

# 1 Strategies for Generating Simulated Noise in Directed Network Data

## 1.1 Assorted Strategies

### 1.1.1 Out-Degree Preserving Perturbation

**Overview:** For  $p \in [0, 1]$ , randomly select  $100p\%$  of observed edges in the network. For each selected edge pair, re-assign the "receiver" node to some other random node provided that:

- the new receiver node is not already an existing receiver of an edge from the origin node.
- the new receiver node is not the origin node.

**Properties:**

- Probability of switching an a given directed edge to a non-edge:  $p$
- Probability of switching a non-connected edge to a directed one:

$$(1 - (1 - p)^{O_i}) \frac{1}{N - O_i}$$

where:

- For each non-existing edge pair,  $(i, j)$ ,  $i$  is the sender,  $j$  is the receiver.
- Assume the sender has an outdegree of  $O_i$ .
  - \* Probability that  $i$  was among the selected edges to be switched is  $(1 - (1 - p)^{O_i})$ .
- Probability of  $E_{ij} = 1$  in the perturbed graph where  $j$  was not among the  $O_i$  original receivers.  $j$  was selected by chance  $\frac{1}{N - O_i}$ .