

# Encryption

CS6025 Data Encoding

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2-24-15

PENGUIN CLASSICS



EDGAR ALLAN POE

THE FALL OF THE  
HOUSE OF USHER  
AND OTHER WRITINGS



# Edgar A. Poe's "The Gold-Bug" (1843)

53†††305))6\*;4826)4†.)4†);806\*;48†8¶(60))85;]8\*:†\*8†83(88)5\*†;  
46(;88\*96\*?;8)\*†(;485);5\*†2:\*†(;4956\*2(5\*—4)8¶8\*;4069285);)6†8)  
4††;I(†9;4808I;8:8†I;48†85;4)485†528806\*8I(†9;48;(88;4(†?34;48)  
4†;I6I;;I88;†?;

“A good glass in the bishop's hostel in the devil's seat twenty-one degrees and thirteen minutes northeast and by north main branch seventh limb east side shoot from the left eye of the death's-head a bee-line from the tree through the shot fifty feet out.”

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The Adventures of Sherlock Holmes:  
Season 1, Episode 2  
**The Dancing Men** (1 May 1984)

TV Episode - 52 min - Crime | Drama | Mystery

**Your rating:** ★★★★★★★★ -/10  
**8.2** Ratings: 8.2/10 from 411 users  
Reviews: 8 user

A gentleman is baffled when the childish drawings of little dancing men terrify his American wife. Sherlock Holmes soon discovers why.

**Director:** John Bruce

**Writers:** Arthur Conan Doyle (by) (as Sir Arthur Conan Doyle) , John Hawkesworth (developed for television by), 2 more credits »

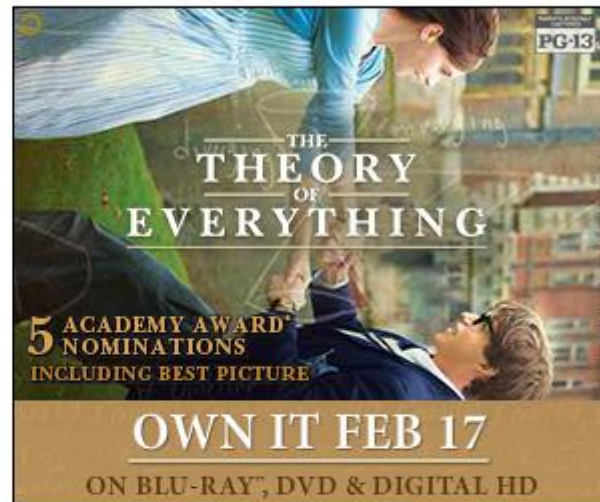
**Stars:** Jeremy Brett, David Burke, Tenniel Evans |  
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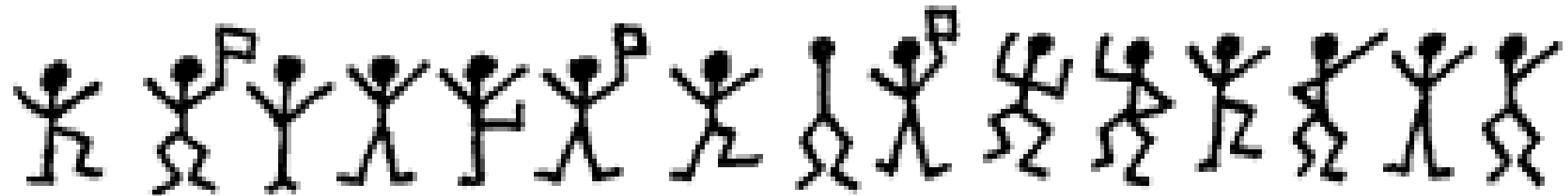
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# The Adventure of the Dancing Men (1898)



# Substitute Bytes (SubBytes) in AES

Table 5.2 AES S-Boxes

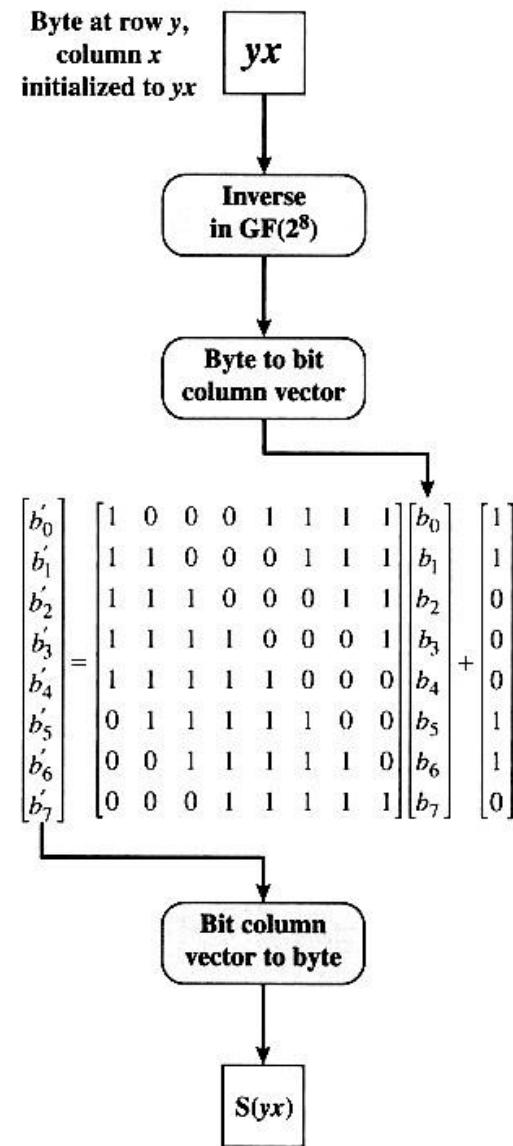
|   |   | y  |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |
|---|---|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|
|   |   | 0  | 1  | 2  | 3  | 4  | 5  | 6  | 7  | 8  | 9  | A  | B  | C  | D  | E  | F  |
| x | 0 | 63 | 7C | 77 | 7B | F2 | 6B | 6F | C5 | 30 | 01 | 67 | 2B | FE | D7 | AB | 76 |
|   | 1 | CA | 82 | C9 | 7D | FA | 59 | 47 | F0 | AD | D4 | A2 | AF | 9C | A4 | 72 | C0 |
|   | 2 | B7 | FD | 93 | 26 | 36 | 3F | F7 | CC | 34 | A5 | E5 | F1 | 71 | D8 | 31 | 15 |
|   | 3 | 04 | C7 | 23 | C3 | 18 | 96 | 05 | 9A | 07 | 12 | 80 | E2 | EB | 27 | B2 | 75 |
|   | 4 | 09 | 83 | 2C | 1A | 1B | 6E | 5A | A0 | 52 | 3B | D6 | B3 | 29 | E3 | 2F | 84 |
|   | 5 | 53 | D1 | 00 | ED | 20 | FC | B1 | 5B | 6A | CB | BE | 39 | 4A | 4C | 58 | CF |
|   | 6 | D0 | EF | AA | FB | 43 | 4D | 33 | 85 | 45 | F9 | 02 | 7F | 50 | 3C | 9F | A8 |
|   | 7 | 51 | A3 | 40 | 8F | 92 | 9D | 38 | F5 | BC | B6 | DA | 21 | 10 | FF | F3 | D2 |
|   | 8 | CD | 0C | 13 | EC | 5F | 97 | 44 | 17 | C4 | A7 | 7E | 3D | 64 | 5D | 19 | 73 |
|   | 9 | 60 | 81 | 4F | DC | 22 | 2A | 90 | 88 | 46 | EE | B8 | 14 | DE | 5E | 0B | DB |
|   | A | E0 | 32 | 3A | 0A | 49 | 06 | 24 | 5C | C2 | D3 | AC | 62 | 91 | 95 | E4 | 79 |
|   | B | E7 | C8 | 37 | 6D | 8D | D5 | 4E | A9 | 6C | 56 | F4 | EA | 65 | 7A | AE | 08 |
|   | C | BA | 78 | 25 | 2E | 1C | A6 | B4 | C6 | E8 | DD | 74 | 1F | 4B | BD | 8B | 8A |
|   | D | 70 | 3E | B5 | 66 | 48 | 03 | F6 | 0E | 61 | 35 | 57 | B9 | 86 | C1 | 1D | 9E |
|   | E | E1 | F8 | 98 | 11 | 69 | D9 | 8E | 94 | 9B | 1E | 87 | E9 | CE | 55 | 28 | DF |
|   | F | 8C | A1 | 89 | 0D | BF | E6 | 42 | 68 | 41 | 99 | 2D | 0F | B0 | 54 | BB | 16 |

(a) S-box

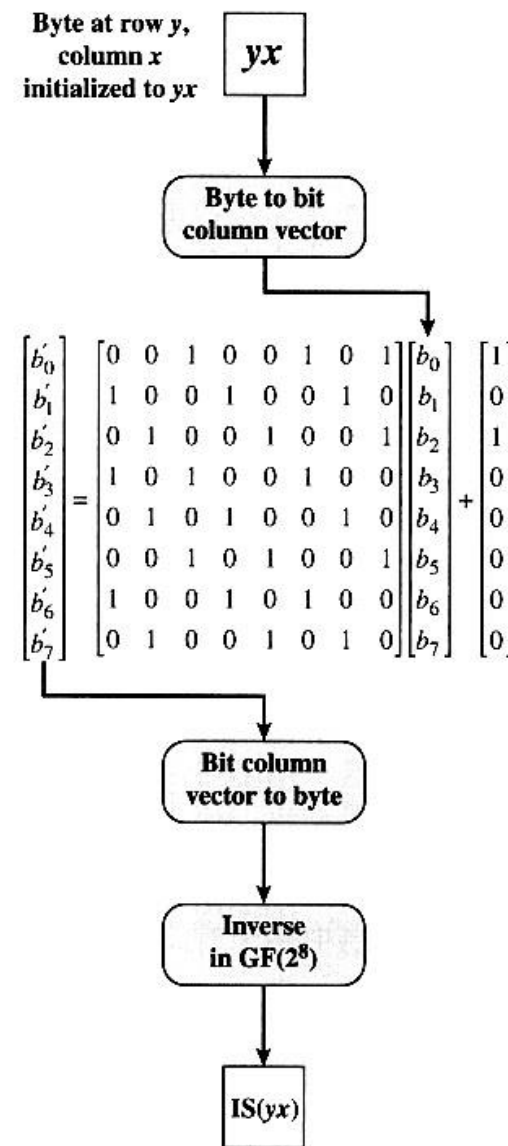
# Inverse S-Box for Decoder

|   |   | y  |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |
|---|---|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|
|   |   | 0  | 1  | 2  | 3  | 4  | 5  | 6  | 7  | 8  | 9  | A  | B  | C  | D  | E  | F  |
| x | 0 | 52 | 09 | 6A | D5 | 30 | 36 | A5 | 38 | BF | 40 | A3 | 9E | 81 | F3 | D7 | FB |
|   | 1 | 7C | E3 | 39 | 82 | 9B | 2F | FF | 87 | 34 | 8E | 43 | 44 | C4 | DE | E9 | CB |
|   | 2 | 54 | 7B | 94 | 32 | A6 | C2 | 23 | 3D | EE | 4C | 95 | 0B | 42 | FA | C3 | 4E |
|   | 3 | 08 | 2E | A1 | 66 | 28 | D9 | 24 | B2 | 76 | 5B | A2 | 49 | 6D | 8B | D1 | 25 |
|   | 4 | 72 | F8 | F6 | 64 | 86 | 68 | 98 | 16 | D4 | A4 | 5C | CC | 5D | 65 | B6 | 92 |
|   | 5 | 6C | 70 | 48 | 50 | FD | ED | B9 | DA | 5E | 15 | 46 | 57 | A7 | 8D | 9D | 84 |
|   | 6 | 90 | D8 | AB | 00 | 8C | BC | D3 | 0A | F7 | E4 | 58 | 05 | B8 | B3 | 45 | 06 |
|   | 7 | D0 | 2C | 1E | 8F | CA | 3F | 0F | 02 | C1 | AF | BD | 03 | 01 | 13 | 8A | 6B |
|   | 8 | 3A | 91 | 11 | 41 | 4F | 67 | DC | EA | 97 | F2 | CF | CE | F0 | B4 | E6 | 73 |
|   | 9 | 96 | AC | 74 | 22 | E7 | AD | 35 | 85 | E2 | F9 | 37 | E8 | 1C | 75 | DF | 6E |
|   | A | 47 | F1 | 1A | 71 | 1D | 29 | C5 | 89 | 6F | B7 | 62 | 0E | AA | 18 | BE | 1B |
|   | B | FC | 56 | 3E | 4B | C6 | D2 | 79 | 20 | 9A | DB | C0 | FE | 78 | CD | 5A | F4 |
|   | C | 1F | DD | A8 | 33 | 88 | 07 | C7 | 31 | B1 | 12 | 10 | 59 | 27 | 80 | EC | 5F |
|   | D | 60 | 51 | 7F | A9 | 19 | B5 | 4A | 0D | 2D | E5 | 7A | 9F | 93 | C9 | 9C | EF |
|   | E | A0 | E0 | 3B | 4D | AE | 2A | F5 | B0 | C8 | EB | BB | 3C | 83 | 53 | 99 | 61 |
|   | F | 17 | 2B | 04 | 7E | BA | 77 | D6 | 26 | E1 | 69 | 14 | 63 | 55 | 21 | 0C | 7D |

(b) Inverse S-box



(a) Calculation of byte at row  $y$ , column  $x$  of S-box



(a) Calculation of byte at row  $y$ , column  $x$  of IS-box

Figure 5.6 Constuction of S-Box and IS-Box



# Global Constants and Variables

```
static final int numberOfBits = 8;
static final int fieldSize = 1 << numberOfBits;
static final int irreducible = 0x11b;
static final int logBase = 3;
static final byte[][] A = new byte[][] {
    {1, 1, 1, 1, 1, 0, 0, 0},
    {0, 1, 1, 1, 1, 1, 0, 0},
    {0, 0, 1, 1, 1, 1, 1, 0},
    {0, 0, 0, 1, 1, 1, 1, 1},
    {1, 0, 0, 0, 1, 1, 1, 1},
    {1, 1, 0, 0, 0, 1, 1, 1},
    {1, 1, 1, 0, 0, 0, 1, 1},
    {1, 1, 1, 1, 0, 0, 0, 1}
};
static final byte[] B = new byte[] { 0, 1, 1, 0, 0, 0, 1, 1};
int[] alog = new int[fieldSize];
int[] log = new int[fieldSize];
int[] S = new int[fieldSize];
```

# Making the S-Box

```
void buildS(){
    int[] bitColumn = new int[8];
    for (int i = 0; i < fieldSize; i++){
        int inverse = i < 2 ? i : multiplicativeInverse(i);
        for (int k = 0; k < 8; k++){
            bitColumn[k] = inverse >> (7 - k) & 1;
        }
        S[i] = 0;
        for (int k = 0; k < 8; k++){
            int bit = B[k];
            for (int l = 0; l < 8; l++){
                if (bitColumn[l] == 1) bit ^= A[k][l];
            }
            S[i] ^= bit << 7 - k;
        }
    }
}
```

# Linear Code

- One step in building the S box is  $b' = Ab + B$ .  $b'$ ,  $b$ , and  $B$  are vectors in  $Z_2$  and  $A$  is a matrix in  $Z_2$ .
- The inverse is  $b = A'b' + B'$ . Substituting  $b' = Ab + B$ , we have  $b = A'Ab + A'B + B'$ . The solution is  $A'A = I$  and  $B' = A'B$ .

# Advanced Encryption Standard (AES)

- Adopted and published by NIST 12/4/2001
- Selected from 15 candidate ciphers
- Invented by Belgians Daemen and Rijmen
- Also named Rijindael
- Plaintext/ciphertext element: 16-byte blocks



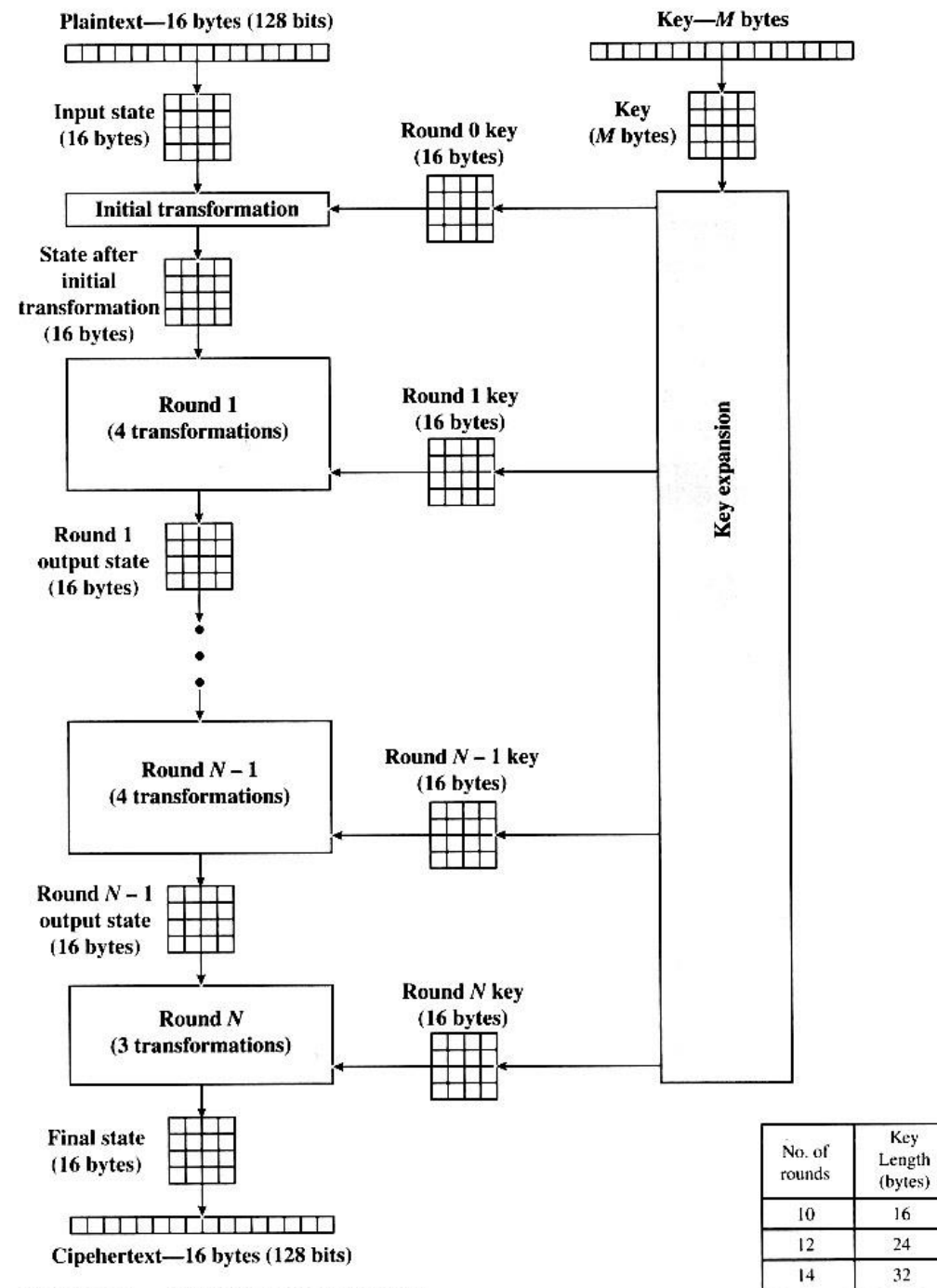


Figure 5.1 AES Encryption Process

# The 16-Byte Block and Key

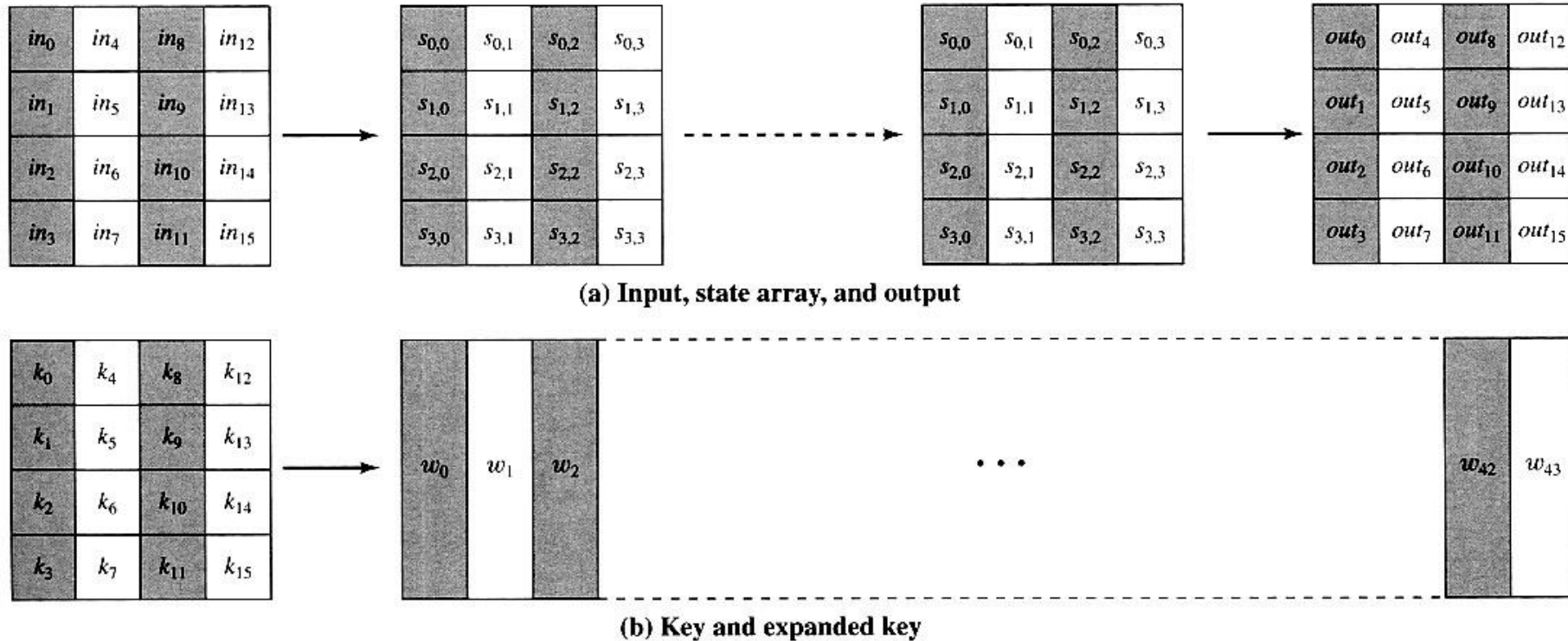


Figure 5.2 AES Data Structures

# Four Transformations and Iteration

- AddRoundKey (A)
- SubBytes (B)
- ShiftRow (S)
- MixColumn (M)
- ABSMAB SMA...BSMA BSA
- Initiation: A
- 10 rounds of BSMA
- Final round: BSA

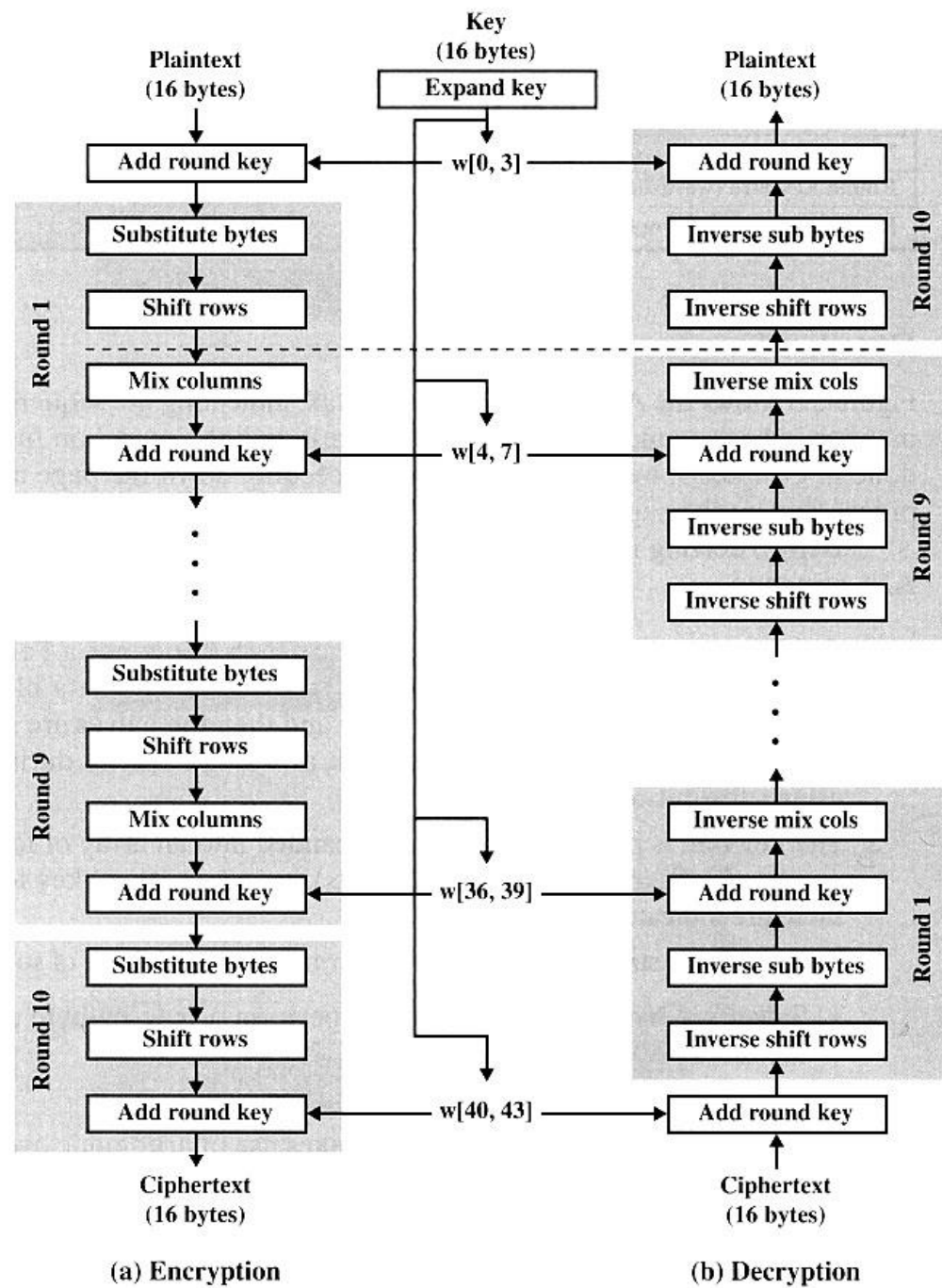


Figure 5.3 AES Encryption and Decryption



```

static final int blockSize = 16;
int[] state = new int[blockSize];

int readBlock(){
    byte[] data = new byte[blockSize];
    int len = 0;
    try {
        len = System.in.read(data);
    } catch (IOException e){
        System.err.println(e.getMessage());
        System.exit(1);
    }
    if (len <= 0) return len;
    for (int i = 0; i < len; i++){
        if (data[i] < 0) state[i] = data[i] + fieldSize;
        else state[i] = data[i];
    }
    for (int i = len; i < blockSize; i++) state[i] = 0;
    return len;
}

void subBytes(){
    for (int i = 0; i < blockSize; i++) state[i] = S[state[i]];
}

```

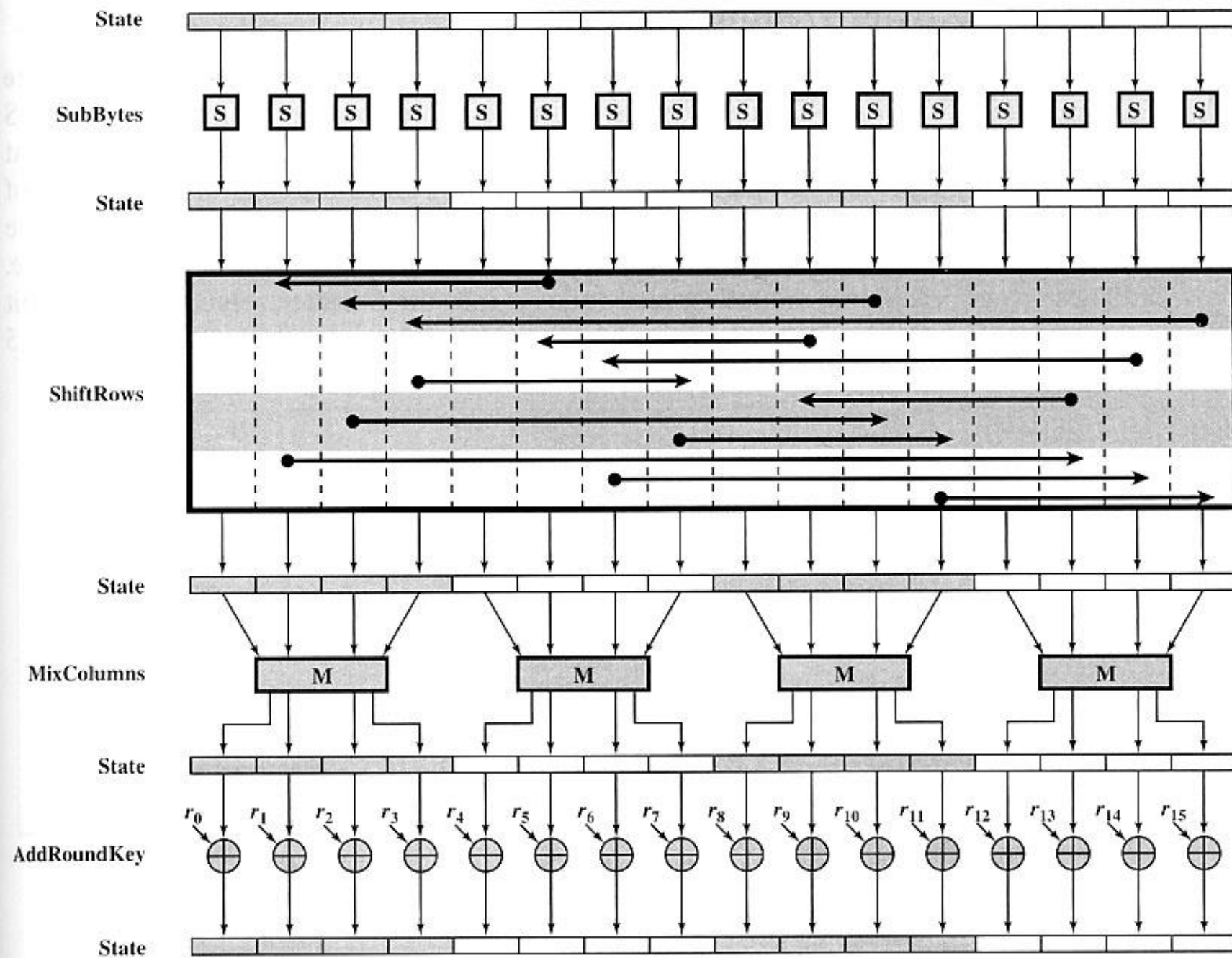


Figure 5.4 AES Encryption Round

# Example of SubBytes

|    |    |    |    |
|----|----|----|----|
| EA | 04 | 65 | 85 |
| 83 | 45 | 5D | 96 |
| 5C | 33 | 98 | B0 |
| F0 | 2D | AD | C5 |

→

|    |    |    |    |
|----|----|----|----|
| 87 | F2 | 4D | 97 |
| EC | 6E | 4C | 90 |
| 4A | C3 | 46 | E7 |
| 8C | D8 | 95 | A6 |

# Example of ShiftRows

|    |    |    |    |
|----|----|----|----|
| 87 | F2 | 4D | 97 |
| EC | 6E | 4C | 90 |
| 4A | C3 | 46 | E7 |
| 8C | D8 | 95 | A6 |

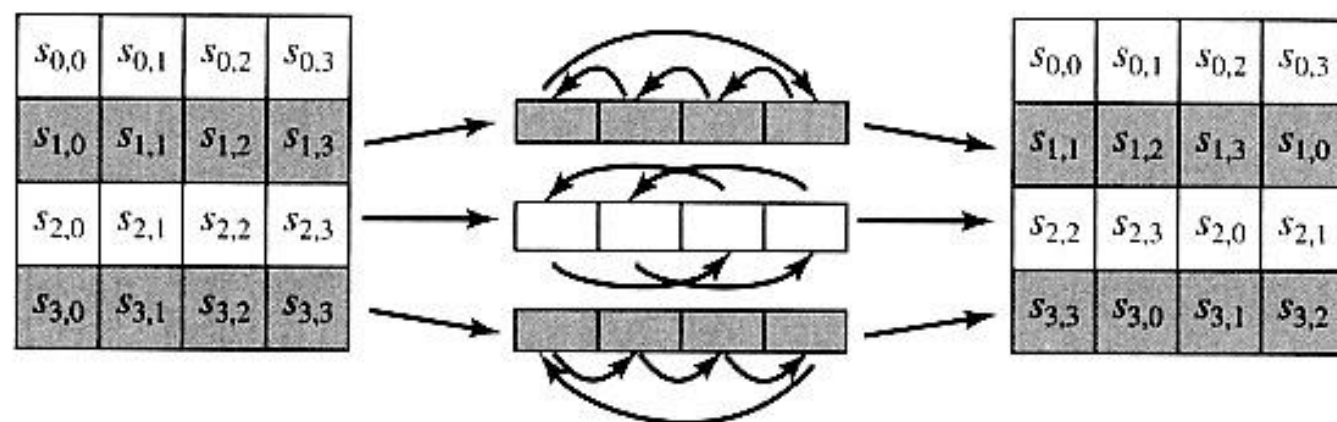
→

|    |    |    |    |
|----|----|----|----|
| 87 | F2 | 4D | 97 |
| 6E | 4C | 90 | EC |
| 46 | E7 | 4A | C3 |
| A6 | 8C | D8 | 95 |

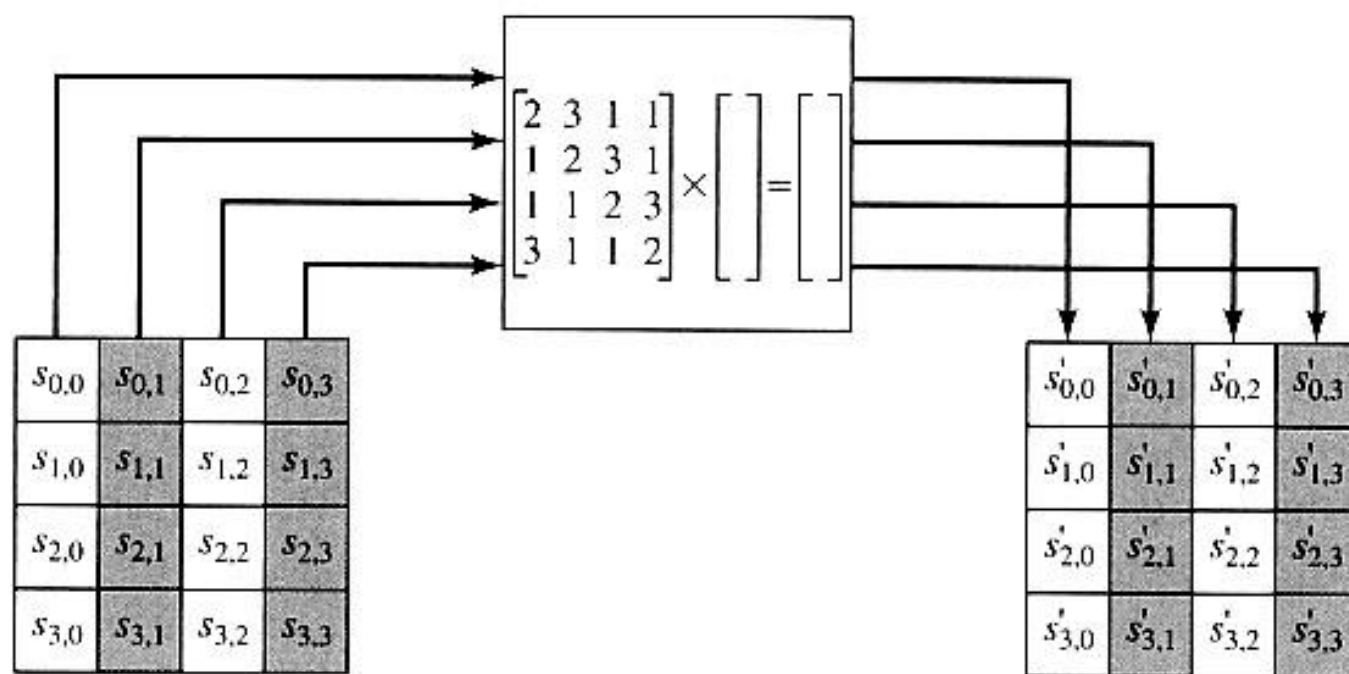


# Shift Rows

```
void shiftRows(){  
    int temp = state[2]; state[2] = state[10]; state[10] = temp;  
    temp = state[6]; state[6] = state[14]; state[14] = temp;  
    temp = state[1]; state[1] = state[5]; state[5] = state[9];  
    state[9] = state[13]; state[13] = temp;  
    temp = state[3]; state[3] = state[15]; state[15] = state[11];  
    state[11] = state[7]; state[7] = temp;  
}
```



(a) Shift row transformation



(b) Mix column transformation

Figure 5.7 AES Row and Column Operations

# MixColumns

$$\begin{bmatrix} 02 & 03 & 01 & 01 \\ 01 & 02 & 03 & 01 \\ 01 & 01 & 02 & 03 \\ 03 & 01 & 01 & 02 \end{bmatrix} \begin{bmatrix} s_{0,0} & s_{0,1} & s_{0,2} & s_{0,3} \\ s_{1,0} & s_{1,1} & s_{1,2} & s_{1,3} \\ s_{2,0} & s_{2,1} & s_{2,2} & s_{2,3} \\ s_{3,0} & s_{3,1} & s_{3,2} & s_{3,3} \end{bmatrix} = \begin{bmatrix} s'_{0,0} & s'_{0,1} & s'_{0,2} & s'_{0,3} \\ s'_{1,0} & s'_{1,1} & s'_{1,2} & s'_{1,3} \\ s'_{2,0} & s'_{2,1} & s'_{2,2} & s'_{2,3} \\ s'_{3,0} & s'_{3,1} & s'_{3,2} & s'_{3,3} \end{bmatrix}$$

# MixColumns

$$s'_{0,j} = (2 \cdot s_{0,j}) \oplus (3 \cdot s_{1,j}) \oplus s_{2,j} \oplus s_{3,j}$$

$$s'_{1,j} = s_{0,j} \oplus (2 \cdot s_{1,j}) \oplus (3 \cdot s_{2,j}) \oplus s_{3,j}$$

$$s'_{2,j} = s_{0,j} \oplus s_{1,j} \oplus (2 \cdot s_{2,j}) \oplus (3 \cdot s_{3,j})$$

$$s'_{3,j} = (3 \cdot s_{0,j}) \oplus s_{1,j} \oplus s_{2,j} \oplus (2 \cdot s_{3,j})$$

The following is an example of MixColumns:

|    |    |    |    |
|----|----|----|----|
| 87 | F2 | 4D | 97 |
| 6E | 4C | 90 | EC |
| 46 | E7 | 4A | C3 |
| A6 | 8C | D8 | 95 |

→

|    |    |    |    |
|----|----|----|----|
| 47 | 40 | A3 | 4C |
| 37 | D4 | 70 | 9F |
| 94 | E4 | 3A | 42 |
| ED | A5 | A6 | BC |



# mixColumns

```
static final byte[][] G = new byte[][] {
    {2, 1, 1, 3},
    {3, 2, 1, 1},
    {1, 3, 2, 1},
    {1, 1, 3, 2}
};

void mixColumns(){
    int[] temp = new int[4];
    for (int k = 0; k < 4; k++){
        for (int i = 0; i < 4; i++){
            temp[i] = 0;
            for (int j = 0; j < 4; j++){
                temp[i] ^= logMultiply(G[j][i], state[k * 4 + j]);
            }
            for (int i = 0; i < 4; i++) state[k * 4 + i] = temp[i];
        }
    }
}
```

# AddRoundKey (A)

- There are 11 roundKeys from expansion of the 128-bit crypt key
- Each roundKey is byte[16]
- AddRoundKey bitwise XORs the roundKey to the block as byte[16].
- AddRoundKey is its own inverse.

```
void addRoundKey(int round){  
    for (int k = 0; k < blockSize; k++)  
        state[k] ^= roundKey[round][k];  
}
```

# AddRoundKey

|    |    |    |    |
|----|----|----|----|
| 47 | 40 | A3 | 4C |
| 37 | D4 | 70 | 9F |
| 94 | E4 | 3A | 42 |
| ED | A5 | A6 | BC |

 $\oplus$ 

|    |    |    |    |
|----|----|----|----|
| AC | 19 | 28 | 57 |
| 77 | FA | D1 | 5C |
| 66 | DC | 29 | 00 |
| F3 | 21 | 41 | 6A |

 $=$ 

|    |    |    |    |
|----|----|----|----|
| EB | 59 | 8B | 1B |
| 40 | 2E | A1 | C3 |
| F2 | 38 | 13 | 42 |
| 1E | 84 | E7 | D6 |

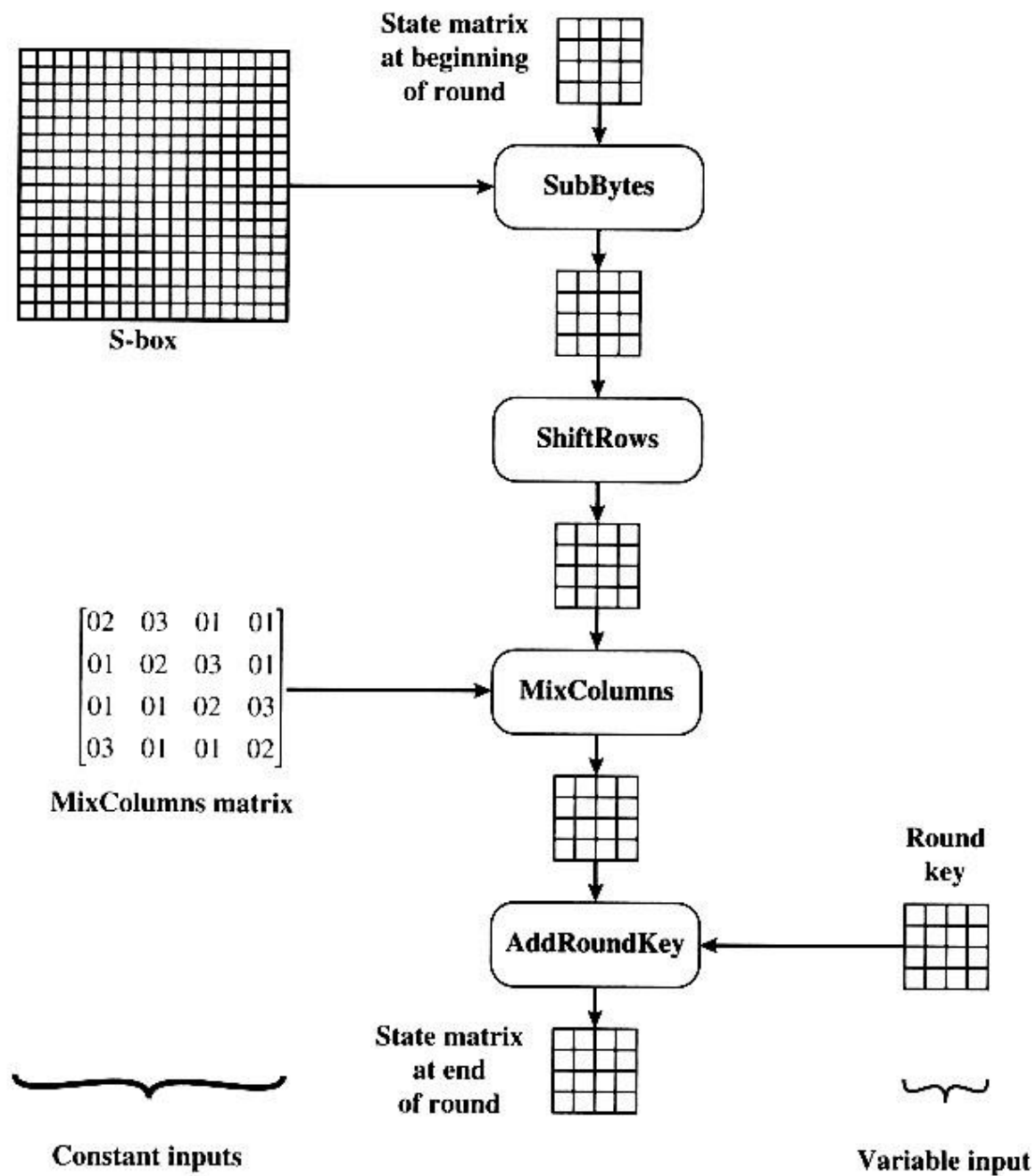
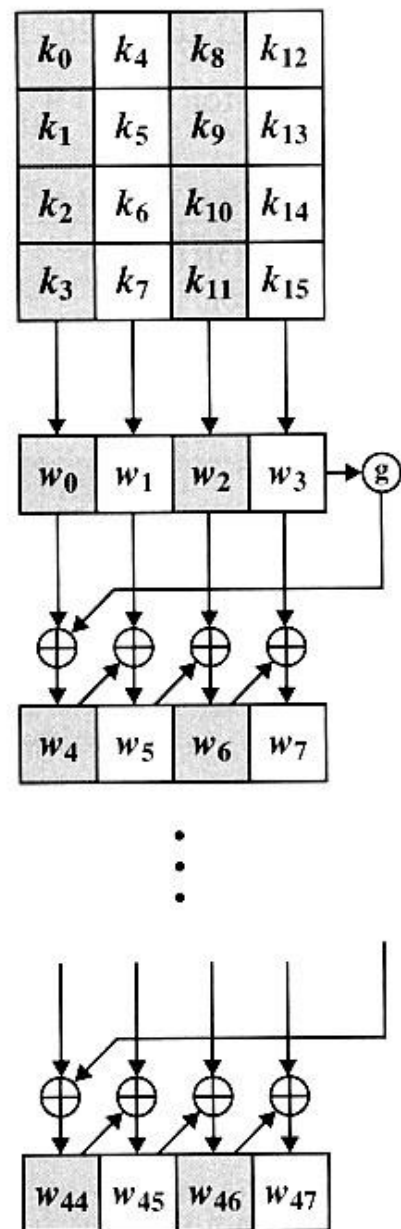


Figure 5.8 Inputs for Single AES Round

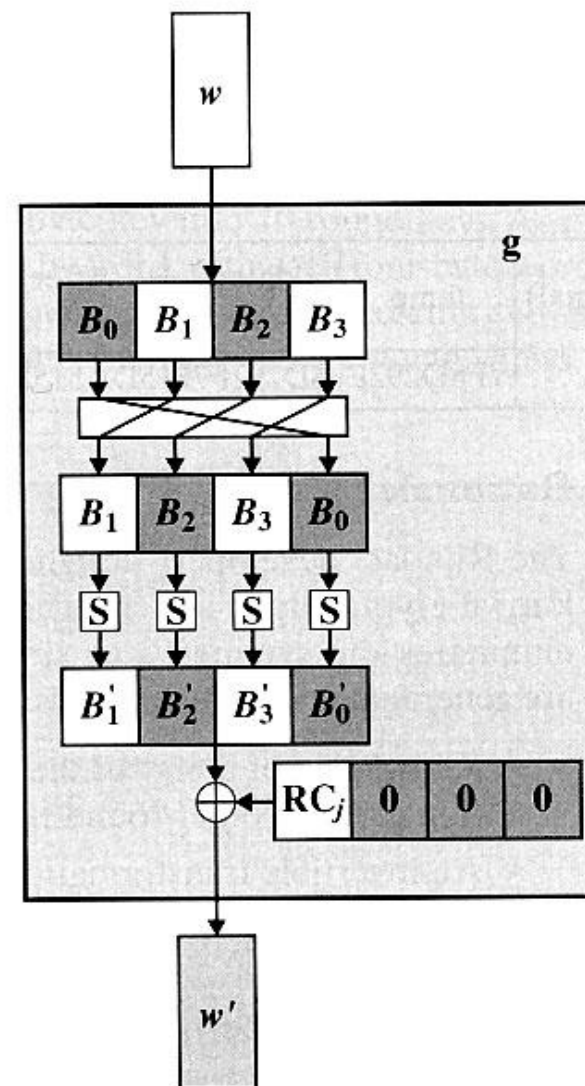
# readKey and stallingskey.txt

```
void readKey(String filename){
    Scanner in = null;
    try {
        in = new Scanner(new File(filename));
    } catch (FileNotFoundException e){
        System.err.println(filename + " not found");
        System.exit(1);
    }
    hexkey = in.nextLine();
    in.close();
}
```

0f1571c947d9e8590cb7add6af7f6798



(a) Overall algorithm



(b) Function  $g$

Figure 5.9 AES Key Expansion

# Key Expansion

1. RotWord performs a one-byte circular left shift on a word. This means that an input word  $[B_0, B_1, B_2, B_3]$  is transformed into  $[B_1, B_2, B_3, B_0]$ .
2. SubWord performs a byte substitution on each byte of its input word, using the S-box (Table 5.2a).
3. The result of steps 1 and 2 is XORed with a round constant,  $Rcon[j]$ .

The round constant is a word in which the three rightmost bytes are always 0. Thus, the effect of an XOR of a word with  $Rcon$  is to only perform an XOR on the leftmost byte of the word. The round constant is different for each round and is defined as  $Rcon[j] = (RC[j], 0, 0, 0)$ , with  $RC[1] = 1$ ,  $RC[j] = 2 \cdot RC[j-1]$  and with multiplication defined over the field  $GF(2^8)$ . The values of  $RC[j]$  in hexadecimal are

| j       | 1  | 2  | 3  | 4  | 5  | 6  | 7  | 8  | 9  | 10 |
|---------|----|----|----|----|----|----|----|----|----|----|
| $RC[j]$ | 01 | 02 | 04 | 08 | 10 | 20 | 40 | 80 | 1B | 36 |

# expandKey

```
void expandKey(){
    for (int i = 0; i < 16; i++) roundKey[0][i] =
        Integer.parseInt(hexkey.substring(i * 2, (i + 1) * 2), 16);
    int rcon = 1;
    for (int i = 1; i < numberOfRounds; i++){
        roundKey[i][0] = S[roundKey[i-1][13]] ^ rcon;
        rcon <<= 1; if (rcon > 0xFF) rcon ^= 0x11B;
        roundKey[i][1] = S[roundKey[i-1][14]];
        roundKey[i][2] = S[roundKey[i-1][15]];
        roundKey[i][3] = S[roundKey[i-1][12]];
        for (int k = 0; k < 4; k++)
            roundKey[i][k] ^= roundKey[i-1][k];
        for (int k = 4; k < 16; k++)
            roundKey[i][k] = roundKey[i][k-4] ^ roundKey[i-1][k];
    }
}
```



# AES Block Cipher

```
void blockCipher(){
    addRoundKey(0);
    for (int i = 1; i < numberOfRounds; i++){
        subBytes();
        shiftRows();
        if (i < numberOfRounds - 1) mixColumns();
        addRoundKey(i);
    }
}
```

# Main and encrypt()

```
void encrypt(){
    while (readBlock() > 0){
        blockCipher();
        writeBlock();
    }
    System.out.flush();
}
```

```
public static void main(String[] args){
    H11A h11 = new H11A();
    h11.makeLog();
    h11.buildS();
    h11.readKey(args[0]);
    h11.expandKey();
    h11.encrypt();
}
```

# Homework 11: due 3-2-15

- Complete the “inverse” versions of the steps in H11B.java so H11B is the inverse of H11A.
- Test your H11B with stallingskey.txt on test11.aes and get the original file to submit along with your code.

```
void decrypt(){
    while (readBlock() > 0){
        blockDecipher();
        writeBlock();
    }
    System.out.flush();
}
```

```
public static void main(String[] args){
    if (args.length < 1){
        System.err.println("Usage: java H11B key < encrypted > original");
        return;
    }
    H11B h11 = new H11B();
    h11.makeLog();
    h11.buildS();
    h11.readKey(args[0]);
    h11.expandKey();
    h11.decrypt();
}
```

# blockDecipher and inverseAddRoundKey

```
void inverseAddRoundKey(int round){
    for (int k = 0; k < blockSize; k++){
        state[k] ^= roundKey[?][k]; // which roundkey?
    }

    void blockDecipher(){
        inverseAddRoundKey(0);
        for (int i = 1; i < numberOfRounds; i++){
            inverseSubBytes();
            inverseShiftRows();
            inverseAddRoundKey(i);
            if (i < numberOfRounds - 1) inverseMixColumns();
        }
    }
}
```

```
void inverseSubBytes(){
    for (int i = 0; i < blockSize; i++)
        state[i] = Si[state[i]];
}
```

```
void inverseShiftRows(){
    // your code here
}
```

```
void inverseMixColumns(){
    int[] temp = new int[4];
    for (int k = 0; k < 4; k++){
        for (int i = 0; i < 4; i++){
            temp[i] = 0;
            for (int j = 0; j < 4; j++){
                temp[i] ^= logMultiply(Gi[j][i], state[k * 4 + j]);
            }
        }
        for (int i = 0; i < 4; i++) state[k * 4 + i] = temp[i];
    }
}
```

# Building Si Table too

```
void buildS(){
    int[] bitColumn = new int[8];
    for (int i = 0; i < fieldSize; i++){
        int inverse = i < 2 ? i : multiplicativeInverse(i);
        for (int k = 0; k < 8; k++)
            bitColumn[k] = inverse >> (7 - k) & 1;
        S[i] = 0;
        for (int k = 0; k < 8; k++){
            int bit = B[k];
            for (int l = 0; l < 8; l++)
                if (bitColumn[l] == 1) bit ^= A[k][l];
            S[i] ^= bit << 7 - k;
        }
        Si[S[i]] = i;
    }
}
```

# Propagation of a Single-Bit Flip (H11C.java)

```
void randomBlock(){
    for (int i = 0; i < blockSize; i++)
        state2[i] = state1[i] = random.nextInt(fieldSize);
    int index = random.nextInt(blockSize);
    state2[index]++; if (state2[index] >= fieldSize) state2[index] = 0;
}
```



# Blockcipher on Two Blocks

```
void blockCipher(){
    addRoundKey(state1, 0); addRoundKey(state2, 0);
    System.out.print("0 "); numberOfFlips();
    for (int i = 1; i < numberOfRounds; i++){
        subBytes(state1); subBytes(state2);
        System.out.print(i + " "); numberOfFlips();
        shiftRows(state1); shiftRows(state2);
        System.out.print(i + " "); numberOfFlips();
        if (i < numberOfRounds - 1){ mixColumns(state1); mixColumns(state2); }
        System.out.print(i + " "); numberOfFlips();
        addRoundKey(state1, i); addRoundKey(state2, i);
    }
}
```

# Comparing Two States

```
void numberOfFlips(){
    int total = 0;
    for (int i = 0; i < blockSize; i++){
        int diff = state1[i] ^ state2[i];
        int nf = 0;
        for (int mask = 0x80; mask > 0; mask >>= 1)
            if ((diff & mask) > 0) nf++;
        total += nf;
        System.out.print(nf + " ");
    }
    System.out.println(total);
}
```

|   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |    |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|----|
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 4 | 0 | 0 | 0 | 0 | 0 | 0 | 4  |
| 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 4 | 0 | 0 | 0 | 0 | 0 | 0 | 4  |
| 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 4 | 0 | 0 | 0 | 0 | 0 | 0 | 4  |
| 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 4 | 4 | 4 | 4 | 0 | 0 | 0 | 16 |
| 2 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 5 | 2 | 5 | 6 | 0 | 0 | 0 | 18 |
| 2 | 0 | 0 | 5 | 0 | 0 | 2 | 0 | 0 | 5 | 0 | 0 | 0 | 0 | 0 | 0 | 6 | 18 |
| 2 | 5 | 3 | 4 | 5 | 4 | 2 | 2 | 2 | 4 | 5 | 5 | 3 | 6 | 6 | 3 | 7 | 66 |
| 3 | 5 | 4 | 4 | 3 | 1 | 4 | 3 | 3 | 5 | 4 | 6 | 2 | 7 | 3 | 2 | 5 | 61 |
| 3 | 5 | 4 | 6 | 5 | 1 | 4 | 2 | 3 | 5 | 3 | 4 | 3 | 7 | 4 | 3 | 2 | 61 |
| 3 | 5 | 0 | 5 | 4 | 2 | 4 | 3 | 5 | 7 | 4 | 4 | 4 | 4 | 7 | 4 | 3 | 65 |
| 4 | 6 | 0 | 3 | 6 | 4 | 1 | 1 | 6 | 2 | 5 | 5 | 6 | 4 | 5 | 4 | 3 | 61 |
| 4 | 6 | 1 | 5 | 3 | 4 | 5 | 4 | 6 | 2 | 5 | 3 | 6 | 4 | 0 | 1 | 6 | 61 |
| 4 | 5 | 6 | 1 | 3 | 7 | 5 | 1 | 6 | 2 | 5 | 4 | 5 | 5 | 4 | 4 | 4 | 67 |
| 5 | 5 | 4 | 5 | 4 | 6 | 5 | 5 | 5 | 5 | 3 | 3 | 3 | 3 | 6 | 6 | 4 | 72 |
| 5 | 5 | 5 | 3 | 4 | 6 | 3 | 6 | 4 | 5 | 6 | 5 | 5 | 3 | 4 | 5 | 3 | 72 |
| 5 | 3 | 3 | 2 | 7 | 3 | 4 | 3 | 5 | 3 | 4 | 4 | 2 | 2 | 3 | 4 | 4 | 56 |
| 6 | 5 | 4 | 4 | 2 | 6 | 1 | 4 | 3 | 5 | 4 | 4 | 7 | 3 | 5 | 4 | 5 | 66 |
| 6 | 5 | 1 | 4 | 5 | 6 | 4 | 4 | 2 | 5 | 5 | 4 | 3 | 3 | 4 | 4 | 7 | 66 |
| 6 | 6 | 4 | 5 | 6 | 5 | 4 | 3 | 2 | 3 | 2 | 6 | 4 | 4 | 6 | 4 | 0 | 64 |

|    |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |   |    |    |
|----|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|----|----|
| 7  | 4 | 4 | 2 | 3 | 3 | 3 | 3 | 3 | 4 | 2 | 4 | 4 | 3 | 6 | 3 | 0 | 51 |    |
| 7  | 4 | 3 | 4 | 0 | 3 | 2 | 3 | 3 | 4 | 6 | 2 | 3 | 3 | 4 | 3 | 4 | 51 |    |
| 7  | 6 | 5 | 2 | 6 | 2 | 3 | 3 | 3 | 5 | 4 | 5 | 3 | 7 | 6 | 4 | 5 | 69 |    |
| 8  | 5 | 2 | 2 | 6 | 5 | 4 | 6 | 4 | 7 | 5 | 4 | 5 | 5 | 5 | 3 | 4 | 72 |    |
| 8  | 5 | 4 | 4 | 4 | 5 | 5 | 3 | 6 | 7 | 5 | 2 | 4 | 5 | 2 | 6 | 5 | 72 |    |
| 8  | 4 | 5 | 3 | 1 | 6 | 4 | 5 | 6 | 4 | 4 | 6 | 4 | 3 | 6 | 3 | 4 | 68 |    |
| 9  | 5 | 3 | 5 | 4 | 5 | 5 | 3 | 4 | 3 | 4 | 4 | 3 | 3 | 5 | 4 | 1 | 61 |    |
| 9  | 5 | 5 | 4 | 1 | 5 | 4 | 4 | 4 | 3 | 5 | 5 | 4 | 3 | 3 | 3 | 3 | 61 |    |
| 9  | 3 | 4 | 5 | 3 | 6 | 6 | 4 | 5 | 4 | 3 | 3 | 5 | 4 | 4 | 3 | 3 | 65 |    |
| 10 | 3 | 3 | 5 | 3 | 6 | 4 | 2 | 5 | 4 | 3 | 5 | 5 | 5 | 5 | 1 | 2 | 4  | 60 |
| 10 | 3 | 4 | 5 | 4 | 6 | 3 | 2 | 3 | 4 | 1 | 5 | 5 | 5 | 5 | 3 | 2 | 5  | 60 |
| 10 | 3 | 4 | 5 | 4 | 6 | 3 | 2 | 3 | 4 | 1 | 5 | 5 | 5 | 5 | 3 | 2 | 5  | 60 |