Lucerne University of Applied Sciences

Time Series Analysis

Final Exam

Last name: Answer Key

Given name:

mscbf_ rm01_ tsa, Fall term

Time Series Analysis
Fall term

Final Exam

Last name: Answer Key
mscbf_ rm01_ tsa

Question 0

1. Hi

0 pts

Question 1 (27 points)

1. I have simulated 1000 observations $\{y_t\}$ from an ARMA(p, q) model. Figure 2 on Page 8 shows a plot of the correlogram. Based on it, what do you think p and q are, and why? | 6 pts

Solution: The PACF becomes insignificant after 2 lags, whereas the ACF decays exponentially. This is indicative of an AR(2) process.

2. The 11th sample autocorrelation (not shown in the graph) is $\hat{\tau}_{11} = -0.045$. Test if $\hat{\tau}_{11}$ is significantly different from zero. | 6 pts

Solution:

- The hypotheses are $H_0 : \tau_{11} = 0$ vs. $H_a : \tau_{11} \neq 0$.
- The test statistic is $\hat{\tau}_{11}$.
- Its distribution is $\mathrm{N}(0, 1/T)$ under H_0.
- The critical value is thus $1.96/\sqrt{1000} = 0.062$.
- The observed test statistic is $\hat{\tau}_{11} = -0.045$.
- Since $|-0.045| < 0.062$, we do not reject H_0.
- Conclusion: τ_{11} is not significantly different from zero.

3. The output in Figure 3 on Page 8 shows the results of regressing Δy_t (`y`) on y_{t-1} (`x1`) and Δy_{t-1} (`x2`). Use it to test if the data are integrated. | 6 pts

Solution:

- The hypotheses are $H_0 : y_t$ is integrated of order 1 vs. $H_a : y_t$ is stationary.
- The test statistic is the estimated coefficient on the lagged level, y_{t-1} (`x1`).
- Its follows a Dickey-Fuller distribution under H_0.
- The critical value is -2.86, because a constant has been included.
- The observed test statistic is -6.52.
- Since $-6.52 < -2.86$, we reject H_0.
- Conclusion: the data are stationary.

4. I have estimated a particular ARMA model. The estimation output is shown in Figure 4 on Page 9. Write down the estimated model in equation form. | 3 pts

Solution: The estimated AR(2) model is

$$y_t = 0.2158 \cdot (1 - 0.5236 - 0.3595) + 0.5236 \cdot y_{t-1} + 0.3595 \cdot y_{t-2}.$$

Note `archmodel`'s peculiar interpretation of the intercept, `const`: in our notation, it corresponds to $\hat{\alpha}/(1-\hat{\phi}_1-\hat{\phi}_2)$, so that $\hat{\alpha} = \texttt{const}\cdot(1-\hat{\phi}_1-\hat{\phi}_2)$. See the exercises.

5. Use the estimated model from the previous question to forecast the value of the series at $t = 1001$. You may need some of the values below. 6 pts

t	y_t	u_t
999	-0.77	-4.18
1000	-0.75	-1.41

Solution: The forecast is

$$\begin{aligned}\widehat{y}_{1001} &= \texttt{const} \cdot (1 - \hat{\phi}_1 - \hat{\phi}_2) + \hat{\phi}_1 \cdot y_{1000} + \hat{\phi}_2 \cdot y_{999} \\ &= 0.2158 \cdot (1 - 0.5236 - 0.3595) + 0.5236 \cdot (-0.75) + 0.3595 \cdot (-0.77) \\ &= -0.644.\end{aligned}$$

Question 2 (27 points) In this exercise, we analyze the daily returns on Tesla stock between 6/29/2010 and 12/14/2022. The returns are shown in Figure 1.

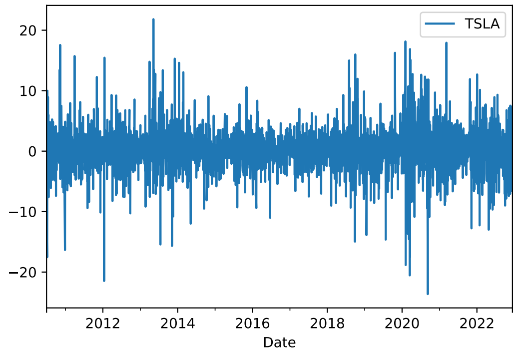

Figure 1: Returns on Tesla stock

The data clearly display volatility clustering, which we want to model using a GARCH model. The output is shown in Figure 5 on Page 9.

1. Write the model down as an equation (only the volatility equation). 3 pts

Solution:

$$\sigma_{t+1}^2 = 0.14 + 0.0323 \cdot u_t^2 + 0.9564 \cdot \sigma_t^2.$$

2. Does your model incorporate a leverage effect? Justify your answer. 6 pts

Solution: No. Since u_t (the demeaned return) only enters as its square, the news impact curve is symmetric: volatility reacts equally strongly to a positive and negative shock of the same magnitude.

3. Explain what the standardized residuals from a GARCH model are, ideally with an equation. Also, explain what their properties should be if the volatility model is correct. 6 pts

Solution: The standardized residuals are defined as $\hat{z}_t = \frac{r_t - \hat{\mu}_t}{\hat{\sigma}_t}$. If the model for σ_t is correct, then they should no longer display volatility clustering. This means that there should be no autocorrelation in their squares.

4. Regressing the squared standardized residuals on an intercept and 5 of their own lags results in an R^2 of 0.00364. Use this to test if the GARCH model has successfully removed the volatility clustering. 6 pts

Solution: This is an ARCH-LM test.

- The hypotheses are H_0 : no remaining volatility clustering vs. H_a : there is remaining volatility clustering.
- The test statistic is $T \cdot R^2$ in the auxiliary regression.
- Its distribution is $\chi^2(5)$ under H_0.
- The critical value is thus 11.07.
- The observed test statistic is $3138 \cdot 0.00364 = 11.42$.

- Since $11.42 > 11.07$, we reject H_0 (although barely).
- Conclusion: Some volatility clustering remains.

5. Use the model to predict the variance σ_{t+1}^2 for 12/15/2022, using the following values on 12/14/2022: $\hat{u}_t = -2.72$ and $\sigma_t^2 = 15.198$. 6 pts

Solution:

$$\begin{aligned} \widehat{\sigma}_{t+1}^2 &= 0.14 + 0.0323 \cdot u_t^2 + 0.9564 \cdot \sigma_t^2 \\ &= 0.14 + 0.0323 \cdot (-2.72)^2 + 0.9564 \cdot 15.198 \\ &= 14.91. \end{aligned}$$

Question 3 (18 points) We now turn our attention to Value at Risk forecasting.

1. Use the model from the previous question to predict the 1% Value at Risk for 12/15/2022. Note: If you weren't successful in predicting the variance in the previous question, you can use the value $\widehat{\sigma}^2_{t+1} = 15.00$. 6 pts

Solution: The VaR forecast is

$$\begin{aligned} VaR^{0.01}_{t+1} &= -\mu_{t+1} - \sigma_{t+1} \cdot \Phi^{-1}_{0.01} \\ &= -\mu_{t+1} - \sigma_{t+1} \cdot \Phi^{-1}_{0.01} \\ &= -0.1118 - \sqrt{14.91} \cdot (-2.326) \\ &= 8.87. \end{aligned}$$

2. In a backtesting exercise, we have created 1% VaR forecasts for the entire sample. From this and the returns, we have created the hit series $\{I_t\}$. 12 pts

(a) (6 pts) Explain how the hit series is defined, and how many times you expect it to equal 1 if the VaR model is correct.

Solution: The hit series equals 1 whenever a VaR violation occurs, i.e.,

$$I_{t+1} = \begin{cases} 1, & \text{if } R_{t+1} < -VaR^p_{t+1}, \\ 0, & \text{if } R_{t+1} > -VaR^p_{t+1}. \end{cases}$$

If the model is correct, we would expect a violation 1% of the time. Since $T = 3138$, we would therefore expect around 31 VaR violations.

(b) (6 pts) Figure 6 on page 10 shows the result of regressing $(I_t - 0.01)$ on an intercept and I_{t-1}. Use it to test the independence of the VaR violations.

Solution:

- The hypotheses are H_0 : the VaR violations are independent vs. H_a : they are not.
- The test statistic is the t statistic for the lagged hit series.
- Its distribution is (asymptotically) standard normal under H_0.
- The critical value is thus 1.96.
- The observed test statistic is 1.187.
- Since $|1.187| < 1.96$, we do not reject H_0.
- Conclusion: there is no evidence of dependence between the VaR violations.

Note: in this case, since I didn't delete the p-value, you could alternatively have answered as follows.

- The hypotheses are H_0 : the VaR violations are independent vs. H_a : they are not.
- The test statistic is the t statistic for the lagged hit series.
- The observed p-value is $0.235 > 0.05$, so we do not reject H_0.
- Conclusion: there is no evidence of dependence between the VaR violations.

Question 4 (18 points) Answer the questions below.

18 pts

1.

(a) (3 pts) Spurious regressions can occur between cointegrated variables.

☐ True ☑ False

(b) (3 pts) In a stationary time series, shocks U_t have a transitory effect on the future of the series.

☑ True ☐ False

(c) (3 pts) An ARCH(q) model for the returns corresponds to an AR(q) for the squared returns.

☑ True ☐ False

(d) (3 pts) The order q of an MA(q) model can be determined from the correlogram.

☑ True ☐ False

(e) (3 pts) In the presence of the leverage effect, the news impact curve is steeper to the right of the origin than to the left.

☐ True ☑ False

(f) (3 pts) A VaR model is correctly specified if no VaR violations occur.

☐ True ☑ False

End of exam.

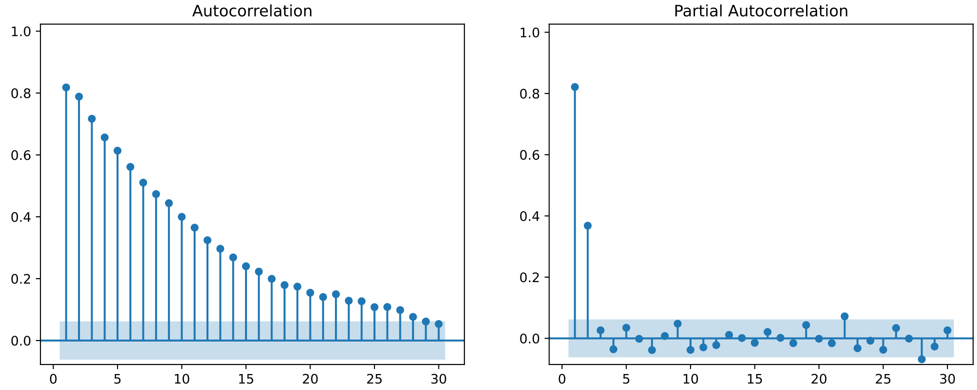

Figure 2: Correlogram of simulated data.

```
                            OLS Regression Results
==============================================================================
Dep. Variable:                      y   R-squared:                       0.208
Model:                            OLS   Adj. R-squared:                  0.207
Method:                 Least Squares   F-statistic:                     130.9
Date:                Mon, 16 Oct 2023   Prob (F-statistic):           3.47e-51
Time:                        15:04:00   Log-Likelihood:                -2086.9
No. Observations:                 998   AIC:                             4180.
Df Residuals:                     995   BIC:                             4195.
Df Model:                           2
Covariance Type:            nonrobust
==============================================================================
                 coef    std err          t      P>|t|      [0.025      0.975]
------------------------------------------------------------------------------
x1            -0.1161      0.018     -6.520      0.000      -0.151      -0.081
x2            -0.3594      0.030    -12.164      0.000      -0.417      -0.301
const          0.0309      0.062      0.497      0.619      -0.091       0.153
==============================================================================
Omnibus:                        0.529   Durbin-Watson:                   2.023
Prob(Omnibus):                  0.768   Jarque-Bera (JB):                0.404
Skew:                           0.003   Prob(JB):                        0.817
Kurtosis:                       3.098   Cond. No.                         3.76
==============================================================================
```

Figure 3: Output for ADF test.


```
                               SARIMAX Results
==============================================================================
Dep. Variable:                      y   No. Observations:                 1000
Model:                 ARIMA(2, 0, 0)   Log Likelihood               -2091.958
Date:                Mon, 16 Oct 2023   AIC                           4191.917
Time:                        15:10:43   BIC                           4211.548
Sample:                             0   HQIC                          4199.378
                               - 1000
Covariance Type:                  opg
==============================================================================
                 coef    std err          z      P>|z|      [0.025      0.975]
------------------------------------------------------------------------------
const          0.2158      0.528      0.409      0.683      -0.818       1.250
ar.L1          0.5236      0.032     16.396      0.000       0.461       0.586
ar.L2          0.3595      0.032     11.326      0.000       0.297       0.422
sigma2         3.8369      0.169     22.770      0.000       3.507       4.167
===================================================================================
Ljung-Box (L1) (Q):                   0.13   Jarque-Bera (JB):                 0.36
Prob(Q):                              0.71   Prob(JB):                         0.83
Heteroskedasticity (H):               0.86   Skew:                             0.00
Prob(H) (two-sided):                  0.18   Kurtosis:                         3.09
===================================================================================
```

Figure 4: Estimated ARMA model.

```
                     Constant Mean - GARCH Model Results
==============================================================================
Dep. Variable:                   TSLA   R-squared:                       0.000
Mean Model:             Constant Mean   Adj. R-squared:                  0.000
Vol Model:                      GARCH   Log-Likelihood:               -8267.70
Distribution:                  Normal   AIC:                           16543.4
Method:            Maximum Likelihood   BIC:                           16567.6
                                        No. Observations:                 3138
Date:                Mon, Oct 16 2023   Df Residuals:                     3137
Time:                        15:33:03   Df Model:                            1
                                 Mean Model
============================================================================
                 coef    std err          t      P>|t|     95.0% Conf. Int.
----------------------------------------------------------------------------
mu             0.1118  5.774e-02      1.936  5.281e-02 [-1.359e-03,  0.225]
                              Volatility Model
=============================================================================
                 coef    std err          t      P>|t|      95.0% Conf. Int.
-----------------------------------------------------------------------------
omega          0.1400      0.117      1.196      0.232  [-8.938e-02,  0.369]
alpha[1]       0.0323  1.341e-02      2.409  1.599e-02 [6.024e-03,5.859e-02]
beta[1]        0.9564  2.205e-02     43.369      0.000      [  0.913,  1.000]
=============================================================================
```

Figure 5: Estimated GARCH model.


```
                            OLS Regression Results
===============================================================================
Dep. Variable:     np.subtract(I, 0.01)   R-squared:                      0.000
Model:                              OLS   Adj. R-squared:                 0.000
Method:                   Least Squares   F-statistic:                    1.409
Date:                  Mon, 16 Oct 2023   Prob (F-statistic):             0.235
Time:                          18:54:56   Log-Likelihood:                1976.8
No. Observations:                  3137   AIC:                           -3950.
Df Residuals:                      3135   BIC:                           -3938.
Df Model:                             1
Covariance Type:              nonrobust
=============================================================================
                 coef    std err          t      P>|t|      [0.025      0.975]
-----------------------------------------------------------------------------
b0             0.0065      0.002      2.817      0.005       0.002       0.011
I.shift(1)     0.0212      0.018      1.187      0.235      -0.014       0.056
=============================================================================
Omnibus:                     4069.691   Durbin-Watson:                   2.001
Prob(Omnibus):                  0.000   Jarque-Bera (JB):           412759.104
Skew:                           7.492   Prob(JB):                         0.00
Kurtosis:                      57.160   Cond. No.                         7.76
=============================================================================
```

Figure 6: Test regression for the Value at Risk backtest.