

1) $A(0,0)$,
 $B(u,0)$
 $C(u,v)$
 $D(0,v)$

$u, v > 0$

$(u,v) \in f(x) = -x^3 + 8$

$S = uv \rightarrow \max$

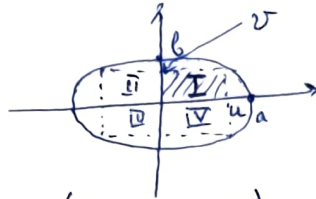
$v = -u^3 + 8$

$\Rightarrow S(u) = uv = u(-u^3 + 8)$

$S'(u) = -4(u^3 - 2) = 0 \Rightarrow \begin{cases} u = 2^{\frac{1}{3}} \\ v = -2 + 8 = 6 \end{cases}$

$S_{\max} = 6\sqrt[3]{2}$

2) $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$



Рассмотрим I четверть ($S = 4 \cdot uv$), $u, v > 0$

$\frac{u^2}{a^2} + \frac{v^2}{b^2} = 1 \Rightarrow v = \frac{b}{a} \sqrt{a^2 - u^2} \Rightarrow S(u) = 4u \cdot \frac{b}{a} \sqrt{a^2 - u^2}$

$S'(u) = \frac{4b}{a} \left(\sqrt{a^2 - u^2} + u \frac{-2u}{2\sqrt{a^2 - u^2}} \right) = \frac{4b}{a} \cdot \frac{a^2 - 2u^2}{\sqrt{a^2 - u^2}}$

$\Rightarrow u = \frac{a}{\sqrt{2}} \quad v = \frac{b}{\sqrt{2}} \Rightarrow S_{\max} = 4uv = 2ab$