

Template of Report for PDM course

Group Names

Abstract—Keep short - the whole paper shall not exceed four pages plus references (Strict).

III. WORKING OF QUADROTORS

I. INTRODUCTION

Objective: Navigation of a quadcopter in a dynamic environment with multiple obstacles moving in a random motion. Obstacles are represented by cylinders which may represent people moving around in an indoor environment.

Environment: We will use PyBulletDrone environment for this project

Assumptions: The instantaneous state of the system, including instantaneous positions and sizes of all obstacles, is available to the motion planning algorithm.

Procedure:

- 1) Start with a vanilla RRT^* algorithm that generates the trajectory in a static environment.
- 2) Sampling is done from a 12D C-space (Position, Velocity) taking the kinodynamics of the rotor into account.
- 3) Then make the quadcopter follow this path using an off the shelf control algorithm.
- 4) Implement different flavours of the RRT algorithm for dynamic environments.
- 5) Compare these algorithm on the basis of their compute time and the path length.

Questions: Should we search for the optimal path in the 3D World space or the 6D C-Space of the robot or 12D Position + Velocity space?

II. ROBOT MODEL

We have decided to use Quadrotor as our robot. Quadrotor is a type of unmanned aerial vehicle (UAV) or drone that is lifted and propelled by four rotors. Each rotor is mounted at the end of a horizontal arm extending from the central body of the vehicle.

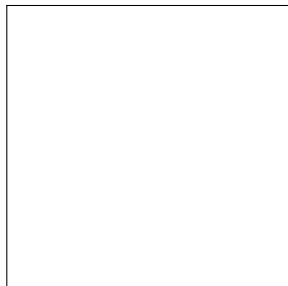


Fig. 1: Quadrotor

The quadrotor is a highly non-linear, six degree-of-freedom and under-actuated system. It is under-actuated because inputs are only 4 from 4 motors and total degree-of-freedom is 6. It is controlled by varying the thrust forces of each rotor and balancing the drag torque. A quadrotor has two sets of counter-rotating propellers, therefore neutralizing the effective aerodynamic drag. It has four principal modes of operation(Figure 2):

- 1) Vertical movement is controlled by simultaneously increasing or decreasing the thrust of all rotors.
- 2) Yaw moment is created by proportionally varying the speeds of counter-rotating parts to have movement with respect to quadrotor's z-axis.
- 3) Roll can be controlled by applying differential thrust forces on opposite rotors of the quadrotor to have movement with respect to quadrotor's x-axis.
- 4) Pitch can be controlled by applying differential thrust forces on opposite rotors of the quadrotor to have movement with respect to quadrotor's y-axis.

IV. QUADROTOR MODEL

A. Model of a rotor

Each rotor rotates with angular velocity ω and generates a lift force F and moment M . Moment is acting opposite to the directing of rotation. (Figure 3)

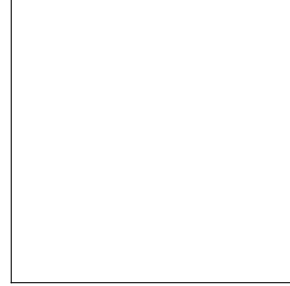


Fig. 3: Quadrotor arm

The lift Force F and moment M of i th rotor can be calculated by:

$$\begin{aligned} F_i &= k_f * \omega_i^2 & \text{and} & & k_f &= \\ k_T * \rho * D^4 & & & & & \\ M_i &= k_m * \omega_i^2 & \text{and} & & k_m &= \\ k_Q * \rho * D^5 & & & & & \end{aligned}$$

where:

k_T is thrust coefficient

k_Q is torque

ρ is fluid density

D is diameter of propeller

B. Equations of Motion

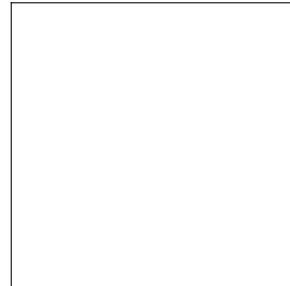


Fig. 4: Quadrotor in inertial and body frames

Total thrust and moment is the sum of individual ones in each of the 4 rotors.

$$\text{Thrust: } F = F_1 + F_2 + F_3 + F_4 - m g a_3$$

Here, F_x are individual lift forces by the propellers and $m * g$ is the one by gravity.

$$\text{Moment: } M = r_1 * F_1 + r_2 * F_2 + r_3 * F_3 + r_4 * F_4 + M_1 + M_2 + M_3 + M_4$$

Here, $r_x * F_x$ are the moments created by forces in quadrotor's centre of gravity and M_x are the

individual moments created by the propellers.

C. State Variables

u, ϕ, p are the linear velocity along the roll-axis direction, angle rotated and the angular velocity about the roll-axis. w, ψ, r is the linear velocity along the yaw-axis direction, angle rotated and the angular velocity about the yaw-axis. v, θ, q is the linear velocity along the pitch-axis direction, angle rotated and the angular velocity about the pitch-axis. x, y, z are the position of quadrotor along a_1, a_2, a_3 axis of the inertial frame.

D. Newton-Euler Equations for Quadrotor

Linear Dynamics Applying Newton's Second Law for system of particles, we get (in inertial frame);

$$F = mass * acceleration$$

$$acceleration(\ddot{r}) = d\dot{r}/dt, \text{ where } \dot{r} = [u, v, w]^T \quad (3.3)$$

Since, w is the yaw-axis in which we calculate thrust, we get;

$$mass * \ddot{r} = \begin{bmatrix} 0 \\ 0 \\ -m * g \end{bmatrix} + R_{\psi} \phi \theta \begin{bmatrix} 0 \\ 0 \\ F_1 + F_2 + F_3 + F_4 \end{bmatrix}$$

Rotational Dynamics Applying Euler's rotation equations, we get (in body frame);

$$M_c = {}^A dH_c^B / dt = {}^B dH_c^B / dt + {}^A \omega^B \times H_c^B$$

where, H_c is the angular momentum and ${}^A \omega^B$ is angular velocity of body B in frame A which is given by $p.b_1 + q.b_2 + r.b_3$

General vector form of Euler's equation is; $M_c = I\dot{\omega} + \omega \times (I\omega)$

For Quadrotor, after rearranging the general vector form;

$$I \begin{bmatrix} \dot{p} \\ \dot{q} \\ \dot{r} \end{bmatrix} = \begin{bmatrix} L(F_2 - F_4) \\ L(F_3 - F_1) \\ M_1 - M_2 + M_3 - M_4 \end{bmatrix} - \begin{bmatrix} p \\ q \\ r \end{bmatrix} \times I \begin{bmatrix} p \\ q \\ r \end{bmatrix}$$

Let $\gamma = k_M/k_F$, $M_i = \gamma F_i$, we get;

$$I \begin{bmatrix} \dot{p} \\ \dot{q} \\ \dot{r} \end{bmatrix} = \begin{bmatrix} 0 & L & 0 & -L \\ -L & 0 & L & 0 \\ \gamma & -\gamma & \gamma & -\gamma \end{bmatrix} \begin{bmatrix} F_1 \\ F_2 \\ F_3 \\ F_4 \end{bmatrix} - \begin{bmatrix} p \\ q \\ r \end{bmatrix} \times I \begin{bmatrix} p \\ q \\ r \end{bmatrix}$$

E. Final Joint Equation

Joint Equation using Linear and Rotational dynamics equations, we get;

$$\mathbf{u} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & L & 0 & -L \\ -L & 0 & L & 0 \\ \gamma & -\gamma & \gamma & -\gamma \end{bmatrix} \begin{bmatrix} F_1 \\ F_2 \\ F_3 \\ F_4 \end{bmatrix}$$

where \mathbf{u} is;

$$\begin{bmatrix} thrust \\ moment \text{ about } x \\ moment \text{ about } y \\ moment \text{ about } z \end{bmatrix}$$

V. CONTROL INPUTS

Control inputs are in the form of motor speeds for controlling thrust and 3 moments.

$$\begin{bmatrix} T \\ \tau_1 \\ \tau_2 \\ \tau_3 \end{bmatrix} = \begin{bmatrix} k_F & k_F & k_F & k_F \\ 0 & Lk_F & 0 & -Lk_F \\ -Lk_F & 0 & Lk_F & 0 \\ k_M & -k_M & k_M & -k_M \end{bmatrix} \begin{bmatrix} \omega_1^2 \\ \omega_2^2 \\ \omega_3^2 \\ \omega_4^2 \end{bmatrix}$$