Exercise 5

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1. (a) Load the data in the file `sp500.csv` and construct the log returns (in percent). Make a time series plot and a histogram of the returns. Then, produce a correlogram of the squared residuals, and test the first 5 autocorrelations of the squared returns for joint significance using the Ljung-Box Q-test. Interpret your results.

   (b) Perform an ARCH-LM test by regressing the returns on an intercept, and feeding the residuals into `sm.stats.diagnostic.het_arch`.

   (c) Compute the historical volatility and plot it, by using a rolling object.

   (d) Compute the EWMA volatility and plot it, by using a ewm object. Also, try and recreate the plot with the EWMA volatilities for different values of λ shown in the slides.

   (e) Find a suitable (G)ARCH or TARCH model. Start with a GARCH(1, 1) or an ARCH(6) model, and determine whether it needs to be adjusted. You may need to install the `arch` package (docs). This can be done by putting `!pip install arch` into a Jupyter cell and executing it.

   (f) Make a plot of the volatility estimates that your model generates. Then, compute the unconditional (or average) volatility, and plot the NIC.

   (g) Forecast the volatility for $T + 1$.

2. (a) Show that
$$\widehat{\sigma}^2_{t+1,EWMA} = \lambda\widehat{\sigma}^2_{t,EWMA} + (1-\lambda)r_t^2, \qquad 0 < \lambda < 1.$$