Physics practical report

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Part IB Physics Report

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Abstract

1 Introduction

Transformers form a core part of modern technology, being used for impedance matching within electronics and electricity transmission on the grid. In both systems, the efficiency and performance are determined by how the transformer responds to an applied magnetic field.

This response is described by the relationship between the magnetic flux density B and the magnetising field H, known as the B-H curve. By altering the core of the transformer its properties can be changed. For ferromagnetic materials, the B-H curve is non-linear and exhibits hysteresis (**Figure 1**) due to magnetic subdomains in the material.

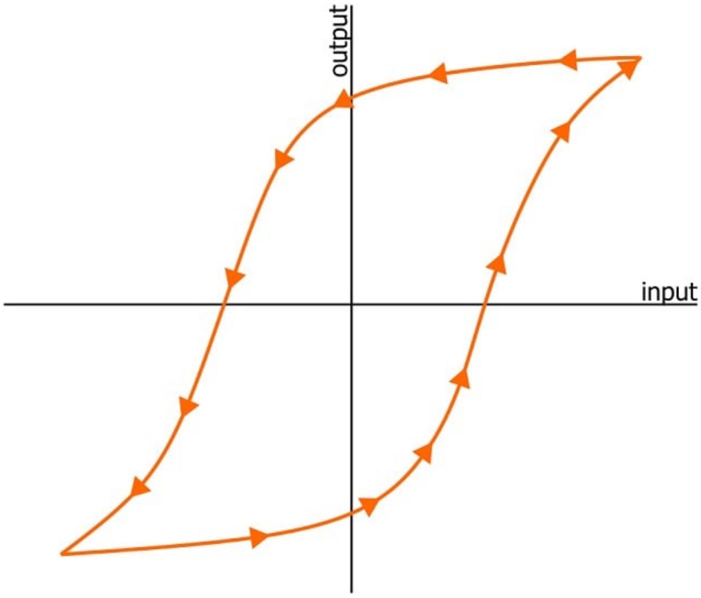

Figure 1: A typical hysteresis curve for a ferromagnetic material, showing the relationship between B and H. [1].

The most efficient transformers are made by having a high magnetic susceptibility (μ_r) and low energy loss per cycle. This experiment aims to quantify the usability of different materials as transformer cores, by measuring the energy loss per cycle per unit volume (given by the area of a hysterysis loop) and the maximum magnetic susceptibility (given by the gradient). Section 2 of this report will go through the relevant theory. Section 3 outlines the experimental setup, and Section 4 shows the results obtained. This data is analysed and compared against theoretical expectations in Section 5. The overall conclusions are then presented in Section 6.

2 Theoretical background

The magnetising field H inside a solenoid is given by

$$H = \frac{nI}{L} \tag{2.1}$$

Where n is the number of turns, I the current, and L the solenoid length.
This gives rise to the magnetic flux density B For solenoids, with non-magnetic media in their core, there is a linear relationship:

$$B = \mu_0(H + M) \tag{2.2}$$

$$M = \chi H \quad \Rightarrow \quad B = \mu_0(1 + \chi)H \tag{2.3}$$

$$\mu_r = 1 + \chi \quad \Rightarrow \quad B = \mu_0 \mu_r H \tag{2.4}$$

However, when using magnetic materials, μ_r is not a constant and so:

$$B = \mu_0(H + M) \tag{2.5}$$

μ_0 is the permeability of free space ($4\pi \times 10^{-7}$ H/m $\approx 1.257 \times 10^{-6}$ H/m),
μ_r is the relative permeability of the material.
M is the magnetisation of the core material.
χ is the volume magnetic susceptibility

By using different materials within the centre of the solenoid different values of mu are used resulting in different hysteresis loops being formed. These B and H values cannot be measured directly. But if you arrange 2 solenoids as in (**Figure 2**).

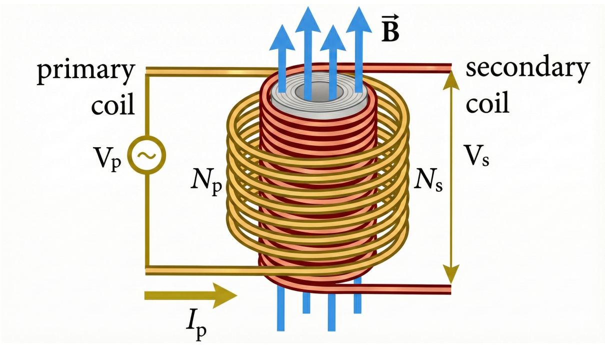

Figure 2: Interlinked solenoids [**?**].

the variance of the magnetic field in one (the primary solenoid) results in an induced emf in the other coil. This is given by Faraday's law of induction:

$$\mathcal{E} = -N_s \frac{d\Phi}{dt} \tag{2.6}$$

Where: $\mathcal{E}$ is the emf induced in the secondary coil N_s is the number of turns of coil And $\frac{d\Phi}{dt}$ is the rate of change of flux
Φ, the magnetic flux given by:

$$\Phi = BA_s \tag{2.7}$$

Where B is the magnetic flux density inside the coil and is provided by the primary coil and A_s is the area of the secondary coil.

A_s is the Crossectional area of the material in the core.

This secondary coil then allows us to calculate B by integrating $\mathcal{E} = -N_s \frac{d\Phi}{dt}$ to give:

$$B = -\frac{1}{N_s A_s} \int \mathcal{E} dt \tag{2.8}$$

Then through the use of an integrator circuit we can let $V_{out} = -\frac{1}{R_i C} \int \mathcal{E} dt$ and then calculate B using:

$$B = -\frac{R_i C}{N_s A_s} V_{out} \tag{2.9}$$

Our loop allows us to calculate the energy loss per cycle from the area under the B-H curve and the μ_r values at different points. In this case, $\mu_r = \frac{1}{\mu_0} \frac{\partial B}{\partial H}$ as linear dependence is assumed in the localised region.

3 Experimental Methods

3.1 Coil assembly and primary circuit

Two coils, one of $\sim$ 500 turns and one of $\sim$ 400 turns, were co-axially wound into a solenoid. The coil with 500 turns was designated the primary coil and connected in series with a $2 \pm 0.1\,\Omega$ resistor (R_p) to reduce the current. This allowed us to calculate the current through the coil and thus the magnetic field strength through the coil ($I = \frac{V}{R_p} \rightarrow H = \frac{nI}{L}$ $(eqn2.1)$) and ensured the coil did not overheat. This part of the circuit was completed by connecting it to a signal generator set to produce an AC voltage of $5 \pm 0.01 V_{pp}$ (Peak to Peak) at 50 ± 0.1 Hz.

The secondary coil was then attached to an integrator circuit.

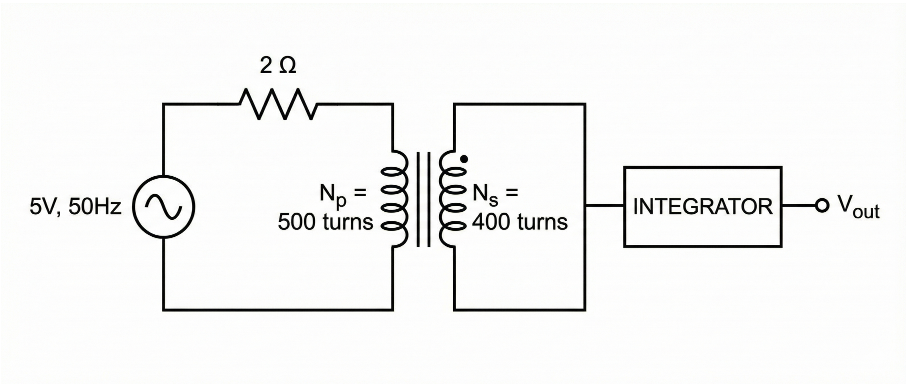

Figure 3: integrator circuit.

3.2 Integrator design and calibration

The integrator circuit was set up as shown below:
R_2 was chosen to be, 1MΩ.
After measuring R_2, it was found to be 991.2$\pm$0.1$K\Omega$

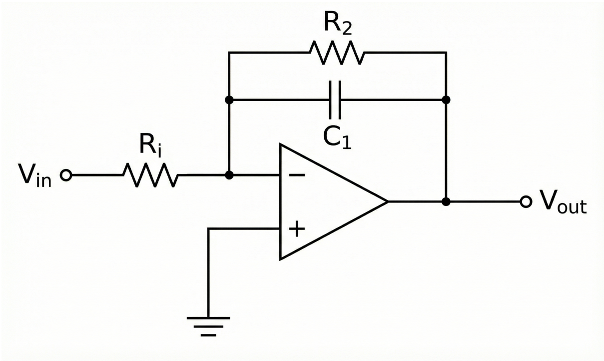

Figure 4: integrator circuit[**?**].

The value of C_1 s chosen such that $Z_i << 1M\Omega$ at 50 Hz.

$$Z_i << 1M\Omega \tag{3.1}$$
$$Z_i = \frac{1}{\omega C_1} \tag{3.2}$$
$$\frac{1}{\omega C_1} << 1M\Omega \tag{3.3}$$
$$\frac{1}{2\pi \times 50 \times C_1} << 10^6 \tag{3.4}$$
$$C_1 >> 3.18nF \tag{3.5}$$

Therefore Picking 100 nF was suitable.
Next R_i needed to be chosen. To make the calculations easier, an R_i value that would give a gain of 1 was chosen.

$$gain = \left|\frac{V_{out}}{V_{in}}\right| = \frac{1}{\omega R_i C_1} \tag{3.6}$$
$$1 = \frac{1}{\omega R_i C_1} \tag{3.7}$$
$$R_i = 31.83K\Omega \tag{3.8}$$
$$R_i \approx 30K\Omega \tag{3.9}$$

the chosen resistor and capacitor were then measured accurately, Giving:

$$R_i = 32,830 \pm 5\Omega \tag{3.10}$$
$$C_1 = 97.8 \pm 0.1nF \tag{3.11}$$
$$\therefore$$
$$theoretical\ gain = \frac{1}{2\pi \times 50 \times 32,830 \times 97.8 \times 10^{-9}} = 0.9913 \tag{3.12}$$

After setting up the circuit, the oscilloscope was attached to the signal generator to measure the input and to measure the output of the circuit.

$$Measured\ gain = \left|\frac{V_{out}}{V_{in}}\right| = \frac{14.38V}{10.49V} = 1.37 \pm 0.005 \tag{3.13}$$

This is a 38% deviation from the theoretical value and may be due to a square wave being

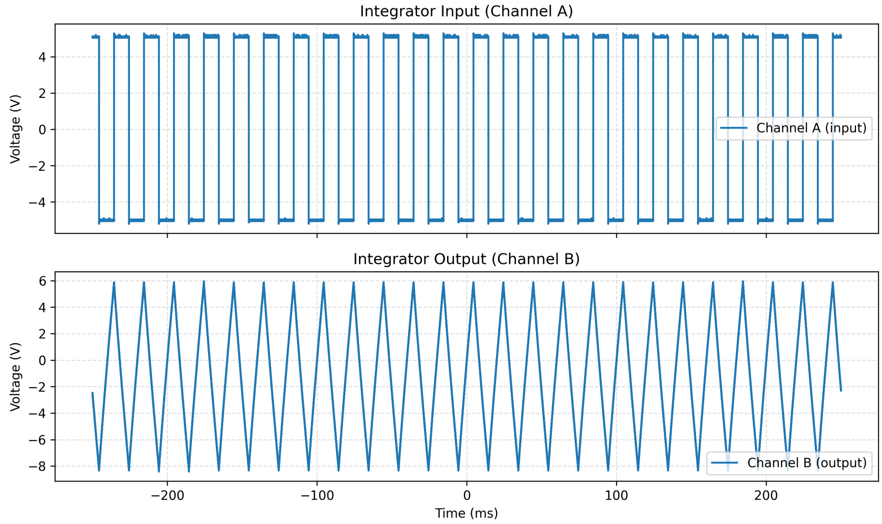

Figure 5: Measurements of the input and output of the circuit figure.

tested. This is composed of sinusoidal waves of many frequencies and so will have some at the odd harmonics of the fundamental frequency of the integrator circuit (e.g. 3f = 150 Hz, 5f = 250 Hz, etc.). This is combined with the op-amp not being ideal and having a limited bandwidth can result in a different gain to the theoretical value. This is treated as a systematic error in the experiment and was used as gain in the subsequent calculations.

3.3 Data acquisition and sample preparation

The samples used were:
- Mild steel
- Transformer iron
- CuNi alloy at 10°C and above 40°C

the coil length and the cross-sectional area of the samples were measured. This was done using a vernier caliper to measure the length and width of the cuboid CuNi alloy and transformer iron samples and the diameter of the cylindrical mild steel sample.
Then the experiment was run: The signal generator was turned on to feed an AC current into the primary coil after Waiting until the circuit reached a steady state the data points for 20 cycles were saved. This was done immediately to ensure the coil didn't heat up and change its resistance during the measurement. The coil was then turned off and this was repeated for the other samples.

First the measurements were performed without a sample (air) to act as a control, then metals.

When measuring the CuNi alloy it was placed into hot water to raise its temperature to above 40°C and then placed into ice water to cool it to 10°C. The low time difference between the samples being removed from the water and being tested results in minor temperature changes during the measurement but the actual temperature was not measured during the hysteresis and thus the temperatures should be taken as approximate.

Voltages across the series 2 Ω resistor (Channel A) and integrator output (Channel B) were recorded using a PicoScope at a sampling rate of 1.95 MHz (in spite of the AC current being used the PicoScope was set to DC. This was to avoid the possibility of some AC current being filtered out by the PicoScope as it is seen as DC offset as a result of the low frequency being used). For each sample 20 cycles at 50 Hz ($\sim$0.32 s) were recorded. Was then exported to be analysed using a Python script that splits up the individual loop and plots them for a visual check during the experiment and allowed for further analysis later in Section 4.

4 Results and Analysis

4.1 Overview

The first step was to record the nominal measurements of the samples used throughout the run, which are summarised in Table 1. Over the course of the experiment five sets of data were collected: the air control experiment, mild steel, transformer iron, copper at approximately 10 °C, and copper at 40 °C. Transformer iron and the elevated-temperature copper measurements were both repeated because the initial readings could not be reconciled with the literature values. The retaken datasets proved indistinguishable from the originals, so the initial measurements were retained for the subsequent analysis.

Table 1: Nominal sample dimensions and cross-sectional areas used to convert the voltage measurements into magnetic quantities. Values marked "–" should be replaced with the recorded data.

Sample	Key measurements	Cross-sectional area / mm^2
Air control	Length = – mm, Diameter = – mm	–
Mild steel	Length = – mm, Diameter = – mm	–
Transformer iron	Length = – mm, Diameter = – mm	–
Copper (10 °C)	Length = – mm, Diameter = – mm	–
Copper (40 °C)	Length = – mm, Diameter = – mm	–

Table 2 summarises the recorded data sets and notes the repeated runs.

Table 2: Summary of the collected data sets.

Data set	Notes
Air control experiment	Baseline linear response, no hysteresis expected.
Mild steel	Hysteresis loop with pronounced end tails.
Transformer iron	Initial loop deviated from literature; repeated run matched original.
Copper at 10 °C	Hysteresis loop with tail artefacts.
Copper at 40 °C	Repeat confirmed the linear response with negligible hysteresis.

Twenty loops of the steady-state response were recorded for each dataset. Using Equations (2.9) and (2.1), the voltages across the 2 Ω resistor and the integrator output were converted into magnetic field strength H and magnetic flux density B for the plots that follow.

4.2 Air calibration

The air control plot shows the expected linear response with no hysteresis, because the permeability should be close to that of free space. The behaviour was modelled by $B = \mu_0\mu_\mathrm{r}H$, and the apparent relative permeability μ_r extracted from the slope is smaller than the literature value. The spread of the measured points does not account for this offset, so the discrepancy is interpreted as a systematic error possibly due to parasitic capacitances or the finite input impedance of the measurement chain.

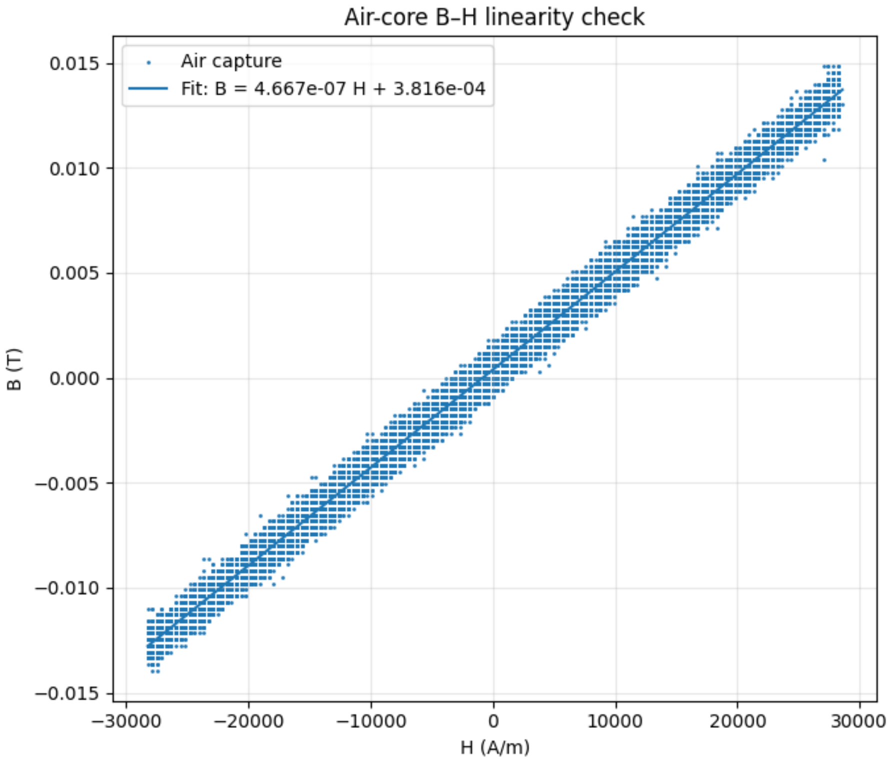

Figure 6: Linear $B(H)$ response obtained from the air control experiment.

4.3 Hysteresis plots

Each solid sample produced hysteresis loops whose end regions exhibited tail-like features, despite reaching steady state. These tails are likely caused by unaccounted resistance and capacitances in the coils or by the finite response time of the integrator. To recover smooth gradients at the loop extremities, the tails were fitted with fifth-order polynomials before extracting the magnetic parameters.

4.3.1 Mild steel

Figure 7 shows the raw hysteresis data for the mild steel sample.

After applying the fifth-order polynomial fit to the tails, we obtained the smoother loop shown in Figure 8. The fit allows the determination of a continuous gradient at the loop ends for the permeability estimates.

4.3.2 Transformer iron

Figure 9 shows the raw transformer iron loops, which also required polynomial smoothing of the tails.

4.3.3 CuNi at 10 °C

The CuNi alloy at 10 °C follows similar behaviour, with the raw data plotted in Figure 11.

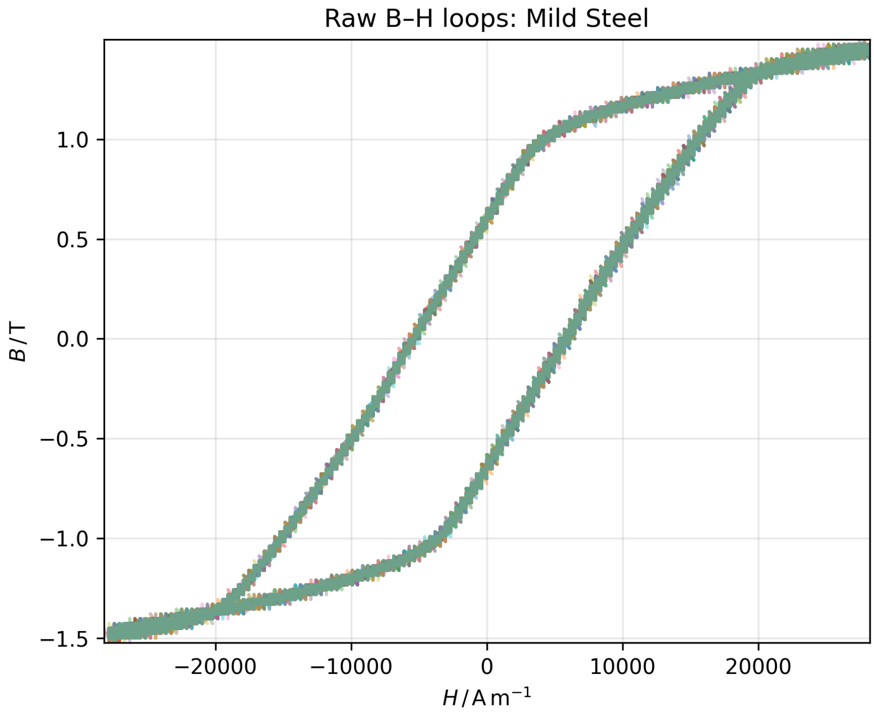

Figure 7: Raw mild steel hysteresis loops.

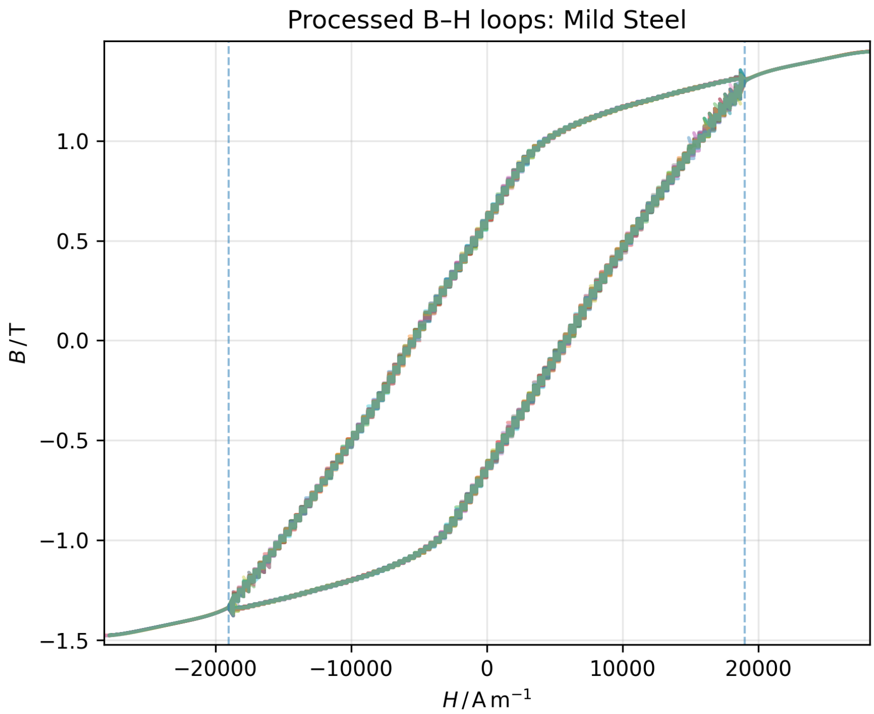

Figure 8: Mild steel hysteresis loop after polynomial tail fitting.

Table 3: Mild steel loop statistics from the polynomial-tail analysis (cell 10 of the notebook).

Quantity	Value	Uncertainty	Units
B range	2.92	0.00034	T
H range	5.61×10^4	31	A/m
Loop area (absolute)	2.76×10^4	8.2	T·A/m
Maximum μ_r (numeric gradient)	123	4.5	—
Minimum μ_r (numeric gradient)	0.00	0.00	—
Tail slope (min)	2.22×10^{-6}	2.4×10^{-7}	T/A/m
Tail slope (max)	1.16×10^{-4}	2.2×10^{-8}	T/A/m
RMS B residual	0.0149	0.000013	T

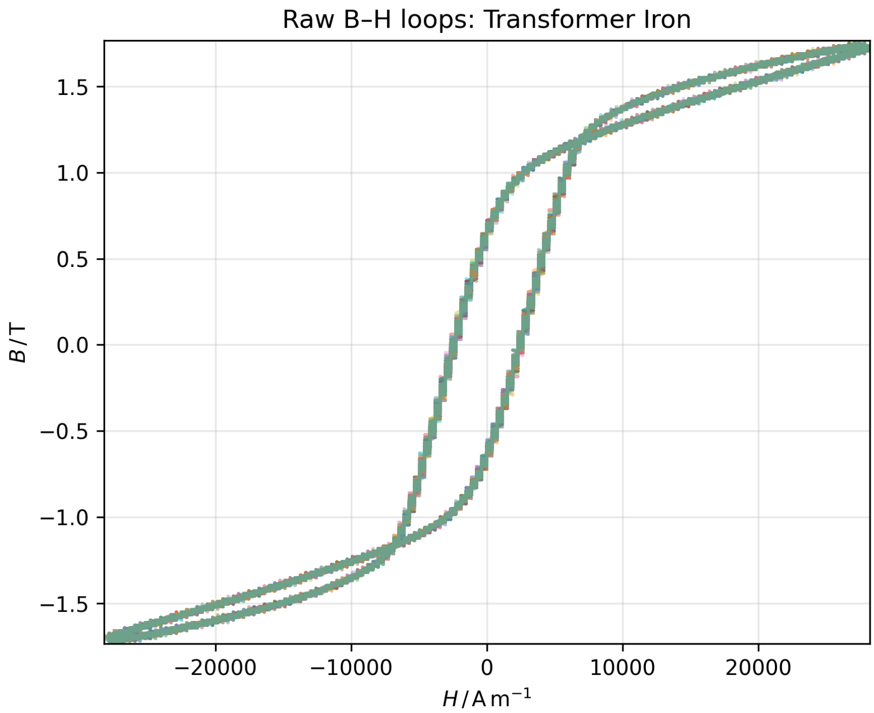

Figure 9: Raw transformer iron hysteresis loops.

Table 4: Transformer iron loop statistics from the polynomial-tail analysis (cells 7 and 9).

Quantity	Value	Uncertainty	Units
B range	3.44	0.00068	T
H range	5.61×10^4	38	A/m
Loop area (absolute)	9.46×10^3	21	T·A/m
Maximum μ_r (numeric gradient)	332	4.2	—
Minimum μ_r (numeric gradient)	13.9	0.025	—
Tail slope (min)	1.74×10^{-5}	3.2×10^{-8}	T/A/m
Tail slope (max)	3.14×10^{-4}	1.9×10^{-7}	T/A/m
RMS B residual	0.0326	0.000013	T

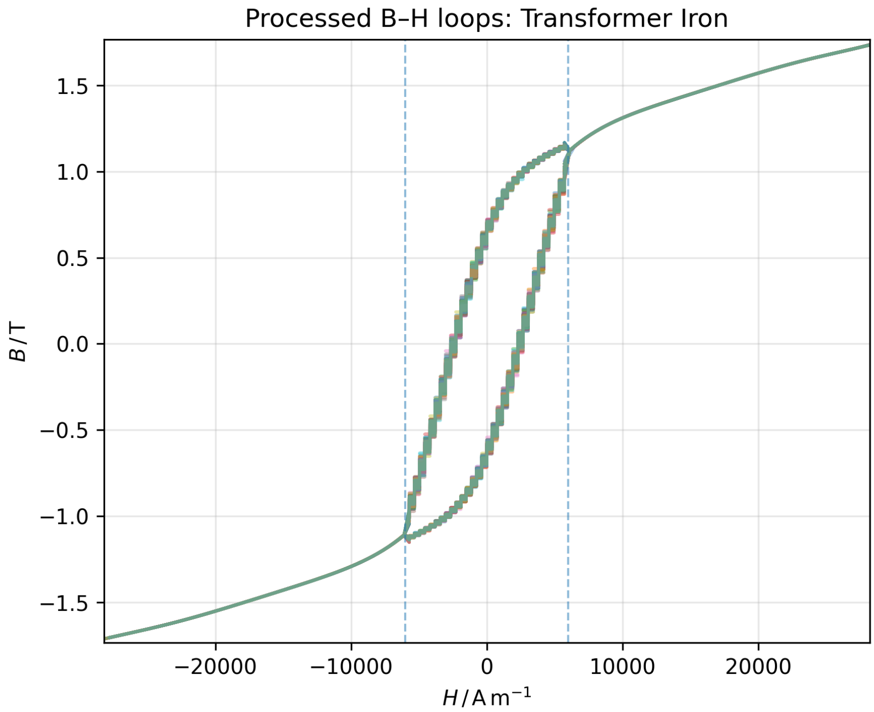

Figure 10: Transformer iron loop after polynomial tail fitting.

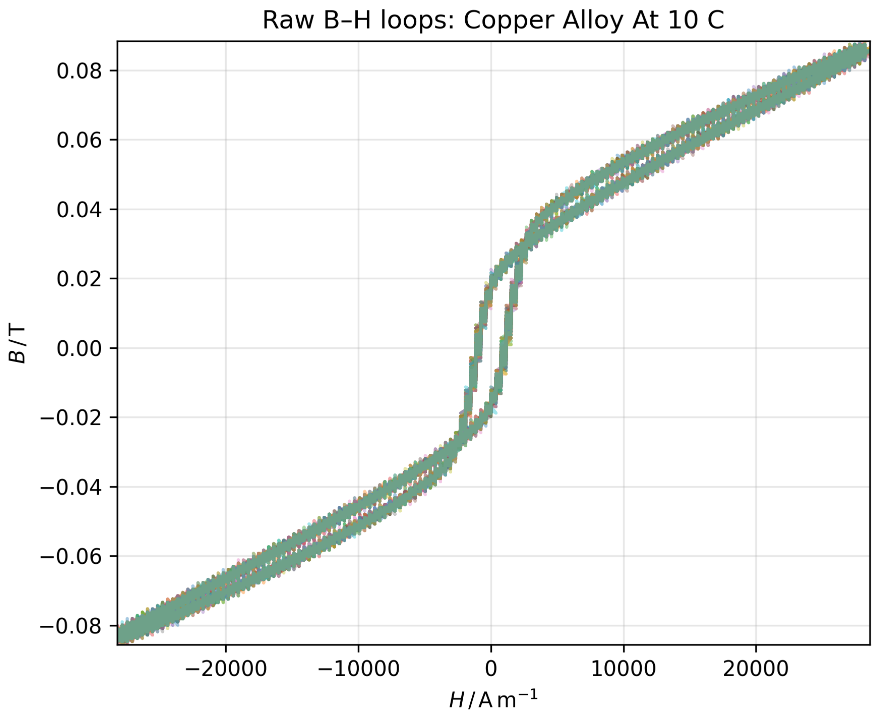

Figure 11: Raw CuNi (10 °C) hysteresis loops.

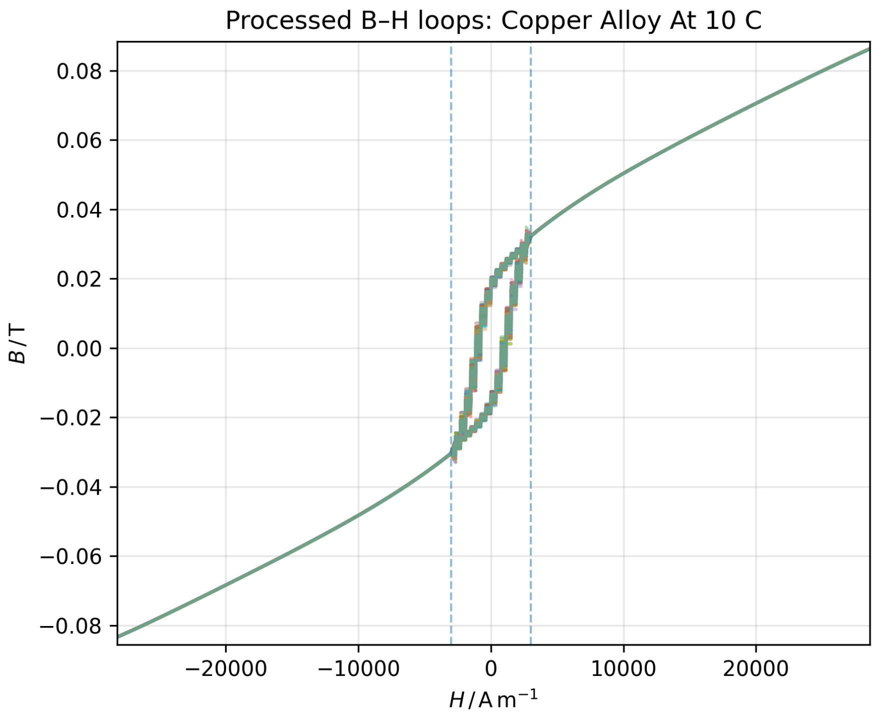

Figure 12: Polynomial-fit CuNi (10 °C) loop.

Table 5: CuNi (10 °C) loop statistics after polynomial tail smoothing (cell 11).

Quantity	Value	Uncertainty	Units
B range	0.17	0.00001	T
H range	5.68×10^{4}	1.7×10^{-12}	A/m
Loop area (absolute)	92.1	0.41	T·A/m
Maximum μ_r (numeric gradient)	7.88	0.0023	—
Minimum μ_r (numeric gradient)	1.35	0.0037	—
Tail slope (min)	1.28×10^{-6}	1.3×10^{-7}	T/A/m
Tail slope (max)	2.19×10^{-5}	5.6×10^{-8}	T/A/m
RMS B residual	0.002	0.0000013	T

4.3.4 CuNi at 40 °C

Heating the CuNi sample to 40 °C suppresses the hysteresis, leaving a near-linear $B(H)$ response (Figure 13). A linear regression therefore replaced the polynomial tail fits for this dataset.

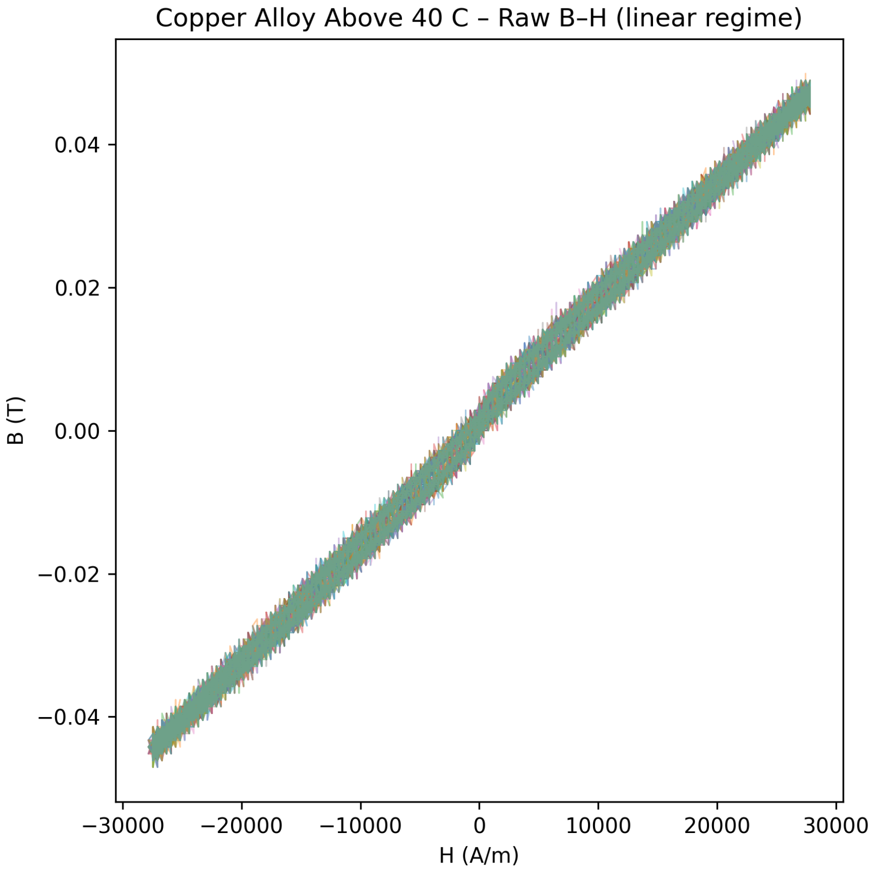

Figure 13: Raw CuNi (40 °C) data showing the predominantly linear response.

Table 6: Linear regression summary for CuNi at 40 °C (cell 13).

Quantity	Value	Uncertainty	Units
B range	0.0954	0.00011	T
H range	5.56×10^4	19	A/m
Slope dB/dH	1.67×10^{-6}	3.36×10^{-10}	T/A/m
Intercept B_0	0.00108	6.58×10^{-6}	T
Relative μ_r (linear fit)	1.33	7.2×10^{-5}	—
Loop area	126	2.4	T·A/m
RMS B residual	0.0013	0.0000017	T

For all samples the raw loops and the fitted/regressed curves feed into the final discussion of permeability versus temperature. The figures above preserve the unaltered data, while the smoothed loops and regressions provide the numerical input used later in the report.

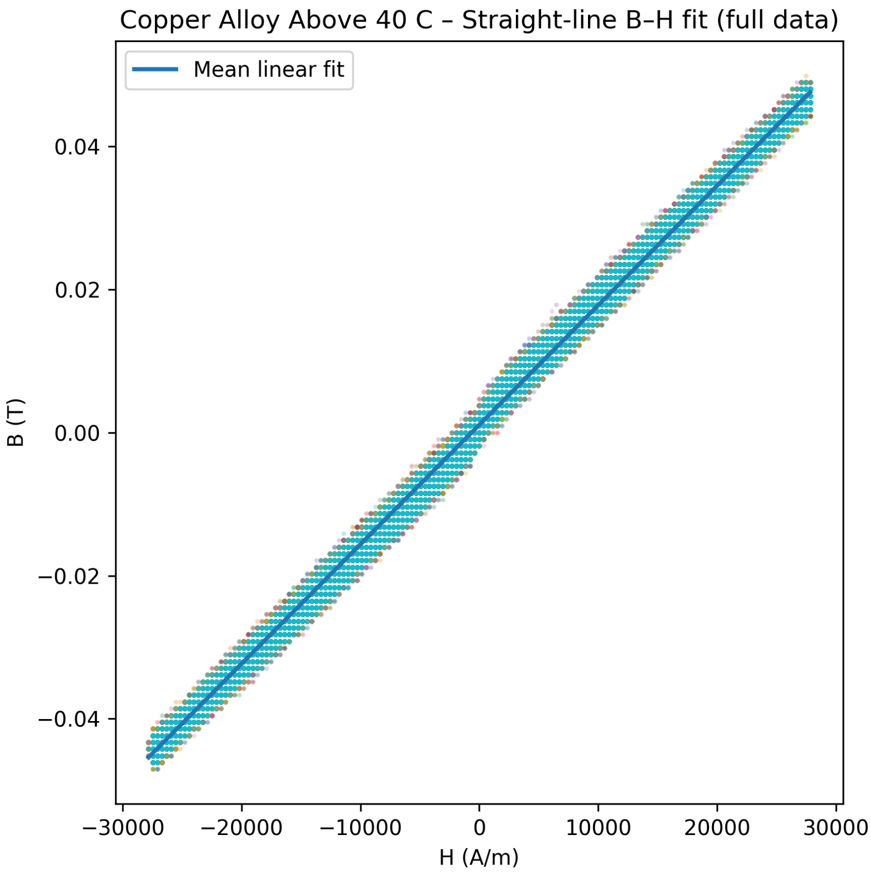

Figure 14: Linear regression for the CuNi (40 °C) run.

5 Discussion

6 Conclusion

References

[1] Robert Keim. What is hysteresis? an introduction for electrical engineers, 2023.