

## Solution to Exercise 5

Simon A. Broda

1. See Jupyter notebook.
2. (a) Splitting out the first term of the sum immediately yields

$$\begin{aligned}\hat{\sigma}_{t+1,EWMA}^2 &= (1 - \lambda) \sum_{j=0}^{\infty} \lambda^j r_{t-j}^2 \\ \hat{\sigma}_{t+1,EWMA}^2 &= (1 - \lambda) \lambda^0 r_{t-0}^2 + (1 - \lambda) \sum_{j=1}^{\infty} \lambda^j r_{t-j}^2 \\ \hat{\sigma}_{t+1,EWMA}^2 &= (1 - \lambda) r_t^2 + (1 - \lambda) \sum_{j=0}^{\infty} \lambda^{j+1} r_{t-1-j}^2 \\ \hat{\sigma}_{t+1,EWMA}^2 &= (1 - \lambda) r_t^2 + \lambda(1 - \lambda) \sum_{j=0}^{\infty} \lambda^j r_{t-1-j}^2 \\ &= (1 - \lambda) r_t^2 + \lambda \hat{\sigma}_{t,EWMA}^2.\end{aligned}$$