

Module Examination MSCBF_RM01_TSA

Course 9.3: Time Series Analysis

Autumn Semester

Mock Exam

Candidate Information

Name:	
First name:	

Exam Information

Duration of exam:	90 min		
Points:	Assignment	Maximum number of points	Number of points awarded
	Question 1	27	
	Question 2	27	
	Question 3	18	
	Question 4	18	
	Total	90	
	Grade		
Aids:	<ul style="list-style-type: none"> ▪ Closed book ▪ Calculator ▪ 8 page summary (4 double sided a4 sheets) 		
Lecturers:	Dr. Simon A. Broda		

Exam Instructions

Instructions:	<ul style="list-style-type: none"> ▪ When performing a statistical test, give the hypotheses, test statistic and its distribution, critical value, observed test statistic, decision (reject/don't reject), and a conclusion in words. ▪ For simplicity, all tests use the 5% significance level.
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Question 1 (27 points)

1. I have simulated 1000 observations $\{y_t\}$ from an ARMA(p, q) model. Figure 2 on Page 8 shows a plot of the correlogram. Based on it, what do you think p and q are, and why?

6 pts

Solution: The PACF becomes insignificant after 2 lags, whereas the ACF decays exponentially. This is indicative of an AR(2) process.

2. The 31st sample autocorrelation (not shown in the graph) is $\hat{\tau}_{31} = -0.045$. Test if $\hat{\tau}_{31}$ is significantly different from zero.

6 pts

Solution:

- The hypotheses are $H_0 : \tau_{31} = 0$ vs. $H_a : \tau_{31} \neq 0$.
- The test statistic is $\hat{\tau}_{31}$.
- Its distribution is $N(0, 1/T)$ under H_0 .
- The critical value is thus $1.96/\sqrt{1000} = 0.062$.
- The observed test statistic is $\hat{\tau}_{31} = -0.045$.
- Since $|-0.045| < 0.062$, we do not reject H_0 .
- Conclusion: τ_{31} is not significantly different from zero.

3. The output in Figure 3 on Page 8 shows the results of regressing Δy_t (y) on y_{t-1} (x1) and Δy_{t-1} (x2). Use it to test if the data are integrated.

6 pts

Solution:

- The hypotheses are $H_0 : y_t$ is integrated of order 1 vs. $H_a : y_t$ is stationary.
- The test statistic is the estimated coefficient on the lagged level, y_{t-1} (x1).
- Its follows a Dickey-Fuller distribution under H_0 .
- The critical value is -2.86, because a constant has been included.
- The observed test statistic is -6.52.
- Since $-6.52 < -2.86$, we reject H_0 .
- Conclusion: the data are stationary.

4. I have estimated a particular ARMA model. The estimation output is shown in Figure 4 on Page 9. Write down the estimated model in equation form.

3 pts

Solution: The estimated AR(2) model is

$$Y_t = 0.2158 \cdot (1 - 0.5236 - 0.3595) + 0.5236 \cdot Y_{t-1} + 0.3595 \cdot Y_{t-2} + U_t.$$

Note `archmodel`'s peculiar interpretation of the intercept, `const`: in our notation, it corresponds to $\hat{\alpha}/(1 - \hat{\phi}_1 - \hat{\phi}_2)$, so that $\hat{\alpha} = \text{const} \cdot (1 - \hat{\phi}_1 - \hat{\phi}_2)$. See the exercises.

5. Use the estimated model from the previous question to forecast the value of the series at $t = 1001$. You may need some of the values below.

6 pts

t	y_t	u_t
999	-0.77	-4.18
1000	-0.75	-1.41

Solution: The forecast is

$$\begin{aligned}
 \hat{y}_{1001} &= \text{const} \cdot (1 - \hat{\phi}_1 - \hat{\phi}_2) + \hat{\phi}_1 \cdot y_{1000} + \hat{\phi}_2 \cdot y_{999} \\
 &= 0.2158 \cdot (1 - 0.5236 - 0.3595) + 0.5236 \cdot (-0.75) + 0.3595 \cdot (-0.77) \\
 &= -0.644.
 \end{aligned}$$

Question 2 (27 points) In this exercise, we analyze the daily returns on Tesla stock between 6/29/2010 and 12/14/2022. The returns are shown in Figure 1.

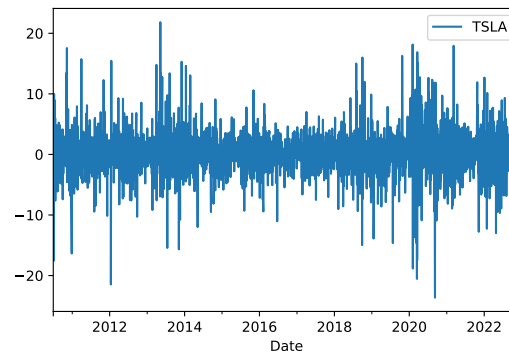


Figure 1: Returns on Tesla stock

The data clearly display volatility clustering, which we want to model using a GARCH model. The output is shown in Figure 5 on Page 9.

1. Write the model down as an equation (only the volatility equation).

Solution:

$$\sigma_{t+1}^2 = 0.14 + 0.0323 \cdot u_t^2 + 0.9564 \cdot \sigma_t^2.$$

3 pts

2. Does your model incorporate a leverage effect? Justify your answer.

Solution: No. Since u_t (the demeaned return) only enters as its square, the news impact curve is symmetric: volatility reacts equally strongly to a positive and negative shock of the same magnitude.

6 pts

3. Explain what the standardized residuals from a GARCH model are, ideally with an equation. Also, explain what their properties should be if the volatility model is correct.

Solution: The standardized residuals are defined as $\hat{z}_t = \frac{r_t - \hat{\mu}_t}{\hat{\sigma}_t}$. If the model for σ_t is correct, then they should no longer display volatility clustering. This means that there should be no autocorrelation in their squares.

6 pts

4. Regressing the squared standardized residuals on an intercept and 5 of their own lags results in an R^2 of 0.00364. Use this to test if the GARCH model has successfully removed the volatility clustering.

Solution: This is an ARCH-LM test.

- The hypotheses are H_0 : no remaining volatility clustering vs. H_a : there is remaining volatility clustering.
- The test statistic is $T \cdot R^2$ in the auxiliary regression.
- Its distribution is $\chi^2(5)$ under H_0 .
- The critical value is thus 11.07.
- The observed test statistic is $3138 \cdot 0.00364 = 11.42$.

6 pts

- Since $11.42 > 11.07$, we reject H_0 (although barely).
- Conclusion: Some volatility clustering remains.

5. Use the model to predict the variance σ_{t+1}^2 for 12/15/2022, using the following values on 12/14/2022: $\hat{u}_t = -2.72$ and $\sigma_t^2 = 15.198$.

6 pts

Solution:

$$\begin{aligned}\hat{\sigma}_{t+1}^2 &= 0.14 + 0.0323 \cdot u_t^2 + 0.9564 \cdot \sigma_t^2 \\ &= 0.14 + 0.0323 \cdot (-2.72)^2 + 0.9564 \cdot 15.198 \\ &= 14.91.\end{aligned}$$

Question 3 (18 points) We now turn our attention to Value at Risk forecasting.

1. Use the model from the previous question to predict the 1% Value at Risk for 12/15/2022. Note: If you weren't successful in predicting the variance in the previous question, you can use the value $\hat{\sigma}_{t+1}^2 = 15.00$.

6 pts

Solution: The VaR forecast is

$$\begin{aligned} VaR_{t+1}^{0.01} &= -\mu_{t+1} - \sigma_{t+1} \cdot \Phi_{0.01}^{-1} \\ &= -\mu_{t+1} - \sigma_{t+1} \cdot \Phi_{0.01}^{-1} \\ &= -0.1118 - \sqrt{14.91} \cdot (-2.326) \\ &= 8.87. \end{aligned}$$

2. In a backtesting exercise, we have created 1% VaR forecasts for the entire sample. From this and the returns, we have created the hit series $\{I_t\}$.

12 pts

- (a) (6 pts) Explain how the hit series is defined, and how many times you expect it to equal 1 if the VaR model is correct.

Solution: The hit series equals 1 whenever a VaR violation occurs, i.e.,

$$I_{t+1} = \begin{cases} 1, & \text{if } R_{t+1} < -VaR_{t+1}^p, \\ 0, & \text{if } R_{t+1} > -VaR_{t+1}^p. \end{cases}$$

If the model is correct, we would expect a violation 1% of the time. Since $T = 3138$, we would therefore expect around 31 VaR violations.

- (b) (6 pts) Figure 6 on page 10 shows the result of regressing $(I_t - 0.01)$ on an intercept and I_{t-1} . Use it to test the independence of the VaR violations.

Solution:

- The hypotheses are H_0 : the VaR violations are independent vs. H_a : they are not.
- The test statistic is the t statistic for the lagged hit series.
- Its distribution is (asymptotically) standard normal under H_0 .
- The critical value is thus 1.96.
- The observed test statistic is 1.187.
- Since $|1.187| < 1.96$, we do not reject H_0 .
- Conclusion: there is no evidence of dependence between the VaR violations.

Note: in this case, since I didn't delete the p -value, you could alternatively have answered as follows.

- The hypotheses are H_0 : the VaR violations are independent vs. H_a : they are not.
- The test statistic is the t statistic for the lagged hit series.
- The observed p -value is $0.235 > 0.05$, so we do not reject H_0 .
- Conclusion: there is no evidence of dependence between the VaR violations.

Question 4 (18 points) Answer the questions below.

1.

- (a) (3 pts) Spurious regressions can occur between cointegrated variables.

☐ True☒ False

- (b) (3 pts) In a stationary time series, shocks
- U_t
- have a transitory effect on the future of the series.

☒ True☐ False

- (c) (3 pts) An ARCH(
- q
-) model for the returns corresponds to an AR(
- q
-) for the squared returns.

☒ True☐ False

- (d) (3 pts) The order
- q
- of an MA(
- q
-) model can be determined from the correlogram.

☒ True☐ False

- (e) (3 pts) In the presence of the leverage effect, the news impact curve is steeper to the right of the origin than to the left.

☐ True☒ False

- (f) (3 pts) A VaR model is correctly specified if no VaR violations occur.

☐ True☒ False

18 pts

End of exam.

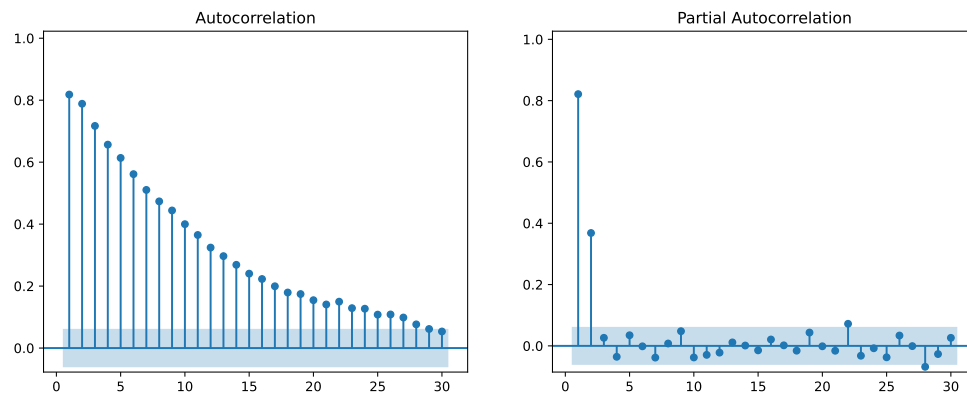


Figure 2: Correlogram of simulated data.

OLS Regression Results						
=====						
Dep. Variable:	y	R-squared:	0.208			
Model:	OLS	Adj. R-squared:	0.207			
Method:	Least Squares	F-statistic:	130.9			
Date:	Mon, 16 Oct 2023	Prob (F-statistic):	3.47e-51			
Time:	15:04:00	Log-Likelihood:	-2086.9			
No. Observations:	998	AIC:	4180.			
Df Residuals:	995	BIC:	4195.			
Df Model:	2					
Covariance Type:	nonrobust					
=====						
	coef	std err	t	P> t	[0.025	0.975]

x1	-0.1161	0.018	-6.520	0.000	-0.151	-0.081
x2	-0.3594	0.030	-12.164	0.000	-0.417	-0.301
const	0.0309	0.062	0.497	0.619	-0.091	0.153
=====						
Omnibus:	0.529	Durbin-Watson:	2.023			
Prob(Omnibus):	0.768	Jarque-Bera (JB):	0.404			
Skew:	0.003	Prob(JB):	0.817			
Kurtosis:	3.098	Cond. No.	3.76			
=====						

Figure 3: Output for ADF test.


```

SARIMAX Results
=====
Dep. Variable:          y      No. Observations:      1000
Model:                ARIMA(2, 0, 0)  Log Likelihood      -2091.958
Date:                Mon, 16 Oct 2023  AIC              4191.917
Time:                15:10:43      BIC              4211.548
Sample:              0      HQIC              4199.378
                  - 1000
Covariance Type:      opg
=====
              coef      std err          z      P>|z|      [0.025      0.975]
-----
const          0.2158         0.528         0.409      0.683      -0.818         1.250
ar.L1          0.5236         0.032        16.396      0.000         0.461         0.586
ar.L2          0.3595         0.032        11.326      0.000         0.297         0.422
sigma2         3.8369         0.169        22.770      0.000         3.507         4.167
=====
Ljung-Box (L1) (Q):          0.13      Jarque-Bera (JB):          0.36
Prob(Q):          0.71      Prob(JB):          0.83
Heteroskedasticity (H):      0.86      Skew:          0.00
Prob(H) (two-sided):      0.18      Kurtosis:          3.09
=====

```

Figure 4: Estimated ARMA model.

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Constant Mean - GARCH Model Results
=====
Dep. Variable:          TSLA      R-squared:          0.000
Mean Model:          Constant Mean  Adj. R-squared:          0.000
Vol Model:          GARCH      Log-Likelihood:      -8267.70
Distribution:          Normal      AIC:          16543.4
Method:          Maximum Likelihood  BIC:          16567.6
                  No. Observations:          3138
Date:          Mon, Oct 16 2023      Df Residuals:          3137
Time:          15:33:03      Df Model:          1
                  Mean Model
=====
              coef      std err          t      P>|t|      95.0% Conf. Int.
-----
mu          0.1118      5.774e-02         1.936      5.281e-02      [-1.359e-03,  0.225]
Volatility Model
=====
              coef      std err          t      P>|t|      95.0% Conf. Int.
-----
omega       0.1400         0.117         1.196         0.232      [-8.938e-02,  0.369]
alpha[1]    0.0323         1.341e-02         2.409         1.599e-02      [6.024e-03, 5.859e-02]
beta[1]     0.9564         2.205e-02        43.369         0.000         [ 0.913,  1.000]
=====

```

Figure 5: Estimated GARCH model.

```

=====
                        OLS Regression Results
=====
Dep. Variable:      np.subtract(I, 0.01)    R-squared:                 0.000
Model:                OLS                   Adj. R-squared:            0.000
Method:              Least Squares          F-statistic:              1.409
Date:                Mon, 16 Oct 2023       Prob (F-statistic):       0.235
Time:                18:54:56              Log-Likelihood:           1976.8
No. Observations:    3137                  AIC:                      -3950.
Df Residuals:        3135                  BIC:                      -3938.
Df Model:             1
Covariance Type:     nonrobust
=====

```

	coef	std err	t	P> t	[0.025	0.975]
b0	0.0065	0.002	2.817	0.005	0.002	0.011
I.shift(1)	0.0212	0.018	1.187	0.235	-0.014	0.056

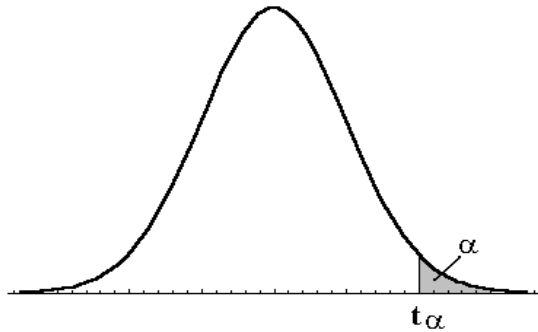
```

=====
Omnibus:                4069.691    Durbin-Watson:              2.001
Prob(Omnibus):           0.000      Jarque-Bera (JB):           412759.104
Skew:                    7.492      Prob(JB):                   0.00
Kurtosis:                 57.160    Cond. No.                   7.76
=====

```

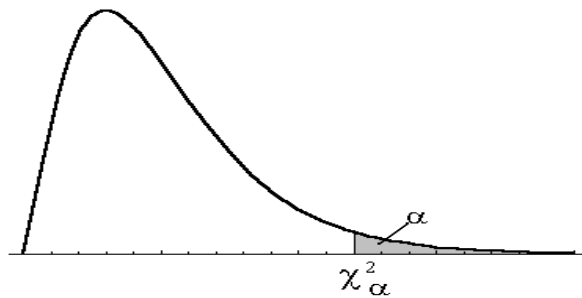
Figure 6: Test regression for the Value at Risk backtest.

Critical Values of t



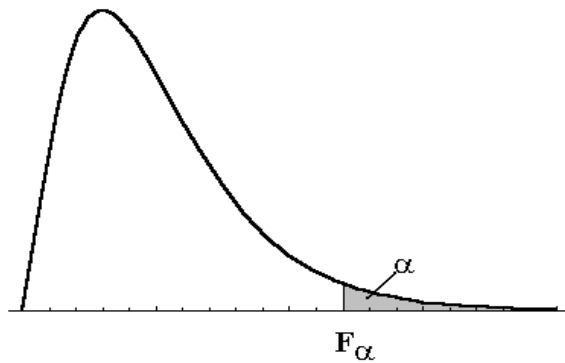
Degrees of freedom	$t_{.10}$	$t_{.05}$	$t_{.025}$	$t_{.01}$	$t_{.005}$	Degrees of freedom	$t_{.10}$	$t_{.05}$	$t_{.025}$	$t_{.01}$	$t_{.005}$
1	3.078	6.314	12.706	31.821	63.657	24	1.318	1.711	2.064	2.492	2.797
2	1.886	2.920	4.303	6.965	9.925	25	1.316	1.708	2.060	2.485	2.787
3	1.638	2.353	3.182	4.541	5.841	26	1.315	1.706	2.056	2.479	2.779
4	1.533	2.132	2.776	3.747	4.604	27	1.314	1.703	2.052	2.473	2.771
5	1.476	2.015	2.571	3.365	4.032	28	1.313	1.701	2.048	2.467	2.763
6	1.440	1.943	2.447	3.143	3.707	29	1.311	1.699	2.045	2.462	2.756
7	1.415	1.895	2.365	2.998	3.499	30	1.310	1.697	2.042	2.457	2.750
8	1.397	1.860	2.306	2.896	3.355	35	1.306	1.690	2.030	2.438	2.724
9	1.383	1.833	2.262	2.821	3.250	40	1.303	1.684	2.021	2.423	2.704
10	1.372	1.812	2.228	2.764	3.169	45	1.301	1.679	2.014	2.412	2.690
11	1.363	1.796	2.201	2.718	3.106	50	1.299	1.676	2.009	2.403	2.678
12	1.356	1.782	2.179	2.681	3.055	60	1.296	1.671	2.000	2.390	2.660
13	1.350	1.771	2.160	2.650	3.012	70	1.294	1.667	1.994	2.381	2.648
14	1.345	1.761	2.145	2.624	2.977	80	1.292	1.664	1.990	2.374	2.639
15	1.341	1.753	2.131	2.602	2.947	90	1.291	1.662	1.987	2.368	2.632
16	1.337	1.746	2.120	2.583	2.921	100	1.290	1.660	1.984	2.364	2.626
17	1.333	1.740	2.110	2.567	2.898	120	1.289	1.658	1.980	2.358	2.617
18	1.330	1.734	2.101	2.552	2.878	140	1.288	1.656	1.977	2.353	2.611
19	1.328	1.729	2.093	2.539	2.861	160	1.287	1.654	1.975	2.350	2.607
20	1.325	1.725	2.086	2.528	2.845	180	1.286	1.653	1.973	2.347	2.603
21	1.323	1.721	2.080	2.518	2.831	200	1.286	1.653	1.972	2.345	2.601
22	1.321	1.717	2.074	2.508	2.819	∞	1.282	1.645	1.960	2.326	2.576
23	1.319	1.714	2.069	2.500	2.807						

Critical Values of χ^2



Degrees of freedom	$\chi^2_{.10}$	$\chi^2_{.05}$	$\chi^2_{.025}$	$\chi^2_{.01}$	$\chi^2_{.005}$
1	2.706	3.841	5.024	6.635	7.879
2	4.605	5.991	7.378	9.210	10.597
3	6.251	7.815	9.348	11.345	12.838
4	7.779	9.488	11.143	13.277	14.860
5	9.236	11.070	12.833	15.086	16.750
6	10.645	12.592	14.449	16.812	18.548
7	12.017	14.067	16.013	18.475	20.278
8	13.362	15.507	17.535	20.090	21.955
9	14.684	16.919	19.023	21.666	23.589
10	15.987	18.307	20.483	23.209	25.188
11	17.275	19.675	21.920	24.725	26.757
12	18.549	21.026	23.337	26.217	28.300
13	19.812	22.362	24.736	27.688	29.819
14	21.064	23.685	26.119	29.141	31.319
15	22.307	24.996	27.488	30.578	32.801
16	23.542	26.296	28.845	32.000	34.267
17	24.769	27.587	30.191	33.409	35.718
18	25.989	28.869	31.526	34.805	37.156
19	27.204	30.144	32.852	36.191	38.582
20	28.412	31.410	34.170	37.566	39.997
21	29.615	32.671	35.479	38.932	41.401
22	30.813	33.924	36.781	40.289	42.796
23	32.007	35.172	38.076	41.638	44.181
24	33.196	36.415	39.364	42.980	45.559
25	34.382	37.652	40.646	44.314	46.928
26	35.563	38.885	41.923	45.642	48.290
27	36.741	40.113	43.195	46.963	49.645
28	37.916	41.337	44.461	48.278	50.993
29	39.087	42.557	45.722	49.588	52.336
30	40.256	43.773	46.979	50.892	53.672
40	51.805	55.758	59.342	63.691	66.766
50	63.167	67.505	71.420	76.154	79.490
60	74.397	79.082	83.298	88.379	91.952
70	85.527	90.531	95.023	100.425	104.215
80	96.578	101.879	106.629	112.329	116.321
90	107.565	113.145	118.136	124.116	128.299
100	118.498	124.342	129.561	135.807	140.169

Critical Values of F $\alpha=.05$



		NUMERATOR DEGREES OF FREEDOM								
		1	2	3	4	5	6	7	8	9
D E N O M I N A T O R D E G R E E S O F F R E E D O M	df _{den}	1	2	3	4	5	6	7	8	9
	1	161.4	199.5	215.7	224.6	230.2	234.0	236.8	238.9	240.5
	2	18.51	19.00	19.16	19.25	19.30	19.33	19.35	19.37	19.38
	3	10.13	9.55	9.28	9.12	9.01	8.94	8.89	8.85	8.81
	4	7.71	6.94	6.59	6.39	6.26	6.16	6.09	6.04	6.00
	5	6.61	5.79	5.41	5.19	5.05	4.95	4.88	4.82	4.77
	6	5.99	5.14	4.76	4.53	4.39	4.28	4.21	4.15	4.10
	7	5.59	4.74	4.35	4.12	3.97	3.87	3.79	3.73	3.68
	8	5.32	4.46	4.07	3.84	3.69	3.58	3.50	3.44	3.39
	9	5.12	4.26	3.86	3.63	3.48	3.37	3.29	3.23	3.18
	10	4.96	4.10	3.71	3.48	3.33	3.22	3.14	3.07	3.02
	11	4.84	3.98	3.59	3.36	3.20	3.09	3.01	2.95	2.90
	12	4.75	3.89	3.49	3.26	3.11	3.00	2.91	2.85	2.80
	13	4.67	3.81	3.41	3.18	3.03	2.92	2.83	2.77	2.71
	14	4.60	3.74	3.34	3.11	2.96	2.85	2.76	2.70	2.65
	15	4.54	3.68	3.29	3.06	2.90	2.79	2.71	2.64	2.59
	16	4.49	3.63	3.24	3.01	2.85	2.74	2.66	2.59	2.54
	17	4.45	3.59	3.20	2.96	2.81	2.70	2.61	2.55	2.49
	18	4.41	3.55	3.16	2.93	2.77	2.66	2.58	2.51	2.46
	19	4.38	3.52	3.13	2.90	2.74	2.63	2.54	2.48	2.42
	20	4.35	3.49	3.10	2.87	2.71	2.60	2.51	2.45	2.39
	21	4.32	3.47	3.07	2.84	2.68	2.57	2.49	2.42	2.37
	22	4.30	3.44	3.05	2.82	2.66	2.55	2.46	2.40	2.34
	23	4.28	3.42	3.03	2.80	2.64	2.53	2.44	2.37	2.32
	24	4.26	3.40	3.01	2.78	2.62	2.51	2.42	2.36	2.30
	25	4.24	3.39	2.99	2.76	2.60	2.49	2.40	2.34	2.28
	26	4.23	3.37	2.98	2.74	2.59	2.47	2.39	2.32	2.27
	27	4.21	3.35	2.96	2.73	2.57	2.46	2.37	2.31	2.25
	28	4.20	3.34	2.95	2.71	2.56	2.45	2.36	2.29	2.24
	29	4.18	3.33	2.93	2.70	2.55	2.43	2.35	2.28	2.22
	30	4.17	3.32	2.92	2.69	2.53	2.42	2.33	2.27	2.21
	40	4.08	3.23	2.84	2.61	2.45	2.34	2.25	2.18	2.12
	60	4.00	3.15	2.76	2.53	2.37	2.25	2.17	2.10	2.04
	120	3.92	3.07	2.68	2.45	2.29	2.18	2.09	2.02	1.96
	∞	3.84	3.00	2.60	2.37	2.21	2.10	2.01	1.94	1.88

Dickey-Fuller critical values for different significance levels α

0.01	0.025	0.05	0.10
τ			
-2.58	-2.23	-1.95	-1.62
τ_{μ}			
-3.43	-3.12	-2.86	-2.57
τ_{τ}			
-3.96	-3.66	-3.41	-3.12

5% critical values of Engle-Granger test vs. number of regressors k (excluding constant)

1	2	3	4	5
-3.41	-3.80	-4.16	-4.49	-4.74