① $\mathbb{E}x = 300$

$P(x > A) \le \frac{\mathbb{E}x}{A}$ – нер-во Маркова

а) $P(x < 400) \le \frac{300}{400} = 0,75$

б) $P(x \le 500) = 1 - P(x < 500) = 1 - \frac{300}{500} = 0,4$

② $n = 1600 \quad p = 0,3, \quad \varepsilon = 50$

$\xi \sim$ Бернулли

$\mathbb{E}\xi = np = 1600 \cdot 0,3 = 480$

$D\xi = np(1-p) = 336$

$P(|\xi - \mathbb{E}\xi| < \varepsilon) \ge 1 - \frac{D\xi}{\varepsilon^2}$ – нер-во Чебышева

$P(|\xi - 480| < 50) \ge 1 - \frac{336}{50^2} = \underline{0,866}$

③ $Dx = 1 \qquad x \in \{9,5,7,7,4,10\} \qquad \underline{\alpha = 0,01}$

$\bar{x} = \frac{\sum_{i=1}^{K} x_i}{K} = \frac{9+5+7+7+4+10}{6} = \frac{42}{6} = 7$

$1 - \frac{\alpha}{2} = 0,995 \qquad z_\alpha = 2,58 \qquad \Delta = \frac{\sigma}{\sqrt{n}} z_\alpha = \frac{2,58}{\sqrt{6}} = 1,053$

99% Довер. инт: $(\bar{x} - \Delta, x + \Delta) = \underline{(5,947; 8,053)}$

④ $X_i \sim N(\mu, \sigma^2)$ ОМП $\hat{\mu}$ и $\hat{\sigma}$

$f_{(\mu,\sigma^2)}(x) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(\frac{-(y-\mu)^2}{2\sigma^2}\right)$

$f(x, \mu, \sigma^2) = \prod_{i=1}^{n} \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(-\frac{(x_i-\mu)^2}{2\sigma^2}\right) = \frac{1}{\left(\sqrt{2\pi\sigma^2}\right)^n} \exp\left(-\frac{\sum (x_i-\mu)^2}{2\sigma^2}\right)$

$L(x, \mu, \sigma^2) = -\ln(2\pi)^{\frac{n}{2}} - \frac{n}{2}\ln\sigma^2 - \frac{\sum (x_i-\mu)^2}{2\sigma^2}$

$\frac{\partial}{\partial\mu} L = \frac{2\sum_{i=1}^{n}(x_i-\mu)}{2\sigma^2} = \frac{n(\bar{x}-\mu)}{\sigma^2} = 0$

$\frac{\partial}{\partial\mu} L = -\frac{n}{2\sigma^2} + \frac{\sum (x_i-\mu)^2}{2\sigma^4} = 0$

$\Rightarrow \hat{\mu} = \bar{x}, \quad \hat{\sigma}^2 = \frac{1}{n}\sum_{i=1}^{n}(x_i - \bar{x})^2$