Solution to Exercise 7

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1. (a) If the relative price of oil expressed in units of gold, $\text{oil}_t/\text{gold}_t$, is stationary, then this implies that $\log(\text{oil}_t/\text{gold}_t) = \log(\text{oil}_t) - \log(\text{gold}_t)$ is also stationary, so $\log(\text{oil}_t)$ and $\log(\text{gold}_t)$ must be cointegrated with cointegrating vector (1, -1) if the individual series are integrated.

   (b) See Jupyter notebook.

2. (a) X_t is a random walk, hence $I(1)$. So no, it is not stationary.

   (b) Y_t depends on X_t if $\beta_2 \neq 0$, so it cannot be stationary.

   (c) Yes, because there exists a linear combination of them that is stationary:

   $$Y_t - \beta_2 X_t = \beta_1 + U_{1,t}.$$

   The cointegrating vector is $(1, -\beta_2)$.

   (d) The goal is to find two equations, one with ΔY_t on the LHS, and one with ΔX_t. Both should have the equilibrium error $Y_{t-1} - \beta_1 - \beta_2 X_{t-1}$ on the RHS.
   For Y_t, we find

   $$\begin{aligned}
   Y_t &= \beta_1 + \beta_2 X_t + U_{1,t} && | - Y_{t-1}\\
   \Delta Y_t &= -Y_{t-1} + \beta_1 + \beta_2 X_t + U_{1,t} && | \pm \beta_2 X_{t-1}\\
   \Delta Y_t &= -(Y_{t-1} - \beta_1 - \beta_2 X_{t-1}) + \beta_2 \Delta X_t + U_{1,t}\\
   \Delta Y_t &= \alpha_1 (Y_{t-1} - \beta_1 - \beta_2 X_{t-1}) + \beta_2 \Delta X_t + U_{1,t},
   \end{aligned}$$

   where $\alpha_1 = -1$. For X_t,

   $$\begin{aligned}
   X_t &= X_{t-1} + U_{2,t} && | - X_{t-1}\\
   \Delta X_t &= U_{2,t}\\
   \Delta X_t &= 0(Y_{t-1} - a X_{t-1}) + U_{2,t}\\
   \Delta X_t &= \alpha_2 (Y_{t-1} - a X_{t-1}) + U_{2,t}
   \end{aligned}$$

   where $\alpha_2 = 0$. This means that we can treat this as a single-equation ECM.