

Merkle Hash Tree





CS 5158/6058 Data Security and Privacy

Spring 2018

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Crypto Currency

- 1545 Different crypto currency
 - As of 03/08/2018, based on coinmarketcap.com
 - Major ones: Bitcoin, Ethereum, etc.

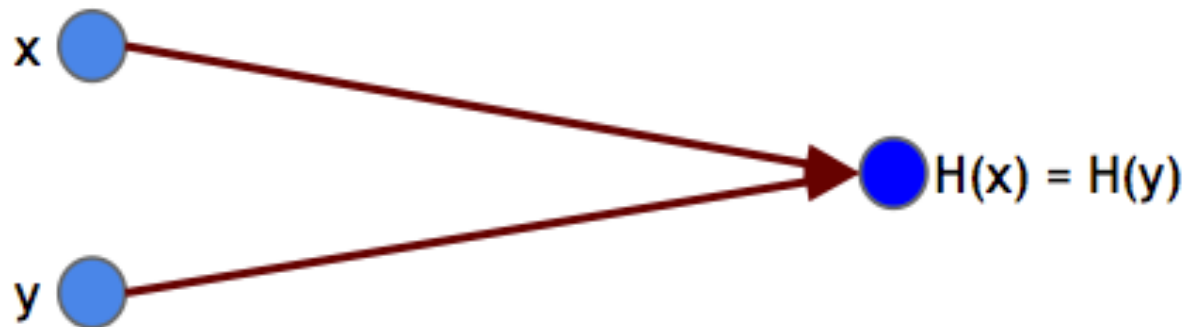
▲ #	Name	Symbol	Market Cap	Price
1	 Bitcoin	BTC	\$167,557,694,218	\$9,910.31
2	 Ethereum	ETH	\$73,453,471,840	\$749.06
3	 Ripple	XRP	\$33,722,246,441	\$0.862639
4	 Bitcoin Cash	BCH	\$18,424,896,270	\$1,083.40

Crypto Currency

- Bitcoin and blockchain are different!
 - Bitcoin is crypto currency
 - Blockchain is a chain of hash values
- We will focus on crypto building blocks in Bitcoin
 - Blockchain (hash chain, merkle hash tree)
 - Mining (Proof of Works)

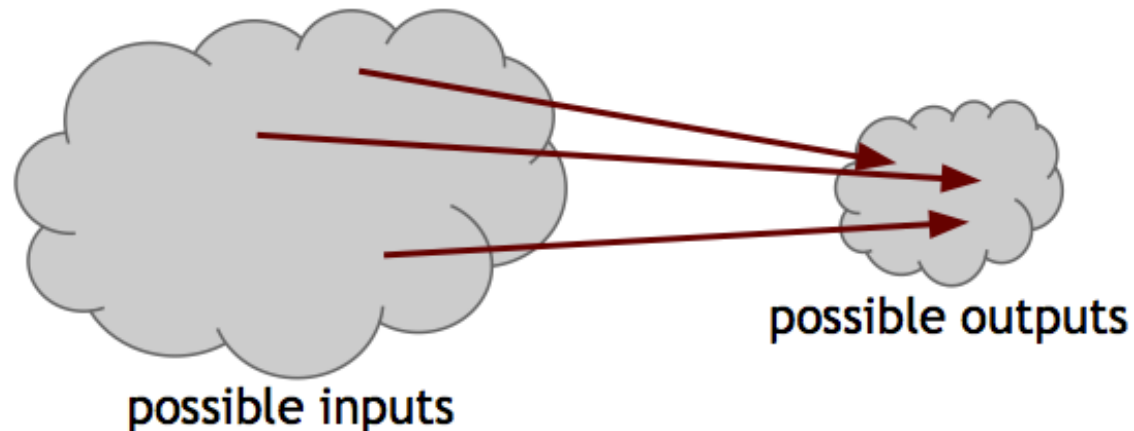
Hash Function Review

- Hash Function (e.g., SHA256):
 - An arbitrary-length input \rightarrow a fixed-length output
 - Deterministic function
 - Efficient to compute, but hard to invert
 - A collision: $x \neq y$, but $H(x) == H(y)$



Hash Function Review

- Hash Function (e.g., SHA256):
 - A collision must exist
 - No. of inputs \gg no. of outputs
 - A collision is hard to find
 - happens with a negligible prob. for PPT algo.



Trans. ID	Sender	Receiver	Amount	Time
t1	Alice	Bob	\$20	03/10/2018
t2	Alice	David	\$100	03/15/2018
t3	Bob	David	\$50	03/21/2018

- A bank has a number of transactions
- Bank wants to ensure all the trans are correct.
 - Trans could be changed by hacker on server
 - Trans could be changed by attacker on channel

Use Hash Function

- Assume a bank has transactions: {t1, t2, t3}
 - Can use a hash function (e.g., SHA256)
 - Take all trans. as input, compute one hash value
 - E.g., $H(t1, t2, t3) = h$
- Bank publishes **h**, anyone can verify {t1, t2, t3}
 - Given {t1, t2, t3}, Alice checks $H(t1, t2, t3) \stackrel{?}{=} h$
 - If identical, all trans are correct;
 - otherwise, some trans is not correct

Use Hash Function

- Assume a bank has transactions: $\{t_1, t_2, t_3\}$
 - Can use a hash function (e.g., SHA256)
 - E.g., $H(t_1, t_2, t_3) = h$
 - Bank publishes **h** , anyone can verify $\{t_1, t_2, t_3\}$
- Attacker changes t_3 to t_3'
 - $H(t_1, t_2, t_3') \neq h$, change can be detected
 - $H(t_1, t_2, t_3') == h$, change will not be detected
 - A collision happens with negligible prob.

Use Hash Function

- Assume a bank has transactions: $\{t1, t2, t3\}$
 - $H(t1, t2, t3) = h$
 - Bank publishes **h**, anyone can verify $\{t1, t2, t3\}$
- Attacker changes the order of trans
 - $\{t1, t2, t3\} \longrightarrow \{t1, t3, t2\}$
 - $H(t1, t3, t2) \neq h$, can be detected
 - $H(t1, t3, t2) = h$, will not be detected
 - A collision happens with negligible prob.

Use Hash Function

- Assume a bank has transactions: $\{t1, t2, t3\}$
 - $H(t1, t2, t3) = h$
 - Bank publishes **h**, anyone can verify $\{t1, t2, t3\}$
- Attacker appends 1 trans
 - $\{t1, t2, t3\} \longrightarrow \{t1, t2, t3, t4\}$
 - $H(t1, t2, t3, t4) \neq h$, can be detected
 - $H(t1, t2, t3, t4) = h$, will not be detected
 - A collision happens with negligible prob.

Use Hash Function

- Assume a bank has transactions: $\{t1, t2, t3\}$
 - $H(t1, t2, t3) = h$
 - Bank publishes **h**, anyone can verify $\{t1, t2, t3\}$
- Can attacker add 1 trans without being detected?
 - $\{t1, t2, t3\} \longrightarrow \{t1, t4, t2, t3\}$
 - $H(t1, t4, t2, t3) \neq h$, can be detected
 - $H(t1, t4, t2, t3) = h$, will not be detected
 - A collision happens with negligible prob.

Use Hash Function

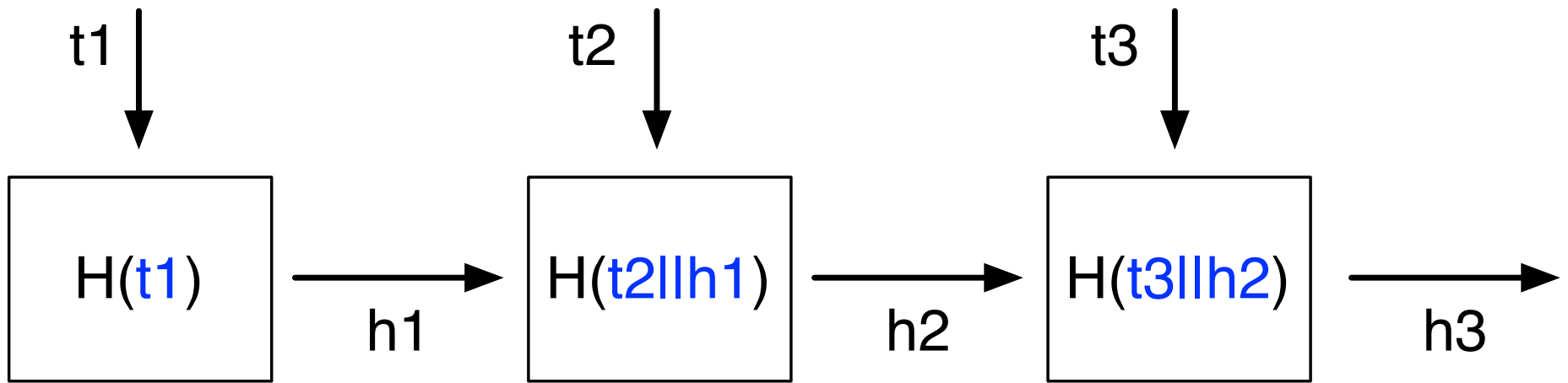
- Assume a bank has transactions: $\{t1, t2, t3\}$
 - $H(t1, t2, t3) = h$
 - Bank publishes **h**, anyone can verify $\{t1, t2, t3\}$
- It is (computationally) hard for attacker to
 - change any trans
 - change order of trans
 - add new trans
- This approach can protect data integrity

Trans. ID	Sender	Receiver	Amount	Time
t1	Alice	Bob	\$20	03/10/2018
t2	Alice	David	\$100	03/15/2018
t3	Bob	David	\$50	03/21/2018
t4	David	Alice	\$80	03/22/2018
t5	Charlie	David	\$10	03/23/2018

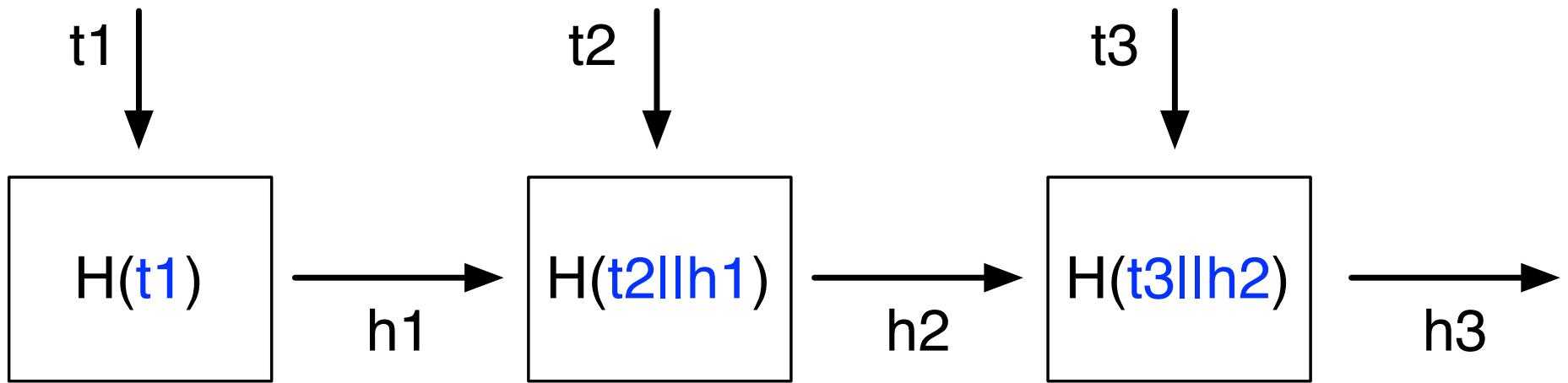
- Bank has new transactions every day
- Bank wants to ensure all the trans are correct.

Update Hash Value

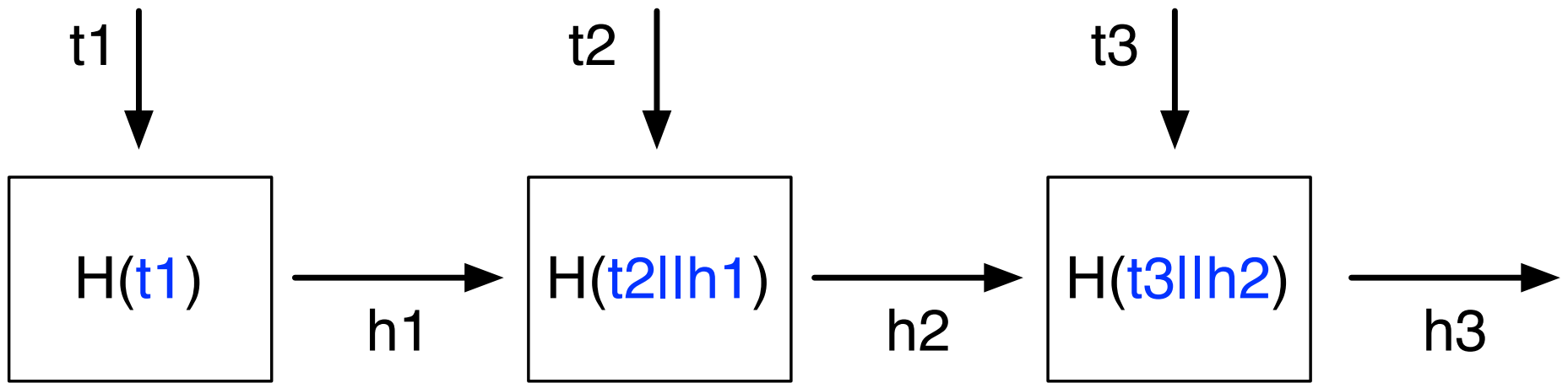
- Bank has new transactions: {t1, t2, t3, t4, t5}
- Recompute/Update a hash value
 - Take all trans. as input, compute one hash value
 - E.g., $H(t1, t2, t3, t4, t5) = h$
 - Bank publishes **h**, anyone can verify all trans
- Inefficient: bank needs to take all the trans as input
 - E.g., reading 1,000,000 trans. takes long time



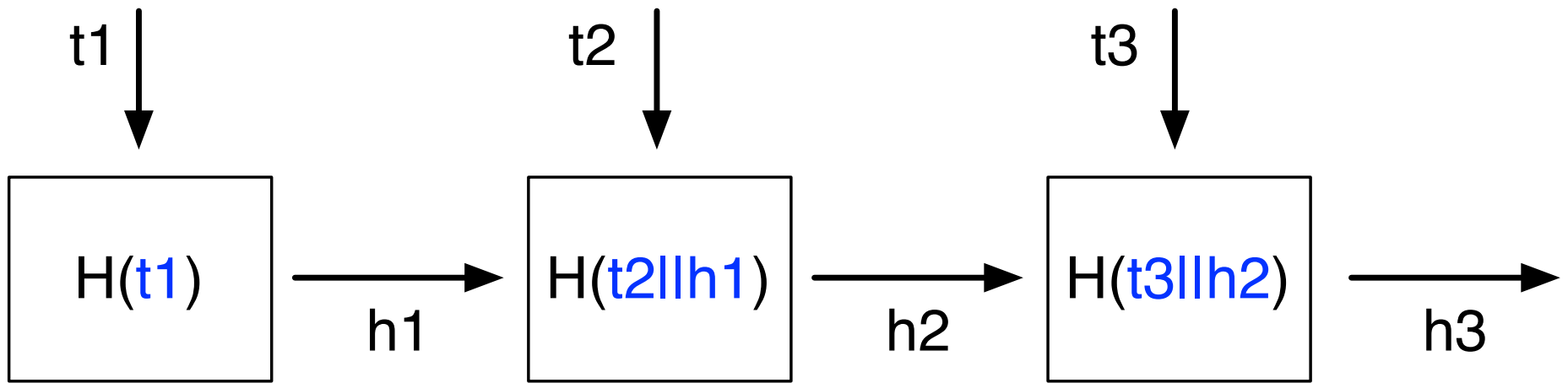
- Hash chain:
 - Given a new trans, bank computes a hash value of new trans with the current hash value
 - $h1$ can prove $\{t1\}$ is correct
 - $h2$ can prove $\{t1, t2\}$ are correct
 - $h3$ can prove $\{t1, t2, t3\}$ are correct
 - Bank always updates the latest hash value



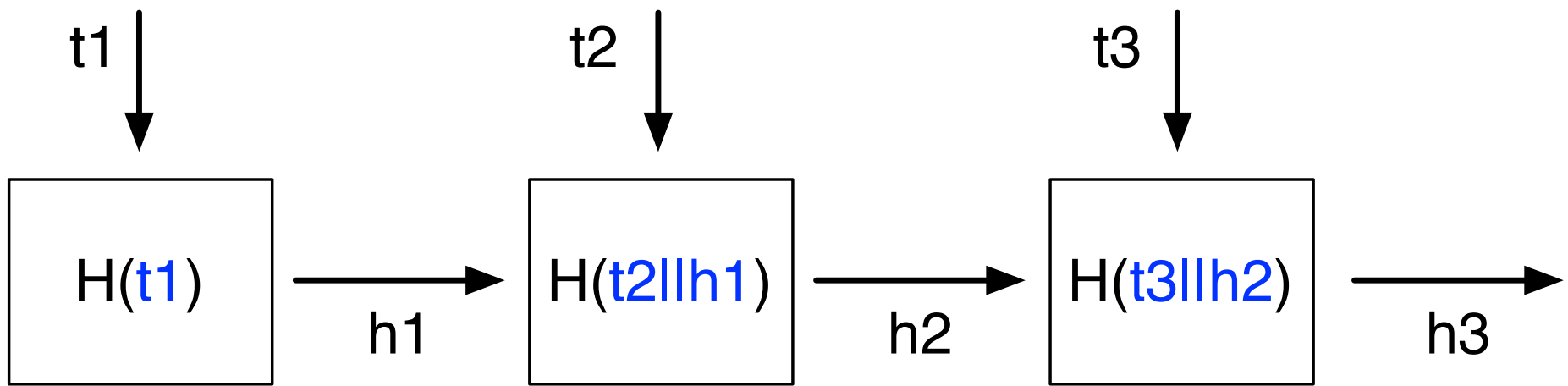
- Hash chain:
 - Bank publishes the latest hash value (e.g., $h3$)
 - Given $\{t1, t2, t3\}$ and $h3$, Alice computes
$$H(t1) = h1; H(t2||h1) = h2; H(t3||h2) \stackrel{?}{=} h3$$
 - If identical, all trans are correct;
 - otherwise, some trans is not correct



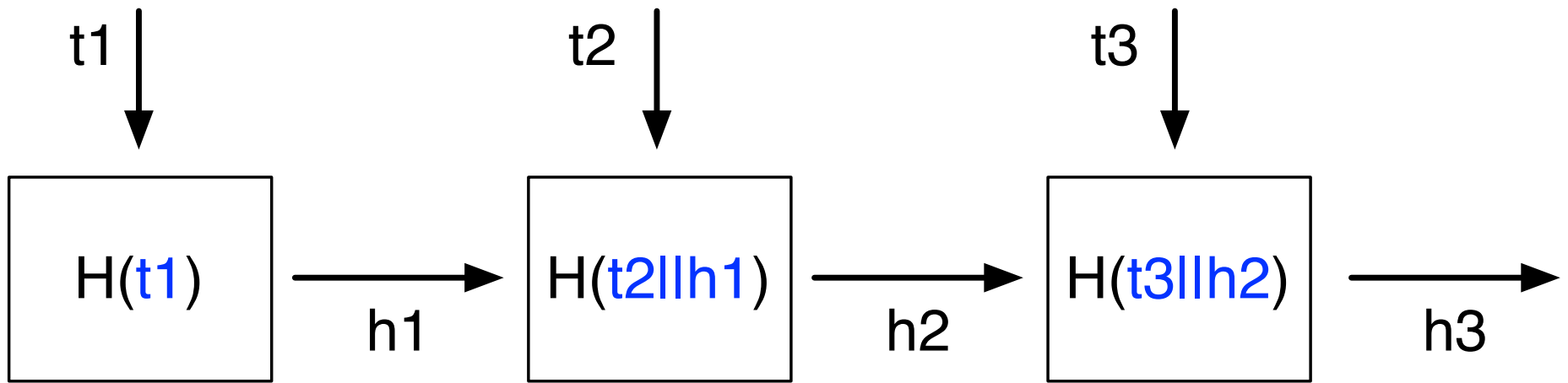
- Attacker changes $t3$ to $t3'$
 - $H(t3' || h2) = h3' \neq h3$,
 - change can be detected
 - $H(t3' || h2) = h3' == h3$,
 - change will not be detected
 - A collision happens with negligible prob.



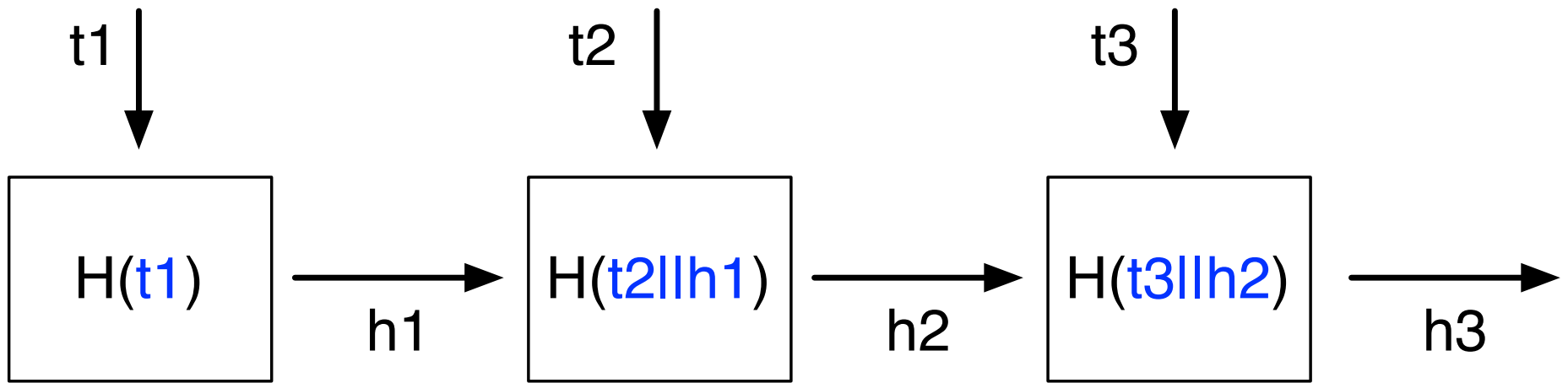
- Attacker changes $t2$ to $t2'$
 - $H(t2' || h1) = h2' \neq h2$
 - Does not find a collision at block 2 for $t2' || h1$
 - Since $h2' \neq h2$, then $H(t3 || h2') = h3' \neq h3$
 - Change can be detected



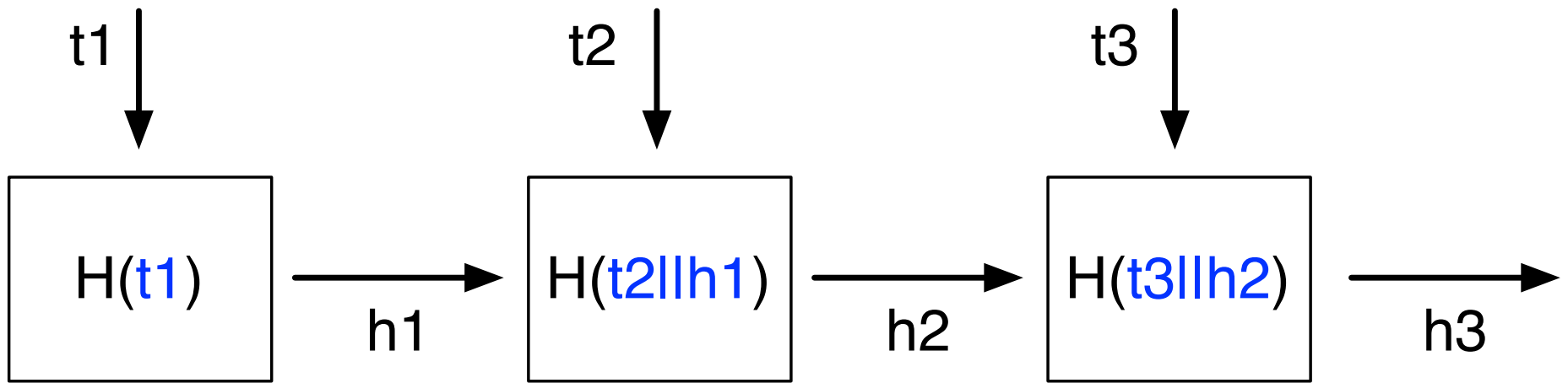
- Attacker changes $t2$ to $t2'$
 - $H(t2' || h1) = h2' \neq h2$,
 - Does not find a collision at block 2 for $t2' || h1$
 - $h2' \neq h2$, $H(t3 || h2') = h3' == h3$
 - Finds a collision at block 3 for $t3 || h2'$
 - Change will not be detected
 - A collision happens with a negligible prob.



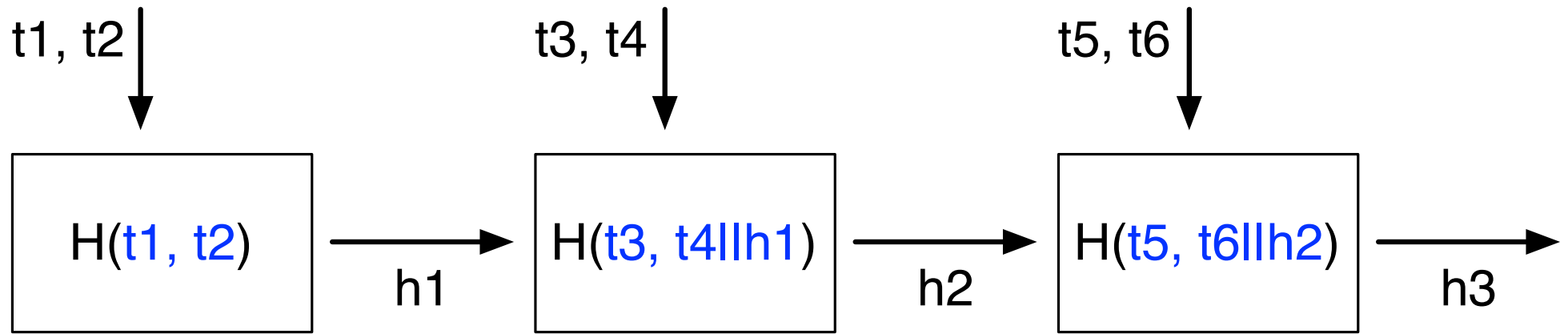
- Attacker changes $t2$ to $t2'$
 - $H(t2' || h1) = h2' == h2$,
 - Finds a collision at block 2 for $t2' || h1$
 - Since $h2' == h2$, then $H(t3 || h2') = h3' == h3$
 - Change will not be detected
 - A collision happens with a negligible prob.



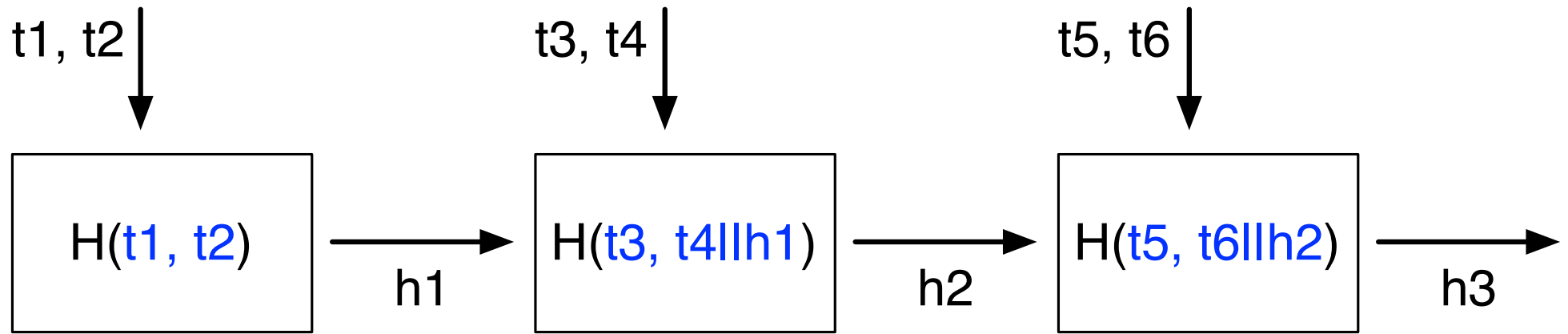
- Bank publishes the latest hash value (e.g., **h_3**)
- Practice 1: Attacker changes the order to $\{t_2, t_3, t_1\}$
 - Given **h_3** , will Alice accept $\{t_2, t_3, t_1\}$?
 - No, finding a collision is hard
- Practice 2: Attacker appends 1 trans $\{t_1, t_2, t_3, t_4\}$
 - Given **h_3** , will Alice accept $\{t_1, t_2, t_3, t_4\}$?
 - No, finding a collision is hard



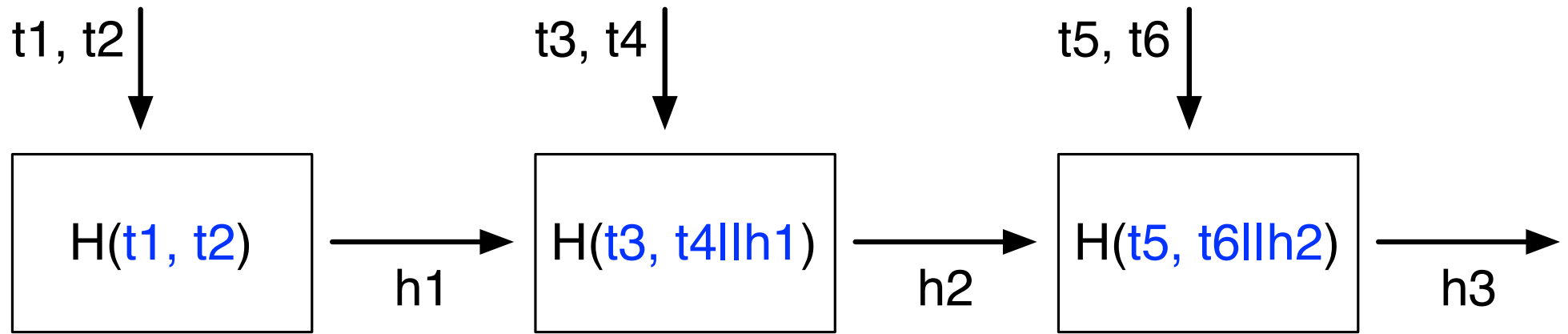
- Bank can efficiently update the latest hash value
 - 1 hash each new trans
- It is (computationally) hard for attacker to change trans, add trans or change the order of trans
- Alice can still verify
 - Reads n trans in total, & computes n hashes (v.s. 1 hashes in previous method)



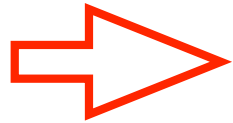
- Bank takes multiple trans each block, can reduce the total number of hash operations
 - $h1$ can prove $\{t1, t2\}$ are correct
 - $h2$ can prove $\{t1, t2, t3, t4\}$ are correct
 - $h3$ can prove $\{t1, t2, t3, t4, t5, t6\}$ are correct
 - 3 hashes v.s. 6 hashes (in previous case)



- Tradeoff:
 - If $t5$ is ready, but $t6$ is not available yet
 - Bank needs to wait to generate next hash value
 - $t5$ cannot be confirmed immediately
- If Bank increases no. of trans in each block
 - less hash operations, more pending trans.

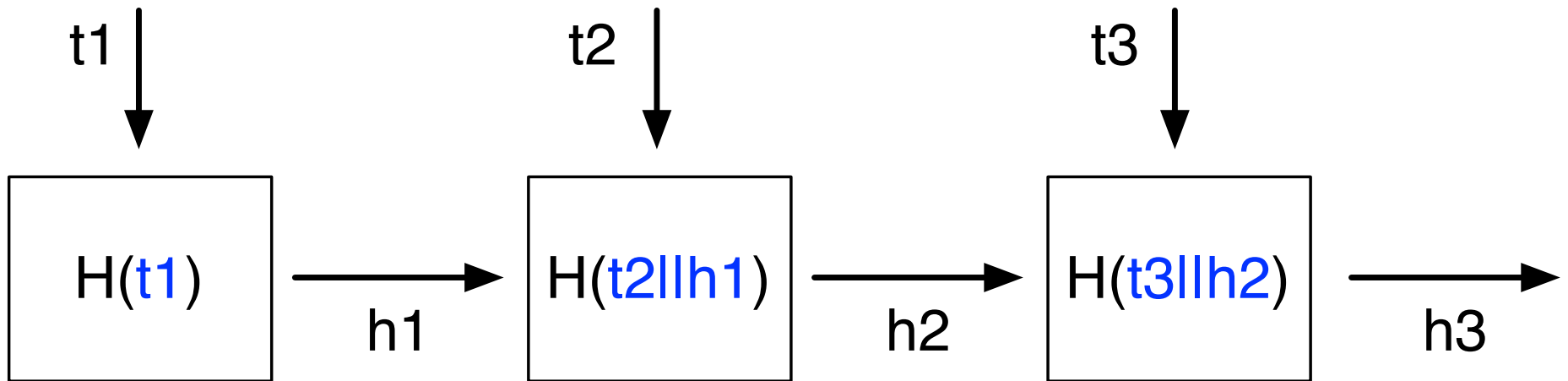


- Attacker changes $t5$ to $t5'$
 - Change can be detected
 - $H(t5', t6||h2) = h3' \neq h3$,
 - Change will not be detected
 - $H(t5', t6||h2) = h3' == h3$,
 - A collision happens with a negligible prob.

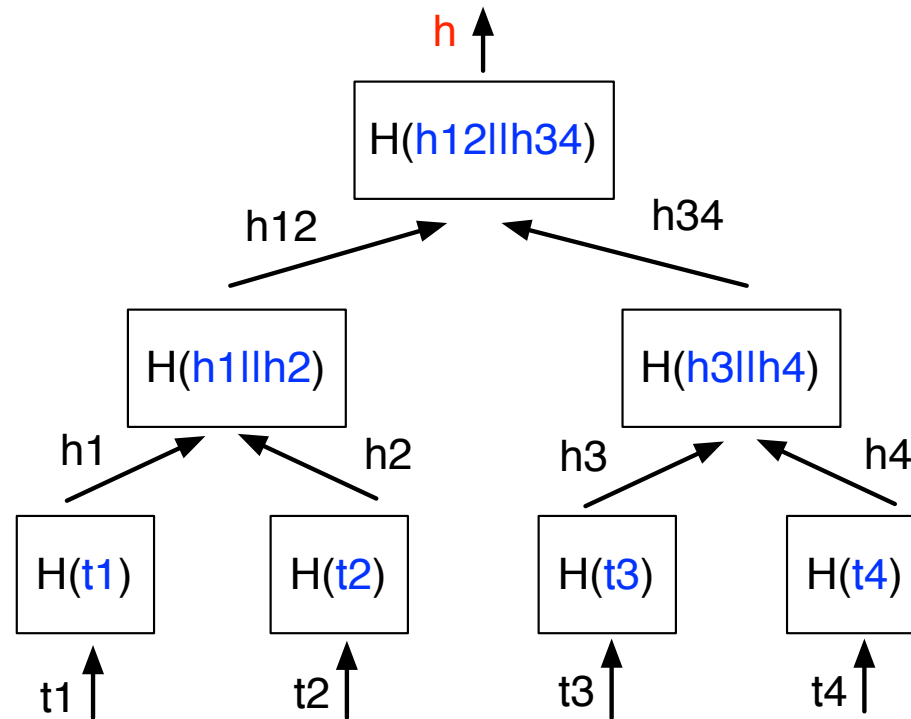


Trans. ID	Sender	Receiver	Amount	Time
t1	Alice	Bob	\$20	03/10/2018
t2	Alice	David	\$1000	03/15/2018
t3	Bob	David	\$50	03/21/2018
t4	David	Alice	\$80	03/22/2018
t5	Charlie	David	\$10	03/23/2018

- Bank has new transactions every day.
- Bank **updates** old transactions.
- Bank wants to ensure all the trans are correct.

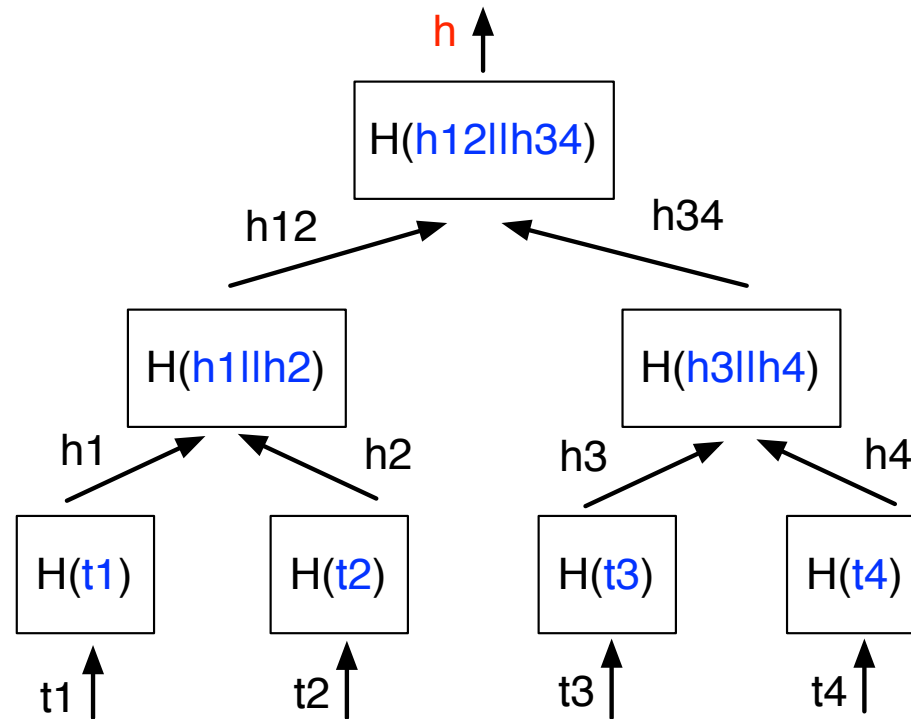


- Bank updates using Hash chain:
 - If $t3$ is updated to $t3'$
 - recompute $h3$, if still has a copy of $h2$; otherwise, recompute $h1, h2, h3$
 - If $t1$ is updated to $t1'$,
 - recompute $h1, h2, h3$
- Recompute $O(N)$ hashes, not very efficient
 - E.g., $N=1,000,000$

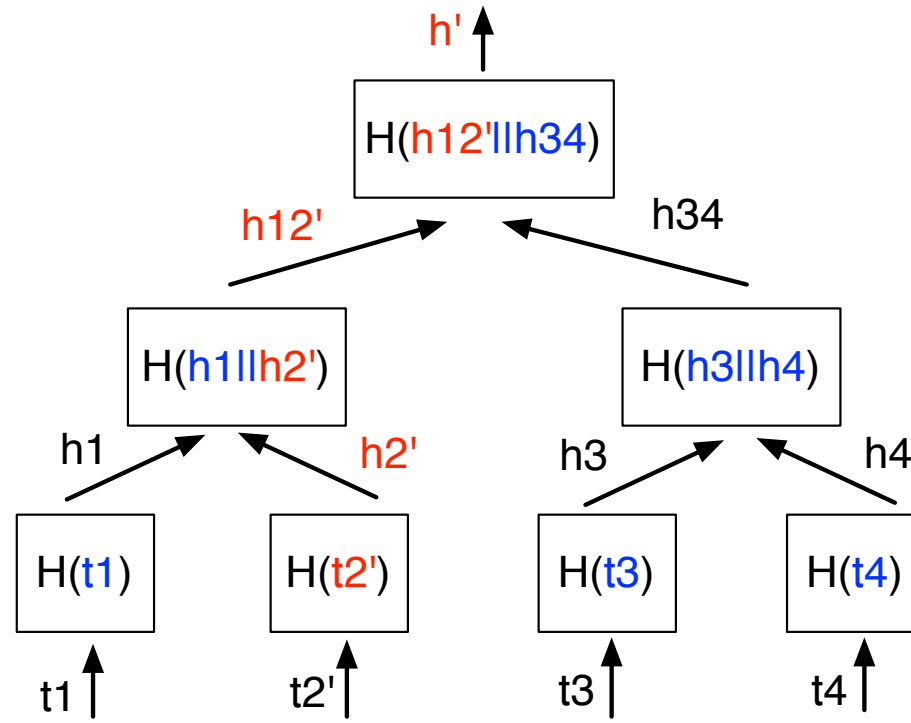


- Merkle hash tree:

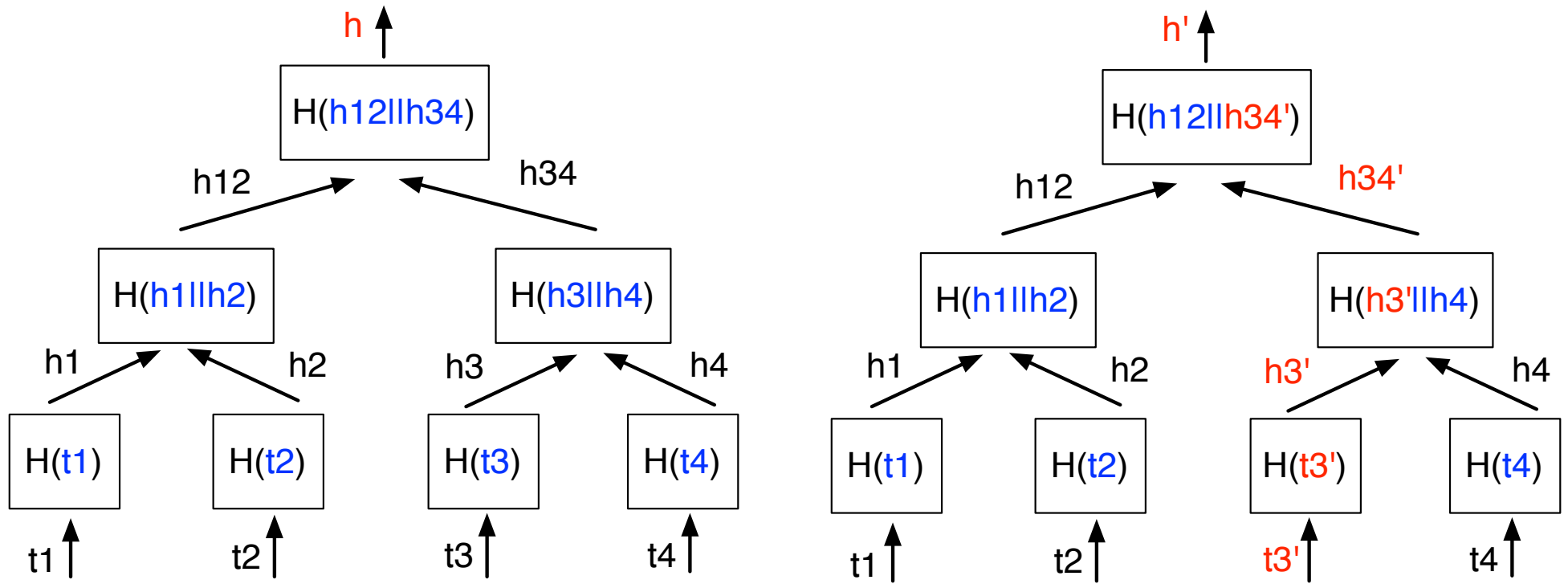
- Bank has $\{t1, t2, t3, t4\}$
- Bank computes a root hash h based on tree
- Each trans is a leaf
- Root hash h proves $\{t1, t2, t3, t4\}$
- Bank publishes h , anyone can verify all trans



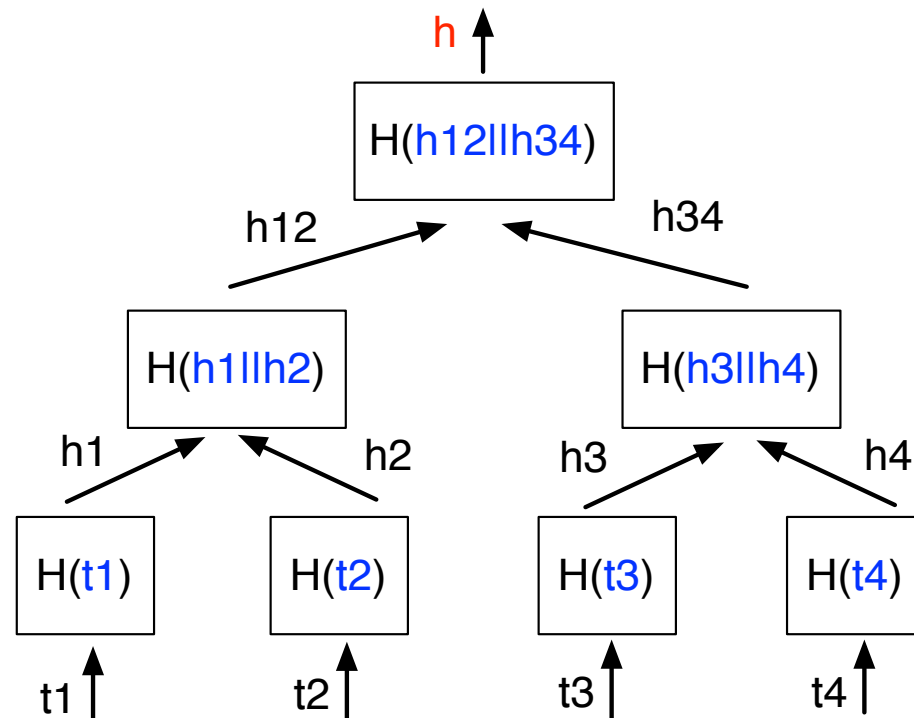
- Bank publishes **h** , anyone can verify all trans
- Given h and $\{t1, t2, t3, t4\}$, Alice recomputes root hash h' , and compare it with **h**
 - $2N-1$ hashes v.s. N hashes in hash chain
 - $N = 4$: 7 hashes v.s 4 hashes



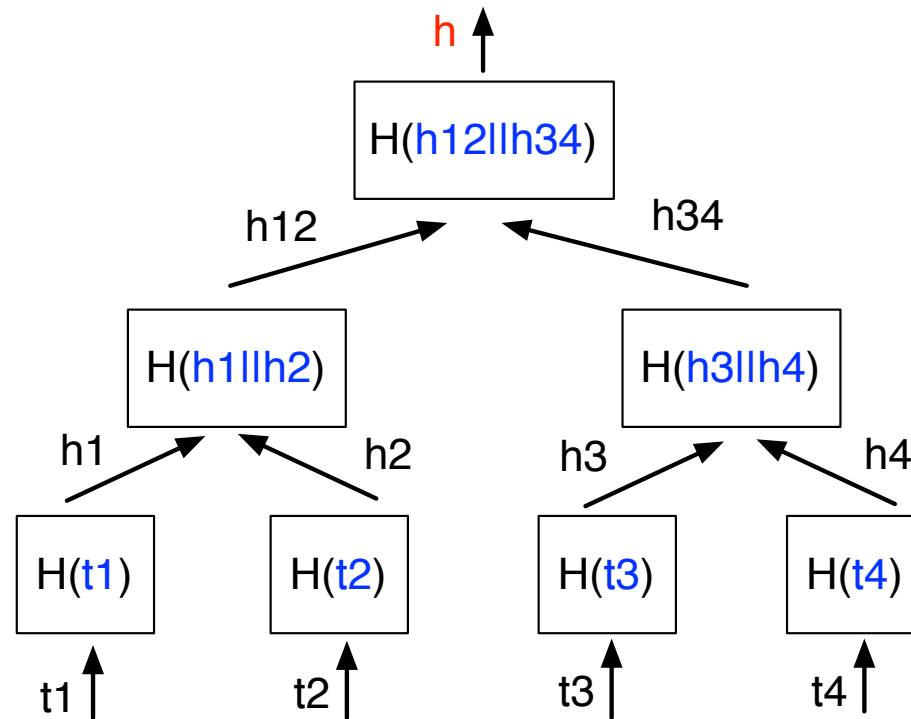
- Bank updates **t2** to **t2'**
 - Recomputes h2', h12', h' in the tree
 - Assume bank maintains all the non-leaf nodes
 - $O(\log N)$ hashes v.s. $O(N)$ in hash chain
- Bank publishes **h'**, anyone can verify all trans



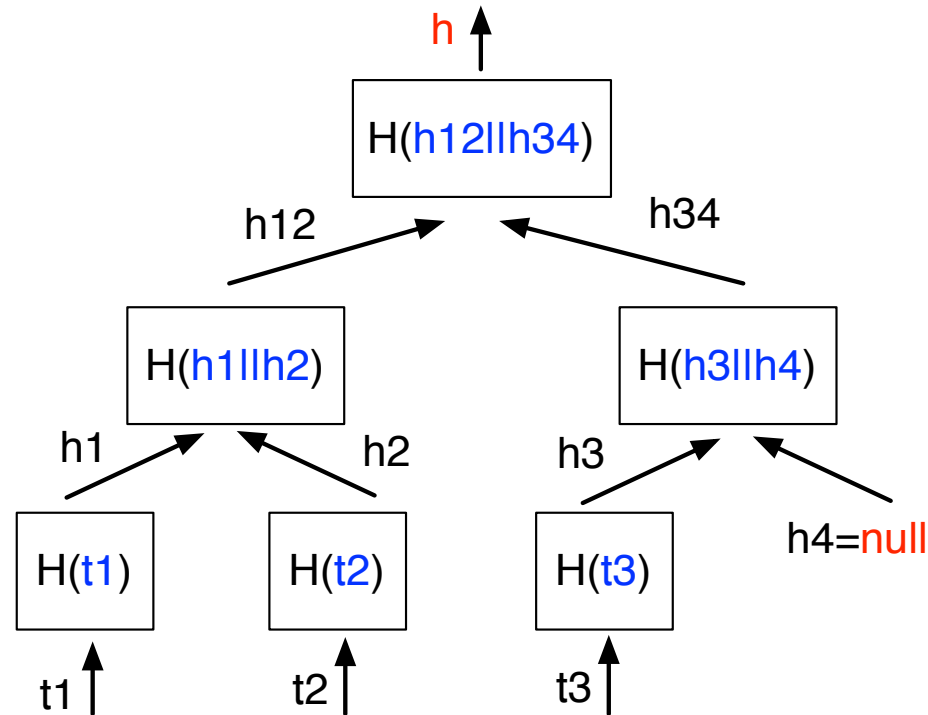
- Practice: if bank updates t_3 to t_3' , which hash values in the tree need to be recomputed?
- Bank recomputes h_3' , h_{34}' , h'



- Bank publishes **h**, anyone can verify all trans
- Attacker changes **t2** to **t2'** without being detected
 - Attacker needs to find 1 collision, i.e., $H(t2') == H(t2)$, s.t. root **h** is still correct
 - Happens with a negligible probability



- Bank publishes **h**, anyone can verify all trans
- Similarly, it is hard for attacker to add new trans or change the order of trans
 - Need to find collisions
 - Happens with a negligible probability



- If the no. of trans is not a power of 2, how does bank build this tree?
- Example: if bank only has 3 trans, {t1, t2, t3}
 - Only 3 leaf nodes in tree
 - Bank leaves hash value h4 as **null**

- Practice: if bank has 5 trans, {t1, t2, t3, t4, t5}
 - How to compute the root hash?
 - Leave h6, h7, h8 as **null**

