

Exercise 3

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1. (a) Open the file `simulations.xlsx`. The sheet “AR(1)” simulates $T = 1000$ observations from an AR(1) process. Play around with α and $-1 < \phi_1 < 1$ and describe your observations.
(b) Also try setting $\phi_1 = 1$ and describe the effect of α .
(c) The file `simulated_data.csv` contains three series simulated using the same spreadsheet, `simulation.xlsx`, one each for an AR(1), an MA(1), and an ARMA(1, 1) process. The AR and ARMA processes use $\phi_1 = 0.7$, and the MA and ARMA processes use $\theta_1 = 0.7$. Plot the sample ACF and PACF for both, and describe your observations.
2. (a) Use the Box-Jenkins approach to model year-on-year real GDP growth. The (quarterly) GDP data can be found in the file `realgdpch.csv`. You will need to transform them into year-on-year growth rates first, by doing

```
df["YoY_GROWTH"] = np.log(df["REAL_GDP"]) \
                    - np.log(df["REAL_GDP"].shift(4))
```


(b) Produce forecasts for 2022Q3 and 2022Q4, both manually and using Python. Do this for
 - i. an MA(3) model, and
 - ii. an AR(1) model.
3. (a) Obtain the mean and variance of a random walk with drift.
(b) Show that the random walk with drift is integrated of order 1.
(c) Derive the expression for the variance of a stationary AR(1) given in the slides.
(d) **Optional:** Find the mean, variance, and ACF of an MA(1).
(e) **Optional:** Find the ACF of a stationary AR(1).