Exercise 2

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1. (a) Open the file `simulations.xlsx`. The sheet “White Noise” simulates $T = 1000$ observations from a (Gaussian) white noise process; i.e., 1000 uncorrelated mean-zero normals. By repeatedly pressing `F9`, you can draw new random numbers. Describe your observations.

   (b) Similarly, the sheet “Random Walk” simulates $T = 1000$ observations from a (Gaussian) random walk. Describe your observations.

2. (a) Open the file `sp500.csv`. Add a column to the resulting dataframe with the name `log_price` containing the log prices, and a column `log_return` containing the continuously compounded returns. Make a time series plot for each, and a histogram of the returns. Describe your findings.

   (b) Compute the skewness and kurtosis of the returns and manually conduct a Jarque-Bera test. Then, use `statsmodels` to do the test.

   (c) Generate a correlogram (ACF and PACF) of the returns and interpret it.

   (d) Test whether the first 10 autocorrelations are jointly significant at the 5% level.

   (e) Generate a correlogram of the log prices and interpret it.

   Hint: Here are some useful functions.

```
plt.hist
sm.stats.stattools.robust_skewness
sm.stats.stattools.robust_kurtosis
sm.stats.stattools.jarque_bera
scipy.stats.chi2.ppf
sm.graphics.tsa.plot_acf
sm.graphics.tsa.plot_pacf
tsa.acf
sm.stats.diagnostic.acorr_ljungbox
```

3. (a) Show that for the random walk $Y_t = Y_{t-1} + U_t$, where U_t is white noise and Y_0 some constant,

$$Y_t = Y_0 + U_1 + U_2 + \cdots + U_t = Y_0 + \sum_{s=1}^{t} U_t.$$

   (b) Building on the result from the previous question, show that

$$\mathbb{E}[Y_t] = Y_0, \quad \text{and}$$
$$\mathrm{var}(Y_t) = \sigma^2 t.$$