results in the following output.

[1] One could argue that there is a breaking trend, upwards until around 1982, downwards thereafter, but this kind of test is not available in EViews.

Null Hypothesis: TBILL3M has a unit root
Exogenous: Constant
Lag Length: 21 (Automatic - based on AIC, maxlag=30)

		t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic		-2.206152	0.2043
Test critical values:	1% level	-3.436395	
	5% level	-2.864098	
	10% level	-2.568183	

*MacKinnon (1996) one-sided p-values.

Augmented Dickey-Fuller Test Equation
Dependent Variable: D(TBILL3M)
Method: Least Squares
Date: 11/17/22 Time: 17:41
Sample (adjusted): 1935M11 2022M10
Included observations: 1044 after adjustments

Variable	Coefficient	Std. Error	t-Statistic	Prob.
TBILL3M(-1)	-0.007083	0.003210	-2.206152	0.0276
D(TBILL3M(-1))	0.428522	0.031217	13.72731	0.0000
D(TBILL3M(-2))	-0.192362	0.033870	-5.679419	0.0000
D(TBILL3M(-3))	0.067953	0.034329	1.979469	0.0480
D(TBILL3M(-4))	-0.058873	0.034422	-1.710351	0.0875
D(TBILL3M(-5))	0.129746	0.034477	3.763218	0.0002
D(TBILL3M(-6))	-0.219951	0.034516	-6.372418	0.0000
D(TBILL3M(-7))	0.015916	0.035015	0.454557	0.6495
D(TBILL3M(-8))	0.058508	0.034911	1.675922	0.0941
D(TBILL3M(-9))	0.122711	0.034920	3.514096	0.0005
D(TBILL3M(-10))	-0.031473	0.034841	-0.903338	0.3666
D(TBILL3M(-11))	0.117373	0.034680	3.384416	0.0007
D(TBILL3M(-12))	-0.137393	0.034856	-3.941757	0.0001
D(TBILL3M(-13))	0.060972	0.034933	1.745389	0.0812
D(TBILL3M(-14))	0.093066	0.034949	2.662912	0.0079
D(TBILL3M(-15))	-0.114104	0.035068	-3.253817	0.0012
D(TBILL3M(-16))	0.124406	0.034513	3.604620	0.0003
D(TBILL3M(-17))	-0.014426	0.034526	-0.417819	0.6762
D(TBILL3M(-18))	0.031963	0.034467	0.927367	0.3540
D(TBILL3M(-19))	-0.066898	0.034435	-1.942716	0.0523
D(TBILL3M(-20))	-0.082596	0.033888	-2.437344	0.0150
D(TBILL3M(-21))	-0.053385	0.031389	-1.700730	0.0893
C	0.026842	0.014594	1.839237	0.0662

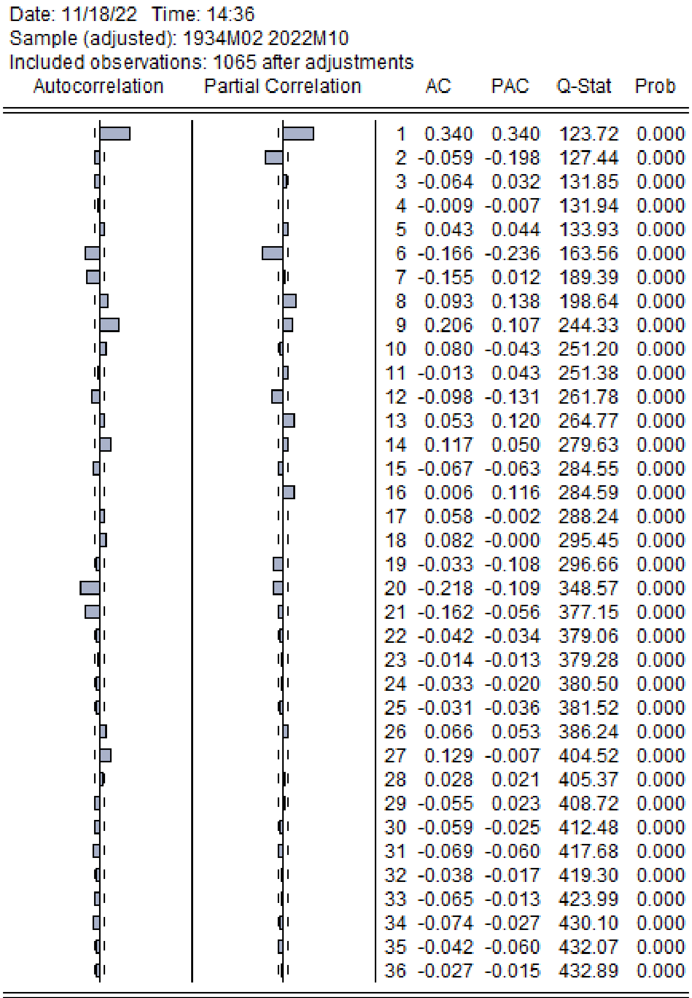
R-squared	0.290413	Mean dependent var	0.003372
Adjusted R-squared	0.275124	S.D. dependent var	0.362986
S.E. of regression	0.309045	Akaike info criterion	0.511124
Sum squared resid	97.51447	Schwarz criterion	0.620194
Log likelihood	-243.8069	Hannan-Quinn criter.	0.552492
F-statistic	18.99390	Durbin-Watson stat	2.001414
Prob(F-statistic)	0.000000		

The observed test statistic is -2.206, larger than the critical value -2.86, so the test does not reject the null that the data are I(1), as expected. The same conclusion can be drawn from the p-value of 0.2043[2].

(b) We start by inspecting the correlogram of the first difference. This can either be done

[2]Please be aware that in an exam, I might delete the top of the EViews output, and give you only the test regression at the bottom. Note that the p-value of the t-statistic there is wrong; it corresponds to the p-value from a standard regression with stationary variables, i.e., it's from a normal distribution instead of the Dickey-Fuller distribution. You can try this yourself by running the ADF regression manually, by entering `d(TBILL3M) c TBILL3M(-1) d(TBILL3M(-1)) ...TBILL3M(-21)` under `Quick→Estimate Equation...`.

by creating a new series via `genr DTBILL3M = d(TBILL3M)` and inspecting its correlogram, or by opening `TBILL3M` (i.e., the series in levels), and then clicking `View`→ `Correlogram...`, and specifying that you want the correlogram of the first difference.



Date: 11/18/22 Time: 14:36
Sample (adjusted): 1934M02 2022M10
Included observations: 1065 after adjustments

	AC	PAC	Q-Stat	Prob
1	0.340	0.340	123.72	0.000
2	-0.059	-0.198	127.44	0.000
3	-0.064	0.032	131.85	0.000
4	-0.009	-0.007	131.94	0.000
5	0.043	0.044	133.93	0.000
6	-0.166	-0.236	163.56	0.000
7	-0.155	0.012	189.39	0.000
8	0.093	0.138	198.64	0.000
9	0.206	0.107	244.33	0.000
10	0.080	-0.043	251.20	0.000
11	-0.013	0.043	251.38	0.000
12	-0.098	-0.131	261.78	0.000
13	0.053	0.120	264.77	0.000
14	0.117	0.050	279.63	0.000
15	-0.067	-0.063	284.55	0.000
16	0.006	0.116	284.59	0.000
17	0.058	-0.002	288.24	0.000
18	0.082	-0.000	295.45	0.000
19	-0.033	-0.108	296.66	0.000
20	-0.218	-0.109	348.57	0.000
21	-0.162	-0.056	377.15	0.000
22	-0.042	-0.034	379.06	0.000
23	-0.014	-0.013	379.28	0.000
24	-0.033	-0.020	380.50	0.000
25	-0.031	-0.036	381.52	0.000
26	0.066	0.053	386.24	0.000
27	0.129	-0.007	404.52	0.000
28	0.028	0.021	405.37	0.000
29	-0.055	0.023	408.72	0.000
30	-0.059	-0.025	412.48	0.000
31	-0.069	-0.060	417.68	0.000
32	-0.038	-0.017	419.30	0.000
33	-0.065	-0.013	423.99	0.000
34	-0.074	-0.027	430.10	0.000
35	-0.042	-0.060	432.07	0.000
36	-0.027	-0.015	432.89	0.000

The first difference looks stationary. After some specification search[3], we find that an ARMA(5, 5) model is the most adequate, even though it doesn't remove the autocorrelation completely. Estimating this model (either as `d(TBILL3M) c AR(1 to 5) MA(1 to 5)`, or if you created a new variable for the difference above, `DTBILL3M c AR(1 to 5) MA(1 to 5)`) yields the output and correlogram below.

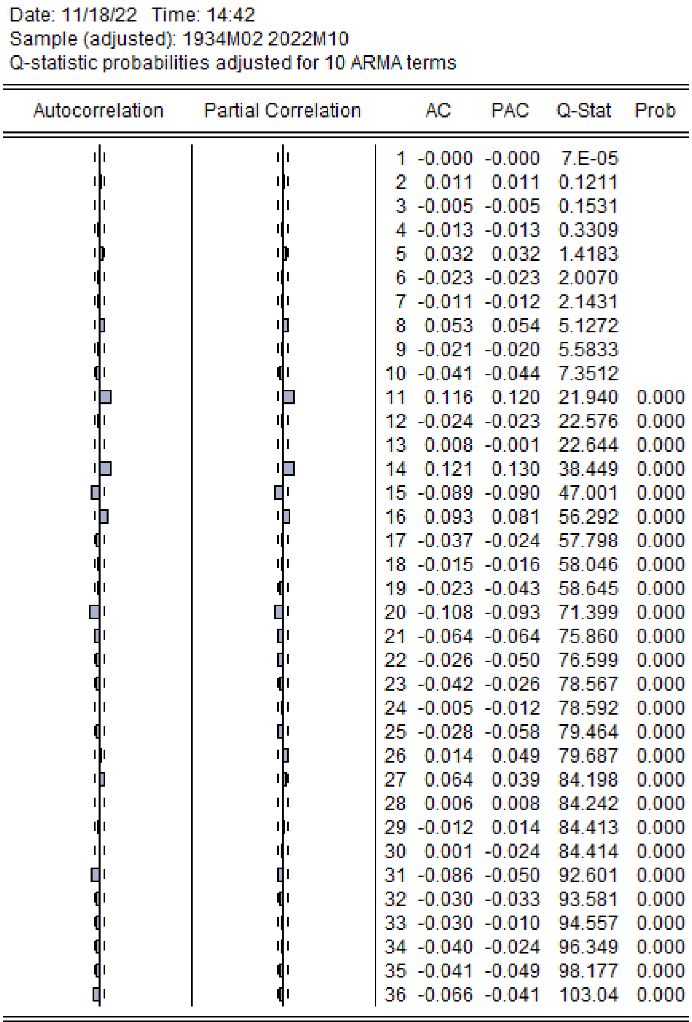
[3] `freeze(mode=overwrite, armatable) tbill3m.autoarma(tform=none, diff=1, select=sic, maxar=10, maxma=10, atable) forec c`

Dependent Variable: D(TBILL3M)
Method: ARMA Maximum Likelihood (OPG - BHHH)
Date: 11/18/22 Time: 14:41
Sample: 1934M02 2022M10
Included observations: 1065
Convergence achieved after 233 iterations
Coefficient covariance computed using outer product of gradients

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.002897	0.013904	0.208373	0.8350
AR(1)	0.922979	0.049452	18.66408	0.0000
AR(2)	-1.062794	0.076221	-13.94354	0.0000
AR(3)	0.002677	0.090163	0.029689	0.9763
AR(4)	0.176621	0.066271	2.665159	0.0078
AR(5)	-0.500188	0.031151	-16.05682	0.0000
MA(1)	-0.511548	0.051091	-10.01258	0.0000
MA(2)	0.662301	0.059644	11.10430	0.0000
MA(3)	0.460269	0.053463	8.609122	0.0000
MA(4)	-0.249555	0.041143	-6.065538	0.0000
MA(5)	0.526464	0.026709	19.71112	0.0000
SIGMASQ	0.099211	0.001604	61.86396	0.0000

R-squared	0.232134	Mean dependent var	0.002817
Adjusted R-squared	0.224113	S.D. dependent var	0.359617
S.E. of regression	0.316767	Akaike info criterion	0.550842
Sum squared resid	105.6592	Schwarz criterion	0.606850
Log likelihood	-281.3231	Hannan-Quinn criter.	0.572063
F-statistic	28.93938	Durbin-Watson stat	1.998371
Prob(F-statistic)	0.000000		

Inverted AR Roots	.65-.65i	.65+.65i	.15-.93i	.15+.93i
	-.66			
Inverted MA Roots	.59-.69i	.59+.69i	.09-.87i	.09+.87i
	-.84			



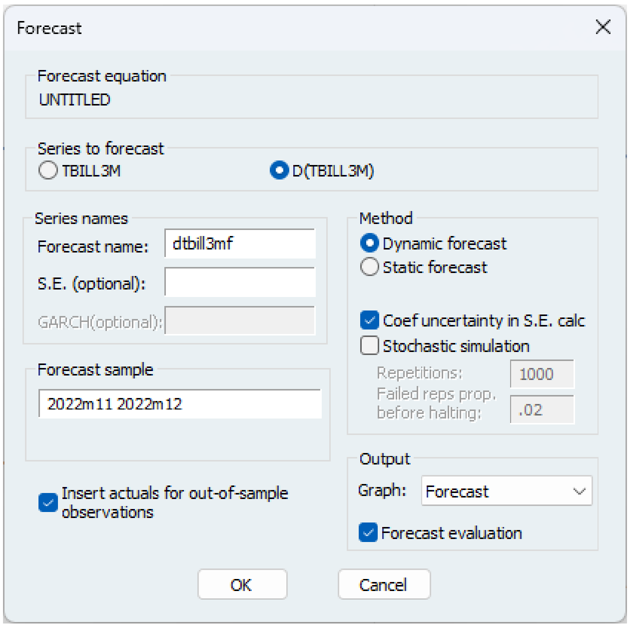
Date: 11/18/22 Time: 14:42
Sample (adjusted): 1934M02 2022M10
Q-statistic probabilities adjusted for 10 ARMA terms

Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob
		1	-0.000	-0.000	7.E-05	
		2	0.011	0.011	0.1211	
		3	-0.005	-0.005	0.1531	
		4	-0.013	-0.013	0.3309	
		5	0.032	0.032	1.4183	
		6	-0.023	-0.023	2.0070	
		7	-0.011	-0.012	2.1431	
		8	0.053	0.054	5.1272	
		9	-0.021	-0.020	5.5833	
		10	-0.041	-0.044	7.3512	
		11	0.116	0.120	21.940	0.000
		12	-0.024	-0.023	22.576	0.000
		13	0.008	-0.001	22.644	0.000
		14	0.121	0.130	38.449	0.000
		15	-0.089	-0.090	47.001	0.000
		16	0.093	0.081	56.292	0.000
		17	-0.037	-0.024	57.798	0.000
		18	-0.015	-0.016	58.046	0.000
		19	-0.023	-0.043	58.645	0.000
		20	-0.108	-0.093	71.399	0.000
		21	-0.064	-0.064	75.860	0.000
		22	-0.026	-0.050	76.599	0.000
		23	-0.042	-0.026	78.567	0.000
		24	-0.005	-0.012	78.592	0.000
		25	-0.028	-0.058	79.464	0.000
		26	0.014	0.049	79.687	0.000
		27	0.064	0.039	84.198	0.000
		28	0.006	0.008	84.242	0.000
		29	-0.012	0.014	84.413	0.000
		30	0.001	-0.024	84.414	0.000
		31	-0.086	-0.050	92.601	0.000
		32	-0.030	-0.033	93.581	0.000
		33	-0.030	-0.010	94.557	0.000
		34	-0.040	-0.024	96.349	0.000
		35	-0.041	-0.049	98.177	0.000
		36	-0.066	-0.041	103.04	0.000

Some autocorrelation remains, but we won't bother to model it. So our final model for Δ TBILL3M is an ARMA(5, 5). This means that the levels TBILL3M follow an ARIMA(5, 1, 5) model.

(c) To forecast an ARIMA model, we first forecast the ARMA model for the first difference. We'll spare us the hassle of doing a manual forecast for this complicated model, and just use EViews for it. Extending the workfile to 2022M12 via `Proc`$\to$`Structure / Resize Current Page...`, we can forecast by clicking `Forecast` on the estimated equation and setting the options as follows[4]:

[4]Note that because I estimated my model using `d(TBILL3M) c AR(1 to 5) MA(1 to 5)`, EViews gives me the option to directly predict the levels, but we'll ignore this here.



The forecasts are $\widehat{\Delta\text{TBILL}}_{2022M11} = 0.1636$ and $\widehat{\Delta\text{TBILL}}_{2022M12} = -0.0328$. The T-Bill rate in 2022M10 was 3.72, so the corresponding forecasts for the levels are

$$\widehat{\text{TBILL}}_{2022M11} = 3.72 + 0.1636 = 3.8836$$

and

$$\widehat{\text{TBILL}}_{2022M12} = 3.8836 - 0.0328 = 3.8508.$$

3. (a) We begin by creating the excess returns Note that the T-Bill rate is quoted in percent and in annualized terms, so we have to convert it to daily log returns first. The commands are

```
genr rf = log(1+dtb3/100)/365
genr r = dlog(ibm)-rf
genr rm = dlog(spx)-rf
```

Running the CAPM regression of `r` on `rm` and an intercept results in the following output.

Dependent Variable: R
Method: Least Squares
Date: 11/18/22 Time: 18:17
Sample (adjusted): 12/29/2015 11/16/2022
Included observations: 1735 after adjustments

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	-0.000279	0.000294	-0.948949	0.3428
RM	0.851947	0.024060	35.40913	0.0000

R-squared	0.419782	Mean dependent var	1.88E-05
Adjusted R-squared	0.419447	S.D. dependent var	0.016051
S.E. of regression	0.012230	Akaike info criterion	-5.968706
Sum squared resid	0.259208	Schwarz criterion	-5.962414
Log likelihood	5179.853	Hannan-Quinn criter.	-5.966379
F-statistic	1253.806	Durbin-Watson stat	1.979212
Prob(F-statistic)	0.000000		

The intercept (usually called α) is insignificant as predicted by the theory, and the CAPM β of IBM is .852.

(b) The DW statistic is 1.9792. This value is between $d_u = 1.78$ and 4, so we don't reject the null of no first order autocorrelation.

(c) In the estimation output, go to `View→Residual Diagnostics→ Serial Correlation LM Test`, and include 5 lags. This results in the output below.

Breusch-Godfrey Serial Correlation LM Test:
Null hypothesis: No serial correlation at up to 5 lags

F-statistic	0.941334	Prob. F(5,1728)	0.4530
Obs*R-squared	4.712901	Prob. Chi-Square(5)	0.4519

Test Equation:
Dependent Variable: RESID
Method: Least Squares
Date: 11/18/22 Time: 18:27
Sample: 12/29/2015 11/16/2022
Included observations: 1735
Presample missing value lagged residuals set to zero.

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	-1.19E-06	0.000294	-0.004058	0.9968
RM	0.001882	0.024087	0.078140	0.9377
RESID(-1)	0.009036	0.024052	0.375695	0.7072
RESID(-2)	-0.029789	0.024061	-1.238089	0.2159
RESID(-3)	-0.020329	0.024062	-0.844837	0.3983
RESID(-4)	-0.016857	0.024059	-0.700651	0.4836
RESID(-5)	-0.033360	0.024081	-1.385292	0.1661

R-squared	0.002716	Mean dependent var	-6.40E-20
Adjusted R-squared	-0.000746	S.D. dependent var	0.012226
S.E. of regression	0.012231	Akaike info criterion	-5.965662
Sum squared resid	0.258504	Schwarz criterion	-5.943639
Log likelihood	5182.212	Hannan-Quinn criter.	-5.957518
F-statistic	0.784445	Durbin-Watson stat	2.000099
Prob(F-statistic)	0.582073		

There are two versions of the test, the F-Test and the LM test. We'll focus on the LM test. The observed test statistic is $T \cdot R^2_{aux} = 1735 \cdot 0.002716 = 4.71$, which doesn't exceed the critical value of 11.07 from the $\chi^2(5)$ distribution, so we don't reject the null of no serial correlation of order 5 or less. The same conclusion can be drawn from the top of the output, but that might get deleted in an exam.

(d) Selecting HAC standard errors in the `Options` tab of the estimation window results in the following.

Dependent Variable: R
Method: Least Squares
Date: 11/18/22 Time: 18:38
Sample (adjusted): 12/29/2015 11/16/2022
Included observations: 1735 after adjustments
HAC standard errors & covariance (Bartlett kernel, Newey-West fixed bandwidth = 8.0000)

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	-0.000279	0.000283	-0.986500	0.3240
RM	0.851947	0.038631	22.05368	0.0000

R-squared	0.419782	Mean dependent var	1.88E-05
Adjusted R-squared	0.419447	S.D. dependent var	0.016051
S.E. of regression	0.012230	Akaike info criterion	-5.968706
Sum squared resid	0.259208	Schwarz criterion	-5.962414
Log likelihood	5179.853	Hannan-Quinn criter.	-5.966379
F-statistic	1253.806	Durbin-Watson stat	1.979212
Prob(F-statistic)	0.000000	Wald F-statistic	486.3648
Prob(Wald F-statistic)	0.000000		

As you can see, the difference isn't that big here, because there was little autocorrelation to begin with. Generally, if there is autocorrelation, then the HAC standard errors will be larger, i.e., the regular standard errors underestimate the variance of the estimates.

4. (a) For $Y_{1,t}$, we have

$$\begin{aligned}\mathbb{E}[\Delta Y_{1,t}] &= \mathbb{E}[Y_{1,t} - Y_{1,t-1}] \\ &= \mathbb{E}[\delta t + U_{1,t} - (\delta(t-1) + U_{1,t-1})] \\ &= \delta + \mathbb{E}[U_{1,t} - U_{1,t-1})] \\ &= \delta.\end{aligned}$$

For $Y_{2,t}$,

$$\begin{aligned}\mathbb{E}[\Delta Y_{2,t}] &= \mathbb{E}[Y_{2,t} - Y_{2,t-1}] \\ &= \mathbb{E}[\delta + Y_{2,t-1} + U_{2,t} - Y_{2,t-1}] \\ &= \mathbb{E}[\delta + U_{2,t}] \\ &= \delta.\end{aligned}$$

(b) Consider the AR(2) process

$$Y_t = \phi_1 Y_{t-1} + \phi_2 Y_{t-2} + U_t.$$

We would like to test the null that $\phi_1 + \phi_2 = 1$ (unit root) vs. $\phi_1 + \phi_2 < 1$ (stationarity). This can be done by rearranging the equation as follows:

$$\begin{aligned}Y_t &= \phi_1 Y_{t-1} + \phi_2 Y_{t-2} + U_t && \mid -Y_{t-1} \\ Y_t - Y_{t-1} &= (\phi_1 - 1)Y_{t-1} + \phi_2 Y_{t-2} + U_t && \mid \pm \phi_2 Y_{t-1} \\ Y_t - Y_{t-1} &= (\phi_1 - 1)Y_{t-1} + \phi_2 Y_{t-1} - \phi_2 Y_{t-1} + \phi_2 Y_{t-2} + U_t \\ Y_t - Y_{t-1} &= (\phi_1 + \phi_2 - 1)Y_{t-1} - \phi_2 \Delta Y_{t-1} + U_t \\ \Delta Y_t &= \psi Y_{t-1} + \alpha_1 \Delta Y_{t-1} + U_t,\end{aligned}$$

where $\psi := (\phi_1 + \phi_2 - 1)$ and $\alpha_1 := -\phi_2$. Thus testing $\phi_1 + \phi_2 = 1$ vs. $\phi_1 + \phi_2 =< 1$ is equivalent to testing $\psi = 0$ vs. $\psi < 0$ in a regression of ΔY_t onto Y_{t-1}, augmented by one lag of ΔY_{t-1}.

Solution to Exercise 5

Simon A. Broda

1. (a) The returns are constructed using

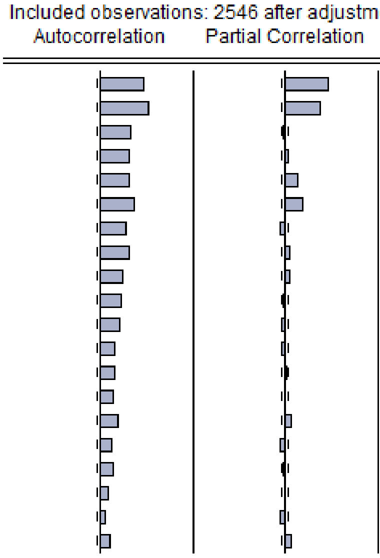
```
genr r = dlog(sp500)
```

as usual. We then construct the squared residuals via

```
genr r2 = r^2
```

and generate the correlogram, shown below.

Date: 11/24/22 Time: 14:51
Sample (adjusted): 10/30/2012 10/27/2022
Included observations: 2546 after adjustments



	AC	PAC	Q-Stat	Prob
1	0.491	0.491	614.81	0.000
2	0.541	0.395	1360.4	0.000
3	0.339	-0.025	1653.4	0.000
4	0.329	0.031	1930.0	0.000
5	0.323	0.145	2196.3	0.000
6	0.380	0.189	2564.3	0.000
7	0.280	-0.065	2764.3	0.000
8	0.324	0.047	3032.4	0.000
9	0.256	0.050	3200.3	0.000
10	0.245	-0.011	3354.3	0.000
11	0.209	-0.032	3465.6	0.000
12	0.169	-0.047	3538.9	0.000
13	0.159	0.018	3604.0	0.000
14	0.154	0.005	3664.4	0.000
15	0.192	0.078	3758.7	0.000
16	0.128	-0.063	3800.8	0.000
17	0.141	-0.011	3851.9	0.000
18	0.088	-0.002	3871.8	0.000
19	0.062	-0.052	3881.6	0.000
20	0.108	0.075	3911.7	0.000

.

Alternatively, we can regress the return on an intercept and inspect the correlogram of the squared residuals (under `Residual Diagnostics...`). There is clear evidence of autocorrelation in the squared returns (all Q-stats are significant), indicative of the presence of volatility clustering. Since the SPACF seems to more or less drop to zero after 6 lags, we might try an ARCH(6) model. Usually, a simple GARCH(1, 1) will do better though.

(b) The ARCH-LM test is only offered after a regression has been estimated, so we start by regressing the returns on an intercept. The test is then available under `View→Residual Diagnostics→Heteroskedasticity Tests`. We choose to include 5 lags (one trading week). The result is shown below.

Heteroskedasticity Test: ARCH

F-statistic	302.2045	Prob. F(5,2535)	0.0000
Obs*R-squared	948.9587	Prob. Chi-Square(5)	0.0000

Test Equation:
Dependent Variable: RESID^2
Method: Least Squares
Date: 11/24/22 Time: 15:11
Sample (adjusted): 11/06/2012 10/27/2022
Included observations: 2541 after adjustments

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	3.14E-05	8.68E-06	3.614959	0.0003
RESID^2(-1)	0.299355	0.019651	15.23388	0.0000
RESID^2(-2)	0.396998	0.020528	19.33915	0.0000
RESID^2(-3)	-0.087729	0.021921	-4.001999	0.0001
RESID^2(-4)	-0.015277	0.020529	-0.744168	0.4568
RESID^2(-5)	0.145365	0.019653	7.396390	0.0000

R-squared	0.373459	Mean dependent var	0.000120
Adjusted R-squared	0.372223	S.D. dependent var	0.000528
S.E. of regression	0.000419	Akaike info criterion	-12.71671
Sum squared resid	0.000444	Schwarz criterion	-12.70292
Log likelihood	16162.59	Hannan-Quinn criter.	-12.71171
F-statistic	302.2045	Durbin-Watson stat	2.054495
Prob(F-statistic)	0.000000		

The null of no heteroskedasticity is clearly rejected; the p-value is essentially zero, and the test observed test statistic $T \cdot R^2_{aux}$ is much larger than the critical value 11.07.

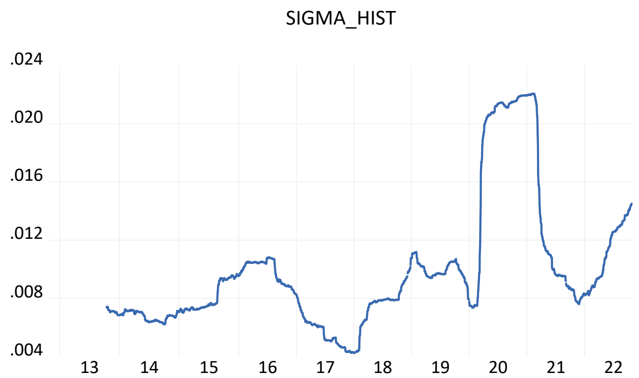
(c) The historical volatility forecasts can be obtained via

```
genr sigma_hist(1) = @sqrt(@movav(r^2, 250))
```

The (1) on the LHS is there because `@movav` includes the observation at time t, and we can only include that in our forecast for time $t+1$. Alternatively, if you don't want to assume a zero mean for the daily returns, you can do

```
genr sigma_hist2(1) = @movstdevp(r, 250)
```

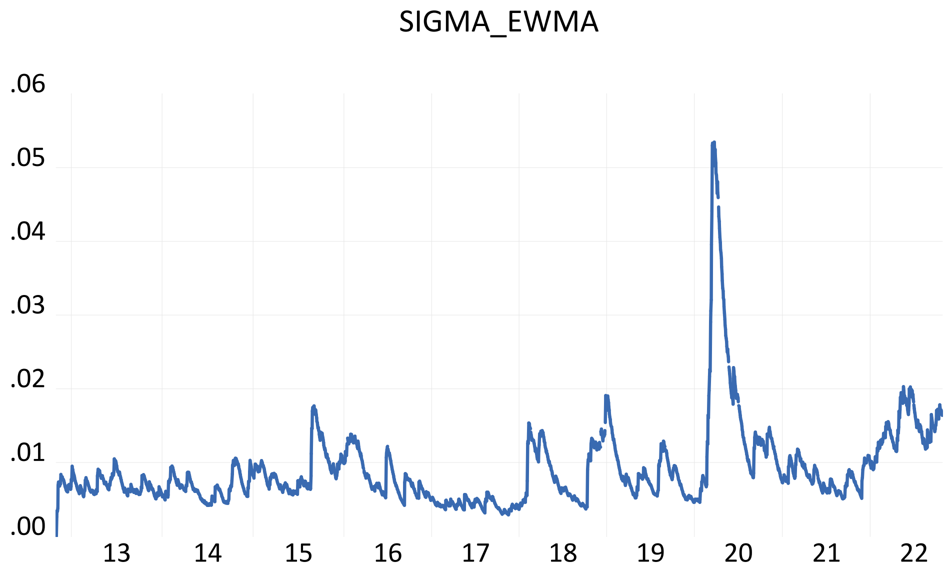
Plotting the historical volatility produces the figure below.



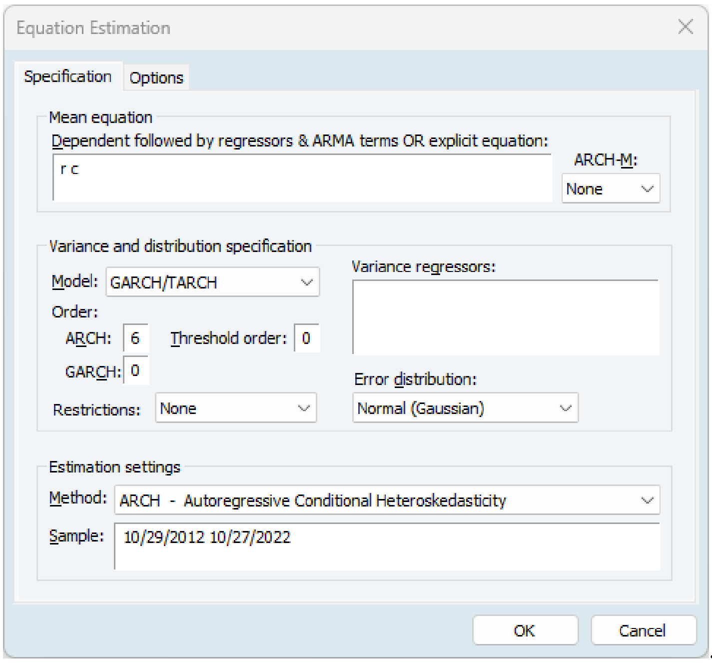
(d) The RiskMetrics volatility with $\lambda = 0.94$ is obtained as follows:

```
scalar lambda = .94
smpl @first+2 @first+2
series sigma_EWMA = r(-1)^2
smpl @first+3 @last
sigma_EWMA = lambda * sigma_EWMA(-1) + (1-lambda) * r(-1)^2
sigma_EWMA = @sqrt(sigma_EWMA)
```

Graphically, it looks as follows.

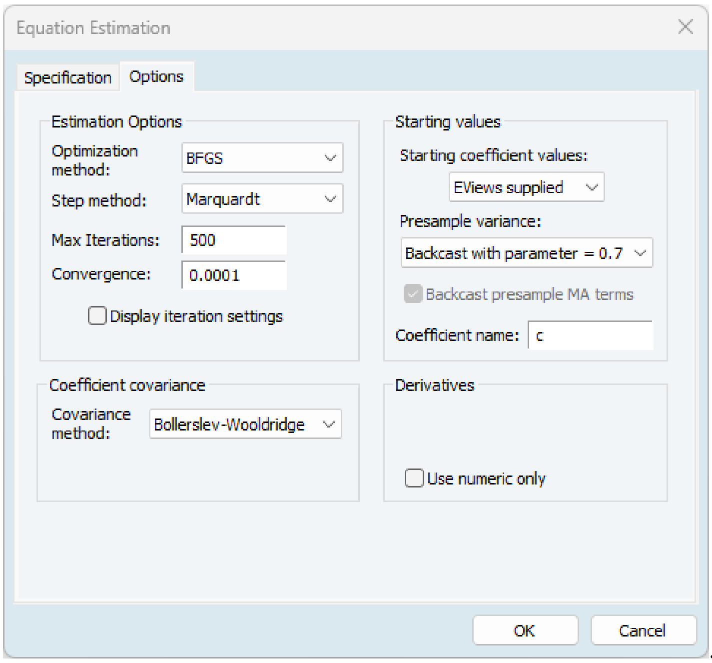


(e) ARCH models can be estimated by clicking `Quick`→`Estimate Equation...` and changing the estimation method to ARCH. To fit an ARCH(6), enter the dependent variable at the top, followed by any regressors (this is called the *mean equation*). We only include an intercept, but it's also possible to include ARMA terms if there is any autocorrelation. Then, specify 6 ARCH lags (this is q), zero GARCH lags (this is p), and a zero threshold order (this is for GJR/TARCH):



.

You should also select Bollerslev-Wooldridge standard errors in the `Options` tab, because we will see later that the standardized residuals will not be normally distributed:



.

The estimated model is

Dependent Variable: R
Method: ML ARCH - Normal distribution (BFGS / Marquardt steps)
Date: 11/25/22 Time: 18:43
Sample (adjusted): 10/30/2012 10/27/2022
Included observations: 2546 after adjustments
Convergence achieved after 23 iterations
Coefficient covariance computed using Bollerslev-Wooldridge QML
sandwich with expected Hessian
Presample variance: backcast (parameter = 0.7)
GARCH = C(2) + C(3)*RESID(-1)^2 + C(4)*RESID(-2)^2 + C(5)*RESID(-3)^2
+ C(6)*RESID(-4)^2 + C(7)*RESID(-5)^2 + C(8)*RESID(-6)^2

Variable	Coefficient	Std. Error	z-Statistic	Prob.
C	0.000809	0.000142	5.687025	0.0000
Variance Equation				
C	1.98E-05	2.36E-06	8.385856	0.0000
RESID(-1)^2	0.216152	0.050172	4.308196	0.0000
RESID(-2)^2	0.177032	0.036417	4.861280	0.0000
RESID(-3)^2	0.178068	0.036320	4.902714	0.0000
RESID(-4)^2	0.160963	0.045330	3.550881	0.0004
RESID(-5)^2	0.066833	0.031598	2.115088	0.0344
RESID(-6)^2	0.064229	0.027097	2.370341	0.0178

R-squared	-0.001467	Mean dependent var	0.000390
Adjusted R-squared	-0.001467	S.D. dependent var	0.010946
S.E. of regression	0.010954	Akaike info criterion	-6.734590
Sum squared resid	0.305352	Schwarz criterion	-6.716233
Log likelihood	8581.134	Hannan-Quinn criter.	-6.727932
Durbin-Watson stat	2.287626		

To see if the model is adequate, we can look at the correlogram of the squared standardized residuals (under `View→Residual Diagnostics→Correlogram Squared Residuals`). It looks as follows.

Date: 11/25/22 Time: 18:47
Sample (adjusted): 10/30/2012 10/27/2022
Included observations: 2546 after adjustments

Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob*
		1	0.005	0.005	0.0673	0.795
		2	0.003	0.003	0.0966	0.953
		3	-0.019	-0.019	1.0342	0.793
		4	-0.005	-0.005	1.0924	0.895
		5	-0.033	-0.033	3.8858	0.566
		6	-0.018	-0.018	4.7484	0.576
		7	-0.004	-0.004	4.7962	0.685
		8	-0.003	-0.004	4.8169	0.777
		9	0.015	0.014	5.3654	0.801
		10	0.048	0.046	11.152	0.346
		11	0.004	0.002	11.194	0.427
		12	-0.010	-0.011	11.460	0.490
		13	0.016	0.018	12.111	0.519
		14	-0.015	-0.013	12.654	0.554
		15	0.005	0.009	12.731	0.623
		16	0.014	0.016	13.204	0.658
		17	0.005	0.005	13.275	0.718
		18	0.000	0.001	13.276	0.775
		19	0.005	0.004	13.330	0.821
		20	0.034	0.032	16.233	0.702

*Probabilities may not be valid for this equation specification.

None of the Q-tests reject, so there is no remaining autocorrelation. Alternatively, we can try a GARCH(1, 1) model. This produces

Dependent Variable: R
Method: ML ARCH - Normal distribution (BFGS / Marquardt steps)
Date: 11/25/22 Time: 18:50
Sample (adjusted): 10/30/2012 10/27/2022
Included observations: 2546 after adjustments
Convergence achieved after 18 iterations
Coefficient covariance computed using Bollerslev-Wooldridge QML
sandwich with expected Hessian
Presample variance: backcast (parameter = 0.7)
GARCH = C(2) + C(3)*RESID(-1)^2 + C(4)*GARCH(-1)

Variable	Coefficient	Std. Error	z-Statistic	Prob.
C	0.000799	0.000142	5.620743	0.0000
Variance Equation				
C	4.50E-06	1.14E-06	3.935962	0.0001
RESID(-1)^2	0.223750	0.030237	7.399998	0.0000
GARCH(-1)	0.741309	0.030859	24.02285	0.0000

R-squared	-0.001398	Mean dependent var	0.000390
Adjusted R-squared	-0.001398	S.D. dependent var	0.010946
S.E. of regression	0.010953	Akaike info criterion	-6.742939
Sum squared resid	0.305331	Schwarz criterion	-6.733760
Log likelihood	8587.761	Hannan-Quinn criter.	-6.739609
Durbin-Watson stat	2.287783		

The estimated coefficients are $\hat{\beta} = 0.74$ and $\hat{\alpha} = 0.22$. This is a bit unusual; typically we find $\hat{\beta}$ around 0.9 for daily returns (c.f. $\lambda = 0.94$ for the RiskMetrics model). The fact that the estimated model is close to the stationarity border (recall that stationarity requires $\alpha + \beta < 1$) is typical, though. The correlogram of the squared standardized residuals looks as follows.

Date: 11/25/22 Time: 18:51
Sample (adjusted): 10/30/2012 10/27/2022
Included observations: 2546 after adjustments

Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob*
		1	0.001	0.001	0.0011	0.973
		2	-0.000	-0.000	0.0015	0.999
		3	-0.003	-0.003	0.0182	0.999
		4	0.024	0.024	1.4430	0.837
		5	-0.028	-0.028	3.4029	0.638
		6	-0.008	-0.008	3.5668	0.735
		7	-0.021	-0.021	4.6583	0.702
		8	-0.017	-0.017	5.3624	0.718
		9	-0.008	-0.006	5.5064	0.788
		10	0.038	0.037	9.1425	0.519
		11	-0.004	-0.004	9.1864	0.605
		12	-0.017	-0.017	9.8904	0.626
		13	0.000	-0.001	9.8906	0.703
		14	-0.018	-0.020	10.676	0.711
		15	-0.002	-0.001	10.686	0.774
		16	0.003	0.004	10.712	0.827
		17	0.002	0.002	10.720	0.871
		18	-0.007	-0.005	10.850	0.901
		19	-0.007	-0.009	10.990	0.924
		20	0.020	0.017	11.993	0.916

*Probabilities may not be valid for this equation specification.

This looks equally good as the ARCH(6). As usual, we prefer smaller models, so we stick with the GARCH(1, 1)[1]. We can confirm that there is no remaining volatility clustering by running an ARCH-LM test on the standardized residuals (under `View`→`Residual Diagnostics`→`ARCH LM Test`).

Heteroskedasticity Test: ARCH

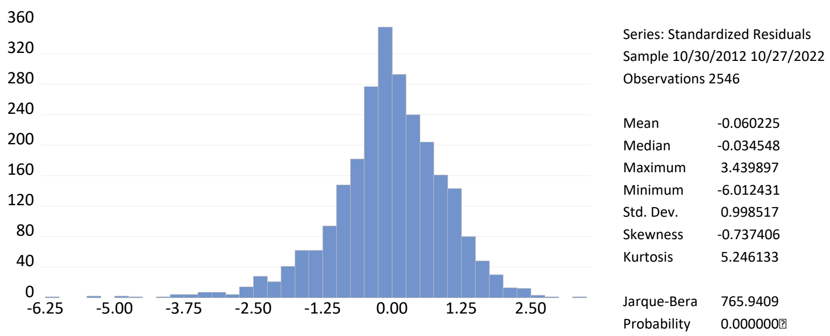
F-statistic	0.676487	Prob. F(5,2535)	0.6413
Obs*R-squared	3.385925	Prob. Chi-Square(5)	0.6407

Test Equation:
Dependent Variable: WGT_RESID^2
Method: Least Squares
Date: 11/25/22 Time: 18:57
Sample (adjusted): 11/06/2012 10/27/2022
Included observations: 2541 after adjustments

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	1.006032	0.060805	16.54509	0.0000
WGT_RESID^2(-1)	0.001390	0.019854	0.069995	0.9442
WGT_RESID^2(-2)	-0.000122	0.019848	-0.006170	0.9951
WGT_RESID^2(-3)	-0.002597	0.019846	-0.130879	0.8959
WGT_RESID^2(-4)	0.023571	0.019845	1.187738	0.2350
WGT_RESID^2(-5)	-0.027765	0.019852	-1.398648	0.1620

R-squared	0.001333	Mean dependent var	1.000523
Adjusted R-squared	-0.000637	S.D. dependent var	2.101669
S.E. of regression	2.102338	Akaike info criterion	4.326336
Sum squared resid	11204.26	Schwarz criterion	4.340126
Log likelihood	-5490.610	Hannan-Quinn criter.	4.331338
F-statistic	0.676487	Durbin-Watson stat	2.000405
Prob(F-statistic)	0.641285		

The test doesn't reject, so it seems that we have successfully modeled the volatility clustering. We can look at a histogram of the standardized residuals by clicking on `View`→`Residual Diagnostics`→`Histogram - Normality Test`. This produces the following plot.



Normality is clearly rejected, another typical finding. This means that we were right to use Bollerslev-Wooldridge standard errors. Alternatively, we could have specified a different error distribution, but we'll reserve that for next week.

EViews doesn't offer a statistical test for the leverage effect, so we'll just go ahead and estimate a TARCH(1, 1, 1) model. The fitted model is

[1]We could also use the BIC to confirm this decision.

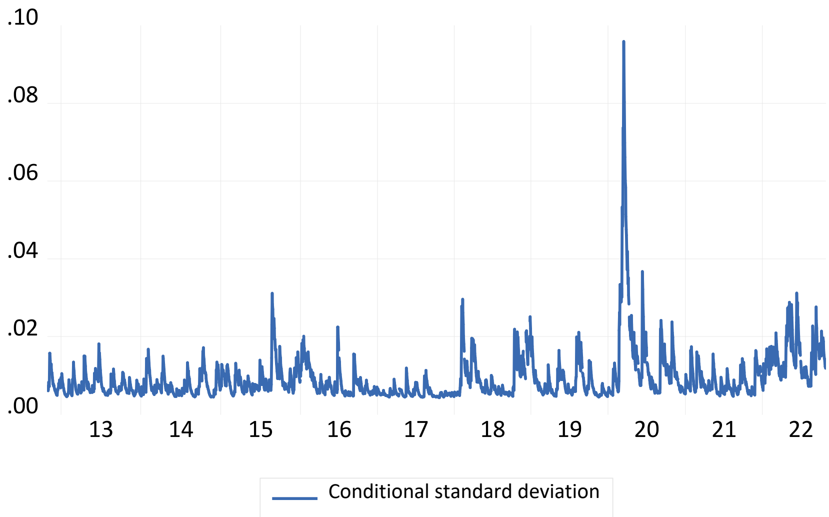
Dependent Variable: R
Method: ML ARCH - Normal distribution (BFGS / Marquardt steps)
Date: 11/25/22 Time: 19:03
Sample (adjusted): 10/30/2012 10/27/2022
Included observations: 2546 after adjustments
Convergence achieved after 26 iterations
Coefficient covariance computed using Bollerslev-Wooldridge QML
sandwich with expected Hessian
Presample variance: backcast (parameter = 0.7)
GARCH = C(2) + C(3)*RESID(-1)^2 + C(4)*RESID(-1)^2*(RESID(-1)<0) +
C(5)*GARCH(-1)

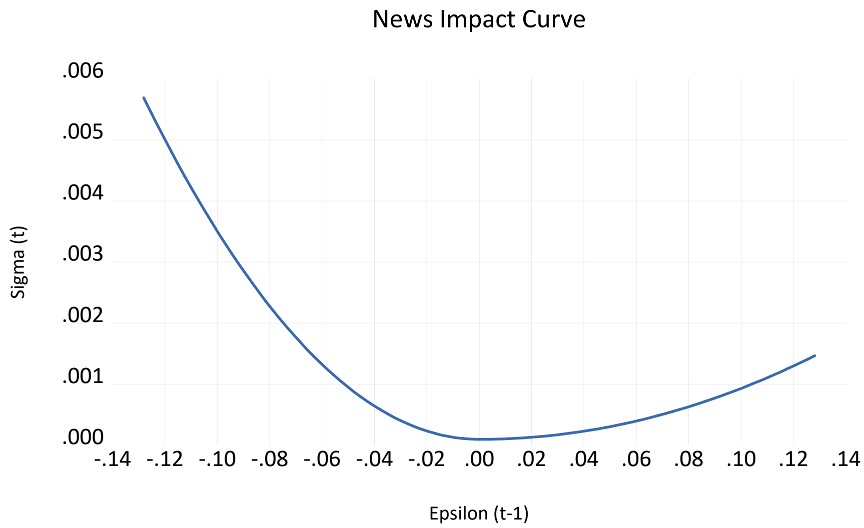
Variable	Coefficient	Std. Error	z-Statistic	Prob.
C	0.000472	0.000132	3.560013	0.0004
Variance Equation				
C	4.18E-06	9.41E-07	4.443218	0.0000
RESID(-1)^2	0.083481	0.072149	1.157069	0.2472
RESID(-1)^2*(RESID(-1)<0)	0.257376	0.063931	4.025838	0.0001
GARCH(-1)	0.755584	0.041655	18.13929	0.0000

R-squared	-0.000056	Mean dependent var	0.000390
Adjusted R-squared	-0.000056	S.D. dependent var	0.010946
S.E. of regression	0.010946	Akaike info criterion	-6.767193
Sum squared resid	0.304922	Schwarz criterion	-6.755719
Log likelihood	8619.636	Hannan-Quinn criter.	-6.763031
Durbin-Watson stat	2.290853		

The asymmetry coefficient $\hat{\gamma} = 0.257$ is clearly significant, so there is clear evidence of leverage. Also note that the ARCH coefficient has become insignificant. This means that volatility shows no significant reaction to good news (positive returns) at all, only to bad news.

(f) The plots can be found under `View→GARCH Graphs` and look as follows.





(g) As usual, we can use EViews for the forecasts, or do it manually. We'll do the latter. To do that, we need the estimated $\widehat{\sigma}_t$ and $\hat{u}_t$. We can get them via `Proc`$\rightarrow$ `Make GARCH Variance Series...` and `Proc`$\rightarrow$ `Make Residual Series....` For the latter, you can specify either the ordinary residuals $\hat{u}_t$ or the standardized residuals $\hat{z}_t$. We need the former; the latter would be needed for forecastin and EGARCH model. We find that the residual for 27/10 is -0.006573, and the corresponding $\widehat{\sigma}_t$ is 0.0001399. This is all we need to produce a forecast, by plugging into our estimated model as follows.

$$\begin{aligned}\widehat{\sigma}^2_{t+1} &= \hat{\omega} + \hat{\alpha}\hat{u}_t^2 + \hat{\gamma}\hat{u}_t^2 \mathrm{I}_t + \hat{\beta}\widehat{\sigma}_t^2 \\ &= 0.00000418 + 0.083\hat{u}_t^2 + 0.257\hat{u}_t^2 \cdot 1 + 0.756\widehat{\sigma}_t^2 \\ &= 0.0^5418 + 0.083 \cdot (-0.006573)^2 + 0.257 \cdot (-0.006573)^2 + 0.756 \cdot 0.0001399 \\ &= 0.0001246;\end{aligned}$$

note that the indicator function is 1 because $\hat{u}_t$ is negative.

2. (a) Splitting out the first term of the sum immediately yields

$$\widehat{\sigma}^2_{t+1,EWMA} = (1-\lambda)\sum_{j=0}^{\infty} \lambda^j r^2_{t-j}$$

$$\widehat{\sigma}^2_{t+1,EWMA} = (1-\lambda)\lambda^0 r^2_{t-0} + (1-\lambda)\sum_{j=1}^{\infty} \lambda^j r^2_{t-j}$$

$$\widehat{\sigma}^2_{t+1,EWMA} = (1-\lambda) r^2_t + (1-\lambda)\sum_{j=0}^{\infty} \lambda^{j+1} r^2_{t-1-j}$$

$$\begin{aligned}\widehat{\sigma}^2_{t+1,EWMA} &= (1-\lambda) r^2_t + \lambda(1-\lambda)\sum_{j=0}^{\infty} \lambda^j r^2_{t-1-j} \\ &= (1-\lambda) r^2_t + \lambda\widehat{\sigma}^2_{t,EWMA}.\end{aligned}$$

(b) Solving $\widehat{\sigma}^2 = \hat{\omega}/(1-\hat{\alpha}-\hat{\beta})$ for $\hat{\omega}$, we find $\hat{\omega} = \widehat{\sigma}^2 \cdot (1-\hat{\alpha}-\hat{\beta})$. Plugging this into the equation

$$\widehat{\sigma}^2_{t+1} = \hat{\omega} + \hat{\alpha}\hat{u}^2_t + \hat{\beta}\widehat{\sigma}^2_t$$

yields

$$\begin{aligned}\widehat{\sigma}^2_{t+1} &= \widehat{\sigma}^2 \cdot (1-\hat{\alpha}-\hat{\beta}) + \hat{\alpha}\hat{u}^2_t + \hat{\beta}\widehat{\sigma}^2_t \\ &= \widehat{\sigma}^2 + \hat{\alpha}(\hat{u}^2_t - \widehat{\sigma}^2) + \hat{\beta}(\widehat{\sigma}^2_t - \widehat{\sigma}^2).\end{aligned}$$

(c) The result of the previous question also implies that

$$\widehat{\sigma}^2_{t+2} = \widehat{\sigma}^2 + \hat{\alpha}(\hat{u}^2_{t+1} - \widehat{\sigma}^2) + \hat{\beta}(\widehat{\sigma}^2_{t+1} - \widehat{\sigma}^2).$$

Replacing the unknown $\hat{u}^2_{t+1}$ with its forecast $\widehat{\sigma}^2_{t+1}$, we see that

$$\begin{aligned}\widehat{\sigma}^2_{t+2} &= \widehat{\sigma}^2 + \hat{\alpha}(\widehat{\sigma}^2_{t+1} - \widehat{\sigma}^2) + \hat{\beta}(\widehat{\sigma}^2_{t+1} - \widehat{\sigma}^2) \\ \widehat{\sigma}^2_{t+2} &= \widehat{\sigma}^2 + (\hat{\alpha}+\hat{\beta})(\widehat{\sigma}^2_{t+1} - \widehat{\sigma}^2). \qquad (1)\end{aligned}$$

Equation (1) also implies that

$$\widehat{\sigma}^2_{t+3} = \widehat{\sigma}^2 + (\hat{\alpha}+\hat{\beta})(\widehat{\sigma}^2_{t+2} - \widehat{\sigma}^2), \qquad (2)$$

and plugging (1) into (2) results in

$$\begin{aligned}\widehat{\sigma}^2_{t+3} &= \widehat{\sigma}^2 + (\hat{\alpha}+\hat{\beta})((\hat{\alpha}+\hat{\beta})(\widehat{\sigma}^2_{t+1} - \widehat{\sigma}^2)) \\ &= \widehat{\sigma}^2 + (\hat{\alpha}+\hat{\beta})^2(\widehat{\sigma}^2_{t+1} - \widehat{\sigma}^2).\end{aligned}$$

Iterating this process produces

$$\widehat{\sigma}^2_{t+s} = \widehat{\sigma}^2 + (\hat{\alpha}+\hat{\beta})^{s-1}(\widehat{\sigma}^2_{t+1} - \widehat{\sigma}^2)$$

as claimed.

Solution to Exercise 6

Simon A. Broda

1. (a) We start by generating the log returns as usual. We also create the negative returns for plotting later.

```
genr r = dlog(sp500)
genr negative_returns = -r
```

The historical VaR is then obtained via

```
genr var_hist = -@quantile(r, 0.01)
```

Notice that this uses the whole sample instead of the last 250 days for each t. Doing that would require us to write a loop.

(b) The normal VaR can be computed as $-\mu-\sigma\cdot\Phi^{-1}_{0.01} = -0.000390-0.010946\cdot(-2.236) = 0.024085$, where the value of $\Phi^{-1}(0.01)$ can be read of the table for the t distribution (the last row)[1]. In EViews, the Normal VaR is generated as

```
genr var_norm = -@mean(r)-@stdev(r)*@qnorm(0.01)
```

(c) For the GARCH VaR, we estimate a GARCH(1, 1) as usual, first using normal errors. We then save the predicted variances by clicking `Proc`→`Make GARCH Variance Series...` and entering `garchn_sig2` as the series name. We then convert the variances into volatilities and compute the VaR, using the commands

```
genr garchn_sig = @sqrt(garchn_sig2)
genr var_garch_n = -c(1)-garchn_sig * @qnorm(0.01)
```

Here, `c(1)` refers to the estimated intercept in the mean equation. We then re-estimate the model using standardized t innovations (just choose it in the GARCH specification dialog). This results in the estimated model below.

[1] Note that the table lists the quantiles with the sign reversed; 2.236 is the 99th percentile, while we need the 1st percentile, -2.236

Dependent Variable: R
Method: ML ARCH - Student's t distribution (BFGS / Marquardt steps)
Date: 12/01/22 Time: 17:14
Sample (adjusted): 10/30/2012 10/27/2022
Included observations: 2546 after adjustments
Convergence achieved after 27 iterations
Coefficient covariance computed using outer product of gradients
Presample variance: backcast (parameter = 0.7)
GARCH = C(2) + C(3)*RESID(-1)^2 + C(4)*GARCH(-1)

Variable	Coefficient	Std. Error	z-Statistic	Prob.
C	0.000904	0.000126	7.176877	0.0000
		Variance Equation		
C	2.92E-06	6.17E-07	4.743725	0.0000
RESID(-1)^2	0.221296	0.026849	8.242232	0.0000
GARCH(-1)	0.773192	0.023233	33.27982	0.0000
T-DIST. DOF	5.275193	0.568869	9.273121	0.0000

R-squared	-0.002211	Mean dependent var	0.000390
Adjusted R-squared	-0.002211	S.D. dependent var	0.010946
S.E. of regression	0.010958	Akaike info criterion	-6.810961
Sum squared resid	0.305579	Schwarz criterion	-6.799488
Log likelihood	8675.354	Hannan-Quinn criter.	-6.806800
Durbin-Watson stat	2.285926		

The estimated degrees of freedom are 5.28, which corresponds to fairly heavy tails. We then save the GARCH variance series as before, choosing the name `garcht_sig2`. Then we convert the variances into volatilities and compute the VaR, using the commands

```
genr garcht_sig = @sqrt(garcht_sig2)
genr var_garch_t = -c(1)-garcht_sig *
                     @sqrt((c(5)-2)/c(5))*@qtdist(0.01, c(5))
```

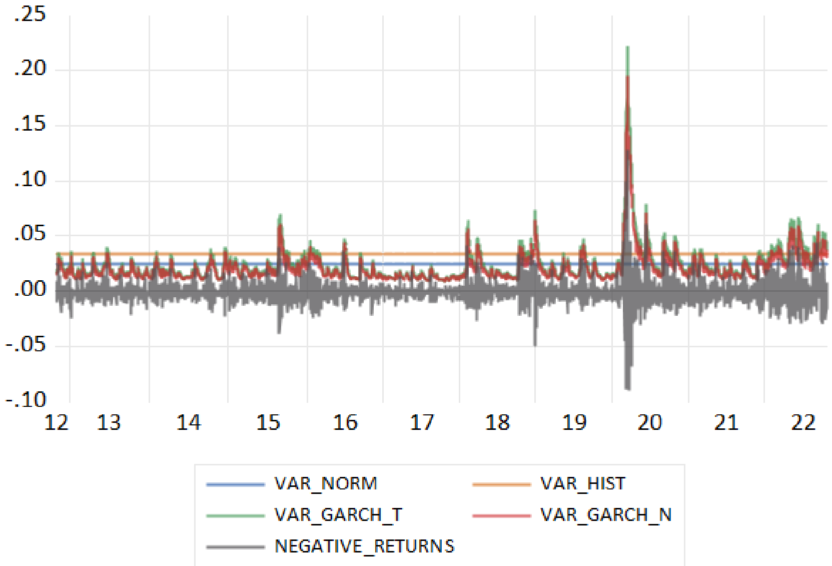
Note that the second command should be entered on a single line. Also, `c(5)` corresponds to the estimated degrees of freedom.

(d) The value $\sigma_t = 0.014875$ for 10/27/2022 can be obtained from the spreadsheet view of `garcht_sig`. The VaR is then computed as

$$\begin{aligned} VaR_t^{0.01} &= -\mu - \sigma_t \sqrt{\frac{5.28-2}{5.28}} t_{0.01}^{-1}(5.28) \\ &\approx -0.000904 - 0.014875 \cdot \sqrt{\frac{3.28}{5.28}} \cdot (-3.365) \\ &= 0.038547. \end{aligned}$$

This is an approximation, because $t_{0.01}^{-1}(5.28)$ cannot be obtained from the t distribution table, so we use $t_{0.01}^{-1}(5) = -3.365$. The exact result, 0.037690, can be found in the spreadsheet view for `var_garch_t`.

(e) The plot below can be generated by opening the four VaR predictions, together with the negative returns, as a group.



2. The four hit series are generated as follows:

```
genr hit_norm = (negative_returns > var_norm)
genr hit_hist = (negative_returns > var_hist)
genr hit_garchn = (negative_returns > var_garch_n)
genr hit_garcht = (negative_returns > var_garch_t)
```

Each of the hit series will be equal to one at time t if the negative return on that day exceeded the respective VaR estimate. We begin by testing, for each hit series, if the proportion of VaR violations is significantly different from zero. For example, for the historical VaR, this works as follows: compute $\hat{\pi} = T_1/T = \frac{1}{T}\sum_{t=1}^{T} I_t$ via

```
scalar p_hist = @mean(hit_hist)
```

This yields $\hat{\pi} = 0.009819$. From this, we can compute the t-statistic

$$t = \frac{\hat{\pi} - p}{\sqrt{\hat{\pi}(1-\hat{\pi})/T}} = \frac{0.009819 - 0.01}{\sqrt{0.009819(1-0.009819)/2547}} = -0.09264.$$

Comparing this with the critical value ± 1.96, we see that $\hat{\pi}$ is not significantly different from p, hence the model has correct unconditional coverage. Note that this is **by construction**, because we defined the the historical VaR as a sample quantile. So by construction, exactly 1% of the negative returns should exceed the VaR. Alteratively, we can just regress $I_t - 0.01$ onto an intercept by entering the regression specification

```
(hit_hist - 0.01) c
```

and test whether the intercept is significant. The output is

Dependent Variable: HIT_HIST-0.01
Method: Least Squares
Date: 12/02/22 Time: 17:59
Sample (adjusted): 10/30/2012 10/27/2022
Included observations: 2546 after adjustments

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	-0.000181	0.001955	-0.092437	0.9264

R-squared	0.000000	Mean dependent var	-0.000181
Adjusted R-squared	0.000000	S.D. dependent var	0.098624
S.E. of regression	0.098624	Akaike info criterion	-1.794608
Sum squared resid	24.75452	Schwarz criterion	-1.792314
Log likelihood	2285.536	Hannan-Quinn criter.	-1.793776
Durbin-Watson stat	1.939040		

We see that the t-statistic obtained this way is almost the same (the two tests are asymptotically equivalent), so we arrive at the same conclusion.

To test for independence, we regress $I_t - 0.01$ on an intercept and I_{t-1}, by entering the regression specification

```
(hit_hist) c hit_hist(-1)
```

The output is

Dependent Variable: HIT_HIST-0.01
Method: Least Squares
Date: 12/02/22 Time: 18:52
Sample (adjusted): 10/31/2012 10/27/2022
Included observations: 2545 after adjustments

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	-0.000476	0.001964	-0.242398	0.8085
HIT_HIST(-1)	0.030476	0.019821	1.537573	0.1243

R-squared	0.000929	Mean dependent var	-0.000177
Adjusted R-squared	0.000536	S.D. dependent var	0.098643
S.E. of regression	0.098617	Akaike info criterion	-1.794362
Sum squared resid	24.73143	Schwarz criterion	-1.789771
Log likelihood	2285.326	Hannan-Quinn criter.	-1.792697
F-statistic	2.364130	Durbin-Watson stat	2.011661
Prob(F-statistic)	0.124278		

Independence can be tested by checking if b_1, the coefficient on the lagged hit, is significant at the 5% level. Here, surprisingly, it isn't, so we don't reject the null of independence of the VaR violations. Finally, we can use an F-test for $H_0 : b_0 = b_1 = 0$ to test the correctness of the conditional coverage. Note that we cannot use the F-statistic provided in the output above, because that is for a test of the null that all coefficients *except the intercept* are significant. Instead, go to `View`→`Coefficient Diagnostics`→`Wald Test`, and enter the restrictions

```
c(1)=0, c(2)=0
```

The result is shown below.

Wald Test:
Equation: Untitled

Test Statistic	Value	df	Probability
F-statistic	1.186156	(2, 2543)	0.3056
Chi-square	2.372311	2	0.3054

Null Hypothesis: C(1)=0, C(2)=0
Null Hypothesis Summary:

Normalized Restriction (= 0)	Value	Std. Err.
C(1)	-0.000476	0.001964
C(2)	0.030476	0.019821

Restrictions are linear in coefficients.

The degrees of freedom of the F-test are 2 and $T - 2$. From the table, we find that the critical value is 3, which is not exceeded by the observed value 1.19. Hence, the null of correct conditional coverage is resoundingly rejected. I should note that this is **highly unusual**; typically, the historical VaR will fail the independence and conditional coverage tests, because the fact that it doesn't react to changes in volatility means that violations will cluster during crises, making them dependent.

Repeating the analysis for the other hit series results in the table below.

	Hist	Norm	GARCHn	GARCHt
$\hat{\pi} \quad (\times 100)$	0.98	2.04	2.59	1.65
$t(\pi = 0.01)$	-0.09	3.72	5.06	2.57
$\hat{b}_1$	0.03	0.08	0.00	0.03
$t(b_1 = 0)$	1.54	3.91	0.23	1.60
$F(b_0 = b_1 = 0)$	1.18	14.60	12.80	4.59

For the normal VaR, all 3 tests reject. The GARCH models pass the independence test, while the conditional and unconditional coverage tests reject.

Solution to Exercise 7

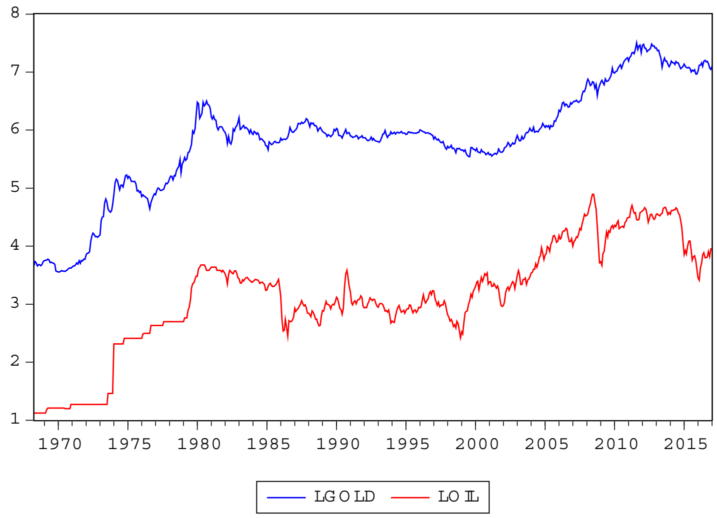
Simon A. Broda

1. (a) If the relative price of oil expressed in units of gold, $\text{oil}_t/\text{gold}_t$, is stationary, then this implies that $\log(\text{oil}_t/\text{gold}_t) = \log(\text{oil}_t) - \log(\text{gold}_t)$ is also stationary, so $\log(\text{oil}_t)$ and $\log(\text{gold}_t)$ must be cointegrated with cointegrating vector (1, -1) if the individual series are integrated.

   (b) We begin by transforming the data to logs and making a plot:

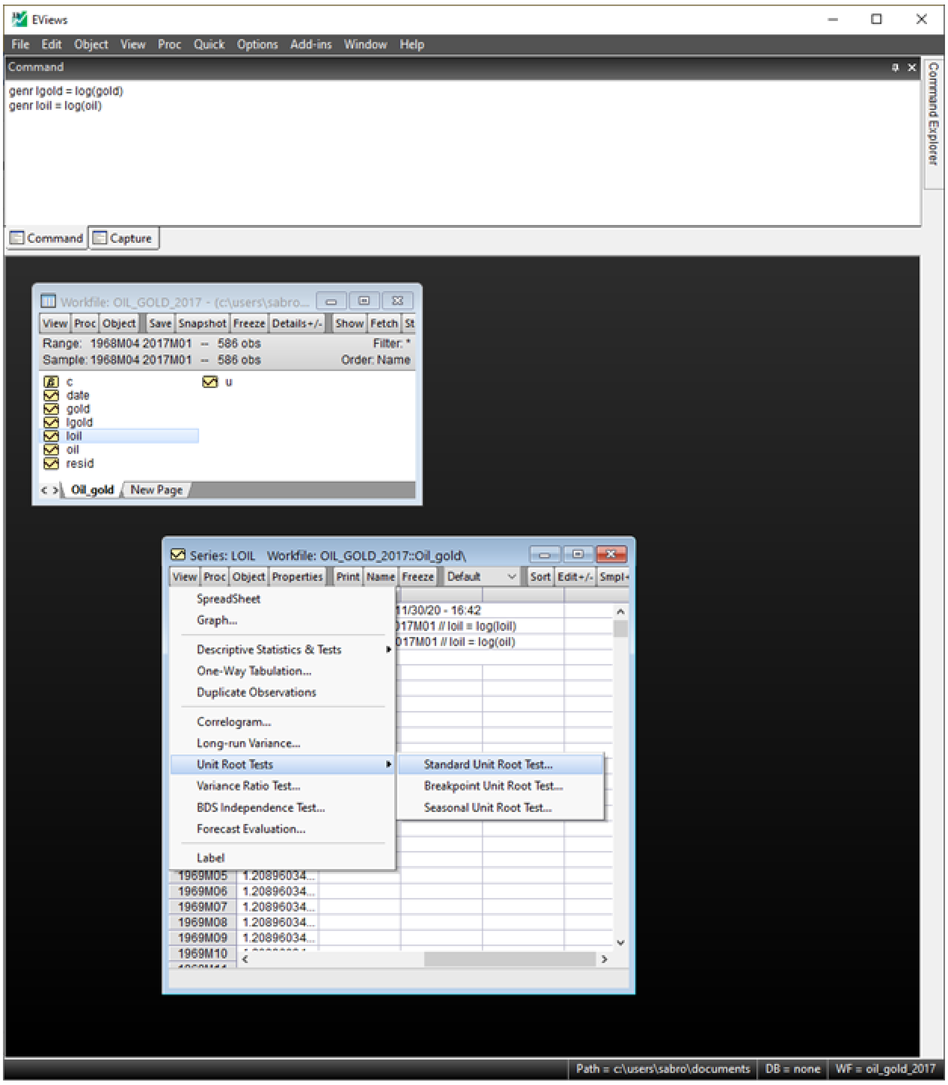
```
genr lgold = log(gold)
genr loil = log(oil)
```

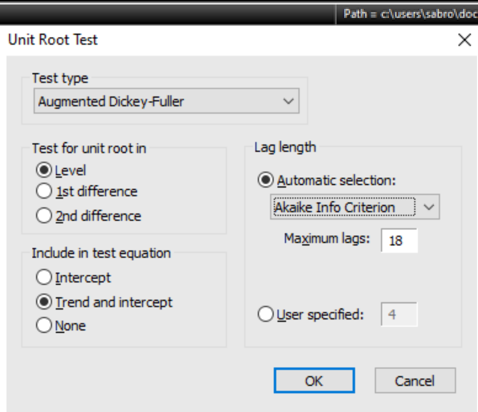
   Plotting the data requires opening the two series as a group. The resulting plot is given below.



   Then we follow the Engle-Granger procedure.

   Step 1: Conduct individual unit root tests to make sure both series are integrated. We include a time trend for both because the data look trending, and choose the lag length automatically by the AIC.





The results is shown below.

Null Hypothesis: LOIL has a unit root
Exogenous: Constant, Linear Trend
Lag Length: 1 (Automatic - based on SIC, maxlag=18)

		t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic		-2.552422	0.3027
Test critical values:	1% level	-3.973847	
	5% level	-3.417533	
	10% level	-3.131184	

*MacKinnon (1996) one-sided p-values.

Null Hypothesis: LGOLD has a unit root
Exogenous: Constant, Linear Trend
Lag Length: 0 (Automatic - based on SIC, maxlag=18)

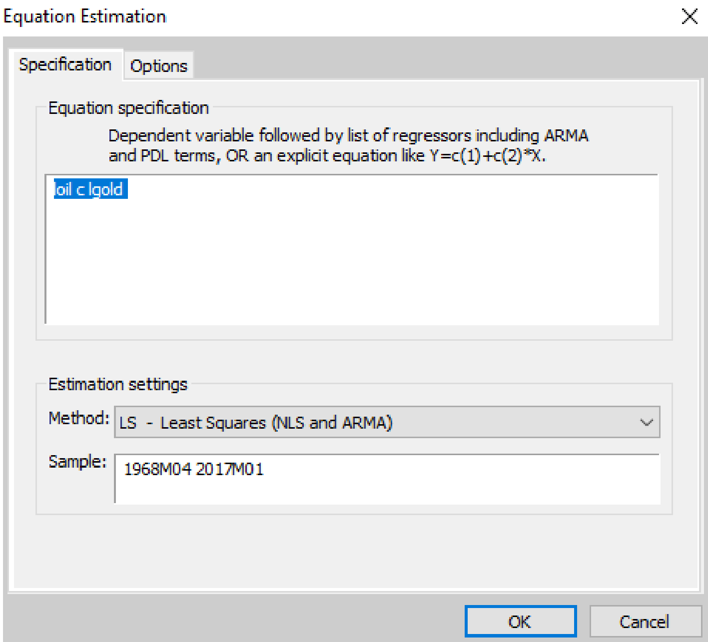
		t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic		-1.820648	0.6937
Test critical values:	1% level	-3.973820	
	5% level	-3.417519	
	10% level	-3.131176	

*MacKinnon (1996) one-sided p-values.

EViews chose to include one lagged difference. Neither test rejects, so the series are I(1).

Step 2: Estimate the long-run relationship (cointegrating relationship)

$$\text{loil}_t = \beta_1 + \beta_2 \text{lgold}_t + U_t.$$



The result is

Dependent Variable: LOIL
Method: Least Squares
Date: 11/30/20 Time: 16:50
Sample: 1968M04 2017M01
Included observations: 586

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	-2.258963	0.086329	-26.16698	0.0000
LGOLD	0.926232	0.014618	63.36354	0.0000

R-squared	0.873014	Mean dependent var	3.141848
Adjusted R-squared	0.872797	S.D. dependent var	0.929639
S.E. of regression	0.331561	Akaike info criterion	0.633398
Sum squared resid	64.20072	Schwarz criterion	0.648324
Log likelihood	-183.5855	Hannan-Quinn criter.	0.639214
F-statistic	4014.938	Durbin-Watson stat	0.074317
Prob(F-statistic)	0.000000		

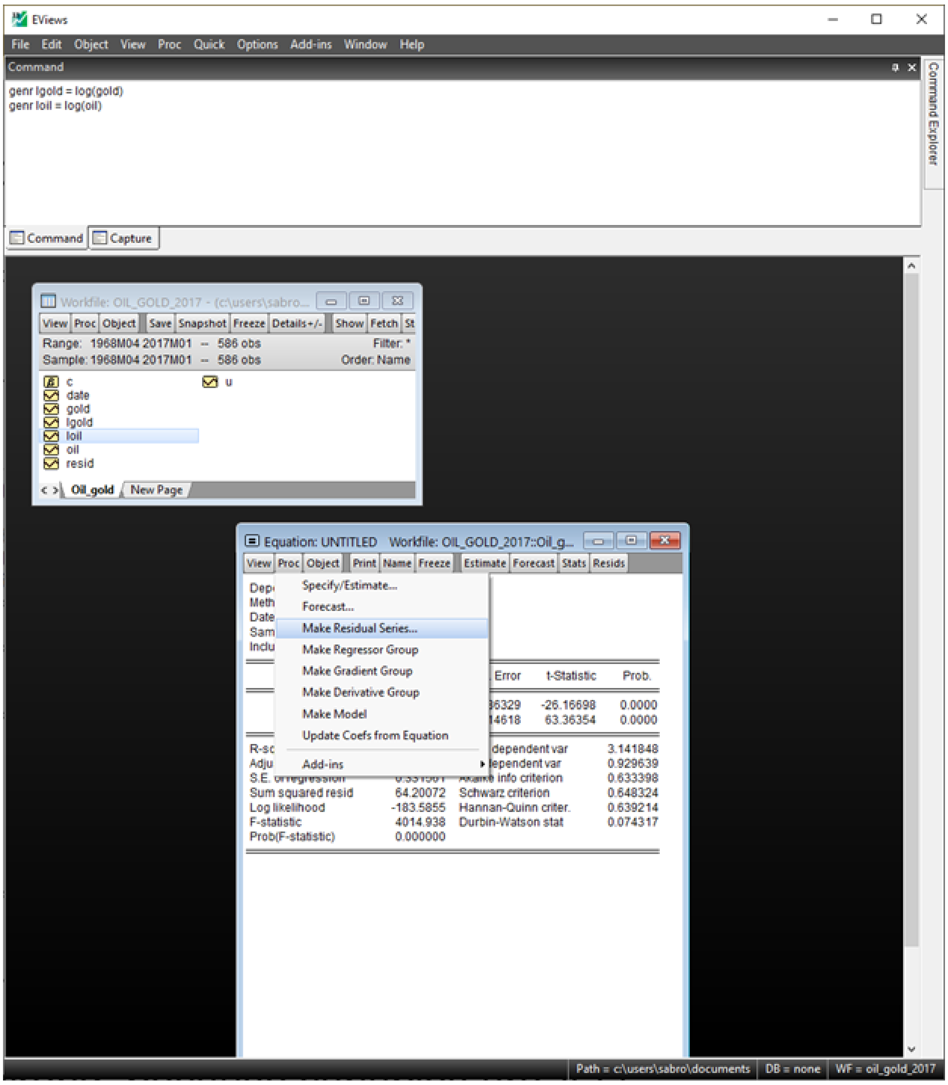
The estimated long-run relationship is

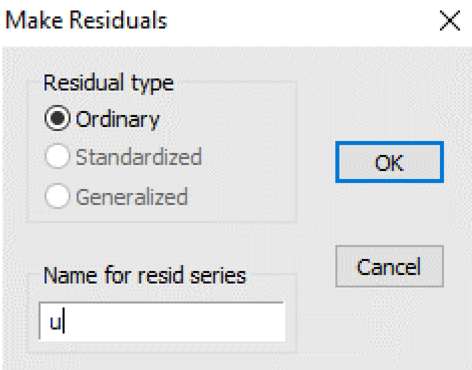
$$\mathrm{loil}_t = -2.26 + 0.926\mathrm{lgold}_t + U_t.$$

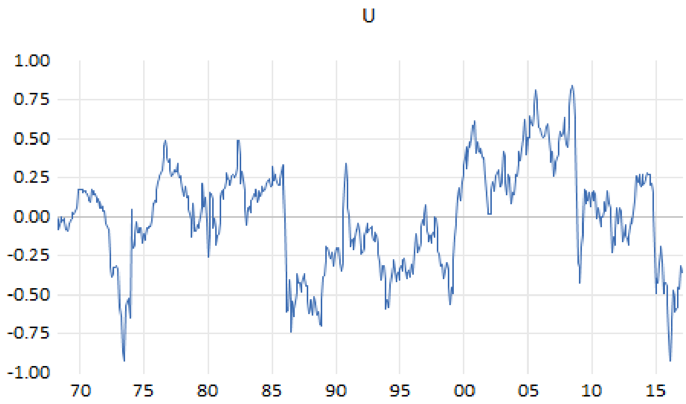
The estimated cointegrating vector (provided we find cointegration) is $(1, -0.926)$, i.e.,

$$\mathrm{loil}_t - 0.926\mathrm{lgold}_t = -2.26 + U_t$$

is stationary. We save the residuals for later use and plot them:


EViews
File Edit Object View Proc Quick Options Add-ins Window Help
Command
genr lgold = log(gold)
genr loil = log(oil)
Command
Capture
Workfile: OIL_GOLD_2017 - (c:\users\sabro...
Range: 1968M04 2017M01 -- 586 obs
Sample: 1968M04 2017M01 -- 586 obs
c
date
gold
lgold
loil
oil
resid
u
Oil_gold
New Page
Equation: UNTITLED Workfile: OIL_GOLD_2017::Oil_g...
Specify/Estimate...
Forecast...
Make Residual Series...
Make Regressor Group
Make Gradient Group
Make Derivative Group
Make Model
Update Coefs from Equation
Add-ins
Sum squared resid 64.20072
Log likelihood -183.5855
F-statistic 4014.938
Prob(F-statistic) 0.000000
Schwarz criterion 0.648324
Hannan-Quinn criter. 0.639214
Durbin-Watson stat 0.074317
Path = c:\users\sabro\documents
DB = none
WF = oil_gold_2017


Make Residuals
Residual type
Ordinary
Standardized
Generalized
OK
Cancel
Name for resid series
u



Step 3: Conduct an ADF test (with just an intercept, no trend!) for the residuals to test H_0 : No Cointegration. Result:

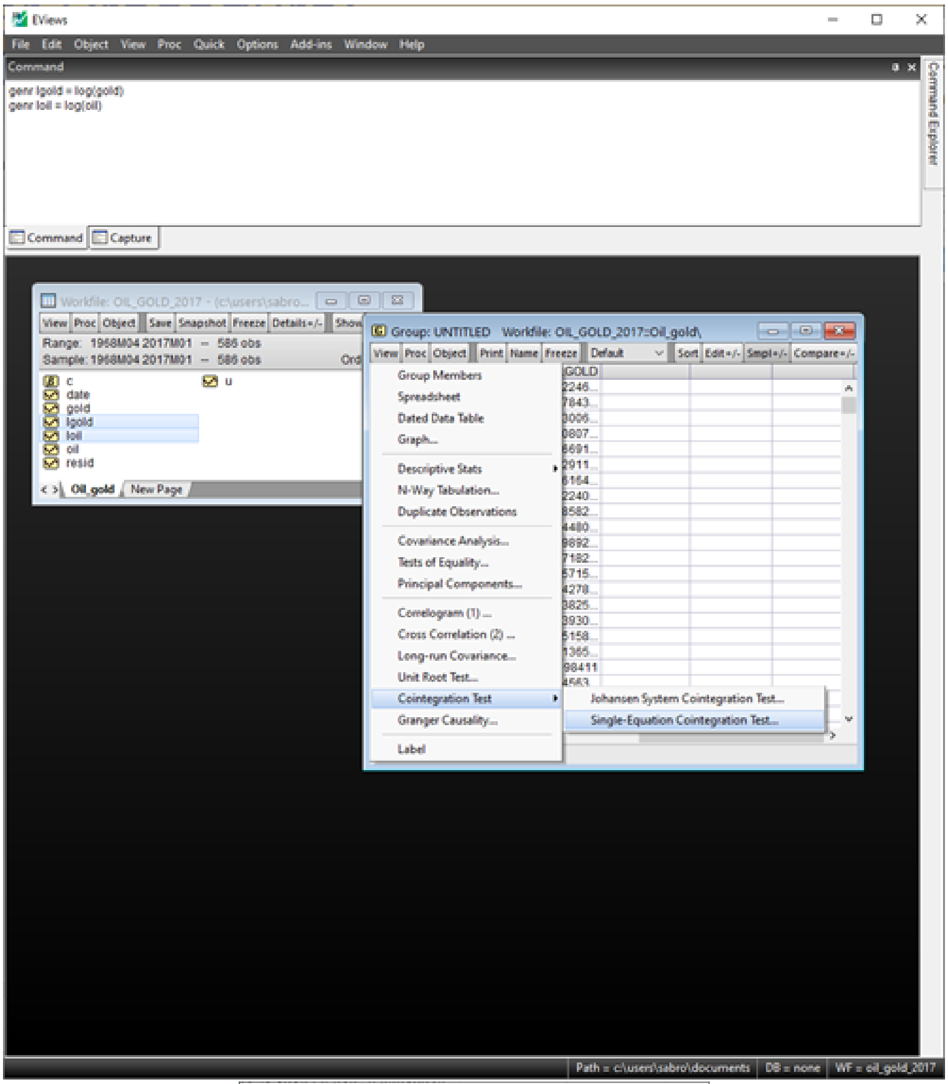
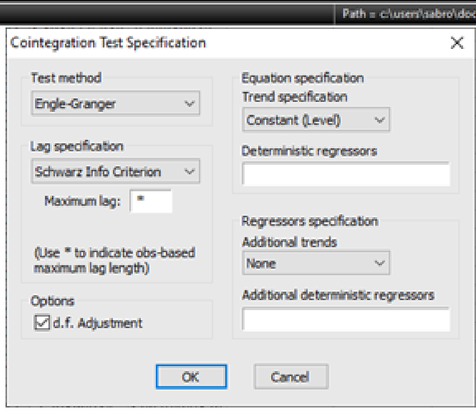
Null Hypothesis: U has a unit root
Exogenous: Constant
Lag Length: 1 (Automatic - based on SIC, maxlag=18)

		t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic		-3.724771	0.0040
Test critical values:	1% level	-3.441318	
	5% level	-2.866270	
	10% level	-2.569348	

*MacKinnon (1996) one-sided p-values.

Be careful to use the Engle-Granger critical value of -3.41 at the 5% level. The test rejects the null. Conclusion: there is indeed cointegration.

Alternative to this manual approach: select `loil` and `lgold` (click on one, press control, click on the other), and open them as a group. Then do


EViews
File Edit Object View Proc Quick Options Add-ins Window Help
Command
genr lgold = log(gold)
genr loil = log(oil)
Command Explorer
Command
Capture
Workfile: OIL_GOLD_2017
View Proc Object Save Snapshot Freeze Details+/- Show
Range: 1968M04 2017M01 -- 586 obs
Sample: 1968M04 2017M01 -- 586 obs
c
date
gold
lgold
loil
oil
resid
u
Oil_gold
New Page
Group: UNTITLED Workfile: OIL_GOLD_2017::Oil_gold\
View Proc Object Print Name Freeze Default Sort Edit+/- Smpl+/- Compare+/-
Group Members
Spreadsheet
Dated Data Table
Graph...
Descriptive Stats
N-Way Tabulation...
Duplicate Observations
Covariance Analysis...
Tests of Equality...
Principal Components...
Correlogram (1) ...
Cross Correlation (2) ...
Long-run Covariance...
Unit Root Test...
Cointegration Test
Granger Causality...
Label
Johansen System Cointegration Test...
Single-Equation Cointegration Test...
LGOLD
Path = c:\users\sabro\documents
DB = none
WF = oil_gold_2017

Cointegration Test Specification
Test method
Engle-Granger
Lag specification
Schwarz Info Criterion
Maximum lag: *
(Use * to indicate obs-based maximum lag length)
Options
d.f. Adjustment
Equation specification
Trend specification
Constant (Level)
Deterministic regressors
Regressors specification
Additional trends
None
Additional deterministic regressors
OK
Cancel

Result:

Date: 11/30/20 Time: 20:28
Series: LOIL LGOLD
Sample: 1968M04 2017M01
Included observations: 586
Null hypothesis: Series are not cointegrated
Cointegrating equation deterministics: C
Automatic lags specification based on Schwarz criterion (maxlag=18)

Dependent	tau-statistic	Prob.*	z-statistic	Prob.*
LOIL	-3.728235	0.0177	-28.50078	0.0087
LGOLD	-3.488501	0.0346	-25.44978	0.0171

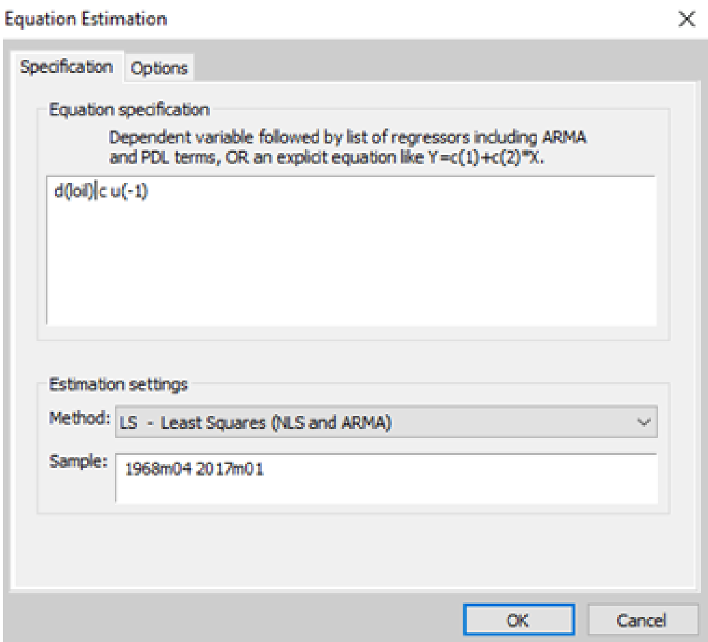
*MacKinnon (1996) p-values.

Same result (first row), and we even get a p-value! Note that EViews does the test "in both directions", once with Y_t as dependent variable and once with X_t. As mentioned in the slides, this doesn't matter asymptotically, but in finite samples it might matter which of the variables we consider endogenous and which one exogenous. In this particular case though, they both give the same answer. Still, it would be better not to have to make a choice here. That's what the Johansen procedure accomplishes.

Step 4: Estimate the VECM

$$\begin{aligned} \Delta Y_t &= c_1 + \alpha_1 (Y_{t-1} - \beta_1 - \beta_2 X_{t-1}) + e_{1t}, \\ \Delta X_t &= c_2 + \alpha_2 (Y_{t-1} - \beta_1 - \beta_2 X_{t-1}) + e_{2t}. \end{aligned}$$

replacing $Y_{t-1} - \beta_1 - \beta_2 X_{t-1}$ by the OLS residual $\hat{u}_{t-1} = Y_{t-1} - \hat{\beta}_1 - \hat{\beta}_2 X_{t-1}$ we saved earlier. Then, we can estimate α_1 and α_2 by OLS. We start with the equation for `LOIL`:

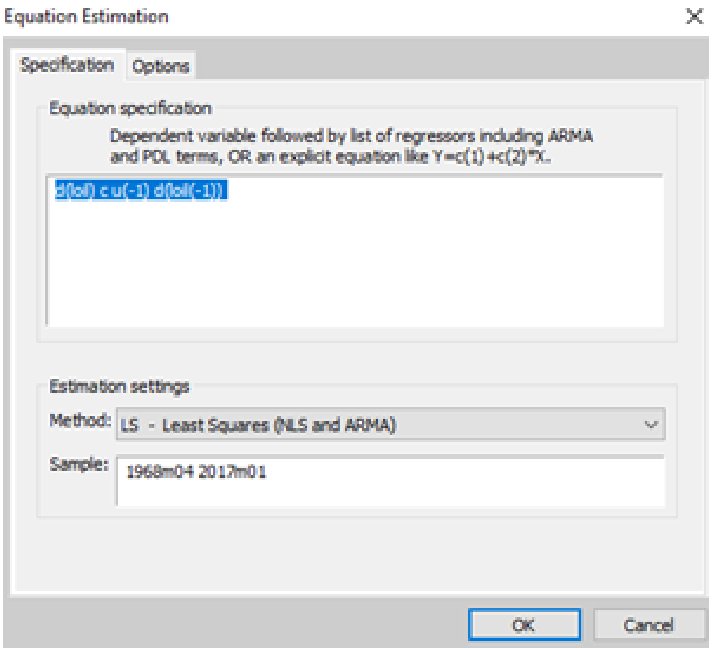


Dependent Variable: D(LOIL)
Method: Least Squares
Date: 11/30/20 Time: 20:16
Sample (adjusted): 1968M05 2017M01
Included observations: 585 after adjustments

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.004870	0.003377	1.442012	0.1498
U(-1)	-0.027903	0.010205	-2.734171	0.0064

R-squared	0.012660	Mean dependent var	0.004853
Adjusted R-squared	0.010967	S.D. dependent var	0.082140
S.E. of regression	0.081688	Akaike info criterion	-2.168397
Sum squared resid	3.890354	Schwarz criterion	-2.153451
Log likelihood	636.2562	Hannan-Quinn criter.	-2.162573
F-statistic	7.475692	Durbin-Watson stat	1.498327
Prob(F-statistic)	0.006444		

We notice that there is autocorrelation (look at the DW stat). We can cure this by adding a lagged difference (like the ADF test did automatically when it selected the lag length).

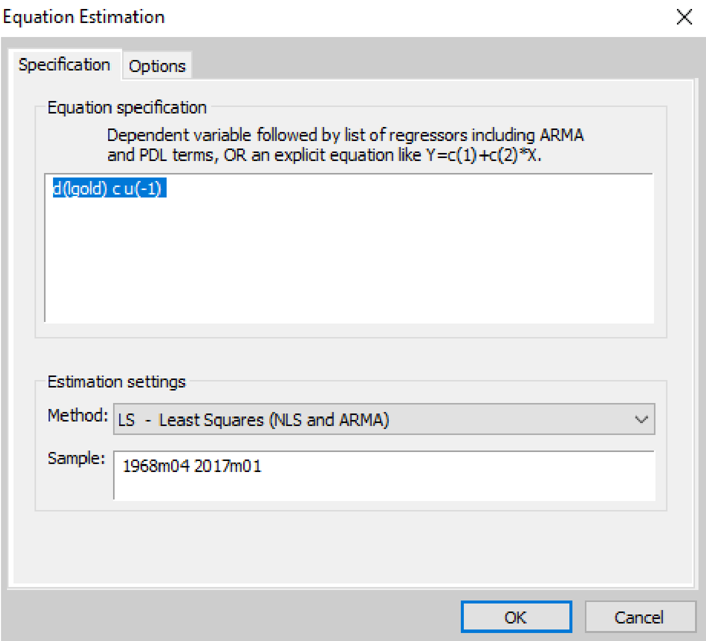


Dependent Variable: D(LOIL)
Method: Least Squares
Date: 11/30/20 Time: 17:09
Sample (adjusted): 1968M06 2017M01
Included observations: 584 after adjustments

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.003641	0.003278	1.110606	0.2672
U(-1)	-0.034971	0.009942	-3.517509	0.0005
D(LOIL(-1))	0.256547	0.040092	6.399038	0.0000

R-squared	0.077666	Mean dependent var	0.004862
Adjusted R-squared	0.074491	S.D. dependent var	0.082210
S.E. of regression	0.079089	Akaike info criterion	-2.231362
Sum squared resid	3.634195	Schwarz criterion	-2.208914
Log likelihood	654.5578	Hannan-Quinn criter.	-2.222613
F-statistic	24.46179	Durbin-Watson stat	2.006622
Prob(F-statistic)	0.000000		

Now it's fine. Let's repeat for the other variable:



Dependent Variable: D(LGOLD)
Method: Least Squares
Date: 11/30/20 Time: 17:17
Sample (adjusted): 1968M05 2017M01
Included observations: 585 after adjustments

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.005866	0.002342	2.504458	0.0125
U(-1)	0.008997	0.007077	1.271298	0.2041

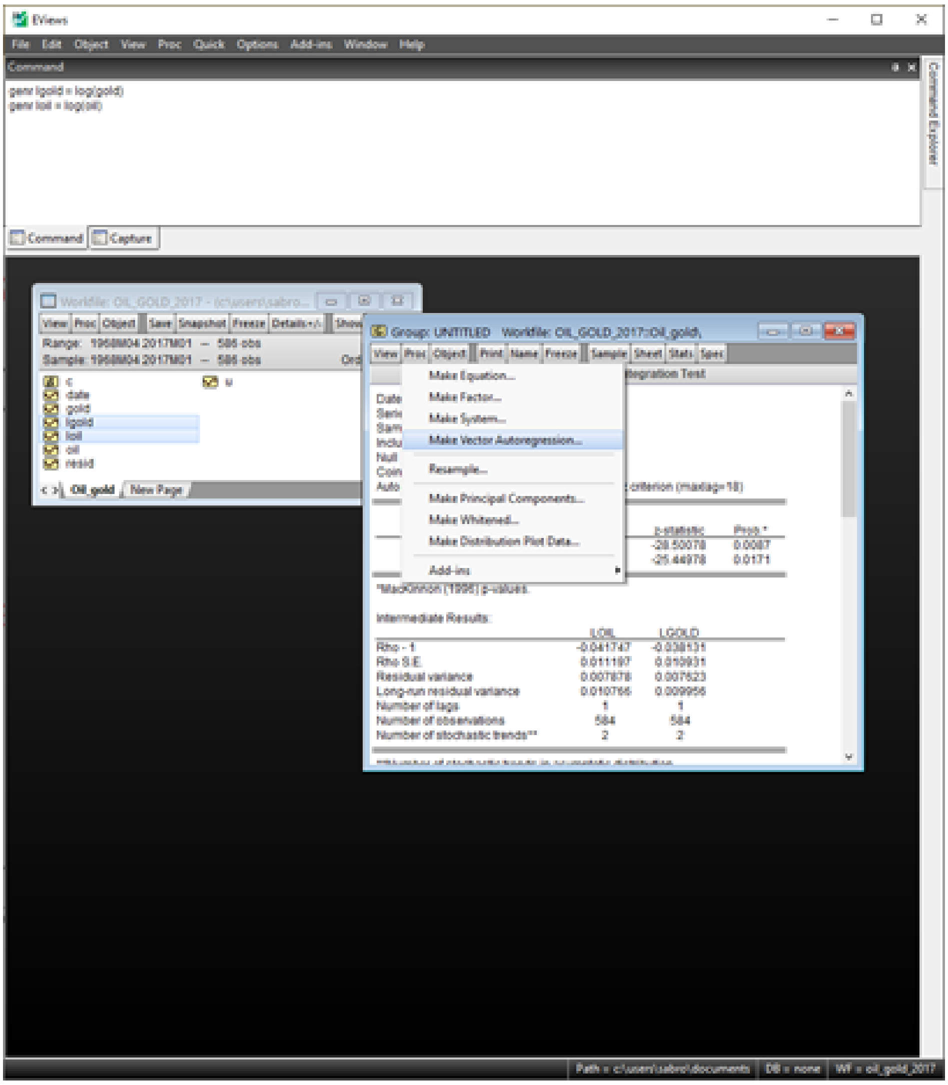
R-squared	0.002765	Mean dependent var	0.005871
Adjusted R-squared	0.001054	S.D. dependent var	0.056676
S.E. of regression	0.056646	Akaike info criterion	-2.900562
Sum squared resid	1.870743	Schwarz criterion	-2.885616
Log likelihood	850.4143	Hannan-Quinn criter.	-2.894737
F-statistic	1.616198	Durbin-Watson stat	1.932704
Prob(F-statistic)	0.204130		

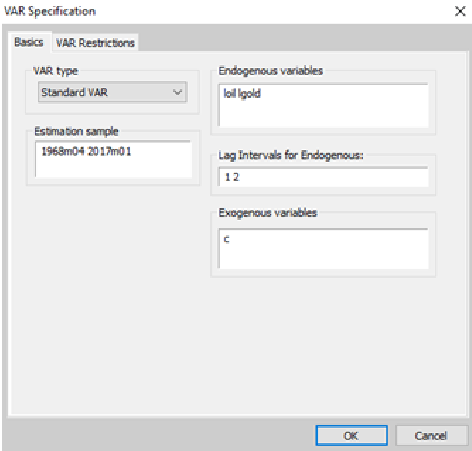
This one seems fine without any lagged differences. We notice that u_{t-1} is insignificant, so we can stick with a single-equation ECM. The final model is

$$\Delta \text{loil}_t = 0.0036 - 0.035(\text{loil}_{t-1} - 0.926\text{lgold}_{t-1} + 2.26) + 0.25\Delta\text{loil}_{t-1} + e_{1t}$$

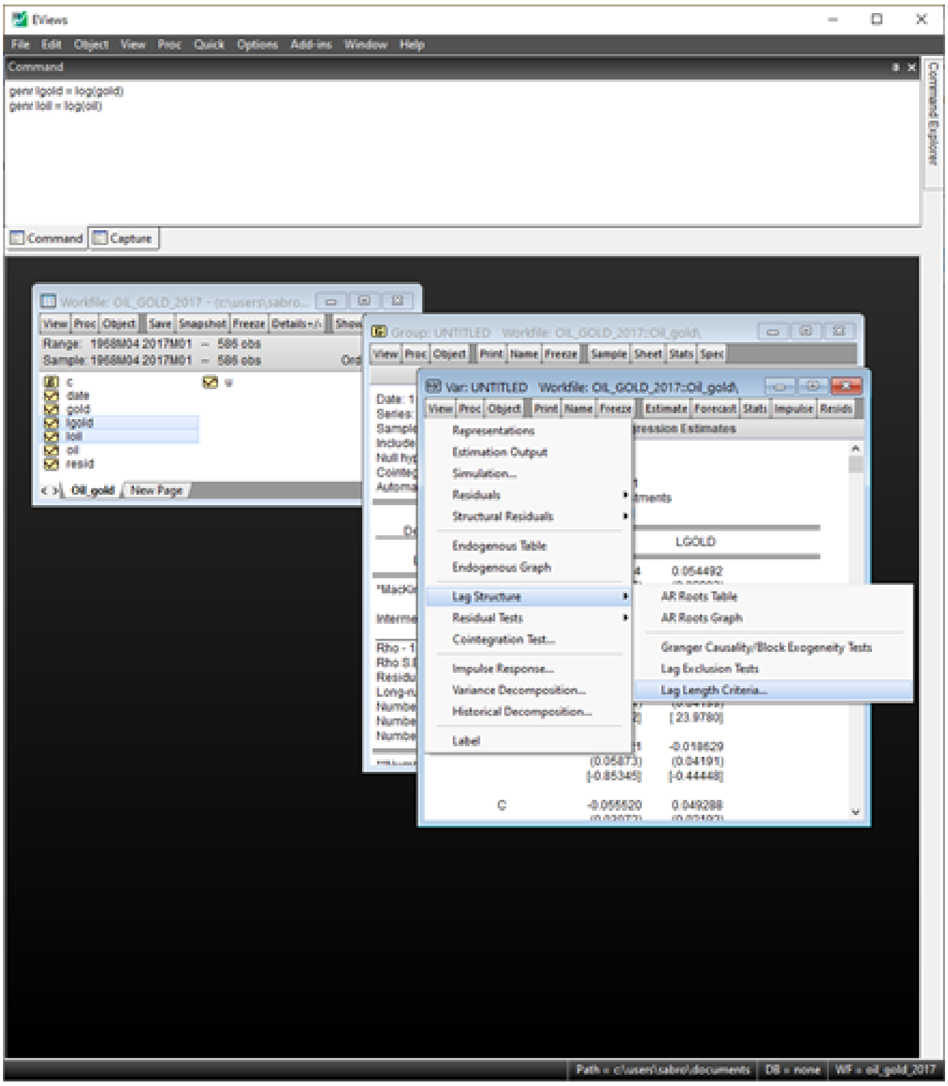
Interpretation: there is an equilibrium relationship between `loil` and `lgold`, with cointegrating vector $(1, -0.926)$. In case of a disequilibrium, `loil` adjusts towards the equilibrium. The adjustment amounts to 3.5% of the disequilibrium per period.

(c) We now repeat the analysis, but using Johansen's procedure, rather than Engle and Granger's.
Step 0 The first step is to pick a lag length. In slides, we didn't do this: we just picked a lag length of 1 because that's what we used in the Engle-Granger procedure. If we hadn't done Engle-Granger and started with Johansen right away, we'd need a way to do this. The easiest way is the following: open the variables as a group like before, then click on `Proc→Make Vector Autoregression`.





Keep the defaults, hit enter, and ignore the output. We're just doing this to get to the multivariate information criteria, by clicking `View→Lag Structure→Lag length criteria`:



This yields

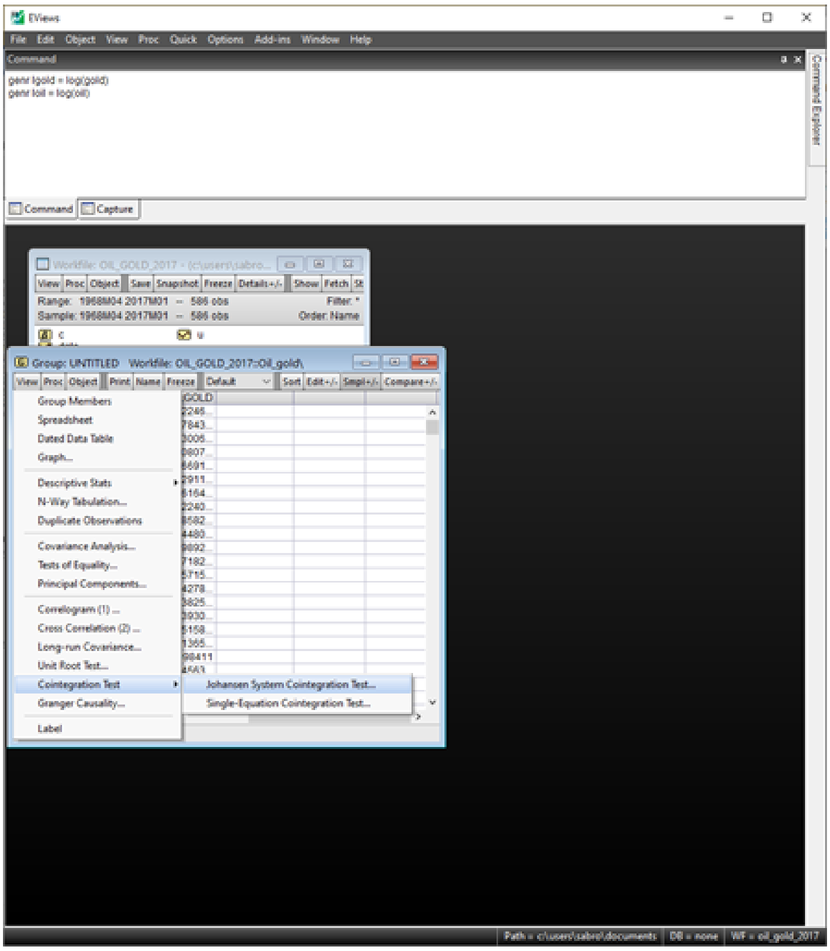
VAR Lag Order Selection Criteria
Endogenous variables: LOIL LGOLD
Exogenous variables: C
Date: 11/30/20 Time: 20:40
Sample: 1968M04 2017M01
Included observations: 578

Lag	LogL	LR	FPE	AIC	SC	HQ
0	-949.7189	NA	0.092309	3.293145	3.308230	3.299027
1	1473.535	4821.353	2.14e-05	-5.077976	-5.032721	-5.060329
2	1493.683	39.94747*	2.02e-05*	-5.133851*	-5.058426*	-5.104441*
3	1494.055	0.735532	2.05e-05	-5.121298	-5.015703	-5.080124
4	1494.302	0.485776	2.07e-05	-5.108311	-4.972546	-5.055373
5	1496.013	3.357075	2.09e-05	-5.100391	-4.934456	-5.035688
6	1498.043	3.969130	2.10e-05	-5.093575	-4.897470	-5.017108
7	1501.239	6.225229	2.11e-05	-5.090792	-4.864516	-5.002561
8	1502.698	2.832684	2.13e-05	-5.082000	-4.825555	-4.982005

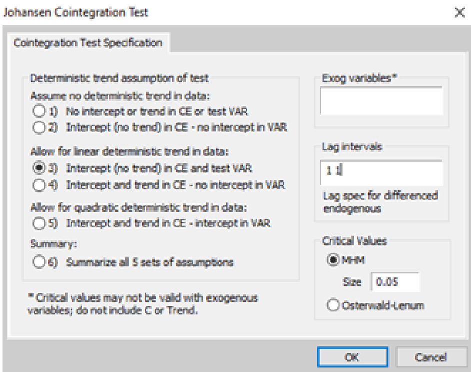
* indicates lag order selected by the criterion
LR: sequential modified LR test statistic (each test at 5% level)
FPE: Final prediction error
AIC: Akaike information criterion
SC: Schwarz information criterion
HQ: Hannan-Quinn information criterion

Most criteria agree on 2 lags. For technical reasons (this is a VAR, not a VECM), this means that we need 1 lag in the Johansen test and in the VECM.

Step 1 Conduct the Johansen test. Open the variables as a group as before, then do



Choose Option 3 to allow for a trend, and specify the lag interval as `1 1` (this means all lagged differences from lag 1 to lag 1), because that is what the information criteria suggested.



Result:

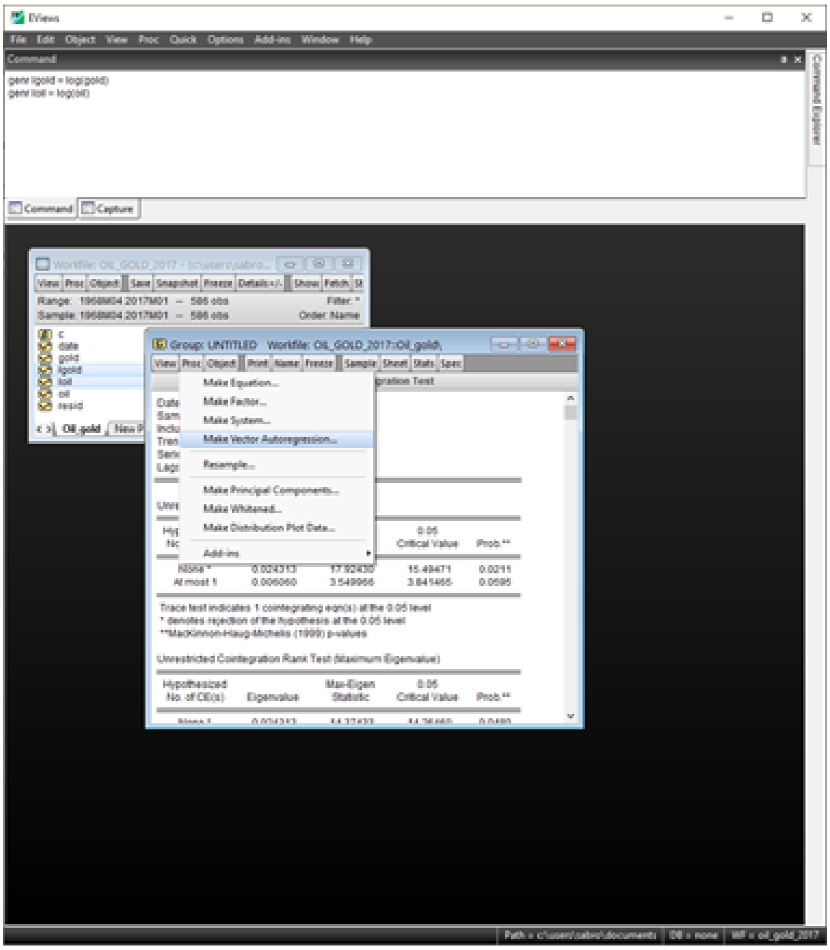
Unrestricted Cointegration Rank Test (Trace)

Hypothesized No. of CE(s)	Eigenvalue	Trace Statistic	0.05 Critical Value	Prob.**
None *	0.024313	17.92430	15.49471	0.0211
At most 1	0.006060	3.549966	3.841465	0.0595

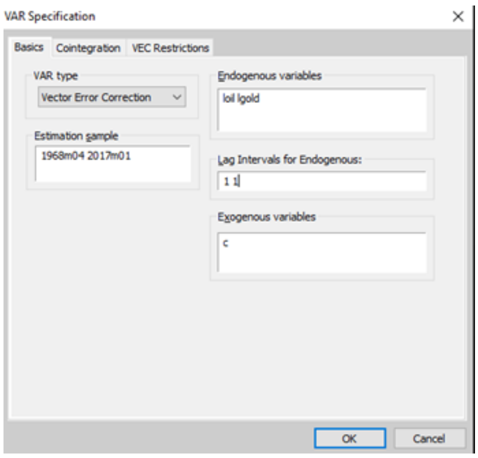
Trace test indicates 1 cointegrating eqn(s) at the 0.05 level
* denotes rejection of the hypothesis at the 0.05 level
**MacKinnon-Haug-Michelis (1999) p-values

The test for H_{00}: “No cointegration”rejects. The test for H_{01}: “1 cointegration relationship” accepts. Conclusion: there is one cointegrating relationship. It the latter test had rejected as well, then that would mean that the model is stationary; i.e., the variables aren’t integrated in the first place.

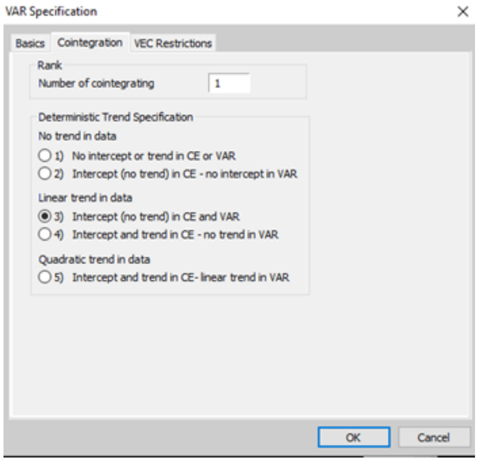
Step 2: Make a vector autoregression. In the group window, click on `Proc`→`Make Vector Autoregression`.



Choose "Vector Error Correction", and set the lag interval to match what the criteria found.



Go to the `Cointegration` tab, choose the same model as for the Johansen test (so 3 in this case), and set the number of cointegrating relationships to what the test found (so 1 in this case).



Result:

Vector Error Correction Estimates
Date: 11/30/20 Time: 20:59
Sample (adjusted): 1968M06 2017M01
Included observations: 584 after adjustments
Standard errors in ( ) & t-statistics in [ ]

Cointegrating Eq:	CointEq1	
LOIL(-1)	1.000000	
LGOLD(-1)	-0.914126	
	(0.09263)	
	[-9.86821]	
C	2.187722	
Error Correction:	**D(LOIL)**	**D(LGOLD)**
CointEq1	-0.034498	0.007463
	(0.00997)	(0.00713)
	[-3.46000]	[1.04641]
D(LOIL(-1))	0.250758	0.048515
	(0.04063)	(0.02906)
	[6.17135]	[1.66924]
D(LGOLD(-1))	0.050704	0.019596
	(0.05870)	(0.04199)
	[0.86374]	[0.46668]
C	0.003354	0.005412
	(0.00329)	(0.00236)
	[1.01839]	[2.29777]
R-squared	0.079176	0.008236
Adj. R-squared	0.074413	0.003106
Sum sq. resids	3.628244	1.856348
S.E. equation	0.079092	0.056574
F-statistic	16.62361	1.605456
Log likelihood	655.0364	850.7165
Akaike AIC	-2.229577	-2.899714
Schwarz SC	-2.199646	-2.869783
Mean dependent	0.004862	0.005761
S.D. dependent	0.082210	0.056662
Determinant resid covariance (dof adj.)		1.96E-05
Determinant resid covariance		1.93E-05
Log likelihood		1512.206
Akaike information criterion		-5.144541
Schwarz criterion		-5.069714
Number of coefficients		10

The red part is the VECM equation for `LOIL`, the green part is that for `LGOLD`, and the blue part is the cointegrating relationship. The cointegrating vector is $(1, -0.914)$. The results are remarkably similar to what we found with the Engle-Grander procedure. We could again ignore the equation for `LGOLD`, because the adjustment coefficient (α_2, in yellow) is insignificant (t-statistic 1.04). The final model is

$$\Delta \text{loil}_t = 0.0034 - 0.034(\text{loil}_{t-1} - 0.914\text{lgold}_{t-1} + 2.19) + 0.25\Delta\text{loil}_{t-1} + 0.05\Delta\text{lgold}_{t-1} + e_{1t}$$
$$\Delta \text{lgold}_t = 0.0054 - 0.007(\text{loil}_{t-1} - 0.914\text{lgold}_{t-1} + 2.19) + 0.049\Delta\text{loil}_{t-1} + 0.019\Delta\text{lgold}_{t-1} + e_{2t}$$

2. (a) X_t is a random walk, hence $I(1)$. So no, it is not stationary.

   (b) Y_t depends on X_t if $\beta_2 \neq 0$, so it cannot be stationary.

   (c) Yes, because there exists a linear combination of them that is stationary:

$$Y_t - \beta_2 X_t = \beta_1 + U_{1,t}.$$

   The cointegrating vector is $(1, -\beta_2)$.

   (d) The goal is to find two equations, one with ΔY_t on the LHS, and one with ΔX_t. Both should have the equilibrium error $Y_{t-1} - \beta_1 - \beta_2 X_{t-1}$ on the RHS.
   For Y_t, we find

$$\begin{aligned}
Y_t &= \beta_1 + \beta_2 X_t + U_{1,t} && | - Y_{t-1} \\
\Delta Y_t &= -Y_{t-1} + \beta_1 + \beta_2 X_t + U_{1,t} && | \pm \beta_2 X_{t-1} \\
\Delta Y_t &= -(Y_{t-1} - \beta_1 - \beta_2 X_{t-1}) + \beta_2 \Delta X_t + U_{1,t} \\
\Delta Y_t &= \alpha_1 (Y_{t-1} - \beta_1 - \beta_2 X_{t-1}) + \beta_2 \Delta X_t + U_{1,t},
\end{aligned}$$

   where $\alpha_1 = -1$. For X_t,

$$\begin{aligned}
X_t &= X_{t-1} + U_{2,t} && | - X_{t-1} \\
\Delta X_t &= U_{2,t} \\
\Delta X_t &= 0(Y_{t-1} - a X_{t-1}) + U_{2,t} \\
\Delta X_t &= \alpha_2 (Y_{t-1} - a X_{t-1}) + U_{2,t}
\end{aligned}$$

   where $\alpha_2 = 0$. This means that we can treat this as a single-equation ECM.